Name	Helpful Notes / Walkthrough	Purpose	Constraints	Runtime
BFS	N/A	Searching/ Traversal	None	$O(V+E)$
DFS	Can be done recursively			
Krskl	Keep picking the cheapest possible valid edge in the ENTIRE graph	Finding MST		$O(E \log V)$
Prm	Start at the cheapest edge Keep picking the cheapest edge leaving the tree			$O((V+E)\log V)$
Dijk	Keep PQ of vertices ordered by $v.d$ Let $v = q.pop()$ where $(v \to (u_{1 \ldots i \ldots k}))$ (until empty) Run $v.relax(u_i)$ for each u_i	Single-Source Shortest Path	Only positive edges	
BllFd	Do the following $V-1$ times For each edge $(v \to u)$, run $v.relax(u)$ Can detect negative cycles (run one more time, if d values change, $\exists$ negative cycle		No negative cycles	$O(VE)$
FldWar	Nodes numbered $\{1 \ldots V\} \supset \{i, j\}$ Semantic: $C[i,j,K]$ is $d(i \twoheadrightarrow j)$ where $(i \twoheadrightarrow j)$ uses only nodes $\{1 \ldots K\}$ Computational: $C[i,j,K] = \begin{cases} 0, i = j \\ \min \begin{cases} C[i,j,K-1] \\ C[i,K,K-1] + C[K,j,K-1] \end{cases} \end{cases}$	All-Pairs Shortest Paths		$O(V^3)$
John	Reweight edges to eliminate negative edges Then run Dijkstra for every node (V times)			$O((V^2+VE)\log V)$
FdFk	While we can find augmentations: Send the maximum possible flow over the augmentation Update edge capacities on the residual graph	Finding Max-Flow	Source has no parents Sink has no children	$O(Ef)$ f = maximum flow value
EdKp	Exactly the same as FF, but use BFS to find augmentations (prefer paths with less edges)			$O(VE^2)$

Simplex Hand Instructions

1. Convert the problem into a slack matrix/tableau
   While R_1 has a negative number
   a. Find the variable x_j with the most negative coefficient a_j in R_1
   b. Find R_i such that $c^i \div K_j^i$ is minimized
   c. Do elementary row operations (Using only R_i) to reduce the column of x_j to a 1-hot vector (hot in R_i) ← zero out basics
3. When R_1 has no negative numbers
   a. Convert the yielded matrix back into equation form, ignoring the non-basic variables, and report the values of $z, x_{1 \ldots D}$

Matrix Form of Slack

$$\begin{array}{c} \text{cols} \\ R_1 \\ R_i \end{array} \begin{bmatrix} [z] & [x_{1 \ldots E}] & [c] \\ 1 & a_{1 \ldots E} & 0 \\ 0 & K^i_{1 \ldots E} & c^i \end{bmatrix}$$

Isolate for constants/zeros of each equation, convert to matrix form, objective on R_1

First/Second	First	Second	How?
Informal (Max) Standard	Max: $(a_{1 \ldots D} \cdot x_{1 \ldots D})$ Sub to: $(K^i_{1 \ldots D} \cdot x_{1 \ldots D} (\geq \mid = \mid \leq) c^i)_{i \in \{1 \ldots \lvert C \rvert\}}$ C = set of constraints	Max: $(a_{1 \ldots D} \cdot x_{1 \ldots D})$ Sub to: $(K^i_{1 \ldots E} \cdot x_{1 \ldots E} \leq c^i)_{i \in \{1 \ldots \lvert C \rvert\}}$ $(x_{1 \ldots E}) \geq (0)_E$ E = # of new constraints	Replace all variables with no non-negativity constraints: $(\neg\exists (x_i \geq 0) \in C) \Rightarrow (x_i = x_i' - x_i'')$ $C \leftarrow C \cup \{x_i' \geq 0, x_i'' \geq 0\}$ Replace equalities with two inequalities: $(=) \to (\leq, \geq)$ Flip all $\geq$ constraints (exclude $x_i \geq 0$): multiply by -1
Standard/ Slack	Max: $(a_{1 \ldots D} \cdot x_{1 \ldots D})$ Sub to: $(K^i_{1 \ldots D} \cdot x_{1 \ldots D} \leq c^i)$, $i \in \{1 \ldots \lvert C \rvert\}$ $(x_{1 \ldots D}) \geq (0)_D$	$z = (a_{1 \ldots D} \cdot x_{1 \ldots D})$ $x_{D+i} = c^i - (K^i_{1 \ldots D} \cdot x_{1 \ldots D})_{i \in \{1 \ldots \lvert C \rvert\}}$	Let basic variables be $x_{1 \ldots D}$ Let non-basic variables be $x_{D+i \in \{1 \ldots \lvert C \rvert\}}$ (the LHS of the new equations) Set z = objective function
Standard/ Matrix	Max: $(a_{1 \ldots D} \cdot x_{1 \ldots D})$ Sub to: $(K^i_{1 \ldots D} \cdot x_{1 \ldots D} \leq c^i)_{i \in \{1 \ldots \lvert C \rvert\}}$ $(x_{1 \ldots D}) \geq (0)_D$	Max: $(c \cdot x)$ Such that: $Ax \leq b, x \geq 0$	$a = \begin{bmatrix} a_1 \\ \vdots \\ a_D \end{bmatrix}, x = \begin{bmatrix} x_1 \\ \vdots \\ x_D \end{bmatrix}$ $b = \begin{bmatrix} c^1 \\ \vdots \\ c^{\lvert C \rvert} \end{bmatrix}$ $A_{\lvert C \rvert \times D} = \begin{bmatrix} K^1_1 & \cdots & K^1_D \\ \vdots & \ddots & \vdots \\ K^{\lvert C \rvert}_1 & \cdots & K^{\lvert C \rvert}_D \end{bmatrix}$
Primal/Dual	Max: $(c \cdot x)$ Such that: $Ax \leq b, x \geq 0$	Min: $(y \cdot b)$ Such that: $y^T A \geq c^T, y \geq 0$	Dual provides an upper bound for optimal primal value
Farkas Lemma	$A \in \mathbb{R}^{m \times n} \wedge b \in \mathbb{R}^n \Rightarrow (\exists x \in \mathbb{R}^n : Ax \leq b) \oplus (\exists y \in \mathbb{R}^m : (y^T A = 0) \wedge (y^T b < 0))$		
Weak Duality	$\bar{x}, \bar{y}$ optimal solutions to primal and dual, respectively $\Rightarrow (c^T \bar{x} \leq \bar{y}^T b) \equiv (c \cdot \bar{x} \leq \bar{y} \cdot b)$ i.e. (Optimal Primal Value $\leq$ Optimal Dual Value (provided both exist)) (Dual Feasible) $\to$ (Primal Bounded) (Primal Feasible) $\Rightarrow$ (Dual Bounded) (Primal Feasible $\wedge$ Bounded) $\Leftrightarrow$ (Dual Feasible $\wedge$ Bounded)		
Strong Duality	(Primal Feasible and Bounded) $\Rightarrow$ (Optimal Primal Value = Optimal Dual Value)		