Component response rate variation underlies the stability of highly complex finite systems

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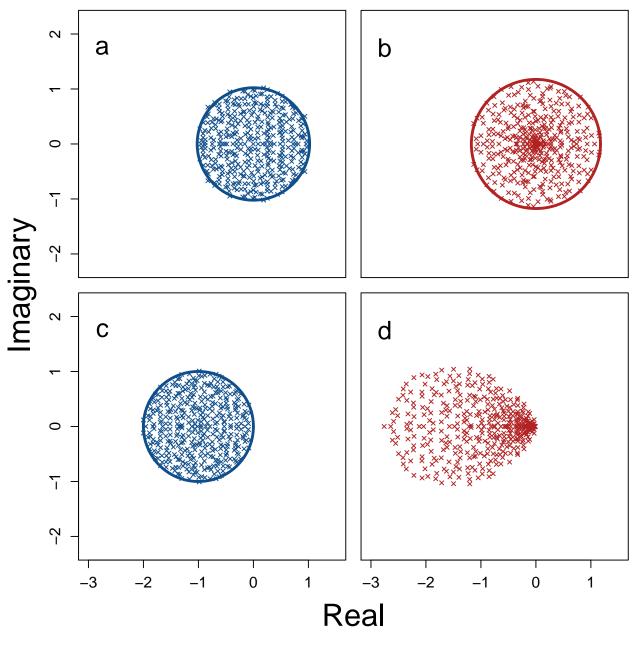
The stability of a complex system generally decreases with increasing system size and interconnectivity, a counterintuitive result of widespread importance across the physical, life, and social sciences. Despite recent interest in the relationship between system properties and stability, the effect of variation in response rate across system components remains unconsidered. Here I vary the component response rates (γ) of randomly generated complex systems. I use numerical simulations to show that when component response rates vary, the potential for system stability increases. These results are robust to common network structures, including small-world and scale-free networks, and cascade food webs. Variation in γ is especially important for stability in highly complex systems, in which the probability of stability would otherwise be negligible. At such extremes of simulated system complexity, the largest stable complex systems would be unstable if not for variation in γ . My results therefore reveal a previously unconsidered aspect of system stability that is likely to be pervasive across all realistic complex systems.

Introduction

In 1972, May¹ first demonstrated that randomly assembled systems of sufficient complexity are almost inevitably unstable given infinitesimally small perturbations. Complexity in this case is defined by the size of the system (i.e., the number of potentially interacting components; S), its connectance (i.e., the probability that one component will interact with another; C), and the variance of interaction strengths $(\sigma^2)^2$. May's finding that the probability of local stability falls to near zero given a sufficiently high threshold of $\sigma\sqrt{SC}$ is broadly relevant for understanding the dynamics and persistence of systems such as ecological neurological^{7,8}, biochemical^{9,10}, and socio-economic^{11–14} networks. As such, identifying general principles that affect stability in complex systems is of wide-ranging importance.

Randomly assembled complex systems can be represented as large square matrices (\mathbf{M}) with S components (e.g., networks of species² or banks¹²). One element of such a matrix, M_{ij} , defines how component j affects component i in the system at a point of equilibrium². Off-diagonal elements ($i \neq j$) therefore define interactions between components, while diagonal elements (i = j) define component self-regulation (e.g., carrying capacity in ecological communities). Traditionally, off-diagonal elements are assigned non-zero values with a probability C, which are sampled from a distribution with variance σ^2 ; diagonal elements are set to $-1^{1,2,5}$. Local system stability is assessed using eigenanalysis on \mathbf{M} , with the system being stable if the real parts of all eigenvalues (λ), and therefore the leading eigenvalue (λ_{max}), are negative ($\Re(\lambda_{max}) < 0$)^{1,2}. In a large system (high S), eigenvalues are distributed uniformly¹⁵ within a circle centred at $\Re = -d$ (-d is the mean value of diagonal elements) and $\Im = 0$, with a radius of $\sigma\sqrt{SC^{1,2,5}}$ (Fig. 1a). Local stability of randomly assembled systems therefore becomes increasingly unlikely as S, C, and σ increase.

Figure 1: Eigenvalue distributions of random complex systems. Each panel shows the real (x-axis) and imaginary (y-axis) parts of S=400 eigenvalues from random $S\times S$ matrices. (a) A system represented by a matrix ${\bf A}$, in which all elements are sampled from a normal distribution with $\mu=0$ and $\sigma_A=1/\sqrt{S}$. Points are uniformly distributed within the blue circle centred at the origin with a radius of $\sigma_A\sqrt{S}=1$. (b) The same system as in a after including variation in the response rates of S components, represented by the diagonal matrix γ , such that ${\bf M}=\gamma{\bf A}$. Elements of γ are randomly sampled from a uniform distribution from min = 0 to max = 2. Eigenvalues of ${\bf M}$ are then distributed non-uniformly within the red circle centred at the origin with a radius of $\sqrt{\sigma_A^2(1+\sigma_\gamma^2)S}\approx 1.15$. (c) A different random system ${\bf A}$ constructed from the same parameters as in ${\bf a}$, except with diagonal element values of -1. (d) The same system ${\bf c}$ after including variation in component response rates, sampled from ${\cal U}(0,2)$ as in ${\bf b}$.



May's^{1,2} stability criterion $\sigma\sqrt{SC} < d$ assumes that the expected response rates (γ) of individual components to perturbations of the system are identical, but this is highly unlikely in any complex system. In ecological communities, for example, the rate at which population density changes following perturbation will depend on the generation time of organisms, which might vary by orders of magnitude among species. Species with short generation times will respond quickly (high γ) to perturbations relative to species with long generation times (low γ). Similarly, the speed at which individual banks respond to perturbations in financial networks, or individuals or institutions respond to perturbations in complex social networks, is likely to vary. The effect of such variance on stability has not been investigated in complex systems theory. Intuitively, variation in γ (σ_{γ}^2) might be expected to decrease system stability by introducing a new source of variation into the system and thereby increasing σ . Here I show that, despite higher σ , realistic complex systems (in which S is high but finite) are actually more likely to be stable if their individual component response rates vary. My results are robust across commonly observed network structures, including random¹, small-world¹⁶, scale-free¹⁷, and cascade food web^{18,19} networks.

Results

Component response rates of random complex systems. Complex systems (**M**) are built from two matrices, one modelling component interactions (**A**), and second modelling component response rates (γ). Both **A** and γ are square $S \times S$ matrices. Rows in **A** define how a given component i is affected by each component j in the system, including itself (where i = j). Off-diagonal elements of **A** are independent and identically distributed (i.i.d), and diagonal elements are set to $A_{ii} = -1$ as in May¹. Diagonal elements of γ are positive, and off-diagonal elements are set to zero (i.e., γ is a diagonal matrix with positive support). The distribution of $diag(\gamma)$ over S components thereby models the distribution of component response rates. The dynamics of the entire system **M** can be defined as follows²⁰,

$$\mathbf{M} = \gamma \mathbf{A}.\tag{1}$$

Equation 1 thereby serves as a null model to investigate how variation in component response rate (σ_{γ}^2) affects complex systems. In the absence of such variation $(\sigma_{\gamma}^2 = 0)$, γ is set to the identity matrix (diagonal elements all equal 1), and $\mathbf{M} = \mathbf{A}$. Under these conditions, eigenvalues of \mathbf{M} are distributed uniformly in a circle centred at (-1,0) with a radius of $\sigma\sqrt{SC}$ (Fig. 1a).

Effect of σ_{γ}^2 on M (co)variation. The value of $\Re(\lambda_{max})$, and therefore system stability, can be estimated from five properties of \mathbf{M}^{21} . These properties include (1) system size (S), (2) mean self-regulation of components (d), (3) mean interaction strength between components (μ), (4) the variance of between component interaction strengths (hereafter σ_M^2 , to distinguish from σ_A^2 and σ_{γ}^2), and (5) the correlation of interaction strengths between components, M_{ij} and M_{ji} (ρ)²². Positive σ_{γ}^2 does not change S, nor does it necessarily change E[d] or $E[\mu]$. What σ_{γ}^2 does change is the total variation in component interaction strengths (σ_M^2), and ρ . Introducing variation in γ increases the total variation in the system. Variation in the off-diagonal elements of \mathbf{M} is described by the joint variation of two random variables,

$$\sigma_M^2 = \sigma_A^2 \sigma_\gamma^2 + \sigma_A^2 E[\gamma_i]^2 + \sigma_\gamma^2 E[A_{ij}]^2. \tag{2}$$

Given $E[\gamma_i] = 1$ and $E[A_{ij}] = 0$, Eq. 2 can be simplified,

$$\sigma_M^2 = \sigma_A^2 (1 + \sigma_\gamma^2).$$

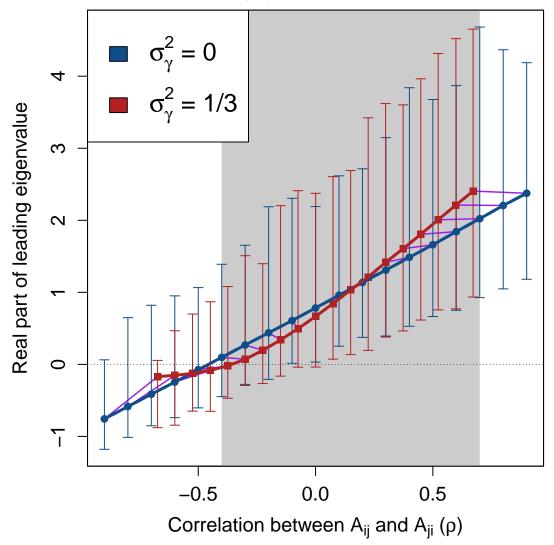
The increase in σ_M^2 caused by σ_γ^2 can be visualised from the eigenvalue spectra of **A** versus $\mathbf{M} = \gamma \mathbf{A}$ (Fig. 1). Given d=0 and C=1, the distribution of eigenvalues of **A** and **M** lie within a circle of a radius $\sigma_A \sqrt{S}$ and $\sigma_M \sqrt{S}$, respectively (Fig. 1a vs. 1b). If $d \neq 0$, positive σ_γ^2 changes the distribution of eigenvalues 23–25, potentially affecting stability (Fig. 1c vs. 1d).

Given $\sigma_{\gamma}^2 = 0$, $\Re(\lambda_{max})$ increases linearly with ρ such that 26 ,

$$\Re(\lambda_{max}) \approx \sigma_M \sqrt{SC} (1+\rho)$$
.

If $\rho < 0$, such as when **M** models a predator-prey system in which M_{ij} and M_{ji} have opposing signs, stability increases². If diagonal elements of γ vary independently, the magnitude of ρ is decreased because σ_{γ}^2 increases the variance of M_{ij} without affecting the expected covariance between M_{ij} and M_{ji} (Figure 2).

Figure 2: Complex system correlation versus stability with and without variation in component response rates. Each point represents 10000 replicate numerical simulations of a random complex system $\mathbf{M} = \gamma \mathbf{A}$ with a fixed correlation between off-diagonal elements A_{ij} and A_{ji} (ρ , x-axis). Where real parts of eigenvalues of \mathbf{M} are negative (y-axis), \mathbf{M} is stable (black dotted line). Blue circles show systems in the absence of variation in component response rates ($\sigma_{\gamma}^2 = 0$). Red squares show systems in which $\sigma_{\gamma}^2 = 1/3$. Arrows show the range of real parts of leading eigenvalues observed. Because γ decreases the magnitude of ρ , purple lines are included to link replicate simulations before (blue circles) and after (red squares) including γ . The range of ρ values in which γ decreases the mean real part of the leading eigenvalue is indicated with grey shading. In all simulations, system size and connectence were set to S = 25 and C = 1, respectively. Off-diagonal elements of \mathbf{A} were randomly sampled from $A_{ij} \sim \mathcal{N}(0, 0.4^2)$, and diagonal elements were set to -1. Elements of γ were sampled, $\gamma \sim \mathcal{U}(0, 2)$.



Numerical simulations of random systems with and without σ_{γ}^2 . I used numerical simulations and eigenanalysis to test how variation in γ affects stability in random matrices with known properties, comparing the stability of \mathbf{A} versus $\mathbf{M} = \gamma \mathbf{A}$. Values of γ were sampled from a uniform distribution where $\gamma \sim \mathcal{U}(0,2)$ and $\sigma_{\gamma}^2 = 1/3$ (see Supplementary Information for other γ distributions, which gave similar results). In all simulations, diagonal elements were standardised to ensure that -d between individual \mathbf{A} and \mathbf{M} pairs were identical (also note that $E[\gamma_i] = 1$). First I focus on the effect of γ across values of ρ , then for increasing system sizes (S) in random and structured networks. By increasing S, the objective is to determine the effect of γ as system complexity increases toward the boundary at which stability is realistic for a finite system.

Simulation of random M across ρ . Numerical simulations revealed that σ_{γ}^2 results in a nonlinear relationship between ρ and $\Re(\lambda_{max})$, which can sometimes increase the stability of the system. Figure 2 shows a comparison of $\Re(\lambda_{max})$ across ρ values for \mathbf{A} ($\sigma_{\gamma}^2 = 0$) versus \mathbf{M} ($\sigma_{\gamma}^2 = 1/3$) given S = 25, C = 1, and $\sigma_A = 0.4$. For $-0.4 \le \rho \le 0.7$ (shaded region of Fig. 2), expected $\Re(\lambda_{max})$ was lower in \mathbf{M} than \mathbf{A} . For $\rho \ge -0.1$, the lower bound of the range of $\Re(\lambda_{max})$ values also decreased given σ_{γ}^2 , resulting in negative $\Re(\lambda_{max})$ in \mathbf{M} for $\rho = -0.1$ and $\rho = 0$. Hence, across a wide range of system correlations, variation in the response rate of system components had a stabilising effect.

The stabilising effect of σ_{γ}^2 across ρ increased with increasing S. Figure 3 shows numerical simulations of \mathbf{M} across increasing S given C=1 and $\sigma_A=0.2$ (σ_A has been lowered here to better illustrate the effect of S; note that now given S=25, $1=\sigma_A\sqrt{SC}$). For relatively small systems ($S\leq25$), σ_{γ}^2 never decreased the expected $\Re(\lambda_{max})$. But as S increased, the curvilinear relationship between ρ and $\Re(\lambda_{max})$ decreased expected $\Re(\lambda_{max})$ for \mathbf{M} given low magnitudes of ρ . In turn, as S increased, and systems became more complex, σ_{γ}^2 increased the proportion of numerical simulations that were observed to be stable (see below).

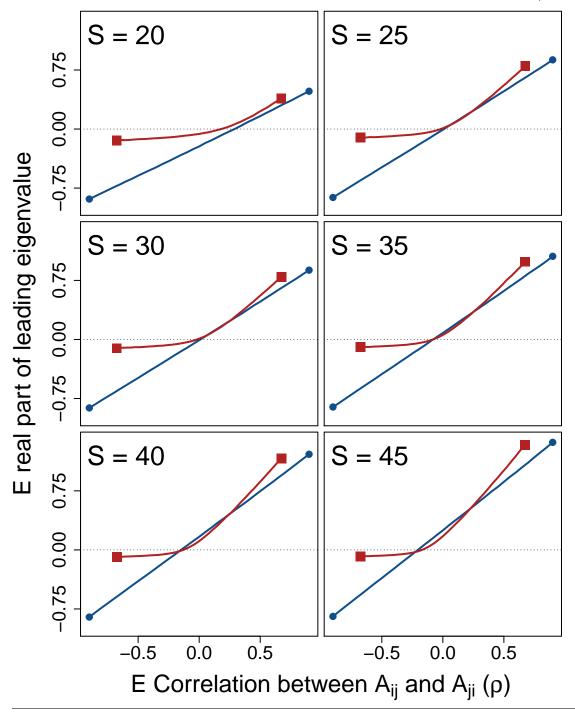
Simulation of random M across S. To investigate the effect of σ_{γ}^2 on stability across systems of increasing complexity, I simulated random $\mathbf{M} = \gamma \mathbf{A}$ matrices at $\sigma_A = 0.4$ and C = 1 across $S = \{2, 3, ..., 49, 50\}$. One million M were simulated for each S, and the stability of A vesus M was assessed given $\gamma \sim \mathcal{U}(0, 2)$ ($\sigma_{\gamma}^2 = 1/3$). For all S > 10, I found that the number of stable random systems was higher in M than A (Fig. 4; see Supplementary Information for full table of results), and that the difference between the probabilities of observing a stable system increased with an increase in S. In other words, the potential for σ_{γ}^2 to affect stability increased with increasing system complexity and was most relevant for systems on the cusp of being too complex to be realistically stable. For the highest values of S, nearly all systems that were stable given varying γ would not have been stable given $\gamma = 1$.

I also simulated 100000 **M** for three types of random networks that are typically interpreted as modelling three types of interspecific ecological interactions^{2,27}. These interaction types are competitive, mutualist, and predator-prey, as modelled by off-diagonal elements that are constrained to be negative, positive, or paired such that if $A_{ij} > 0$ then $A_{ji} < 0$, respectively² (but are otherwise identical to the purely random **A**). As S increased, a higher number of stable **M** relative to **A** was observed for competitor and predator-prey, but not mutualist, systems. A higher number of stable systems was observed whenever S > 12 and S > 40 for competitive and predator-prey systems, respectively (note that $\rho < 0$ for predator-prey systems, making stability more likely overall). The stability of mutualist systems was never affected by σ_{γ}^2 .

The effect of σ_{γ}^2 on stability did not change qualitatively across values of C, σ_A , or for different distributions of γ (see Supporting Information).

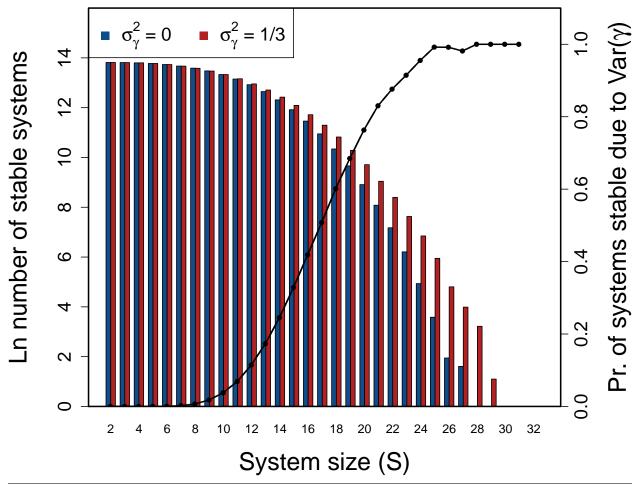
Simulation of structured M across S. To investigate how σ_{γ}^2 affects the stability of commonly observed network structures, I simulated one million $\mathbf{M} = \gamma \mathbf{A}$ for small-world¹⁶, scale-free¹⁷, and cascade food web^{18,19} networks. In all of these networks, rules determining the presence or absence of an interaction between components i and j constrain the overall structure of the network. In small-world networks, interactions between components are constrained so that the expected degree of separation between any two components increases in proportion to $\log(S)^{16}$. In scale-free networks, the distribution of the number of components with which a focal component interacts follows a power law; a few components have many interactions while most components have few interactions¹⁷. In cascade food webs, species are ranked and interactions are constrained such that a species i can only feed on j if the rank of i > j.

Figure 3: System correlation versus stability across different system sizes. In each panel, 10000 random complex systems $\mathbf{M} = \gamma \mathbf{A}$ are simulated for each correlation $\rho = \{-0.90, -0.85, ..., 0.85, 0.90\}$ between off-diagonal elements A_{ij} and A_{ji} . Lines show the expected real part of the leading eigenvalues of \mathbf{M} (red squares; $\sigma_{\gamma}^2 = 1/3$) versus \mathbf{A} (blue circles; $\sigma_{\gamma}^2 = 0$) across ρ , where negative values (below the dotted black line) indicate system stability. Differences between lines thereby show the effect of component response rate variation (γ) on system stability across system correlations and sizes (S). For all simulations, system connectance was C = 1. Off-diagonal elements of \mathbf{A} were randomly sampled from $A_{ij} \sim \mathcal{N}(0, 0.2^2)$, and diagonal elements were set to -1. Elements of γ were sampled such that $\gamma \sim \mathcal{U}(0, 2)$, so $\sigma_{\gamma}^2 = 1/3$.



Network structure did not strongly modulate the effect that σ_{γ}^2 had on stability. For comparable magnitudes of complexity, structured networks still had a higher number of stable \mathbf{M} than \mathbf{A} . For random networks, σ_{γ}^2 increased stability given S > 10 ($\sigma_A = 0.4$ and C = 1), and therefore complexity $\sigma_A \sqrt{SC} \gtrsim 1.26$. This threshold of complexity, above which more \mathbf{M} than \mathbf{A} were stable, was comparable for small-world networks, and slightly lower for scale-free networks (note that algorithms for generating small-world and scale-free networks necessarily led to varying C; see methods). Varying γ increased stability in cascade food webs for S > 27, and therefore at a relatively low complexity magnitudes compared to random predator-prey networks (S > 40). Overall, network structure did not greatly change the effect that σ_{γ}^2 had on increasing the upper bound of complexity within which stability might reasonably be observed.

Figure 4: Stability of large complex systems with and without variation in component response rate (γ) . The log number of systems that are stable across different system sizes $(S = \{2, 3, ..., 49, 50\})$ given C = 1, and the proportion of systems for which variation in γ is critical for system stability. For each S, 1 million complex systems are randomly generated. Stability of each complex system is tested given variation in γ by randomly sampling $\gamma \sim \mathcal{U}(0,2)$. Stability given $\sigma_{\gamma}^2 > 0$ is then compared to stability in an otherwise identical system in which $\gamma_i = E[\mathcal{U}(0,2)]$ for all components. Blue and red bars show the number of stable systems in the absence and presence of σ_{γ}^2 , respectively. The black line shows the proportion of systems that are stable when $\sigma_{\gamma}^2 > 0$, but would be unstable if $\sigma_{\gamma}^2 = 0$ (i.e., the conditional probability that \mathbf{A} is unstable given that \mathbf{M} is stable).



System feasibility given σ_{γ}^2 For complex systems in which individual system components represent the density of some tangible quantity, it is relevant to consider the feasibility of the system. Feasibility assumes that values of all components are positive at equilibrium^{6,28,29}. This is of particular interest for ecological communities because population density (n) cannot take negative values, meaning that ecological systems

need to be feasible for stability to be biologically realistic²⁸. While my results are intended to be general to all complex systems, and not restricted to species networks, I have also performed a feasibility analysis on all matrices tested for stability. I emphasise that γ is not interpreted as population density in this analysis, but instead as a fundamental property of species life history such as expected generation time. Feasibility was unaffected by σ_{γ}^2 and instead occurred with a fixed probability of $1/2^S$, consistent with a recent proof by Serván et al.³⁰ (see Supplementary Information). Hence, for pure interacting species networks, variation in component response rate (i.e., species generation time) does not affect stability at biologically realistic species densities.

Targeted manipulation of γ . To further investigate the potential of σ_{γ}^2 to be stabilising, I used a genetic algorithm. Genetic algorithms are heuristic tools that mimic evolution by natural selection, and are useful when the space of potential solutions (in this case, possible combinations of γ values leading to stability in a complex system) is too large to search exhaustively³¹. Generations of selection on γ value combinations to minimise $\Re(\lambda_{max})$ demonstrated the potential for σ_{γ}^2 to increase system stability. Across $S = \{2, 3, ..., 39, 40\}$, sets of γ values were found that resulted in stable systems with probabilities that were up to four orders of magnitude higher than when $\gamma = 1$ (see Supplementary Information), meaning that stability could often be achieved by manipulating S γ values rather than $S \times S$ M elements (i.e., by manipulating component response rates rather than interactions between components).

Discussion

I have shown that the stability of complex systems might often be contigent upon variation in the response rates of their individual components, meaning that factors such as rate of trait evolution (in biological networks), transaction speed (in economic networks), or communication speed (in social networks) need to be considered when investigating the stability of complex systems. Variation in component response rate is more likely to be critical for stability in systems that are especially complex, and it can ultimately increase the probability that system stability is observed above that predicted by May's¹ classically derived $\sigma\sqrt{SC}$ criterion. The logic outlined here is general, and potentially applies to any complex system in which individual system components can vary in their reaction rates to system perturbation.

It is important to recognise that variation in component response rate is not stabilising per se; that is, adding variation in component response rates to a particular system does not increase the probability that the system will be stable. Rather, highly complex systems that are observed to be stable are more likely to have varying component response rates, and for this variation to be critical to their stability (Fig. 4). This is caused by the shift to a non-uniform distribution of eigenvalues that occurs by introducing variation in γ (Fig. 1), which can sometimes cause all of the real components of the eigenvalues of the system matrix to become negative, but might also increase the real components of eigenvalues.

My focus here is distinct from Gibbs et al.²⁴, who applied the same mathematical framework to investigate how a diagonal matrix \mathbf{X} (equivalent to γ in my model) affects the stability of a community matrix \mathbf{M} given an interaction matrix \mathbf{A} within a generalised Lotka-Volterra model, where $\mathbf{M} = \mathbf{X}\mathbf{A}$. Gibbs et al.²⁴ analytically demonstrated that the effect of \mathbf{X} on system stability decreases exponentially as system size becomes arbitrarily large $(S \to \infty)$ for a given magnitude of complexity $\sigma \sqrt{SC}$. My numerical results do not contradict this prediction because I did not scale $\sigma = 1/\sqrt{S}$, but instead fixed σ and increased S to thereby increase total system complexity (see Supplemental Information for results simulated across σ and C). Overall, I show that component response rate variation increases the upper bound of complexity at which stability can be realistically observed, meaning that highly complex systems are more likely than not to vary in their component response rates, and for this variation to be critical for system stability.

Interestingly, while complex systems were more likely to be stable given variation in component response rate, they were not more likely to be feasible, meaning that stability was not increased when component values were also restricted to being positive at equilibrium. Feasibility is important to consider, particularly for the study of ecological networks of species^{6,25,28,30} because population densities cannot realistically be negative. My results therefore suggest that variation in the rate of population responses to perturbation (e.g., due to differences in generation time among species) is unlikely to be critical to the stability of purely multi-species

interaction networks (see also Supplementary Information). Nevertheless, ecological interactions do not exist in isolation in empirical systems²⁰, but instead interact with evolutionary, abiotic, or social-economic systems. The relevance of component response rate for complex system stability should therefore not be ignored in the broader context of ecological communities.

The potential importance of component response rate variation was most evident from the results of simulations in which the genetic algorithm was used in attempt to maximise the probability of system stability. The probability that some combination of component response rates could be found to stabilise the system was shown to be up to four orders of magnitude higher than the background probabilities of stability in the absence of any component response rate variation. Instead of manipulating the $S \times S$ interactions between system components, it might therefore be possible to manipulate only the S response rates of individual system components to achieve stability. Hence, managing the response rates of system components in a targeted way could potentially facilitate the stabilisation of complex systems through a reduction in dimensionality.

A general mathematical framework encompassing shifts in eigenvalue distributions caused by a diagonal matrix γ has been investigated²³ and recently applied to questions concerning species density and feasibility^{24,25}, but γ has not been interpreted as rates of response of individual system components to perturbation. My model focuses on component response rates for systems of a finite size, in which complexity is high but not yet high enough to make the probability of stability unrealistically low for actual empirical systems. For this upper range of system size, randomly assembled complex systems are more likely to be stable if their component response rates vary (e.g., 10 < S < 30 for parameter values in Fig. 4). Variation in component response rate might therefore be critical for maintaining stability in many highly complex empirical systems. These results are broadly applicable for understanding the stability of complex networks across the physical, life, and social sciences.

Methods

Component response rate (γ) variation. In a synthesis of eco-evolutionary feedbacks on community stability, Patel et al.²⁰ model a system that includes a vector of potentially changing species densities (\mathbf{n}) and a vector of potentially evolving traits (\mathbf{x}) . For any species i or trait j, change in species density (n_i) or trait value (x_j) with time (t) is a function of the vectors \mathbf{n} and \mathbf{x} ,

$$\frac{dn_i}{dt} = n_i f_i(\mathbf{n}, \mathbf{x}),$$

$$\frac{dx_j}{dt} = \epsilon g_j(\mathbf{n}, \mathbf{x}).$$

In the above, f_i and g_j are functions that define the effects of all species densities and trait values on the density of a species i and the value of trait j, respectively. Patel et al.²⁰ were interested in stability when the evolution of traits was relatively slow or fast in comparison with the change in species densities, and this is modulated in the above by the scalar ϵ . The value of ϵ thereby determines the timescale separation between ecology and evolution, with high ϵ modelling relatively fast evolution and low ϵ modelling relatively slow evolution²⁰.

I use the same principle that Patel et al.²⁰ use to modulate the relative rate of evolution to modulate rates of component responses for S components. Following May^{1,32}, the value of a component i at time t ($v_i(t)$) is affected by the value of j ($v_i(t)$) and j's marginal effect on i (a_{ij}), and by i's response rate (γ_i),

$$\frac{dv_i(t)}{dt} = \gamma_i \sum_{j=1}^{S} a_{ij} v_j(t).$$

In matrix notation 32 ,

$$\frac{d\mathbf{v}(t)}{dt} = \gamma \mathbf{A}\mathbf{v}(t).$$

In the above, γ is a diagonal matrix in which elements correspond to individual component response rates. Therefore, $\mathbf{M} = \gamma \mathbf{A}$ defines the change in values of system components and can be analysed using the techniques of May^{1,23,32}. In these analyses, row means of \mathbf{A} are expected to be identical, but variation around this expectation will naturally arise due to random sampling of \mathbf{A} off-diagonal elements and finite S. In simulations, the total variation in \mathbf{M} row means that is attributable to \mathbf{A} is small relative to that attributable to γ , especially at high S. Variation in γ specifically isolates the effects of differing component response rates, hence causing differences in expected \mathbf{M} row means.

Construction of random and structured networks. I used the R programming language for all numerical simulations and analyses³³. Purely random networks were generated by sampling off-diagonal elements from an i.i.d $A_{ij} \sim \mathcal{N}(0, 0.4^2)$ with a probability C (unsampled elements were set to zero). Diagonal elements A_{ii} were set to -1. Elements of γ were simulated i.i.d. from a distribution with positive support (typically $\gamma \sim \mathcal{U}(0,2)$). Random \mathbf{A} matrices with correlated elements A_{ij} and A_{ji} were built using Cholesky decomposition. Competitor networks in which off-diagonal elements $A_{ij} \leq 0$ were constructed by first building a random \mathbf{A} , then flipping the sign of elements where $A_{ij} < 0$. Predator-prey networks were constructed by first building a random \mathbf{A} , then flipping the sign of either A_{ij} or A_{ji} if $A_{ij} \times A_{ji} > 0$.

Small-world networks were constructed using the method of Watts and Strogatz¹⁶. First, a regular network¹⁶ was created such that components were arranged in a circle. Each component was initially set to interact with its k/2 closest neighbouring components on each side, where k was an even natural number (e.g., for k=2 the regular network forms a ring in which each component interacts with its two adjacent neighbours; see Supplemental Material for examples). Each interaction between a focal component and its neighbour was then removed and replaced with with a probability of β . In replacement, a new component was randomly selected to interact with the focal component; selection was done with equal probability among all but the focal component. The resulting small-world network was represented by a square $S \times S$ binary matrix **B** in which 1s represented interactions between components and 0s represented the absence of an interaction. A new random matrix **J** was then generated with elements J_{ij} sampled i.i.d. from $\mathcal{N}(0, 0.4^2)$. To build the interaction matrix **A**, I used element-wise multiplication $\mathbf{A} = \mathbf{J} \odot \mathbf{B}$, then set $diag(\mathbf{A}) = -1$. I set k = S/12 and simulated small-world networks across all combinations of $S = \{24, 48, 72, 96, 120, 144, 168\}$ and $\beta = \{0, 0.01, 0.1, 0.25, 1\}$.

Scale-free networks were constructed using the method of Albert and Barabási¹⁷. First, a saturated network (all components interact with each other) of size $m \leq S$ was created. New components were then added sequentially to the network; each newly added component was set to interact with m randomly selected existing components. When the system size reached S, the distribution of the number of total interactions that components had followed a power-law tail¹⁷. The resulting network was represented by an $S \times S$ binary matrix G, where 1s and 0s represent the presence and absence of an interaction, respectively. As with small-world networks, a random matrix J was generated, and $A = J \odot G$. Diagonal elements were set to -1. I simulated scale-free networks across all combinations of $S = \{24, 48, 72, 96, 120\}$ and $m = \{2, 3, ..., 11, 12\}$.

Cascade food webs were constructed following Solow and Beet¹⁸. First, a random matrix **A** was generated with off-diagonal elements sampled i.i.d so that $A_{ij} \sim \mathcal{N}(0, 0.4^2)$. Each component in the system was ranked from 1 to S. If component i had a higher rank than component j and $A_{ij} < 0$, then A_{ij} was multiplied by -1. If i had a lower rank than j and $A_{ji} < 0$, then A_{ji} was multiplied by -1. In practice, this resulted in a matrix **A** with negative and positive values in the lower and upper triangles, respectively. Diagonal elements of **A** were set to -1 and C = 1. I simulated cascade food webs for $S = \{2, 3, ..., 59, 60\}$.

System feasibility. Dougoud et al. 28 identify the following feasibility criteria for ecological systems characterised by S interacting species with varying densities in a generalised Lotka-Volterra model,

$$\mathbf{n}^* = -\left(\theta \mathbf{I} + (CS)^{-\delta} \mathbf{J}\right)^{-1} \mathbf{r}.$$

In the above, \mathbf{n}^* is the vector of species densities at equilibrium. Feasibility is satisfied if all elements in \mathbf{n}^* are positive. The matrix \mathbf{I} is the identity matrix, and the value θ is the strength of intraspecific competition (diagonal elements). Diagonal values are set to -1, so $\theta = -1$. The variable δ is a normalisation parameter that modulates the strength of interactions (σ) for \mathbf{J} . Implicitly, here $\delta = 0$ underlying strong interactions. Hence, $(CS)^{-\delta} = 1$, so in the above, a diagonal matrix of -1s ($\theta \mathbf{I}$) is added to \mathbf{J} , which has a diagonal of all zeros and an off-diagonal affecting species interactions (i.e., the expression $(CS)^{-\delta}$ relates to May's¹ stability criterion²⁸ by $\frac{\sigma}{(CS)^{-\delta}}\sqrt{SC} < 1$, and hence for my purposes $(CS)^{-\delta} = 1$). Given $\mathbf{A} = \theta \mathbf{I} + \mathbf{J}$, the above criteria is therefore reduced to the below (see also Serván et al.³⁰),

$$\mathbf{n}^* = -\mathbf{A}^{-1}\mathbf{r}.$$

To check the feasibility criteria for $\mathbf{M} = \gamma \mathbf{A}$, I therefore evaluated $-\mathbf{M}^{-1}\mathbf{r}$ (\mathbf{r} elements were sampled i.i.d. from $r \sim \mathcal{N}(0, 0.4^2)$). Feasibility is satisfied if all of the elements of the resulting vector are positive.

Genetic algorithm. Ideally, to investigate the potential of σ_{γ}^2 for increasing the proportion of stable complex systems, the search space of all possible $diag(\gamma)$ vectors would be evaluated for each unique $\mathbf{M} = \gamma \mathbf{A}$. This is technically impossible because γ_i can take any real value between 0-2, but even rounding γ_i to reasonable values would result in a search space too large to practically explore. Under these conditions, genetic algorithms are highly useful tools for finding practical solutions by mimicking the process of biological evolution³¹. In this case, the practical solution is finding vectors of $diag(\gamma)$ that decrease the most positive real eigenvalue of \mathbf{M} . The genetic algorithm used achieves this by initialising a large population of 1000 different potential $diag(\gamma)$ vectors and allowing this population to evolve through a process of mutation, crossover (swaping γ_i values between vectors), selection, and reproduction until either a $diag(\gamma)$ vector is found where all $\Re(\lambda) < 0$ or some "giving up" critiera is met.

For each $S = \{2, 3, ..., 39, 40\}$, the genetic algorithm was run for 100000 random $\mathbf{M} = \gamma \mathbf{A}$ ($\sigma_A = 0.4, C = 1$). The genetic algorithm was initialised with a population of 1000 different $diag(\gamma)$ vectors with elements sampled i.i.d from $\gamma \sim \mathcal{U}(0,2)$. Eigenanalysis was performed on the \mathbf{M} resulting from each γ , and the 20 $diag(\gamma)$ vectors resulting in \mathbf{M} with the lowest $\Re(\lambda_{max})$ each produced 50 clonal offspring with subsequent random mutation and crossover between the resulting new generation of 1000 $diag(\gamma)$ vectors. Mutation of each γ_i in a $diag(\gamma)$ vector occurred with a probability of 0.2, resulting in a mutation effect of size $\mathcal{N}(0,0.02^2)$ being added to generate the newly mutated γ_i (any γ_i values that mutated below zero were multiplied by -1, and any values that mutated above 2 were set to 2). Crossover occurred between two sets of 100 $diag(\gamma)$ vectors paired in each generation; vectors were randomly sampled with replacement among but not within sets. Vector pairs selected for crossover swapped all elements between and including two γ_i randomly selected with replacement (this allowed for reversal of vector element positions during crossover; e.g., $\{\gamma_4, \gamma_5, \gamma_6, \gamma_7\} \rightarrow \{\gamma_7, \gamma_6, \gamma_5, \gamma_4\}$). The genetic algorithm terminated if a stable \mathbf{M} was found, 20 generations occurred, or if the mean γ fitness increase between generations was less than 0.01 (where fitness was defined as $W_{\gamma} = -\Re(\lambda_{max})$ for \mathbf{M}).

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Supplementary Information: Full tables of stability results for simulations across different system size (S) values, ecological community types, connectance (C) values, interaction strengths (σ) , and γ distributions are provided as supplementary material. An additional table also shows results for how feasibility changes across S. All code and simulation outputs are publicly available as part of the RandomMatrixStability package on GitHub (https://github.com/bradduthie/RandomMatrixStability).

Additional Information: The author declares no competing interests. All work was carried out by A. Bradley Duthie, and all code and data are accessible on GitHub.

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Supplemental Information

This supplemental information supports the manuscript "Component response rate variation underlies the stability of complex systems" with additional analyses to support its conclusions. All text, code, and data underlying this manuscript are publicly available on GitHub as part of the RandomMatrixStability R package.

The RandomMatrixStability package includes all functions and tools for recreating the text, this supplemental information, and running all code; additional documentation is also provided for package functions. The RandomMatrixStability package is available on GitHub; to download it, the devtools library is needed.

```
install.packages("devtools");
library(devtools);
```

The code below installs the RandomMatrixStability package using devtools.

```
install_github("bradduthie/RandomMatrixStability");
```

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Stability across increasing S

Figure 4 of the main text reports the number of stable random complex systems found over 1 million iterations. The table below shows the results for all simulations of random \mathbf{M} matrices at $\sigma_A = 0.4$ and C = 1 given a range of $S = \{2, 3, ..., 49, 50\}$. In this table, the A refers to A matrices where $\gamma = 1$, while M refers to M matrices after σ_{γ}^2 is added and $\gamma \sim \mathcal{U}(0,2)$. Each row summarises data for a given S over 1 million randomly simulated \mathbf{M} . The column A_unstable shows the number of A matrices that are unstable, and the column A_stable shows the number of A matrices that are stable (these two columns sum to 1 million). Similarly, the column M_unstable shows the number of M matrices that are unstable and M_stable shows the number that are stable. The columns A_stablised and A_destabilised show how many M matrices were stabilised or destabilised, respectively, by σ_{γ}^2 .

| \overline{S} | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----------------|------------|----------|------------|----------|--------------|----------------|
| 2 | 293 | 999707 | 293 | 999707 | 0 | 0 |
| 3 | 3602 | 996398 | 3609 | 996391 | 0 | 7 |
| 4 | 14937 | 985063 | 15008 | 984992 | 0 | 71 |
| 5 | 39289 | 960711 | 39783 | 960217 | 36 | 530 |
| 6 | 78845 | 921155 | 80207 | 919793 | 389 | 1751 |
| 7 | 133764 | 866236 | 136904 | 863096 | 1679 | 4819 |
| 8 | 204112 | 795888 | 208241 | 791759 | 5391 | 9520 |
| 9 | 288041 | 711959 | 291775 | 708225 | 12619 | 16353 |
| 10 | 384024 | 615976 | 384931 | 615069 | 23153 | 24060 |
| 11 | 485975 | 514025 | 481019 | 518981 | 35681 | 30725 |
| 12 | 590453 | 409547 | 577439 | 422561 | 48302 | 35288 |
| 13 | 689643 | 310357 | 669440 | 330560 | 57194 | 36991 |
| 14 | 777496 | 222504 | 751433 | 248567 | 60959 | 34896 |
| 15 | 850159 | 149841 | 821613 | 178387 | 58567 | 30021 |
| 16 | 905057 | 94943 | 877481 | 122519 | 51255 | 23679 |
| 17 | 943192 | 56808 | 919536 | 80464 | 40854 | 17198 |
| 18 | 969018 | 30982 | 949944 | 50056 | 30102 | 11028 |
| 19 | 984301 | 15699 | 970703 | 29297 | 20065 | 6467 |
| 20 | 992601 | 7399 | 983507 | 16493 | 12587 | 3493 |
| 21 | 996765 | 3235 | 991532 | 8468 | 7030 | 1797 |
| 22 | 998693 | 1307 | 995567 | 4433 | 3884 | 758 |
| 23 | 999503 | 497 | 997941 | 2059 | 1883 | 321 |
| 24 | 999861 | 139 | 999059 | 941 | 899 | 97 |
| 25 | 999964 | 36 | 999617 | 383 | 380 | 33 |
| 26 | 999993 | 7 | 999878 | 122 | 121 | 6 |
| 27 | 999995 | 5 | 999946 | 54 | 53 | 4 |
| 28 | 1000000 | 0 | 999975 | 25 | 25 | 0 |
| 29 | 1000000 | 0 | 999997 | 3 | 3 | 0 |
| 30 | 1000000 | 0 | 999999 | 1 | 1 | 0 |
| 31 | 1000000 | 0 | 999999 | 1 | 1 | 0 |
| 32 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 33 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 34 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 35 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 36 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 37 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 38 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 39 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 40 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 41 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----|------------|----------|------------|----------|--------------|----------------|
| 42 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 43 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 44 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 45 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 46 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 47 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 48 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 49 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 50 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |

Overall, the ratio of stable **A** matrices to stable **M** matrices found is greater than 1 whenever S > 10 (compare column 3 to column 5), and this ratio increases with increasing S (column 1). Hence, more randomly created complex systems (**M**) are stable given variation in γ than when $\gamma = 1$. Note that feasibility results were omitted for the table above, but are reported below.

Stability of random ecological networks

While the foundational work of May¹ applies broadly to complex networks, much attention has been given specifically to ecological networks of interacting species. In these networks, the matrix \mathbf{A} is interpreted as a community matrix and each row and column is interpreted as a single species. The per capita effect that the density of any species i has on the population dynamics of species j is found in A_{ij} , meaning that \mathbf{A} holds the effects of pair-wise interactions between S species^{2,3}. While May's original work¹ considered only randomly assembled communities, recent work has specifically looked at more restricted ecological communities including competitive networks (all off-diagonal elements of \mathbf{A} are negative), mutualist networks (all off-diagonal elements of \mathbf{A} are positive), and predator-prey networks (for any pair of i and j, the effect of i on j is negative and j on i is positive, or vice versa)^{2,3}. In general, competitor and mutualist networks tend to be unstable, while predator-prey networks tend to be highly stabilising².

I investigated competitor, mutualist, and predator-prey networks following Allesina et al.². To create these networks, I first generated a random matrix \mathbf{A} , then changed the elements of \mathbf{A} accordingly. If \mathbf{A} was a competitive network, then the sign of any positive off-diagonal elements was reversed to be negative. If \mathbf{A} was a mutualist network, then the sign of any positive off-diagonal elements was reversed to be positive. And if \mathbf{A} was a predator-prey network, then all i and j pairs of elements were checked; any pairs of the same sign were changed so that one was negative and the other was positive.

The number of stable $\mathbf{M} = \gamma \mathbf{A}$ systems was calculated exactly as it was for random matrices for values of S from 2 to 50 (100 in the case of the relatively more stable predator-prey interactions), except that only 100000 random \mathbf{M} were generated instead of 1 million.

The following tables for restricted ecological communities can therefore be compared with the random \mathbf{M} results above (but note that counts from systems with comparable probabilities of stability will be an order of magnitude lower in the tables below due to the smaller number of \mathbf{M} matrices generated). As with the results above, in the tables below, \mathbf{A} refers to matrices \mathbf{A} when $\gamma=1$ and \mathbf{M} refers to matrices after σ_{γ}^2 is added. The column \mathbf{A} _unstable shows the number of \mathbf{A} matrices that are unstable, and the column \mathbf{A} _stable shows the number of \mathbf{A} matrices that are stable (these two columns sum to 100000). Similarly, the column \mathbf{M} _unstable shows the number of \mathbf{M} matrices that are unstable and \mathbf{M} _stable shows the number that are stable. The columns \mathbf{A} _stabilised and \mathbf{A} _destabilised show how many \mathbf{A} matrices were stabilised or destabilised, respectively, by σ_{γ}^2 .

Competition

Results for competitor interaction networks are shown below

| \overline{S} | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----------------|------------|----------|------------|----------|--------------|----------------|
| 2 | 48 | 99952 | 48 | 99952 | 0 | 0 |
| 3 | 229 | 99771 | 231 | 99769 | 0 | 2 |
| 4 | 701 | 99299 | 704 | 99296 | 0 | 3 |
| 5 | 1579 | 98421 | 1587 | 98413 | 0 | 8 |
| 6 | 3218 | 96782 | 3253 | 96747 | 6 | 41 |
| 7 | 5519 | 94481 | 5619 | 94381 | 23 | 123 |
| 8 | 9062 | 90938 | 9237 | 90763 | 77 | 252 |
| 9 | 13436 | 86564 | 13729 | 86271 | 230 | 523 |
| 10 | 18911 | 81089 | 19303 | 80697 | 505 | 897 |
| 11 | 25594 | 74406 | 25961 | 74039 | 1011 | 1378 |
| 12 | 33207 | 66793 | 33382 | 66618 | 1724 | 1899 |
| 13 | 41160 | 58840 | 41089 | 58911 | 2655 | 2584 |
| 14 | 50575 | 49425 | 49894 | 50106 | 3777 | 3096 |
| 15 | 59250 | 40750 | 57892 | 42108 | 4824 | 3466 |
| 16 | 67811 | 32189 | 65740 | 34260 | 5634 | 3563 |
| 17 | 75483 | 24517 | 73056 | 26944 | 5943 | 3516 |
| 18 | 82551 | 17449 | 79878 | 20122 | 5780 | 3107 |
| 19 | 88030 | 11970 | 85204 | 14796 | 5417 | 2591 |
| 20 | 92254 | 7746 | 89766 | 10234 | 4544 | 2056 |
| 21 | 95233 | 4767 | 93002 | 6998 | 3695 | 1464 |
| 22 | 97317 | 2683 | 95451 | 4549 | 2803 | 937 |
| 23 | 98508 | 1492 | 97122 | 2878 | 1991 | 605 |
| 24 | 99240 | 760 | 98407 | 1593 | 1216 | 383 |
| 25 | 99669 | 331 | 99082 | 918 | 739 | 152 |
| 26 | 99871 | 129 | 99490 | 510 | 452 | 71 |
| 27 | 99938 | 62 | 99732 | 268 | 240 | 34 |
| 28 | 99985 | 15 | 99888 | 112 | 108 | 11 |
| 29 | 99990 | 10 | 99951 | 49 | 46 | 7 |
| 30 | 100000 | 0 | 99981 | 19 | 19 | 0 |
| 31 | 100000 | 0 | 99993 | 7 | 7 | 0 |
| 32 | 100000 | 0 | 99996 | 4 | 4 | 0 |
| 33 | 100000 | 0 | 99998 | 2 | 2 | 0 |
| 34 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| | | | | | | |
| 50 | 100000 | 0 | 100000 | 0 | 0 | 0 |

 ${\bf Mutualism}$ Results for mutualist interaction networks are shown below

| S | A _unstable | A_stable | $M_{unstable}$ | M_stable | $A_stabilised$ | A_destabilised |
|----|---------------|------------|----------------|------------|----------------|----------------|
| 2 | 56 | 99944 | 56 | 99944 | 0 | 0 |
| 3 | 3301 | 96699 | 3301 | 96699 | 0 | 0 |
| 4 | 34446 | 65554 | 34446 | 65554 | 0 | 0 |
| 5 | 86520 | 13480 | 86520 | 13480 | 0 | 0 |
| 6 | 99683 | 317 | 99683 | 317 | 0 | 0 |
| 7 | 99998 | 2 | 99998 | 2 | 0 | 0 |
| 8 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| 9 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| 10 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| 11 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| 12 | 100000 | 0 | 100000 | 0 | 0 | 0 |

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----|------------|----------|------------|----------|--------------|----------------|
| 50 | 100000 | 0 | 100000 | 0 | 0 | 0 |

Predator-prey

Results for predator-prey interaction networks are shown below

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----|------------|----------|------------|----------|--------------|----------------|
| 2 | 0 | 100000 | 0 | 100000 | 0 | 0 |
| 3 | 0 | 100000 | 0 | 100000 | 0 | 0 |
| 4 | 0 | 100000 | 0 | 100000 | 0 | 0 |
| 5 | 1 | 99999 | 1 | 99999 | 0 | 0 |
| 6 | 4 | 99996 | 4 | 99996 | 0 | 0 |
| 7 | 2 | 99998 | 2 | 99998 | 0 | 0 |
| 8 | 5 | 99995 | 5 | 99995 | 0 | 0 |
| 9 | 20 | 99980 | 21 | 99979 | 0 | 1 |
| 10 | 20 | 99980 | 22 | 99978 | 0 | 2 |
| 11 | 38 | 99962 | 39 | 99961 | 0 | 1 |
| 12 | 64 | 99936 | 66 | 99934 | 0 | 2 |
| 13 | 87 | 99913 | 91 | 99909 | 0 | 4 |
| 14 | 157 | 99843 | 159 | 99841 | 0 | 2 |
| 15 | 215 | 99785 | 227 | 99773 | 0 | 12 |
| 16 | 293 | 99707 | 310 | 99690 | 0 | 17 |
| 17 | 383 | 99617 | 408 | 99592 | 0 | 25 |
| 18 | 443 | 99557 | 473 | 99527 | 3 | 33 |
| 19 | 642 | 99358 | 675 | 99325 | 4 | 37 |
| 20 | 836 | 99164 | 887 | 99113 | 7 | 58 |
| 21 | 1006 | 98994 | 1058 | 98942 | 10 | 62 |
| 22 | 1153 | 98847 | 1228 | 98772 | 20 | 95 |
| 23 | 1501 | 98499 | 1593 | 98407 | 30 | 122 |
| 24 | 1841 | 98159 | 1996 | 98004 | 40 | 195 |
| 25 | 2146 | 97854 | 2316 | 97684 | 58 | 228 |
| 26 | 2643 | 97357 | 2809 | 97191 | 119 | 285 |
| 27 | 3034 | 96966 | 3258 | 96742 | 158 | 382 |
| 28 | 3690 | 96310 | 3928 | 96072 | 201 | 439 |
| 29 | 4257 | 95743 | 4532 | 95468 | 290 | 565 |
| 30 | 4964 | 95036 | 5221 | 94779 | 424 | 681 |
| 31 | 5627 | 94373 | 5978 | 94022 | 452 | 803 |
| 32 | 6543 | 93457 | 6891 | 93109 | 666 | 1014 |
| 33 | 7425 | 92575 | 7777 | 92223 | 818 | 1170 |
| 34 | 8540 | 91460 | 8841 | 91159 | 1071 | 1372 |
| 35 | 9526 | 90474 | 9842 | 90158 | 1337 | 1653 |
| 36 | 10617 | 89383 | 10891 | 89109 | 1624 | 1898 |
| 37 | 12344 | 87656 | 12508 | 87492 | 2021 | 2185 |
| 38 | 13675 | 86325 | 13877 | 86123 | 2442 | 2644 |
| 39 | 15264 | 84736 | 15349 | 84651 | 2870 | 2955 |
| 40 | 17026 | 82974 | 17053 | 82947 | 3363 | 3390 |
| 41 | 18768 | 81232 | 18614 | 81386 | 3905 | 3751 |
| 42 | 20791 | 79209 | 20470 | 79530 | 4579 | 4258 |
| 43 | 23150 | 76850 | 22754 | 77246 | 5217 | 4821 |
| 44 | 25449 | 74551 | 24184 | 75816 | 6285 | 5020 |
| 45 | 27702 | 72298 | 26464 | 73536 | 6754 | 5516 |

| 46 30525 69475 28966 71034 7646 6087 47 32832 67168 31125 68875 8487 6780 48 36152 68348 33865 66135 9479 7192 49 38714 61286 36242 63758 10125 7653 50 41628 58372 38508 61492 11036 7916 51 44483 55517 41023 58977 11704 8244 52 48134 51866 44287 55713 12573 8726 53 51138 48862 46721 53279 13223 8806 54 54261 45739 49559 50441 13757 9055 55 57647 42353 52403 47597 14324 9080 56 60630 39370 55293 44707 14669 932 57 63647 36353 57877 42 | S | A | unstable | A_ | stable | M | unstable | M | stable | A | stabilised | A | destabilised |
|--|----|---|----------|----|--------|---|----------|---|--------|---|------------|---|--------------|
| 47 32832 67168 31125 68875 9479 7192 48 36152 63848 33865 66135 9479 7192 49 33714 61286 36242 63758 10125 7653 50 44628 58372 38508 61492 11036 7916 51 44483 55517 41023 58977 11704 8244 52 48134 51866 44287 55713 12573 8726 53 51138 48862 46721 53279 13223 8806 54 54261 45739 49559 50441 13757 9055 55 57647 42353 52403 47597 14324 9080 56 60630 39370 55293 44707 14669 9332 57 63647 33039 60439 39561 15450 8928 59 69968 30032 63708 | 46 | | 30525 | | 69475 | | 28966 | | 71034 | | 7646 | | 6087 |
| 48 36152 63848 33865 66135 9479 7192 49 38714 61286 36242 63758 10125 7653 50 41628 58372 38508 61492 11036 7916 51 44483 55517 41023 58977 11704 8244 52 48134 51866 44287 55713 12573 8266 53 51138 48862 46721 53279 13223 8806 54 54261 45739 49559 50441 13757 9055 55 57647 42353 52403 47597 14324 9080 56 60630 39370 55293 44707 14669 9332 57 63647 36353 57787 42213 15103 9243 58 66961 33039 60439 39561 15450 8986 60 72838 27162 66270 <td< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<> | | | | | | | | | | | | | |
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| 91 99982 18 99839 161 155 12 92 99988 12 99865 135 135 12 93 99994 6 99885 115 115 6 94 99993 7 99911 89 88 6 95 99998 2 99953 47 47 2 96 99999 1 99965 35 35 1 | | | 99976 | | | | 99675 | | | | | | |
| 92 99988 12 99865 135 135 12 93 99994 6 99885 115 115 6 94 99993 7 99911 89 88 6 95 99998 2 99953 47 47 2 96 99999 1 99965 35 35 1 | | | 99977 | | | | 99756 | | | | | | |
| 93 99994 6 99885 115 115 6 94 99993 7 99911 89 88 6 95 99998 2 99953 47 47 2 96 99999 1 99965 35 35 1 | | | | | | | | | | | | | |
| 94 99993 7 99911 89 88 6 95 99998 2 99953 47 47 2 96 99999 1 99965 35 35 1 | | | | | | | | | | | | | |
| 95 99998 2 99953 47 47 2 96 99999 1 99965 35 35 1 | | | | | | | | | | | | | |
| 96 99999 1 99965 35 35 | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| 97 99999 1 99979 21 21 1 | | | | | | | | | | | | | |
| | 97 | | 99999 | | 1 | | 99979 | | 21 | | 21 | | 1 |

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|-----|------------|----------|------------|----------|--------------|----------------|
| 98 | 100000 | 0 | 99973 | 27 | 27 | 0 |
| 99 | 100000 | 0 | 99984 | 16 | 16 | 0 |
| 100 | 100000 | 0 | 99989 | 11 | 11 | 0 |

Overall, as expected², predator-prey communities are relatively stable while mutualist communities are highly unstable. But interestingly, while σ_{γ}^2 stabilises predator-prey and competitor communities, it does not stabilise mutualist communities. This is unsurprising because purely mutualist communities are characterised by a very positive² leading $\Re(\lambda)$, and it is highly unlikely that σ_{γ}^2 alone will shift all real parts of eigenvalues to negative values.

Sensitivity of connectance (C) values

In the main text, for simplicity, I assumed connectance values of C=1, meaning that all off-diagonal elements of a matrix \mathbf{M} were potentially nonzero and sampled from a normal distribution $\mathcal{N}(0,\sigma_A^2)$ where $\sigma_A=0.4$. Here I present four tables showing the number of stable communities given $C=\{0.3,0.5,0.7,0.9\}$. In all cases, uniform variation in component response rate $(\gamma \sim \mathcal{U}(0,2))$ led to a higher number of stable communities than when γ did not vary $(\gamma=1)$. In contrast to the main text, 100000 rather than 1 million \mathbf{M} were simulated. As with the results on stability with increasing S shown above, in the tables below \mathbf{A} refers to \mathbf{A} matrices when $\gamma=1$, and \mathbf{M} refers to \mathbf{M} matrices after σ_{γ}^2 is added. The column \mathbf{A} _unstable shows the number of \mathbf{A} matrices that are unstable, and the column \mathbf{A} _stable shows the number of \mathbf{M} matrices that are unstable and \mathbf{M} _stable shows the number that are stable. The columns \mathbf{A} _stabilised and \mathbf{A} _destabilised show how many \mathbf{A} matrices were stabilised or destabilised, respectively, by σ_{γ}^2 .

Connectance C = 0.3

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----|------------|----------|------------|----------|--------------|----------------|
| 2 | 5 | 99995 | 5 | 99995 | 0 | 0 |
| 3 | 6 | 99994 | 6 | 99994 | 0 | 0 |
| 4 | 24 | 99976 | 24 | 99976 | 0 | 0 |
| 5 | 59 | 99941 | 59 | 99941 | 0 | 0 |
| 6 | 98 | 99902 | 98 | 99902 | 0 | 0 |
| 7 | 160 | 99840 | 161 | 99839 | 0 | 1 |
| 8 | 290 | 99710 | 293 | 99707 | 0 | 3 |
| 9 | 430 | 99570 | 434 | 99566 | 0 | 4 |
| 10 | 648 | 99352 | 653 | 99347 | 1 | 6 |
| 11 | 946 | 99054 | 957 | 99043 | 0 | 11 |
| 12 | 1392 | 98608 | 1415 | 98585 | 4 | 27 |
| 13 | 2032 | 97968 | 2065 | 97935 | 5 | 38 |
| 14 | 2627 | 97373 | 2688 | 97312 | 10 | 71 |
| 15 | 3588 | 96412 | 3647 | 96353 | 35 | 94 |
| 16 | 5019 | 94981 | 5124 | 94876 | 51 | 156 |
| 17 | 6512 | 93488 | 6673 | 93327 | 79 | 240 |
| 18 | 8444 | 91556 | 8600 | 91400 | 165 | 321 |
| 19 | 10416 | 89584 | 10667 | 89333 | 244 | 495 |
| 20 | 13254 | 86746 | 13477 | 86523 | 425 | 648 |
| 21 | 16248 | 83752 | 16481 | 83519 | 642 | 875 |
| 22 | 19497 | 80503 | 19719 | 80281 | 929 | 1151 |
| 23 | 23654 | 76346 | 23776 | 76224 | 1368 | 1490 |
| 24 | 28485 | 71515 | 28389 | 71611 | 1914 | 1818 |

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|-----------------|------------|----------|------------|------------|--------------|----------------|
| $\overline{25}$ | 32774 | 67226 | 32483 | 67517 | 2428 | 2137 |
| 26 | 38126 | 61874 | 37411 | 62589 | 3221 | 2506 |
| 27 | 43435 | 56565 | 42418 | 57582 | 3828 | 2811 |
| 28 | 49333 | 50667 | 47840 | 52160 | 4565 | 3072 |
| 29 | 55389 | 44611 | 53381 | 46619 | 5329 | 3321 |
| 30 | 60826 | 39174 | 58388 | 41612 | 5918 | 3480 |
| 31 | 66820 | 33180 | 64043 | 35957 | 6345 | 3568 |
| 32 | 72190 | 27810 | 69036 | 30964 | 6685 | 3531 |
| 33 | 77053 | 22947 | 73587 | 26413 | 6826 | 3360 |
| 34 | 81816 | 18184 | 78157 | 21843 | 6673 | 3014 |
| 35 | 85651 | 14349 | 82041 | 17959 | 6383 | 2773 |
| 36 | 88985 | 11015 | 85657 | 14343 | 5721 | 2393 |
| 37 | 92072 | 7928 | 88805 | 11195 | 5180 | 1913 |
| 38 | 94329 | 5671 | 91444 | 8556 | 4451 | 1566 |
| 39 | 95912 | 4088 | 93295 | 6705 | 3804 | 1187 |
| 40 | 97232 | 2768 | 95201 | 4799 | 2967 | 936 |
| 41 | 98179 | 1821 | 96506 | 3494 | 2356 | 683 |
| 42 | 98826 | 1174 | 97489 | 2511 | 1786 | 449 |
| 43 | 99275 | 725 | 98312 | 1688 | 1251 | 288 |
| 44 | 99583 | 417 | 98872 | 1128 | 903 | 192 |
| 45 | 99776 | 224 | 99339 | 661 | 576 | 139 |
| 46 | 99865 | 135 | 99518 | 482 | 413 | 66 |
| 47 | 99938 | 62 | 99744 | 256 | 226 | 32 |
| 48 | 99956 | 44 | 99824 | 176 | 151 | 19 |
| 49 | 99980 | 20 | 99914 | 86 | 85 | 19 |
| 50 | 99993 | 7 | 99950 | 50 | 46 | 3 |
| 51 | 99998 | 2 | 99971 | 29 | 28 | 1 |
| 52 | 99998 | 2 | 99986 | 14 | 14 | 2 |
| 53 | 99999 | 1 | 99992 | 8 | 7 | 0 |
| 54 | 100000 | 0 | 99997 | 3 | 3 | 0 |
| 55 | 100000 | 0 | 99999 | 1 | 1 | 0 |
| 56 | 100000 | 0 | 99998 | 2 | 2 | 0 |
| 57 | 100000 | 0 | 99999 | 1 | 1 | 0 |
| 58 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| | | | | | | |
| 100 | 100000 | 0 | 100000 | 0 | 0 | 0 |

$Connectance\ C=0.5$

| S | A _unstable | A_stable | $M_{unstable}$ | M_stable | $A_stabilised$ | $A_{destabilised}$ |
|----|---------------|------------|----------------|------------|----------------|--------------------|
| 2 | 7 | 99993 | 7 | 99993 | 0 | 0 |
| 3 | 32 | 99968 | 32 | 99968 | 0 | 0 |
| 4 | 122 | 99878 | 122 | 99878 | 0 | 0 |
| 5 | 320 | 99680 | 321 | 99679 | 0 | 1 |
| 6 | 667 | 99333 | 673 | 99327 | 0 | 6 |
| 7 | 1233 | 98767 | 1252 | 98748 | 0 | 19 |
| 8 | 2123 | 97877 | 2156 | 97844 | 3 | 36 |
| 9 | 3415 | 96585 | 3471 | 96529 | 16 | 72 |
| 10 | 5349 | 94651 | 5450 | 94550 | 30 | 131 |
| 11 | 7990 | 92010 | 8185 | 91815 | 81 | 276 |
| 12 | 11073 | 88927 | 11301 | 88699 | 219 | 447 |

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----|------------|----------|------------|----------|--------------|----------------|
| 13 | 14971 | 85029 | 15204 | 84796 | 445 | 678 |
| 14 | 19754 | 80246 | 19992 | 80008 | 764 | 1002 |
| 15 | 25020 | 74980 | 25239 | 74761 | 1185 | 1404 |
| 16 | 30860 | 69140 | 30938 | 69062 | 1902 | 1980 |
| 17 | 37844 | 62156 | 37562 | 62438 | 2758 | 2476 |
| 18 | 44909 | 55091 | 44251 | 55749 | 3595 | 2937 |
| 19 | 52322 | 47678 | 51011 | 48989 | 4573 | 3262 |
| 20 | 60150 | 39850 | 58295 | 41705 | 5382 | 3527 |
| 21 | 67147 | 32853 | 64895 | 35105 | 5925 | 3673 |
| 22 | 74177 | 25823 | 71358 | 28642 | 6310 | 3491 |
| 23 | 80297 | 19703 | 77034 | 22966 | 6507 | 3244 |
| 24 | 85372 | 14628 | 82039 | 17961 | 6209 | 2876 |
| 25 | 89719 | 10281 | 86539 | 13461 | 5562 | 2382 |
| 26 | 92947 | 7053 | 90141 | 9859 | 4707 | 1901 |
| 27 | 95436 | 4564 | 92950 | 7050 | 3844 | 1358 |
| 28 | 97196 | 2804 | 95171 | 4829 | 2999 | 974 |
| 29 | 98300 | 1700 | 96842 | 3158 | 2115 | 657 |
| 30 | 99103 | 897 | 98033 | 1967 | 1466 | 396 |
| 31 | 99502 | 498 | 98665 | 1335 | 1068 | 231 |
| 32 | 99745 | 255 | 99185 | 815 | 696 | 136 |
| 33 | 99881 | 119 | 99572 | 428 | 375 | 66 |
| 34 | 99955 | 45 | 99788 | 212 | 191 | 24 |
| 35 | 99979 | 21 | 99900 | 100 | 95 | 16 |
| 36 | 99995 | 5 | 99950 | 50 | 50 | 5 |
| 37 | 99997 | 3 | 99970 | 30 | 28 | 1 |
| 38 | 99998 | 2 | 99986 | 14 | 13 | 1 |
| 39 | 99999 | 1 | 99991 | 9 | 9 | 1 |
| 40 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| 41 | 100000 | 0 | 99999 | 1 | 1 | 0 |
| 42 | 100000 | 0 | 99999 | 1 | 1 | 0 |
| 43 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| | 100000 | | 100000 | | | |
| 50 | 100000 | 0 | 100000 | 0 | 0 | 0 |

$Connectance\ C=0.7$

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----|------------|----------|------------|----------|--------------|----------------|
| 2 | 7 | 99993 | 7 | 99993 | 0 | 0 |
| 3 | 106 | 99894 | 106 | 99894 | 0 | 0 |
| 4 | 395 | 99605 | 397 | 99603 | 0 | 2 |
| 5 | 1117 | 98883 | 1123 | 98877 | 0 | 6 |
| 6 | 2346 | 97654 | 2367 | 97633 | 6 | 27 |
| 7 | 4314 | 95686 | 4388 | 95612 | 16 | 90 |
| 8 | 7327 | 92673 | 7456 | 92544 | 61 | 190 |
| 9 | 11514 | 88486 | 11792 | 88208 | 150 | 428 |
| 10 | 16247 | 83753 | 16584 | 83416 | 415 | 752 |
| 11 | 22481 | 77519 | 22759 | 77241 | 884 | 1162 |
| 12 | 29459 | 70541 | 29729 | 70271 | 1548 | 1818 |
| 13 | 37631 | 62369 | 37567 | 62433 | 2419 | 2355 |
| 14 | 46317 | 53683 | 45696 | 54304 | 3548 | 2927 |
| 15 | 54945 | 45055 | 53695 | 46305 | 4671 | 3421 |

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----|------------|----------|------------|----------|--------------|----------------|
| 16 | 63683 | 36317 | 61643 | 38357 | 5567 | 3527 |
| 17 | 72004 | 27996 | 69375 | 30625 | 6124 | 3495 |
| 18 | 79220 | 20780 | 76158 | 23842 | 6413 | 3351 |
| 19 | 85286 | 14714 | 82283 | 17717 | 5982 | 2979 |
| 20 | 90240 | 9760 | 87181 | 12819 | 5398 | 2339 |
| 21 | 93676 | 6324 | 91077 | 8923 | 4468 | 1869 |
| 22 | 96203 | 3797 | 94045 | 5955 | 3425 | 1267 |
| 23 | 97866 | 2134 | 96161 | 3839 | 2496 | 791 |
| 24 | 98842 | 1158 | 97633 | 2367 | 1713 | 504 |
| 25 | 99433 | 567 | 98630 | 1370 | 1079 | 276 |
| 26 | 99760 | 240 | 99259 | 741 | 655 | 154 |
| 27 | 99895 | 105 | 99576 | 424 | 377 | 58 |
| 28 | 99950 | 50 | 99790 | 210 | 194 | 34 |
| 29 | 99981 | 19 | 99915 | 85 | 80 | 14 |
| 30 | 99994 | 6 | 99952 | 48 | 47 | 5 |
| 31 | 99998 | 2 | 99972 | 28 | 28 | 2 |
| 32 | 99999 | 1 | 99992 | 8 | 8 | 1 |
| 33 | 100000 | 0 | 99997 | 3 | 3 | 0 |
| 34 | 100000 | 0 | 99999 | 1 | 1 | 0 |
| 35 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| | | | | | | |
| 50 | 100000 | 0 | 100000 | 0 | 0 | 0 |

Connectance C = 0.9

| 2 3 | A_unstable | A_stable | $M_{unstable}$ | M_stable | A_stabilised | $A_{destabilised}$ |
|-----|------------|------------|----------------|------------|--------------|--------------------|
| | 14 | 99986 | 14 | 99986 | 0 | 0 |
| 4 | 240 | 99760 | 240 | 99760 | 0 | 0 |
| 4 | 1008 | 98992 | 1016 | 98984 | 0 | 8 |
| 5 | 2708 | 97292 | 2729 | 97271 | 2 | 23 |
| 6 | 5669 | 94331 | 5755 | 94245 | 13 | 99 |
| 7 | 9848 | 90152 | 10057 | 89943 | 91 | 300 |
| 8 | 15903 | 84097 | 16201 | 83799 | 336 | 634 |
| 9 | 22707 | 77293 | 23110 | 76890 | 765 | 1168 |
| 10 | 30796 | 69204 | 31122 | 68878 | 1526 | 1852 |
| 11 | 40224 | 59776 | 40082 | 59918 | 2649 | 2507 |
| 12 | 49934 | 50066 | 49288 | 50712 | 3773 | 3127 |
| 13 | 60138 | 39862 | 58803 | 41197 | 4984 | 3649 |
| 14 | 69100 | 30900 | 67110 | 32890 | 5755 | 3765 |
| 15 | 77607 | 22393 | 74884 | 25116 | 6273 | 3550 |
| 16 | 84663 | 15337 | 81780 | 18220 | 5975 | 3092 |
| 17 | 90075 | 9925 | 87290 | 12710 | 5209 | 2424 |
| 18 | 93944 | 6056 | 91419 | 8581 | 4271 | 1746 |
| 19 | 96650 | 3350 | 94530 | 5470 | 3287 | 1167 |
| 20 | 98160 | 1840 | 96698 | 3302 | 2191 | 729 |
| 21 | 99111 | 889 | 98133 | 1867 | 1389 | 411 |
| 22 | 99588 | 412 | 98905 | 1095 | 903 | 220 |
| 23 | 99837 | 163 | 99480 | 520 | 452 | 95 |
| 24 | 99932 | 68 | 99744 | 256 | 228 | 40 |
| 25 | 99976 | 24 | 99863 | 137 | 133 | 20 |
| 26 | 99995 | 5 | 99950 | 50 | 49 | 4 |

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----|------------|----------|------------|----------|--------------|----------------|
| 27 | 99996 | 4 | 99986 | 14 | 13 | 3 |
| 28 | 100000 | 0 | 99993 | 7 | 7 | 0 |
| 29 | 100000 | 0 | 99996 | 4 | 4 | 0 |
| 30 | 100000 | 0 | 99998 | 2 | 2 | 0 |
| 31 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| | | | | | | • • • |
| 50 | 100000 | 0 | 100000 | 0 | 0 | 0 |

Sensitivity of interaction strength (σ_A) values

Results below show stability results given varying interaction strengths (σ_A) for C=0.05 (note that system size S values are larger and increase by 10 with increasing rows). In the tables below (as above), A and M refers to matrices for $\gamma=1$ and σ_{γ}^2 , respectively.

Interaction strength $\sigma_A = 0.3$

| \overline{S} | A_ | _unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----------------|----|-----------|----------|------------|----------|--------------|----------------|
| 10 | | 0 | 100000 | 0 | 100000 | 0 | 0 |
| 20 | | 0 | 100000 | 0 | 100000 | 0 | 0 |
| 30 | | 0 | 100000 | 0 | 100000 | 0 | 0 |
| 40 | | 0 | 100000 | 0 | 100000 | 0 | 0 |
| 50 | | 0 | 100000 | 0 | 100000 | 0 | 0 |
| 60 | | 2 | 99998 | 2 | 99998 | 0 | 0 |
| 70 | | 4 | 99996 | 4 | 99996 | 0 | 0 |
| 80 | | 6 | 99994 | 6 | 99994 | 0 | 0 |
| 90 | | 5 | 99995 | 5 | 99995 | 0 | 0 |
| 100 | | 11 | 99989 | 11 | 99989 | 0 | 0 |
| 110 | | 12 | 99988 | 13 | 99987 | 0 | 1 |
| 120 | | 23 | 99977 | 23 | 99977 | 0 | 0 |
| 130 | | 40 | 99960 | 40 | 99960 | 0 | 0 |
| 140 | | 62 | 99938 | 65 | 99935 | 0 | 3 |
| 150 | | 162 | 99838 | 165 | 99835 | 0 | 3 |
| 160 | | 325 | 99675 | 329 | 99671 | 2 | 6 |
| 170 | | 829 | 99171 | 851 | 99149 | 6 | 28 |
| 180 | | 1817 | 98183 | 1860 | 98140 | 31 | 74 |
| 190 | | 3927 | 96073 | 3989 | 96011 | 143 | 205 |
| 200 | | 8084 | 91916 | 8048 | 91952 | 557 | 521 |
| 210 | | 15558 | 84442 | 15147 | 84853 | 1534 | 1123 |
| 220 | | 26848 | 73152 | 25342 | 74658 | 3625 | 2119 |
| 230 | | 43386 | 56614 | 39535 | 60465 | 6992 | 3141 |
| 240 | | 62734 | 37266 | 56684 | 43316 | 9815 | 3765 |
| 250 | | 80128 | 19872 | 73080 | 26920 | 10128 | 3080 |
| 260 | | 92206 | 7794 | 86619 | 13381 | 7490 | 1903 |
| 270 | | 97946 | 2054 | 94824 | 5176 | 3797 | 675 |
| 280 | | 99659 | 341 | 98534 | 1466 | 1265 | 140 |
| 290 | | 99962 | 38 | 99696 | 304 | 281 | 15 |
| 300 | | 99994 | 6 | 99964 | 36 | 34 | 4 |

Interaction strength $\sigma_A = 0.4$

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|-----|------------|----------|------------|----------|--------------|----------------|
| 10 | 3 | 99997 | 3 | 99997 | 0 | 0 |
| 20 | 15 | 99985 | 15 | 99985 | 0 | 0 |
| 30 | 48 | 99952 | 48 | 99952 | 0 | 0 |
| 40 | 85 | 99915 | 85 | 99915 | 0 | 0 |
| 50 | 163 | 99837 | 163 | 99837 | 0 | 0 |
| 60 | 280 | 99720 | 282 | 99718 | 0 | 2 |
| 70 | 561 | 99439 | 566 | 99434 | 3 | 8 |
| 80 | 1009 | 98991 | 1029 | 98971 | 6 | 26 |
| 90 | 2126 | 97874 | 2175 | 97825 | 31 | 80 |
| 100 | 4580 | 95420 | 4653 | 95347 | 142 | 215 |
| 110 | 9540 | 90460 | 9632 | 90368 | 465 | 557 |
| 120 | 19090 | 80910 | 18668 | 81332 | 1676 | 1254 |
| 130 | 35047 | 64953 | 33220 | 66780 | 4172 | 2345 |
| 140 | 56411 | 43589 | 52439 | 47561 | 7297 | 3325 |
| 150 | 78003 | 21997 | 72574 | 27426 | 8477 | 3048 |
| 160 | 92678 | 7322 | 88438 | 11562 | 5901 | 1661 |
| 170 | 98614 | 1386 | 96670 | 3330 | 2397 | 453 |
| 180 | 99839 | 161 | 99418 | 582 | 499 | 78 |
| 190 | 99990 | 10 | 99945 | 55 | 52 | 7 |
| 200 | 100000 | 0 | 99995 | 5 | 5 | 0 |
| 210 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| | | | | | | |
| 300 | 100000 | 0 | 100000 | 0 | 0 | 0 |

Interaction strength $\sigma_{\rm A}=0.5$

| \mathbf{S} | $A_{unstable}$ | A_stable | $M_{unstable}$ | $\mathbf{M}_\mathbf{stable}$ | $A_stabilised$ | $A_destabilised$ |
|--------------|----------------|------------|----------------|-------------------------------|----------------|-------------------|
| 10 | 36 | 99964 | 36 | 99964 | 0 | 0 |
| 20 | 195 | 99805 | 195 | 99805 | 0 | 0 |
| 30 | 519 | 99481 | 523 | 99477 | 0 | 4 |
| 40 | 1096 | 98904 | 1101 | 98899 | 2 | 7 |
| 50 | 2375 | 97625 | 2397 | 97603 | 9 | 31 |
| 60 | 4898 | 95102 | 4968 | 95032 | 83 | 153 |
| 70 | 10841 | 89159 | 10916 | 89084 | 432 | 507 |
| 80 | 22281 | 77719 | 21988 | 78012 | 1622 | 1329 |
| 90 | 42010 | 57990 | 39998 | 60002 | 4458 | 2446 |
| 100 | 67289 | 32711 | 63098 | 36902 | 7153 | 2962 |
| 110 | 88137 | 11863 | 84023 | 15977 | 6108 | 1994 |
| 120 | 97678 | 2322 | 95557 | 4443 | 2740 | 619 |
| 130 | 99795 | 205 | 99304 | 696 | 578 | 87 |
| 140 | 99989 | 11 | 99948 | 52 | 49 | 8 |
| 150 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| | | | | | | |
| 300 | 100000 | 0 | 100000 | 0 | 0 | 0 |

Interaction strength $\sigma_{\rm A}=0.6$

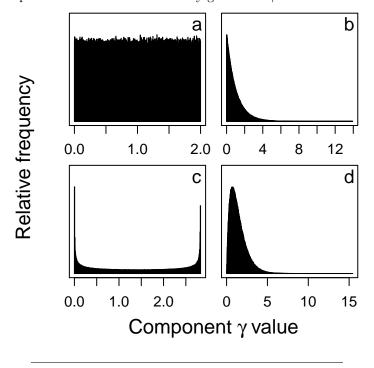
| \overline{S} | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----------------|------------|----------|------------|----------|--------------|----------------|
| 10 | 162 | 99838 | 162 | 99838 | 0 | 0 |
| 20 | 798 | 99202 | 799 | 99201 | 0 | 1 |

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|-----|------------|----------|------------|----------|--------------|----------------|
| 30 | 2273 | 97727 | 2289 | 97711 | 6 | 22 |
| 40 | 5259 | 94741 | 5298 | 94702 | 70 | 109 |
| 50 | 12084 | 87916 | 12054 | 87946 | 446 | 416 |
| 60 | 26072 | 73928 | 25511 | 74489 | 1810 | 1249 |
| 70 | 50121 | 49879 | 47747 | 52253 | 4748 | 2374 |
| 80 | 77806 | 22194 | 73810 | 26190 | 6421 | 2425 |
| 90 | 94862 | 5138 | 92069 | 7931 | 3842 | 1049 |
| 100 | 99527 | 473 | 98822 | 1178 | 870 | 165 |
| 110 | 99984 | 16 | 99912 | 88 | 80 | 8 |
| 120 | 100000 | 0 | 99998 | 2 | 2 | 0 |
| 130 | 100000 | 0 | 100000 | 0 | 0 | 0 |
| | | | | | | |
| 300 | 100000 | 0 | 100000 | 0 | 0 | 0 |

Sensitivity of distribution of γ

In the main text, I considered a uniform distribution of component response rates $\gamma \sim \mathcal{U}(0,2)$. The number of unstable and stable M matrices are reported in a table above across different values of S. Here I show complementary results for three different distributions including an exponential, beta, and gamma distribution of γ values. The shape of these distributions is shown in the figure below.

Distributions of component response rate (γ) values in complex systems. The stabilities of simulated complex systems with these γ distributions are compared to identical systems in which $\gamma=1$ across different system sizes (S; i.e., component numbers) given a unit γ standard deviation $(\sigma_{\gamma}=1)$ for b-d. Distributions are as follows: (a) uniform, (b) exponential, (c) beta $(\alpha=0.5)$ and $(\alpha=0.5)$, and (d) gamma $(\alpha=0.5)$ and $(\alpha=0.5)$. Each panel shows 1 million randomly generated $(\alpha=0.5)$ values.



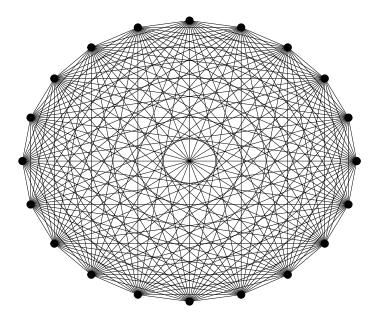
The stability of **A** versus **M** was investigated for each of the distributions of γ shown in panels b-d above. The table below shows the number of **A** versus **M** that were stable for the exponential (exp), beta, and gamma distributions.

| \overline{S} | exp_A | exp_M | beta_A | beta_M | gamma_A | gamma_M |
|----------------|-------|-------|--------|--------|---------|---------|
| 2 | 99965 | 99965 | 99974 | 99974 | 99977 | 99977 |
| 3 | 99636 | 99635 | 99650 | 99648 | 99628 | 99628 |
| 4 | 98576 | 98564 | 98482 | 98470 | 98508 | 98492 |
| 5 | 96053 | 95971 | 96156 | 96096 | 96068 | 96004 |
| 6 | 92036 | 91867 | 92104 | 91927 | 92233 | 92029 |
| 7 | 86667 | 86333 | 86456 | 86070 | 86604 | 86161 |
| 8 | 79670 | 79153 | 79392 | 78822 | 79393 | 78771 |
| 9 | 71389 | 70911 | 70998 | 70529 | 71070 | 70548 |
| 10 | 61674 | 61609 | 61794 | 61586 | 61265 | 61093 |
| 11 | 51150 | 51935 | 51352 | 51924 | 51313 | 51951 |
| 12 | 41209 | 42925 | 40954 | 42670 | 40708 | 42183 |
| 13 | 30827 | 33462 | 30969 | 33770 | 31046 | 33522 |
| 14 | 22203 | 25767 | 22208 | 25629 | 22342 | 25435 |
| 15 | 15003 | 18877 | 15206 | 18913 | 15025 | 18464 |
| 16 | 9613 | 13372 | 9504 | 13357 | 9418 | 12737 |
| 17 | 5579 | 8967 | 5570 | 8976 | 5719 | 8487 |
| 18 | 3104 | 5833 | 3048 | 5853 | 3060 | 5447 |
| 19 | 1516 | 3578 | 1553 | 3633 | 1600 | 3185 |
| 20 | 717 | 2067 | 799 | 2179 | 769 | 1862 |
| 21 | 312 | 1196 | 310 | 1200 | 331 | 1039 |
| 22 | 129 | 643 | 128 | 654 | 135 | 510 |
| 23 | 48 | 321 | 48 | 359 | 57 | 242 |
| 24 | 11 | 161 | 19 | 159 | 20 | 120 |
| 25 | 1 | 59 | 5 | 81 | 7 | 45 |
| 26 | 0 | 30 | 0 | 48 | 0 | 22 |
| 27 | 0 | 10 | 0 | 16 | 0 | 6 |
| 28 | 1 | 3 | 2 | 2 | 0 | 3 |
| 29 | 0 | 2 | 0 | 0 | 0 | 0 |
| 30 | 0 | 0 | 0 | 1 | 0 | 0 |
| 31 | 0 | 0 | 0 | 1 | 0 | 0 |
| 32 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | |
| 50 | 0 | 0 | 0 | 0 | 0 | 0 |

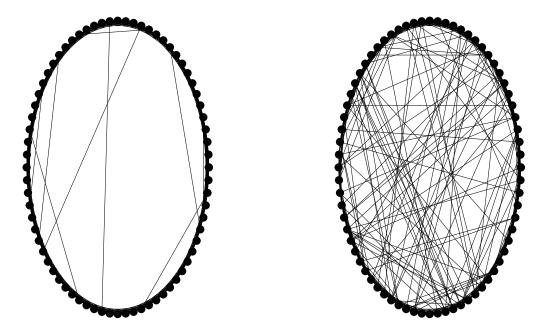
In comparison to the uniform distribution (a), proportionally fewer random systems are found with the exponential distribution (b), while more are found with the beta (c) and gamma (d) distributions.

Stability of structured networks

I tested the stability of one million random, small-world, scale-free, and cascade food web networks for different network parameters. Each of these networks is structured differently. In the main text, the random networks and cascade food webs that I built were saturated (C=1), meaning that every component was connected to, and interacted with, every other component (see immediately below).



Small-world networks, in contrast, are not saturated. They are instead defined by components that interact mostly with other closely neighbouring components, but have a proportion of interactions (β) that are instead between non-neighbours⁴. Two small-world networks are shown below.



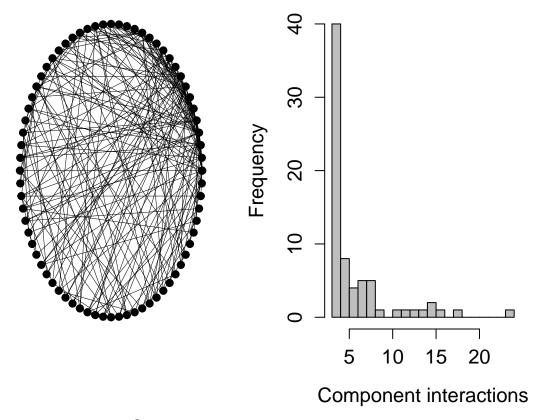
The small-world network on the left shows a system in which $\beta=0.01$, while the small-world network on the right shows one in which $\beta=0.1$. At the extremes of $\beta=0$ and $\beta=1$, networks are regular and random, respectively. The table below shows how σ_{γ}^2 affects stability in small world networks across different values of S and β .

| C | $complex_M$ | $complex_A$ | M_{stable} | $M_{unstable}$ | A_stable | $A_{unstable}$ | \mathbf{S} | beta |
|-----------|--------------|--------------|--------------|----------------|------------|----------------|--------------|------|
| 0.1304348 | 0.6582632 | 0.5748066 | 982554 | 17446 | 982612 | 17388 | 24 | 0.00 |
| 0.1063830 | 0.9294192 | 0.8073918 | 739421 | 260579 | 741976 | 258024 | 48 | 0.00 |
| 0.0985915 | 1.1364805 | 0.9860840 | 277361 | 722639 | 284964 | 715036 | 72 | 0.00 |
| 0.0947368 | 1.3110263 | 1.1369395 | 37212 | 962788 | 38566 | 961434 | 96 | 0.00 |

| beta | S | A_unstable | A_stable | M_unstable | M_stable | complex_A | complex_M | C |
|------|-----|------------|----------|------------|----------|-----------|-----------|-----------|
| 0.00 | 120 | 999008 | 992 | 998857 | 1143 | 1.2700387 | 1.4649832 | 0.0924370 |
| 0.00 | 144 | 999997 | 3 | 999994 | 6 | 1.3903192 | 1.6041216 | 0.0909091 |
| 0.00 | 168 | 1000000 | 0 | 1000000 | 0 | 1.5010334 | 1.7320676 | 0.0898204 |
| 0.01 | 24 | 17673 | 982327 | 17720 | 982280 | 0.5747156 | 0.6581503 | 0.1304319 |
| 0.01 | 48 | 255038 | 744962 | 257647 | 742353 | 0.8073388 | 0.9292952 | 0.1063800 |
| 0.01 | 72 | 708892 | 291108 | 716829 | 283171 | 0.9859457 | 1.1363940 | 0.0985884 |
| 0.01 | 96 | 960635 | 39365 | 961876 | 38124 | 1.1370640 | 1.3112193 | 0.0947337 |
| 0.01 | 120 | 999040 | 960 | 998794 | 1206 | 1.2698715 | 1.4648280 | 0.0924338 |
| 0.01 | 144 | 999997 | 3 | 999994 | 6 | 1.3901601 | 1.6039285 | 0.0909060 |
| 0.01 | 168 | 1000000 | 0 | 1000000 | 0 | 1.5009490 | 1.7319739 | 0.0898173 |
| 0.10 | 24 | 20382 | 979618 | 20455 | 979545 | 0.5742520 | 0.6573563 | 0.1302974 |
| 0.10 | 48 | 237747 | 762253 | 240370 | 759630 | 0.8066604 | 0.9284434 | 0.1062311 |
| 0.10 | 72 | 679874 | 320126 | 685575 | 314425 | 0.9849695 | 1.1352553 | 0.0984349 |
| 0.10 | 96 | 961984 | 38016 | 960128 | 39872 | 1.1358912 | 1.3097957 | 0.0945788 |
| 0.10 | 120 | 999546 | 454 | 999275 | 725 | 1.2687142 | 1.4634587 | 0.0922779 |
| 0.10 | 144 | 1000000 | 0 | 1000000 | 0 | 1.3890356 | 1.6025900 | 0.0907489 |
| 0.10 | 168 | 1000000 | 0 | 1000000 | 0 | 1.4994818 | 1.7302649 | 0.0896598 |
| 0.25 | 24 | 23654 | 976346 | 23775 | 976225 | 0.5722185 | 0.6546853 | 0.1296712 |
| 0.25 | 48 | 228318 | 771682 | 231208 | 768792 | 0.8033257 | 0.9244966 | 0.1055259 |
| 0.25 | 72 | 666982 | 333018 | 669104 | 330896 | 0.9808676 | 1.1304109 | 0.0977066 |
| 0.25 | 96 | 966456 | 33544 | 961545 | 38455 | 1.1307841 | 1.3039452 | 0.0938392 |
| 0.25 | 120 | 999749 | 251 | 999507 | 493 | 1.2632327 | 1.4571506 | 0.0915316 |
| 0.25 | 144 | 1000000 | 0 | 1000000 | 0 | 1.3827642 | 1.5953248 | 0.0899987 |
| 0.25 | 168 | 1000000 | 0 | 1000000 | 0 | 1.4926700 | 1.7224506 | 0.0889064 |
| 1.00 | 24 | 26331 | 973669 | 26478 | 973522 | 0.5561013 | 0.6356655 | 0.1249651 |
| 1.00 | 48 | 211199 | 788801 | 214154 | 785846 | 0.7720342 | 0.8881302 | 0.0991370 |
| 1.00 | 72 | 613621 | 386379 | 615771 | 384229 | 0.9394912 | 1.0825566 | 0.0908153 |
| 1.00 | 96 | 943191 | 56809 | 936396 | 63604 | 1.0812364 | 1.2466510 | 0.0867047 |
| 1.00 | 120 | 999157 | 843 | 998396 | 1604 | 1.2065026 | 1.3916458 | 0.0842561 |
| 1.00 | 144 | 1000000 | 0 | 999997 | 3 | 1.3199179 | 1.5227509 | 0.0826325 |
| 1.00 | 168 | 1000000 | 0 | 1000000 | 0 | 1.4243560 | 1.6434386 | 0.0814738 |

In the above, the complexity of **A** and **M**, and the mean C, are also shown. For similar magnitudes of complexity as in random networks of $\sigma\sqrt{SC} \gtrsim 1.26$, variation in γ typically results in more stable than unstable systems.

Scale-free networks are also not saturated, but are defined by an interaction frequency distribution that follows a power law. In other words, a small number of components interact with many other components, while most components interact with only a small number of other components. Scale-free networks can be built by adding new components, one by one, to an existing system, with each newly added component interacting with a randomly selected subset of m existing components. The network on the left below shows an example of a scale-free network in which m=3. The histogram on the right shows the number of other components with which each component interacts.



The table below shows how σ_{γ}^2 affects stability across different scale-free networks with different S and m values.

| m | S | A_unstable | A_stable | M_unstable | M_stable | complex_A | complex_M | С |
|----|----|------------|----------|------------|----------|-----------|-----------|-----------|
| 2 | 24 | 152791 | 847209 | 156034 | 843966 | 0.7891257 | 0.9034663 | 0.1648551 |
| 3 | 24 | 320481 | 679519 | 326351 | 673649 | 0.9566487 | 1.0967499 | 0.2409420 |
| 4 | 24 | 504433 | 495567 | 504826 | 495174 | 1.0922870 | 1.2532761 | 0.3134058 |
| 5 | 24 | 670676 | 329324 | 660426 | 339574 | 1.2073054 | 1.3857169 | 0.3822464 |
| 6 | 24 | 798637 | 201363 | 779345 | 220655 | 1.3067095 | 1.5004508 | 0.4474638 |
| 7 | 24 | 884082 | 115918 | 862215 | 137785 | 1.3942577 | 1.6013368 | 0.5090580 |
| 8 | 24 | 936190 | 63810 | 915630 | 84370 | 1.4722315 | 1.6908563 | 0.5670290 |
| 9 | 24 | 964868 | 35132 | 948297 | 51703 | 1.5414455 | 1.7707292 | 0.6213768 |
| 10 | 24 | 981460 | 18540 | 967911 | 32089 | 1.6030044 | 1.8417459 | 0.6721014 |
| 11 | 24 | 989838 | 10162 | 980232 | 19768 | 1.6586511 | 1.9059313 | 0.7192029 |
| 12 | 24 | 994393 | 5607 | 987436 | 12564 | 1.7081503 | 1.9628898 | 0.7626812 |
| 2 | 48 | 303963 | 696037 | 310053 | 689947 | 0.7946875 | 0.9132519 | 0.0828901 |
| 3 | 48 | 577855 | 422145 | 579996 | 420004 | 0.9685494 | 1.1141445 | 0.1227837 |
| 4 | 48 | 810001 | 189999 | 799132 | 200868 | 1.1122992 | 1.2799335 | 0.1617908 |
| 5 | 48 | 938004 | 61996 | 924613 | 75387 | 1.2369960 | 1.4236817 | 0.1999113 |
| 6 | 48 | 984975 | 15025 | 976433 | 23567 | 1.3478291 | 1.5514420 | 0.2371454 |
| 7 | 48 | 997160 | 2840 | 994005 | 5995 | 1.4473792 | 1.6663763 | 0.2734929 |
| 8 | 48 | 999584 | 416 | 998590 | 1410 | 1.5385445 | 1.7716359 | 0.3089539 |
| 9 | 48 | 999955 | 45 | 999707 | 293 | 1.6227742 | 1.8687074 | 0.3435284 |
| 10 | 48 | 999992 | 8 | 999939 | 61 | 1.7006157 | 1.9583879 | 0.3772163 |
| 11 | 48 | 999999 | 1 | 999990 | 10 | 1.7731759 | 2.0420990 | 0.4100177 |
| 12 | 48 | 1000000 | 0 | 999999 | 1 | 1.8410402 | 2.1203112 | 0.4419326 |
| 2 | 72 | 427243 | 572757 | 434600 | 565400 | 0.7964226 | 0.9166566 | 0.0553599 |
| 3 | 72 | 741345 | 258655 | 739020 | 260980 | 0.9723446 | 1.1195788 | 0.0823552 |

| m | S | A_unstable | A_stable | M_unstable | M_stable | complex_A | complex_M | C |
|----|-----|------------|----------|------------|----------|-----------|-----------|-----------|
| 4 | 72 | 931043 | 68957 | 921145 | 78855 | 1.1188220 | 1.2888100 | 0.1089593 |
| 5 | 72 | 989644 | 10356 | 984372 | 15628 | 1.2466268 | 1.4361875 | 0.1351721 |
| 6 | 72 | 999131 | 869 | 997914 | 2086 | 1.3604666 | 1.5674966 | 0.1609937 |
| 7 | 72 | 999946 | 54 | 999804 | 196 | 1.4642496 | 1.6872501 | 0.1864241 |
| 8 | 72 | 999999 | 1 | 999988 | 12 | 1.5596340 | 1.7974044 | 0.2114632 |
| 9 | 72 | 1000000 | 0 | 999999 | 1 | 1.6482181 | 1.8994441 | 0.2361111 |
| 10 | 72 | 1000000 | 0 | 1000000 | 0 | 1.7307859 | 1.9947150 | 0.2603678 |
| 11 | 72 | 1000000 | 0 | 1000000 | 0 | 1.8086766 | 2.0847262 | 0.2842332 |
| 12 | 72 | 1000000 | 0 | 1000000 | 0 | 1.8817533 | 2.1689764 | 0.3077074 |
| 2 | 96 | 527633 | 472367 | 535188 | 464812 | 0.7974024 | 0.9183557 | 0.0415570 |
| 3 | 96 | 842274 | 157726 | 837756 | 162244 | 0.9741293 | 1.1224709 | 0.0619518 |
| 4 | 96 | 975834 | 24166 | 969478 | 30522 | 1.1220115 | 1.2931371 | 0.0821272 |
| 5 | 96 | 998391 | 1609 | 996991 | 3009 | 1.2511287 | 1.4422331 | 0.1020833 |
| 6 | 96 | 999955 | 45 | 999838 | 162 | 1.3669903 | 1.5757699 | 0.1218202 |
| 7 | 96 | 999999 | 1 | 999996 | 4 | 1.4725862 | 1.6977057 | 0.1413377 |
| 8 | 96 | 1000000 | 0 | 1000000 | 0 | 1.5699145 | 1.8099762 | 0.1606360 |
| 9 | 96 | 1000000 | 0 | 1000000 | 0 | 1.6606162 | 1.9146804 | 0.1797149 |
| 10 | 96 | 1000000 | 0 | 1000000 | 0 | 1.7457971 | 2.0129344 | 0.1985746 |
| 11 | 96 | 1000000 | 0 | 1000000 | 0 | 1.8260368 | 2.1055559 | 0.2172149 |
| 12 | 96 | 1000000 | 0 | 1000000 | 0 | 1.9018608 | 2.1929362 | 0.2356360 |
| 2 | 120 | 609563 | 390437 | 616036 | 383964 | 0.7979355 | 0.9194404 | 0.0332633 |
| 3 | 120 | 904064 | 95936 | 899040 | 100960 | 0.9753815 | 1.1243251 | 0.0496499 |
| 4 | 120 | 991710 | 8290 | 988410 | 11590 | 1.1239922 | 1.2957520 | 0.0658964 |
| 5 | 120 | 999781 | 219 | 999477 | 523 | 1.2539362 | 1.4458518 | 0.0820028 |
| 6 | 120 | 999999 | 1 | 999981 | 19 | 1.3707937 | 1.5806987 | 0.0979692 |
| 7 | 120 | 1000000 | 0 | 999999 | 1 | 1.4775366 | 1.7038860 | 0.1137955 |
| 8 | 120 | 1000000 | 0 | 1000000 | 0 | 1.5762636 | 1.8177236 | 0.1294818 |
| 9 | 120 | 1000000 | 0 | 1000000 | 0 | 1.6680647 | 1.9238257 | 0.1450280 |
| 10 | 120 | 1000000 | 0 | 1000000 | 0 | 1.7545110 | 2.0233838 | 0.1604342 |
| 11 | 120 | 1000000 | 0 | 1000000 | 0 | 1.8363882 | 2.1178385 | 0.1757003 |
| 12 | 120 | 1000000 | 0 | 1000000 | 0 | 1.9135798 | 2.2069806 | 0.1908263 |

As in small-world networks, the mean C is shown, along with the mean complexities of \mathbf{A} and \mathbf{M} . Like all other networks, σ_{γ}^2 increases the stability of scale-free networks given sufficiently high complexity.

Cascade food webs are saturated, and similar to predator-prey random networks. What distinguishes them from predator-prey networks is that cascade food webs are also defined by intactions in which components are ranked such that if the rank of i > j, then $A_{ij} < 0$ and $A_{ji} > 0^{6,7}$. In other words, if interpreting components as ecological species, species can only feed off of a species of lower rank. The table below shows how σ_{γ}^2 affects stability across system sizes in cascade food webs.

| S | A_unstable | A_stable | M_unstable | M_stable | complex_A | complex_M |
|----|------------|----------|------------|----------|-----------|-----------|
| 2 | 0 | 1000000 | 0 | 1000000 | 0.6378839 | 0.6381485 |
| 3 | 1 | 999999 | 1 | 999999 | 0.7055449 | 0.7525143 |
| 4 | 2 | 999998 | 2 | 999998 | 0.8060500 | 0.8826100 |
| 5 | 17 | 999983 | 17 | 999983 | 0.8974749 | 0.9967594 |
| 6 | 42 | 999958 | 43 | 999957 | 0.9821323 | 1.0999762 |
| 7 | 124 | 999876 | 124 | 999876 | 1.0600906 | 1.1938910 |
| 8 | 303 | 999697 | 309 | 999691 | 1.1329713 | 1.2807302 |
| 9 | 653 | 999347 | 661 | 999339 | 1.2009135 | 1.3616372 |
| 10 | 1401 | 998599 | 1413 | 998587 | 1.2661142 | 1.4387567 |
| 11 | 2534 | 997466 | 2566 | 997434 | 1.3276636 | 1.5113096 |

| S | A_unstable | A_stable | M_unstable | M_stable | complex_A | complex_M |
|----|------------|----------|------------|----------|-----------|-----------|
| 12 | 4514 | 995486 | 4597 | 995403 | 1.3865754 | 1.5804005 |
| 13 | 7570 | 992430 | 7722 | 992278 | 1.4424479 | 1.6462780 |
| 14 | 12223 | 987777 | 12502 | 987498 | 1.4970134 | 1.7102322 |
| 15 | 18433 | 981567 | 18879 | 981121 | 1.5498812 | 1.7719564 |
| 16 | 26973 | 973027 | 27712 | 972288 | 1.6002970 | 1.8310447 |
| 17 | 38272 | 961728 | 39499 | 960501 | 1.6494195 | 1.8884211 |
| 18 | 52397 | 947603 | 54099 | 945901 | 1.6975099 | 1.9443860 |
| 19 | 69986 | 930014 | 72342 | 927658 | 1.7439233 | 1.9987398 |
| 20 | 92851 | 907149 | 95776 | 904224 | 1.7893524 | 2.0514394 |
| 21 | 117487 | 882513 | 121095 | 878905 | 1.8335974 | 2.1030121 |
| 22 | 147852 | 852148 | 151989 | 848011 | 1.8761874 | 2.1527108 |
| 23 | 183501 | 816499 | 187888 | 812112 | 1.9186092 | 2.2019827 |
| 24 | 222592 | 777408 | 226021 | 773979 | 1.9591518 | 2.2491948 |
| 25 | 267691 | 732309 | 269822 | 730178 | 1.9999089 | 2.2963949 |
| 26 | 316090 | 683910 | 316371 | 683629 | 2.0396325 | 2.3427211 |
| 27 | 369830 | 630170 | 366550 | 633450 | 2.0785319 | 2.3879356 |
| 28 | 426407 | 573593 | 419136 | 580864 | 2.1169703 | 2.4324407 |
| 29 | 485068 | 514932 | 473666 | 526334 | 2.1545265 | 2.4759539 |
| 30 | 544300 | 455700 | 527568 | 472432 | 2.1912376 | 2.5187795 |
| 31 | 605803 | 394197 | 584385 | 415615 | 2.2271037 | 2.5603818 |
| 32 | 664689 | 335311 | 638047 | 361953 | 2.2626270 | 2.6016360 |
| 33 | 718848 | 281152 | 689172 | 310828 | 2.2979241 | 2.6424881 |
| 34 | 770790 | 229210 | 737639 | 262361 | 2.3327303 | 2.6828460 |
| 35 | 817531 | 182469 | 783112 | 216888 | 2.3666720 | 2.7221952 |
| 36 | 858750 | 141250 | 823548 | 176452 | 2.3998286 | 2.7608037 |
| 37 | 893017 | 106983 | 859194 | 140806 | 2.4332806 | 2.7994470 |
| 38 | 921268 | 78732 | 890177 | 109823 | 2.4658414 | 2.8372307 |
| 39 | 943551 | 56449 | 915655 | 84345 | 2.4974678 | 2.8741350 |
| 40 | 961088 | 38912 | 936883 | 63117 | 2.5301278 | 2.9116114 |
| 41 | 973664 | 26336 | 953645 | 46355 | 2.5616210 | 2.9481298 |
| 42 | 982829 | 17171 | 967044 | 32956 | 2.5925309 | 2.9841081 |
| 43 | 989464 | 10536 | 977033 | 22967 | 2.6228949 | 3.0191690 |
| 44 | 993622 | 6378 | 984470 | 15530 | 2.6534626 | 3.0548439 |
| 45 | 996221 | 3779 | 989678 | 10322 | 2.6832092 | 3.0890543 |
| 46 | 997963 | 2037 | 993318 | 6682 | 2.7130588 | 3.1236201 |
| 47 | 998818 | 1182 | 995957 | 4043 | 2.7423480 | 3.1575904 |
| 48 | 999422 | 578 | 997446 | 2554 | 2.7714223 | 3.1912463 |
| 49 | 999746 | 254 | 998532 | 1468 | 2.7999596 | 3.2244020 |
| 50 | 999864 | 136 | 999132 | 868 | 2.8285547 | 3.2574510 |
| 51 | 999934 | 66 | 999561 | 439 | 2.8566907 | 3.2900943 |
| 52 | 999970 | 30 | 999761 | 239 | 2.8844703 | 3.3222721 |
| 53 | 999985 | 15 | 999873 | 127 | 2.9122645 | 3.3544290 |
| 54 | 999999 | 1 | 999935 | 65 | 2.9395400 | 3.3859103 |
| 55 | 1000000 | 0 | 999971 | 29 | 2.9665996 | 3.4173273 |
| 56 | 999999 | 1 | 999988 | 12 | 2.9936263 | 3.4486027 |
| 57 | 1000000 | 0 | 999989 | 11 | 3.0199283 | 3.4789408 |
| 58 | 1000000 | 0 | 999998 | 2 | 3.0460952 | 3.5094530 |
| 59 | 1000000 | 0 | 999999 | 1 | 3.0728115 | 3.5401634 |
| 60 | 1000000 | 0 | 1000000 | 0 | 3.0983367 | 3.5698067 |

Cascade food webs are more likely to be stable than small-world or scale-free networks at equivalent magnitudes of complexity (note C=1 for all above rows). A higher number of stable ${\bf M}$ than ${\bf A}$ was found given $S\geq 27$.

Feasibility of complex systems

When feasibility was evaluated with and without variation in γ , there was no increase in stability for M where γ varied as compared to where $\gamma = 1$. Results below illustrate this result, which was general to all other simulations performed.

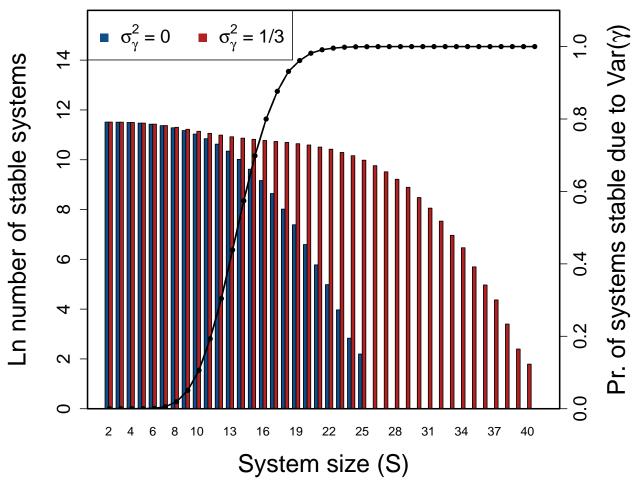
| \overline{S} | A_infeasible | A_feasible | M_infeasible | M_feasible | A_made_feasible | A_made_infeasible |
|-----------------|--------------------|------------|--------------------|------------|-----------------|----------------------|
| 2 | 749978 | 250022 | 749942 | 250058 | 35552 | 35516 |
| 3 | 874519 | 125481 | 874296 | 125704 | 36803 | 36580 |
| 4 | 937192 | 62808 | 937215 | 62785 | 26440 | 26463 |
| 5 | 968776 | 31224 | 968639 | 31361 | 16319 | 16182 |
| 6 | 984313 | 15687 | 984463 | 15537 | 9006 | 9156 |
| 7 | 992149 | 7851 | 992161 | 7839 | 4991 | 5003 |
| 8 | 996124 | 3876 | 996103 | 3897 | 2644 | 2623 |
| 9 | 998014 | 1986 | 998027 | 1973 | 1361 | 1374 |
| 10 | 999031 | 969 | 999040 | 960 | 698 | 707 |
| 11 | 999546 | 454 | 999514 | 486 | 377 | 345 |
| 12 | 999764 | 236 | 999792 | 208 | 160 | 188 |
| 13 | 999883 | 117 | 999865 | 135 | 105 | 87 |
| 14 | 999938 | 62 | 999945 | 55 | 40 | 47 |
| 15 | 999971 | 29 | 999964 | 36 | 31 | 24 |
| 16 | 999988 | 12 | 999991 | 9 | 8 | 11 |
| 17 | 999996 | 4 | 999991 | 9 | 8 | 3 |
| 18 | 999997 | 3 | 999999 | 1 | 1 | 3 |
| 19 | 999998 | 2 | 999997 | 3 | 3 | 2 |
| 20 | 1000000 | 0 | 999999 | 1 | 1 | 0 |
| 21 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 22 | 999999 | 1 | 1000000 | 0 | 0 | 1 |
| 23 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 24 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 25 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 26 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 27 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 28 29 | 1000000 1000000 | 0 | 1000000 1000000 | 0 | 0 | 0 |
| $\frac{29}{30}$ | 1000000 | 0 | 1000000 | $0 \\ 0$ | $0 \\ 0$ | $0 \\ 0$ |
| 31 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| $\frac{31}{32}$ | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 33 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 34 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 35 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 36 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 37 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 38 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 39 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 40 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 41 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 42 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 43 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 44 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 45 | 1000000 | 0 | 1000000 | 0 | 0 | $\overset{\circ}{0}$ |
| 46 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 47 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| | 200000 | O | 2000000 | O | 0 | O . |

| S | A_infeasible | A_feasible | M_infeasible | M_feasible | A_made_feasible | A_made_infeasible |
|----|--------------|------------|--------------|------------|-----------------|-------------------|
| 48 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 49 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |
| 50 | 1000000 | 0 | 1000000 | 0 | 0 | 0 |

Hence, in general, σ_{γ}^2 does not appear to affect feasibility in pure species interaction networks⁸.

Stability given targeted manipulation of γ (genetic algorithm)

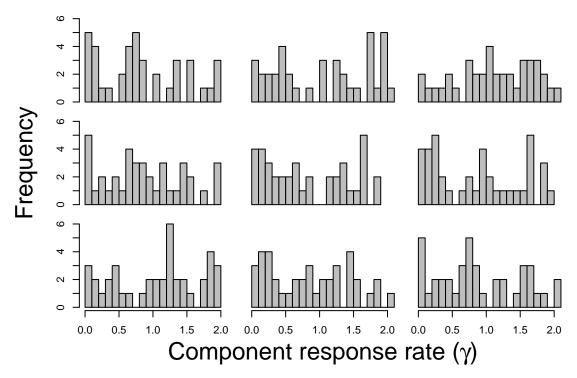
The figure below compares the stability of large complex systems given $\gamma=1$ versus targeted manipulation of γ elements. For each S, 100000 complex systems are randomly generated. Stability of each complex system is tested given variation in γ using a genetic algorithm to maximise the effect of γ values on increasing stability, as compared to stability in an otherwise identical system in which γ is the same for all components. Blue bars show the number of stable systems in the absence of component response rate variation, while red bars show the number of stable systems that can be generated if component response rate is varied to maximise system stability. The black line shows the proportion of systems that are stable when component response rate is targeted to increase stability, but would not be stable if $\sigma_{\gamma}^2=0$. The y-axis shows the ln number of systems that are stable across $S=\{1,2,...,39,40\}$ for C=1, and the proportion of systems wherein a targeted search of γ values successfully resulted in system stability.



Stability results are also shown in the table below. Results for A indicate systems in which $\gamma = 1$, while M refers to systems in which the genetic algorithm searched for a set of γ values that stabilised the system.

| 2 26 99974 26 99974 0 3 358 99642 358 99642 0 4 1505 98495 1505 98495 0 5 3995 96005 3982 96018 13 6 8060 91940 7956 92044 104 7 13420 86580 12953 87047 468 8 20518 79482 18940 81060 1578 9 28939 71061 25148 74852 3793 10 38241 61759 30915 69085 7327 11 48682 51318 36398 63602 12286 12 58752 41248 40710 59290 18043 13 68888 31112 44600 55400 24289 14 77651 22349 47528 52472 30124 15 84912 15088 4 | \overline{S} | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|--|----------------|------------|----------|------------|----------|--------------|----------------|
| 3 358 99642 358 99642 0 4 1505 98495 1505 98495 0 5 3995 96005 3982 96018 13 6 8060 91940 7956 92044 104 7 13420 86580 12953 87047 468 8 20518 79482 18940 81060 1578 9 28939 71061 25148 74852 3793 10 38241 61759 30915 69085 7327 11 48682 51318 36398 63602 12286 12 58752 41248 40710 59290 18043 13 68888 31112 44600 55400 24289 14 77651 22349 47528 52472 30124 15 84912 15088 49971 50029 34942 16 90451 9549 | 2 | 26 | 99974 | 26 | 99974 | 0 | 0 |
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| 9 28939 71061 25148 74852 3793 10 38241 61759 30915 69085 7327 11 48682 51318 36398 63602 12286 12 58752 41248 40710 59290 18043 13 68888 31112 44600 55400 24289 14 77651 22349 47528 52472 30124 15 84912 15088 49971 50299 34942 16 90451 9549 52274 47726 38178 17 94332 5668 54124 45876 40209 18 96968 3032 55831 44169 41139 19 98384 1616 58079 41921 40305 20 99269 731 60181 39819 39088 21 99677 323 63338 36662 36339 22 99854 146 66350 33650 33504 23 99947 5 | 8 | 20518 | 79482 | 18940 | 81060 | 1578 | 0 |
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| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | 11 | 48682 | 51318 | 36398 | 63602 | 12286 | 2 |
| 14 77651 22349 47528 52472 30124 15 84912 15088 49971 50029 34942 16 90451 9549 52274 47726 38178 17 94332 5668 54124 45876 40209 18 96968 3032 55831 44169 41139 19 98384 1616 58079 41921 40305 20 99269 731 60181 39819 39088 21 99677 323 63338 36662 36339 22 99854 146 66350 33650 33504 23 99947 53 70478 29522 29469 24 99983 17 74121 25879 25862 25 99991 9 78364 21636 21627 26 99999 1 82635 17365 17364 27 100000 0 86433 13567 13567 28 100000 0 < | 12 | 58752 | | 40710 | 59290 | 18043 | 1 |
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| 17 94332 5668 54124 45876 40209 18 96968 3032 55831 44169 41139 19 98384 1616 58079 41921 40305 20 99269 731 60181 39819 39088 21 99677 323 63338 36662 36339 22 99854 146 66350 33650 33504 23 99947 53 70478 29522 29469 24 99983 17 74121 25879 25862 25 99991 9 78364 21636 21627 26 99999 1 82635 17365 17364 27 100000 0 86433 13567 13567 28 100000 0 92716 7284 7284 30 100000 0 95171 4829 4829 31 100000 0 96844 3156 3156 32 100000 0 98128 | 15 | 84912 | 15088 | 49971 | 50029 | 34942 | 1 |
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| 19 98384 1616 58079 41921 40305 20 99269 731 60181 39819 39088 21 99677 323 63338 36662 36339 22 99854 146 66350 33650 33504 23 99947 53 70478 29522 29469 24 99983 17 74121 25879 25862 25 99991 9 78364 21636 21627 26 99999 1 82635 17365 17364 27 100000 0 86433 13567 13567 28 100000 0 89951 10049 10049 29 100000 0 92716 7284 7284 30 100000 0 95171 4829 4829 31 100000 0 96844 3156 3156 32 100000 0 98128 1872 1872 33 100000 0 99358 | 17 | 94332 | 5668 | 54124 | 45876 | 40209 | 1 |
| 20 99269 731 60181 39819 39088 21 99677 323 63338 36662 36339 22 99854 146 66350 33650 33504 23 99947 53 70478 29522 29469 24 99983 17 74121 25879 25862 25 99991 9 78364 21636 21627 26 99999 1 82635 17365 17364 27 100000 0 86433 13567 13567 28 100000 0 89951 10049 10049 29 100000 0 92716 7284 7284 30 100000 0 95171 4829 4829 31 100000 0 96844 3156 3156 32 100000 0 98128 1872 1872 33 100000 0 99358 642 642 | 18 | 96968 | 3032 | 55831 | 44169 | 41139 | 2 |
| 21 99677 323 63338 36662 36339 22 99854 146 66350 33650 33504 23 99947 53 70478 29522 29469 24 99983 17 74121 25879 25862 25 99991 9 78364 21636 21627 26 99999 1 82635 17365 17364 27 100000 0 86433 13567 13567 28 100000 0 89951 10049 10049 29 100000 0 92716 7284 7284 30 100000 0 95171 4829 4829 31 100000 0 96844 3156 3156 32 100000 0 98128 1872 1872 33 100000 0 98941 1059 1059 34 100000 0 99358 642 642 | 19 | 98384 | 1616 | 58079 | 41921 | 40305 | 0 |
| 22 99854 146 66350 33650 33504 23 99947 53 70478 29522 29469 24 99983 17 74121 25879 25862 25 99991 9 78364 21636 21627 26 99999 1 82635 17365 17364 27 100000 0 86433 13567 13567 28 100000 0 89951 10049 10049 29 100000 0 92716 7284 7284 30 100000 0 95171 4829 4829 31 100000 0 96844 3156 3156 32 100000 0 98128 1872 1872 33 100000 0 98941 1059 1059 34 100000 0 99358 642 642 | 20 | 99269 | 731 | 60181 | 39819 | 39088 | 0 |
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| 26 99999 1 82635 17365 17364 27 100000 0 86433 13567 13567 28 100000 0 89951 10049 10049 29 100000 0 92716 7284 7284 30 100000 0 95171 4829 4829 31 100000 0 96844 3156 3156 32 100000 0 98128 1872 1872 33 100000 0 98941 1059 1059 34 100000 0 99358 642 642 | 24 | 99983 | 17 | 74121 | 25879 | 25862 | 0 |
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| 30 100000 0 95171 4829 4829 31 100000 0 96844 3156 3156 32 100000 0 98128 1872 1872 33 100000 0 98941 1059 1059 34 100000 0 99358 642 642 | 28 | 100000 | 0 | 89951 | 10049 | 10049 | 0 |
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| 32 100000 0 98128 1872 1872 33 100000 0 98941 1059 1059 34 100000 0 99358 642 642 | 30 | 100000 | 0 | 95171 | 4829 | 4829 | 0 |
| 33 100000 0 98941 1059 1059 34 100000 0 99358 642 642 | 31 | 100000 | 0 | 96844 | 3156 | 3156 | 0 |
| 34 100000 0 99358 642 642 | 32 | 100000 | 0 | 98128 | 1872 | 1872 | 0 |
| | 33 | 100000 | 0 | 98941 | 1059 | 1059 | 0 |
| 35 100000 0 99702 298 298 | 34 | 100000 | 0 | 99358 | 642 | 642 | 0 |
| | 35 | 100000 | 0 | 99702 | 298 | 298 | 0 |
| 36 100000 0 99856 144 144 | 36 | 100000 | 0 | 99856 | 144 | 144 | 0 |
| 37 100000 0 99921 79 79 | 37 | 100000 | 0 | 99921 | 79 | 79 | 0 |
| 38 100000 0 99970 30 30 | 38 | 100000 | 0 | 99970 | 30 | 30 | 0 |
| 39 100000 0 99989 11 11 | 39 | 100000 | 0 | 99989 | 11 | 11 | 0 |
| 40 100000 0 99994 6 6 | 40 | 100000 | 0 | 99994 | 6 | 6 | 0 |

The distributions of nine γ vectors from the highest S values are shown below. This comparison shows the high number of stable $\mathbf M$ that can be produced through a targeted search of γ values, and suggests that many otherwise unstable systems could potentially be stabilised by an informed manipulation of their component response times. Such a possibility might conceivably reduce the dimensionality of problems involving stability in social-ecological or economic systems.



The distribution of γ values found by the genetic algorithm is uniform. A uniform distribution was used to initialise γ values, so there is therefore no evidence that a particular distribution of γ is likely to be found to stabilise a matrix \mathbf{M} .

Consistency with Gibbs et al. (2018)

The question that I address in the main text is distinct from that of Gibbs et al.⁹, who focused instead on the effect of a diagonal matrix of biological species densities \mathbf{X} on a community matrix \mathbf{M} given a species interaction matrix \mathbf{A} . This is modelled as below,

$$\mathbf{M} = \mathbf{X}\mathbf{A}$$
.

Mathematically, the above is identical to my model in the main text where the system \mathbf{M} is defined by component interaction strengths \mathbf{A} and individual component response rates $\boldsymbol{\gamma}$,

$$\mathbf{M} = \gamma \mathbf{A}$$
.

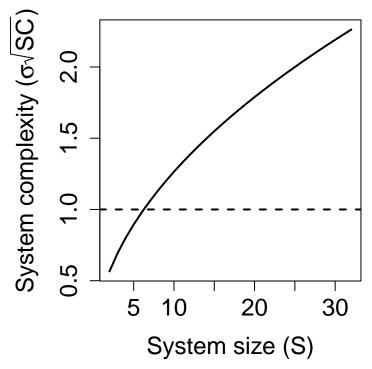
I focused on the probability of observing a stable versus unstable system given variation in γ as system complexity $(\sigma\sqrt{SC})$ increased. I increased system complexity by holding C and σ constant and incrementally increasing S to obtain numerical results. In contrast, Gibbs et al.⁹ applied analytical techniques to instead focus on a different question concerning the effect of γ on the stability of \mathbf{M} given \mathbf{A} as $S \to \infty$, with σ scaled so that $\sigma = 1/\sqrt{S}$. Under such scaling, Gibbs et al.⁹ showed that the effect of γ on stability should decrease exponentially as S increases, which I demonstrate below by running simulations in which $\sigma = 1/\sqrt{S}$.

| \mathbf{S} | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | $A_{destabilised}$ |
|--------------|------------|----------|------------|----------|--------------|--------------------|
| 2 | 3111 | 96889 | 3111 | 96889 | 0 | 0 |
| 3 | 5203 | 94797 | 5237 | 94763 | 1 | 35 |
| 4 | 6743 | 93257 | 6818 | 93182 | 6 | 81 |

| S | A_unstable | A_stable | M_unstable | M_stable | A_stabilised | A_destabilised |
|----|------------|----------|------------|----------|--------------|----------------|
| 5 | 7889 | 92111 | 8005 | 91995 | 20 | 136 |
| 6 | 8834 | 91166 | 8991 | 91009 | 55 | 212 |
| 7 | 9885 | 90115 | 10072 | 89928 | 81 | 268 |
| 8 | 10516 | 89484 | 10764 | 89236 | 108 | 356 |
| 9 | 11135 | 88865 | 11383 | 88617 | 145 | 393 |
| 10 | 11819 | 88181 | 12095 | 87905 | 181 | 457 |
| 11 | 12414 | 87586 | 12700 | 87300 | 213 | 499 |
| 12 | 12865 | 87135 | 13136 | 86864 | 283 | 554 |
| 13 | 13530 | 86470 | 13836 | 86164 | 324 | 630 |
| 14 | 13745 | 86255 | 14042 | 85958 | 362 | 659 |
| 15 | 14401 | 85599 | 14720 | 85280 | 387 | 706 |
| 16 | 14793 | 85207 | 15123 | 84877 | 428 | 758 |
| 17 | 15004 | 84996 | 15356 | 84644 | 444 | 796 |
| 18 | 15361 | 84639 | 15735 | 84265 | 472 | 846 |
| 19 | 16062 | 83938 | 16303 | 83697 | 592 | 833 |
| 20 | 15814 | 84186 | 16184 | 83816 | 566 | 936 |
| 21 | 16171 | 83829 | 16492 | 83508 | 640 | 961 |
| 22 | 16671 | 83329 | 17049 | 82951 | 641 | 1019 |
| 23 | 17000 | 83000 | 17291 | 82709 | 718 | 1009 |
| 24 | 17411 | 82589 | 17666 | 82334 | 765 | 1020 |
| 25 | 17414 | 82586 | 17742 | 82258 | 783 | 1111 |
| 26 | 17697 | 82303 | 18027 | 81973 | 806 | 1136 |
| 27 | 18010 | 81990 | 18316 | 81684 | 880 | 1186 |
| 28 | 18584 | 81416 | 18735 | 81265 | 1008 | 1159 |
| 29 | 18401 | 81599 | 18572 | 81428 | 942 | 1113 |
| 30 | 18497 | 81503 | 18754 | 81246 | 952 | 1209 |
| 31 | 18744 | 81256 | 18942 | 81058 | 991 | 1189 |
| 32 | 18936 | 81064 | 19194 | 80806 | 1022 | 1280 |
| 33 | 19174 | 80826 | 19346 | 80654 | 1113 | 1285 |
| 34 | 19477 | 80523 | 19632 | 80368 | 1120 | 1275 |
| 35 | 19659 | 80341 | 19777 | 80223 | 1206 | 1324 |
| 36 | 19883 | 80117 | 19929 | 80071 | 1275 | 1321 |
| 37 | 20275 | 79725 | 20348 | 79652 | 1308 | 1381 |
| 38 | 20067 | 79933 | 20190 | 79810 | 1275 | 1398 |
| 39 | 20416 | 79584 | 20516 | 79484 | 1340 | 1440 |
| 40 | 20370 | 79630 | 20489 | 79511 | 1359 | 1478 |
| 41 | 20295 | 79705 | 20430 | 79570 | 1382 | 1517 |
| 42 | 20767 | 79233 | 20839 | 79161 | 1418 | 1490 |
| 43 | 20688 | 79312 | 20705 | 79295 | 1471 | 1488 |
| 44 | 21049 | 78951 | 21028 | 78972 | 1555 | 1534 |
| 45 | 21114 | 78886 | 21034 | 78966 | 1572 | 1492 |
| 46 | 21163 | 78837 | 21195 | 78805 | 1463 | 1495 |
| 47 | 21373 | 78627 | 21353 | 78647 | 1535 | 1515 |
| 48 | 21338 | 78662 | 21285 | 78715 | 1632 | 1579 |
| 49 | 21547 | 78453 | 21566 | 78434 | 1575 | 1594 |
| 50 | 21738 | 78262 | 21633 | 78367 | 1636 | 1531 |
| 51 | 21967 | 78033 | 21892 | 78108 | 1698 | 1623 |

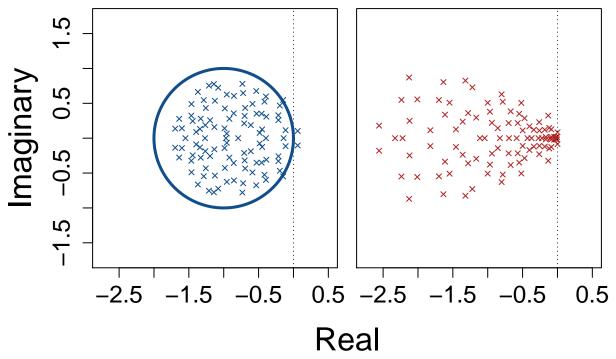
Above table results can be compared to those of the main results. Note that 100000 (not 1 million), simulations are run to confirm consistency with Gibbs et al.⁹. The difference between my model and Gibbs et al.⁹ is that in the latter, $\sigma\sqrt{SC}=1$ remains constant with increasing S. In the former, $\sigma\sqrt{SC}$ increases with S, so the expected complexity of the system also increases accordingly. Consequently, for the scaled σ in the table

above, systems are not more likely to be stabilised by γ as S increases, consistent with Gibbs et al.⁹. Note that overall stability does decrease with increasing S due to the increased density of eigenvalues (see below).

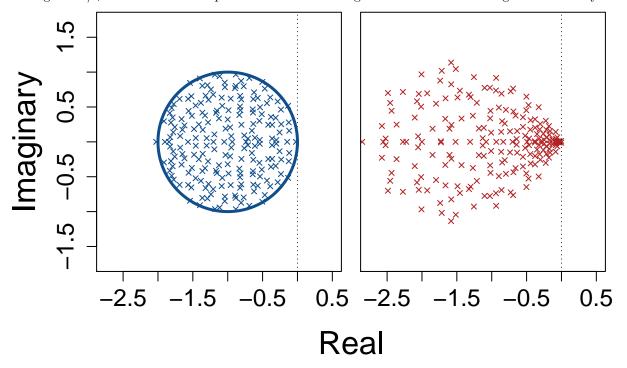


Complexity as a function of S in the main text (solid) versus in Gibbs et al.⁹ (dashed).

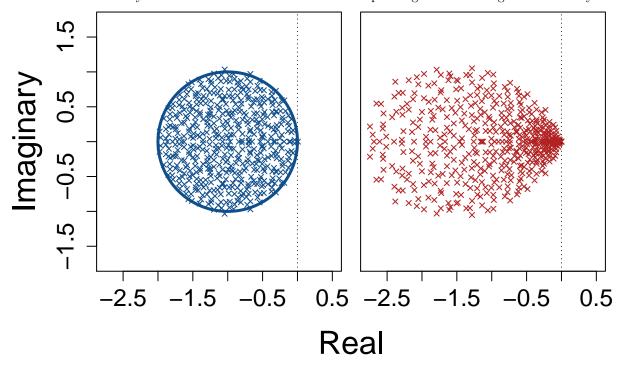
When the complexity is scaled to $\sigma\sqrt{SC}=1$, an increase in S increases the eigenvalue density within a circle with a unit radius centred at (-1,0) on the complex plane. As $S\to\infty$, this circle becomes increasingly saturated. Gibbs et al.⁹ showed that a diagonal matrix γ will have an exponentially decreasing effect on stability with increasing S. Increasing S is visualised below, first with a system size S=100.



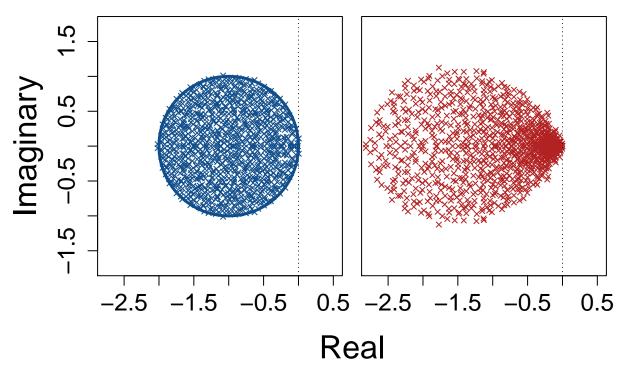
The left panel above shows the distribution of eigenvalues; the blue ellipse shows the unit radius within which eigenvalues are expected to be contained. The right panel shows how eigenvalue distributions change given $\gamma \sim \mathcal{U}(0,2)$. The vertical dotted line shows the threshold of stability, $\Re = 0$. Increasing to S = 200, the scaling $\sigma = 1/\sqrt{S}$ maintains the expected distribution of eigenvalues but increases eigenvalue density.



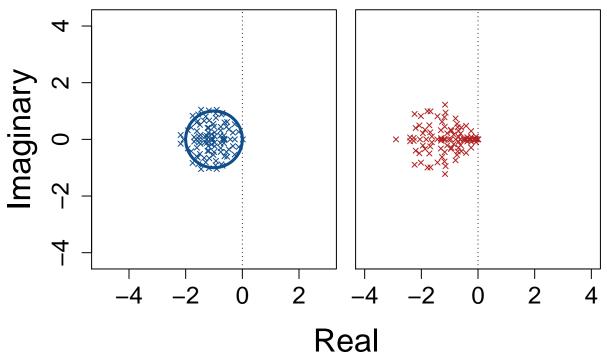
We can increase the system size to S = 500 and see the corresponding increase in eigenvalue density.



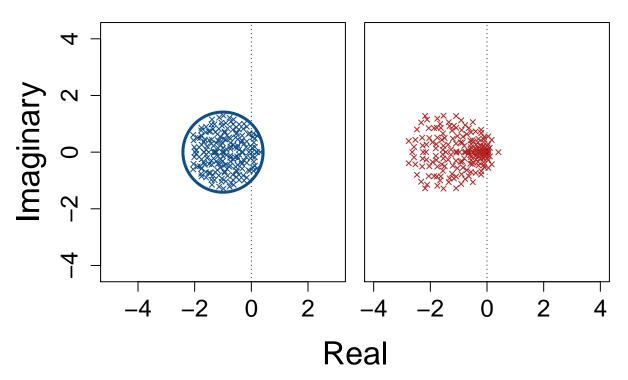
Finally, below shows a increase in system size to S = 1000.



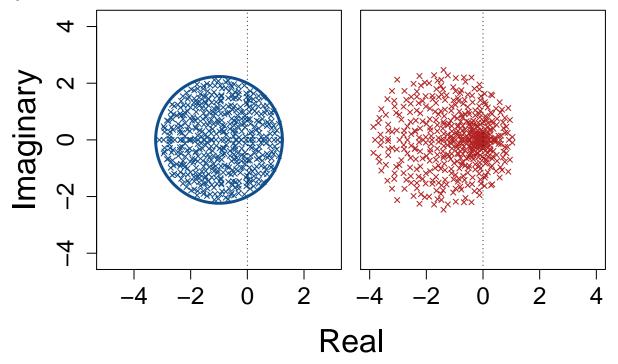
In contrast, in the model of the main text, the complexity of system is not scaled to $\sigma\sqrt{SC}=1$. Rather, the density of eigenvalues within a circle centred at (-1,0) with a radius $\sigma\sqrt{SC}$ is held constant such that there are $S/\pi(\sigma\sqrt{SC})^2$ eigenvalues per unit area of the circle. As S increases, so does the expected complexity of the system, but the density of eigenvalues remains finite causing error around this expectation. Below shows a system where S=100, C=0.0625, and $\sigma=0.4$, where $\sigma\sqrt{SC}=1$ (identical to the first example distribution above in which S=100 and $\sigma=1/\sqrt{S}$).



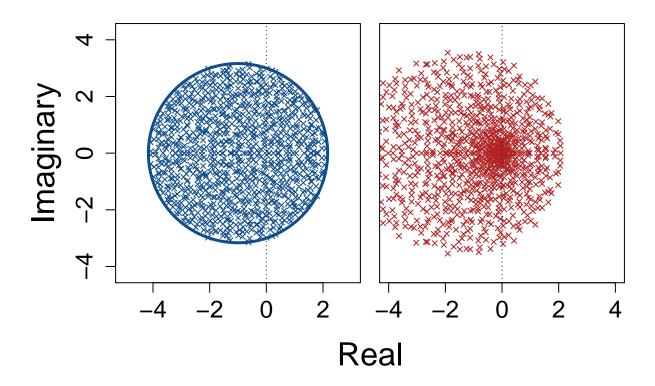
Now when S is increased to 200 while keeping C = 0.0625 and $\sigma = 0.4$, the area of the circle within which eigenvalues are contained increases to keep the density of eigenvalues constant.



Note that the expected distribution of eigenvalues increases so that the threshold $\Re=0$ is exceeded. Below, system size is increased to S=500.



Finally, S = 1000 is shown below. Again, the density of eigenvalues per unit remains constant at ca 2, but the system has increased in complexity such that some real components of eigenvalues are almost assured to be greater than zero.



Reproducing simulation results

All results in the main text and the literature cited can be reproduced using the RandomMatrixStability R package, which can be downloaded as instructed at the beginning of this Supplemental Information document. The most relevant R functions for reproducing simulations include the following:

- 1. rand_gen_var: Simulates random complex systems and cascade food webs
- 2. rand_rho_var: Simulates random complex systems across a fixed correlation of $\rho = cor(A_{ij}, A_{ji})$
- 3. rand_gen_swn: Simulates randomly generated small-world networks
- 4. rand_gen_sfn: Simulates randomly generated scale-free networks
- 5. Evo_rand_gen_var: Use a genetic algorithm to find stable random complex systems

For the functions 1-4 above, R output will be a table of results. Below describes the headers of this table to more clearly explain what is being reported.

| Header | Description | ${\it Header_cont.}$ | Description_cont. |
|-----------------------|-----------------------------------|-----------------------|--|
| S | The system size | A_rho | Corr. between elements A[ij] and A[ji] |
| $A_{unstable}$ | No. of A that were unstable | M_rho | Corr. between elements M[ij] and M[ji] |
| A_stable | No. of A that were stable | rho_diff | Diff. between A and M rho values |
| $M_{unstable}$ | No. of M that were unstable | rho_abs | Diff. between A and M rho magnitudes |
| M_{stable} | No. of M that were stable | $complex_A$ | Complexity of A |
| A_stabilised | No. of A stabilised by gamma | $complex_M$ | Complexity of M |
| A_destabilised | No. of A destabilised by gamma | A_eig | Expected real part of leading A eigenvalue |
| A_infeasible | No. of A that were infeasible | M_eig | Expected real part of leading M eigenvalue |
| A_feasible | No. of A that were feasible | LR_A | Lowest obs. real part of leading A eigenvalue |
| $M_{infeasible}$ | No. of M that were infeasible | UR_A | Highest obs. real part of leading A eigenvalue |
| M_{feasible} | No. of M that were feasible | LR_M | Lowest obs. real part of leading M eigenvalue |
| $A_{made_feasible}$ | No. of A made feasible by gamma | UR_M | Highest obs. real part of leading M eigenvalue |
| $A_made_infeasible$ | No. of A made infeasible by gamma | $^{\mathrm{C}}$ | Obs. network connectance |

Note that output from $Evo_rand_gen_var$ only includes the first seven rows of the table above, and $rand_gen_var$ does not include C (which can be defined as an argument). All results presented here and in

the main text are available in the inst/extdata folder of the RandomMatrixStability R package.

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