| | ♀ | ♂ |
|---|---|---|
| ♀ | 0, 0 | 1, 1 |
| ♂ | 1, 1 | 0, 0 |

simple game, choose to play either female or male.

We can work out the mixed strategy from Maynard Smith (1982) p. 16

(Take the left as the focal strategy)

| | ♀ | ♂ |
|---|---|---|
| ♀ | a | b |
| ♂ | c | d |

$$p = \frac{(b-d)}{(b+c-a-d)}$$

$$p = \frac{1-0}{1+1-0-0} = \frac{1}{2}$$

I think that this recovers the 1:1 sex ratio assuming that individuals interact at random and produce an offspring when they encounter a member of the opposite sex. Note that it is trivial to generalise payoffs to any value 'W' and recover $p = \frac{1}{2}$

Let $F$ represent the female strategy,
$N$ represent a male with a nuptual gift
and $D$ represent a male without a gift.

Let $W(F)$, $W(N)$, and $W(D)$ be the fitness of each strategy.

Let $E(X, Y)$ be the payoff of an individual $X$ against a $Y$ opponent, as in Maynard Smith (1982).

Let $p_F$, $p_N$, and $p_D$ be the frequencies of $F$, $N$, and $D$, respectively, in the population.

I think we want to take the approach of assuming each $p$ is very low to see if it can invade.

The 3x3 payoff matrix will look something like the below

| | F | N | D |
|---|---|---|---|
| F | 0, 0 | | |
| N | | 0, 0 | 0, 0 |
| D | | 0, 0 | 0, 0 |

The empty boxes are the only areas with potentially nonzero fitness. F should thereby have positive fitness payoff when interacting with N or D (but not necessarily equal fitness), but N and D should only have positive fitness when interacting with F.

The intuition is that the stable $p_F = 0.5$, with $p_N$ & $p_D$ dependent on the details

Note from Bishop & Cannings (1978) and Maynard Smith 1982 (p. 15), a stable mixed strategy I requires: $E(I, I) = E(F, I) = E(N, I) = E(D, I)$

I think that we need to set the following equality,

$$p_F E(F,F) + p_N E(F,N) + p_D E(F,D) =$$
$$p_F E(N,F) + p_N E(N,N) + p_D E(N,D) =$$
$$p_F E(D,F) + p_N E(D,N) + p_D E(D,D).$$

For $W(F)$, perhaps we can assume the female gets some baseline reproductive output 'r', plus some added bonus 'b' if given a nuptual gift, 'b'.

For $W(N)$, assume that the reproductive output r comes at a cost 'c' (for the nuptual gift.

For $W(D)$, assume, for the moment, no cost.

Intuitively, we will end up with $p_N = 0$ and $p_F = p_D = \frac{1}{2}$, but let's work it out.

| | F | N | D |
|---|---|---|---|
| F | 0, 0 | r+b, r−c | r, r |
| N | r−c, r+b | 0, 0 | 0, 0 |
| D | r, r | 0, 0 | 0, 0 |

Therefore,

$$p_F(0) + p_N(r+b) + p_D(r) =$$

$$p_F(r-c) + p_N(0) + p_D(0) =$$

$$p_F(r) + p_N(0) + p_D(0).$$ So,

$$p_N(r+b) + p_D(r) = p_F(r-c) = p_F(r)$$

It is clear that $p_F(r-c) \neq p_F(r)$ unless $c=0$, so the nuptual strategy cannot be stable under these conditions, and we are back to

$$p_D(r) = p_F(r), \text{ so } p_D = p_F = \frac{1}{2}.$$

It seems that we need something else for a nuptual strategy to be stable — perhaps either a condition dependent strategy, or maybe a second type of female.

Maybe a 'satiated' versus 'hungry' female, the latter of which will eat the D male but not the N male, removing some future fitness component, 'd'?

My intuition is that a mixed strategy could arise depending on the relative values of b, c, and d.

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Working this out now, let's add the 'H' strategy to represent an especially hungry female (could have gone with F1 & F2 and M1 & M2 instead).

Here is the new 4x4 matrix,

| | F | H | N | D |
|---|---|---|---|---|
| F | 0, 0 | 0, 0 | r+b, r−c | r, r |
| H | 0, 0 | 0, 0 | r+b, r−c | r+m, r−δ |
| N | r−c, r+b | r−c, r+b | 0, 0 | 0, 0 |
| D | r, r | r−δ, r+m | 0, 0 | 0, 0 |

Note that all of the same 9 elements from the previous table involving interactions among F, N, and D are retain. The matrix just adds interactions with H.

- H does not have any fitness consequences from interactions with other females F or H.
- H interacts with nuptual providing males N in the same way that F does.
- H attempts to eat defecting D males, getting a meal benefit 'm', while these males suffer a fitness loss δ.

We now need to work out the below

$$p_F(0) + p_H(0) + p_N(r+b) + p_D(r) =$$
$$p_F(0) + p_H(0) + p_N(r+b) + p_D(r+m) =$$
$$p_F(r-c) + p_H(r-c) + p_N(0) + p_D(0) =$$
$$p_F(r) + p_H(r-\delta) + p_N(0) + p_D(0).$$

We can get rid of most terms

$$p_N(r+b) + p_D(r) = p_N(r+b) + p_D(r+m) =$$
$$p_F(r-C) + p_H(r-c) = p_F(r) + p_H(r-\delta)$$

I should have seen this coming. There is no cost, and only a benefit, of adopting H over F, so there cannot be a stable mixed strategy with both.

That's okay, we can come back to this later, if it seems beneficial to do so, perhaps by having some cost (e.g., risk), associated with the H strategy.

For now, we can just get rid of it,

| | H | N | D |
|---|---|---|---|
| H | 0, 0 | r+b, r−c | r+m, r−δ |
| N | r−c, r+b | 0, 0 | 0, 0 |
| D | r−δ, r+m | 0, 0 | 0, 0 |

I think that we can already simplify the above, but let's just calculate as before.

$$p_H(0) + p_N(r+b) + p_D(r+m) =$$
$$p_H(r-c) + p_N(0) + p_D(0) =$$
$$p_H(r-\delta) + p_N(0) + p_D(0).$$

So we have,

$$p_N(r+b) + p_D(r+m) = p_H(r-c) = p_H(r-\delta)$$

For the above to hold, it is obvious that $c=\delta$ (fitness loss of nuptial equals fitness loss from cannibalism risk), now...

$$p_N(r+b) + p_D(r+m) = p_H(r-c)$$

$$p_N r + p_N b + p_D r + p_D m = p_H r - p_H c$$

$$p_N r + p_D r - p_H r = -p_H c - p_N b - p_D m$$

$$p_H r - p_N r - p_D r = p_H c + p_N b + p_D m$$

$$r(p_H - p_N - p_D) = p_H c + p_N b + p_D m$$

$$r = \frac{p_H c + p_N b + p_D m}{p_H - p_N - p_D}$$

That's not terribly useful. Will try $c$ (also $\delta$) and $m$

$$p_N r + p_N b + p_D r + p_D m = p_H r - p_H c$$

$$p_H c = p_H r - p_N r - p_D r - p_D m - p_N b$$

$$p_H c = r(p_H - p_N - p_D) - p_D m - p_N b$$

$$c = \frac{r(p_H - p_N - p_D) - p_D m - p_N b}{p_H}$$

Not much use yet

$$p_N(r+b) + p_D(r+m) = p_H(r-c)$$

$$p_N r + p_N b + p_D r + p_D m = p_H r - p_H c$$

$$p_N b = p_H r - p_N r - p_D r - p_H c - p_D m$$

$$p_N b = r(p_H - p_N - p_D) - p_H c - p_D m$$

$$b = \frac{r(p_H - p_N - p_D) - p_H c - p_D m}{p_N}$$

This has not helped much either, and m will be similar. Lets try to isolate $p_H$

$$p_H = \frac{p_N(r+b) + p_D(r+m)}{r-c}.$$

Maybe we can make use of

$$1 = p_H + p_N + p_D,$$

$$p_H = 1 - p_N - p_D,$$

So

$$p_N(r+b) + p_D(r+m) = (1 - p_N - p_D)(r-c)$$

$$\cancel{p_N r} + p_N b + \cancel{p_D r} + p_D M = r - c + \cancel{p_N r} - p_N c + \cancel{p_D r} - p_D c$$

$$p_N b + p_D M = r - c - p_N c - p_D c$$

$$c(1 + p_N + p_D) = r - p_N b - p_D M$$

Let's isolate $p_N$, actually

$$c + p_N c + p_D c = r - p_N b - p_D m$$

$$p_N c + p_N b = r - p_D m - p_D c - c$$

$$p_N = \frac{r - p_D m - p_D c - c}{(c+b)}.$$

This seems like a mess still. One more attempted trick. What if we instead set $p_N = 1 - p_H - p_D$

$$(r+b)(1 - p_H - p_D) + p_D(r+m) = p_H(r-c)$$

$$(1-p_H-p_D)(r+b)+p_D(r+m) = p_H(r-c)$$

$$r - p_H r - \cancel{p_D r} + b - p_H b - p_D b + \cancel{p_D r} + p_D m = p_H r - p_H c$$

$$r + b - p_H r - p_H b - p_D b + p_D m = p_H r - p_H c$$

$$r + b - p_D b + p_D m = 2p_H r - p_H c + p_H b$$

$$p_H(2r - c + b) = r + b - p_D b + p_D m$$

$$p_H = \frac{r + b - p_D b + p_D m}{2r - c + b}$$

I really wanted that to be obviously $1/2$, in which case

$$2r - c + b = 2(r + b - p_D b + p_D m)$$

$$2r - c + b = 2r + 2b - 2p_D b + 2p_D m$$

$$2p_D b - 2p_D m = 2r - 2r + 2b - b + c$$

$$2p_D(b-m) = b + c$$

$$2 p_D (b-m) = b + c$$

$$p_D = \frac{b+c}{2(b-m)}$$

This would seem to imply that b must necessarily be greater than m, which doesn't make sense, but could be a consequence of forcing $p_t = \frac{1}{2}$ (or just bad maths).

The benefit of a nuptial must exceed that of a male meal can't be correct, so I will start over again.

I think m is actually completely extraneous, actually, so let's remove it.

| | H | N | D |
|---|---|---|---|
| H | 0, 0 | r+b, r−c | r, r−δ |
| N | r−c, r+b | 0, 0 | 0, δ |
| D | r−δ, r | 0, 0 | 0, 0 |

Once more now,

$$p_H(0) + p_N(r+b) + p_D(r) =$$
$$p_H(r-c) + p_N(0) + p_D(0) =$$
$$p_H(r-\delta) + p_N(0) + p_D(0)$$

$$p_N(r+b) + p_D(r) = p_H(r-c) = p_H(r-\delta)$$

We again get $c = \delta$, so

$$p_N(r+b) + p_D(r) = p_H(r-c)$$

Again, note:

$$p_D = 1 - p_N - p_H$$

$$p_N(r+b) + (1-p_N-p_H)(r) = p_H(r-c)$$

$$\cancel{p_N r} + p_N b + r - \cancel{p_N r} - p_H r = p_H r - p_H c$$

$$p_N b + r - p_H r = p_H r - p_H c$$

$$p_N b = 2p_H r - p_H c - r$$

$$p_N = \frac{p_H(2r-c) - r}{b}$$

Something isn't right here.
Can't we just set $r=1$?

$$p_N = \frac{2p_H - p_H c - 1}{b} = \frac{p_H(2-c)-1}{b}$$

Need to regroup and start again. I think r can be standardised mating fitness of 1, leaving b and c, which might not get much.