The Illusion of Certainty in Meta-Analysis

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Packages

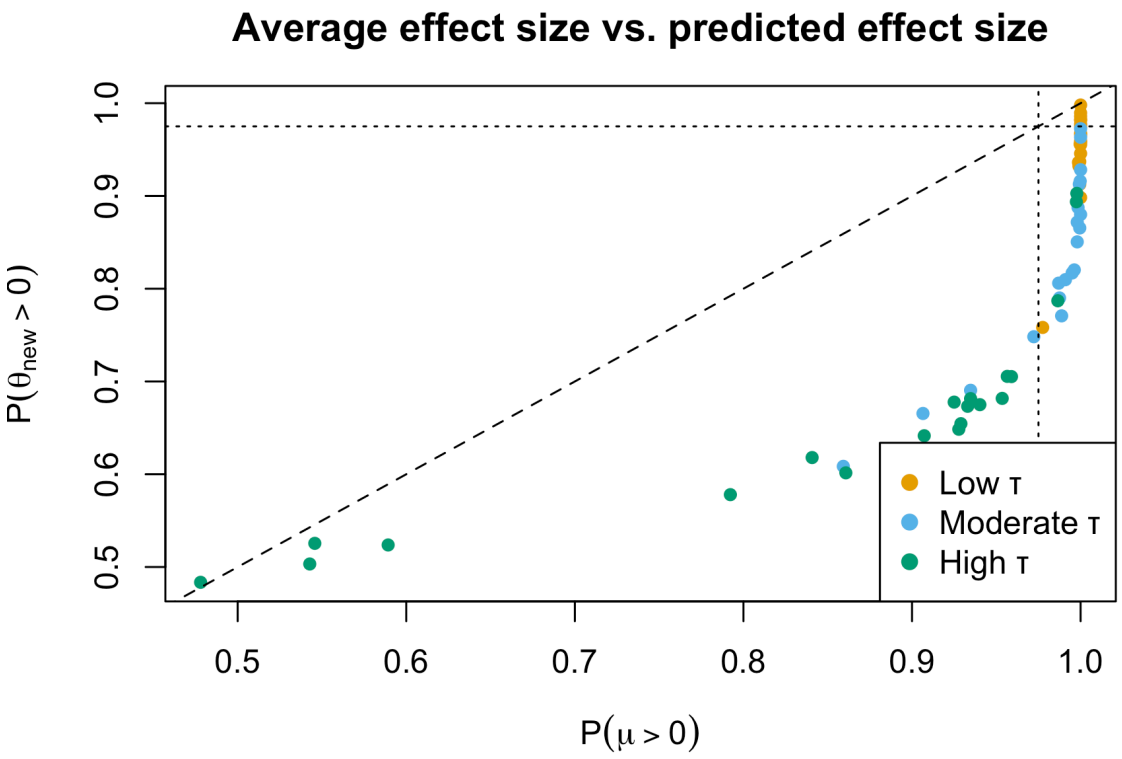
```
library(tidyverse)
library(here)
```

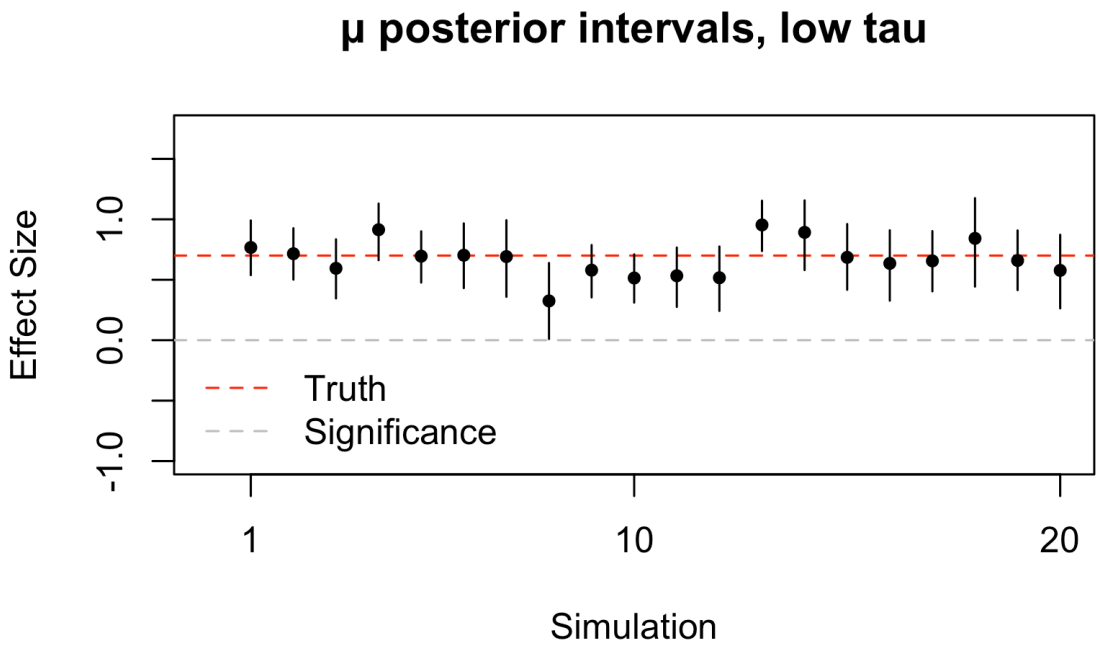
Stan model

```
data {
  int<lower=0> J; // num studies
  array[J] int<lower=0> n_t; // num cases, treatment
  array[J] int<lower=0> r_t; // num events, treatment
  array[J] int<lower=0> n_c; // num cases, control
  array[J] int<lower=0> r_c; // num events, control

  int<lower=0> estimate_posterior; // switch for estimating posterior vs running prior predi
  int<lower=0> priors; // switch for checking sensitivity of posterior to alternative specif
}
transformed data {
  array[J] real y; // log odds ratio for each study
  for (j in 1:J) {
    y[j] = log(r_t[j]) - log(n_t[j] - r_t[j])
    - (log(r_c[j]) - log(n_c[j] - r_c[j]));
  }

  array[J] real<lower=0> se; // standard error of y (inverse variance method)
  for (j in 1:J) {
    se[j] = sqrt(1.0 / r_t[j] + 1.0 / (n_t[j] - r_t[j])
    + 1.0 / r_c[j] + 1.0 / (n_c[j] - r_c[j]));
  }
```

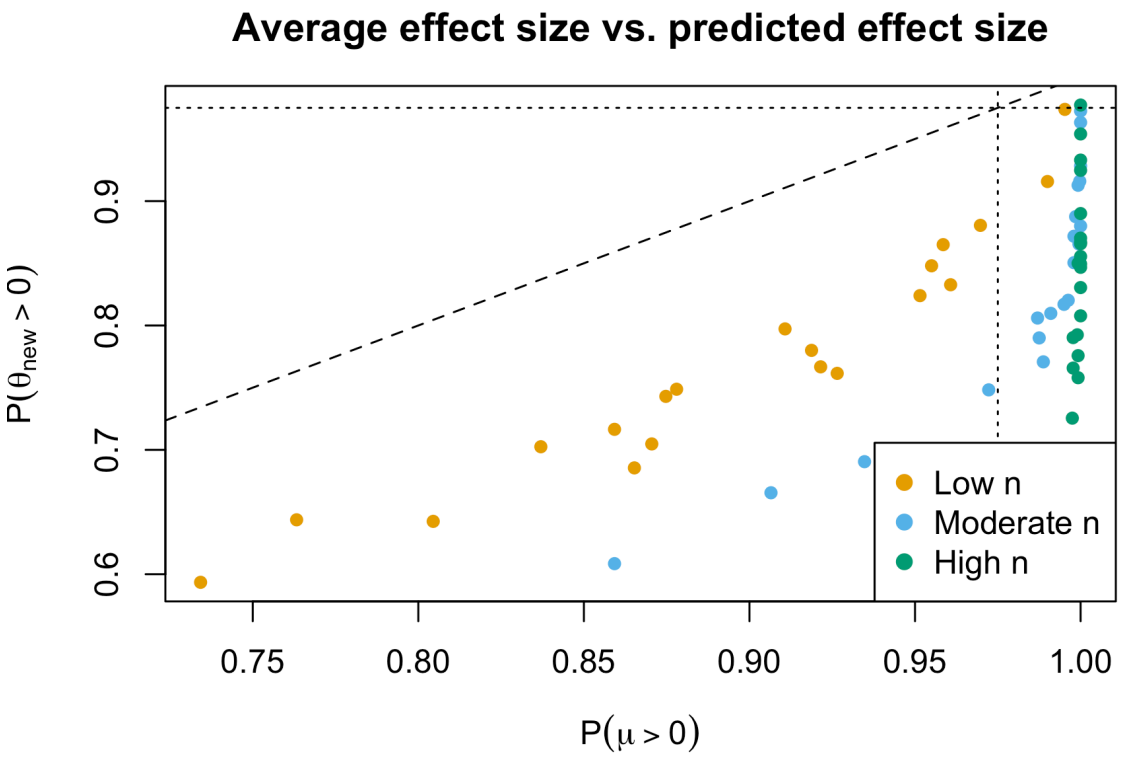
```
}
parameters {
  real mu; // mean treatment effect
  real<lower=0> tau; // deviation of treatment effects from the mean
  vector<offset=mu,multiplier=tau>[J] theta; // trial-specific treatment effects
}
model {
  if (estimate_posterior == 1) {
  y[1:J] ~ normal(theta[1:J], se[1:J]);
  }

  theta[1:J] ~ normal(mu, tau);
  if (priors == 1) { // standard normal
    mu ~ std_normal();
    tau ~ std_normal();
  } else { // CDSR
    mu ~ student_t(3.8, 0, 0.48);
    tau ~ lognormal(-1.44, 0.79);
  }
}
generated quantities {
  // pooling metrics
  vector[J] se2 = square(to_vector(se)); // approximate sampling variance for each study
  real se2_hat = sum(se2) / J; // average approximate sampling variance across all studies
  real<lower=0> i2 = square(tau) / (square(tau) + se2_hat); // proportion of total variance i
  vector[J] p = 1 - (square(tau) / (square(tau) + se2)); // proportion of variance in the tri

  // posterior predictive distribution
  real theta_new = normal_rng(mu, tau);

  // event probabilities
  real mu_gt_0 = mu > 0;
  real theta_new_gt_0 = theta_new > 0;

  // Universe of 100 possible future studies
  array[100] real theta_100;
  for (i in 1:100) {
    theta_100[i] = normal_rng(mu, tau);
  }
}
```

Experiment 1: variance

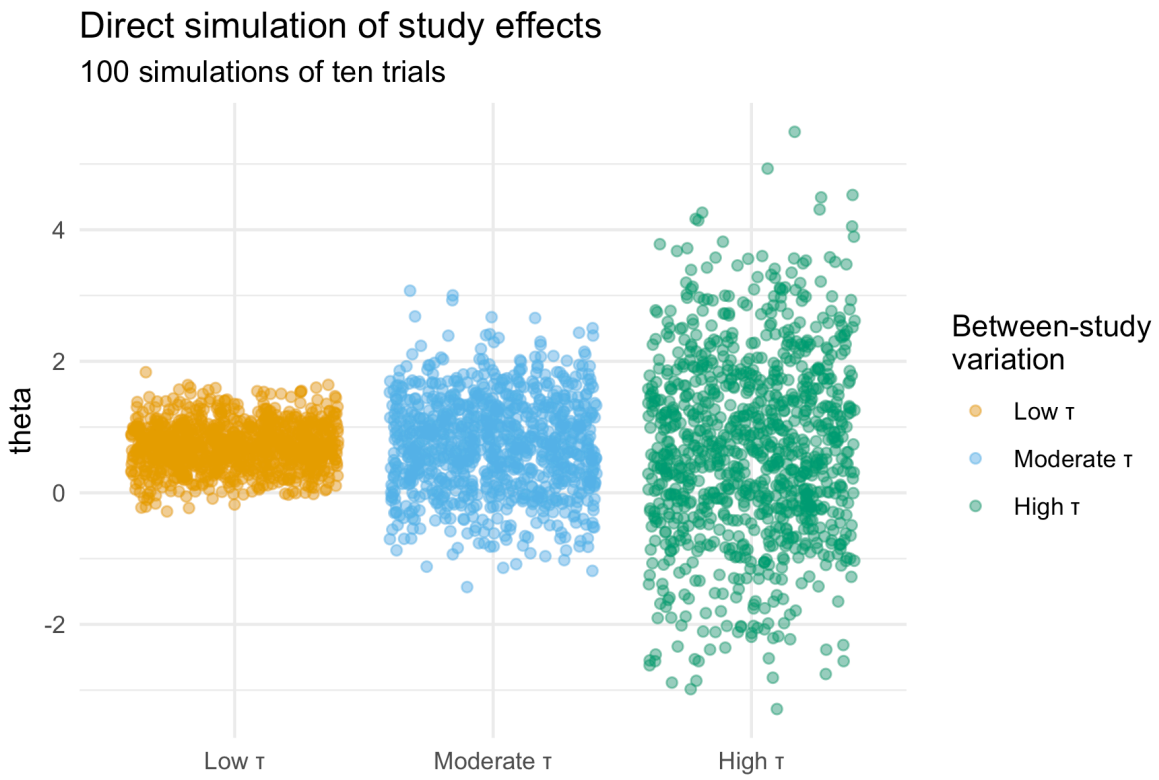


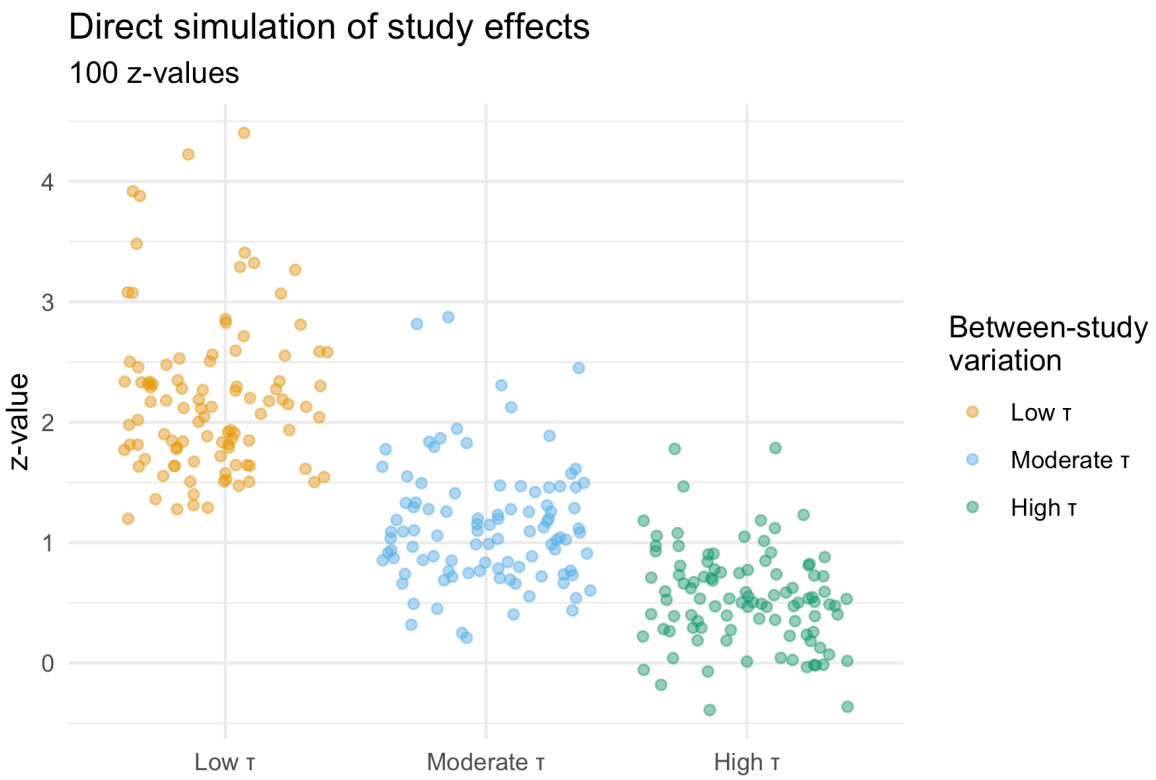

μ posterior intervals, low tau
Effect Size
1.0
0.0
-1.0
Truth
Significance
1
10
20
Simulation

Experiment 2: trials



Experiment 3: forward simulation




Direct simulation of study effects
100 z-values
z-value
4
3
2
1
0
Low τ
Moderate τ
High τ
Between-study variation
Low τ
Moderate τ
High τ