CS663 Assign 1

Question 3 : Part(a)
Part(b)

3) a) $\int_0^a h(x)\,dx = \alpha_e$

$\int_a^1 h(x)\,dx = 1 - \alpha_e$

$h_1(x) = h(x)\,\mathbb{1}_{[0,a]}$

$h_2(x) = h(x)\,\mathbb{1}_{(a,1]}$

where $\mathbb{1}_A = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$

let, $h_{1eq}(x) = HE(h_1(x)) = k_1\,\mathbb{1}_{[0,a]}$

$h_{2eq}(x) = HE(h_2(x)) = k_2\,\mathbb{1}_{[a,1]}$

where $k_1 \Rightarrow$ constant parameter for the uniform distribution obtained after equalization of $h_1(x)$

$k_2 \Rightarrow$ constant parameter for the uniform distribution obtained after equalization of $h_2(x)$

$\int_0^a k_1\,dx = \alpha_e$ (mass is preserved)

$\int_a^1 k_2\,dx = (1-\alpha_e)$ (mass is preserved)

$\Rightarrow k_1 = \frac{\alpha_e}{a}, \quad k_2 = \left(\frac{1-\alpha_e}{1-a}\right)$

final mean intensity of equalized histogram $= \dfrac{\int_0^a x.\,h_{1eq}(x)\,dx + \int_a^1 x.\,h_{2eq}(x)\,dx}{\int_0^a h_{1eq}(x)\,dx + \int_a^1 h_{2eq}(x)\,dx}$

$\cancel{\left(\frac{1-\alpha_e+a}{2}\right)}$

$$= \frac{\int_0^a x \frac{k_e}{a} dx + \int_a^1 \left(\frac{1-k_e}{1-a}\right) x \, dx}{1}$$

$$= \frac{k_e a}{2} + \frac{(1-k_e)(1+a)}{2}$$

$$= \frac{1-k_e+a}{2}$$

b) ~~mode~~ median is the value where CDF becomes $1/2$

so, if 'a' is median,

$$\int_0^a h(x) \, dx = 1/2$$

$$\Rightarrow k_e = \frac{1}{2}$$

So, mean intensity of equalized image

$$= \frac{1-\left(\frac{1}{2}\right)+a}{2}$$

$$= \frac{1+2a}{4} = \frac{1}{2}\left(a+\frac{1}{2}\right)$$

Scanned by CamScanner