

CASE STUDY ON VOGEL'S APPROXIMATION METHOD (VAM)

Introduction:

Vogel's Approximation Method (VAM) is one of the methods used to calculate the initial basic feasible solution to a transportation problem. However, VAM is an iterative procedure such that in each step, we should find the penalties for each available row and column by taking the least cost and second least cost.

Problem:

A defence firm has three branches located in three different regions, Kochi, Vishakpatnam and Mumbai. The company has to export Air Defence Systems to three destinations that is Bangalore, West Bengal and Pune. The availability from Kochi, Vishakpatnam and Mumbai is 40, 60 and 70 Batteries respectively. The demand at Bangalore, West Bengal and Pune are 70, 40 and 60 respectively. Use the Vogel's Approximation Method to find a basic feasible solution (BFS).

	Bangalore	West Bengal	Pune	Supply
Kochi	4	5	1	40
Vishakpatnam	3	4	3	60
Mumbai	6	2	8	70
Demand	70	40	60	

Steps:

Step 1: Identify the two lowest costs in each row and column of the given cost matrix and then write the absolute row and column difference. These differences are called penalties.

Step 2: Identify the row or column with the maximum penalty and assign the corresponding cell's min(supply, demand). If two or more columns or rows have the same maximum penalty, then we can choose one among them as per our convenience.

Step 3: If the assignment in the previous satisfies the supply at the origin, delete the corresponding row. If it satisfies the demand at that destination, delete the corresponding column.

Step 4: Stop the procedure if supply at each origin is 0, i.e., every supply is exhausted, and demand at each destination is 0, i.e., every demand is satisfying. If not, repeat the above steps, i.e., from step 1.

Solution:

	Bangalore	West Bengal	Pune	Supply	Rows Penalty
Kochi	4	5	1	30	$3=4-1$
Vishakpatnam	3	4	3	50	$0=3-3$
Mumbai	6	2	8	60	$4=6-2$
Demand	70	40	60		
Column Penalty	$1=4-3$	$2=4-2$	$2=3-1$		

The maximum penalty, 4, occurs in row Mumbai.

The minimum c_{ij} in this row is $c_{32}=2$.

The maximum allocation in this cell is $\min(70,40) = 40$.

It satisfy demand of West Bengal and adjust the supply of Mumbai from 70 to 30 ($70 - 40=30$).

	Bangalore	West Bengal	Pune	Supply	Rows Penalty
Kochi	4	5	1	40	$3=4-1$
Vishakpatnam	3	4	3	60	$0=3-3$
Mumbai	6	2(40)	8	30	$2=8-6$
Demand	70	0	60		
Column Penalty	$1=4-3$	--	$2=3-1$		

The maximum penalty, 3, occurs in row Kochi.

The minimum c_{ij} in this row is $c_{13}=1$.

The maximum allocation in this cell is $\min(40,60) = 40$.

It satisfy supply of Kochi and adjust the demand of Pune from 60 to 20 ($60 - 40=20$).

	Bangalore	West Bengal	Pune	Supply	Rows Penalty
Kochi	4	5	1(40)	0	--
Vishakpatnam	3	4	3	60	0=3-3
Mumbai	6	2(40)	8	30	2=8-6
Demand	70	0	20		
Column Penalty	3=6-3	--	5=8-3		

The maximum penalty, 5, occurs in column Pune.

The minimum c_{ij} in this column is $c_{23}=3$.

The maximum allocation in this cell is $\min(60,20) = 20$.

It satisfy demand of Pune and adjust the supply of Vishakpatnam from 60 to 40 ($60 - 20=40$).

	Bangalore	West Bengal	Pune	Supply	Rows Penalty
Kochi	4	5	1(40)	0	--
Vishakpatnam	3	4	3(20)	40	3
Mumbai	6	2(40)	8	30	6
Demand	70	0	0		
Column Penalty	3=6-3	--	--		

The maximum penalty, 6, occurs in row Mumbai.

The minimum c_{ij} in this row is $c_{31}=6$.

The maximum allocation in this cell is $\min(30,70) = 30$.

It satisfy supply of Mumbai and adjust the demand of Bangalore from 70 to 40 ($70 - 30=40$).

	Bangalore	West Bengal	Pune	Supply	Rows Penalty
Kochi	4	5	1(40)	0	--
Vishakpatnam	3	4	3(20)	40	3
Mumbai	6(30)	2(40)	8	0	--
Demand	40	0	0		
Column Penalty	3	--	--		

The maximum penalty, 3, occurs in row Vishakpatnam.

The minimum cij in this row is $c_{21}=3$.

The maximum allocation in this cell is $\min(40,40) = 40$.

It satisfy supply of Vishakpatnam and demand of Bangalore.

	Bangalore	West Bengal	Pune	Supply	Rows Penalty
Kochi	4	5	1(40)	40	3 3 -- -- --
Vishakpatnam	3(40)	4	3(20)	60	0 0 0 3 3
Mumbai	6(30)	2(40)	8	70	4 2 2 6 --
Demand	70	40	60		
Column Penalty	1 1 3 3 3	2 -- -- -- --	2 2 5 -- --		

Transportation cost = $1 \times 40 + 3 \times 40 + 3 \times 20 + 6 \times 30 + 2 \times 40 = 480$

Code:

```
library(lpSolve)
costs <- matrix(c(4,5,1,
                 3,4,3,
                 6,2,8), nrow = 3, byrow = TRUE)
colnames(costs) <- c("Bangalore", "West Bengal", "Pune")
rownames(costs) <- c("Kochi", "Vishakpatnam", "Mumbai")
row.signs <- rep("<=", 3)
row.rhs <- c(40, 60, 70)
col.signs <- rep(">=", 3)
col.rhs <- c(70, 40, 60)
TotalCost <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)
lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)$solution
print(TotalCost)
```

Output:

```
      [,1] [,2] [,3]
[1,]    0    0  40
[2,]   40    0  20
[3,]   30   40    0
Success: the objective function is 480
```