**CSC411 Homework 7**

**Question 1: Representer Theorem**

**Part 1A**

Formula for reference (expanded )

Expanding

Decomposing according to hint: , where is the projection of onto and is orthogonal to

Since is orthogonal to and every row vector in , so their dot products reduce to 0

Thus, we have where is the projection of onto .

Therefore, is a linear combination of , which are the rows of .

Therefore, must belong to the row space of .**Part 1B**

**Final Answer:**

**Proof:**

Formula for reference:

Rephrasing using definition of squared magnitude

Subbing in )

Simplifying using gram matrix

Removing fraction for simplicity (won’t affect finding minimizing )

Rearranging addition

Factoring out for first two terms:

Dividing entire equation by 2

Using hint from handout:

Where:

Then:

**Question 2: Compositional Kernels**

**Part 2A**

**Final Answer**

Where: A, B are the respective lengths of feature maps

**Proof**

Formulae for reference

Want: such that

Expanding (with respective lengths )

Express the above as the dot product of the concatenation of the feature vectors

**Part 2B**

**Final Answer**

**Proof**

Formulae for reference

Want: such that

Expanding (with respective lengths )

Merging summations and rearranging multiplication

Note that the terms associated with are the same as the terms associated with (multiplication of a element then a element).

Clearly, the -th element of the desired kernel (represented as a matrix of size ) must be:

We can represent the whole kernel as a vectorization (**flattening**) of the feature mappings’ matrix multiplication: