ROB 456: Intelligent Robots

Week 7, Lecture 1
Path Planning

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Today

- Motion planning
- Graph search methods
- Sampling-based methods
- Planning in the real world

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MOTION PLANNING

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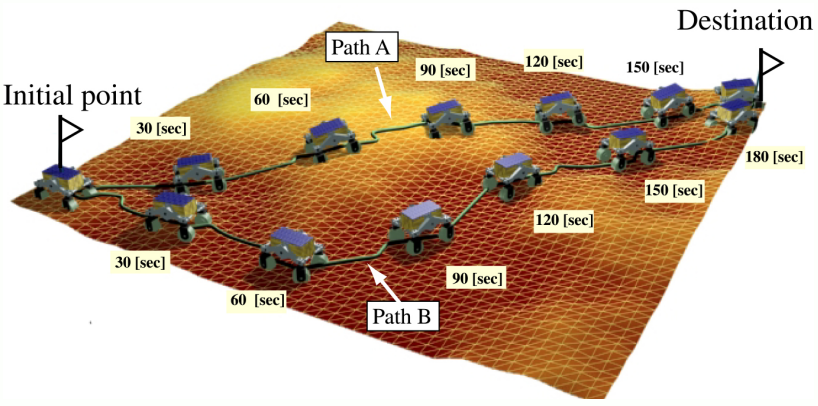
What is Path Planning?



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Planning

- How to get from the current robot state to the desired robot state
- Possible when robot possesses a **representation** of the world state and can measure the **utility** of its actions
- Challenging because the space of possible plans grows exponentially with the plan duration, dimension, etc.

Motion Planning

- Given start point S, end point G:
  - Find controls to take robot from S to G.
- What are challenges:
  - Obstacles?
  - Dynamics?
  - Feasibility?
- Typically cast as an optimization problem

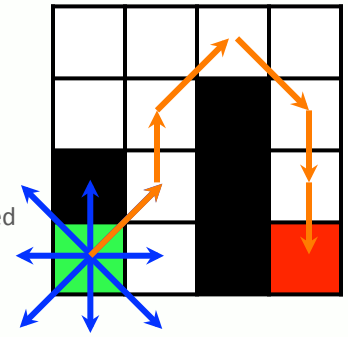
What is Path Planning?

- Path planning: detailing a motion task into discrete motions
  - Robot must get from starting location to defined waypoint
  - Robot must not run into walls, fall down stairs, etc.
  - Ideally, path taken is most efficient (shortest) path
- Inputs to algorithm:
  - Description of task (starting and ending locations)
  - Constraints (robot's physical limitations, environmental obstacles, etc.)
  - Uncertainty about robot and environment
- Outputs of algorithm:
  - Physical path
    - Set of waypoints
    - Speed and turning commands sent to robot's wheels at each time step

GRAPH SEARCH METHODS

Example 1: Discretizing the World

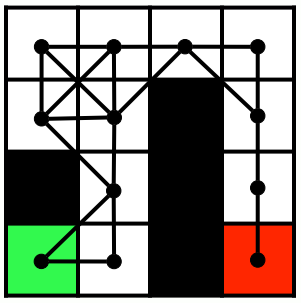
- Move from green cell to red cell
- 8-connected grid motion
- Shortest path?
- What if robot motion was restricted to 4-connected grid motion?



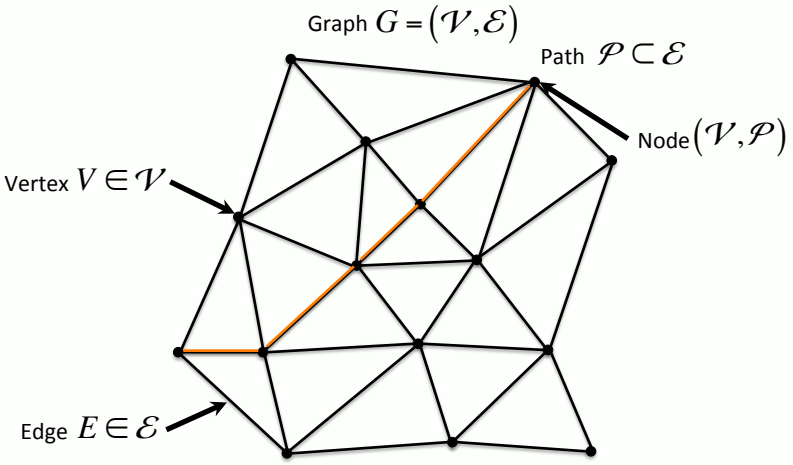
Assumptions about robot motion

Example 1: Discretizing the World

- Search over the underlying graph
- Solve for paths from any point to any other point
- Assume all edge transitions are dynamically feasible



Some Terminology



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Graph Search

- Need to keep track of path costs
- Expand only the necessary nodes

- Graph search methods
  - Breadth-first search (BFS)
  - Depth-first search (DFS)
  - Dijkstra's Algorithm
  - A*

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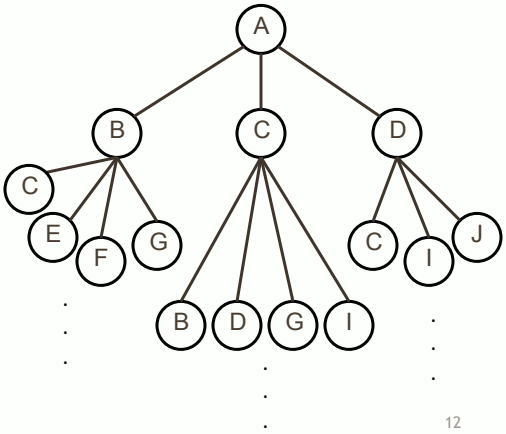
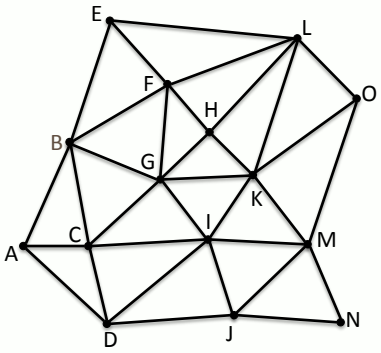
Search Trees

- We construct a "tree" through which we can search for optimal paths through the environment





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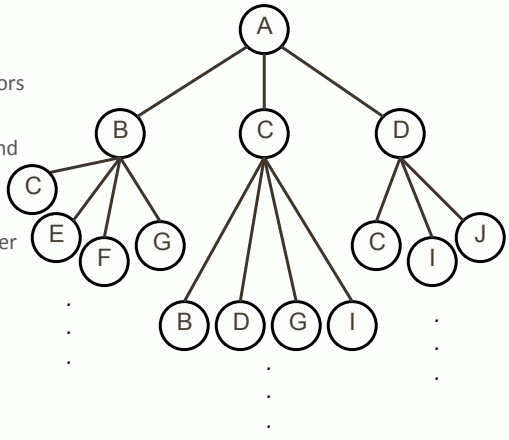
Search Trees

- A search tree:
  - Start state at the root node
  - Children correspond to successors
  - Nodes contain states, correspond to PLANS to those states
  - For most problems, we can never actually build the whole tree
  - Use search algorithms to efficiently traverse tree



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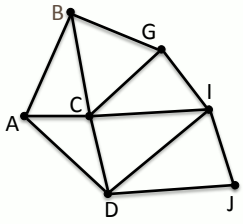
Basics of Search

- Expand out possible plans
- Try to expand as few tree nodes as possible

- **Priority queue** maintains a frontier of unexpanded plans
  - **Open** set keeps track of what nodes to expand next

- **Closed** set keeps track of nodes that have been expanded

Breadth-First Search Example



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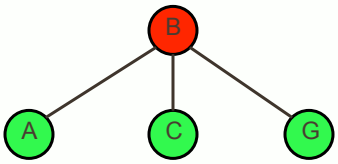
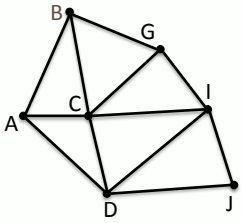
Breadth-First Search Example





Opened: {A,C,G}

Closed: {B}

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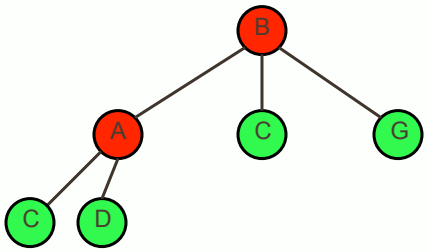
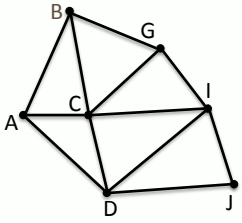
Breadth-First Search Example





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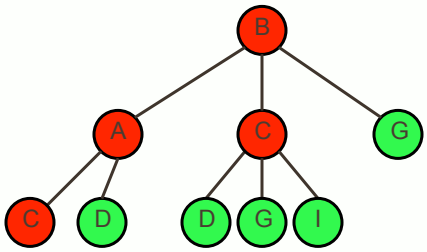
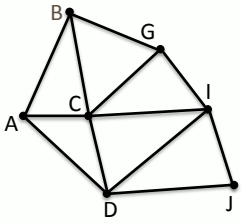
Breadth-First Search Example





Opened: {G,D,I}

Closed: {B,A,C}

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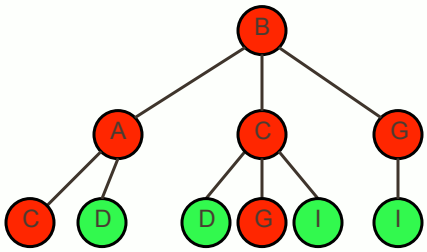
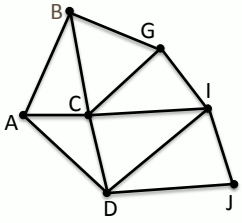
Breadth-First Search Example





Opened: {D,I}

Closed: {B,A,C,G}

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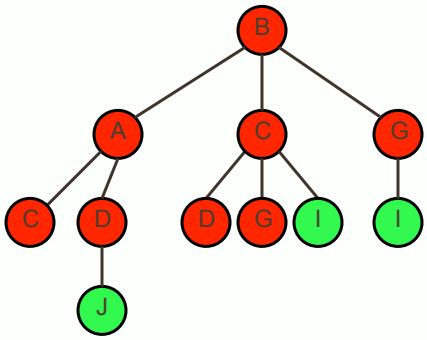
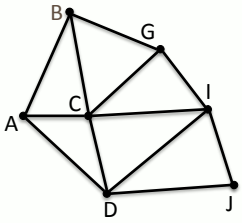
Breadth-First Search Example





Opened: {I,J}

Closed: {B,A,C,G,D}

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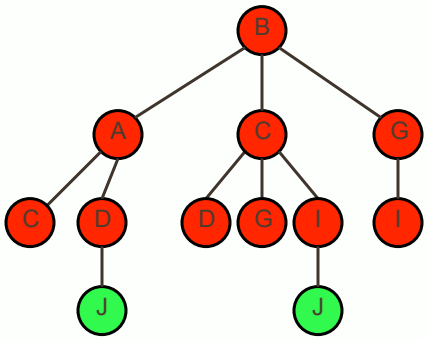
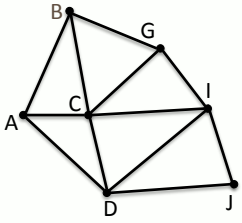
Breadth-First Search Example





Opened: {J}

Closed: {B,A,C,G,D,I}

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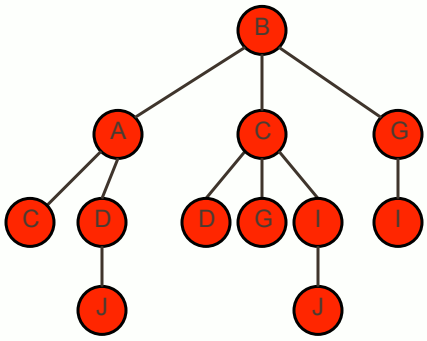
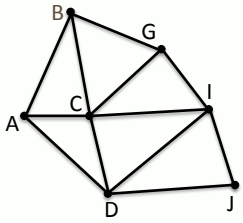
Breadth-First Search Example





Opened:
{}

Closed:
{B,A,C,G,D,I,J}

Final path solution: B→A→D→J

Other solutions exist but also have the same number of transitions

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Breadth-First Search

- BFS is guaranteed to find the optimal path through the search tree
  - First solution found is the optimal path
- Unfortunately, BFS is computationally inefficient
- Time complexity of O(V+E)
  - Worst case is that each node is visited: computationally expensive!
- Consider another approach: Depth-first search

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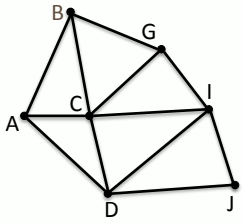
Depth-First Search Example (Iterative Deepening)

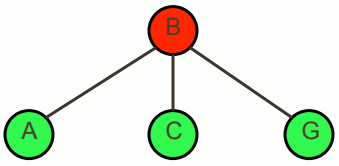
(Keep track of the least cost path to vertex)

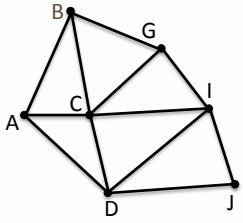


Opened: {B}

Closed: {}

Depth-First Search Example (Iterative Deepening)

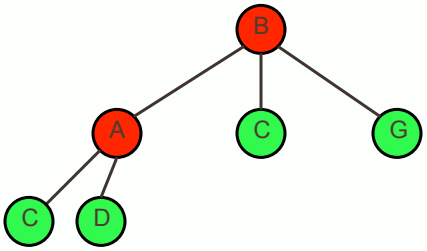


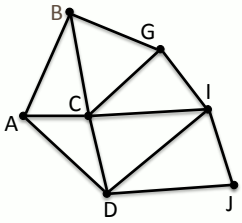


Opened: {A,C,G}

Closed: {B}

Depth-First Search Example (Iterative Deepening)





Opened: {C,G,D}

Closed: {B,A}

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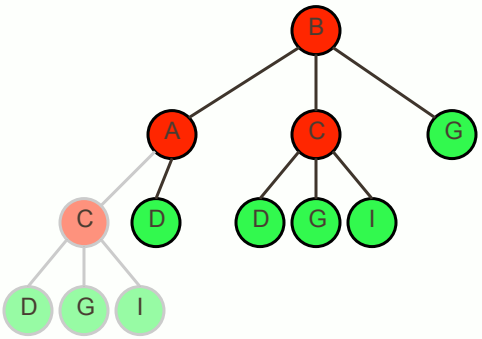
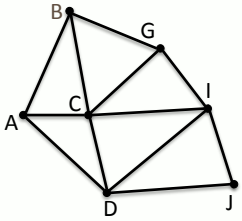
Depth-First Search Example (Iterative Deepening)





Opened: {G,D,I}

Closed: {B,A,C}

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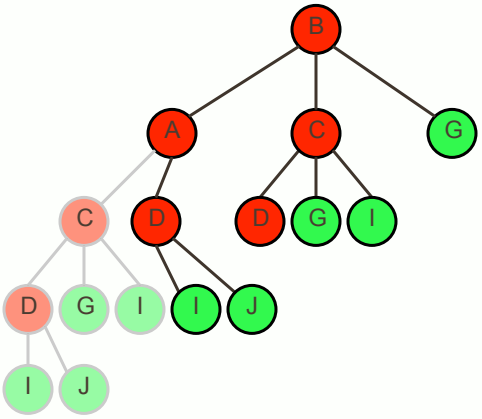
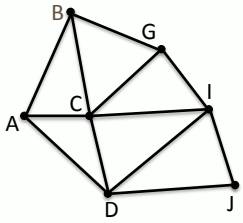
Depth-First Search Example (Iterative Deepening)





Opened: {G,I,J}

Closed: {B,A,C,D}

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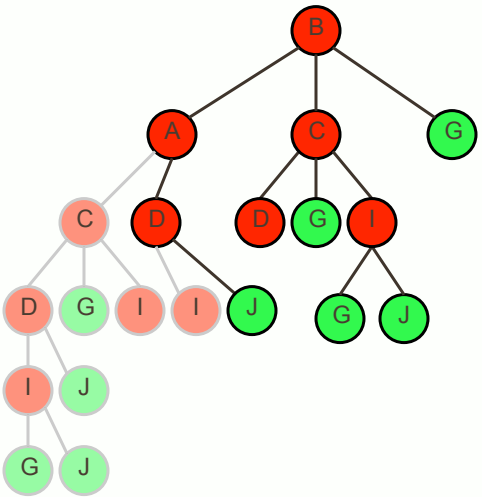
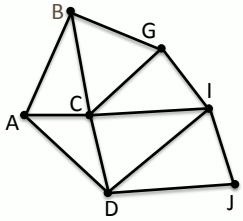
Depth-First Search Example (Iterative Deepening)





Opened: {G,J}

Closed: {B,A,C,D,I}

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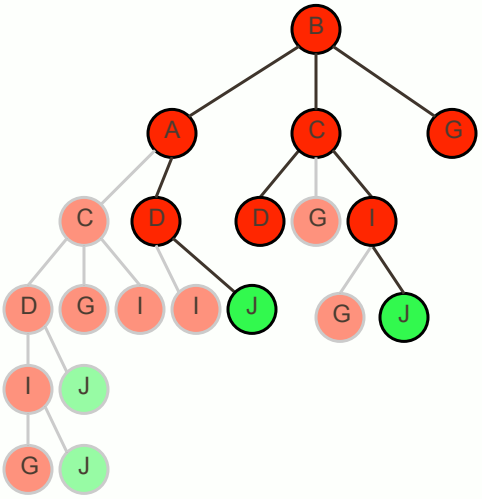
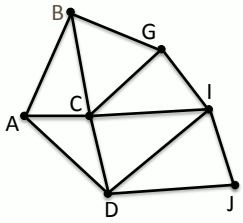
Depth-First Search Example (Iterative Deepening)





Opened: {J}

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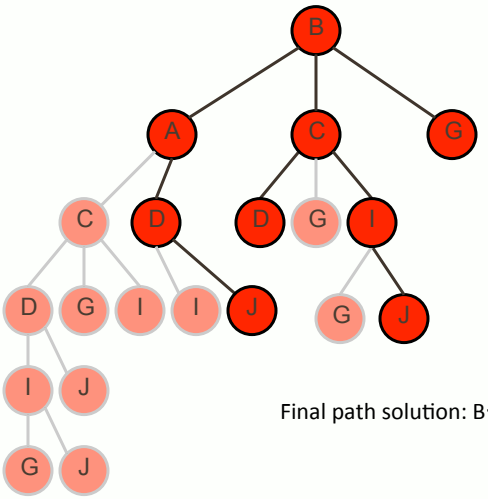
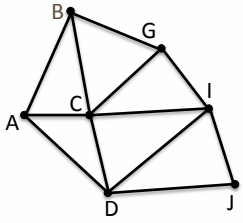
Depth-First Search Example (Iterative Deepening)





Opened: {}

Closed: {B,A,C,D,I,G,J}

Final path solution: B→A→D→J

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Depth-First Search

- Depth First Search (DFS) is a search algorithm where the search starts at the root node and explores as far as possible along each branch before backtracking
- Like BFS, DFS is guaranteed to find the optimal path in the tree
- Problem with DFS: graph is often too large to visit in its entirety, and DFS may not terminate when path lengths are infinite (looping through grid)
  - Therefore we typically only search to limited depth

BFS vs. DFS

- Time bounds of DFS and BFS are equivalent: in the worst case, each algorithm searches the entire tree
- Space bounds of DFS are better than BFS: DFS takes less memory than BFS, since it isn't necessary to store all children at each level of the tree search
- Which algorithm is "better" is more dependent on the domain than computational complexity of the algorithms:
  - If solution is higher up the tree (closer to the root node), then BFS will perform better
  - If solution is farther down the tree (farther from the root node), then DFS will perform better
- Both BFS and DFS are simple to implement, however are computationally inefficient. More complex algorithms are faster, but more difficult to implement

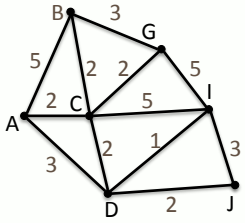
Costs on Actions

- Both BFS and DFS find the shortest path in terms of number of transitions.
  - What about costs?
- Dijkstra's Algorithm and A* search

Dijkstra's Algorithm

- Developed by Edsger Dijkstra in 1959
- Breadth-first search with edge costs
- One of the most commonly used routing algorithms in graph traversal problems
- Asymptotically the fastest known single-source shortest path algorithm for arbitrary directed graphs
- **Open** queue is ordered according to cost to arrive

Dijkstra's Algorithm Example



Open: Queue: {B(0),

Unvisited: A(∞), C(∞), D(∞), G(∞), I(∞), J(∞)}

Closed: {}

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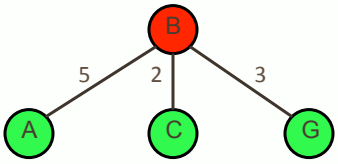
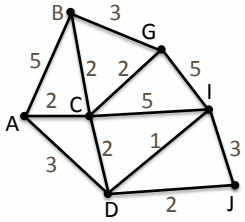
Dijkstra's Algorithm Example





Open: Queue: {C(2), G(3), A(5),

Unvisited: D(∞), I(∞), J(∞)}

Closed: {B(0)}

Ordered according to cost to arrive

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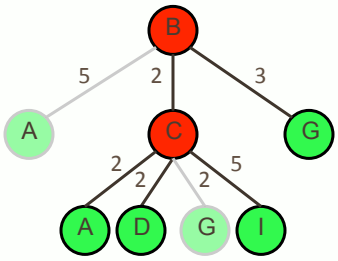
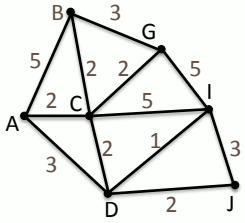
Dijkstra's Algorithm Example





Queue: {G(3), A(4), D(4), I(7), J(∞)}

Open: {G(3), A(4), D(4), I(7),

Unvisited: J(∞)}

Closed: {B(0), C(2)}

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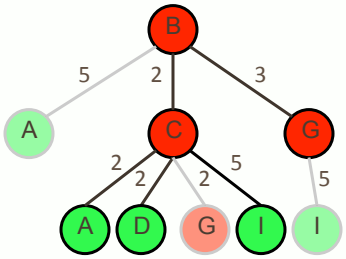
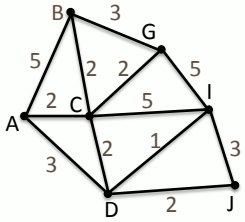
Dijkstra's Algorithm Example





Queue: {A(4), D(4), I(7), J(∞)}

Open: {A(4), D(4), I(7),

Unvisited: J(∞)}

Closed: {B(0), C(2), G(3)}

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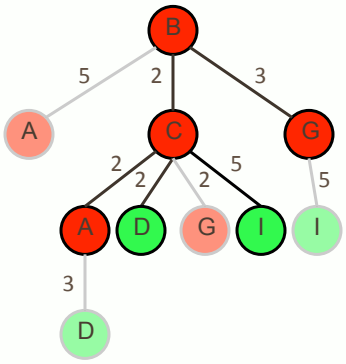
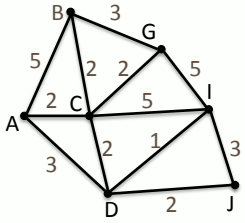
Dijkstra's Algorithm Example





Queue:
Open: {D(4), I(7),
Unvisited: J(∞)}

Closed:
{B(0), C(2), G(3), A(4)}

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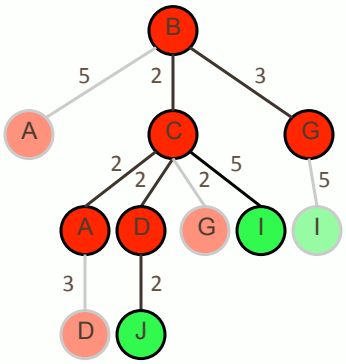
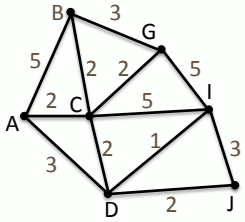
Dijkstra's Algorithm Example





Queue:
Open: {J(6), I(7)}

Closed:
{B(0), C(2), G(3), A(4), D(4)}

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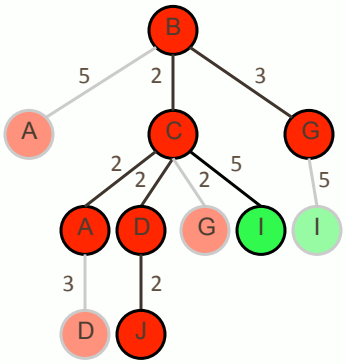
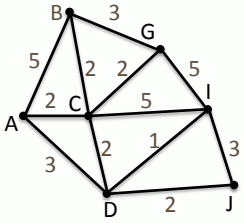
Dijkstra's Algorithm Example





Queue:
{I(7)}

Closed:
{B(0),
C(2),
G(3),
A(4),
D(4),
J(6)}

Final path solution: B→C→D→J
with path cost 6

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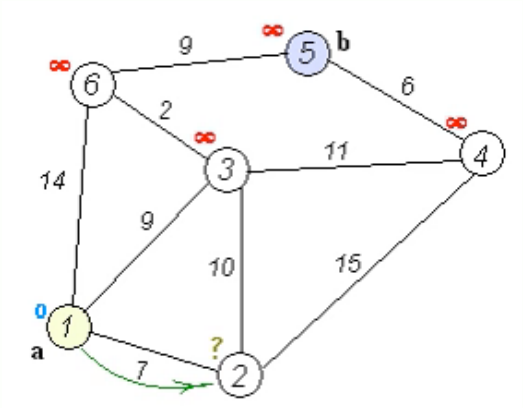
Dijkstra's Algorithm Overview



Courtesy wikipedia (http://en.wikipedia.org/wiki/Dijkstra%27s_algorithm)

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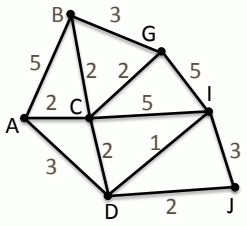
A* Heuristic Search

- Heuristic:
  - Any *estimate* of how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance

- A* : Cost is f(n) = g(n) + h(n)

Cost to arrive Heuristic cost to goal

A* Example

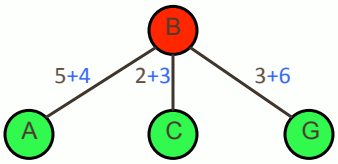


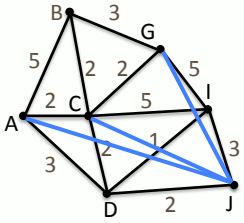
Open: Queue: {B(0),

Unvisited: A(∞), C(∞), D(∞), G(∞), I(∞), J(∞)}

Closed: {}

A* Example

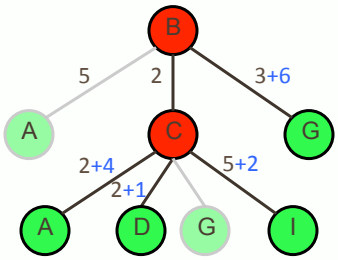


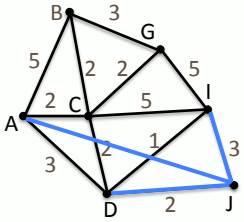


Queue: Closed:
Open: {C(2+3), {B(0)}
G(3+6),
A(5+4),
Unvisited: D(∞),
I(∞),
J(∞)}

Ordered according to A* cost

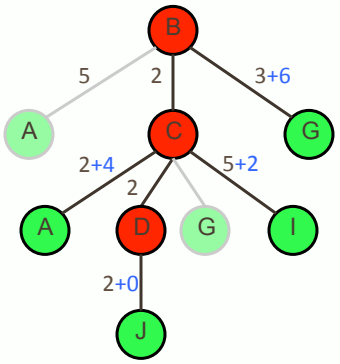
A* Example

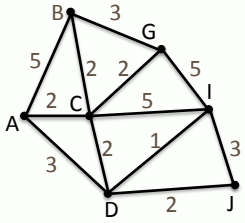




Queue: Closed:
Open: {D(4+1), {B(0),
A(4+4), C(2)}
G(3+6),
I(7+2),
Unvisited: J(∞)}

A* Example





Open: Queue: {J(6+0, A(4+4), G(3+6), I(7+2)}

Closed: {B(0), C(2), D(4)}

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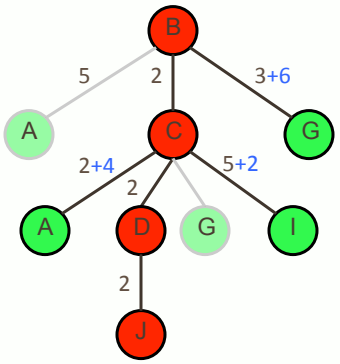
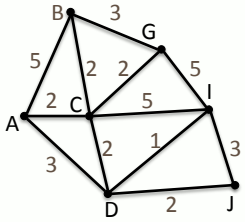
A* Example





Open: Queue: {A(4+4), G(3+6), I(7+2)}

Closed: {B(0), C(2), D(4), J(6)}

Final path solution: B→C→D→J
with path cost 6

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A* Heuristic

- The heuristic must be ***admissible***
  - It never overestimates the cost

$$h(x) \leq d(x, goal)$$

True cost to goal

- The heuristic must be ***consistent***
  - For any pair of adjacent nodes x and y, where d(x,y) is the cost of edge between them

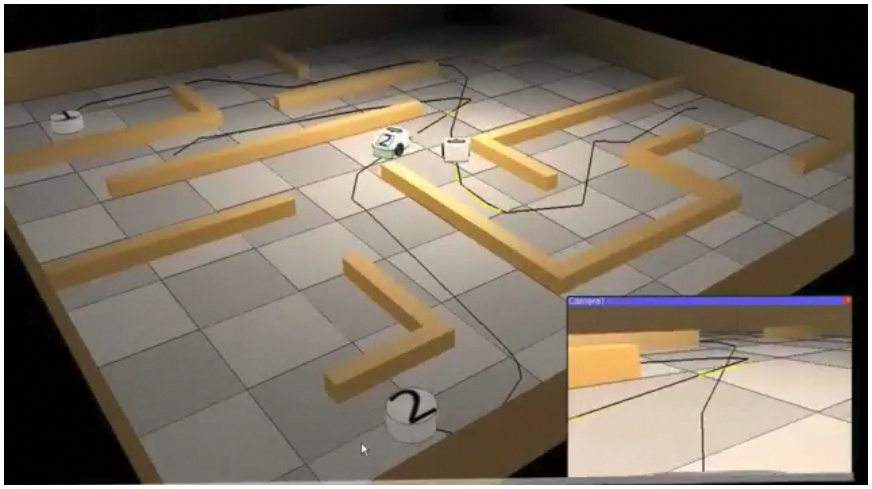
$$h(x) \leq d(x, y) + h(y)$$

- Typical valid heuristics:
  - Euclidean distance
  - Manhattan distance
  - Zero (Dijkstra's algorithm)

A* Search Algorithm

- A* is an extension of Dijkstra's algorithm, and achieves faster performance by using heuristics
- Uses best-first search: A* traverses a graph following a path of lowest expected total cost or distance
- Uses a knowledge plus heuristic cost function to determine the order in which the search visits nodes in the tree.
- The cost function is a sum of two functions:
  - Past path-cost function, which is known cost from the starting node to the current node
  - Future path-cost function, which is a "heuristic estimate" of the distance from the current node to the goal

Planning Example



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Overview of Planning So Far

- We have explained how to discretize world into a grid, and then use search algorithms to find shortest paths from starting point to endpoint in that grid.
- We covered BFS, DFS, Dijkstra's algorithm, and A* search

- Assumptions:
  - World is static
  - Paths on graphs are dynamically feasible
  - Robot has control actions that allow it to follow paths exactly

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SAMPLING-BASED PLANNING

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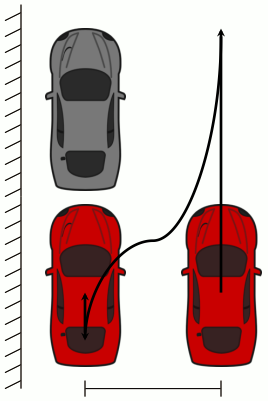
Planning Space

- Reverse parallel parking example:



$$d_{workspace} = \|\mathbf{x}_1 - \mathbf{x}_0\|$$

1. Drive straight forward
2. Reverse and turn steering wheel to left
3. Reverse and go straight
4. Reverse and turn steering wheel to right
5. Drive straight forward

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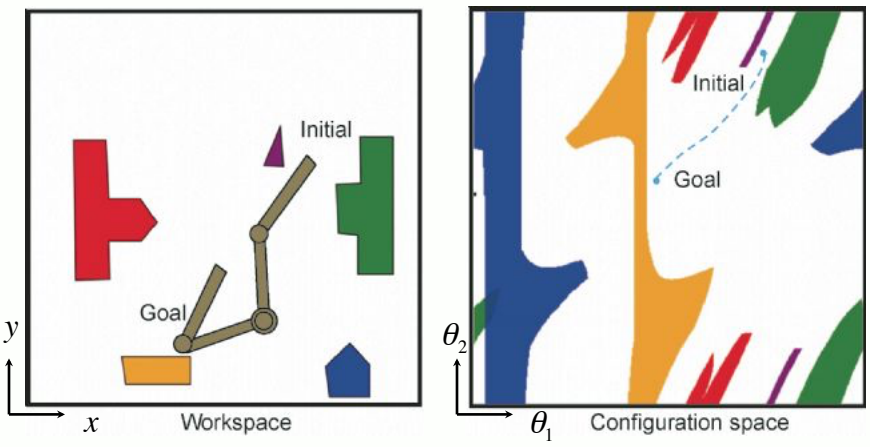
Planning Space: Robot Arm Example



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Sampling-Based Planning

- Sample available configurations and control actions
- Create a tree or graph of valid transitions
- Find the best path that gets you closest to the goal
- Methods:
  - Rapidly-exploring random trees (RRTs)
  - Probabilistic roadmaps (PRMs)

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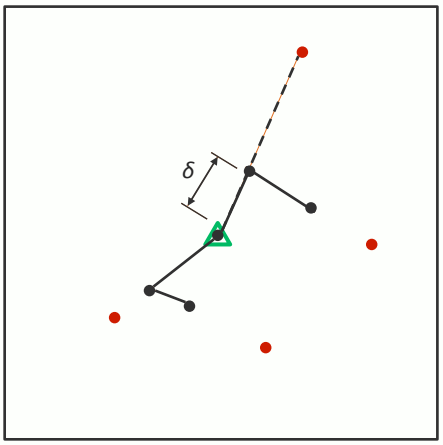
Rapidly-Exploring Random Trees (RRTs)



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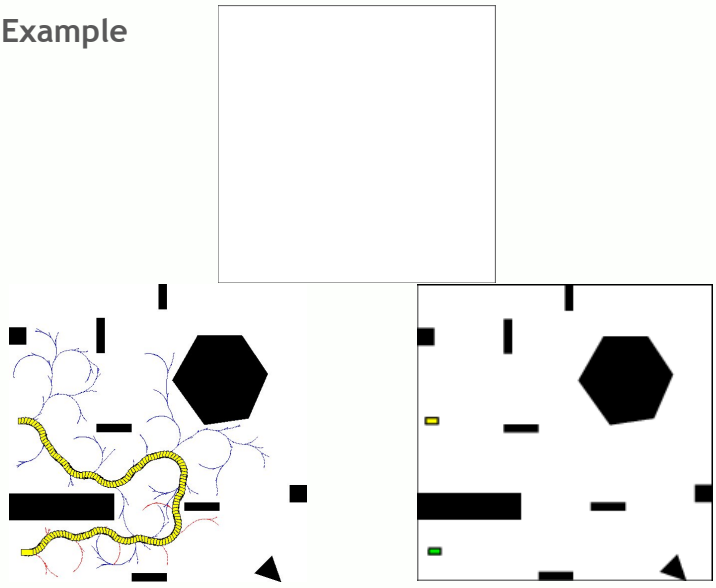
RRT Example



Steve LaValle (http://msl.cs.uiuc.edu/rrt/gallery.html)

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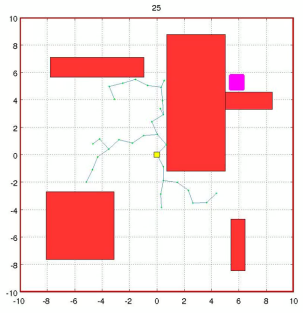
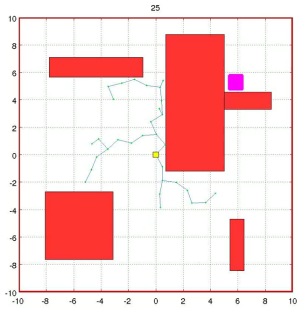
RRT* Comparison

RRT



RRT*



https://www.youtube.com/watch?v=FAFw8DoKvik

https://www.youtube.com/watch?v=YKiQTJpPFkA

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RRT Summary

- Paths are dynamically feasible
- RRT* is ***probabilistically complete***
  - As the number of points you sample goes to infinity, you will find a path from start to goal
- RRT* is ***asymptotically optimal***
  - As the number of points you sample goes to infinity, your path solution converges to the optimal path
- Single query path planning

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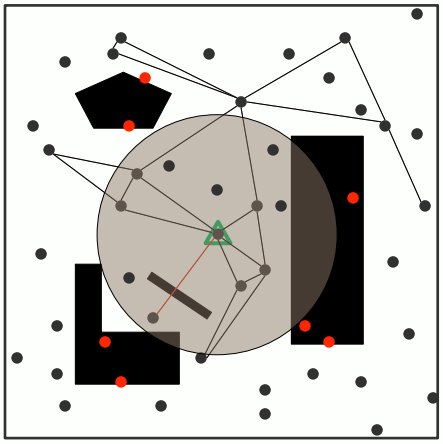
Probabilistic Roadmaps (PRMs)

- Step 1: Roadmap construction
- Step 2: Graph search
  - Pick your favourite
- PRM*
  - Connection radius as function of number of vertices

Configuration space



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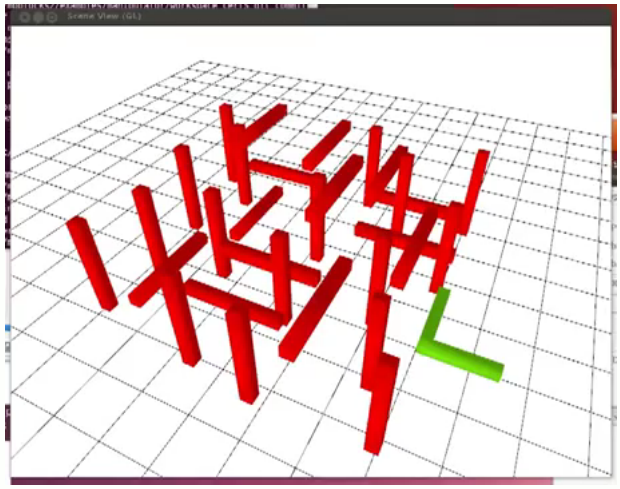
PRM* Example: Piano Mover's Problem



https://www.youtube.com/watch?v=3_S3GPxAMYA

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PRM Summary

- Paths are feasible in configuration space, therefore feasible in workspace
- PRM* is ***probabilistically complete***
  - As the number of points you sample goes to infinity, you will find a path from start to goal
- PRM* is ***asymptotically optimal***
  - As the number of points you sample goes to infinity, your path solution converges to the optimal path
- Multi-query path planning

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PLANNING IN THE REAL WORLD

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Difficulties in Planning

- Dynamic environments
- Uncertainty

When the environment changes or we don't know everything about the robot/environment, planned paths may need to be altered

Uncertainty always complicates robotics!

Let's look at these problems in detail

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Planning in Non-Static Maps

- What if the environment isn't static?
  - People (obstacles) moving around in the world
  - Doors opening and closing (paths which used to exist may no longer exist, and vice versa)
  - In disaster exploration missions, parts of buildings may collapse, closing off previously existing paths
- How do we account for environment changing after we have already found an optimal path?
- Two main ways to deal with dynamic maps:
  - Keep original plan, and deviate from plan as little as possible
  - Replan a new path based on new environmental conditions

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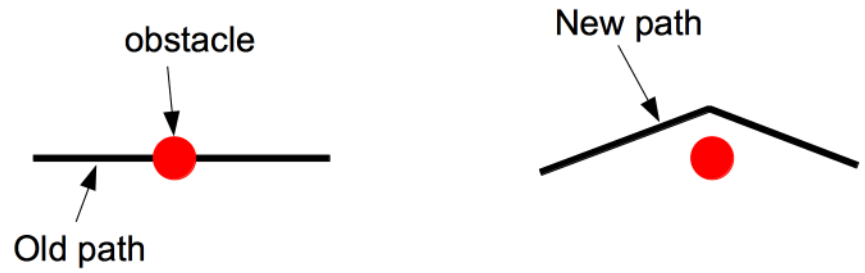
Keep Original Plan with Small Deviation



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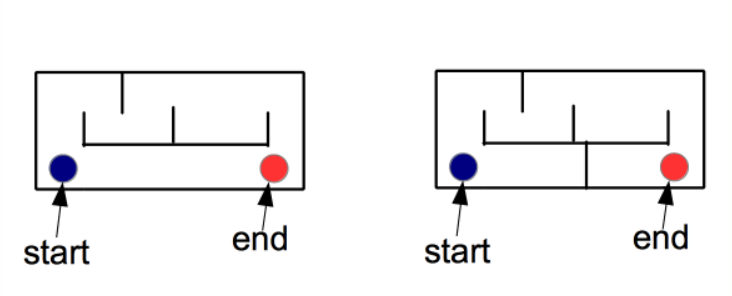
Keep Original Plan with Small Deviation

- This works fine if a person gets in the way and the robot simply needs to move around them
- What if a door is closed, or part of a hallway collapses?
  - The "small deviation" in the path is no longer well defined



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Replan a New Map

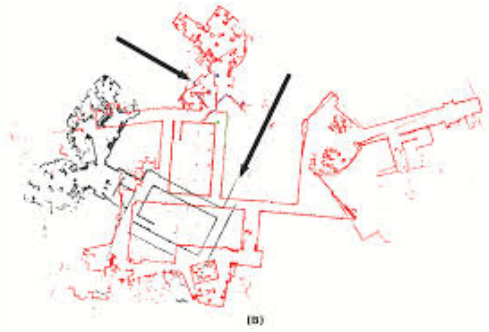
- If the environment changes, construct a new search tree and re-run the search algorithm to find the new best path based on changing environmental conditions
- What are the problems with this approach?
  - Search algorithms are slow, often difficult to compute in real-time
  - If we have to replan many times, this could slow down the mission substantially
- Path replanning algorithms exist, such as D* (dynamic A*), to deal with these types of problems

Which to Use?

- So, do we deviate slightly from paths or plan new paths?
  - There is no canonical answer to this question.
  - It depends on nature of how the environment changes
- As a general rule of thumb:
  - If dynamics are minor and we can get away with small deviation, do it
  - If major change in environment occurs and it isn't clear what a "small deviation" to path is, then replan
- Dynamic environments are very difficult to deal with
  - If environment is static, then we can easily find and execute a solution
  - If environment is dynamic, then we often have to adjust solution during execution to deal with new environmental conditions

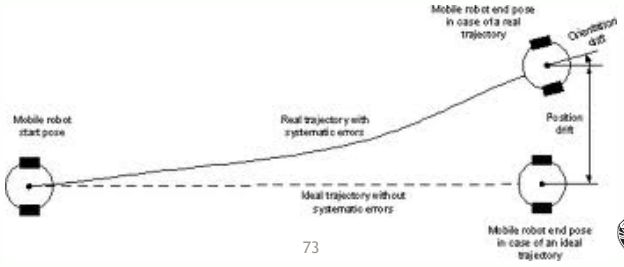
Planning with Uncertainty

- There are many types of uncertainty in robotics
  - Motion uncertainty
  - Missing information about environment
- How do these different types of uncertainty affect planning?



Motion Uncertainty

- A path planning algorithm gives a series of commands for a robot to move from an initial to ending point
- What if a command calls for a forward motion of 10 cm, but results in a forward motion of 9.7 cm?
- What if a command calls for a turn of 1 radian, but results in a turn of 1.2 radians?
- These errors will propagate over time, resulting in a large deviation from the planned course



Motion Uncertainty

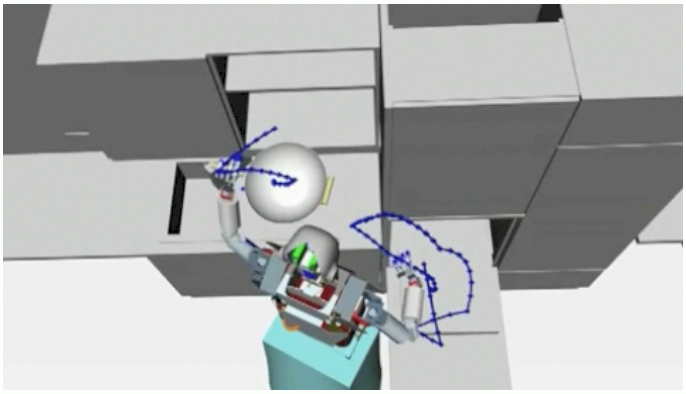
- We need to perform localization during execution of the planned path
- When the robot deviates from planned path, we need to adjust commands sent to wheels to get back to desired path
- Key idea: the planned path works perfectly in simulation, but in hardware, we need to ensure motion uncertainty doesn't cause us to deviate from this path
  - Real robots never work exactly like their simulated counterparts!

Missing Information About Environment

- Very commonly, we do not have all the information about an environment
  - Consider the case where we send a robot into a building after an earthquake to assess damage, damage may cause building map to be incorrect
  - What if we send a robot into a previously unknown (or only partially known) environment, such as Mars?
- We can plan paths based on the limited/incorrect knowledge we have, but the plan will probably need to be modified during execution
  - As more information about the environment is discovered, we will need to replan a path
- Missing and incorrect information has similar effects when compared to dynamic environments. An unexpected aspect of the environment is discovered, and the path is changed to account for this new information

Other Applications

- Path planning is not limited to determining how a mobile robot will move across an environment…



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PR2 Arm Planning Example

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PR2 Arm Planning Example

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A Note on Computational Complexity

- We have covered robot localization (state estimation), grid mapping, SLAM, and path planning
- All of these algorithms are computationally expensive
- Computational complexity grows with size and dimensionality of the problem

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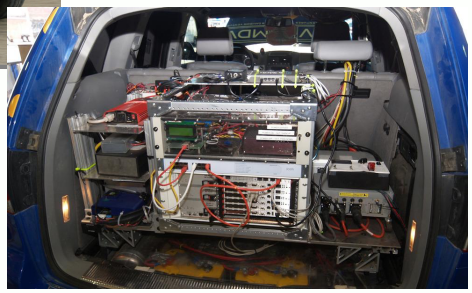
Overall Computational Complexity

- We never run one of these algorithms by itself
- If we are path planning, we also need localization to figure out where the robot is (to account for actuation noise), and possibly a mapping algorithm if the map isn't static
- If we have a camera on the robot, we need to run an image processing algorithm, which typically consider each pixel in the image as an input
- This requires a MASSIVE amount of computation

Robots in the Real World

The more capabilities…

…the more computers



A Note on Computational Complexity

- The computationally complexity of problems in robotics are not intuitive, but you need to keep it in mind when designing algorithms
- If you can get away with ignoring information, do it!
  - In mapping and localization, we generally don't consider every possible grid cell
- Use the simplest algorithm that works!
  - Roomba navigation algorithm is very low level:
    - Follow walls
    - Zig zag randomly through rooms
    - Spiral around dirty areas

Hierarchical Planning

- Basic idea: Create high level plans first, then provide detail as needed:
  - Incrementally creates a plan by refining more abstract plan steps, expanding them into more detailed subplans.
  - Exploits knowledge captured in the form of a library of subplans: rather than constructing a plan directly as a search through primitive executable actions, retrieve and combine subplans that have been prebuilt to achieve typical (sub)goals.
  - The planning process is complete when the plan is refined down to the level of executable actions.

Summary

- Path planning: given an environment, a starting location, and an ending location, find the optimal path for the robot to take
  - Graph search methods
  - Sampling-based methods
- Process:
  - Enumerate all possible plans in a tree
  - Search through the tree to find the best path from initial to final location
- When it isn't possible to follow previously planned path:
  - Take a slight deviation around obstacle and return to planned path, or
  - Plan a new path
- Difficulties:
  - Dynamic environments, motion uncertainty
  - Missing/incorrect information about environment
- Keep computational complexity in mind when designing algorithms

Next Time

- Final project code setup
- Reinforcement Learning