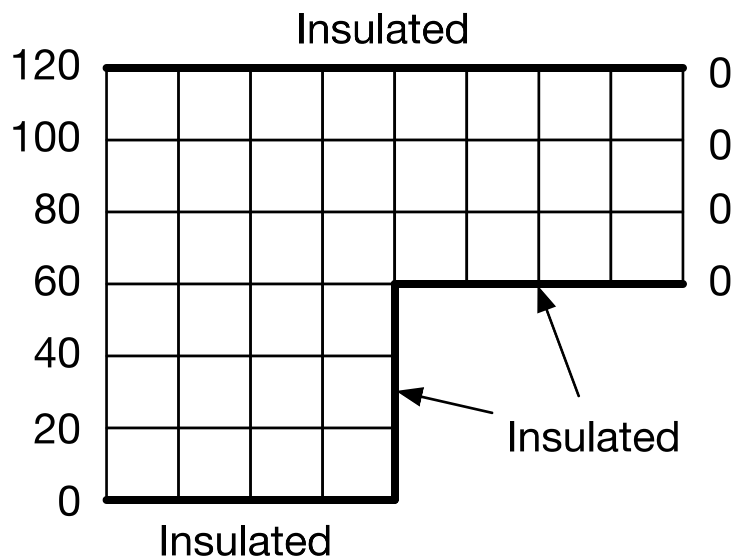
HOMEWORK 3

ME 373: Mechanical Engineering Methods, Fall 2017

Due **Friday, December 1** at **11:59 pm**

100 points total. This assignment can be submitted as a group effort with **three members or less.** Submit **all** work (analytical, plots, code) to Gradescope.

1. (50 pts) Solve the (elliptic) Laplace equation to determine the steady-state temperature distribution in the Γ-shaped plate shown. The numbers indicate the temperature at each location. Include a node map (meaning, label the figure with the indices of your nodes). Report your results both with printed numbers and a contour plot (i.e., heat plot).



2. (50 pts) A wall 0.1 m thick having a thermal diffusivity of $\alpha = 1.5\times10^{-6}$ m^2/s is initially at a uniform temperature of 85°C. At time = 0 s, the face at $x = 0$ is lowered to a temperature of 20°C, while the other face ($x = L$) is insulated.

   Using the Crank–Nicolson finite-difference numerical method,

   a) Determine the temperature distribution after 45 minutes using space and time increments of $\Delta x = 25$ mm and $\Delta t = 300$ s, respectively. Print your results in table form.
   b) Using space and time increments of $\Delta x = 10$ mm and $\Delta t = 30$ s, respectively, plot the temperature distribution as a function of distance x at 5, 30, 60, and 120 minutes on the same graph.
   c) Plot the temperature at the insulated side of the wall as a function of time in the domain $0 \le t \le 180$ min.
   d) For the Fourier number(s) considered in parts (a) and (b), discuss and compare/contrast the stability of your Crank–Nicolson method with purely explicit and implicit methods.