

Gravitational Collapse in Anti-de Sitter Space

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of Manitoba**



THE UNIVERSITY OF
WINNIPEG

Outline

- Gravitational Collapse
- Massive Scalars in AdS_5
 - Scalar Field Collapse in AdS
 - Classifying Phases
 - Phase Diagram
- High-Temperature QP Solutions in AdS_4
 - The Two-Time Formalism (TTF)
 - Quasi-Periodic Solutions
 - High-Temperature Families
- Driven Scalars in AdS
 - Extending TTF to Driven Scalars
 - Resonant Contributions
 - Special Values of Non-normalizable Frequencies
- Conclusions

Gravitational Collapse

- ▶ AdS/CFT¹ \rightarrow thermal quench in gauge theory \Leftrightarrow formation of black hole in gravitational theory
- ▶ Massless scalar fields in AdS: unstable against generic initial data, no minimum amplitude² \rightarrow c.f. Minkowski³
- ▶ Stability for specific initial data below critical amplitude
- ▶ **Nonlinear theory:** continue⁴ with exploration of phase space
- ▶ **Perturbative theory:** effects of truncation, space of solutions, time-dependent boundary conditions

¹Maldacena [hep-th/9711200]

²Bizoń & Rostworowski [1104.3702]

³Choptuik PRL70 9 (1993)

⁴Deppe & Frey [1508.02709]

B Cownden, N Deppe, and AR Frey, *Phase Diagram of Stability for Massive Scalars in Anti-de Sitter Spacetime*, Phys.Rev.D 102 (2020) 026015, [1711.00454].

Scalar Field Collapse in AdS

- ▶ Minimally-coupled scalar field in AdS₅ (dual to 4D CFT)
- ▶ Spherical symmetry, Schwarzschild-like coordinates $\rightarrow A(t, x), \delta(t, x)$
- ▶ Horizon formation when $A(t_H, x_H) \ll 1$

$$ds^2 = \frac{\ell^2}{\cos^2(x/\ell)} \left(-Ae^{-2\delta} dt^2 + A^{-1} dx^2 + \sin^2(x/\ell) d\Omega^{d-1} \right)$$

Scalar Field Collapse in AdS

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- ▶ Spherical symmetry, Schwarzschild-like coordinates $\rightarrow A(t, x), \delta(t, x)$
- ▶ Horizon formation when $A(t_H, x_H) \ll 1$
- ▶ Einstein equations \Rightarrow constraints
- ▶ Klein-Gordon equations \Rightarrow dynamics
- ▶ Examine behaviour near critical amplitude for different **masses**, **widths**

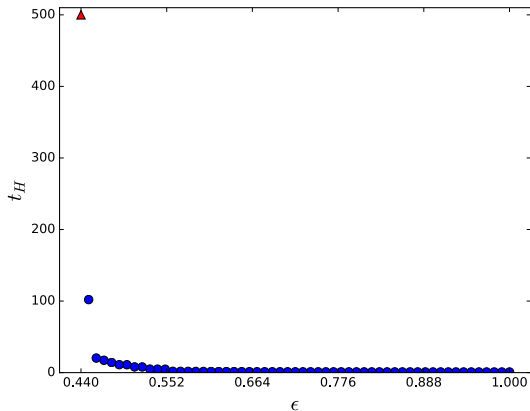
$$\partial_x M_{\text{ADM}} = \frac{\tan^{d-1}(x)}{2} \left(A(\Pi^2 + \Phi^2) + \frac{\mu^2 \phi^2}{\cos^2(x)} \right)$$

$$\Pi(t=0, x) = \epsilon \exp \left(-\frac{\tan^2(x)}{\sigma^2} \right) \quad \text{Phase space: } (\mu, \sigma)$$

Stable vs Unstable Profiles

Blue dot = collapse detected, red triangle = no collapse detected for $t \leq t_{max}$

- Stable: abrupt jump in t_H when $\epsilon < \epsilon_{crit}$

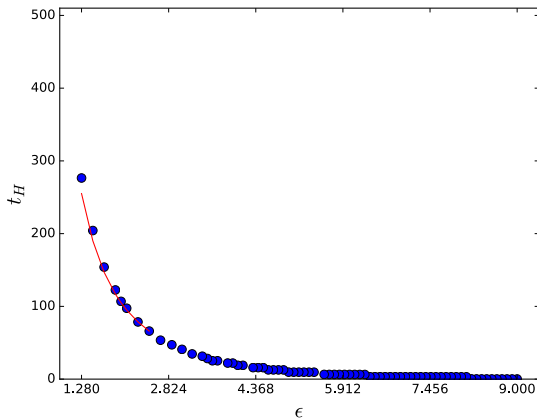


$$\mu = 0, \sigma = 1.5$$

Stable vs Unstable Profiles

Blue dot = collapse detected, red triangle = no collapse detected for $t \leq t_{max}$

- ▶ Stable: abrupt jump in t_H when $\epsilon < \epsilon_{crit}$
- ▶ Unstable: fit $t_H \approx a\epsilon^{-p} + b$ for $t_H \geq 60 \rightarrow$ perturbatively unstable when $p \approx 2$ (TTF)

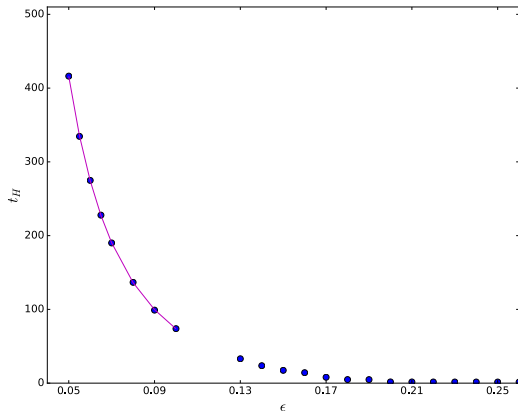


$$\mu = 5, \sigma = 0.25$$

Metastable & Irregular Profiles

Blue dot = collapse detected, red triangle = no collapse detected for $t \leq t_{max}$

- Metastable: fit
 $t_H \approx a\epsilon^{-p} + b$ for $t_H \geq 60$
 $\rightarrow p > 2$

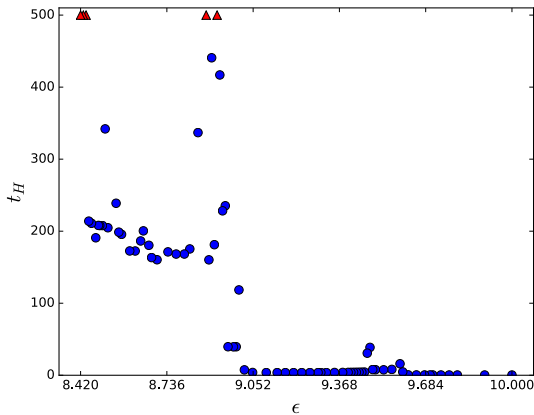


$$\mu = 5, \sigma = 1.7$$

Metastable & Irregular Profiles

Blue dot = collapse detected, red triangle = no collapse detected for $t \leq t_{max}$

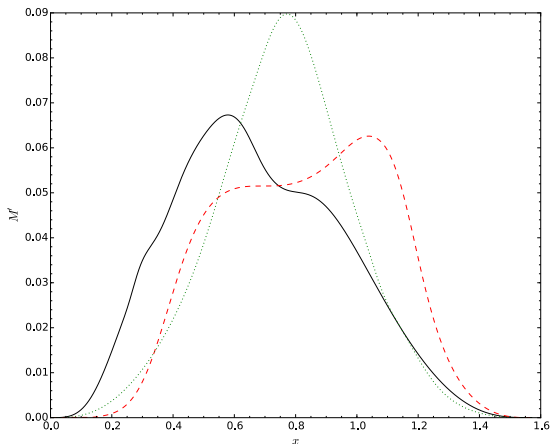
- ▶ Metastable: fit
 $t_H \approx a\epsilon^{-p} + b$ for $t_H \geq 60$
 $\rightarrow p > 2$
- ▶ Irregular: no scaling



$$\mu = 20, \sigma = 0.16$$

Observations of Chaotic Behaviour

- ▶ Possible chaotic evolution
→ scalar self-interaction
- ▶ Previous chaotic evolution
only seen in thin-shell
interactions⁵ in AdS, scalar
collapse in Gauss-Bonnet
gravity⁶



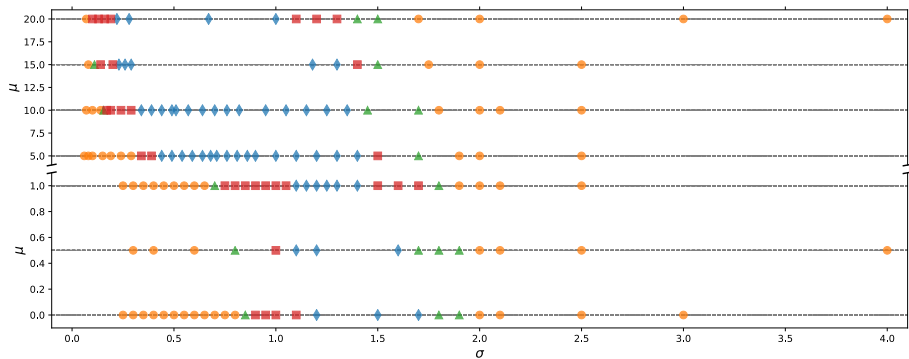
$$\mu = 0, \sigma = 1.1, \epsilon = 1.01$$

$$t = 60, 62, 64$$

⁵Brito *et al.* [1602.03535]

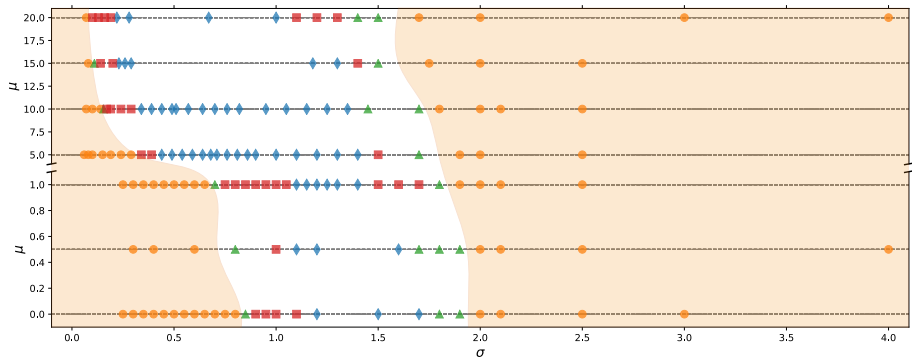
⁶Deppe, Kolly, *et al.* [1608.05402]

Phase Diagram



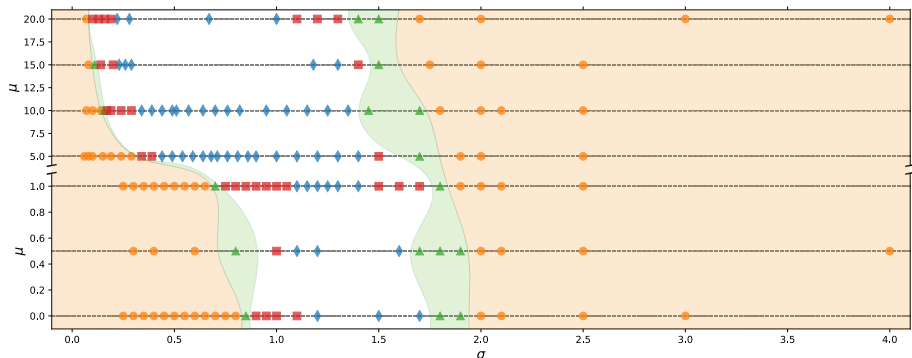
Phase Diagram

► Unstable,



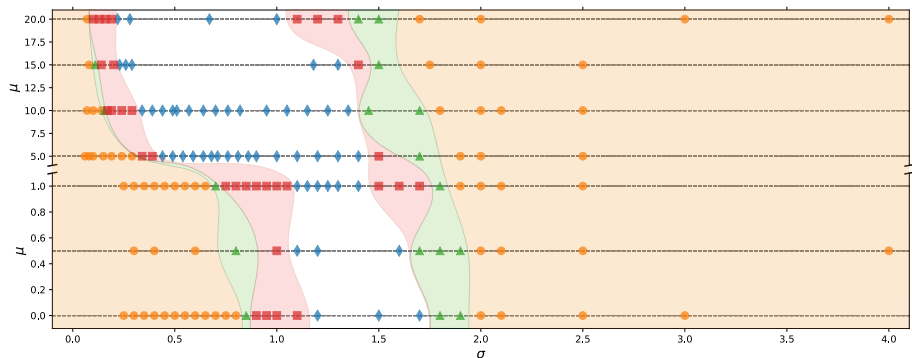
Phase Diagram

► Unstable, metastable,



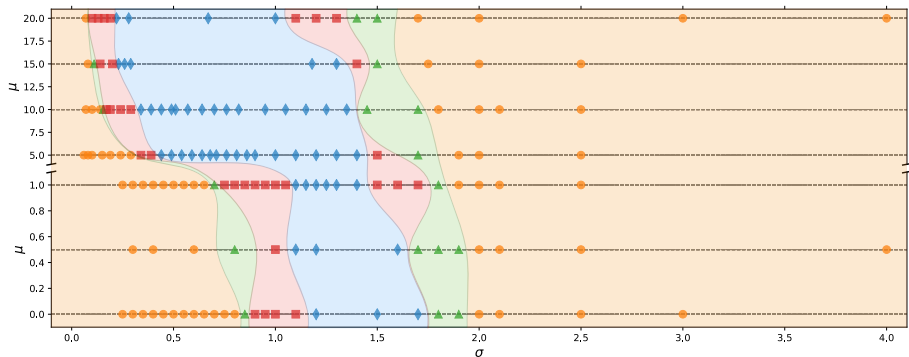
Phase Diagram

► Unstable, metastable, irregular,



Phase Diagram

- Unstable, metastable, irregular, and stable initial data



Results

- ▶ First full phase diagram of stability in $\text{AdS}_5 \rightarrow$ islands of stability and “shorelines”
- ▶ Evidence of metastable and irregular phases at finite ϵ
- ▶ Fate of metastable phase as $\epsilon \rightarrow 0$ yet to be determined
- ▶ Irregular phase contains quasi-stable initial data^{7,8} \rightarrow first evidence for weakly chaotic evolution in massless, spherically-symmetry scalars in AdS
- ▶ Metastable and irregular data to be studied in multiscale perturbation theory

⁷Deppe & Frey [1508.02709]

⁸Buchel *et al.* [1304.4166]

B Cownden, N Deppe, and AR Frey, *On the Stability of High-Temperature, Quasi-Periodic Solutions for Massless Scalars in AdS_4* , In progress.

The Two-Time Formalism (TTF)

- ▶ Small perturbations in AdS_4 : expand scalar field, metric functions in ϵ
- ▶ $\mathcal{O}(\epsilon)$: ϕ_1 in terms of eigenfunctions of AdS , $e_j(x)$
- ▶ Integer eigenvalues $\omega_j = (2j + d) \rightarrow$ fully resonant spectrum

$$\phi_1(t, x) = \sum_{j=0}^{\infty} \left(A_j(t) e^{i\omega_j t} + \bar{A}_j(t) e^{-i\omega_j t} \right) e_j(x)$$

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- ▶ $\mathcal{O}(\epsilon)$: ϕ_1 in terms of eigenfunctions of AdS, $e_j(x)$
- ▶ Integer eigenvalues $\omega_j = (2j + d) \rightarrow$ fully resonant spectrum
- ▶ Secular growth of resonant contributions \rightarrow scalar field collapse
- ▶ $\mathcal{O}(\epsilon^3)$: **source term** for resonant contributions
- ▶ Complex amplitudes vary with “slow time” $\tau \rightarrow$ flow equation to absorb resonances⁹

$$-2i\omega_\ell \frac{dA_\ell(\tau)}{d\tau} = \sum_{i,j,k} f_{ijk}^{(\ell)} \bar{A}_i A_j A_k$$

⁹Balasubramanian *et al.* [1403.6471]

Quasi-Periodic Solutions I

- ▶ Renormalization flow techniques to cancel an infinite number of resonances
→ express non-vanishing ones analytically¹⁰
- ▶ Need to truncate number of modes to find solutions: $j_{max} < \infty$ (must be robust as $j_{max} \rightarrow \infty$)

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- ▶ Need to truncate number of modes to find solutions: $j_{max} < \infty$ (must be robust as $j_{max} \rightarrow \infty$)
- ▶ Quasi-periodic¹¹ solutions $A_j = \alpha_j e^{i\beta_j \tau}$ with $\alpha_j, \beta_j \in \mathbb{R} \rightarrow$ TTF equations become time-independent when $\beta_j = \beta_0 + j(\beta_1 - \beta_0)$
- ▶ TTF: conserved quantities¹² (E, N) \rightarrow classify solutions by $T \equiv E/N$
- ▶ Solve QP equation using Newton-Raphson method

$$2\omega_\ell \alpha_\ell \beta_\ell = T_\ell \alpha_\ell^3 + \sum_{i \neq \ell} R_{i\ell} \alpha_i^2 \alpha_\ell + \sum_{i \neq \ell} \sum_{j \neq \ell}^{\ell \leq i+j} S_{ij(i+j-\ell)\ell} \alpha_i \alpha_j \alpha_{i+j-\ell}$$

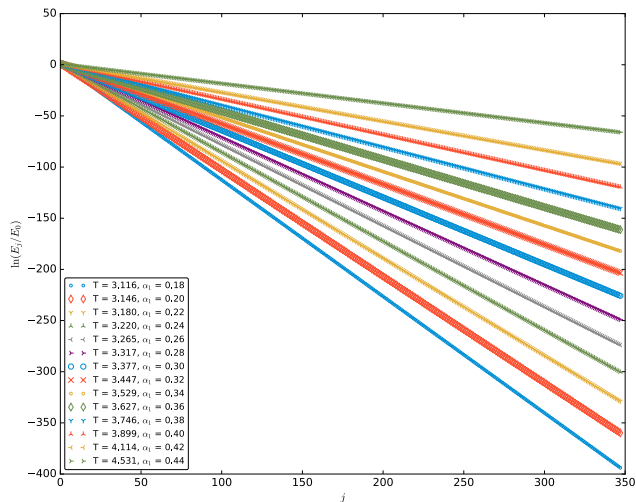
¹⁰Craps *et al.* [1407.6273]

¹¹Balasubramanian *et al.* [1403.6471]

¹²Craps *et al.* [1412.3249]

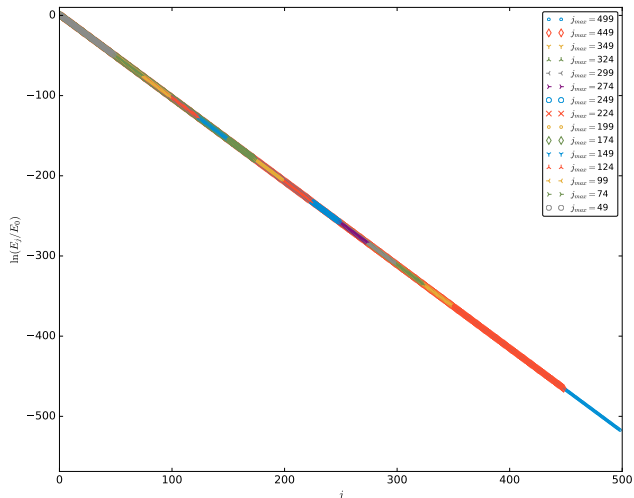
Quasi-Periodic Solutions II

- Solutions found for
 $3 \leq T \lesssim 5.5$



Quasi-Periodic Solutions II

- Solutions found for $3 \leq T \lesssim 5.5$
- Able to extend existing solutions from $j_{max} \sim 100$ to $j_{max} = 500$
- Robust in $j_{max} \rightarrow \infty$ limit



High-Temperature Families I

- ▶ Perturb by $\delta E \rightarrow$ new solutions have energy $E + \delta E$, N , and $T + \delta T$
- ▶ Solve for updated values of $\alpha_j + \delta\alpha_j, \beta_j + \delta\beta_j$
- ▶ Use updated values as seeds to resolve QP equation
- ▶ Repeat process up to T_{max} ¹³

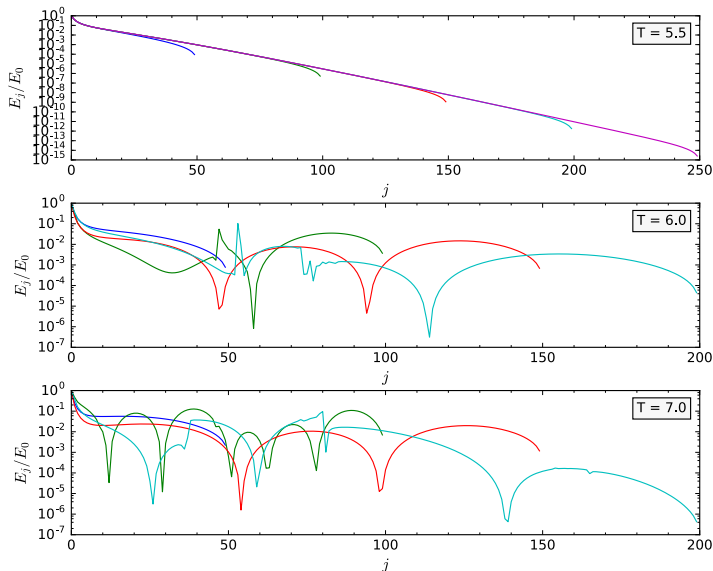
¹³Green *et al.* [1507.08261]

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- ▶ Use updated values as seeds to resolve QP equation
- ▶ Repeat process up to T_{max} ¹³
- ▶ **Issue:** $\delta\alpha_j, \delta\beta_j$ become larger than α_j, β_j at high temperatures
- ▶ No solutions that remain robust as j_{max} increases

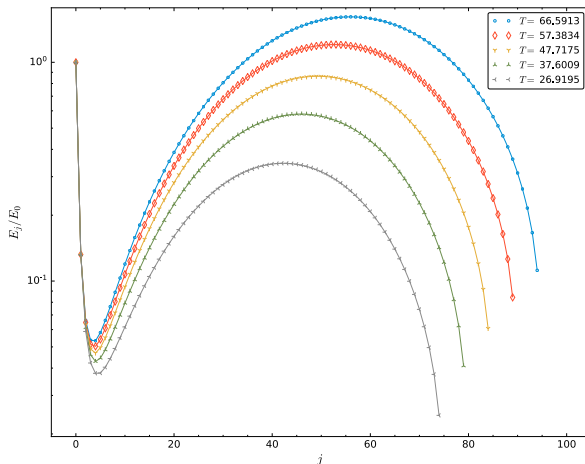
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High-Temperature Families II



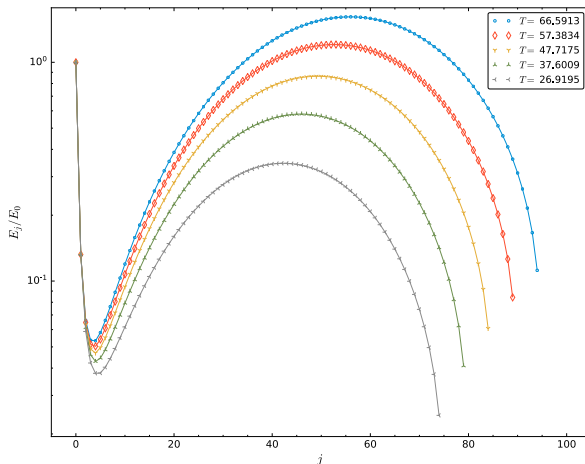
High-Temperature Families III

- ▶ Alternative methods for finding high-T solutions explored
- ▶ E.g. fit low j_{max} , high-T data to generate seeds for Newton-Raphson solver



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Results

- ▶ Low-T QP solutions robust as j_{max} increases
- ▶ Not able to find evidence that high-T solutions continued to exist at large $j_{max} \rightarrow$ possible reduction of space of QP
- ▶ **Caveat:** focused on configurations where $\alpha_0 = 1 \rightarrow$ free to set dominant energy in any $\alpha_j \rightarrow$ other configurations required for high temperatures?
- ▶ Motivation for temperature limit of $T \sim 5.5$?
- ▶ Perturbative system: massless scalar, static boundary conditions at $x = \pi/2$
- ▶ Extend to massive scalars, time-dependent boundary conditions \rightarrow activation of non-normalizable modes

B Cownden, *Examining Instabilities Due to Driven Scalars in AdS*, JHEP_252P_0420 , [1912.07143].

Extending TTF to Driven Scalars

- ▶ Driven scalars \rightarrow equation for ϕ_1 has inhomogeneous term at $x = \pi/2$

$$\partial_t^2 \phi_1 + \hat{L} \phi_1 = \delta(x - \pi/2) \mathcal{F}(t)$$

Extending TTF to Driven Scalars

- ▶ Driven scalars \rightarrow equation for ϕ_1 has inhomogeneous term at $x = \pi/2$
- ▶ Examine scaling behaviour as $x \rightarrow \pi/2$: $\Phi^+(x) \sim (\cos x)^{\Delta^+}$ and $\Phi^-(x) \sim (\cos x)^{\Delta^-}$

$$\Phi^+(x) \equiv \text{“normalizable”} \qquad \Phi^-(x) \equiv \text{“non-normalizable”}$$

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- ▶ Scalar field is linear combination of both kinds of modes
- ▶ $e_j(x)$ are same eigenfunctions of AdS & have eigenvalues $\omega_j = (2j + \Delta^+)$

$$\phi_1(t, x) = \sum_{j=0}^{\infty} a_j(t) \cos(\omega_j t + b_j(t)) e_j(x) + \sum_{\alpha=0}^{\infty} \mathcal{A}_\alpha(t) \cos(\omega_\alpha t + \mathcal{B}_\alpha) E_\alpha(x)$$

$$\Delta^\pm = \frac{d}{2} \pm \frac{1}{2} \sqrt{d^2 + 4m^2}$$

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- ▶ Scalar field is linear combination of both kinds of modes
- ▶ $e_j(x)$ are same eigenfunctions of AdS & have eigenvalues $\omega_j = (2j + \Delta^+)$
- ▶ $E_\alpha(x)$ are hypergeometric functions with frequencies ω_α from $\mathcal{F}(t)$

$$\phi_1(t, x) = \sum_{j=0}^{\infty} a_j(t) \cos(\omega_j t + b_j(t)) e_j(x) + \sum_{\alpha=0}^{\infty} \mathcal{A}_\alpha(t) \cos(\omega_\alpha t + \mathcal{B}_\alpha) E_\alpha(x)$$

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Resonant Contributions I

- ▶ $\mathcal{O}(\epsilon^3)$: source terms for resonant contributions \rightarrow examine resonance conditions

$$\omega_i + \omega_j + \omega_k = \omega_\ell$$

$$\omega_i - \omega_j - \omega_k = \omega_\ell$$

$$\omega_i + \omega_j - \omega_k = \omega_\ell$$

Resonant Contributions I

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- ▶ **Unforced**: restrictions on indices and mass value

$$\omega_i + \omega_j + \omega_k = \omega_\ell \quad \Rightarrow \quad i + j + k = \ell - \Delta^+ \in \mathbb{Z}^+$$

$$\omega_i - \omega_j - \omega_k = \omega_\ell \quad \Rightarrow \quad i - j - k = \ell + \Delta^+ \in \mathbb{Z}^+$$

$$\omega_i + \omega_j - \omega_k = \omega_\ell \quad \Rightarrow \quad i + j = k + \ell \in \mathbb{Z}^+$$

Resonant Contributions I

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Resonant Contributions I

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- ▶ **Unforced**: restrictions on indices and mass value \rightarrow two channels vanish numerically
- ▶ One **non-vanishing** channel

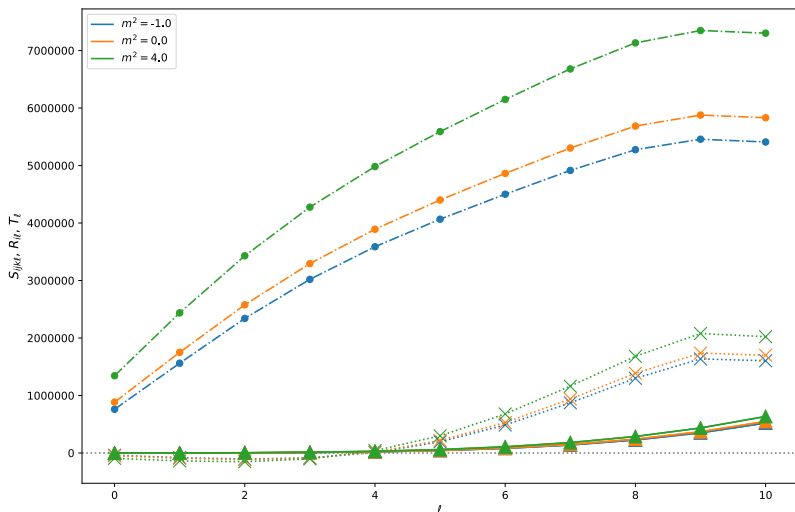
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Resonant Contributions II

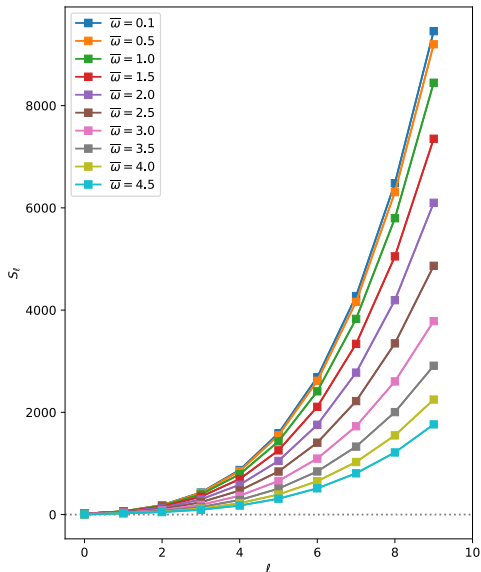
- Sum over i, j with $i + j \leq \ell$



Special Values of Non-normalizable Frequencies

- **Forced:** ω_α set by driving term
- **Single frequency:** $\omega_\alpha = \bar{\omega}$
→ one channel

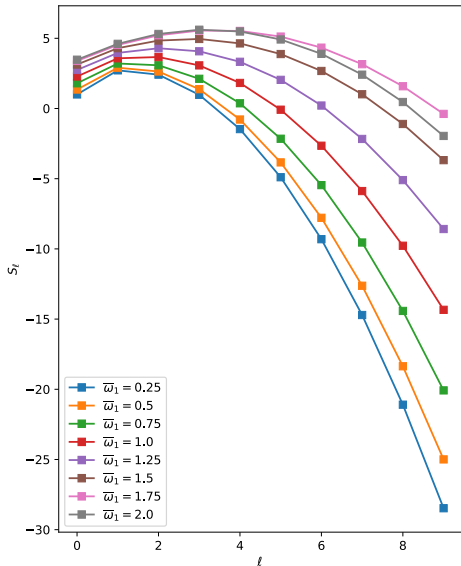
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Special Values of Non-normalizable Frequencies

- **Forced:** ω_α set by driving term
- Single frequency: $\omega_\alpha = \bar{\omega}$
→ one channel
- **Add to integer:**
 $\bar{\omega}_1 + \bar{\omega}_2 = 2n \rightarrow$ three channels

$$\begin{aligned}
 (++) : \omega_i + 2n &= \omega_\ell \quad \forall \ell \geq n \\
 (+-) : \omega_i - 2n &= \omega_\ell \quad \forall n \\
 (-+) : -\omega_i + 2n &= \omega_\ell \quad \forall n \geq \ell + d
 \end{aligned}$$



Flow Equations

- ▶ Source terms give flow equations for amplitude/phase of ϕ_1 , e.g. single frequency \rightarrow equations decouple

$$\frac{2\omega_\ell}{\epsilon^2} \frac{da_\ell}{dt} = 0$$
$$\frac{2\omega_\ell}{\epsilon^2} \frac{db_\ell}{dt} = -S_\ell \bar{A}_{\bar{\omega}}^2$$

Results

- ▶ Confirm two of three resonant channels vanish for massive scalar (all normalizable)¹⁴
- ▶ First TTF formulation with time-dependent boundary conditions
- ▶ No naturally-vanishing source terms \rightarrow sum resonant channels
- ▶ Some flow equations decouple amplitude/phase variables $a_\ell(t)$, $b_\ell(t)$
- ▶ **Note:** normalizable modes are still present \rightarrow sum all contributions
- ▶ **Further work:** quasi-periodic solutions¹⁵? Conserved quantities? Energy cascades?

¹⁴Biasi *et al.* [1810.04753]

¹⁵Carracedo *et al.* [1612.07701]

Conclusions

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- ▶ First evidence of weakly chaotic evolution of scalars in AdS_5

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- ▶ Verified low-T QP solutions are robust in the $j_{max} \rightarrow \infty$ limit
- ▶ No evidence of high-T QP solutions for $\alpha_0 = 1$ family
- ▶ Developed perturbative theory for massive scalars with time-dependent boundary conditions
- ▶ Derived flow equations for amplitude/phase variables for some choices of driving term \rightarrow evaluated source terms numerically

Thanks

- ▶ Supervisor: Andrew Frey
- ▶ PhD Committee: Derek Krepski, Gabor Kunstatter, Robert Mann, Khodr Shamseddine
- ▶ Co-authors: Nils Deppe
- ▶ University of Winnipeg and University of Manitoba
- ▶ Westgrid & Compute Canada

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