Q2)

Part 1) For a 1-D Image.

Given the equation,

$$g(x) = (h * f)(x)$$

Applying the fourier transform, we get

$$G(u) = H(u)\, F(u)$$

$$\Rightarrow \quad F(u) = \frac{G(u)}{H(u)} \qquad — ①$$

$$\Rightarrow \quad f(x) = \underbrace{F^{-1}}_{\text{fourier inverse}}(F(u))$$

The problem with eq ① is that, $h(x)$ is a convolution kernel for representing gradient operation, which implies $H(u)$ is a high pass filter.

So, at low frequencies $H(u) \approx 0$, then the value of $F(u)$ will increase which results in increase in noise in $f(x)$.

Part-2) For 2-D Image.

let g_x is the gradient of the 2-D Image in x-direction.

g_y is the gradient in y-direction.

h_x is the convolution kernel for gradient operation in X-direction

Similarly, h_y be for Y-direction.

$\therefore$ We have,

$$\cancel{g_x(x,y) = (h_x * f)(x,y)}$$

$$\cancel{g_y(x,y) = (h_y * f)(x,y)}$$

$$g_x(x,y) = (h_x * f)(x,y)$$

$$g_y(x,y) = (h_y * f)(x,y)$$

Applying fourier transform, we get

$$G_x(x,y) = H_x(u,v)\,F(u,v)$$

$$G_y(u,v) = H_y(u,v)\,F(u,v)$$

$$\Rightarrow \quad F(u,v) = \underbrace{\frac{G_x(u,v)}{H_x(u,v)}}_{\text{eq-1}} = \underbrace{\frac{G_y(u,v)}{H_y(u,v)}}_{\text{eq-2}}$$

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Now, $h_x(x,y)$ and $h_y(x,y)$ are both gradient kernel in X and Y- directions respectively. So, both $H_x(u,v)$ will be a high-pass filter in u and $H_y(u,v)$ will be a high-pass filter in v.

So, when u is small, $H_x(u,v)$ will be small, which implies $F(u,v)$ ~~wh~~ will significantly increase ~~which results~~ if we use eq ① for calculating $F(u,v)$ which will increase noise in $f(x,y)$

Similarly, when v is small, $H_y(u,v)$ will be small, which implies $F(u,v)$ will significantly increase if we use eq-② for calculating $F(u,v)$ which will increase nose in $f(x,y)$.

If both u and v are small, then both the ~~functions~~ equations will blow up.

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