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A VISION-BASED METHOD FOR ESTIMATING CONTACT FORCES IN INTRACARDIAC CATHETERS

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ABSTRACT

Atrial fibrillation is a kind of cardiac arrhythmia in which the electrical signals of the heart are uncoordinated. The prevalence of this disease is increasing globally and the curative treatment for this problem is catheter ablation therapy. The adequate contact force between the tip of a catheter and cardiac tissue significantly can increase the efficiency and sustainability of the mentioned treatment. To satisfy the need of cardiologists for haptic feedback during the surgery and increase the efficacy of ablation therapy, in this paper a sensor-free method is proposed in such a way that the system is able to estimate the force directly from image data. To this end, a mechanical setup is designed and implemented to imitate the real ablation procedure. A novel vision-based feature extraction algorithm is also proposed to obtain catheter's bending variations obtained from the setup. Using the extracted feature, machine learning algorithms are responsible of estimating the forces. The results revealed $MAE < 0.0041$ and the proposed system is able to estimate the force precisely.

***Index Terms*—** Cardiac catheter, machine learning, regression, artificial neural network, force estimation, machine vision.

1. INTRODUCTION

Cardiovascular Diseases (CVDs) as one of the main reasons of global mortality are caused by disorders of the cardiovascular system [1]. According to heart disease statistics, the annual worldwide deaths associated with CVDs are over 17 million [2]. Atrial Fibrillation (AF) is a common heart arrhythmia which occurs due to erratic electrical impulse of the heart and has affected at least 3 to 6 million people in the United States [3].

Catheter ablation is a well-known minimally invasive treatment for AF to locally heat and destroy (ablate) arrhythmogenic cardiac tissue [4, 5, 6]. To perform the ablation treatment, a long flexible tube called catheter, is inserted into the vascular system to deliver some source of energy to the arrhythmia spots of the heart under X-ray fluoroscopy or MRI monitoring [7].

Adequate catheter-tissue Contact Force (CF) is known as a procedural success factor that leads to a sustainable effect of catheter therapy [8]. Accordingly, the force sensing system plays a significant role in cardiac catheterization [9]. In accordance with experimental studies, CF between $0.1N$ and $0.3N$ is a safe and effective range [10]. Besides, image-based position tracking of the catheter shaft and tip is considered as an important feature in terms of accuracy of guidance [11].

Sensor-based and sensor-free approaches are proposed methods for measuring the CF of a catheter's distal tip [12, 13]. Despite the fact that sensor-based methods are providing accurate measurement, implementation of tactile sensors in catheters has some challenges including high-end cost, physical issues ,and also difficulties in data acquisition systems in the unstructured environment [14]. Accordingly, as an alternative, sensor-free methods have caught attentions in the literature [12].

One approach of the sensor-free method is the analysis of catheter shape in which a parameter called "force index" is identified to address the force range [15]. However, based on experimental results, this method is not capable of detecting the full range of forces. Another approach is model-based techniques consisting of beam theory models, Cosserat-type rod theories ,and multi-body dynamics [16, 17, 18]. Although the mentioned model-based manners in some cases provide an accurate estimation of the force, the main effective factor for this accuracy is the optimal model parameters [19]. In another study, Runge *et al.* [20], used a finite element model to train an artificial neural network for soft robotic application. However, this method requires accurate finite element modeling in which the material parameters and manufacturing aspects of the soft robot should be considered carefully.

Overall, the accuracy of the model-based methods highly depends on the model parameters. In addition, they are computationally expensive, especially when it comes to detection of a real catheter from the operating room's monitor as these models need information from the image of the catheter.

In this study, we proposed a new vision-based solution in collaboration with machine learning methods to address the CF issue in ablation catheters. This system is functional as

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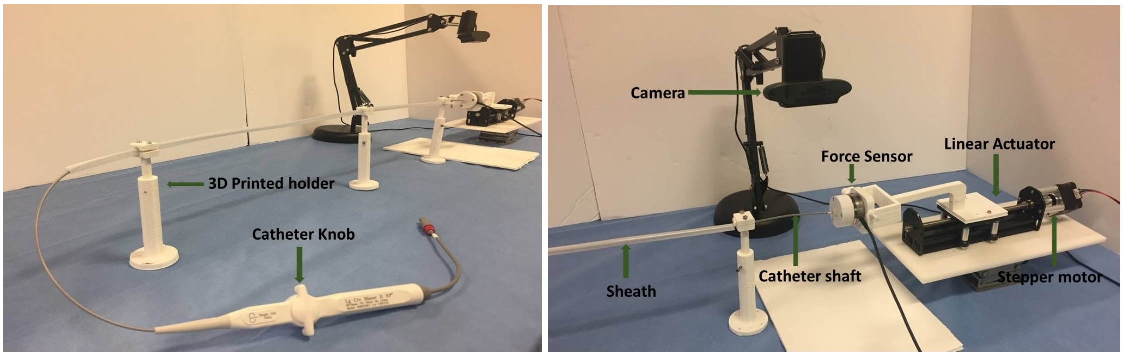


(a) Overall view of setup (b) A view of Catheter, Sensor, Actuator and Camera

Fig. 1: Experimental Setup

a sensor-free and real-time method in which the CF can be estimated directly from the image data. As the deflectable distal shaft of the catheter has unique bending under applied forces, this behaviour can be considered as a distinguishable factor of force estimation [15]. Accordingly, in this work, a mechanical setup has been designed and implemented in order to simulate the authentic Operation Room (OR) along with the available tools for performing an ablation task. Using the data captured from the aforementioned setup, a machine vision algorithm is devised as a feature extractor to find the points on the catheter's deflectable distal shaft within images. These points are deemed as the features which translate the catheter's tip into a numerical feature space. Subsequently, multiple architectures for Artificial Neural Networks (ANN) have been designed and implemented so as to model the extracted features and map every catheter's image to its corresponding contact force. In addition to ANNs, we model the data using Support Vector Regression (SVR) with the aim of making a benchmark. These models are considered as a system which maps the features to the CF in x and y direction. In the next section, the developed experimental setup and data compilation will be explained. Then, the methodology including the feature extraction algorithm as well as the modeling methods will be elaborated. The paper will be concluded in the last section.

2. EXPERIMENTAL SETUP AND PROCEDURE

The experimental setup used for data collection is shown in Fig. 1. In this setup, the camera plays the role of X-ray fluoroscopy machine in a real OR. In addition, a motorized linear actuator simulates the heart motion (one direction) in which an attached force sensor is recording the CF.

2.1. Experimental setup design

The experimental setup is designed in an attempt to collect images from the deflectable shaft of the catheter and corresponding forces. Fig. 1 presents the setup consisting of a 1-DOF (Degree Of Freedom) linear actuator equipped with a 2-phase stepper motor (17HS4401-S 40mm Nema) powered by a micro-step driver (HANPOSE TB660), a Camera (C920 for 640×480 pixels resolution), a 6-DOF Force sensor (ATI Mini40), a Bi-directional catheter (Boston Scientific Blazer II XP), 3D printed parts for holders and a plastic sheath. In this setup, the catheter is passed through a plastic sheath in a straight path in which the sheath is fixed by 3 holders and the Knob of the catheter is configured at zero degrees. The deflectable section at its base point where the body of the catheter is connected to the bending section is fixed by the 3rd holder. Hence, it cannot move inside the sheath. The force sensor attached to the 1-DOF motorized actuator is used to measure the applied forces at the tip of the catheter. In addition, the camera is implemented perpendicular to the bending section to capture a planar image for every sample.

2.2. Data Collection

Fig. 2 depicts the interaction between the software and the hardware in the experimental setup to collect a dataset comprising of 2000 sample images from deflected shaft under the applied forces by the actuator. After calibration of the sensor, data collection is done by following steps for each sample:

1. A Computer program developed in Python sends a command to an Arduino UNO to manipulate the motorized linear actuator.

2. The Arduino and stepper motor driver control the stepper motor rotation for three micro-steps equivalent to 0.6 degree (the driver is set to 1600 pulse per revolution to reach 0.2 degree per pulse).

3. Afterward, the Arduino sends an acknowledge signal to the computer.

4. The image of the deflected shaft of the catheter is captured.

5. The force data (two directions) is recorded from a 16-bit data acquisition device (USB 6210, National Instruments, Austin, TX).

This procedure is repeated 2000 times to build a dataset that contains deflected shaft images from the initial shape of the bendable shaft to fully deflected formation and their corresponding forces.

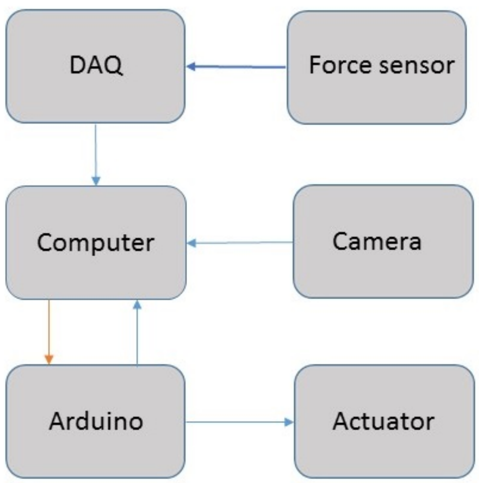


Fig. 2: Software and hardware interaction

3. METHODOLOGY

3.1. Feature extraction

Monitoring the catheter's deflection (distal shaft) during the ablation process provides information about CF [15]. In this regard, a real-time machine vision algorithm is required to extract features and translate the deflection in a numerical feature space. In this study, we have proposed an algorithm to obtain ordered points as features on the body of the catheter's deflectable section. The **Algorithm 1** shows the image processing procedure to extract ordered points from the base point of the deflectable shaft to the tip of catheter. An image captured by the camera is presented in Fig. (3a). This image is converted to the gray scale format and then a binary threshold operation is applied to segment the catheter in the image [see Fig. (3b)]. Subsequently, the algorithm vertically searches through the matrix of the 2D binary image until it finds the body of the catheter. At this point, the Cartesian coordinates (x and y) are stored as the first feature. This procedure continues until the last possible vertical search path is met. The distance between each vertical search path is a hyper-parameter that defined as the skip point. This criteria denotes the number of features. For instance, every image is represented in a 106-dimension feature space, if the skip point is equal to 5. Fig. (3c) depicts the overall procedure of vertical search where blue lines indicate the search path. Fig.(3d) shows the extracted ordered points (features) on the catheter in which the tip as the last recorded point is detected.

The aforementioned algorithm is applied to 2000 images and multiple datasets have been generated with different values for the skip point. Using the compiled datasets, the machine learning models are responsible of estimating the forces.

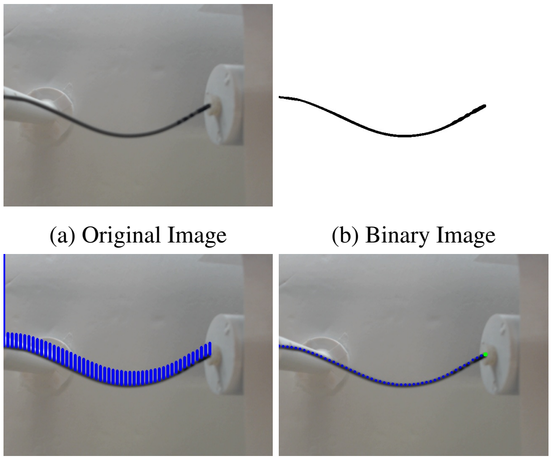
Algorithm 1: Feature Extraction Algorithm

```
Input:RGB image from the Camera
Output:Ordered points on the body of catheter
GrayscaleFunction(RGBimage)
ThresholdFunction(Grayimage)
Flag =False
j=0
for i = 0; i <= width; i = i + 5 do
    for j; j <= height; j+ = 1 do
        Flag =False
        pixel location=[i,j]
        if pixel value ==0 then
            Record i,j location of the point
            Flag=True
            j=horizontal location of the point+30
            Break the Vertical(height)search loop
        else
            Continue the Vertical(height) search
        end
    end
    if Flag==False then
        Break the Horizontal(width) step loop
    else
        Continue the Horizontal(width) search
    end
end
```

3.2. Machine Learning Models

Having the output data of the feature extraction phase, a modeling method is required in between so as to map every single record of the dataset to its corresponding force value. Since the goal is to estimate the forces directly from the images, the modeling technique deals with continuous values. In contrast to the modeling of quantitative values as a classification problem, in the current work, the system strives to create a regression over the data. With this in mind, two more popular modeling approaches have been intended with multiple configurations in order to approximate the forces in x and y direction: Artificial Neural Network (ANN) and Support Vector Regression (SVR) [21, 22].

The ANN is considered to receive a feature vector $x \in$



(a) Original Image (b) Binary Image

(c) Search Path (d) Points

Fig. 3: Features Extraction

R^m where m is the number of features depending on the the settings of the feature extraction phase. It is also expected to output the result in a two-dimensional space in order to estimate the actual forces in x and y direction. To this end, a combination of the following stacked layers is deemed so as to design the architecture of the ANN. The input feature vector x goes into a dense layer:

$$n_l = \sigma(W_l n_{l-1} + b_l) \tag{1}$$

Where W and b are the weights and biases (neurons) respectively. l denotes the layer of the network and n_l is the output of the current layer l. In addition, n_{l-1} indicates the output of the previous layer while for the first layer $n_{l-1} = x$. Also, σ is the activation function in such a way that for the intended architecture the $ReLU$ is preferred:

$$\sigma(z) = max(0, z) \tag{2}$$

Obviously, the activation function above squeezes the negative value and replace them with 0. Moreover, it is common in the Deep Learning (DL) and ANN to equip the model with the regularization term, e.g., ”L1 or L2-Regularization” with the aim of preventing the model from over-fitting. Here in this design, the Dropout is opted as the alternative to the regularization method above. In this method, a coefficient δ is multiplied by every component of the W and b matrices and calculated as follows [23]:

$$\delta = \begin{cases} w_j, b_j & \text{with P(r),} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

In fact, the component j of the parameter matrix W or b is remained with the probability $P(r)$ where $P \backsim Bernoulli(r)$ and in this architecture $r = 0.1$. It is worth noting that, Batch Normalization (BN) method is widely used in ANN in order to accelerate the optimization process [24]. In this work, the design incorporates the BN to the graph prior to applying the activation function. The last layer of the network outputs a vector with two elements corresponding to x and y forces. To optimize the model and train the neurons, Mean Absolute Error (MAE) is selected as the loss function of the model:

$$MAE(n_l, f) = \frac{\sum_{i=1}^{n} |n_{l,i} - f_i|}{n} \tag{4}$$

In the equation above, f denotes the actual forces that the model is supposed to estimate. The designed network can be optimized using Adam optimizer [25].

For the sake of benchmarking the eclectic number of modeling methods for the regression problem upon the extracted features, a linear Support Vector Regression (SVR) has been utilized in order to generate the forces out of the the given features [22]. Since the SVR is a well-known traditional Machine Learning (ML) algorithm for which it has widely contributed to a broad range of applications and also there are valuable references in the literature about it, in this work, we will not explain the method in details. Two separate SVR models are considered for this problem: the SVR for the force in x direction and the other one for y direction. It is worth to say that, the ANN and all derived configurations have been implemented on Python using Tensorflow 2.4 while the implemented API of Sklearn has been utlized for the SVR models [26, 27]. In contrast to the ANN models, no further modification or implementations have been done for the SVR models.

4. RESULTS AND DISCUSSION

In this section, the performance of the proposed system has been investigated in both the feature extraction and the modelling phase. To this end, a diverse range of configurations has been designed and the results have been compared using three main metrics: MAE, MSE and R2. The feature extraction module was fed by a dataset containing 2000 images of the catheter’s tip. The dataset obtained from the feature extraction phase was normalized and divided into three subdatasets: a training set encompassing 1280 samples, a testing set containing 400 samples and a validation including 320 samples. As reported in Table 1, 10 different configurations have been design so as to compare the performance of the proposed method accurately. For all NN-based methods every batch of the dataset includes 16 samples and the system was trained in 200 epochs while the learning rate $lr = 0.001$. The first configuration is a graph of 9 dense layers as a feedforward NN in which the layers contains the following neurons respectively: 256, 256, 128, 128, 64, 64, 32, 32, 2. This model trained on the dataset comprising 36 features acquired from the feature extraction algorithm with 15 skip points. The

Table 1: This table compares the performance of multiple modeling methods with different configurations.

No.	method	configuration	layers	feature dim	skip points	*MAE*	*MSE*	*R2*
1	DNN	dense	9 layers	36	15	0.0042	3.4868e-05	0.984
2	ANN	dense	[128, 64, 2]	36	15	0.0041	3.5274e-05	0.986
3	ANN	dense	[128, 64, 2]	106	5	0.0040	4.4928e-05	0.978
4	ANN	dense	[128, 64, 2]	22	24	0.0043	3.0972e-05	0.986
5	ANN	dense + dropout	[128, 64, 2]	36	15	0.0606	0.00515	0.218
6	ANN	dense + batch	[128, 64, 2]	36	15	0.0044	5.0931e-05	0.974
7	ANN	dense + batch + dropout	[128, 64, 2]	36	15	0.0392	0.0022	0.328
8	SVR	linear	-	22	24	0.0068	3.0705e-04	0.913
9	SVR	linear	-	36	15	0.0058	1.9133e-04	0.9449
10	SVR	linear	-	106	5	0.0046	6.2738e-05	0.982

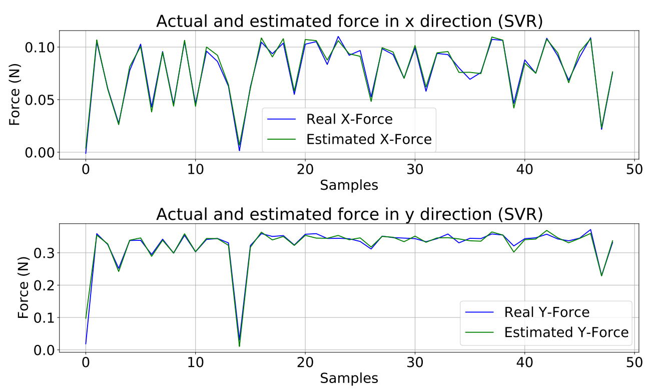


Fig. 4: The performance of the SVR for approximating the forces in both x and y direction.

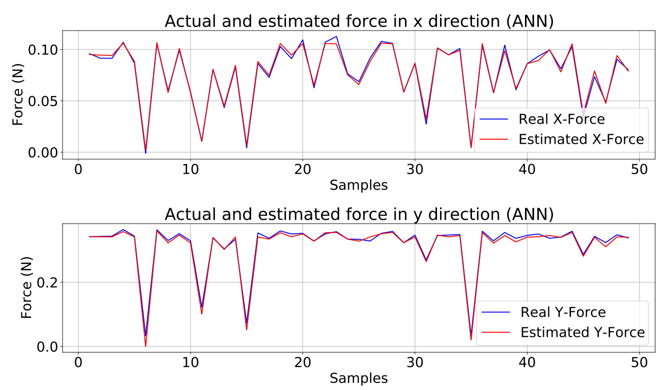


Fig. 5: This plot shows the performance of the designed ANN in estimating the forces in x and y direction.

second configuration has a shallow architecture while the layers are analogous to the previous Deep NN in terms of layer's type. This NN showed a better accuracy whereas the parameters was considerably decreased. One reason is that the complexity of the captured data was not significant so a shallow NN was capable of extracting the model properly. However, the deeper network needs more epochs to be trained completely. Configuration 2 to 4 investigated the impact of feature extraction phase and the feature dimension in the modeling. As it can be seen, given the fact that the training epochs was the same for all configurations, the representation of the catheter images in higher dimensions did not reached to an acceptable performance. However, the data with more features not only needs more parameter and epochs to be trained but also it requires a deeper NN to obtain the relationship between every dimension. Configuration 5 to 7 inspected the influence of adding batch normalization and dropout to the network's graph. The dropout layer prevent the model from reaching to the convergence point. It is worth noting that, the training loss was tracked on the validation set during the training phase and no evidence of over-fitting was caught.

The last three configurations in Table 1 reported the performance of a linear SVR model on the datasets. For every dataset, two separate SVR models were trained: one for the estimation of forces in x direction and the other one for y direction. The reported results are the average of corresponding metrics for x and y models. The tolerance for stopping the training procedure was set to 0.001. Given the tolerance value, the system keeps learning until the tolerance becomes satisfied. For this reason, the impact of representing data in different dimension size can be tangibly evaluated. The SVR showed a better regression's performance on the dataset with higher dimensions while the over performance of the ANN methods surpassed. Fig. 4 and 5 demonstrate the performance of configuration 2 and 10 respectively and show the proposed system can estimate the actual forces accurately.

5. CONCLUSIONS

In this work, a sensor-free method has been proposed for estimating the force of the catheters' tip with the aim of contributing to the catheter ablation treatment for cardiovascular diseases. The method is capable of approximating the forces directly from the images. To this end, a mechanical setup has been designed and implemented in order to imitate an authentic operation room for catheter ablation. Using the setup, the system compiled a dataset containing the images of a catheter's tip and the forces associated to every image. A novel feature extraction algorithm has been proposed to extract the variation of catheter's deflection within the images and represent them in the multi-dimensional feature space. Having the dataset of extracted features, different feed-forward neural network has been designed and implemented to make a regression over the data. Besides, Support Vector Regression as a conventional machine learning method was deployed to model the data as well. The output of the proposed feature extraction collaborating with the implemented modeling methods estimated the forces precisely. As the future work, we will extend the current system in such a way that the 3D forces can be estimated directly from the images without a feature extraction phase.

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