

Gravitational Collapse in Anti-de Sitter Space

Brad Cownden
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University of Manitoba



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WINNIPEG

- Gravitational Collapse
- Massive Scalars in AdS_5 [arXiv:1711.00454]
 - Scalar Field Collapse in AdS
 - Classifying Phases
 - Phase Diagram & Energy Cascades
- High-Temperature QP Solutions in AdS_4
 - The Two-Time Formalism (TTF)
 - Quasi-Periodic Solutions
 - High-Temperature Families
- Examining Instabilities Due to Driven Scalars in AdS [arXiv:1912.07143]
 - Extending TTF to Driven Scalars
 - Resonant Contributions
 - Special Values of Non-normalizable Frequencies
- Conclusions

Gravitational Collapse

- ▶ Numerical studies of gravitational collapse in Minkowski spacetime: horizon size = power law¹, mass gap
- ▶ AdS/CFT \rightarrow thermal quench in gauge theory \Leftrightarrow formation of black hole in gravitational theory
- ▶ Massless scalar fields in AdS: unstable against generic initial data, no mass gap² \rightarrow c.f. Minkowski
- ▶ Stability for specific initial data below critical energy
- ▶ Perturbative theory for stable/nearly-stable solutions
- ▶ **Nonlinear theory:** continue with exploration of phase space³
- ▶ **Perturbative theory:** effects of truncation, space of solutions, evolution of nearly-stable solutions

¹Choptuik PRL70 9 (1993)

²Bizoń & Rostworowski [1104.3702]

³Deppe & Frey [1508.02709]

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Scalar Field Collapse in AdS

- ▶ Minimally-coupled scalar field in AdS₅ (dual to 4D CFT)
- ▶ Spherical symmetry, Schwarzschild-like coordinates → $A(t, x)$, $\delta(t, x)$

$$ds^2 = \frac{\ell^2}{\cos^2(x/\ell)} \left(-Ae^{-2\delta} dt^2 + A^{-1} dx^2 + \sin^2(x/\ell) d\Omega^{d-1} \right)$$

Scalar Field Collapse in AdS

- ▶ Minimally-coupled scalar field in AdS₅ (dual to 4D CFT)
- ▶ Spherical symmetry, Schwarzschild-like coordinates $\rightarrow A(t, x), \delta(t, x)$
- ▶ Einstein + Klein-Gordon \Rightarrow constraint equations
- ▶ Interior gauge $\delta(t, x = 0) = 0$, spherical symmetry $\partial_x \phi(t, x = 0) = 0$
- ▶ Horizon formation when $A(t_H, x_H) \ll 1$

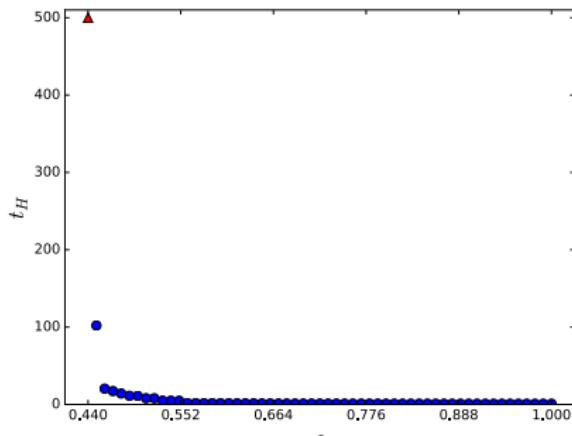
$$\partial_x \delta = -(\Pi^2 + \Phi^2) \sin(x) \cos(x)$$

$$\partial_x M = \frac{\tan^{d-1}(x)}{2} \left(A(\Pi^2 + \Phi^2) + \frac{\mu^2 \phi^2}{\cos^2(x)} \right)$$

$$\Pi(t = 0, x) = \epsilon \exp \left(-\frac{\tan^2(x)}{\sigma^2} \right) \quad \text{Phase space: } \mu, \sigma$$

Stable vs Unstable Profiles

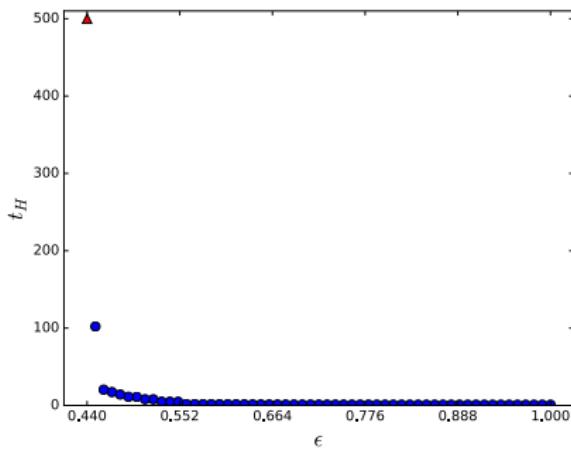
- ▶ Stable: abrupt jump in t_H when $\epsilon < \epsilon_{crit}$



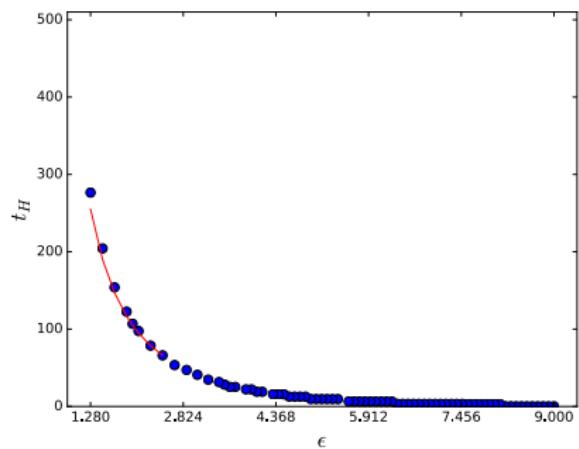
$$\mu = 0, \sigma = 1.5$$

Stable vs Unstable Profiles

- ▶ Stable: abrupt jump in t_H when $\epsilon < \epsilon_{crit}$
- ▶ Unstable: fit $t_H \approx a\epsilon^{-p} + b$ for $t_H \geq 60 \rightarrow$ unstable when $p \approx 2$



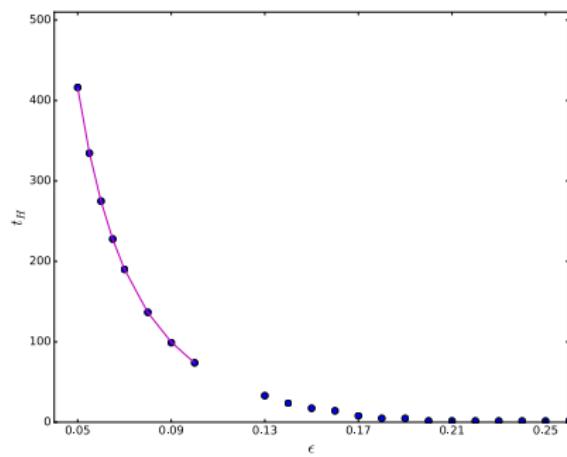
$$\mu = 0, \sigma = 1.5$$



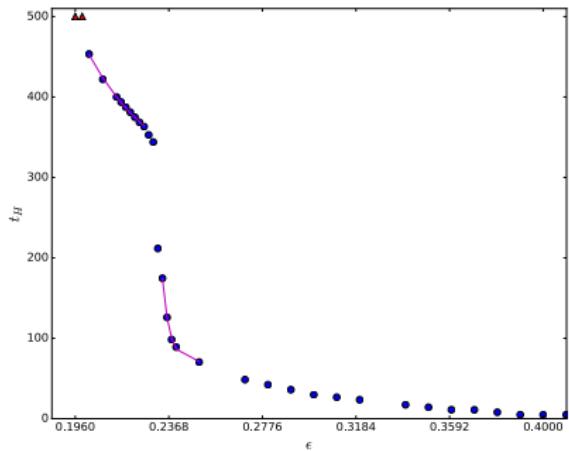
$$\mu = 5, \sigma = 0.25$$

Metastable Profiles

- ▶ Scaling of $p > 2$



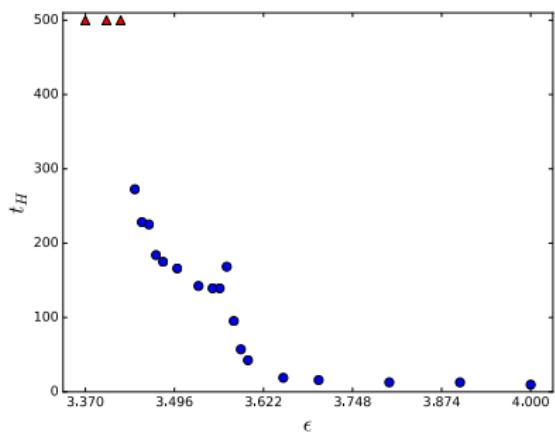
$$\mu = 5, \sigma = 1.7$$



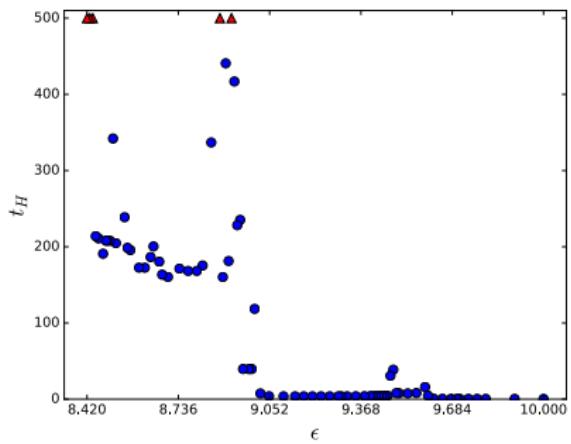
$$\mu = 0.5, \sigma = 1.7$$

Irregular Profiles I

- ▶ No scaling



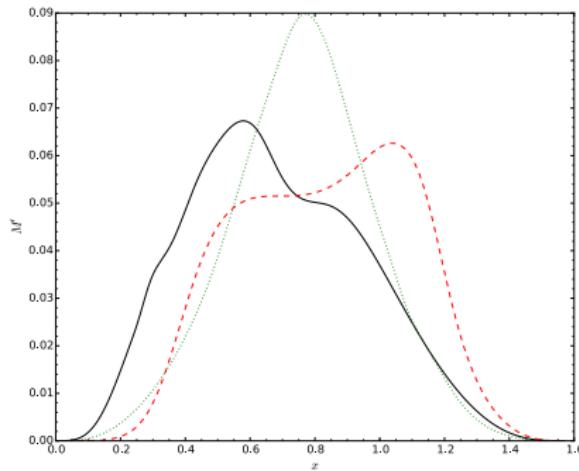
$$\mu = 5, \sigma = 0.34$$



$$\mu = 20, \sigma = 0.16$$

Irregular Profiles II

- ▶ Evidence of chaotic behaviour → possible self-interaction
- ▶ Previous chaotic evolution only seen in thin-shell interactions⁴ in AdS, scalar collapse in Gauss-Bonnet gravity⁵



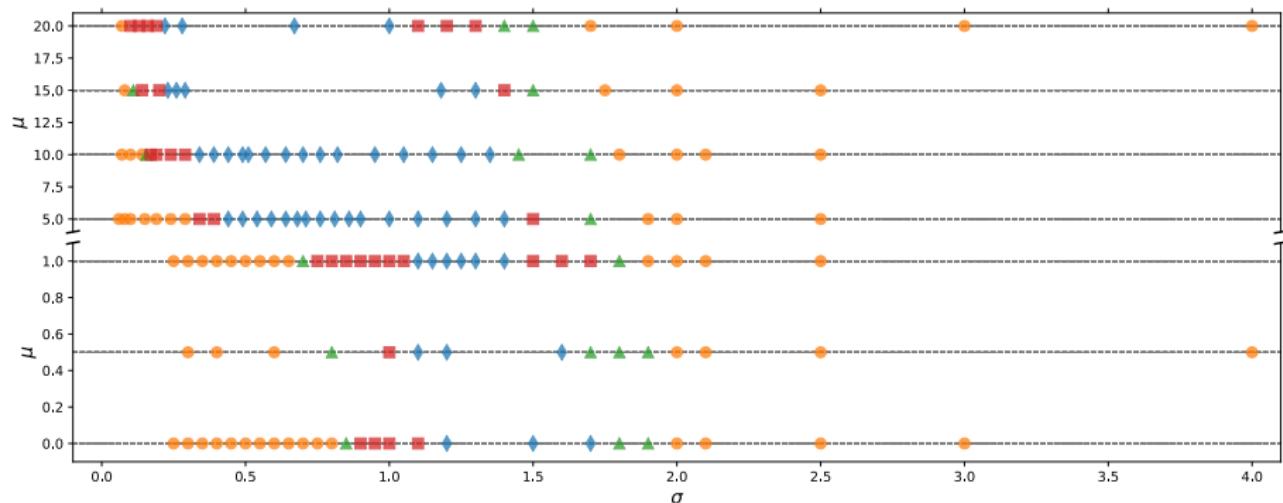
$t = 60, 62, 64$

$$\mu = 0, \sigma = 1.1, \epsilon = 1.01$$

⁴ Brito *et al.* [1602.03535]

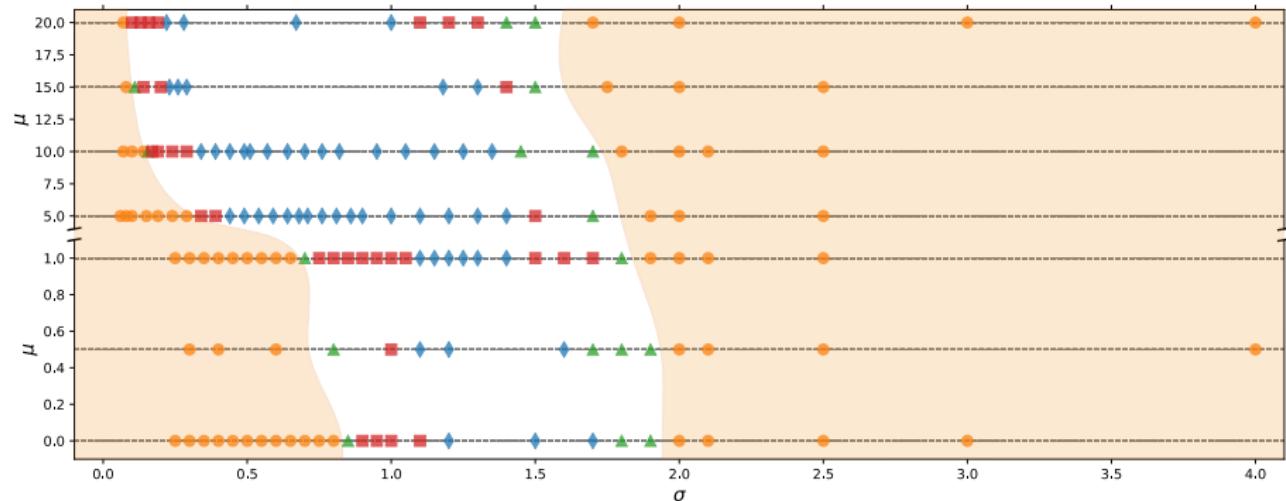
⁵ Deppe, Kolly, *et al.* [1608.05402]

Phase Diagram



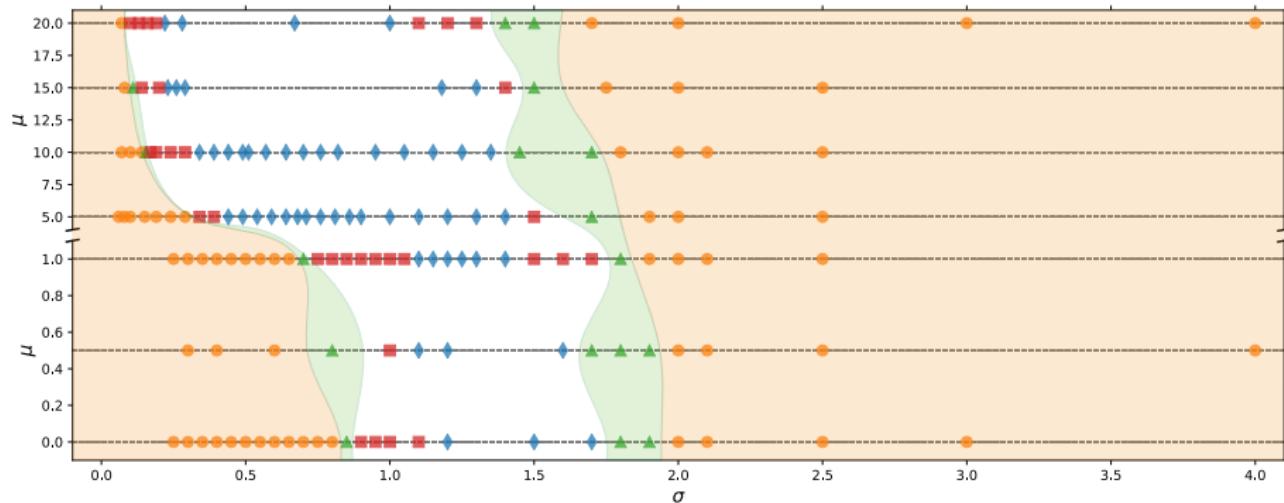
Phase Diagram

► Unstable



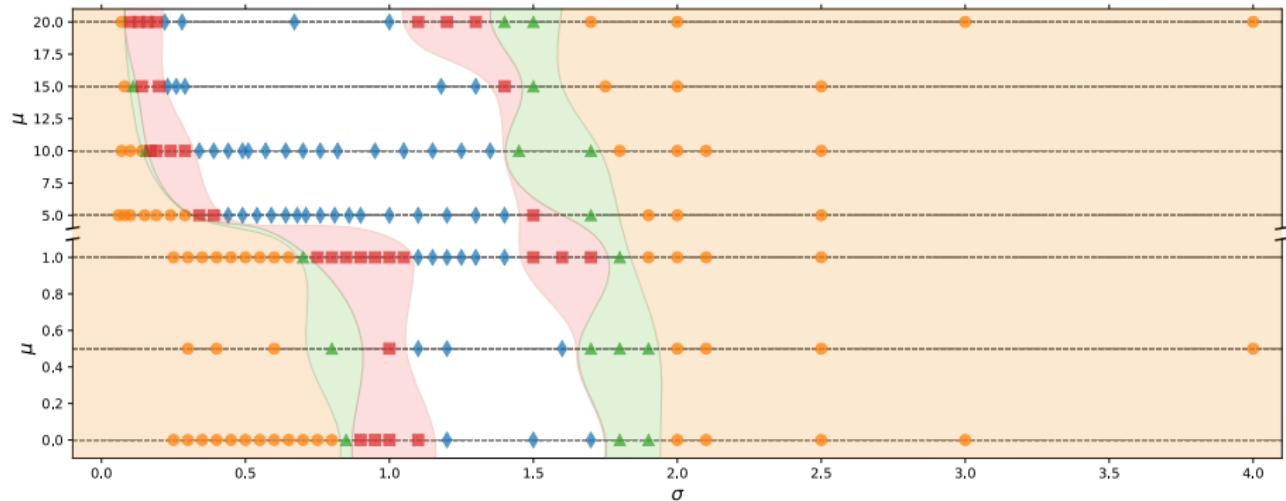
Phase Diagram

- Unstable, metastable,



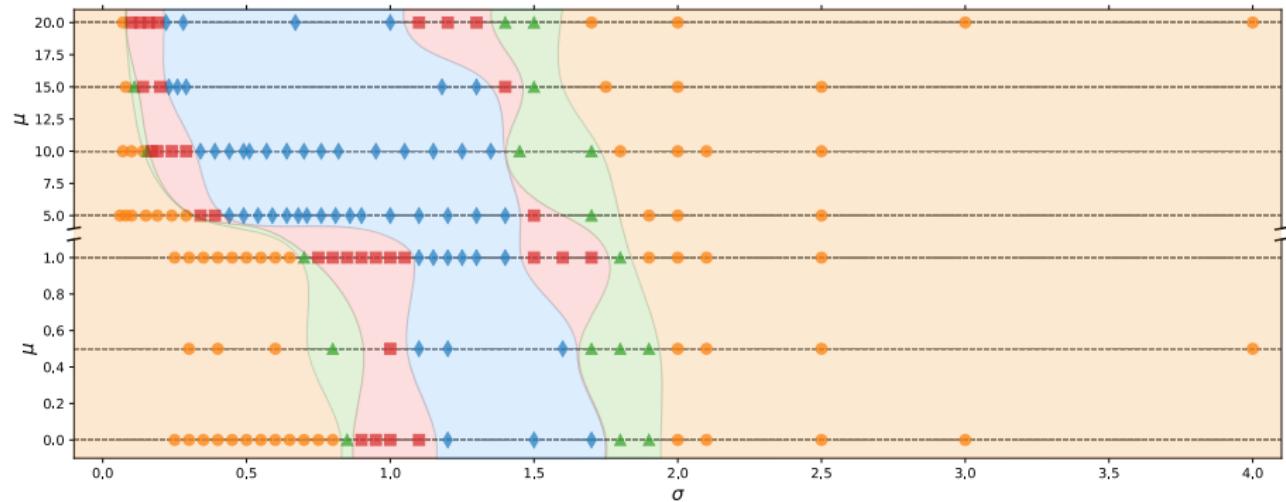
Phase Diagram

- Unstable, metastable, irregular,



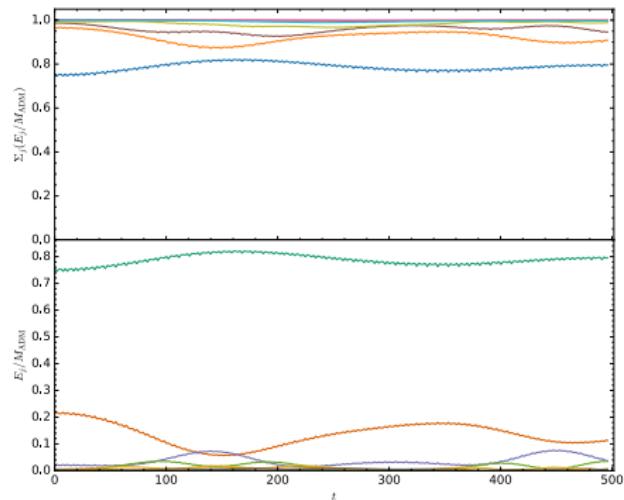
Phase Diagram

- Unstable, metastable, irregular, and stable initial data

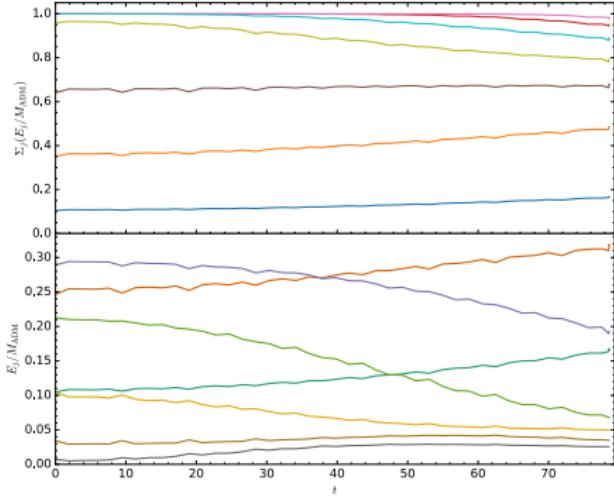


Energy Cascades

- ▶ Stable solutions: **direct** and **inverse** energy cascades



$\mu = 0, \sigma = 1.8, \epsilon = 0.13$
Stable



$\mu = 0, \sigma = 0.25, \epsilon = 2.28$
Unstable

Results

- ▶ First full phase diagram of stability in AdS₅ → islands of stability and “shorelines”
- ▶ Evidence of metastable and irregular phases at finite ϵ
- ▶ Fate of metastable phase as $\epsilon \rightarrow 0$ yet to be determined
- ▶ Irregular phase contains quasi-stable initial data^{6,7} → first evidence for weakly chaotic evolution in massless, spherically-symmetry scalars in AdS
- ▶ Metastable and irregular data to be studied in multiscale perturbation theory

⁶Deppe & Frey [1508.02709]

⁷Buchel *et al.* [1304.4166]

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The Two-Time Formalism (TTF) I

- ▶ Small perturbations in AdS₄: expand scalar field, metric functions in ϵ
- ▶ $\mathcal{O}(\epsilon)$: ϕ_1 in terms of eigenfunctions of AdS, $e_j(x)$
- ▶ Integer eigenvalues $\omega_j = (2j + d) \rightarrow$ fully resonant spectrum
- ▶ $\mathcal{O}(\epsilon^2)$: backreaction on metric in terms of ϕ_1

$$\phi_1(t, x) = \sum_{j=0}^{\infty} A_j(t) \cos(\omega_j t + B_j(t)) e_j(x)$$

The Two-Time Formalism (TTF) I

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- ▶ $\mathcal{O}(\epsilon^2)$: backreaction on metric in terms of ϕ_1
- ▶ $\mathcal{O}(\epsilon^3)$: **source terms** for resonant contributions
- ▶ Define slow time $\tau \equiv \epsilon^2 t$

$$-2\omega_\ell \frac{dA_\ell(\tau)}{d\tau} = \sum_{i \neq \ell} \sum_{j \neq \ell}^{\ell \leq i+j} f_1(A_i, A_j, A_{i+j-\ell}, B_i, B_j, B_{i+j-\ell})$$

$$-2\omega_\ell A_\ell \frac{dB_\ell(\tau)}{d\tau} = \sum_{i \neq \ell} \sum_{j \neq \ell}^{\ell \leq i+j} f_2(A_i, A_j, A_{i+j-\ell}, B_i, B_j, B_{i+j-\ell})$$

The Two-Time Formalism (TTF) II

- ▶ Energy exchange between modes through slowly varying amplitude $A_j(\tau)$ and phase $B_j(\tau)$ to resist collapse⁸
- ▶ Resummation techniques absorb resonances into amplitude/phase variables⁹
- ▶ Solve by truncating series to $j_{max} < \infty$
- ▶ Solutions must be robust as $j_{max} \rightarrow \infty$
- ▶ Examine quasi-periodic families of solutions¹⁰
- ▶ Develop numerical techniques for extending $j_{max} \gtrsim 100$
- ▶ Verify families of solutions remain valid as j_{max} increases

⁸Balasubramanian et al. [1403.6471]

⁹Craps et al. [1407.6273]

¹⁰Green et al. [1507.08261]

Quasi-Periodic Solutions I

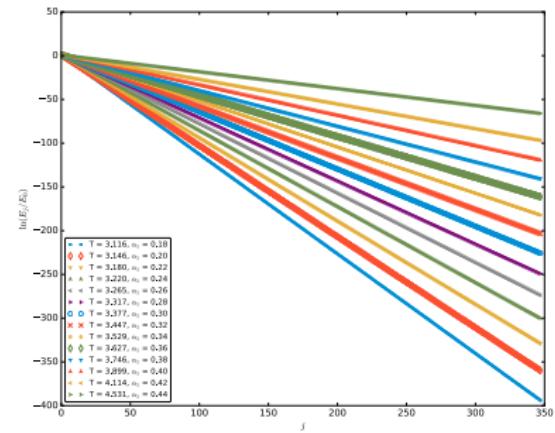
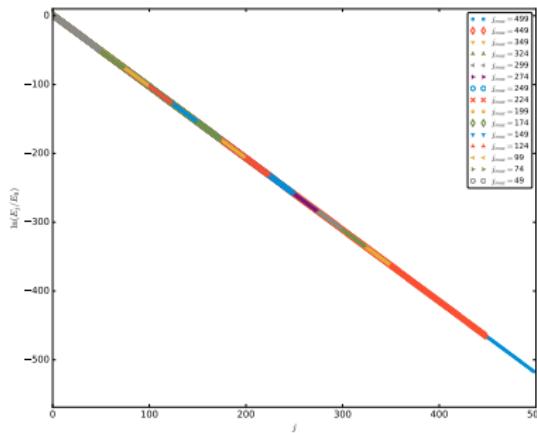
- ▶ Quasi-periodic ansatz $A_j = \alpha_j e^{i\beta_j \tau} \rightarrow$ TTF equations become time-independent when $\beta_j = \beta_0 + j(\beta_1 - \beta_0)$
- ▶ Solve QP equation with Newton-Raphson \rightarrow seed equation $\alpha_j \propto e^{-j}$, $j \neq 0$ for low j_{max}
- ▶ TTF: conserved quantities¹¹ $E, N \rightarrow$ classify solutions by $T \equiv E/N$
- ▶ T_i, R_{ij}, S_{ijkl} calculated numerically

$$2\omega_\ell \alpha_\ell \beta_\ell = T_\ell \alpha_\ell^3 + \sum_{i \neq \ell} R_{i\ell} \alpha_i^2 \alpha_\ell + \sum_{i \neq \ell} \sum_{j \neq \ell}^{\ell \leq i+j} S_{ij(i+j-\ell)\ell} \alpha_i \alpha_j \alpha_{i+j-\ell}$$

¹¹Craps et al. [1412.3249]

Quasi-Periodic Solutions II

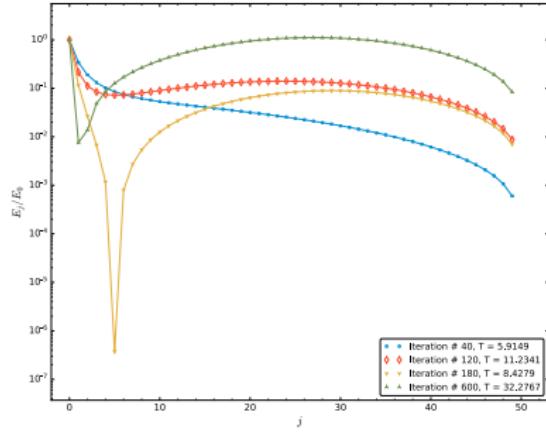
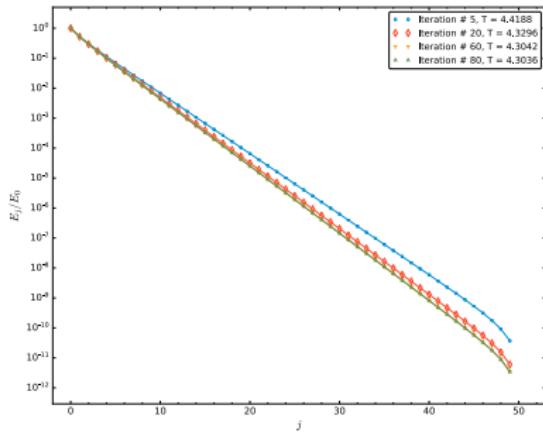
- ▶ Solutions found for $3 \geq T \gtrsim 4.6$
- ▶ Able to extend existing solutions from $j_{max} \sim 100$ to $j_{max} = 500$
- ▶ Robust in $j_{max} \rightarrow \infty$ limit



BC, Deppe, & Frey: In progress

High-Temperature Solutions I

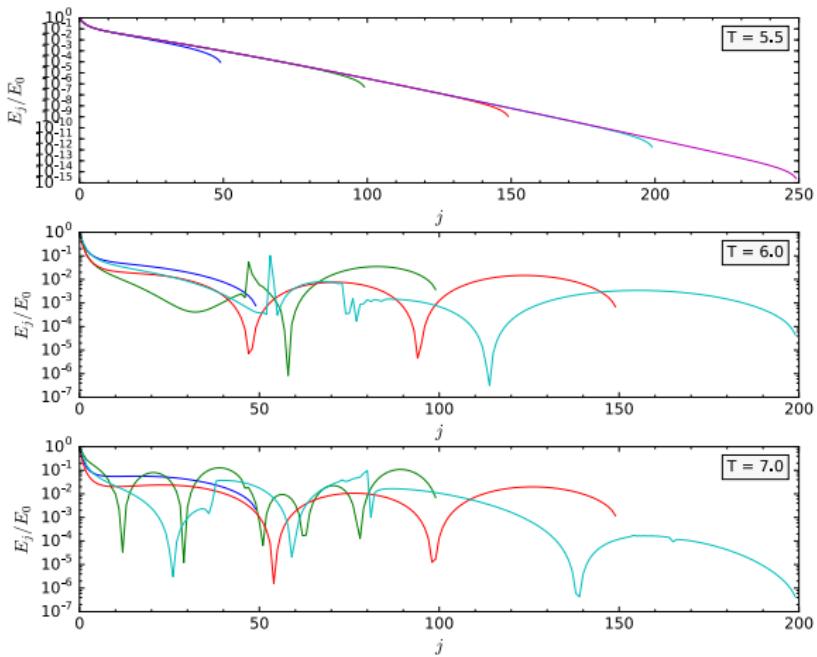
- ▶ Perturb by $\delta E \rightarrow$ new solutions have energy $E + \delta E$, N , and $T + \delta T$
- ▶ Repeat process to $T_{max} = (2j_{max} + d)$
- ▶ Project back to QP solution surface at constant α_1 or T
- ▶ Loss of smooth profile above a certain temperature at **constant α_1**



Cownden, Deppe, & Frey: In progress

High-Temperature Solutions II

- And at constant T



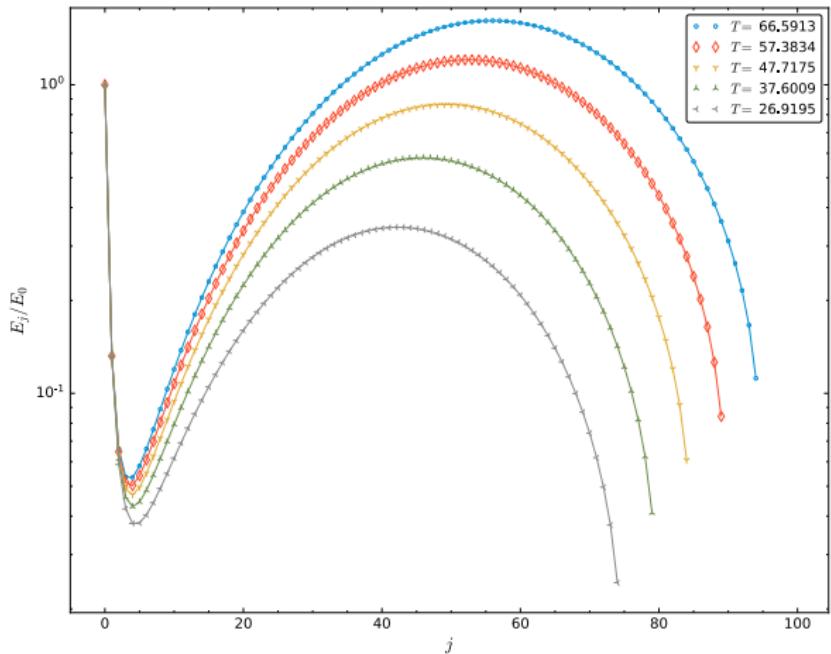
Cownden, Deppe, & Frey: In progress

High-Temperature Solutions III

- ▶ Alternative methods for finding high-T solutions

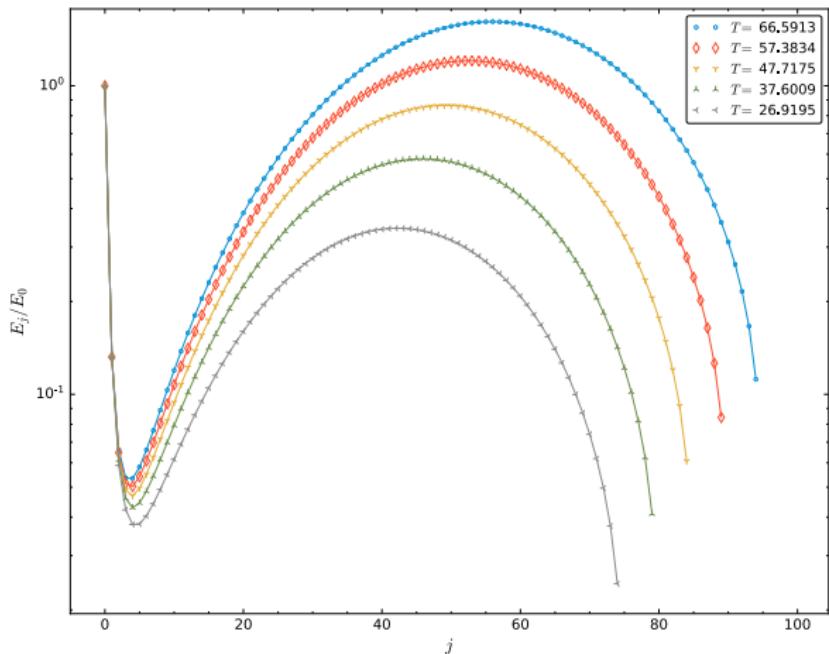
High-Temperature Solutions III

- ▶ Alternative methods for finding high-T solutions
- ▶ Fit low j_{max} , high-T data to generate seeds for Newton-Raphson solver



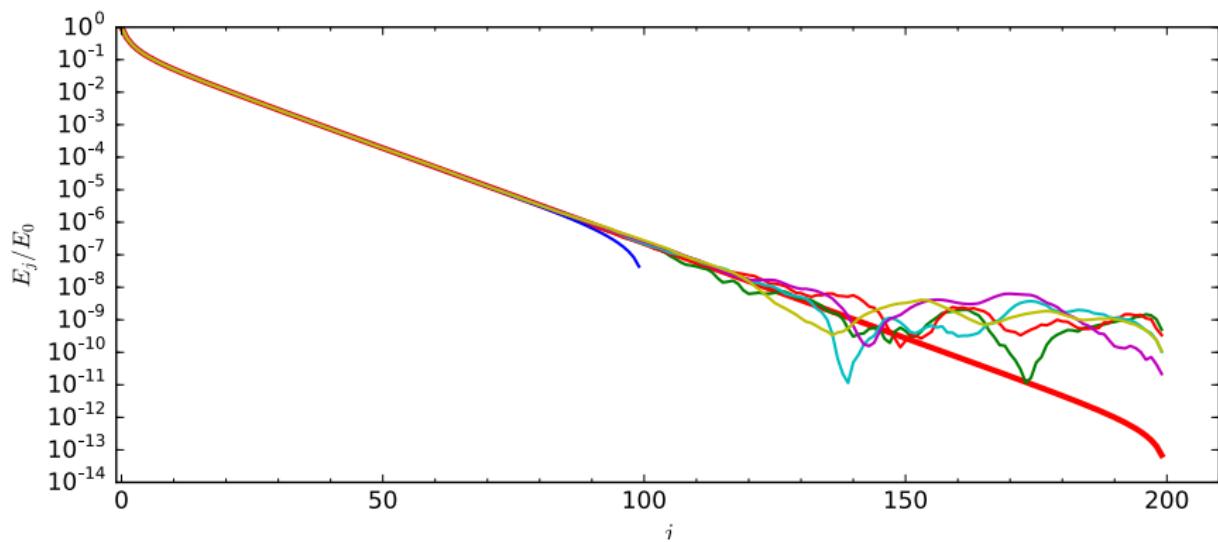
High-Temperature Solutions III

- ▶ Alternative methods for finding high-T solutions
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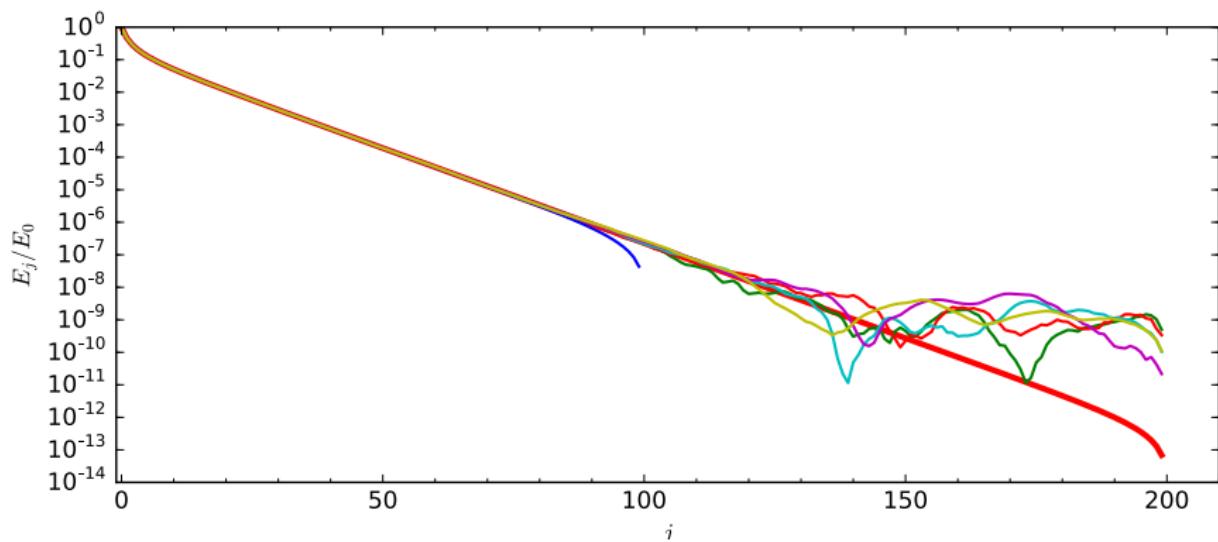
High-Temperature Solutions III

- ▶ Alternative methods for finding high-T solutions
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- ▶ Pad low j_{max} data with zeros, evolve within the TTF



High-Temperature Solutions III

- ▶ Alternative methods for finding high-T solutions
- ▶ Fit low j_{max} , high-T data to generate seeds for Newton-Raphson solver → not robust as j_{max} increases
- ▶ Pad low j_{max} data with zeros, evolve within the TTF → **isothermal evolution does not match known solution**



Results

- ▶ Low-T QP solutions robust as j_{max} increases
- ▶ Not able to find evidence that high-T solutions continued to exist at large $j_{max} \rightarrow$ possible reduction of space of QP solutions up to $T_{max} = 2j_{max} + d$
- ▶ **Caveat:** focused on configurations where $\alpha_0 = 1 \rightarrow$ free to set dominant energy in any α_j
- ▶ Motivation for temperature limit of $T \sim 5.5?$
- ▶ Perturbative system: massless scalar, static boundary conditions at $x = \pi/2$
- ▶ Extend to massive scalars, time-dependent boundary conditions \rightarrow activation of non-normalizable modes

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Extending TTF to Driven Scalars

- ▶ Driven scalars → scalars with time-dependent boundary conditions at $x = \pi/2$
- ▶ $\mathcal{O}(\epsilon)$: $\phi_1(t, x = \pi/2) = f(t)$

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- ▶ $\mathcal{O}(\epsilon)$: $\phi_1(t, x = \pi/2) = f(t)$
- ▶ Examine scaling behaviour as $x \rightarrow \pi/2$: $\Phi^+(x) \sim (\cos x)^{\Delta^+}$ and $\Phi^-(x) \sim (\cos x)^{\Delta^-}$

$\Phi^+(x) \equiv$ “normalizable” $\Phi^-(x) \equiv$ “non-normalizable”

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- ▶ Examine scaling behaviour as $x \rightarrow \pi/2$: $\Phi^+(x) \sim (\cos x)^{\Delta^+}$ and $\Phi^-(x) \sim (\cos x)^{\Delta^-}$
- ▶ Scalar field is linear combination of both kinds of modes
- ▶ $e_j(x)$ are same eigenfunctions of AdS & have eigenvalues $\omega_j = (2j + \Delta^+)$

$$\phi_1(t, x) = \sum_{j=0}^{\infty} A_j(t) \cos(\omega_j t + B_j(t)) e_j(x) + \sum_{\alpha=0}^{\infty} \bar{A}_{\alpha}(t) \cos(\omega_{\alpha} t + \bar{B}_{\alpha}) E_{\alpha}(x)$$
$$\Delta^{\pm} = \frac{d}{2} \pm \frac{1}{2} \sqrt{d^2 - 4m^2}$$

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- ▶ Scalar field is linear combination of both kinds of modes
- ▶ $e_j(x)$ are same eigenfunctions of AdS & have eigenvalues $\omega_j = (2j + \Delta^+)$
- ▶ $E_\alpha(x)$ are hypergeometric functions & have continuous eigenvalues ω_α

$$\phi_1(t, x) = \sum_{j=0}^{\infty} A_j(t) \cos(\omega_j t + B_j(t)) e_j(x) + \sum_{\alpha=0}^{\infty} \bar{A}_\alpha(t) \cos(\omega_\alpha t + \bar{B}_\alpha) E_\alpha(x)$$
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Resonant Contributions

- ▶ $\mathcal{O}(\epsilon^2)$: backreaction on metric in terms of ϕ_1
- ▶ $\mathcal{O}(\epsilon^3)$: source terms for resonant contributions → examine resonance conditions

Resonant Contributions

- ▶ $\mathcal{O}(\epsilon^2)$: backreaction on metric in terms of ϕ_1
- ▶ $\mathcal{O}(\epsilon^3)$: source terms for resonant contributions → examine resonance conditions
- ▶ All normalizable: restrictions on indices **and** mass value

$$\omega_i + \omega_j + \omega_k = \omega_\ell \Rightarrow i + j + k = \ell - \Delta^+ \in \mathbb{Z}^+$$

$$\omega_i - \omega_j - \omega_k = \omega_\ell \Rightarrow i - j - k = \ell + \Delta^+ \in \mathbb{Z}^+$$

$$\omega_i + \omega_j - \omega_k = \omega_\ell \Rightarrow i + j = k + \ell \in \mathbb{Z}^+$$

Special Values of Non-normalizable Frequencies

Conclusions

- ▶ Collapse of scalar field in AdS \Leftrightarrow thermalization of dual CFT
- ▶ **Nonlinear theory:** “islands of stability”, metastable & irregular phases, chaotic behaviour from self-interaction
- ▶ Weakly turbulent energy cascade to short length scales \rightarrow TTF for inverse cascades
- ▶ **Perturbative theory:** QP solutions robust in $j_{max} \rightarrow \infty$, high-T solutions are not; space of stable solutions is restricted by T_{th}
- ▶ **Next steps**
 - ▶ Physical interpretation of T_{th}
 - ▶ Ways to construct robust QP solutions with $T > T_{th}$
- ▶ **Future:** develop theory for massive TTF \rightarrow less symmetry in equations \therefore fewer cancelations of resonant terms; TTF in AdS₅; time-dependent boundary conditions

Thanks

- ▶ Supervisor: Andrew Frey (University of Winnipeg)
- ▶ PhD Committee: (), (), (), ()
- ▶ Co-authors: Nils Deppe (Cornell), Brayden Yarish (junior collaborator)
- ▶ University of Winnipeg and University of Manitoba
- ▶ Westgrid & Compute Canada

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