Variation in the rate at which individual system components respond to perturbation underlies the stability of complex systems.

Component response rate variation underlies the stability of complex systems

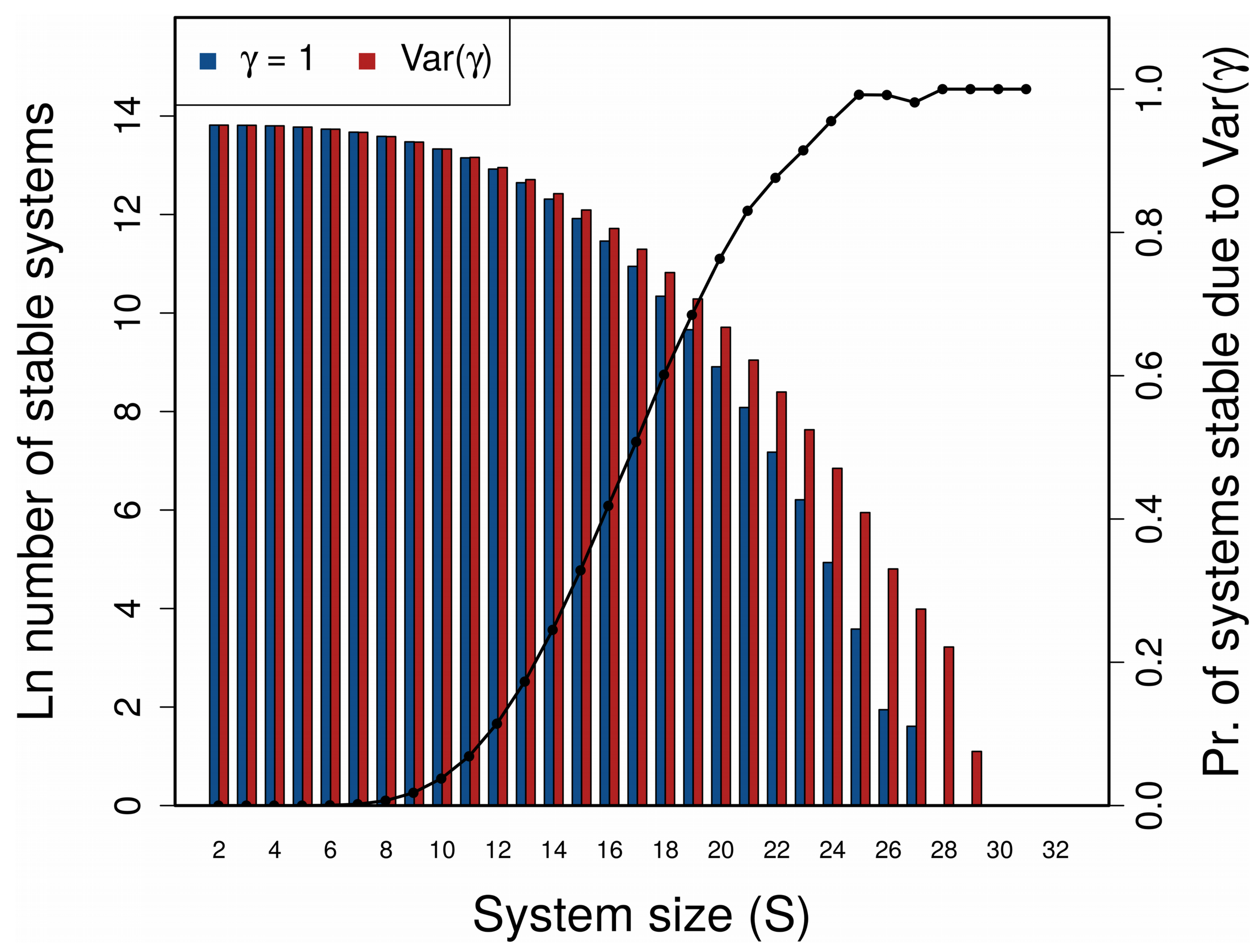
Duthie, A. Bradley   http://bradduthie.github.io   @bradduthie

BACKGROUND

- The stability of a complex system decreases with increasing size (S), inter-connectivity (C), and variance in interaction strength (σ^2).
- Stability occurs if $\sigma\sqrt{(SC)}$ is sufficiently low.
- The effect of variation in the response rates of system components (γ) is unknown.

MODELLING

- Generated random matrices (**M**) with known S, C, and σ^2.
- Eigenanalysis to assess the stability of **M** across. increasingly high S values.
- Compare stability with and without variation in γ.



System stability with (red) and without (blue) variation in component response rate (γ), across 1 million M per S.

DISCUSSION

- Studies of complex systems should account for component response rate variation
- **Note:** Results do not imply that variation in γ *causes* stability in complex systems
- Results relevant for life and social sciences:
  - Trait evolution (biological networks)
  - Transaction speed (economic networks)
  - Communication speed (social networks)

Mathematical details. Following May[1,2], the value of a component i at time t ($v_i(t)$) is affected by the value of j ($v_j(t)$) and j's marginal effect on i ($a_{i,j}$), and by i's response rate (γ_i),

$$\frac{dv_i(t)}{dt} = \gamma_i \sum_{j=1}^{S} a_{ij} v_j(t).$$

In matrix notation[2],

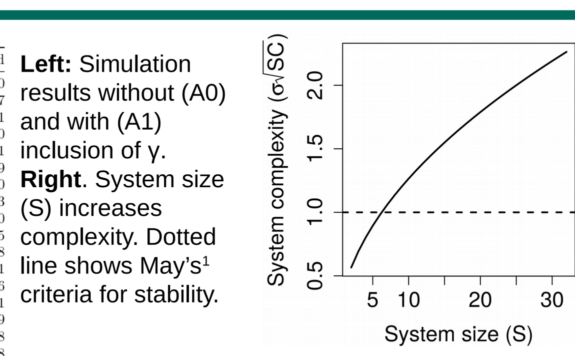
$$\frac{d\mathbf{v}(t)}{dt} = \gamma \mathbf{A} \mathbf{v}(t).$$

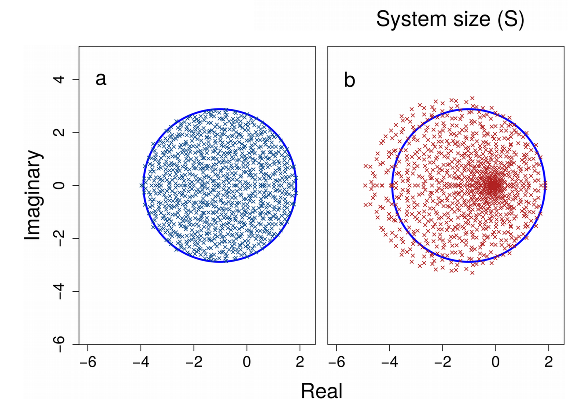
In the above, $\boldsymbol{\gamma}$ is a diagonal matrix in which elements correspond to individual component response rates. Therefore, $\mathbf{M} = \boldsymbol{\gamma}\mathbf{A}$ defines system component values and can be analysed using the techniques of May[1-3]. Row means of **A** are expected to be identical, but variation around this expectation will naturally arise due to random sampling of **A** off-diagonal elements ($a_{i,j} \sim N(0, \sigma^2)$ if $i \neq j$; $a_{i,j} = -1$ if $i = j$) and finite S. In simulations, the total variation in **M** row means attributable to **A** is small relative to that attributable to $\boldsymbol{\gamma}$, especially at high S. Variation in γ specifically isolates the effects of differing component response rates, hence causing differences in expected **M** row means.

Under default parameter values, C = 1, σ = 0.4, and $\gamma_i \sim$ U(0, 2).

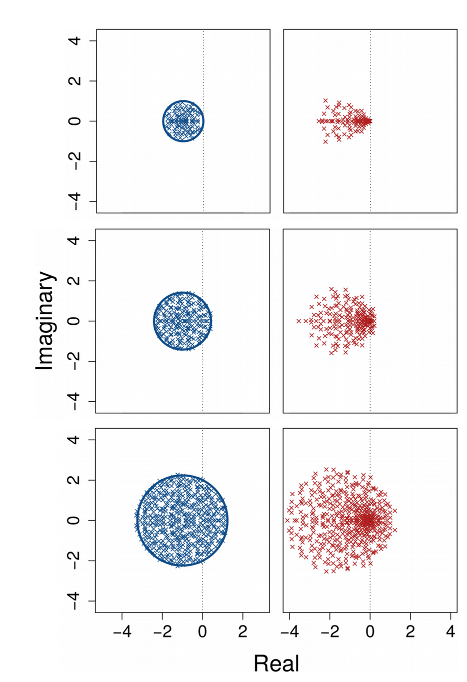
S	A0_unstable	A0_stable	A1_unstable	A1_stable	A1_stabilised	A1_destabilised
2	293	999707	293	999707	0	0
3	3602	996398	3609	996391	0	7
4	14937	985063	15008	984992	0	71
5	39289	960711	39783	960217	36	530
6	78845	921155	80207	919793	389	1751
7	133764	866236	136904	863096	1679	4819
8	204112	795888	208241	791759	5391	9520
9	288041	711959	291775	708225	12619	16353
10	384024	615976	384931	615069	23153	24060
11	485975	514025	481019	518981	35681	30725
12	590453	409547	577439	422561	48302	35288
13	689643	310357	669440	330560	57194	36991
14	777496	222504	751433	248567	60959	34896
15	850159	149841	821613	178387	58567	30021
16	905057	94943	877481	122519	51255	23679
17	943192	56808	919536	80464	40854	17198
18	969018	30982	949944	50056	30102	11028
19	984301	15699	970703	29297	20065	6467
20	992601	7399	983507	16493	12587	3493
21	996765	3235	991532	8468	7030	1797
22	998693	1307	995567	4433	3884	758
23	999503	497	997941	2059	1883	321
24	999861	139	999059	941	899	97
25	999964	36	999617	383	380	33
26	999993	7	999878	122	121	6
27	999995	5	999946	54	53	4
28	1000000	0	999975	25	25	0
29	1000000	0	999997	3	3	0
30	1000000	0	999999	1	1	0
31	1000000	0	999999	1	1	0
32	1000000	0	1000000	0	0	0
33	1000000	0	1000000	0	0	0
34	1000000	0	1000000	0	0	0

Left: Simulation results without (A0) and with (A1) inclusion of γ. **Right**. System size (S) increases complexity. Dotted line shows May's[1] criteria for stability.





Above. Eigenvalues before (a) & after (b) γ



Additional notes. I also found the following results:

- System feasibility (e.g., in an ecological community) was not affected by γ given a generalised Lotka-Volterra model and random species intrinsic growth rates (r_i).
- A targeted manipulation of γ using an evolutionary algorithm increased system stability.
- For ecological communities, γ increased stability in competitive and predator-prey networks, but not mutualist networks.
- Results were general across different C and σ, and for different distributions of γ values.

Code available on GitHub
https://github.com/bradduthie/RandomMatrixStability

References.
1. May, R. M. Will a large complex system be stable? *Nature* **238**, 413-414 (1972).
2. May, R. M. Qualitative stability in model ecosystems. *Ecology* **54**, 638–641 (1973).
3. Ahmadian, Y., Fumarola, F. & Miller, K. D. Properties of networks with partially structured and partially random connectivity. *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics* **91**, 012820 (2015).

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