Component response rate variation drives stability in large complex systems

Supplemental Information

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This supplemental information supports the manuscript "Component response rate variation drives stability in large complex systems" with all of the code required to recreate the analysis in the main text, and with additional analyses to support its conclusions. All text, code, and data underlying this manuscript are publicly available on GitHub as part of the RandomMatrixStability package.

The RandomMatrixStability package includes all functions and tools for recreating the text, this supplemental information, and running all code; additional documentation is also provided for functions as part of the package. The RandomMatrixStability package is available on GitHub; to download it, the `devtools` library is needed.

```
install.packages("devtools");
library(devtools);
```

The code below installs the RandomMatrixStability package using devtools.

```
install_github("bradduthie/RandomMatrixStability");
```

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Stability across increasing S

Figure 3 of the main text reports the number of stable random complex systems found over 1 million iterations. The table below shows the results for all simulations of random M matrices at $\sigma = 0.4$ and $C = 1$ given a range of $S = \{2, 3, ..., 49, 50\}$. In this table, the `A0` refers to matrices where $\gamma = 1$, while `A1` refers to matrices after $Var(\gamma)$ is added and $\gamma \sim \mathcal{U}(0, 2)$. Each row summarises data for a given S over 1 million randomly simulated M (`A0` and `A1`). The column `A0_unstable` shows the number of `A0` matrices that are unstable, and the column `A0_stable` shows the number of `A0` matrices that are stable (these two columns sum to 1 million). Similarly, the column `A1_unstable` shows the number of `A1` matrices that are unstable and `A1_stable` shows the number that are stable. The columns `A1_stabilised` and `A1_destabilised` show how many `A0` matrices were stabilised or destabilised, respectively, by $Var(\gamma)$.

S	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
2	293	999707	293	999707	0	0
3	3602	996398	3609	996391	0	7
4	14937	985063	15008	984992	0	71
5	39289	960711	39783	960217	36	530
6	78845	921155	80207	919793	389	1751
7	133764	866236	136904	863096	1679	4819
8	204112	795888	208241	791759	5391	9520
9	288041	711959	291775	708225	12619	16353
10	384024	615976	384931	615069	23153	24060
11	485975	514025	481019	518981	35681	30725
12	590453	409547	577439	422561	48302	35288
13	689643	310357	669440	330560	57194	36991
14	777496	222504	751433	248567	60959	34896
15	850159	149841	821613	178387	58567	30021
16	905057	94943	877481	122519	51255	23679
17	943192	56808	919536	80464	40854	17198
18	969018	30982	949944	50056	30102	11028
19	984301	15699	970703	29297	20065	6467
20	992601	7399	983507	16493	12587	3493
21	996765	3235	991532	8468	7030	1797
22	998693	1307	995567	4433	3884	758
23	999503	497	997941	2059	1883	321
24	999861	139	999059	941	899	97
25	999964	36	999617	383	380	33
26	999993	7	999878	122	121	6
27	999995	5	999946	54	53	4
28	1000000	0	999975	25	25	0
29	1000000	0	999997	3	3	0
30	1000000	0	999999	1	1	0
31	1000000	0	999999	1	1	0
32	1000000	0	1000000	0	0	0
33	1000000	0	1000000	0	0	0
34	1000000	0	1000000	0	0	0
35	1000000	0	1000000	0	0	0
36	1000000	0	1000000	0	0	0
37	1000000	0	1000000	0	0	0
38	1000000	0	1000000	0	0	0
39	1000000	0	1000000	0	0	0
40	1000000	0	1000000	0	0	0
41	1000000	0	1000000	0	0	0

S	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
42	1000000	0	1000000	0	0	0
43	1000000	0	1000000	0	0	0
44	1000000	0	1000000	0	0	0
45	1000000	0	1000000	0	0	0
46	1000000	0	1000000	0	0	0
47	1000000	0	1000000	0	0	0
48	1000000	0	1000000	0	0	0
49	1000000	0	1000000	0	0	0
50	1000000	0	1000000	0	0	0

Overall, the ratio of stable `A1` matrices to stable `A0` matrices found is greater than 1 (compare column 5 to column 3), and this ratio increases with increasing S (column 1). Hence, more randomly created complex systems (M) are generated given variation in γ than when $\gamma = 1$. Note that feasibility results were ommited for the table above, but are reported below.

Stability given targetted manipulation of γ (genetic algorithm)

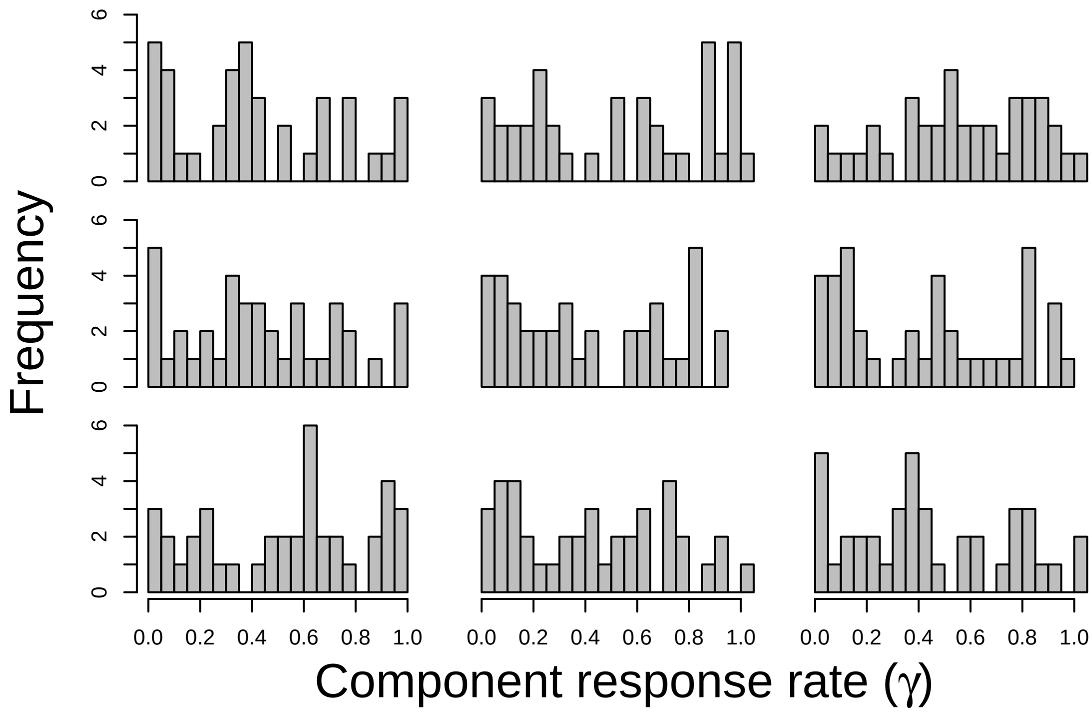
Figure 4 of the main text reports the number of stable random complex systems found over 100000 using the genetic algorithm to maximise stability with a vector γ. Stability results for 100000 M for each S from 2-40 are shown below. Results for `A0` indicate systems in which $\gamma = 1$, while `A1` refers to systems in which the genetic algorithm searched for a set of γ values that stabilised the system.

S	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
2	26	99974	26	99974	0	0
3	358	99642	358	99642	0	0
4	1505	98495	1505	98495	0	0
5	3995	96005	3982	96018	13	0
6	8060	91940	7956	92044	104	0
7	13420	86580	12953	87047	468	1
8	20518	79482	18940	81060	1578	0
9	28939	71061	25148	74852	3793	2
10	38241	61759	30915	69085	7327	1
11	48682	51318	36398	63602	12286	2
12	58752	41248	40710	59290	18043	1
13	68888	31112	44600	55400	24289	1
14	77651	22349	47528	52472	30124	1
15	84912	15088	49971	50029	34942	1
16	90451	9549	52274	47726	38178	1
17	94332	5668	54124	45876	40209	1
18	96968	3032	55831	44169	41139	2
19	98384	1616	58079	41921	40305	0
20	99269	731	60181	39819	39088	0
21	99677	323	63338	36662	36339	0
22	99854	146	66350	33650	33504	0
23	99947	53	70478	29522	29469	0
24	99983	17	74121	25879	25862	0
25	99991	9	78364	21636	21627	0
26	99999	1	82635	17365	17364	0
27	100000	0	86433	13567	13567	0

S	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
28	100000	0	89951	10049	10049	0
29	100000	0	92716	7284	7284	0
30	100000	0	95171	4829	4829	0
31	100000	0	96844	3156	3156	0
32	100000	0	98128	1872	1872	0
33	100000	0	98941	1059	1059	0
34	100000	0	99358	642	642	0
35	100000	0	99702	298	298	0
36	100000	0	99856	144	144	0
37	100000	0	99921	79	79	0
38	100000	0	99970	30	30	0
39	100000	0	99989	11	11	0
40	100000	0	99994	6	6	0

The distributions of nine γ vectors from the highest S values are shown below. This comparison shows the high number of stable M that can be produced through a targetted search of γ values, and suggests that many otherwise unstable systems could potentially be stabilised by an informed manipulation of their component response times. Such a possibility might conceivably reduce the dimensionality of problems involving stability in social-ecological or economic systems.

Distributions of γ values in vectors for the highest values of S are shown below.



The distribution of γ values found by the genetic algorithm is uniform. A uniform distribution was used to initialise γ values, so there is therefore no evidence that a particular distribution of γ is likely to be found to stabilise a matrix M.

Stability of ecological networks

While the foundational work of May (1) applies broadly to complex networks, much attention has been given specifically to ecological networks of interacting species. In these networks, the matrix M is interpreted as a community matrix and each row and column is interpreted as a single species. The effect that the density of any species i has on the population dynamics of species j is found in M_{ij}, meaning that M holds the effects of pair-wise interactions between S species (2, 4, 20). While May's original work (1) considered only randomly assembled communities, recent work has specifically looked at more restricted ecological communities including competitive networks (all off-diagonal elements of M are negative), mutualist networks (all off-diagonal elements of M are positive), and predator-prey networks (for any pair of i and j, the effect of i on j is negative and j on i is positive, or vice versa) (2, 4, 17, 20). In general, competitor and mutualist networks tend to be unstable, while predator-prey networks tend to be highly stabilising.

I investigated competitor, mutualist, and predator-prey networks following Allesina et al. (2). To create these networks, I first generated a random matrix M, then changed the elements of M accordingly. If M was a competitive network, then the sign of any positive off-diagonal elements was reversed to be negative. If M was a mutualist network, then the sign of any positive off-diagonal elements was reversed to be positive. And if M was a predator-prey network, then all i and j pairs of elements were checked; any pairs of the same sign were changed so that one was negative and the other was positive.

The number of stable M matrices was estimated exactly as it was in the main text for random matrices for values of S from 2 to 50 (100 in the case of the relatively more stable predator-prey interactions), except that only 100000 random M were generated instead of 1 million.

The following tables for restricted ecological communities can therefore be compared with the random M results above (but note that counts from systems with comparable probabilities of stability will be an order of magnitude lower in the tables below due to the smaller number of M matrices generated). As with the results above, in the tables below, `A0` refers to matrices when $\gamma = 1$ and `A1` refers to matrices after $Var(\gamma)$ is added. The column `A0_unstable` shows the number of `A0` matrices that are unstable, and the column `A0_stable` shows the number of `A0` matrices that are stable (these two columns sum to 100000). Similarly, the column `A1_unstable` shows the number of `A1` matrices that are unstable and `A1_stable` shows the number that are stable. The columns `A1_stabilised` and `A1_destabilised` show how many `A0` matrices were stabilised or destabilised, respectively, by $Var(\gamma)$.

Competition

Results for competitor interaction networks are shown below

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised
2	48	99952	48	99952	0
3	229	99771	231	99769	0
4	701	99299	704	99296	0
5	1579	98421	1587	98413	0
6	3218	96782	3253	96747	6
7	5519	94481	5619	94381	23
8	9062	90938	9237	90763	77
9	13436	86564	13729	86271	230
10	18911	81089	19303	80697	505
11	25594	74406	25961	74039	1011
12	33207	66793	33382	66618	1724
13	41160	58840	41089	58911	2655
14	50575	49425	49894	50106	3777
15	59250	40750	57892	42108	4824
16	67811	32189	65740	34260	5634
17	75483	24517	73056	26944	5943
18	82551	17449	79878	20122	5780

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised
19	88030	11970	85204	14796	5417
20	92254	7746	89766	10234	4544
21	95233	4767	93002	6998	3695
22	97317	2683	95451	4549	2803
23	98508	1492	97122	2878	1991
24	99240	760	98407	1593	1216
25	99669	331	99082	918	739
26	99871	129	99490	510	452
27	99938	62	99732	268	240
28	99985	15	99888	112	108
29	99990	10	99951	49	46
30	100000	0	99981	19	19
31	100000	0	99993	7	7
32	100000	0	99996	4	4
33	100000	0	99998	2	2
34	100000	0	100000	0	0
...	...	...	...	...	...
50	100000	0	100000	0	0

Mutualism

Results for mutualist interaction networks are shown below

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised
2	56	99944	56	99944	0
3	3301	96699	3301	96699	0
4	34446	65554	34446	65554	0
5	86520	13480	86520	13480	0
6	99683	317	99683	317	0
7	99998	2	99998	2	0
8	100000	0	100000	0	0
9	100000	0	100000	0	0
10	100000	0	100000	0	0
11	100000	0	100000	0	0
12	100000	0	100000	0	0
...	...	...	...	...	...
50	100000	0	100000	0	0

Predator-prey

Results for predator-prey interaction networks are shown below

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised
2	0	100000	0	100000	0
3	0	100000	0	100000	0
4	0	100000	0	100000	0
5	1	99999	1	99999	0
6	4	99996	4	99996	0
7	2	99998	2	99998	0
8	5	99995	5	99995	0
9	20	99980	21	99979	0

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised
10	20	99980	22	99978	0
11	38	99962	39	99961	0
12	64	99936	66	99934	0
13	87	99913	91	99909	0
14	157	99843	159	99841	0
15	215	99785	227	99773	0
16	293	99707	310	99690	0
17	383	99617	408	99592	0
18	443	99557	473	99527	3
19	642	99358	675	99325	4
20	836	99164	887	99113	7
21	1006	98994	1058	98942	10
22	1153	98847	1228	98772	20
23	1501	98499	1593	98407	30
24	1841	98159	1996	98004	40
25	2146	97854	2316	97684	58
26	2643	97357	2809	97191	119
27	3034	96966	3258	96742	158
28	3690	96310	3928	96072	201
29	4257	95743	4532	95468	290
30	4964	95036	5221	94779	424
31	5627	94373	5978	94022	452
32	6543	93457	6891	93109	666
33	7425	92575	7777	92223	818
34	8540	91460	8841	91159	1071
35	9526	90474	9842	90158	1337
36	10617	89383	10891	89109	1624
37	12344	87656	12508	87492	2021
38	13675	86325	13877	86123	2442
39	15264	84736	15349	84651	2870
40	17026	82974	17053	82947	3363
41	18768	81232	18614	81386	3905
42	20791	79209	20470	79530	4579
43	23150	76850	22754	77246	5217
44	25449	74551	24184	75816	6285
45	27702	72298	26464	73536	6754
46	30525	69475	28966	71034	7646
47	32832	67168	31125	68875	8487
48	36152	63848	33865	66135	9479
49	38714	61286	36242	63758	10125
50	41628	58372	38508	61492	11036
51	44483	55517	41023	58977	11704
52	48134	51866	44287	55713	12573
53	51138	48862	46721	53279	13223
54	54261	45739	49559	50441	13757
55	57647	42353	52403	47597	14324
56	60630	39370	55293	44707	14669
57	63647	36353	57787	42213	15103
58	66961	33039	60439	39561	15450
59	69968	30032	63708	36292	15246
60	72838	27162	66270	33730	15177
61	75609	24391	68873	31127	15006

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised
62	77999	22001	71318	28682	14538
63	80616	19384	73517	26483	14510
64	83089	16911	76209	23791	13784
65	85150	14850	78086	21914	13412
66	86908	13092	80437	19563	12477
67	88671	11329	82379	17621	11718
68	90537	9463	84483	15517	10878
69	91969	8031	86233	13767	10033
70	93181	6819	87914	12086	9070
71	94330	5670	89200	10800	8401
72	95324	4676	90833	9167	7359
73	96143	3857	91805	8195	6726
74	96959	3041	93065	6935	5900
75	97543	2457	93987	6013	5222
76	97969	2031	94900	5100	4481
77	98497	1503	95756	4244	3809
78	98744	1256	96442	3558	3269
79	99045	955	96942	3058	2837
80	99276	724	97528	2472	2329
81	99481	519	97996	2004	1894
82	99556	444	98321	1679	1597
83	99691	309	98722	1278	1227
84	99752	248	98943	1057	1015
85	99833	167	99144	856	837
86	99895	105	99346	654	642
87	99925	75	99461	539	530
88	99945	55	99566	434	428
89	99976	24	99675	325	324
90	99977	23	99756	244	243
91	99982	18	99839	161	155
92	99988	12	99865	135	135
93	99994	6	99885	115	115
94	99993	7	99911	89	88
95	99998	2	99953	47	47
96	99999	1	99965	35	35
97	99999	1	99979	21	21
98	100000	0	99973	27	27
99	100000	0	99984	16	16
100	100000	0	99989	11	11

Overall, as expected[1], predator-prey communities are relatively stable while mutualist communties are highly unstable. But interestingly, while $Var(\gamma)$ stabilises predator-prey and competitor communities, it does not stabilise mutualist communities. This is unsurprising because purely mutualist communities are characterised by a very positive (2) leading $\Re(\lambda)$, and it is highly unlikely that $Var(\gamma)$ alone will shift all real parts of eigenvalues to negative values.

Sensitivity of inter-connectivity (C) values

In the main text, for simplicity, I assumed inter-connectivity values of $C = 1$, meaning that all off-diagonal elements of a matrix M were potentially nonzero and sampled from a normal distribution $\mathcal{N}(0, \sigma^2)$ where

$\sigma = 0.4$. Here I present four tables showing the number of stable communities given $C = \{0.3, 0.5, 0.7, 0.9\}$. In all cases, uniform variation in component response time ($\gamma \sim \mathcal{U}(0, 2)$) led to a higher number of stable communities than when γ did not vary ($\gamma = 1$). In contrast to the main text, 100000 rather than 1 million M were simulated. As with the results on stability with increasing S shown above, in the tables below `A0` refers to matrices when $\gamma = 1$, and `A1` refers to matrices after $Var(\gamma)$ is added. The column `A0_unstable` shows the number of `A0` matrices that are unstable, and the column `A0_stable` shows the number of `A0` matrices that are stable (these two columns sum to 100000). Similarly, the column `A1_unstable` shows the number of `A1` matrices that are unstable and `A1_stable` shows the number that are stable. The columns `A1_stabilised` and `A1_destabilised` show how many `A0` matrices were stabilised or destabilised, respectively, by $Var(\gamma)$.

Connectance C = 0.3

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
2	5	99995	5	99995	0	0
3	6	99994	6	99994	0	0
4	24	99976	24	99976	0	0
5	59	99941	59	99941	0	0
6	98	99902	98	99902	0	0
7	160	99840	161	99839	0	1
8	290	99710	293	99707	0	3
9	430	99570	434	99566	0	4
10	648	99352	653	99347	1	6
11	946	99054	957	99043	0	11
12	1392	98608	1415	98585	4	27
13	2032	97968	2065	97935	5	38
14	2627	97373	2688	97312	10	71
15	3588	96412	3647	96353	35	94
16	5019	94981	5124	94876	51	156
17	6512	93488	6673	93327	79	240
18	8444	91556	8600	91400	165	321
19	10416	89584	10667	89333	244	495
20	13254	86746	13477	86523	425	648
21	16248	83752	16481	83519	642	875
22	19497	80503	19719	80281	929	1151
23	23654	76346	23776	76224	1368	1490
24	28485	71515	28389	71611	1914	1818
25	32774	67226	32483	67517	2428	2137
26	38126	61874	37411	62589	3221	2506
27	43435	56565	42418	57582	3828	2811
28	49333	50667	47840	52160	4565	3072
29	55389	44611	53381	46619	5329	3321
30	60826	39174	58388	41612	5918	3480
31	66820	33180	64043	35957	6345	3568
32	72190	27810	69036	30964	6685	3531
33	77053	22947	73587	26413	6826	3360
34	81816	18184	78157	21843	6673	3014
35	85651	14349	82041	17959	6383	2773
36	88985	11015	85657	14343	5721	2393
37	92072	7928	88805	11195	5180	1913
38	94329	5671	91444	8556	4451	1566
39	95912	4088	93295	6705	3804	1187
40	97232	2768	95201	4799	2967	936
41	98179	1821	96506	3494	2356	683
42	98826	1174	97489	2511	1786	449

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
43	99275	725	98312	1688	1251	288
44	99583	417	98872	1128	903	192
45	99776	224	99339	661	576	139
46	99865	135	99518	482	413	66
47	99938	62	99744	256	226	32
48	99956	44	99824	176	151	19
49	99980	20	99914	86	85	19
50	99993	7	99950	50	46	3
51	99998	2	99971	29	28	1
52	99998	2	99986	14	14	2
53	99999	1	99992	8	7	0
54	100000	0	99997	3	3	0
55	100000	0	99999	1	1	0
56	100000	0	99998	2	2	0
57	100000	0	99999	1	1	0
58	100000	0	100000	0	0	0
...	...	...	...	...	...	...
100	100000	0	100000	0	0	0

Connectance C = 0.5

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
2	7	99993	7	99993	0	0
3	32	99968	32	99968	0	0
4	122	99878	122	99878	0	0
5	320	99680	321	99679	0	1
6	667	99333	673	99327	0	6
7	1233	98767	1252	98748	0	19
8	2123	97877	2156	97844	3	36
9	3415	96585	3471	96529	16	72
10	5349	94651	5450	94550	30	131
11	7990	92010	8185	91815	81	276
12	11073	88927	11301	88699	219	447
13	14971	85029	15204	84796	445	678
14	19754	80246	19992	80008	764	1002
15	25020	74980	25239	74761	1185	1404
16	30860	69140	30938	69062	1902	1980
17	37844	62156	37562	62438	2758	2476
18	44909	55091	44251	55749	3595	2937
19	52322	47678	51011	48989	4573	3262
20	60150	39850	58295	41705	5382	3527
21	67147	32853	64895	35105	5925	3673
22	74177	25823	71358	28642	6310	3491
23	80297	19703	77034	22966	6507	3244
24	85372	14628	82039	17961	6209	2876
25	89719	10281	86539	13461	5562	2382
26	92947	7053	90141	9859	4707	1901
27	95436	4564	92950	7050	3844	1358
28	97196	2804	95171	4829	2999	974
29	98300	1700	96842	3158	2115	657
30	99103	897	98033	1967	1466	396

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
31	99502	498	98665	1335	1068	231
32	99745	255	99185	815	696	136
33	99881	119	99572	428	375	66
34	99955	45	99788	212	191	24
35	99979	21	99900	100	95	16
36	99995	5	99950	50	50	5
37	99997	3	99970	30	28	1
38	99998	2	99986	14	13	1
39	99999	1	99991	9	9	1
40	100000	0	100000	0	0	0
41	100000	0	99999	1	1	0
42	100000	0	99999	1	1	0
43	100000	0	100000	0	0	0
...	...	...	...	...	...	...
50	100000	0	100000	0	0	0

Connectance C = 0.7

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
2	7	99993	7	99993	0	0
3	106	99894	106	99894	0	0
4	395	99605	397	99603	0	2
5	1117	98883	1123	98877	0	6
6	2346	97654	2367	97633	6	27
7	4314	95686	4388	95612	16	90
8	7327	92673	7456	92544	61	190
9	11514	88486	11792	88208	150	428
10	16247	83753	16584	83416	415	752
11	22481	77519	22759	77241	884	1162
12	29459	70541	29729	70271	1548	1818
13	37631	62369	37567	62433	2419	2355
14	46317	53683	45696	54304	3548	2927
15	54945	45055	53695	46305	4671	3421
16	63683	36317	61643	38357	5567	3527
17	72004	27996	69375	30625	6124	3495
18	79220	20780	76158	23842	6413	3351
19	85286	14714	82283	17717	5982	2979
20	90240	9760	87181	12819	5398	2339
21	93676	6324	91077	8923	4468	1869
22	96203	3797	94045	5955	3425	1267
23	97866	2134	96161	3839	2496	791
24	98842	1158	97633	2367	1713	504
25	99433	567	98630	1370	1079	276
26	99760	240	99259	741	655	154
27	99895	105	99576	424	377	58
28	99950	50	99790	210	194	34
29	99981	19	99915	85	80	14
30	99994	6	99952	48	47	5
31	99998	2	99972	28	28	2
32	99999	1	99992	8	8	1
33	100000	0	99997	3	3	0

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
34	100000	0	99999	1	1	0
35	100000	0	100000	0	0	0
...	...	...	...	...	...	...
50	100000	0	100000	0	0	0

Connectance C = 0.9

N	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
2	14	99986	14	99986	0	0
3	240	99760	240	99760	0	0
4	1008	98992	1016	98984	0	8
5	2708	97292	2729	97271	2	23
6	5669	94331	5755	94245	13	99
7	9848	90152	10057	89943	91	300
8	15903	84097	16201	83799	336	634
9	22707	77293	23110	76890	765	1168
10	30796	69204	31122	68878	1526	1852
11	40224	59776	40082	59918	2649	2507
12	49934	50066	49288	50712	3773	3127
13	60138	39862	58803	41197	4984	3649
14	69100	30900	67110	32890	5755	3765
15	77607	22393	74884	25116	6273	3550
16	84663	15337	81780	18220	5975	3092
17	90075	9925	87290	12710	5209	2424
18	93944	6056	91419	8581	4271	1746
19	96650	3350	94530	5470	3287	1167
20	98160	1840	96698	3302	2191	729
21	99111	889	98133	1867	1389	411
22	99588	412	98905	1095	903	220
23	99837	163	99480	520	452	95
24	99932	68	99744	256	228	40
25	99976	24	99863	137	133	20
26	99995	5	99950	50	49	4
27	99996	4	99986	14	13	3
28	100000	0	99993	7	7	0
29	100000	0	99996	4	4	0
30	100000	0	99998	2	2	0
31	100000	0	100000	0	0	0
...	...	...	...	...	...	...
50	100000	0	100000	0	0	0

Sensitivity of interaction strength (σ) values

ADD TEXT ABOUT COMPARISON WITH ALLESINA AND VALUES ACROSS SIGMA

Interaction strength $\sigma = 0.3$

S	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
10	0	100000	0	100000	0	0

S	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
20	0	100000	0	100000	0	0
30	0	100000	0	100000	0	0
40	0	100000	0	100000	0	0
50	0	100000	0	100000	0	0
60	2	99998	2	99998	0	0
70	4	99996	4	99996	0	0
80	6	99994	6	99994	0	0
90	5	99995	5	99995	0	0
100	11	99989	11	99989	0	0
110	12	99988	13	99987	0	1
120	23	99977	23	99977	0	0
130	40	99960	40	99960	0	0
140	62	99938	65	99935	0	3
150	162	99838	165	99835	0	3
160	325	99675	329	99671	2	6
170	829	99171	851	99149	6	28
180	1817	98183	1860	98140	31	74
190	3927	96073	3989	96011	143	205
200	8084	91916	8048	91952	557	521
210	15558	84442	15147	84853	1534	1123
220	26848	73152	25342	74658	3625	2119
230	43386	56614	39535	60465	6992	3141
240	62734	37266	56684	43316	9815	3765
250	80128	19872	73080	26920	10128	3080
260	92206	7794	86619	13381	7490	1903
270	97946	2054	94824	5176	3797	675
280	99659	341	98534	1466	1265	140
290	99962	38	99696	304	281	15
300	99994	6	99964	36	34	4

Interaction strength $\sigma = 0.4$

S	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
10	3	99997	3	99997	0	0
20	15	99985	15	99985	0	0
30	48	99952	48	99952	0	0
40	85	99915	85	99915	0	0
50	163	99837	163	99837	0	0
60	280	99720	282	99718	0	2
70	561	99439	566	99434	3	8
80	1009	98991	1029	98971	6	26
90	2126	97874	2175	97825	31	80
100	4580	95420	4653	95347	142	215
110	9540	90460	9632	90368	465	557
120	19090	80910	18668	81332	1676	1254
130	35047	64953	33220	66780	4172	2345
140	56411	43589	52439	47561	7297	3325
150	78003	21997	72574	27426	8477	3048
160	92678	7322	88438	11562	5901	1661
170	98614	1386	96670	3330	2397	453
180	99839	161	99418	582	499	78

S	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
190	99990	10	99945	55	52	7
200	100000	0	99995	5	5	0
210	100000	0	100000	0	0	0
...	...	...	...	...	...	...
300	100000	0	100000	0	0	0

Interaction strength $\sigma = 0.5$

S	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
10	36	99964	36	99964	0	0
20	195	99805	195	99805	0	0
30	519	99481	523	99477	0	4
40	1096	98904	1101	98899	2	7
50	2375	97625	2397	97603	9	31
60	4898	95102	4968	95032	83	153
70	10841	89159	10916	89084	432	507
80	22281	77719	21988	78012	1622	1329
90	42010	57990	39998	60002	4458	2446
100	67289	32711	63098	36902	7153	2962
110	88137	11863	84023	15977	6108	1994
120	97678	2322	95557	4443	2740	619
130	99795	205	99304	696	578	87
140	99989	11	99948	52	49	8
150	100000	0	100000	0	0	0
...	...	...	...	...	...	...
300	100000	0	100000	0	0	0

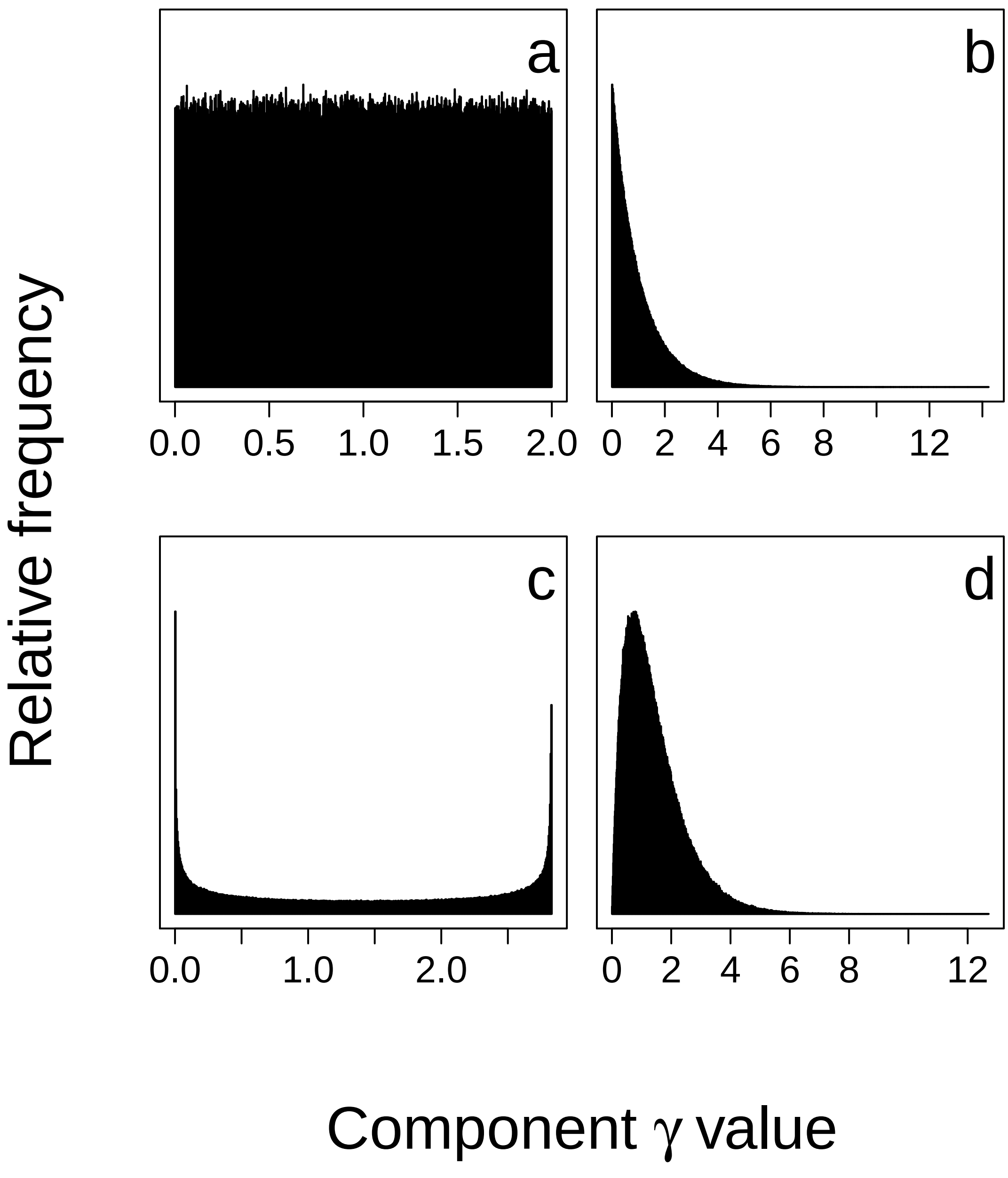
Interaction strength $\sigma = 0.6$

S	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
10	162	99838	162	99838	0	0
20	798	99202	799	99201	0	1
30	2273	97727	2289	97711	6	22
40	5259	94741	5298	94702	70	109
50	12084	87916	12054	87946	446	416
60	26072	73928	25511	74489	1810	1249
70	50121	49879	47747	52253	4748	2374
80	77806	22194	73810	26190	6421	2425
90	94862	5138	92069	7931	3842	1049
100	99527	473	98822	1178	870	165
110	99984	16	99912	88	80	8
120	100000	0	99998	2	2	0
130	100000	0	100000	0	0	0
...	...	...	...	...	...	...
300	100000	0	100000	0	0	0

Sensitivity of distribution of γ

In the main text, I considered a uniform distribution of component response rates $\gamma \sim \mathcal{U}(0, 2)$. The number of unstable and stable M matrices are reported in a table above across different values of S. Here I show complementary results for three different distributions including an exponential, beta, and gamma distribution of γ values. The shape of these distributions is shown in the figure below.

Distributions of component response rate (γ) values in complex systems. The stabilities of simulated complex systems with these γ distributions are compared to otherwise identical complex systems with a fixed component response rate of $\gamma = 1$ across different system sizes (S; i.e., component numbers) given a unit γ standard deviation ($\sigma_\gamma = 1$) for b-d. Distributions are as follows: (a) uniform, (b) exponential, (c) beta ($\alpha = 0.5$ and $\beta = 0.5$), and (d) gamma ($k = 2$ and $\theta = 2$). Each panel shows 1 million randomly generated γ values.



The same 100000 M matrices were used to investigate stability when applying each of these different distributions of γ values. The table below shows the number of M that were unstable (_unst) and stable (_stbl) for the exponential (Exp), beta, and gamma distributions.

```
fourdists <- read.csv(file = "sim_results/different_distr/four_distr_rand.csv");
kable(fourdists);
```

S	Exp_unst	Exp_stbl	beta_unst	beta_stbl	gamma_unst	gamma_stbl
2	30	99970	30	99970	30	99970
3	355	99645	355	99645	355	99645
4	1506	98494	1512	98488	1516	98484
5	3930	96070	3971	96029	4006	95994
6	7738	92262	7844	92156	7918	92082
7	13606	86394	13889	86111	13990	86010
8	20535	79465	21002	78998	21114	78886
9	28614	71386	29060	70940	29110	70890
10	38375	61625	38388	61612	38441	61559
11	48616	51384	48211	51789	47957	52043
12	59254	40746	58025	41975	57473	42527
13	68816	31184	66753	33247	66127	33873
14	77721	22279	75149	24851	74222	25778
15	84842	15158	82030	17970	81040	18960
16	90365	9635	87809	12191	86600	13400
17	94171	5829	91756	8244	90668	9332
18	96978	3022	94977	5023	94176	5824
19	98376	1624	97018	2982	96268	3732
20	99218	782	98357	1643	97765	2235
21	99678	322	99124	876	98746	1254
22	99864	136	99599	401	99323	677
23	99954	46	99783	217	99668	332
24	99978	22	99920	80	99821	179
25	99996	4	99967	33	99911	89
26	99999	1	99979	21	99960	40
27	99999	1	99990	10	99983	17
28	100000	0	99999	1	99991	9
29	100000	0	99999	1	99999	1
30	100000	0	100000	0	100000	0
31	100000	0	100000	0	99999	1
32	100000	0	100000	0	100000	0
…	…	…	…	…	…	…
50	100000	0	100000	0	100000	0

In comparison to the uniform distribution (a), proportionally fewer random systems are found with the exponential distribution (b), while more are found with the beta (c) and gamma (d) distributions.

Feasibility of complex systems

When feasibility was evaluated with and without variation in γ, there was no increase in stability for M where γ varied as compared to where $\gamma = 1$. Results below illustrate this result, which was general to all other simulations performed.

S	A0_infeasible	A0_feasible	A1_infeasible	A1_feasible	A1_made_feasible	A1_made_infeasible
2	749978	250022	749942	250058	35552	35516
3	874519	125481	874296	125704	36803	36580
4	937192	62808	937215	62785	26440	26463
5	968776	31224	968639	31361	16319	16182
6	984313	15687	984463	15537	9006	9156
7	992149	7851	992161	7839	4991	5003
8	996124	3876	996103	3897	2644	2623
9	998014	1986	998027	1973	1361	1374
10	999031	969	999040	960	698	707
11	999546	454	999514	486	377	345
12	999764	236	999792	208	160	188
13	999883	117	999865	135	105	87
14	999938	62	999945	55	40	47
15	999971	29	999964	36	31	24
16	999988	12	999991	9	8	11
17	999996	4	999991	9	8	3
18	999997	3	999999	1	1	3
19	999998	2	999997	3	3	2
20	1000000	0	999999	1	1	0
21	1000000	0	1000000	0	0	0
22	999999	1	1000000	0	0	1
23	1000000	0	1000000	0	0	0
24	1000000	0	1000000	0	0	0
25	1000000	0	1000000	0	0	0
26	1000000	0	1000000	0	0	0
27	1000000	0	1000000	0	0	0
28	1000000	0	1000000	0	0	0
29	1000000	0	1000000	0	0	0
30	1000000	0	1000000	0	0	0
31	1000000	0	1000000	0	0	0
32	1000000	0	1000000	0	0	0
33	1000000	0	1000000	0	0	0
34	1000000	0	1000000	0	0	0
35	1000000	0	1000000	0	0	0
36	1000000	0	1000000	0	0	0
37	1000000	0	1000000	0	0	0
38	1000000	0	1000000	0	0	0
39	1000000	0	1000000	0	0	0
40	1000000	0	1000000	0	0	0
41	1000000	0	1000000	0	0	0
42	1000000	0	1000000	0	0	0
43	1000000	0	1000000	0	0	0
44	1000000	0	1000000	0	0	0
45	1000000	0	1000000	0	0	0
46	1000000	0	1000000	0	0	0
47	1000000	0	1000000	0	0	0
48	1000000	0	1000000	0	0	0
49	1000000	0	1000000	0	0	0
50	1000000	0	1000000	0	0	0

Hence, in general, $Var(\gamma)$ does not appear to affect feasibility in pure species interaction networks.

References

1. Allesina, S. & Tang, S. Stability criteria for complex ecosystems. *Nature* **483,** 205–208 (2012).