Component response rate variation underlies the stability of complex systems (revision notes)

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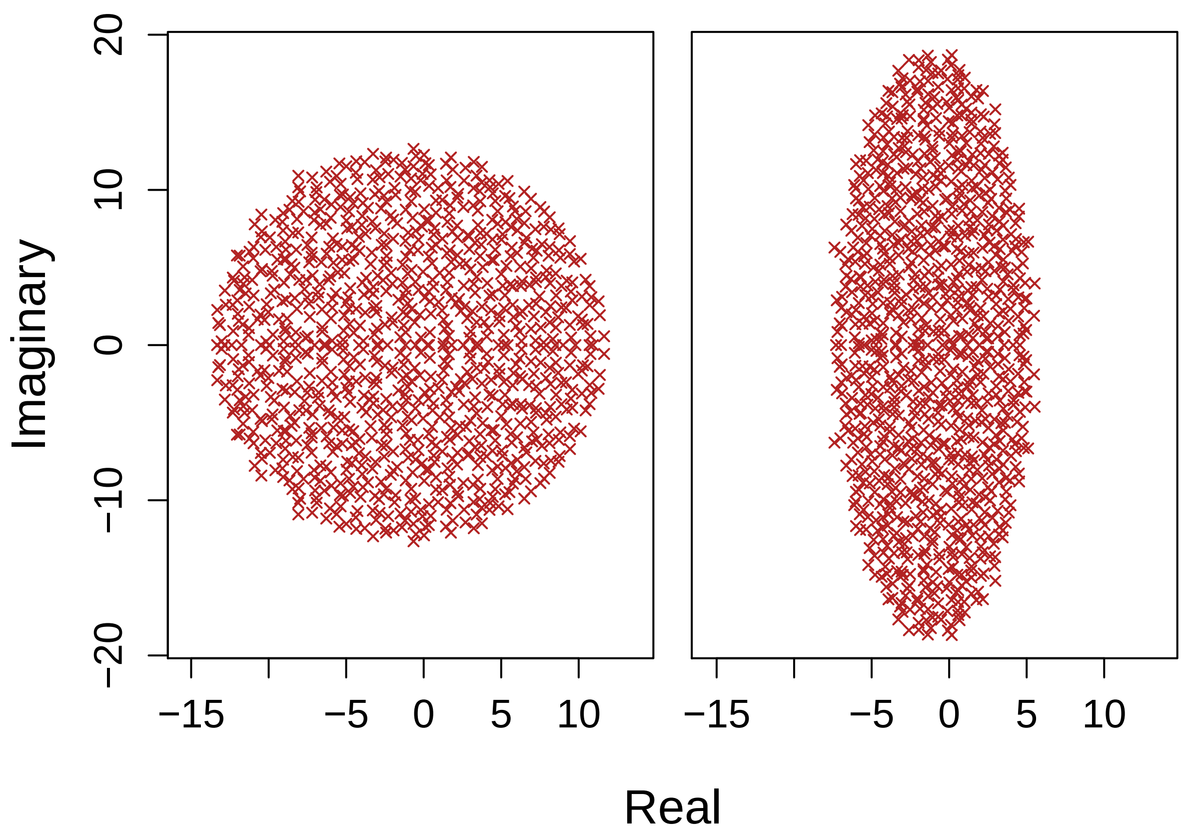
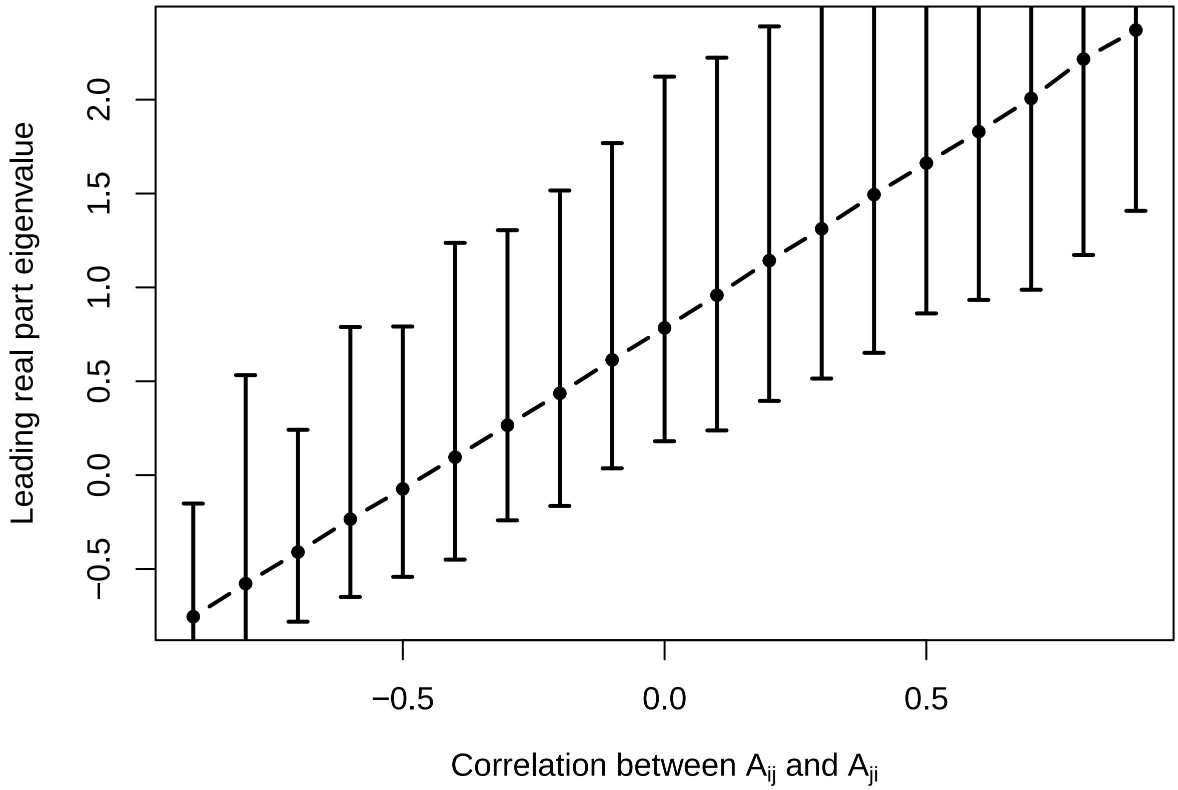
Role of correlated matrices in stabilisation

In complex systems represented by large random matrices, correlation between matrix elements A_{ij} and A_{ji} affects the distribution of eigenvalues and therefore local stability. As the correlation between matrix elements (ρ) decreases, the eigenvalue spectra changes such that more variation falls along the imaginary axis. The figure panels below compare a random matrix in which $\rho = 0$ (left) to one in which (ρ = -0.5). In both cases, complex systems include $S = 1000$ components, with diagonal elements of -1 and off-diagonal elements drawn from a normal distribution with a mean of 0 and $\sigma = 0.4$.



Because of this affect of ρ on the eigenvalue spectra, decreasing values of ρ will also decrease the rightmost eigenvalue of the matrix A. This makes it more likely that the complex system represented by A is locally stable, as stability occurs when all real parts of eigenvalues are negative. Note that this elongation along the imaginary axis is also characteristic of predator-prey communities (in which, by definition A_{ij} and A_{ji} have opposing signs). Also note that as ρ increases such that $\rho > 0$, the same elongation happens along the real axis, making random complex systems less likely to be stable.

A simple numerical analysis illustrates the linear relationship between ρ and the expected value of the real part of the leading eigenvalue $\max(\Re(\lambda))$. Below, I have run 1000 simulations across values of ρ from -0.9 to 0.9 for complex systems with $S = 25$ components.



Error bars show 95% bootstrapped confidence intervals.