Response to Referee Report
JHEP_252P_0420_EDREP032460520

I would like to thank the referee for their detailed and constructive report, and I hope the changes I have outlined here have addressed their concerns. Because the report covered several topics, I will address each of these through the numbering given in the report.

1. (a) The issue of the existance of both types of modes being present at all times has been expanded upon at the end of section 2 following equation (2.19).

   (b) If I have misunderstood the referee's comment here, I apologize and would be happy to reconsider their comment. However, I believe the referee may be mistaken on this point. Let us allow for integer frequency values for the non-normalizable solutions, and examine the general solution for a massless scalar in d dimensions (as per the example put forward in the report). The spatial part of the scalar field is given by

$$E_I(x) = K_I\,(\cos x)^{\Delta^+}\,{}_2F_1\left(\frac{\Delta^+ + \omega_I}{2}, \frac{\Delta^+ - \omega_I}{2}; \frac{d}{2}; \sin^2 x\right) \tag{1}$$

   where K_I is a constant, and $\Delta^+ = d$ for a massless field. Now, we consider values of $\omega_I = 2i + d$ and examine E_I the origin:

$$E_I(0) = K_I\,{}_2F_1\left(i + d, -i; \frac{d}{2}; 0\right) \tag{2}$$

   According to the DLMF, the hypergeometric function ${}_2F_1(a, b; c; z)$ exists on the disk $|z| < 1$ provided that $c \neq 0, -1, -2, \ldots$ which the number of dimensions, d, always satisfies. Furthermore, we can write

$${}_2F_1(a, b; c; z) = \sum_{s=0}^{\infty} \frac{(a)_s (b)_s}{\Gamma(c + s) s!} z^s \tag{3}$$

   for all values of c. This clearly demonstrates that (1) does not diverge at the origin and therefore constitutes a valid field decomposition.