



AP[®] Calculus AB Practice Exam

From the 2013 Administration

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Contents

Exam Instructions
Student Answer Sheet for the Multiple-Choice Section
Section I: Multiple-Choice Questions
Section II: Free-Response Questions
Multiple-Choice Answer Key
Free-Response Scoring Guidelines
Scoring Worksheet

Note: This publication shows the page numbers that appeared in the *2012–13 AP Exam Instructions* book and in the actual exam. This publication was not repaginated to begin with page 1.

Exam Instructions

The following contains instructions taken from
the ***2012–13 AP Exam Instructions*** book.

AP® Calculus AB Exam

Regularly Scheduled Exam Date: Wednesday morning, May 8, 2013

Late-Testing Exam Date: Thursday morning, May 23, 2013

Section I: At a Glance

Total Time: 1 hour, 45 minutes

Number of Questions: 45

Percent of Total Score: 50%

Writing Instrument:

Pencil required

Part A:

Number of Questions: 28

Time: 55 minutes

Electronic Device:

None allowed

Part B:

Number of Questions: 17

Time: 50 minutes

Electronic Device:

Graphing calculator required

Section I: Multiple Choice Booklet Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely.

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Section II: At a Glance

Total Time: 1 hour, 30 minutes

Number of Questions: 6

Percent of Total Score: 50%

Writing Instrument:

Either pencil or pen with black or dark blue ink

Weight:

The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A:

Number of Questions: 2

Time: 30 minutes

Electronic Device:

Graphing calculator required

Percent of Section II Score:

33.3%

Part B:

Number of Questions: 4

Time: 60 minutes

Electronic Device:

None allowed

Percent of Section II Score:

66.6%

Section II: Free Response Booklet Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as $\text{fnInt}(X^2, X, 1, 5)$.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

What Proctors Need to Bring to This Exam

- Exam packets
- Answer sheets
- AP Student Packs
- *2012-13 AP Coordinator's Manual*
- This book — *AP Exam Instructions*
- School Code and Home-School/Self-Study Codes
- Extra graphing calculators
- Pencil sharpener
- Extra No. 2 pencils with erasers
- Extra pens with black or dark blue ink
- Extra paper
- Stapler
- Watch
- Signs for the door to the testing room
 - “Exam in Progress”
 - “Cell phones are prohibited in the testing room”

If you are giving the regularly scheduled AP Calculus AB or BC Exam:

- You may seat students **four feet (approximately 1.2 meters) apart** because these exams have scrambled multiple-choice sections. This allows you to test more students in fewer testing rooms.
- See page 10 for a sample seating plan, including form codes and serial numbers, that shows how exams should be distributed to ensure that students seated next to each other are not given the same form of the exam.
- Administrators and proctors must continue to be vigilant about opportunities for cheating.

If you are giving the alternate AP Calculus AB or BC Exam for late testing:

- You must seat students **no less than five feet (approximately 1.5 meters) apart** because these exams do not have scrambled multiple-choice sections.

Graphing calculators are required to answer some of the questions on the AP Calculus Exams. Before starting the exam administration, make sure each student has a graphing calculator from the approved list on page 42 of the *2012-13 AP Coordinator's Manual*. If a student does not have a graphing calculator from the approved list, you may provide one from your supply. If the student does not want to use the calculator you provide or does not want to use a calculator at all, he or she must hand copy, date, and sign the release statement on page 40 of the *2012-13 AP Coordinator's Manual*.

During the administration of Section I, Part B, and Section II, Part A, students may have no more than two graphing calculators on their desks. Calculators may not be shared. **Calculator memories do not need to be cleared before or after the exam.** Students with Hewlett-Packard 48–50 Series and Casio FX-9860 graphing calculators may use cards designed for use with these calculators. Proctors should make sure infrared ports (Hewlett-Packard) are not facing each other. **Since graphing calculators can be used to store data, including text, proctors should monitor that students are using their calculators appropriately. Attempts by students to use**

the calculator to remove exam questions and/or answers from the room may result in the cancellation of AP Exam scores.

The AP Calculus AB Exam and the AP Calculus BC Exam should be administered simultaneously. They may be administered in separate rooms, or in the same room if it is more convenient.

SECTION I: Multiple Choice

- Do not begin the exam instructions below until you have completed the appropriate General Instructions for your group.**

These exams include survey questions. The time allowed for the survey questions is in addition to the actual test-taking time.

Make sure you begin the exams at the designated time.

If you are giving the regularly scheduled exam, say:

It is Wednesday morning, May 8, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

If you are giving the alternate exam for late testing, say:

It is Thursday morning, May 23, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

In a moment, you will open the packet that contains your exam materials.

By opening this packet, you agree to all of the AP Program's policies and procedures outlined in the *2012-13 Bulletin for AP Students and Parents*.

Please check to make sure you have the correct exam: Calculus AB or Calculus BC. Raise your hand if you do not have the correct exam. . . .

You may now remove the shrinkwrap from your exam packet and take out the Section I booklet, but do not open the booklet or the shrinkwrapped Section II materials. Put the white seals aside. . . .

Carefully remove the AP Exam label found near the top left of your exam booklet cover. Now place it on page 1 of your answer sheet on the dark blue box near the top right-hand corner that reads "AP Exam Label."

If students accidentally place the exam label in the space for the number label or vice versa, advise them to leave the labels in place. They should not try to remove the label; their exam will be processed correctly.

Read the statements on the front cover of Section I and look up when you have finished. . . .

Sign your name and write today's date. Look up when you have finished. . . .

Now print your full legal name where indicated. Are there any questions? . . .

Turn to the back cover and read it completely. Look up when you have finished. . . .

Are there any questions? . . .

Section I is the multiple-choice portion of the exam. You may never discuss these specific multiple-choice questions at any time in any form with anyone, including your teacher and other students. If you disclose these questions through any means, your AP Exam score will be canceled.

Are there any questions? . . .

You must complete the answer sheet using a No. 2 pencil only. Mark all of your responses beginning on page 2 of your answer sheet, one response per question. Completely fill in the circles. If you need to erase, do so carefully and completely. No credit will be given for anything written in the exam booklet. Scratch paper is not allowed, but you may use the margins or any blank space in the exam booklet for scratch work.

Section I is divided into two parts. Each part is timed separately, and you may work on each part only during the time allotted for it. Calculators are not allowed in Part A. Please put your calculators under your chair. Are there any questions? . . .

You have 55 minutes for Part A. Part A questions are numbered 1 through 28. Mark your responses for these questions on page 2 of your answer sheet. Open your Section I booklet and begin.



Note Start Time here _____. Note Stop Time here _____. Check that students are marking their answers in pencil on page 2 of their answer sheets and that they are not looking beyond Part A. The line of A's at the top of each page will assist you in monitoring students' work. After 55 minutes, say:

Stop working on Part A and turn to page 22 in your Section I booklet. . . .

On that page, you should see an area marked "PLACE SEAL HERE." Making sure all of your other exam materials, including your answer sheet, are out of the way, take one of your seals and press it on that area and then fold the seal over the open edge to the front cover. Be sure you don't seal the Part B section of the booklet or let the seal touch anything except the marked areas. . . .

After all students have sealed Part A, say:

Graphing calculators are required for Part B. You may get your calculators from under your chair and place them on your desk. Part B questions are numbered 76 through 92. Fold your answer sheet so only page 3 is showing and mark your responses for these questions on that page. You have 50 minutes for Part B. You may begin.



Note Start Time here _____. Note Stop Time here _____. Check that students have sealed their booklets properly and are now working on Part B. The large B's in an alternating shaded pattern at the top of each page will assist you in monitoring their work. Proctors should

make sure that students are using their calculators appropriately. Proctors should also make sure Hewlett-Packard calculators' infrared ports are not facing each other. After 50 minutes, say:

Stop working and turn to page 38. You have 3 minutes to answer Questions 93–96. These are survey questions and will not affect your score. You may not go back to work on any of the exam questions. . . .

Give students approximately 3 minutes to answer the survey questions. Then say:

Close your booklet and put your answer sheet on your desk, face up. Make sure you have your AP number label and an AP Exam label on page 1 of your answer sheet. I will now collect your answer sheet.

Collect an answer sheet from each student. Check that each answer sheet has an AP number label and an AP Exam label. Then say:

Now you must seal your Section I booklet. Remove the remaining white seals from the backing and press one on each area of your exam booklet cover marked “PLACE SEAL HERE.” Fold each seal over the back cover. When you have finished, place the booklet on your desk, face up. I will now collect your Section I booklet. . . .

Collect a Section I booklet from each student. Check that each student has signed the front cover of the sealed Section I booklet.

There is a 10-minute break between Sections I and II. When all Section I materials have been collected and accounted for and you are ready for the break, say:

Please listen carefully to these instructions before we take a 10-minute break. Everything you placed under your chair at the beginning of the exam must stay there. Leave your shrinkwrapped Section II packet on top of your desk during the break. You are not allowed to consult teachers, other students, or textbooks about the exam during the break. You may not make phone calls, send text messages, use your calculators, check email, use a social networking site, or access any electronic or communication device. Remember, you are not allowed to discuss the multiple-choice section of this exam. If you do not follow these rules, your score could be canceled. Are there any questions? . . .



You may begin your break. Testing will resume at _____.

SECTION II: Free Response

After the break, say:

May I have everyone’s attention? Place your Student Pack on your desk. . . .

You may now remove the shrinkwrap from the Section II packet, but do not open the Section II exam booklet until you are told to do so. . . .

Read the bulleted statements on the front cover of the exam booklet. Look up when you have finished. . . .

Now place an AP number label on the shaded box. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .

Read the last statement. . . .

Using your pen, print the first, middle and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the statements on the front cover. . . .

Turn to the back cover and complete Item 1 under "Important Identification Information." Print the first two letters of your last name and the first letter of your first name in the boxes. Look up when you have finished. . . .

In Item 2, print your date of birth in the boxes. . . .

In Item 3, write the school code you printed on the front of your Student Pack in the boxes. . . .

Read Item 4. . . .

Are there any questions? . . .

I need to collect the Student Pack from anyone who will be taking another AP Exam. You may keep it only if you are not taking any other AP Exams this year. If you have no other AP Exams to take, place your Student Pack under your chair now. . . .

While Student Packs are being collected, read the information on the back cover of the exam booklet, paying careful attention to the bulleted statements in the instructions. Do not open the exam booklet or break the seals in the exam booklet until you are told to do so. Look up when you have finished. . . .

Collect the Student Packs. Then say:

Are there any questions? . . .

Section II also has two parts that are timed separately. You are responsible for pacing yourself, and may proceed freely from one question to the next within each part. Graphing calculators are required for Part A, so you may keep your calculators on your desk. You must write your answers in the appropriate space in the exam booklet using a No. 2 pencil or a pen with black or dark blue ink. Do not break the seals for Part B at this time.

Are there any questions? . . .

You have 30 minutes to answer the questions in Part A. If you need more paper during the exam, raise your hand. At the top of each extra piece of paper you use, be sure to write only your AP number and the number of the question you are working on. Do not write your name. Open your exam booklet and begin.



Note Start Time here _____. Note Stop Time here _____. Check that students are working on Part A only and writing their answers in their exam booklets using pencils or pens with black or dark blue ink. The pages for the Part A questions are marked with large 1s or 2s at the top of each page to assist you in monitoring their work. After 20 minutes, say:

There are 10 minutes remaining in Part A.

After 10 minutes, say:

Stop working on Part A. Calculators are not allowed for Part B. Please put all of your calculators under your chair. . . .

Turn to page 13. You have 1 hour for Part B. During this time you may go back to Part A, but you may not use your calculator. Remember to write your answer to each part of each problem in the appropriate space in the exam booklet. Are there any questions? . . .

Using your finger, break open the seals on Part B. Do not peel the seals away from the booklet. You may begin Part B.



Note Start Time here _____. Note Stop Time here _____. After 50 minutes, say:

There are 10 minutes remaining in Part B.

After 10 minutes, say:

Stop working and close your exam booklet. Place it on your desk, face up. . . .

If any students used extra paper for the free-response section, have those students staple the extra sheet/s to the first page corresponding to that question in their exam booklets. Then say:

Remain in your seat, without talking, while the exam materials are collected. . . .

Collect a Section II exam booklet from each student. Check for the following:

- Exam booklet front cover: The student placed an AP number label on the shaded box, and printed his or her initials and today's date.
- Exam booklet back cover: The student completed the "Important Identification Information" area.

When all exam materials have been collected and accounted for, return to students any electronic devices you may have collected before the start of the exam.

If you are giving the regularly scheduled exam, say:

You may not discuss these specific free-response questions with anyone unless they are released on the College Board website in about two days. Your AP score results will be delivered online in July.

If you are giving the alternate exam for late testing, say:

None of the questions in this exam may ever be discussed or shared in any way at any time. Your AP score results will be delivered online in July.

If any students completed the AP number card at the beginning of this exam, say:

Please remember to take your AP number card with you. You will need the information on this card to view your scores and order AP score reporting services online.

Then say:

You are now dismissed.

All exam materials should be put in secure storage until they are returned to the AP Program after your school's last administration. Before storing materials, check the "School Use Only" section on page 1 of the answer sheet and:

- Fill in the appropriate section number circle in order to access a separate AP Instructional Planning Report (for regularly scheduled exams only) or subject score roster at the class section or teacher level. See "Post-Exam Activities" in the *2012-13 AP Coordinator's Manual*.
- Check your list of students who are eligible for fee reductions and fill in the appropriate circle on their registration answer sheets.

Student Answer Sheet for the Multiple-Choice Section

Use this section to capture student responses. (Note that the following answer sheet is a sample, and may differ from one used in an actual exam.)

COMPLETE THIS AREA AT EACH EXAM (IF APPLICABLE)

P. SURVEY QUESTIONS — Answer the survey questions in the AP Student Pack. Do not put responses to exam questions in this section.

1 A B C D E F G H I
 2 A B C D E F G H I
 3 A B C D E F G H I

4 A B C D E F G H I
 5 A B C D E F G H I
 6 A B C D E F G H I

7 A B C D E F G H I
 8 A B C D E F G H I
 9 A B C D E F G H I

Q. LANGUAGE — Do not complete this section unless instructed to do so.

If this answer sheet is for the French Language and Culture, German Language and Culture, Italian Language and Culture, Spanish Language, or Spanish Literature and Culture Exam, please answer the following questions. Your responses will not affect your score.

1. Have you lived or studied for one month or more in a country where the language of the exam you are now taking is spoken?

Yes No

2. Do you regularly speak or hear the language at home?

Yes No

QUESTIONS 1–75

Indicate your answers to the exam questions in this section (pages 2 and 3). Use a No. 2 pencil only. Mark only one response per question. If a question has only four answer options, do not mark option E. Answers written in the multiple-choice booklet will not be scored.

Your answer sheet will be scored by machine. Any improper gridding may affect your score.

- Completely fill in the circle for your response next to the number of the question you are answering.
- Erase carefully and completely. Stray marks and smudges could be read as answers.

1 A B C D E

26 A B C D E

51 A B C D E

2 A B C D E

27 A B C D E

52 A B C D E

3 A B C D E

28 A B C D E

53 A B C D E

4 A B C D E

29 A B C D E

54 A B C D E

5 A B C D E

30 A B C D E

55 A B C D E

6 A B C D E

31 A B C D E

56 A B C D E

7 A B C D E

32 A B C D E

57 A B C D E

8 A B C D E

33 A B C D E

58 A B C D E

9 A B C D E

34 A B C D E

59 A B C D E

10 A B C D E

35 A B C D E

60 A B C D E

11 A B C D E

36 A B C D E

61 A B C D E

12 A B C D E

37 A B C D E

62 A B C D E

13 A B C D E

38 A B C D E

63 A B C D E

14 A B C D E

39 A B C D E

64 A B C D E

15 A B C D E

40 A B C D E

65 A B C D E

16 A B C D E

41 A B C D E

66 A B C D E

17 A B C D E

42 A B C D E

67 A B C D E

18 A B C D E

43 A B C D E

68 A B C D E

19 A B C D E

44 A B C D E

69 A B C D E

20 A B C D E

45 A B C D E

70 A B C D E

21 A B C D E

46 A B C D E

71 A B C D E

22 A B C D E

47 A B C D E

72 A B C D E

23 A B C D E

48 A B C D E

73 A B C D E

24 A B C D E

49 A B C D E

74 A B C D E

25 A B C D E

50 A B C D E

75 A B C D E



DO NOT WRITE IN THIS AREA

QUESTIONS 76–120

Be sure each mark is dark and completely fills the circle. If a question has only four answer options, do not mark option E.

- | | | | | | |
|----|---------------------|-----|---------------------|-----|---------------------|
| 76 | (A) (B) (C) (D) (E) | 91 | (A) (B) (C) (D) (E) | 106 | (A) (B) (C) (D) (E) |
| 77 | (A) (B) (C) (D) (E) | 92 | (A) (B) (C) (D) (E) | 107 | (A) (B) (C) (D) (E) |
| 78 | (A) (B) (C) (D) (E) | 93 | (A) (B) (C) (D) (E) | 108 | (A) (B) (C) (D) (E) |
| 79 | (A) (B) (C) (D) (E) | 94 | (A) (B) (C) (D) (E) | 109 | (A) (B) (C) (D) (E) |
| 80 | (A) (B) (C) (D) (E) | 95 | (A) (B) (C) (D) (E) | 110 | (A) (B) (C) (D) (E) |
| 81 | (A) (B) (C) (D) (E) | 96 | (A) (B) (C) (D) (E) | 111 | (A) (B) (C) (D) (E) |
| 82 | (A) (B) (C) (D) (E) | 97 | (A) (B) (C) (D) (E) | 112 | (A) (B) (C) (D) (E) |
| 83 | (A) (B) (C) (D) (E) | 98 | (A) (B) (C) (D) (E) | 113 | (A) (B) (C) (D) (E) |
| 84 | (A) (B) (C) (D) (E) | 99 | (A) (B) (C) (D) (E) | 114 | (A) (B) (C) (D) (E) |
| 85 | (A) (B) (C) (D) (E) | 100 | (A) (B) (C) (D) (E) | 115 | (A) (B) (C) (D) (E) |
| 86 | (A) (B) (C) (D) (E) | 101 | (A) (B) (C) (D) (E) | 116 | (A) (B) (C) (D) (E) |
| 87 | (A) (B) (C) (D) (E) | 102 | (A) (B) (C) (D) (E) | 117 | (A) (B) (C) (D) (E) |
| 88 | (A) (B) (C) (D) (E) | 103 | (A) (B) (C) (D) (E) | 118 | (A) (B) (C) (D) (E) |
| 89 | (A) (B) (C) (D) (E) | 104 | (A) (B) (C) (D) (E) | 119 | (A) (B) (C) (D) (E) |
| 90 | (A) (B) (C) (D) (E) | 105 | (A) (B) (C) (D) (E) | 120 | (A) (B) (C) (D) (E) |

For Students Taking AP Biology

**Write your answer in the boxes at the top of the griddable area and fill in the corresponding circles.
Mark only one circle in any column. You will receive credit only if the circles are filled in correctly.**

ETS USE ONLY

SELECTED MEDIA EXAMS	R	W	O	OTHER EXAMS	R	W	O
PT02				TOTAL			
PT03				Subscore (if applicable)			
PT04				Subscore (if applicable)			

Exam	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Exam	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)



COMPLETE THIS AREA ONLY ONCE.

Use the address abbreviations from your AP Student Pack. Fill in only one circle per column. Indicate a space in your address by leaving a blank box; do not grid that column.

Section I: Multiple-Choice Questions

This is the multiple-choice section of the 2013 AP exam. It includes cover material and other administrative instructions to help familiarize students with the mechanics of the exam. (Note that future exams may differ in look from the following content.)

AP[®] Calculus AB Exam

SECTION I: Multiple Choice

2013

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 45 minutes

Number of Questions
45

Percent of Total Score
50%

Writing Instrument
Pencil required

Part A

Number of Questions
28

Time
55 minutes

Electronic Device
None allowed

Part B

Number of Questions
17

Time
50 minutes

Electronic Device
Graphing calculator
required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question Sample Answer

- Chicago is a (A) state (B) city (C) country (D) continent (E) village

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

PLACE SEAL HERE

Form I

Form Code 4JBP4-Q-S

66

PLACE SEAL HERE

DO NOT seal answer sheet inside

A A

**CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28**

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

- (1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
- (2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1} x = \arcsin x$).

A A

1. $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4}$ is
- (A) $-\frac{1}{4}$ (B) 0 (C) 1 (D) $\frac{5}{4}$ (E) nonexistent
-

2. If $f(x) = x^3 - x^2 + x - 1$, then $f'(2) =$
- (A) 10 (B) 9 (C) 7 (D) 5 (E) 3

A A

3. Which of the following definite integrals has the same value as $\int_0^4 xe^{x^2} dx$?

(A) $\frac{1}{2} \int_0^4 e^u du$

(B) $\frac{1}{2} \int_0^{16} e^u du$

(C) $2 \int_0^2 e^u du$

(D) $2 \int_0^4 e^u du$

(E) $2 \int_0^{16} e^u du$

A A

4. Which of the following is an equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point $(1, -3)$?

(A) $y + 3 = -11(x - 1)$

(B) $y + 3 = -\frac{7}{3}(x - 1)$

(C) $y + 3 = \frac{1}{3}(x - 1)$

(D) $y + 3 = \frac{7}{3}(x - 1)$

(E) $y + 3 = \frac{11}{3}(x - 1)$

A A

5. If g is the function given by $g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 70x + 5$, on which of the following intervals is g decreasing?

- (A) $(-\infty, -10)$ and $(7, \infty)$
- (B) $(-\infty, -7)$ and $(10, \infty)$
- (C) $(-\infty, 10)$
- (D) $(-10, 7)$
- (E) $(-7, 10)$

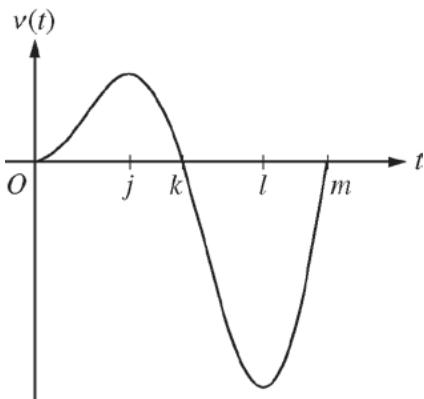
6. $\int_2^4 \frac{dx}{5-3x} =$

- (A) $-\ln 7$
- (B) $-\frac{\ln 7}{3}$
- (C) $\frac{\ln 7}{3}$
- (D) $\ln 7$
- (E) $3\ln 7$

A A

7. Let f be the function given by $f(x) = x^3 - 6x^2 + 8x - 2$. What is the instantaneous rate of change of f at $x = 3$?

- (A) -5 (B) $-\frac{15}{4}$ (C) -1 (D) 6 (E) 17



8. A particle moves along a straight line. The graph of the particle's velocity $v(t)$ at time t is shown above for $0 \leq t \leq m$, where j , k , l , and m are constants. The graph intersects the horizontal axis at $t = 0$, $t = k$, and $t = m$ and has horizontal tangents at $t = j$ and $t = l$. For what values of t is the speed of the particle decreasing?

- (A) $j \leq t \leq l$
(B) $k \leq t \leq m$
(C) $j \leq t \leq k$ and $l \leq t \leq m$
(D) $0 \leq t \leq j$ and $k \leq t \leq l$
(E) $0 \leq t \leq j$ and $l \leq t \leq m$

A A

9. Let f be the function given by $f(x) = \frac{(x-2)^2(x+3)}{(x-2)(x+1)}$. For which of the following values of x is f not continuous?

- (A) -3 and -1 only
- (B) -3 , -1 , and 2
- (C) -1 only
- (D) -1 and 2 only
- (E) 2 only

10. A particle moves along the x -axis with velocity given by $v(t) = 3t^2 - 4$ for time $t \geq 0$. If the particle is at position $x = -2$ at time $t = 0$, what is the position of the particle at time $t = 3$?

- (A) 13
- (B) 15
- (C) 16
- (D) 17
- (E) 25

A A

11. Let f be the function defined by $f(x) = \int_0^x (2t^3 - 15t^2 + 36t) dt$. On which of the following intervals is the graph of $y = f(x)$ concave down?

- (A) $(-\infty, 0)$ only
- (B) $(-\infty, 2)$
- (C) $(0, \infty)$
- (D) $(2, 3)$ only
- (E) $(3, \infty)$ only

A A

12. For which of the following does $\lim_{x \rightarrow \infty} f(x) = 0$?

I. $f(x) = \frac{\ln x}{x^{99}}$

II. $f(x) = \frac{e^x}{\ln x}$

III. $f(x) = \frac{x^{99}}{e^x}$

- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) I and III only

A A

13. Let f be a differentiable function such that $f(0) = -5$ and $f'(x) \leq 3$ for all x . Of the following, which is not a possible value for $f(2)$?

(A) -10 (B) -5 (C) 0 (D) 1 (E) 2

$$f(x) = \begin{cases} x + b & \text{if } x \leq 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

14. Let f be the function given above. What are all values of a and b for which f is differentiable at $x = 1$?

(A) $a = \frac{1}{2}$ and $b = -\frac{1}{2}$
(B) $a = \frac{1}{2}$ and $b = \frac{3}{2}$
(C) $a = \frac{1}{2}$ and b is any real number
(D) $a = b + 1$, where b is any real number
(E) There are no such values of a and b .

A A

$f(3)$	$g(3)$	$f'(3)$	$g'(3)$
-1	2	5	-2

15. The table above gives values for the functions f and g and their derivatives at $x = 3$. Let k be the function given by $k(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$. What is the value of $k'(3)$?

(A) $-\frac{5}{2}$ (B) -2 (C) 2 (D) 3 (E) 8

-
16. If $y = 5x\sqrt{x^2 + 1}$, then $\frac{dy}{dx}$ at $x = 3$ is

(A) $\frac{5}{2\sqrt{10}}$ (B) $\frac{15}{\sqrt{10}}$ (C) $\frac{15}{2\sqrt{10}} + 5\sqrt{10}$ (D) $\frac{45}{\sqrt{10}} + 5\sqrt{10}$ (E) $\frac{45}{\sqrt{10}} + 15\sqrt{10}$

A A

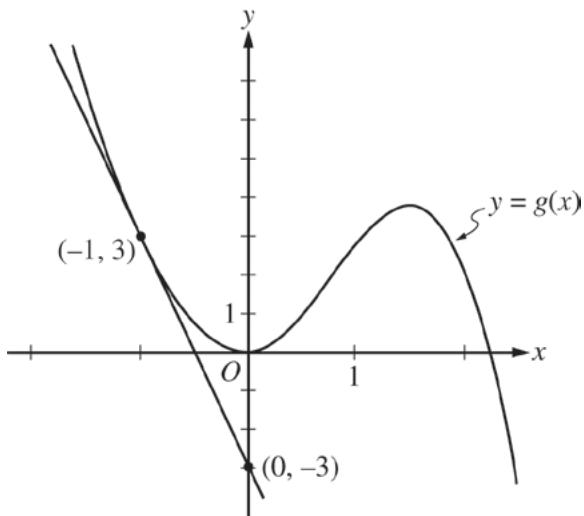
17. If $\lim_{h \rightarrow 0} \frac{\arcsin(a+h) - \arcsin(a)}{h} = 2$, which of the following could be the value of a ?

- (A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\frac{1}{2}$ (E) 2
-

18. If $\ln(2x+y) = x+1$, then $\frac{dy}{dx} =$

- (A) -2 (B) $2x+y-2$ (C) $2x+y$ (D) $4x+2y-2$ (E) $y-\frac{y}{x}$

A A



19. The figure above shows the graph of the function g and the line tangent to the graph of g at $x = -1$. Let h be the function given by $h(x) = e^x \cdot g(x)$. What is the value of $h'(-1)$?

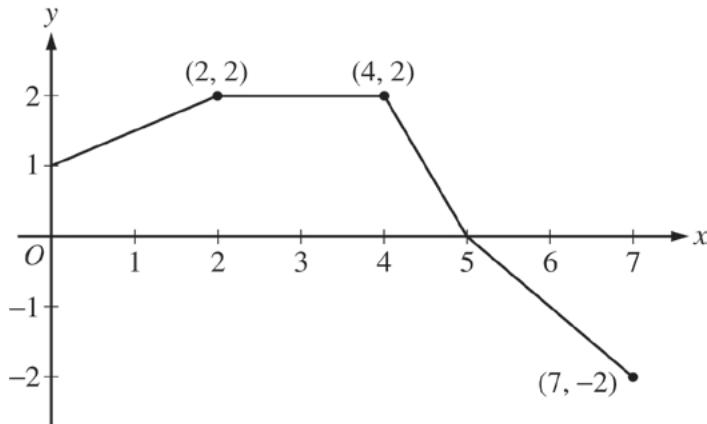
- (A) $\frac{9}{e}$ (B) $\frac{-3}{e}$ (C) $\frac{-6}{e}$ (D) $\frac{-6}{e} - \frac{3}{e^2}$ (E) -6

A A

20. For $x > 0$, $\frac{d}{dx} \left(\int_0^{2x} \ln(t^3 + 1) dt \right) =$

- (A) $\ln(x^3 + 1)$
- (B) $\ln(8x^3 + 1)$
- (C) $2\ln(x^3 + 1)$
- (D) $2\ln(8x^3 + 1)$
- (E) $24x^2 \ln(8x^3 + 1)$

A A



Graph of f

21. The graph of a function f is shown above. What is the value of $\int_0^7 f(x) dx$?
- (A) 6 (B) 8 (C) 10 (D) 14 (E) 18

A A

22. The function f is continuous for all real numbers, and the average rate of change of f on the closed interval $[6, 9]$ is $-\frac{3}{2}$. For $6 < c < 9$, there is no value of c such that $f'(c) = -\frac{3}{2}$. Of the following, which must be true?

- (A) $\frac{1}{3} \int_6^9 f(x) dx = -\frac{3}{2}$
- (B) $\int_6^9 f(x) dx$ does not exist.
- (C) $\frac{f'(6) + f'(9)}{2} = -\frac{3}{2}$
- (D) $f'(x) < 0$ for all x in the open interval $(6, 9)$.
- (E) f is not differentiable on the open interval $(6, 9)$.

23. Let f be the function defined by $f(x) = 2x + e^x$. If $g(x) = f^{-1}(x)$ for all x and the point $(0, 1)$ is on the graph of f , what is the value of $g'(1)$?

- (A) $\frac{1}{2+e}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 3 (E) $2+e$

A A

24. The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of g on the closed interval $[-2, 1]$?

- (A) -7 (B) $-\frac{3}{4}$ (C) 0 (D) 2 (E) 6
-

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

- (A) $y = -x - \ln 4$
(B) $y = x - \ln 4$
(C) $y = -\ln(-e^x + 5)$
(D) $y = -\ln(e^x + 3)$
(E) $y = \ln(e^x + 3)$

A A

26. Which of the following is an antiderivative of $f(x) = \sqrt{1 + x^3}$?

(A) $\frac{2}{3}(1 + x^3)^{3/2}$

(B) $\frac{\frac{2}{3}(1 + x^3)^{3/2}}{3x^2}$

(C) $\int_0^{1+x^3} \sqrt{t} dt$

(D) $\int_0^{x^3} \sqrt{1+t} dt$

(E) $\int_0^x \sqrt{1+t^3} dt$

27. For time $t \geq 0$, the height h of an object suspended from a spring is given by $h(t) = 16 + 7\cos\left(\frac{\pi t}{4}\right)$. What is the average height of the object from $t = 0$ to $t = 2$?

(A) 16

(B) $\frac{39}{2}$

(C) $16 - \frac{14}{\pi}$

(D) $16 + \frac{14}{\pi}$

(E) $32 + \frac{28}{\pi}$

A A

28. The function f is defined by $f(x) = \sin x + \cos x$ for $0 \leq x \leq 2\pi$. What is the x -coordinate of the point of inflection where the graph of f changes from concave down to concave up?

- (A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$ (E) $\frac{9\pi}{4}$

END OF PART A OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART A ONLY.**

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

B**B****B****B****B****B****B****B****B****CALCULUS AB****SECTION I, Part B****Time—50 minutes****Number of questions—17**

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON
THIS PART OF THE EXAM.

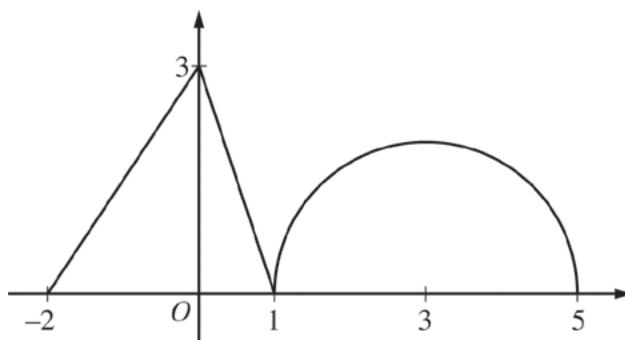
Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

**BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO
QUESTIONS NUMBERED 76–92.**

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

- (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
- (3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1} x = \arcsin x$).

B**B****B****B****B****B****B****B****B**Graph of f

76. The graph of the function f shown above consists of two line segments and a semicircle. Let g be defined by

$$g(x) = \int_0^x f(t) dt.$$

- What is the value of $g(5)$?
- (A) 0 (B) $-1.5 + 2\pi$ (C) 2π (D) $1.5 + 2\pi$ (E) $4.5 + 2\pi$

B**B****B****B****B****B****B****B****B**

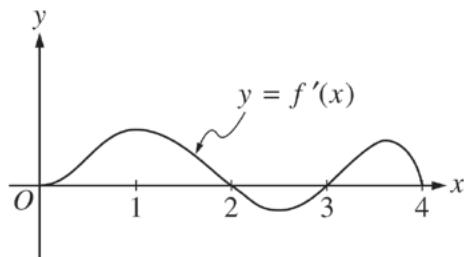
77. The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere?

(The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

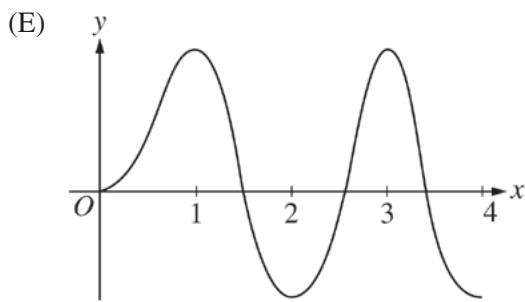
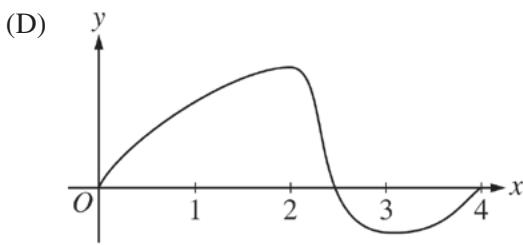
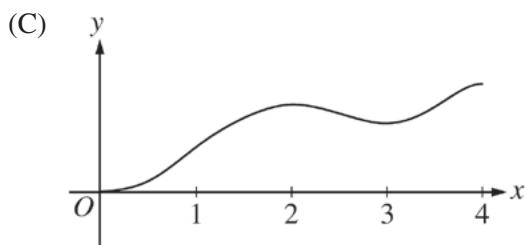
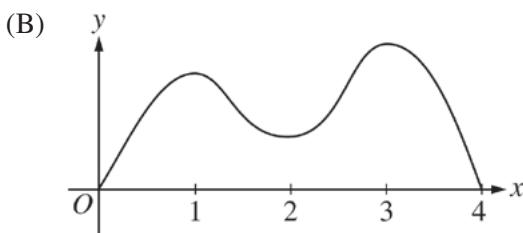
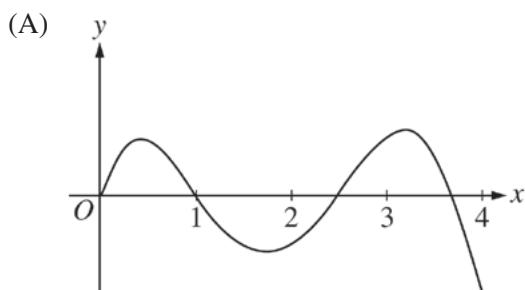
- (A) 0.141 cm (B) 0.244 cm (C) 0.250 cm (D) 0.489 cm (E) 0.977 cm

-
78. Let f and g be continuous functions such that $\int_0^{10} f(x) dx = 21$, $\int_0^{10} \frac{1}{2}g(x) dx = 8$, and $\int_3^{10} (f(x) - g(x)) dx = 2$. What is the value of $\int_0^3 (f(x) - g(x)) dx$?

- (A) 3 (B) 7 (C) 11 (D) 15 (E) 19

B**B****B****B****B****B****B****B****B**

79. The figure above shows the graph of f' , the derivative of the function f . If $f(0) = 0$, which of the following could be the graph of f ?



B**B****B****B****B****B****B****B****B**

80. For time $t \geq 0$, the position of a particle traveling along a line is given by a differentiable function s . If s is increasing for $0 \leq t < 2$ and s is decreasing for $t > 2$, which of the following is the total distance the particle travels for $0 \leq t \leq 5$?

(A) $s(0) + \int_0^2 s'(t) dt - \int_2^5 s'(t) dt$

(B) $s(0) + \int_2^5 s'(t) dt - \int_0^2 s'(t) dt$

(C) $\int_2^5 s'(t) dt - \int_0^2 s'(t) dt$

(D) $\left| \int_0^5 s'(t) dt \right|$

(E) $\int_0^5 |s'(t)| dt$

B**B****B****B****B****B****B****B****B**

81. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit ($^{\circ}\text{F}$). If the initial temperature of the tea, at time $t = 0$ minutes, is 200°F and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

- (A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

B**B****B****B****B****B****B****B****B**

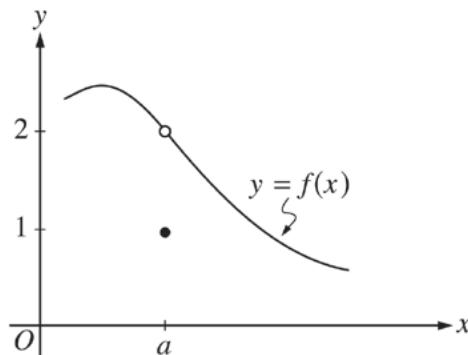
82. The derivative of the function f is given by $f'(x) = x^3 - 4\sin(x^2) + 1$. On the interval $(-2.5, 2.5)$, at which of the following values of x does f have a relative maximum?

- (A) -1.970 and 0
 - (B) -1.467 and 1.075
 - (C) -0.475 , 0.542 , and 1.396
 - (D) -0.475 and 1.396 only
 - (E) 0.542 only
-

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0	4	10	18	28	40	54

83. The table above gives selected values for a continuous function f . If f is increasing over the closed interval $[0, 3]$, which of the following could be the value of $\int_0^3 f(x)dx$?

- (A) 50
- (B) 62
- (C) 77
- (D) 100
- (E) 154

B**B****B****B****B****B****B****B****B**

84. The graph of a function f is shown in the figure above. Which of the following statements is true?

- (A) $f(a) = 2$
- (B) f is continuous at $x = a$.
- (C) $\lim_{x \rightarrow a} f(x) = 1$
- (D) $\lim_{x \rightarrow a} f(x) = 2$
- (E) $\lim_{x \rightarrow a} f(x)$ does not exist.

85. A particle moves along the x -axis so that at time $t \geq 0$ its position is given by $x(t) = \cos \sqrt{t}$. What is the velocity of the particle at the first instance the particle is at the origin?

- (A) -1
- (B) -0.624
- (C) -0.318
- (D) 0
- (E) 0.065

B**B****B****B****B****B****B****B****B**

86. If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x , which of the following could be a table of values for f ?

(A)

x	$f(x)$
-1	4
0	3
1	1

(B)

x	$f(x)$
-1	4
0	4
1	4

(C)

x	$f(x)$
-1	4
0	5
1	6

(D)

x	$f(x)$
-1	4
0	5
1	7

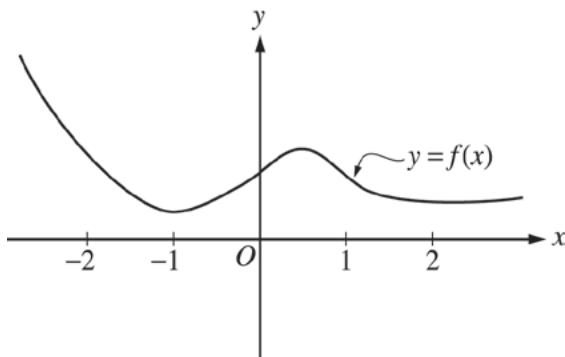
(E)

x	$f(x)$
-1	4
0	6
1	7

B**B****B****B****B****B****B****B****B**

87. Let f be the function with first derivative given by $f'(x) = (3 - 2x - x^2)\sin(2x - 3)$. How many relative extrema does f have on the open interval $-4 < x < 2$?

- (A) Two (B) Three (C) Four (D) Five (E) Six

B**B****B****B****B****B****B****B****B**

88. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

- (A) $f'(-1) < f'(1) < f'(0)$
- (B) $f'(-1) < f'(0) < f'(1)$
- (C) $f'(0) < f'(-1) < f'(1)$
- (D) $f'(1) < f'(-1) < f'(0)$
- (E) $f'(1) < f'(0) < f'(-1)$

B**B****B****B****B****B****B****B****B**

89. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y - 2}$ and the lines $x = 0$ and $y = 5$ is revolved about the y -axis?

- (A) 3.464 (B) 4.500 (C) 7.854 (D) 10.883 (E) 14.137

90. The population P of a city grows according to the differential equation $\frac{dP}{dt} = kP$, where k is a constant and t is measured in years. If the population of the city doubles every 12 years, what is the value of k ?

- (A) 0.058 (B) 0.061 (C) 0.167 (D) 0.693 (E) 8.318

B**B****B****B****B****B****B****B****B**

91. The function f is continuous and $\int_0^8 f(u) \, du = 6$. What is the value of $\int_1^3 xf(x^2 - 1) \, dx$?

- (A) $\frac{3}{2}$ (B) 3 (C) 6 (D) 12 (E) 24

92. The function f is defined for all x in the closed interval $[a, b]$. If f does not attain a maximum value on $[a, b]$, which of the following must be true?

- (A) f is not continuous on $[a, b]$.
(B) f is not bounded on $[a, b]$.
(C) f does not attain a minimum value on $[a, b]$.
(D) The graph of f has a vertical asymptote in the interval $[a, b]$.
(E) The equation $f'(x) = 0$ does not have a solution in the interval $[a, b]$.

B

B

B

B

B

B

B

B

B

END OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART B ONLY.**

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET
- WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET
- TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET

**AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND
ANSWER QUESTIONS 93–96.**

Section II: Free-Response Questions

This is the free-response section of the 2013 AP exam. It includes cover material and other administrative instructions to help familiarize students with the mechanics of the exam. (Note that future exams may differ in look from the following content.)

AP® Calculus AB Exam

SECTION II: Free Response

2013

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes

Number of Questions
6

Percent of Total Score
50%

Writing Instrument
Either pencil or pen with black or dark blue ink

Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2

Time
30 minutes

Electronic Device
Graphing calculator required

Percent of Section II Score
33.3%

Part B

Number of Questions
4

Time
60 minutes

Electronic Device
None allowed

Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name

First letter of your first name

2. Date of birth

Month Day Year

3. Six-digit school code

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.

No, I do not grant the College Board these rights.

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as $\text{fnInt}(X^2, X, 1, 5)$.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I

Form Code Z-3YBP2-S

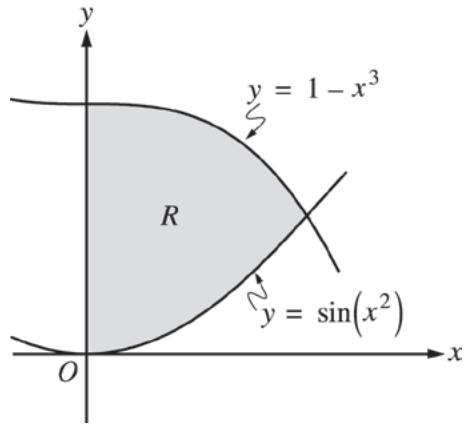
CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

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1. Let R be the shaded region in the first quadrant enclosed by the y -axis and the graphs of $y = 1 - x^3$ and $y = \sin(x^2)$, as shown in the figure above.
- (a) Find the area of R .

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- (b) A horizontal line, $y = k$, is drawn through the point where the graphs of $y = 1 - x^3$ and $y = \sin(x^2)$ intersect. Find k and determine whether this line divides R into two regions of equal area. Show the work that leads to your conclusion.

-
- (c) Find the volume of the solid generated when R is revolved about the line $y = -3$.

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2. The penguin population on an island is modeled by a differentiable function P of time t , where $P(t)$ is the number of penguins and t is measured in years, for $0 \leq t \leq 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by

$$B(t) = 1000e^{0.06t} \text{ penguins per year}$$

and the death rate for the penguins on the island is modeled by

$$D(t) = 250e^{0.1t} \text{ penguins per year.}$$

- (a) What is the rate of change of the penguin population on the island at time $t = 0$?

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- (b) To the nearest whole number, what is the penguin population on the island at time $t = 40$?

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- (c) To the nearest whole number, what is the average rate of change of the penguin population on the island for $0 \leq t \leq 40$?

-
- (d) To the nearest whole number, find the absolute minimum penguin population and the absolute maximum penguin population on the island for $0 \leq t \leq 40$. Show the analysis that leads to your answers.

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END OF PART A OF SECTION II

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON
PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.**

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

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NO CALCULATOR ALLOWED

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

3. The twice-differentiable function W models the volume of water in a reservoir at time t , where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \leq t \leq 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

- (a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

-
- (b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t) dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

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- (c) Explain why there must be at least one time t , other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

-
- (d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A , in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A , in square kilometers per day, with respect to t when $t = 30$ days.

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NO CALCULATOR ALLOWED

4. Let f be the function given by $f(x) = (x^2 - 2x - 1)e^x$.

(a) Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$.

$$\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$$

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(b) Find the intervals on which f is increasing. Show the analysis that leads to your answer.

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NO CALCULATOR ALLOWED

- (c) Find the intervals on which the graph of f is concave down. Show the analysis that leads to your answer.

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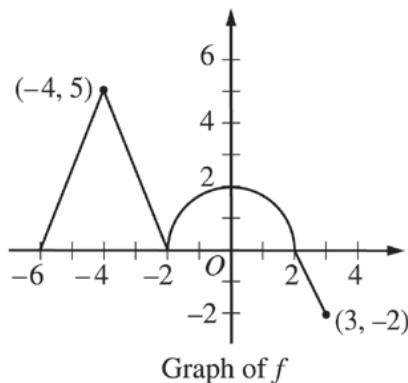
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NO CALCULATOR ALLOWED



5. The graph of the continuous function f , consisting of three line segments and a semicircle, is shown above.

Let g be the function given by $g(x) = \int_{-2}^x f(t) dt$.

(a) Find $g(-6)$ and $g(3)$.

(b) Find $g'(0)$.

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NO CALCULATOR ALLOWED

- (c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

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- (d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

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NO CALCULATOR ALLOWED

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6. Let f be a function with $f(2) = -8$ such that for all points (x, y) on the graph of f , the slope is given by $\frac{3x^2}{y}$.

- (a) Write an equation of the line tangent to the graph of f at the point where $x = 2$ and use it to approximate $f(1.8)$.

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NO CALCULATOR ALLOWED

- (b) Find an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = \frac{3x^2}{y}$ with the initial condition $f(2) = -8$.

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STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Multiple-Choice Answer Key

The following contains the answers to the multiple-choice questions in this exam.

**Answer Key for AP Calculus AB
Practice Exam, Section I**

Question 1: D	Question 24: A
Question 2: B	Question 25: C
Question 3: B	Question 26: E
Question 4: E	Question 27: D
Question 5: D	Question 28: B
Question 6: B	Question 76: D
Question 7: C	Question 77: E
Question 8: C	Question 78: A
Question 9: D	Question 79: C
Question 10: A	Question 80: E
Question 11: D	Question 81: A
Question 12: E	Question 82: E
Question 13: E	Question 83: B
Question 14: A	Question 84: D
Question 15: C	Question 85: C
Question 16: D	Question 86: E
Question 17: B	Question 87: E
Question 18: B	Question 88: D
Question 19: B	Question 89: E
Question 20: D	Question 90: A
Question 21: A	Question 91: B
Question 22: E	Question 92: A
Question 23: B	

Free-Response Scoring Guidelines

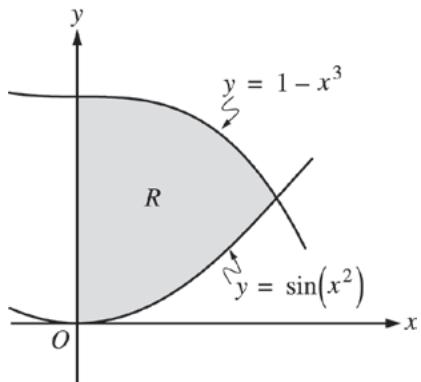
The following contains the scoring guidelines for the free-response questions in this exam.

**AP[®] CALCULUS AB
2013 SCORING GUIDELINES**

Question 1

Let R be the shaded region in the first quadrant enclosed by the y -axis and the graphs of $y = 1 - x^3$ and $y = \sin(x^2)$, as shown in the figure above.

- (a) Find the area of R .
- (b) A horizontal line, $y = k$, is drawn through the point where the graphs of $y = 1 - x^3$ and $y = \sin(x^2)$ intersect. Find k and determine whether this line divides R into two regions of equal area. Show the work that leads to your conclusion.
- (c) Find the volume of the solid generated when R is revolved about the line $y = -3$.



The graphs of $y = 1 - x^3$ and $y = \sin(x^2)$ intersect in the first quadrant at the point $(A, B) = (0.764972, 0.552352)$.

$$\begin{aligned} \text{(a)} \quad \text{Area} &= \int_0^A (1 - x^3 - \sin(x^2)) \, dx \\ &= 0.533 \text{ (or } 0.534\text{)} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad k &= B = 0.552352 \\ \int_0^A (1 - x^3 - k) \, dx &= 0.257 \text{ (or } 0.256\text{)} \\ \int_0^A (k - \sin(x^2)) \, dx &= 0.277 \text{ (or } 0.276\text{)} \end{aligned}$$

The two regions have unequal areas.

$$\begin{aligned} \text{(c)} \quad \text{Volume} &= \pi \int_0^A \left((1 - x^3 + 3)^2 - (\sin(x^2) + 3)^2 \right) \, dx \\ &= 11.841 \text{ (or } 11.840\text{)} \end{aligned}$$

1 : correct limits in an integral in (a), (b), or (c)

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 1 : \text{integral(s) with } k \text{ value} \\ 1 : \text{value(s) of integral(s)} \\ 1 : \text{conclusion tied to part (a)} \\ \text{or comparison of two integrals} \end{cases}$

Note: Stating k value only does not earn a point.

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

**AP[®] CALCULUS AB
2013 SCORING GUIDELINES**

Question 2

The penguin population on an island is modeled by a differentiable function P of time t , where $P(t)$ is the number of penguins and t is measured in years, for $0 \leq t \leq 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by

$$B(t) = 1000e^{0.06t} \text{ penguins per year}$$

and the death rate for the penguins on the island is modeled by

$$D(t) = 250e^{0.1t} \text{ penguins per year.}$$

- (a) What is the rate of change of the penguin population on the island at time $t = 0$?
- (b) To the nearest whole number, what is the penguin population on the island at time $t = 40$?
- (c) To the nearest whole number, what is the average rate of change of the penguin population on the island for $0 \leq t \leq 40$?
- (d) To the nearest whole number, find the absolute minimum penguin population and the absolute maximum penguin population on the island for $0 \leq t \leq 40$. Show the analysis that leads to your answers.

(a) $P'(0) = B(0) - D(0) = 1000 - 250 = 750$ penguins per year

1 : answer

$$\begin{aligned} (b) \quad P(40) &= 100000 + \int_0^{40} (B(t) - D(t)) dt \\ &= 100000 + 33057.56459 \end{aligned}$$

3 : $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

There are 133,058 penguins on the island.

(c) $\frac{1}{40} \int_0^{40} (B(t) - D(t)) dt = 826.439$

1 : answer

OR

$$\frac{P(40) - P(0)}{40 - 0} = \frac{133058 - 100000}{40} = 826.45$$

The average rate of change is 826 penguins per year.

(d) $B(t) - D(t) = 0$

$$1000e^{0.06t} = 250e^{0.1t} \Rightarrow t = A = \frac{\ln 4}{0.04} = 34.657359$$

The absolute minimum and absolute maximum occur at a critical point or at an endpoint.

$$P(0) = 100000$$

$$P(A) = 100000 + \int_0^A (B(t) - D(t)) dt = 139166.667$$

$$P(40) = 133058$$

The minimum population is 100,000 and the maximum population is 139,167 penguins.

4 : $\begin{cases} 1 : B(t) - D(t) = 0 \\ 1 : \text{solves for } t \\ 1 : \text{minimum value} \\ 1 : \text{maximum value} \end{cases}$

**AP[®] CALCULUS AB
2013 SCORING GUIDELINES**

Question 3

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

The twice-differentiable function W models the volume of water in a reservoir at time t , where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \leq t \leq 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

- (a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.
- (b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t) dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.
- (c) Explain why there must be at least one time t , other than $t = 10$, such that $W'(t) = 0.7$ GL/day.
- (d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A , in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A , in square kilometers per day, with respect to t when $t = 30$ days.

(a) An equation of the tangent line is $y = 0.5(t - 30) + 125$.
 $W(32) \approx 0.5(32 - 30) + 125 = 126$

(b) $\int_0^{30} W'(t) dt \approx (10)(0.6) + (12)(0.7) + (8)(1.0) = 22.4$
 $W(0) = W(30) - \int_0^{30} W'(t) dt = 125 - 22.4 = 102.6$

(c) W' is differentiable $\Rightarrow W'$ is continuous.

$$W'(30) = 0.5 < 0.7 < 1.0 = W'(22)$$

By the Intermediate Value Theorem, there must be at least one time t , $22 \leq t \leq 30$, such that $W'(t) = 0.7$.

(d)
$$\frac{dA}{dt} = (0.3) \frac{2}{3} W^{-1/3} \cdot \frac{dW}{dt} = \frac{0.2}{\sqrt[3]{W}} \cdot \frac{dW}{dt}$$

$$\left. \frac{dA}{dt} \right|_{t=30} = \frac{0.2}{\sqrt[3]{125}} \cdot 0.5 = 0.02$$

1 : answer

3 :
$$\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{answer} \end{cases}$$

2 : explanation

3 :
$$\begin{cases} 2 : \frac{dA}{dt} \\ 1 : \text{answer} \end{cases}$$

**AP[®] CALCULUS AB
2013 SCORING GUIDELINES**

Question 4

Let f be the function given by $f(x) = (x^2 - 2x - 1)e^x$.

- (a) Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$.
- (b) Find the intervals on which f is increasing. Show the analysis that leads to your answer.
- (c) Find the intervals on which the graph of f is concave down. Show the analysis that leads to your answer.

(a) $\lim_{x \rightarrow \infty} f(x) = \infty$ or does not exist

1 : answers

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

(b)
$$\begin{aligned}f''(x) &= (2x - 2)e^x + (x^2 - 2x - 1)e^x \\&= (x^2 - 3)e^x\end{aligned}$$

4 :
$$\begin{cases} 2 : f'(x) \\ 1 : \text{analysis} \\ 1 : \text{intervals} \end{cases}$$

$f''(x) = 0$ when $x = -\sqrt{3}, x = \sqrt{3}$

$f''(x) > 0$ for $-\infty < x < -\sqrt{3}$ and $\sqrt{3} < x < \infty$.

f is increasing on the intervals $-\infty < x \leq -\sqrt{3}$ and $\sqrt{3} \leq x < \infty$.

(c)
$$\begin{aligned}f''(x) &= 2xe^x + (x^2 - 3)e^x \\&= (x^2 + 2x - 3)e^x = (x + 3)(x - 1)e^x\end{aligned}$$

4 :
$$\begin{cases} 2 : f''(x) \\ 1 : \text{analysis} \\ 1 : \text{interval} \end{cases}$$

$f''(x) < 0$ for $-3 < x < 1$

The graph of f is concave down on the interval $-3 < x < 1$.

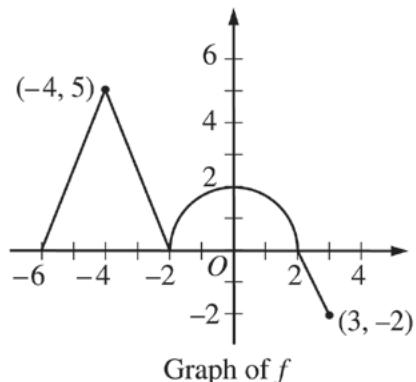
**AP[®] CALCULUS AB
2013 SCORING GUIDELINES**

Question 5

The graph of the continuous function f , consisting of three line segments and a semicircle, is shown above. Let g be the function given by

$$g(x) = \int_{-2}^x f(t) dt.$$

- (a) Find $g(-6)$ and $g(3)$.
- (b) Find $g'(0)$.
- (c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
- (d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.



(a) $g(-6) = \int_{-2}^{-6} f(t) dt = -\int_{-6}^{-2} f(t) dt = -\frac{1}{2} \cdot 4 \cdot 5 = -10$

$$g(3) = \int_{-2}^3 f(t) dt = \frac{1}{2}\pi \cdot 2^2 - \frac{1}{2} \cdot 1 \cdot 2 = 2\pi - 1$$

(b) $g'(0) = f(0) = 2$

- (c) The graph of g has a horizontal tangent at $x = -2$ and $x = 2$ where $g'(x) = f(x) = 0$.

The graph of g has neither a local maximum nor a local minimum at $x = -2$ because $g'(x) = f(x)$ does not change sign at $x = -2$.

The graph of g has a local maximum at $x = 2$ because $g'(x) = f(x)$ changes sign from positive to negative at $x = 2$.

- (d) The graph of g has a point of inflection at $x = -4$, $x = -2$, and $x = 0$.

$g'(x) = f(x)$ changes from increasing to decreasing at $x = -4$ and $x = 0$, and changes from decreasing to increasing at $x = -2$.

OR

$g''(x) = f'(x)$ changes from positive to negative at $x = -4$ and $x = 0$, and changes from negative to positive at $x = -2$.

2 : $\begin{cases} 1 : g(-6) \\ 1 : g(3) \end{cases}$

1 : $g'(0)$

3 : $\begin{cases} 1 : \text{horizontal tangent at } x = -2 \\ \quad \text{and } x = 2 \\ 2 : \text{answers with justifications} \end{cases}$

3 : $\begin{cases} 2 : \text{values of } x \\ 1 : \text{explanation} \end{cases}$

**AP[®] CALCULUS AB
2013 SCORING GUIDELINES**

Question 6

Let f be a function with $f(2) = -8$ such that for all points (x, y) on the graph of f , the slope is given by $\frac{3x^2}{y}$.

- (a) Write an equation of the line tangent to the graph of f at the point where $x = 2$ and use it to approximate $f(1.8)$.
- (b) Find an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = \frac{3x^2}{y}$ with the initial condition $f(2) = -8$.

(a) Slope $= \frac{(3)(4)}{-8} = -\frac{3}{2}$

An equation for the tangent line is $y = -\frac{3}{2}(x - 2) - 8$.

$$f(1.8) \approx -\frac{3}{2}(1.8 - 2) - 8 = -7.7$$

(b) $\int y \, dy = \int 3x^2 \, dx$

$$\frac{1}{2}y^2 = x^3 + C$$

$$\frac{1}{2}(-8)^2 = 2^3 + C \Rightarrow C = 24$$

$$y^2 = 2(x^3 + 24) = 2x^3 + 48$$

$$y = -\sqrt{2x^3 + 48}$$

Note: This solution is valid for $x > -\sqrt[3]{24}$.

3 : $\begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

6 : $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

Scoring Worksheet

The following provides a worksheet and conversion table used for calculating a composite score of the exam.

2013 AP Calculus AB Scoring Worksheet

Section I: Multiple Choice

$$\frac{\text{_____}}{\text{Number Correct}} \times 1.2000 = \frac{\text{_____}}{\text{Weighted Section I Score}} \\ (\text{out of 45}) \qquad \qquad \qquad (\text{Do not round})$$

Section II: Free Response

$$\text{Question 1} \quad \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Question 2} \quad \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Question 3} \quad \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Question 4} \quad \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Question 5} \quad \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Question 6} \quad \frac{\text{_____}}{\text{(out of 9)}} \times 1.0000 = \frac{\text{_____}}{\text{(Do not round)}}$$

$$\text{Sum} = \frac{\text{_____}}{\text{Weighted}} \\ \text{Section II} \\ \text{Score} \\ (\text{Do not round})$$

Composite Score

$$\frac{\text{_____}}{\text{Weighted Section I Score}} + \frac{\text{_____}}{\text{Weighted Section II Score}} = \frac{\text{_____}}{\text{Composite Score}} \\ (\text{Round to nearest whole number})$$

AP Score Conversion Chart
Calculus AB

Composite Score Range	AP Score
69-108	5
55-68	4
44-54	3
36-43	2
0-35	1

AP Calculus AB

The College Board

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