Gravitational Collapse in Anti-de Sitter Space

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PhD Thesis Defence

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University of Manitoba

- Gravitational Collapse
- Massive Scalars in AdS_5 [arXiv:1711.00454]
  - Phase Diagram & Islands of Stability
  - Classifying Phases
  - Spectra
  - Phases
- High-Temperature QP Solutions in AdS_4
  - The Two-Time Formalism (TTF)
  - Quasi-Periodic Solutions
  - Outstanding Questions
- Examining Instabilities Due to Driven Scalars in AdS [arXiv:1912.07143]
  - Time-dependent Boundary Conditions
  - Resonant Contributions
  - Special Values of Non-normalizable Frequencies
- Conclusions

Gravitational Collapse

- Numerical studies of gravitational collapse in Minkowski spacetime: horizon size = power law[1], mass gap
- AdS/CFT $\rightarrow$ thermal quench in gauge theory $\Leftrightarrow$ formation of black hole in gravitational theory
- Massless scalar fields in AdS: unstable against generic initial data, no mass gap[2] $\rightarrow$ c.f. Minkowski
- Stability for specific initial data below critical energy
- Perturbative theory for stable/nearly-stable solutions
- **Nonlinear theory:** continue with exploration of phase space[3]
- **Perturbative theory:** effects of truncation, space of solutions, evolution of nearly-stable solutions

[1] Choptuik PRL70 9 (1993)
[2] Bizoń & Rostworowski [1104.3702]
[3] Deppe & Frey [1508.02709]

Outline

- Manuscript thesis

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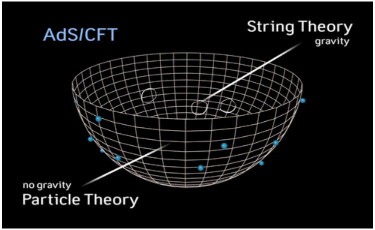
Phase Diagram & Islands of Stability

- Classical gravity in AdS$_{d+1}$ $\Leftrightarrow$ CFT in d-dimensions[4]
- Scalar field in the bulk $\Leftrightarrow$ expectation value of operator in the boundary
- Heating vacuum state
- Black hole in AdS $\Leftrightarrow$ CFT in thermal equilibrium
- Stability against collapse $\Leftrightarrow$ no thermalization



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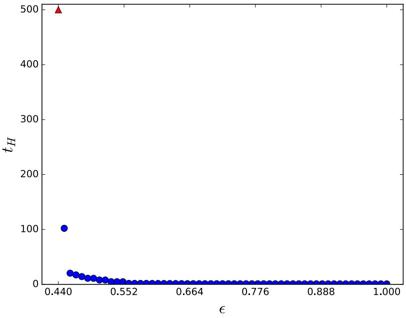
[4] Maldecena [hep-th/9711200]

Scalar Fields in Anti-de Sitter Spacetime

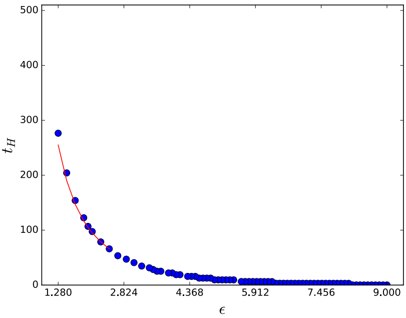
- Massless scalars: reflected off the boundary multiple times ▷, or collapse immediately ▷
- Massive scalars: can be gravitationally refocused ▷, or also collapse immediately ▷

Scalar Fields in Anti-de Sitter Spacetime

- Massless scalars: reflected off the boundary multiple times ▷, or collapse immediately ▷
- Massive scalars: can be gravitationally refocused ▷, or also collapse immediately ▷
- Classify as stable or unstable



$\mu = 0, \sigma = 1.5$



$\mu = 0, \sigma = 0.25$

Classifying Phases

- Minimally-coupled scalar field in AdS_5 (dual to 4D CFT)
- Spherical symmetry, Schwarzschild-like coordinates $\rightarrow A(t,x)$, $\delta(t,x)$

$$ds^2 = \frac{\ell^2}{\cos^2(x/\ell)}\left(-Ae^{-2\delta}dt^2 + A^{-1}dx^2 + \sin^2(x/\ell)d\Omega^{d-1}\right)$$

Classifying Phases

- Minimally-coupled scalar field in AdS$_5$ (dual to 4D CFT)
- Spherical symmetry, Schwarzschild-like coordinates $\to A(t,x)$, $\delta(t,x)$
- Einstein + Klein-Gordon $\Rightarrow$ constraint equations
- Interior gauge $\delta(t,x=0)=0$, mass conservation at $x=\pi/2$, $\phi(t=0,x)=0$
- Canonical momentum $\Pi(t,x)=e^{\delta}A^{-1}\partial_t\phi$, define $\Phi(t,x)\equiv\partial_x\phi$
- Horizon formation when $A(t_H,x_H)\ll 1$

$$\partial_x\delta = -\left(\Pi^2+\Phi^2\right)\sin(x)\cos(x))$$

$$\partial_x M = \frac{\tan^{d-1}(x)}{2}\left(A(\Pi^2+\Phi^2)+\frac{\mu^2\phi^2}{\cos^2(x)}\right)$$

$$A = 1-\frac{2M\sin^2(x)}{(d-1)\tan^d(x)}$$

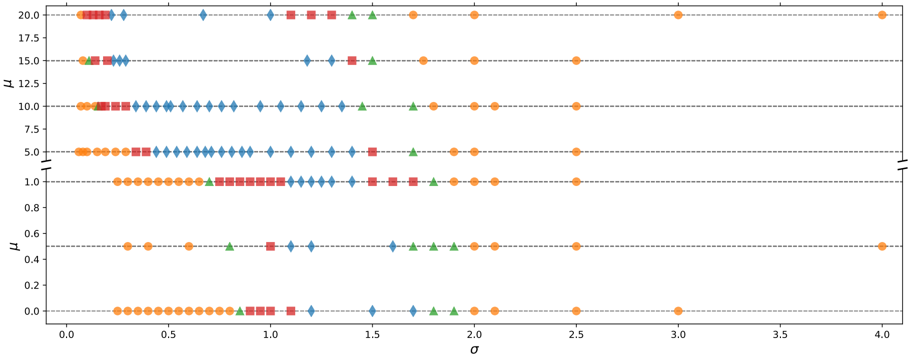
$$\Pi(t=0,x)=\epsilon\exp\left(-\frac{\tan^2(x)}{\sigma^2}\right)$$

Phase space: μ, σ

Spectra

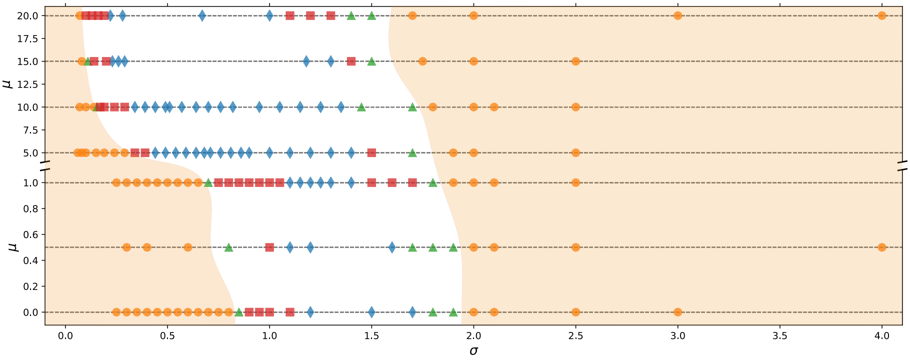
-

Phase Diagram



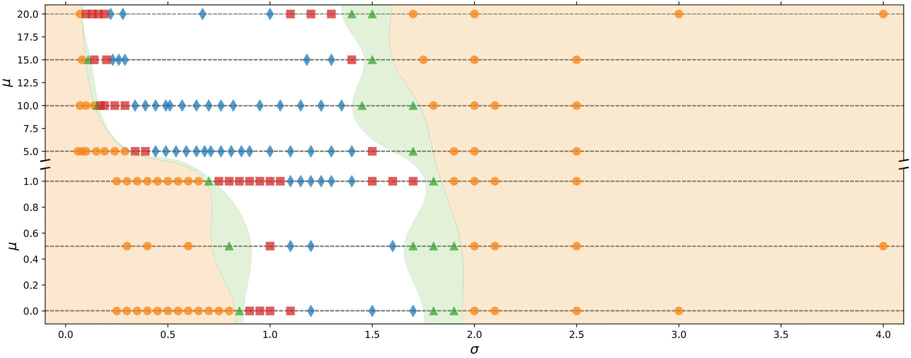
Phase Diagram

- Unstable



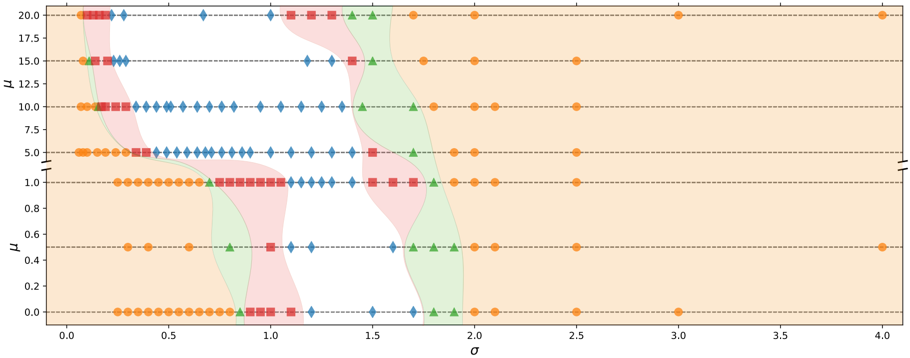
Phase Diagram

- Unstable, metastable,



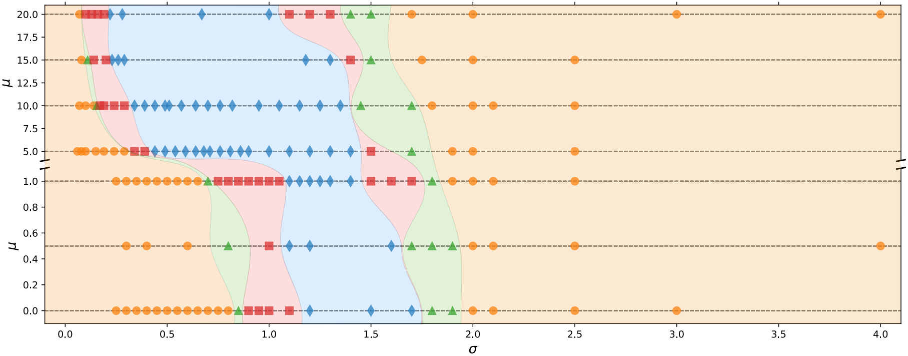
Phase Diagram

- Unstable, metastable, irregular,



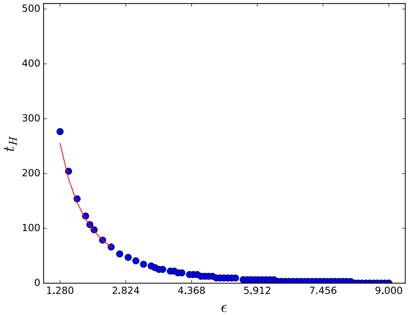
Phase Diagram

- Unstable, metastable, irregular, and stable initial data



Unstable Profiles

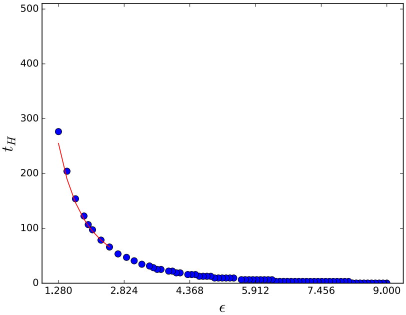
- Fit $t_H \approx a\epsilon^{-p} + b \rightarrow$ unstable when $p \approx 2$



$\mu = 5$, $\sigma = 0.25$

Unstable Profiles

- Fit $t_H \approx a\epsilon^{-p} + b \rightarrow$ unstable when $p \approx 2$
- $\sigma < 1$: $a \sim \sigma^{-2}$, mass independent



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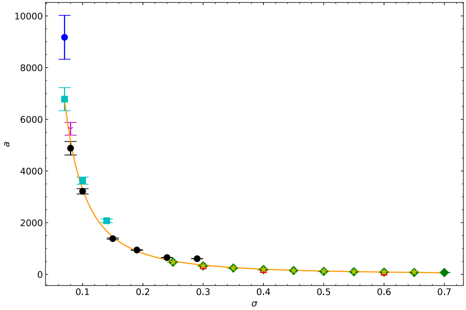
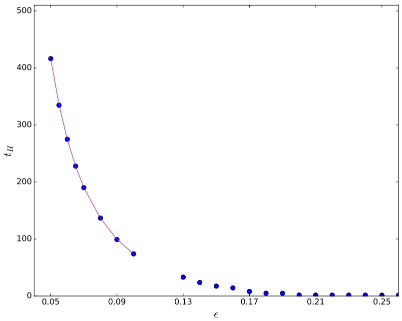
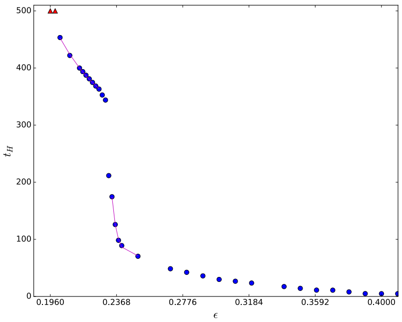


Metastable Profiles

- Scaling of $p > 2$



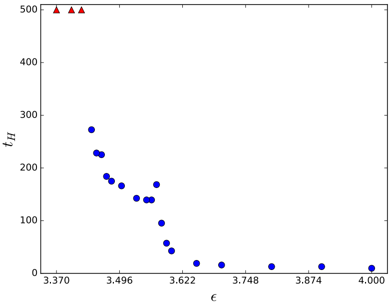
$\mu = 5,\ \sigma = 1.7$



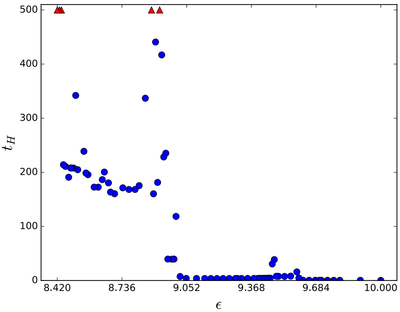
$\mu = 0.5,\ \sigma = 1.7$

Irregular Profiles

- No scaling



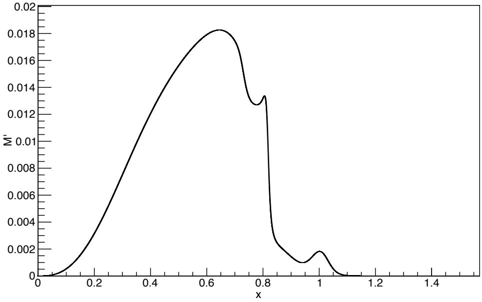
$\mu = 5,\ \sigma = 0.34$



$\mu = 20,\ \sigma = 0.16$

Irregular Profiles

- Evidence of chaotic behaviour $\rightarrow$ possible self-interaction
- Previous chaotic evolution only seen in thin-shell interactions[5] in AdS, scalar collapse in Gauss-Bonnet gravity[6]



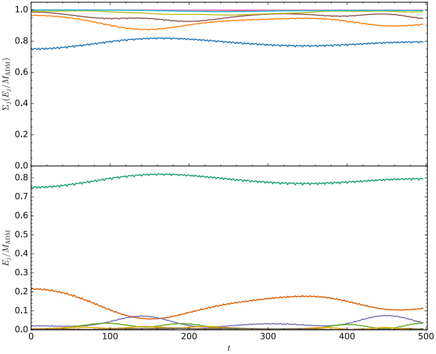
$$\mu = 5,\ \sigma = 0.34,\ \epsilon = 3.52,\ t = 137$$

[5] Brito *et al.* [1602.03535]

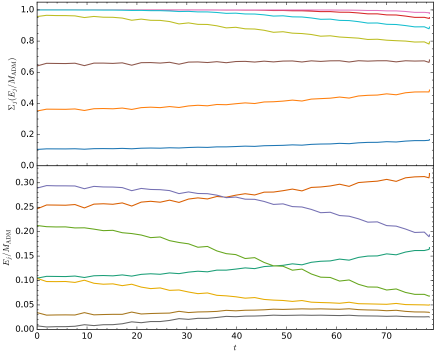
[6] Deppe, Kolly, *et al.* [1608.05402]

Energy Cascades

- Stable solutions: direct and inverse energy cascades



$\mu = 0$, $\sigma = 1.8$, $\epsilon = 0.13$
Stable



$\mu = 0$, $\sigma = 0.25$, $\epsilon = 2.28$
Unstable

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The Two-Time Formalism (TTF) I

- Perturbative description of massless scalar field collapse $\rightarrow$ expand $\phi(t,x)$, $A(t,x)$, $\delta(t,x)$ in powers of small amplitude, ϵ
- Consider TTF in AdS_4
- $\mathcal{O}(\epsilon)$: solution in terms of eigenfunctions of AdS, $e_j(x)$
- Truncate series to j_{max}
- Solutions must be robust as $j_{max} \rightarrow \infty$
- Develop numerical techniques for extending $j_{max} \gtrsim 100$ & quantify effects of truncation

$$\phi_1(t,x) = \sum_{j=0}^{\infty} A_j(t) \cos(\omega_j t + B_j(t))\, e_j(x)$$

The Two-Time Formalism (TTF) II

- Integer eigenvalues $\omega_j = (2j + d) \rightarrow$ fully resonant spectrum
- $\mathcal{O}(\epsilon^2)$: backreaction on metric
- $\mathcal{O}(\epsilon^3)$: source terms for resonant contributions $\rightarrow$ resummation techniques absorb remaining resonances into amplitude/phase[7]
- Energy exchange between modes through slowly varying amplitude $A_j(\tau)$ and phase $B_j(\tau)$ to resist collapse[8]
- Define slow time $\tau \equiv \epsilon^2 t$

$$-2\omega_l \frac{dA_l(\tau)}{d\tau} = \sum_{i\neq l}^{l\leq i+j} \sum_{j\neq l} f_1 \left(A_i, A_j, A_{i+j-l}, B_i, B_j, B_{i+j-l}\right)$$

$$-2\omega_l A_l \frac{dB_l(\tau)}{d\tau} = \sum_{i\neq l}^{l\leq i+j} \sum_{j\neq l} f_2 \left(A_i, A_j, A_{i+j-l}, B_i, B_j, B_{i+j-l}\right)$$

[7] Craps *et al.* [1407.6273]

[8] Balasubramanian *et al.* [1403.6471]

Quasi-Periodic Solutions

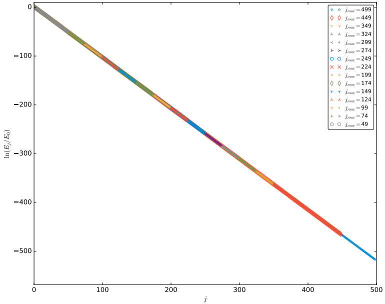
- Quasi-periodic[9] ansatz $A_j = \alpha_j e^{i\beta_j\tau}$ $\rightarrow$ TTF equations become time-independent when $\beta_j = \beta_0 + j(\beta_1 - \beta_0)$
- $\alpha_j > \alpha_{j+1} \, \forall \, j$
- Scaling symmetry (full **and** TTF theory) $A(\tau) \rightarrow \epsilon A(\tau/\epsilon^2)$ allows for $\alpha_0 = 1$ $\rightarrow$ families of solutions in α_1
- Origin of collapse time $t_H \approx a\epsilon^{-p} + b$ in nonlinear profiles
- Solve QP equation with Newton-Raphson $\rightarrow$ seed equation $\alpha_j \propto e^{-j}$ for low j_{max}
- T_i, R_{ij}, S_{ijkl} calculated numerically

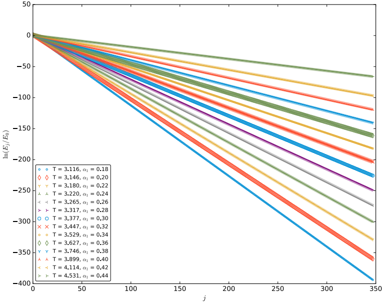
$$2\omega_l\alpha_l\beta_l = T_l\alpha_l^3 + \sum_{i\neq l} R_{il}\alpha_i^2\alpha_l + \sum_{i\neq l}^{l\leq i+j}\sum_{j\neq l} S_{ij(i+j-l)l}\alpha_i\alpha_j\alpha_{i+j-l}$$

[9] Green *et al.* [1507.08261]

Quasi-Periodic Solutions

- TTF: conserved quantities[10] E, N $\rightarrow$ classify solutions by $T \equiv E/N$
- Solutions found for $3 \geq T \gtrsim 4.6$
- Able to extend existing solutions from $j_{max} \sim 100$ to $j_{max} = 500$
- Robust in $j_{max} \rightarrow \infty$ limit



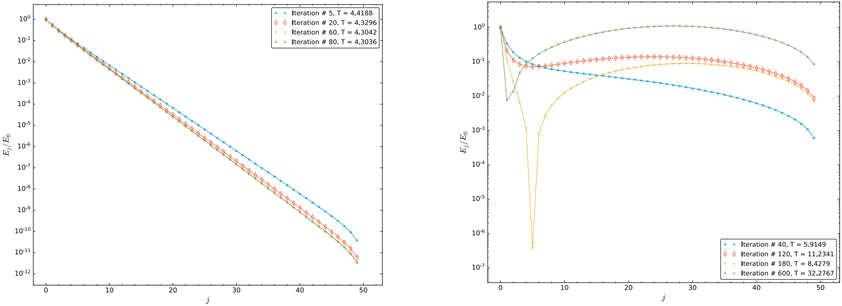


BC, Deppe, & Frey: In progress

[10] Craps *et al.* [1412.3249]

High-Temperature Solutions

- Perturb by $\delta E \to$ new solutions have energy $E + \delta E$, N, and $T + \delta T$
- Repeat process to $T_{max} = (2j_{max} + d)$
- Vary projection frequency back to solution plane $\to$ threshold temperature T_{th}

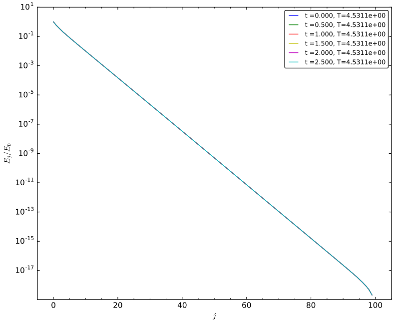


Cownden, Deppe, & Frey: In progress

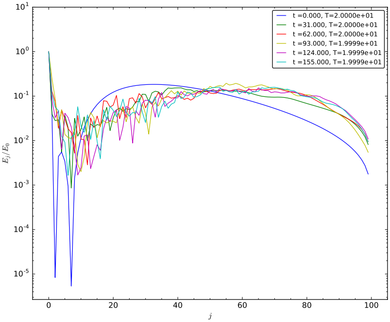
- If $T > T_{th}$, not robust in $j_{max} \to \infty$
- $T_{th} << T_{max}$

Evolving TTF Solutions

- Evolution of: low-temperature QP ▷ high-temperature QP ▷



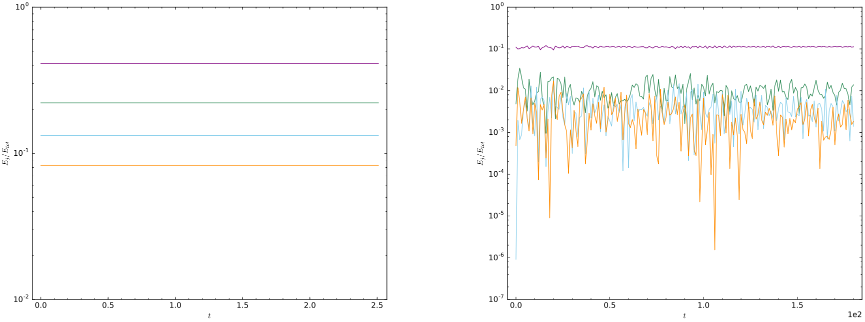
Low-temperature



High-temperature

Evolving TTF Solutions

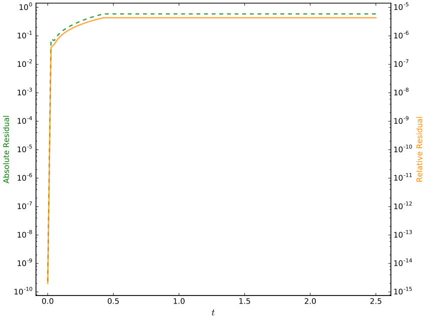
- Evolution of: low-temperature QP ▷ high-temperature QP ▷
- Energy in $j = 0, 1, 2, 3$ modes (purple, green, blue, orange)



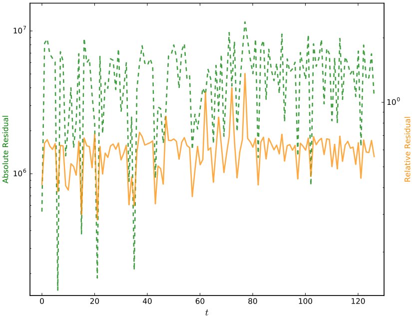
Low-temperature High-temperature

Evolving TTF Solutions

- Evolution of: low-temperature QP ▷ high-temperature QP ▷
- Energy in $j = 0, 1, 2, 3$ modes (purple, green, blue, orange)
- Residuals of QP equation



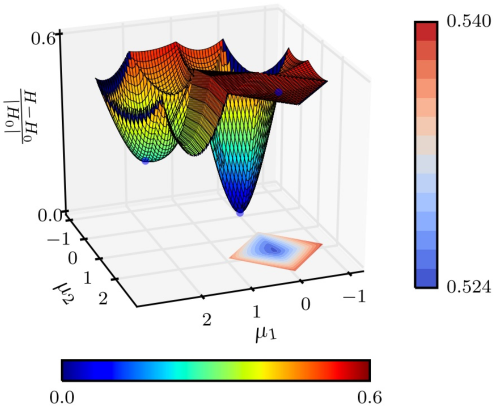
Low-temperature



High-temperature

Outstanding Questions

- Physical interpretation for T_{th}?
- Is $|T_{QP} - T_{th}|$ a function of AdS parameters? What does this mean for boundary CFT?
- Compare perturbative evolution to full numerical $\rightarrow$ establish absolute timescale for approximation
- Use high-temperature QP solutions as initial data in nonlinear evolution $\rightarrow$ metastable/irregular? What is mechanism of collapse?



Green *et al.* [1507.08261]

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Time-dependent Boundary Conditions

Resonant Contributions

Special Values of Non-normalizable Frequencies

Conclusions

- Collapse of scalar field in AdS $\Leftrightarrow$ thermalization of dual CFT
- **Nonlinear theory:** "islands of stability", metastable & irregular phases, chaotic behaviour from self-interaction
- Weakly turbulent energy cascade to short length scales $\rightarrow$ TTF for inverse cascades
- **Perturbative theory:** QP solutions robust in $j_{max} \rightarrow \infty$, high-T solutions are not; space of stable solutions is restricted by T_{th}
- **Next steps**
  - Physical interpretation of T_{th}
  - Ways to construct robust QP solutions with $T > T_{th}$
- **Future:** develop theory for *massive* TTF $\rightarrow$ less symmetry in equations $\therefore$ fewer cancelations of resonant terms; TTF in AdS_5; time-dependent boundary conditions

Thanks

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- PhD Committee: (), (), (), ()
- Co-authors: Nils Deppe (Cornell), Brayden Yarish (junior collaborator)
- University of Winnipeg and University of Manitoba

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