

N.
Q1) Sol)

Given equations are,

$$g_1(x, y) = f_1(x, y) + (h_2 * f_2)(x, y) \quad - (1)$$

and $g_2(x, y) = (h_1 * f_1)(x, y) + f_2(x, y) \quad - (2)$

Fourier transformation of the equations, we get

$$G_1(u, v) = F_1(u, v) + H_2(u, v) F_2(u, v) \quad - (3)$$

$$G_2(u, v) = F_2(u, v) + H_1(u, v) F_1(u, v) \quad - (4)$$

Solving, eq (4) $\times H_2(u, v)$ we get,

$$H_2(u, v) G_2(u, v) = F_2(u, v) H_2(u, v) + H_1(u, v) F_1(u, v) H_2(u, v)$$

$$\Rightarrow G_1(u, v) - H_2(u, v) G_2(u, v) = F_1(u, v) - H_1(u, v) F_1(u, v) H_2(u, v)$$

$$\therefore F_1(u, v) = \frac{G_1(u, v) - H_2(u, v) G_2(u, v)}{1 - H_1(u, v) H_2(u, v)}$$

$$\Rightarrow f_1(x, y) = \underbrace{F^{-1}(F_1(u, v))}_{\text{fourier inverse}}$$

Similarly we get,

$$F_2(u, v) = \frac{G_2(u, v) - H_1(u, v)G_1(u, v)}{1 - H_1(u, v)H_2(u, v)}$$

$$\therefore f_2(x, y) = F^{-1}(F_2(u, v)).$$

The main problem with this formulae is, It is given that both $h_1(x, y)$, $h_2(x, y)$ are blur kernels, which implies both $H_1(u, v)$, $H_2(u, v)$ are low pass filters and at low frequencies low pass filters tend to 1,

$$\Rightarrow H_1(u, v)H_2(u, v) \approx 1$$

$$\Rightarrow 1 - H_1(u, v)H_2(u, v) \approx 0$$

So, for low ω we cannot use this approach to extract low frequency parts of $H_1(u, v)F_1(u, v)$ and $F_2(u, v)$. If the value of,

$1 - H_1(u, v)H_2(u, v) \approx 0$, then both $F_1(u, v)$ and $F_2(u, v)$ will increase which results in noise in $f_1(x, y)$ and $f_2(x, y)$ to get increased.