Međuispit iz Elektromehanike

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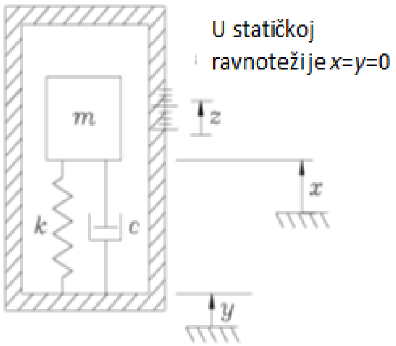
Zadatak

Mogući konceptualni model uređaja za mjerenje brzine tijela koji se giba prikazan je slikom.



Odredite:

- Jednadžbu kojom određujemo brzinu pokazivača mase m
- Analogni električni krug
- Ako se brzina tijela mijenja harmonički s vremenom $\frac{dy}{dt} = v\cos\omega t$ odredite odziv pokazivača u stacionarnom stanju i komentirajte primjenjivost ovakvog koncepta.

$$m\frac{d^2y}{dt^2} = F_{vanjska}$$

$$F_{vanjska} = -F_y$$

$$F_{vanjska} - c\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

$$F_{vanjska} = 0$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx$$

$$\frac{dx}{dt} = e^{st}$$

$$s_{1,2} = \frac{-c \pm \sqrt{c^2 - 4km}}{2m}$$

Zapisemo rješenje u obliku PT2 člana:

$$\frac{d^2x}{dt^2} + 2\gamma\omega_0\frac{dx}{dt} + {\omega_0}^2 kx = 0$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\gamma = \frac{c}{2\sqrt{km}}$$

$$s_{1,2} = \frac{-2\gamma\omega_0 \pm \sqrt{(2\gamma\omega_0)^2 - 4{\omega_0}^2}}{2}$$

$$s_{1,2} = -\gamma\omega_0 \pm \omega_0 j\sqrt{1-\gamma^2}$$

$$x(t)_{Homogeno} = Ae^{-\gamma\omega_0 t} sin(\omega_0 t\sqrt{1-\gamma^2} \ + \ \rho)$$

$$x(t)_{Partikularno} = Različito\ za\ različiti\ obilk\ F_0$$

$$x(t) = x(t)_{Homogeno} + x(t)_{Partikularno}$$

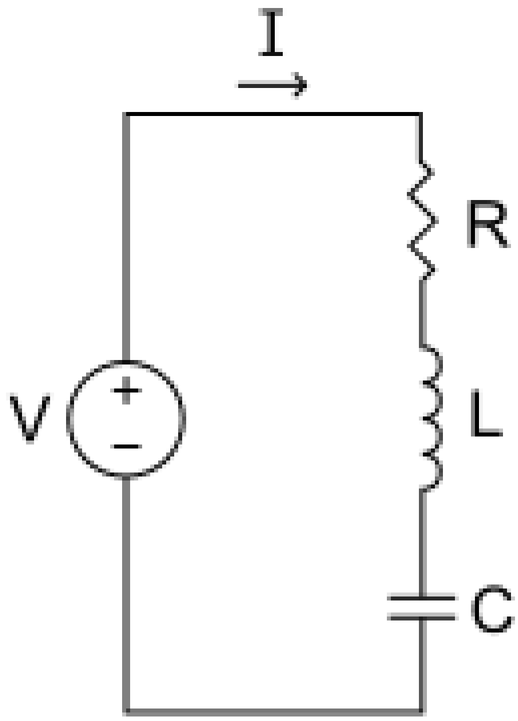
z(t)= $\frac{dx}{dt}$

Za F(t)=0

z(t)= $\frac{dx}{dt} = -\gamma\omega_0 Ae^{-\gamma\omega_0 t} sin\left(\omega_0 t\sqrt{1-\gamma^2} \ + \ \rho\right) + \omega_0\sqrt{1-\gamma^2} Ae^{-\gamma\omega_0 t} sin(\omega_0 t\sqrt{1-\gamma^2} \ + \ \rho)$

Uviđamo da svi elementi sustava imaju jednaku brzinu.
Električki bi to bila struja.

$$RI(t) + L\frac{dI}{dt} + \frac{1}{C}\int_{-\infty}^{t} Idt \;= V(t)$$



Analogno :

$$\frac{dx}{dt} = I(t)$$

$$L = m$$

R= c

$$\frac{1}{C} = k$$

V(t)= $F(t)$

Pobuda oblika:

$$\frac{dy}{dt} = \mathrm{vcos}(\omega \mathrm{t})$$

Tražimo partikularno rješenje:

$$-m\frac{d}{dt}(\frac{dy}{dt}) - c\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

$$mv\omega\sin(\omega t) - c\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

Prelazimo na Laplace –a uz početne uvjete jednake nuli:

$$mv\omega\frac{\omega}{s^2+\omega^2} - csX(s) - kX(s) = ms^2X(s)$$

$$X(s) = \frac{mv\omega^2}{(s^2+\omega^2)(ms^2+cs+k)}$$

Matlab:

```
syms s m v w c k t x(t) A B
tf = 's';
x1 = (-c+sqrt(c^2 -4*m*k))/(2*m);
x2 = (-c-sqrt(c^2 -4*m*k))/(2*m);
Fhomo= A*exp(t*x1)+B*exp(t*x2);
F = (m*v*w^2)/((s^2 + w^2)*(s-x1)*(s-x2));
%pretty(F);
L = ilaplace(F);
%pretty(L)
ukupno = Fhomo + L;
L0 = subs(ukupno, t, 0);


z = diff(ukupno,t);
L1=subs(z, t, 0);
eqn = L1 == 0;
eqn1 = L0 == 0;
rj = solve(eqn, A);
rj1 = solve(eqn1, A);
eq2 = rj == rj1;
B1 = solve(eq2, B);
A1 = subs(rj1, B, B1);
ukupno =subs(ukupno, A, A1);
ukupno = subs(ukupno, B, B1);
z_ukupno = diff(ukupno,t);
z_stac = (k*m^2*v*w^2*cos(t*w) - m^3*v*w^4*cos(t*w) +
c*m^2*v*w^3*sin(t*w))/(c^2*w^2 + k^2 - 2*k*m*w^2 + m^2*w^4);
```

$$
\frac{k\, m^{2}\, v\, w^{2} \cos(t\, w) - m^{3}\, v\, w^{4} \cos(t\, w) + c\, m^{2}\, v\, w^{3} \sin(t\, w)}{\#11} - \frac{\#2\, (c - \mathrm{sqrt}(\#12)) \left(\frac{\#1}{\#4} - \frac{\#8}{\#5} + \frac{\#8}{\#6} + \#7 \right)}{2\, m} + \frac{\#3\, (c + \mathrm{sqrt}(\#12))\, \#1}{m\, \#4\, 2}
$$

$$
+ \frac{m^{3}\, v\, w^{2}\, \#3\, (c + \mathrm{sqrt}(\#12))\, 2}{\#6} - \frac{m^{3}\, v\, w^{2}\, \#2\, (c - \mathrm{sqrt}(\#12))\, 2}{\#5}
$$

$$
\#1 == \frac{m \left(\frac{v\, m^{3}\, w^{4} - k\, v\, m^{2}\, w^{2}}{\#11} + \frac{m^{3}\, v\, w^{2}\, (c - \mathrm{sqrt}(\#12))\, 2}{\#5} - \frac{2\, m^{3}\, v\, w^{2}\, (c + \mathrm{sqrt}(\#12))}{\#6} \right) 2}{c - \mathrm{sqrt}(\#12)} - \frac{\#8}{\#5} + \frac{\#8}{\#6} + \#7
$$

$$
\#2 == \exp\left(- \frac{t\, (c - \mathrm{sqrt}(\#12))}{2\, m} \right)
$$

$$
\#3 == \exp\left(- \frac{t\, (c + \mathrm{sqrt}(\#12))}{2\, m} \right)
$$

$$
\#4 == \frac{c + \mathrm{sqrt}(\#12)}{c - \mathrm{sqrt}(\#12)} - 1
$$

$$
\#5 == \#12^{3/2} - 2\, c^{3} + \#10 + 8\, c\, k\, m + \#9
$$

```
         3/2      3
#6 == #12    + 2 c  + #10 - 8 c k m + #9

         2    2
      c m  v w
#7 == ---------
         #11

         4    2
#8 == 4 m  v w

       2  2
#9 == m  w  sqrt(#12) 4

        2
#10 == c  sqrt(#12)

        2  2    2          2    2  4
#11 == c  w  + k  - 2 k m w  + m  w

        2
#12 == c  - 4 k m
```

Stacionarno stanje ($lim_{t \to \infty}$)uz uvjet: $c - \sqrt{c^2 - 4km} > 0 \ \& \ c^2 - 4km$

$$Z_{stacionarno} = \frac{km^2 v\omega^2 \cos(t\omega) - m^3 v\omega^4 \cos(t\omega) + cm^2 v\omega^3 \sin(t\omega)}{c^2\omega^2 + k^2 - 2km\omega^2 + m^2\omega^4}$$

Imamo oscilatorno gibanje u stacionarnom stanju.Moramo pripazit na raspored polova i nula kod prijenosne funkcije kako bi imali stabilan sustav(polovi u lijevoj poluravnini)

Problemi

- Starenje komponenti
- Precizna skala(odnosno precizne vrijednosti m, c, k)
- Max sila(jer je kučište fizički ograničeno)
- Visoka cijena izrade i njegova isplativost(velika preciznost kod izrade mehaničkih sklopova podiže drastično cijenu, opcija elektronički sklop s istom funkcijom)