Foundations of community ecology: Supporting Information 1

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We start with the fundamental equation that provides a complete description of eco-evolutionary change in any system,

In the main text, we derived both the Price equation and the birth-death model from the above. Here we integrate interactions between individuals and demonstrate how this affects population and evolutionary dynamics simultaneously. We can do this in a way that clarifies model assumptions by allowing an individual to modulate the birth or death of the focal individual . We will first illustrate how this leads to two separate discrete time models of density depedent population growth, then illustrate how the same starting point can be used to recover Hamilton’s rule in evolutionary ecology.

# Density-dependent population growth

There are two potential ways to model the incorporation of density dependence into population growth. First note that here we set , and and for all individuals. We can define as an individual growth rate for ([Lion 2018](#ref-Lion2018); [Lion, Sasaki, and Boots 2023](#ref-Lion2023)). In this case,

Mathematically, the most general approach here would be to define individual growth as a function of the entire system , , where is a vector with elements including any parameters potentially relevant to . Taking this approach would recover a version of eqn 2 in Lion ([2018](#ref-Lion2018)) and permit any relationship between the system and a focal individual’s growth. Focusing on effects of other individuals () and assuming that the effects of these individuals are additive, let be the effect of individual on the growth rate attributable to conditioned on all other individuals within the population such that defines the realised growth rate of ,

Assuming that individual effects of on are also independent, we can remove the condition,

Further assuming that all individuals have the same per capita effect such that for any and pair (as might be reasonable given resource competition in a well-mixed population),

If values are identical,

Equation S3 therefore recovers a classic version of a discrete time logistic growth by making assumptions from an exact model of eco-evolutionary change. An alternative approach would be to define model the effects an individual on the fitness of (), thereby replacing eqn S1 with and replacing eqn S2 with,

By making the same assumptions of additivity, independence, and identical effects such that for all on , and assuming fitness is equal (), we can derive,

# Evolution of altruism

Here we following Lehtonen ([2020](#ref-Lehtonen2020a)). Starting again from eqn 1, we can define groups with individual characteristics () now summed across groups,

# References

Lehtonen, Jussi. 2020. “The Price Equation and the Unity of Social Evolution Theory.” *Philosophical Transactions of the Royal Society B: Biological Sciences* 375: 20190362. <https://doi.org/10.1098/rstb.2019.0362>.

Lion, Sébastien. 2018. “Theoretical approaches in evolutionary ecology: environmental feedback as a unifying perspective.” *American Naturalist* 191 (1). <https://doi.org/10.1086/694865>.

Lion, Sébastien, Akira Sasaki, and Mike Boots. 2023. “Extending Eco-Evolutionary Theory with Oligomorphic Dynamics.” *Ecology Letters* 26 (September): S22–46. <https://doi.org/10.1111/ele.14183>.