

SCIU4T4: Central tendency and measures of spread

Descriptive versus inferential statistics

Descriptive Statistics

- ▶ Summarise observations
- ▶ E.g., average monthly temperature

Inferential Statistics

- ▶ Make estimates or predictions
- ▶ E.g., predict temperature from latitude

Descriptive statistics in jamovi

Descriptives

Variables: Soil organic carbon (g C / kg soil)

Descriptives | Variables across columns Frequency tables

Statistics

Sample Size
 N Missing

Percentile Values
 Cut points for 4 equal groups
 Percentiles 25,50,75

Dispersion
 Std. deviation Minimum
 Variance Maximum
 Range IQR

Mean Dispersion
 Std. error of Mean
 Confidence interval for Mean 95 %

Central Tendency
 Mean Median Mode Sum

Distribution
 Skewness Kurtosis

Normality
 Shapiro-Wilk

Outliers
 Most extreme 5 values

Results

Descriptives

	Soil organic carbon (g C / kg soil)
N	34
Missing	0
Mean	6.52353
Median	5.80000
Mode	2.40000 *
Standard deviation	4.49701
Variance	20.22307
IQR	7.27500
Range	15.60000
Minimum	0.60000
Maximum	16.20000
Skewness	0.55655
Std. error skewness	0.40305
Kurtosis	-0.73034
Std. error kurtosis	0.78790
25th percentile	2.42500
50th percentile	5.80000
75th percentile	9.70000

* More than one mode exists, only the first is reported

Properties of distributions

- ▶ Central tendency
- ▶ Spread
- ▶ Skew & Kurtosis

**Will focus on *samples*
rather than populations**

Descriptive statistics: Central tendency

The screenshot shows the SPSS 'Descriptives' dialog box and the resulting 'Results' table.

Descriptives Dialog Box:

- Variables:** Soil organic carbon (g C / kg soil)
- Descriptives:** Selected
- Statistics:** Sample Size (N, Missing), Percentile Values (Percentiles 25, 50, 75), Central Tendency (Mean, Median, Mode), Dispersion (Std. deviation, Variance, Range), Distribution (Skewness, Kurtosis), Normality (Shapiro-Wilk), Outliers (Most extreme 5 values).

Results Table:

	Soil organic carbon (g C / kg soil)
N	34
Missing	0
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^a More than one mode exists, only the first is reported

Mean, median, and mode

Arithmetic mean

Add values, divide by number (N)

For example, $N = 3$ temperatures:

- ▶ $12.5 \text{ } ^\circ C$
- ▶ $13.4 \text{ } ^\circ C$
- ▶ $14.0 \text{ } ^\circ C$

$$\bar{x} = \frac{12.5 + 13.4 + 14.0}{3} = 13.3$$

Calculating the mean of 7 temperatures ($^{\circ}\text{C}$)

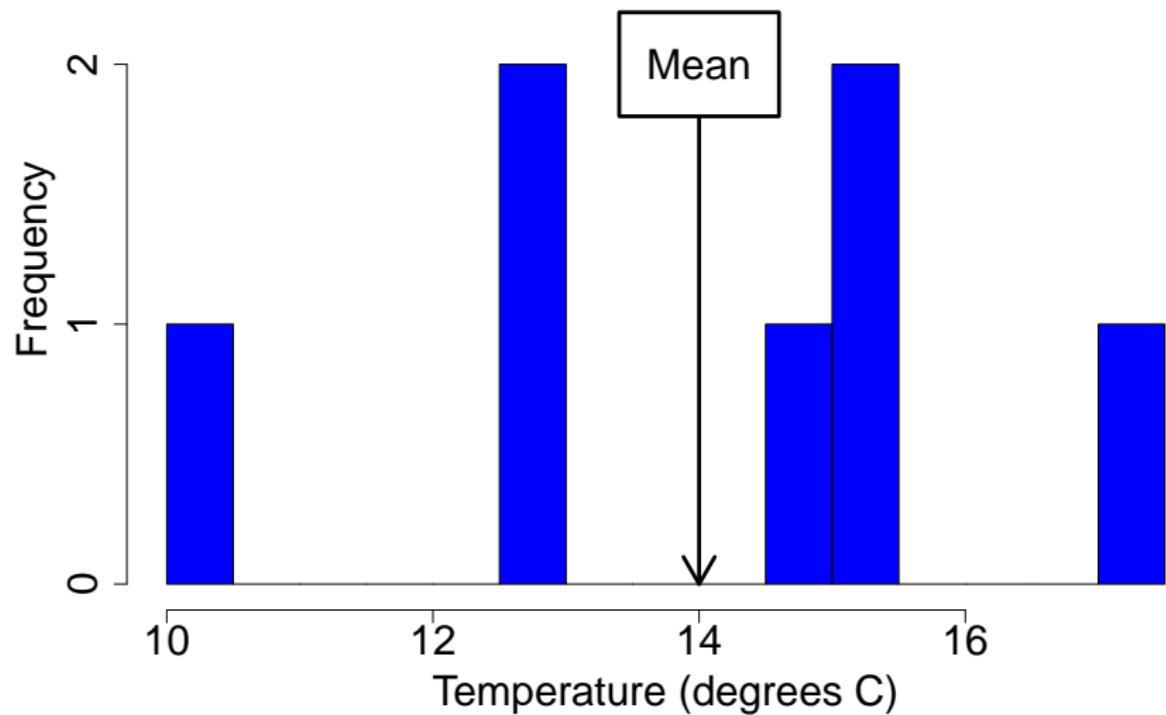
Table 1: Seven values (x) of soil temperature ($^{\circ}\text{C}$) at a site

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

$$\bar{x} = \frac{17.1 + 15.2 + 14.9 + 12.6 + 15.2 + 10.3 + 12.7}{7}$$

$$\bar{x} = 14$$

Arithmetic mean visualisation (histogram)



General formula for arithmetic mean

- ▶ Sample mean: \bar{x} (or $\hat{\mu}_x$)
- ▶ Sample size: N

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_{N-1} + x_N}{N}$$

General formula for arithmetic mean

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_{N-1} + x_N}{N}$$

$$\sum_{i=1}^N x_i = x_1 + x_2 + \dots + x_{N-1} + x_N$$

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

The mode

Most frequently occurring observation

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

Also applies to categorical data

x_1	x_2	x_3	x_4	x_5	x_6
dog	cat	bird	cat	cat	dog

Visualising the mode

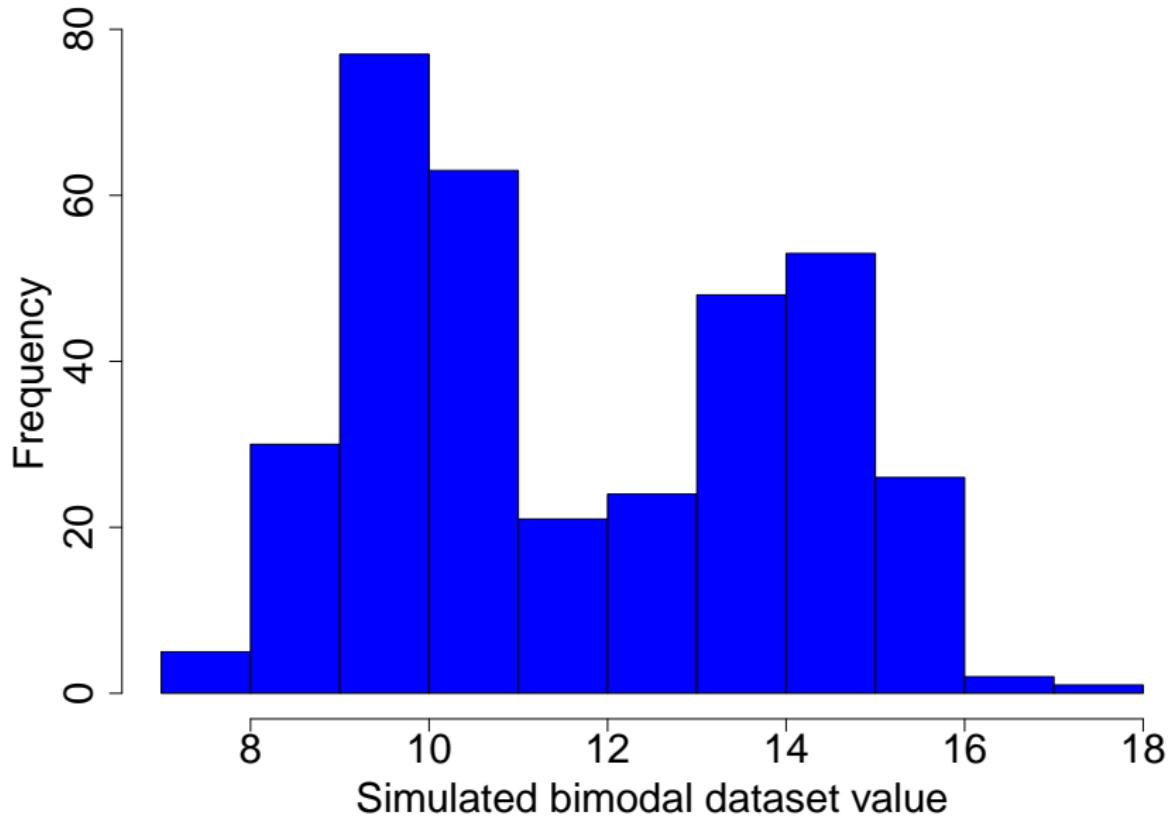


Figure 1: Hypothetical dataset that has a bimodal distribution.

The median

- ▶ Observation in the middle when the observations are arranged in ascending order
- ▶ There are an equal number of observations lower and higher than the median

The median

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

Sorting the data:

x_6	x_4	x_7	x_3	x_2	x_5	x_1
10.3	12.6	12.7	14.9	15.2	15.2	17.1

The median

Median is a type of **quantile** (50%)

- ▶ Can break distribution into other quantiles
 - ▶ First **quartile** (25% quantile)
 - ▶ Third **quartile** (75% quantile)
- ▶ Quantiles also called 'percentiles'

x_1	x_2	x_3	x_4	x_5
2	4	5	6	8

The median

If there is no middle value

x_1	x_2	x_3	x_4	x_5	x_6
3.1	3.5	3.8	4.0	4.2	4.2

Take mean of middle values:

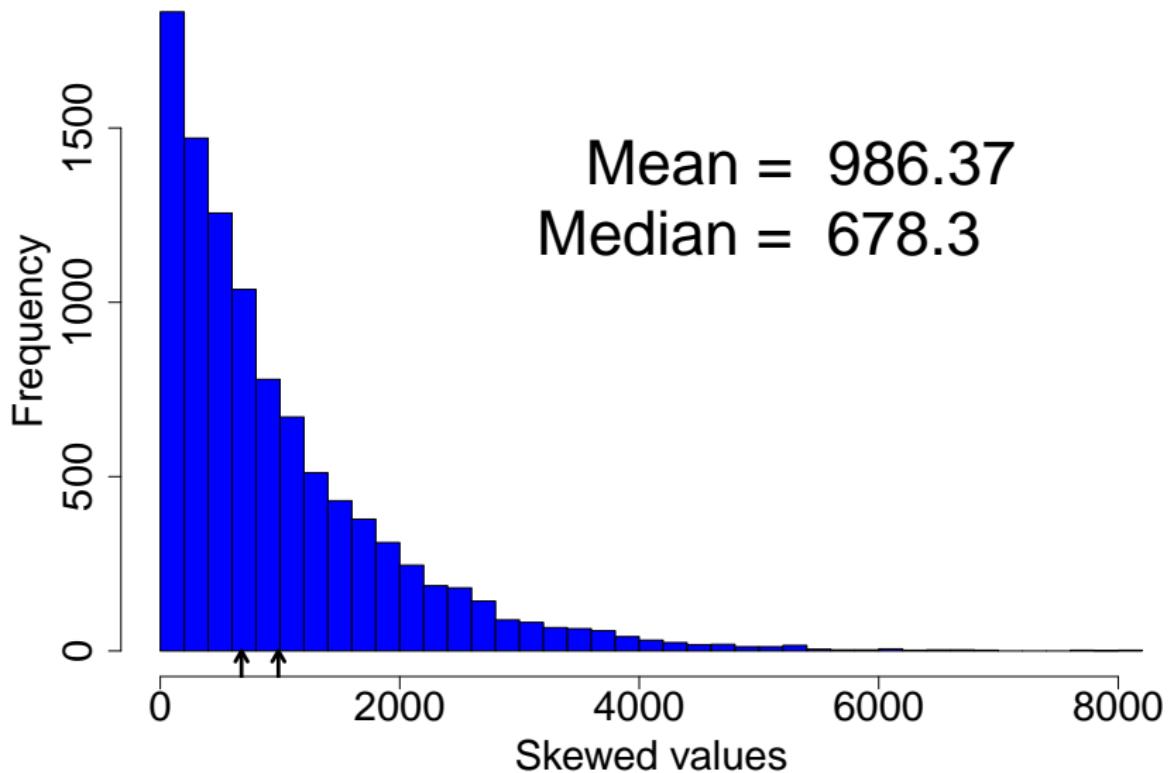
$$\frac{3.8 + 4.0}{2} = 3.9$$

The median

- ▶ Multiple valid ways to calculate quantiles¹
- ▶ No one ‘right’ way
- ▶ Jamovi’s approach might differ from other software

¹Hyndman, RJ, & Y Fan. 1996. American Statistician 50:361–65.

Median more robust to outliers



Measures of spread

- ▶ Range
- ▶ Interquartile range (IQR)
- ▶ Variance (s^2)
- ▶ Standard deviation (s)
- ▶ Coefficient of variation (CV)

Measures of spread

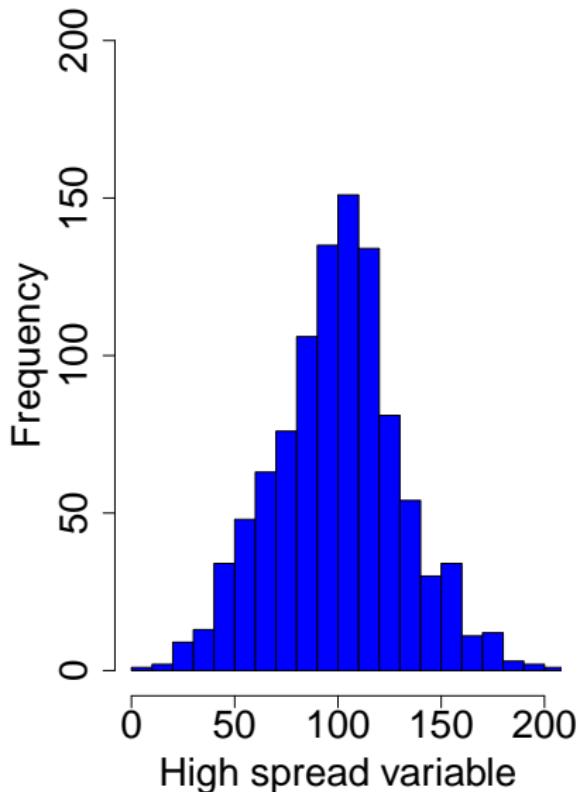
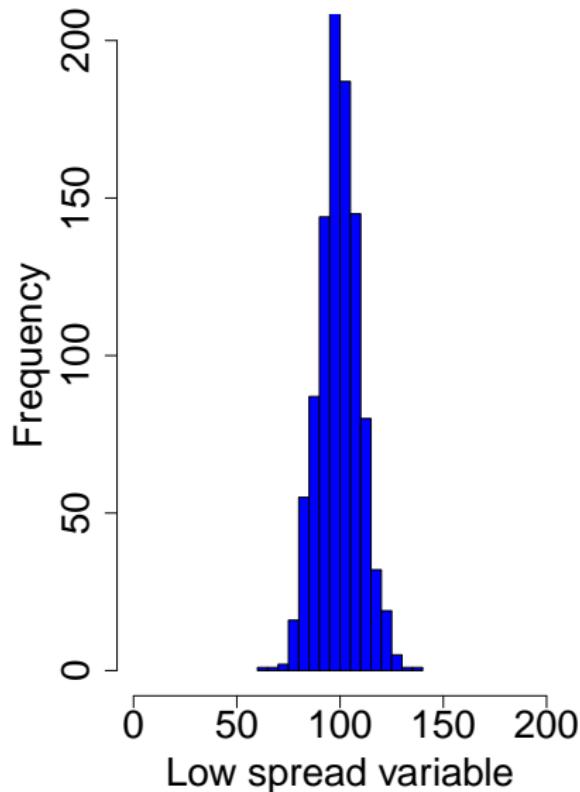
The screenshot shows the SPSS 'Descriptives' dialog box. In the 'Variables' list, 'Soil organic carbon (g C / kg soil)' is selected. The 'Central Tendency' section includes 'Mean', 'Median', 'Mode', and 'Sum'. The 'Dispersion' section, highlighted with a red box, includes 'Std. deviation', 'Variance', 'Range', 'Minimum', 'Maximum', and 'IQR'. The 'Distribution' section includes 'Skewness' and 'Kurtosis'. The 'Normality' section includes 'Shapiro-Wilk'. The 'Outliers' section includes 'Most extreme' and 'values'. The 'Results' panel displays descriptive statistics for 'Soil organic carbon (g C / kg soil)'.

Descriptives	Soil organic carbon (g C / kg soil)
N	34
Missing	0
Mean	6.52353
Median	5.80000
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Standard deviation	4.49701
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Range, IQR, s^2 , s , CV

Measures of spread



Measures of spread: Range

$$\text{Range}(X) = \text{Maximum}(X) - \text{Minimum}(X)$$

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

$$\text{Range}(X) = 17.1 - 10.3 = 6.8$$

Measures of spread: Interquartile Range

$$IQR(X) = Q_3(X) - Q_1(X)$$

x_1	x_2	x_3	x_4	x_5
2	4	5	6	8

$$IQR(X) = 6 - 4 = 2$$

Measures of spread: Variance (s^2)

- ▶ Expected squared deviation from mean
- ▶ More useful than range or IQR
- ▶ Less intuitive than range or IQR¹
- ▶ Jamovi will calculate this for us

$$s^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2.$$

We can break this down step by step!

¹<https://bradduthie.github.io/stats/app/forest/>

Measures of spread: Variance (s^2)

$$s^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2.$$

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

1. Take x_1 minus mean, squared $(17.1 - 14)^2 = 9.61$
2. Repeat step 1 for x_2, x_3, \dots, x_N
3. Sum up all these $(x_i - \bar{x})^2$ values
4. Multiply the sum by $1/(N - 1)$

Measures of spread: Variance (s^2)

$$s^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2.$$

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

$$\begin{aligned}SS &= (17.1 - 14)^2 + (15.2 - 14)^2 + \cdots + (12.7 - 14)^2 \\&= (3.1)^2 + (1.2)^2 + \cdots + (-1.3)^2 \\&= 30.64\end{aligned}$$

$$s^2 = \frac{1}{7-1} \times 30.64 = 5.1067 \text{ } {}^\circ C^2$$

Measures of spread: Standard deviation (s)

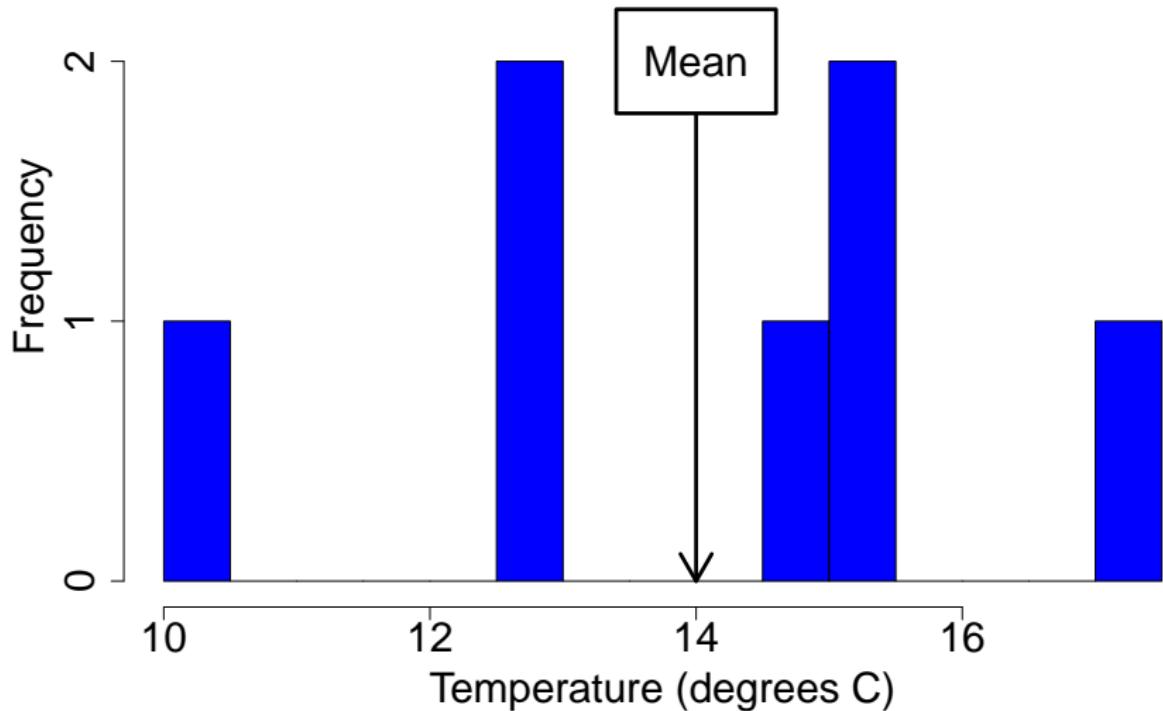
- ▶ Mean deviation from the mean
- ▶ Square-root of the variance
- ▶ Gets back to original units

$$s^2 = 5.1067 \text{ } ^\circ C^2$$

$$s = \sqrt{5.1067} = 2.2598 \text{ } ^\circ C$$

²https://bradduthie.github.io/stats/app/normal_pos_neg/

Standard deviation of the mean: does it look right?



Standard deviation of the mean

$$s = \sqrt{\frac{1}{N - 1} \sum_{i=1}^N (x_i - \bar{x})^2}.$$

- ▶ One checkbox in jamovi
- ▶ Spread of a variable

Coefficient of variation (CV)

Standard deviation divided by the mean

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

$$CV = \frac{s}{\bar{x}} = \frac{2.2598 \text{ } ^\circ C}{14 \text{ } ^\circ C} = 0.1614$$

Note that the units cancel out.

Coefficient of variation (CV)

Often expressed as a percentage

x_1	x_2	x_3	x_4	x_5	x_6	x_7
17.1	15.2	14.9	12.6	15.2	10.3	12.7

$$CV = \frac{2.2598 \text{ } ^\circ\text{C}}{14 \text{ } ^\circ\text{C}} \times 100\% = 16.14\%$$

Useful for comparing variation across categories (e.g., species)

Descriptive statistics: Skew and kurtosis

Descriptives

Variables: Soil organic carbon (g C / kg soil)

Split by:

Descriptives: Variables across columns, Frequency tables

Sample Size: N, Missing

Percentile Values: Cut points for 4 equal groups, Percentiles 25,50,75

Dispersion: Std. deviation, Variance, Range, Minimum, Maximum, IQR

Mean Dispersion: Std. error of Mean, Confidence interval for Mean 95 %

Central Tendency: Mean, Median, Mode, Sum

Distribution: Skewness, Kurtosis

Normality: Shapiro-Wilk

Outliers: Most extreme 5 values

Results

Descriptives

	Soil organic carbon (g C / kg soil)
N	34
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Skew is the asymmetry of a distribution

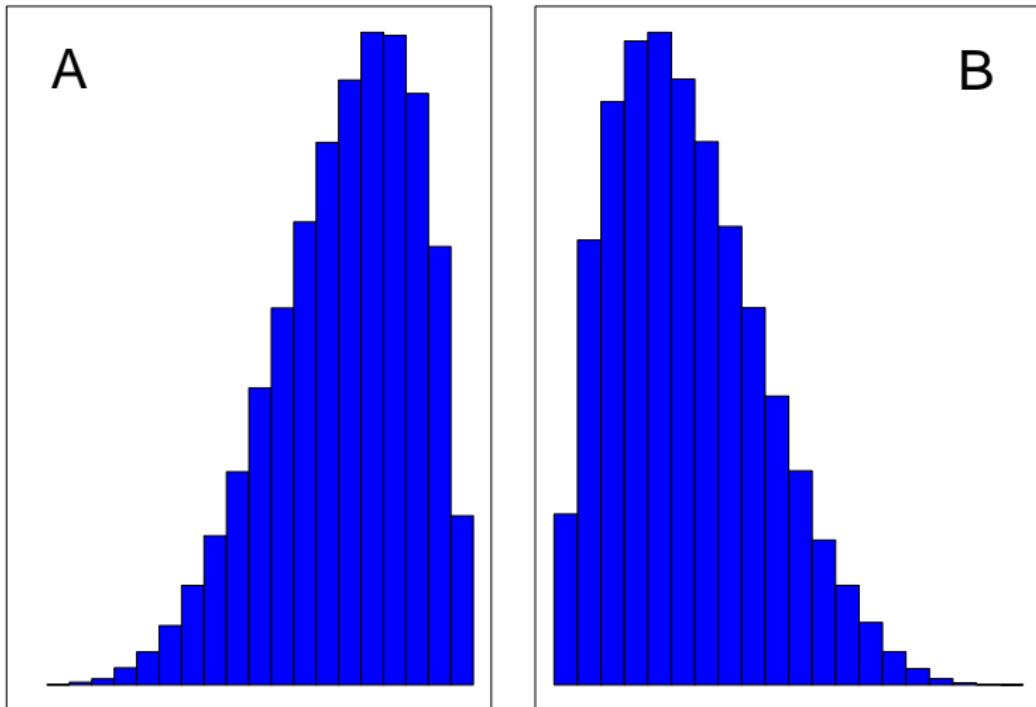


Figure 2: Histograms showing a (A) distribution that has a negative (i.e., 'left') skew and (B) distribution that has a positive (i.e., 'right') skew.

Kurtosis is the flatness of a distribution

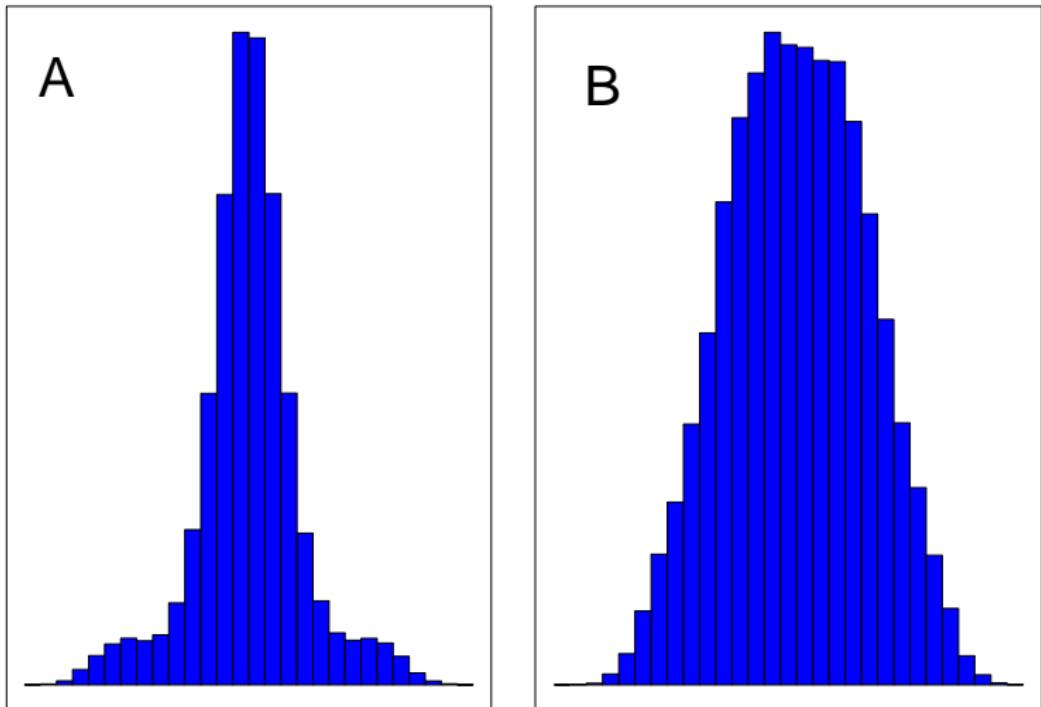


Figure 3: Histograms showing a (A) leptokurtic distribution and (B) platykurtic distribution.

Statistical moments

1. Mean
2. Variance
3. Skew
4. Kurtosis

Mathematically, deviations from mean raised to some power give the shape of a distribution.

Descriptive statistics in jamovi

Descriptives

Variables: Soil organic carbon (g C / kg soil)

Descriptives | Variables across columns Frequency tables

Statistics

Sample Size
 N Missing

Percentile Values
 Cut points for 4 equal groups
 Percentiles 25,50,75

Dispersion
 Std. deviation Minimum
 Variance Maximum
 Range IQR

Mean Dispersion
 Std. error of Mean
 Confidence interval for Mean 95 %

Central Tendency
 Mean Median Mode Sum

Distribution
 Skewness Kurtosis

Normality
 Shapiro-Wilk

Outliers
 Most extreme 5 values

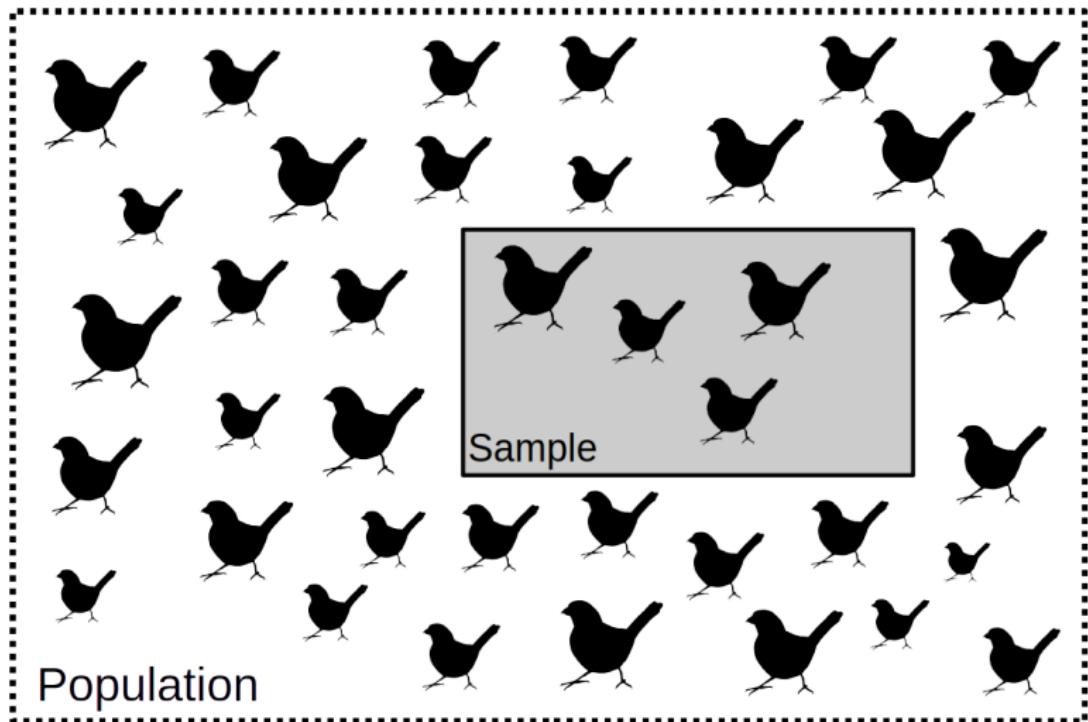
Results

Descriptives

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The standard error: Sample means and population mean



The standard error: Sample means and population mean

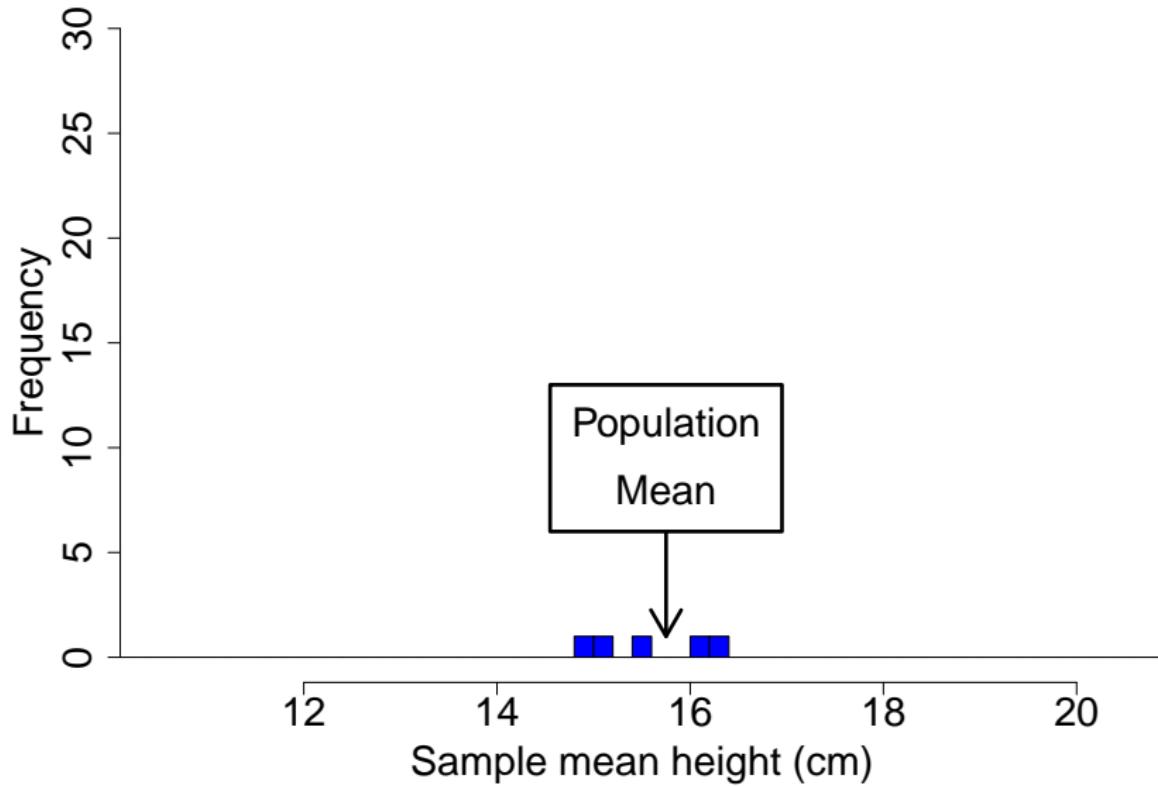
<u>Samples</u>	<u>Average height</u>
	15.53 cm
	16.21 cm
	16.09 cm
	14.88 cm
	15.06 cm

Repeated re-sampling

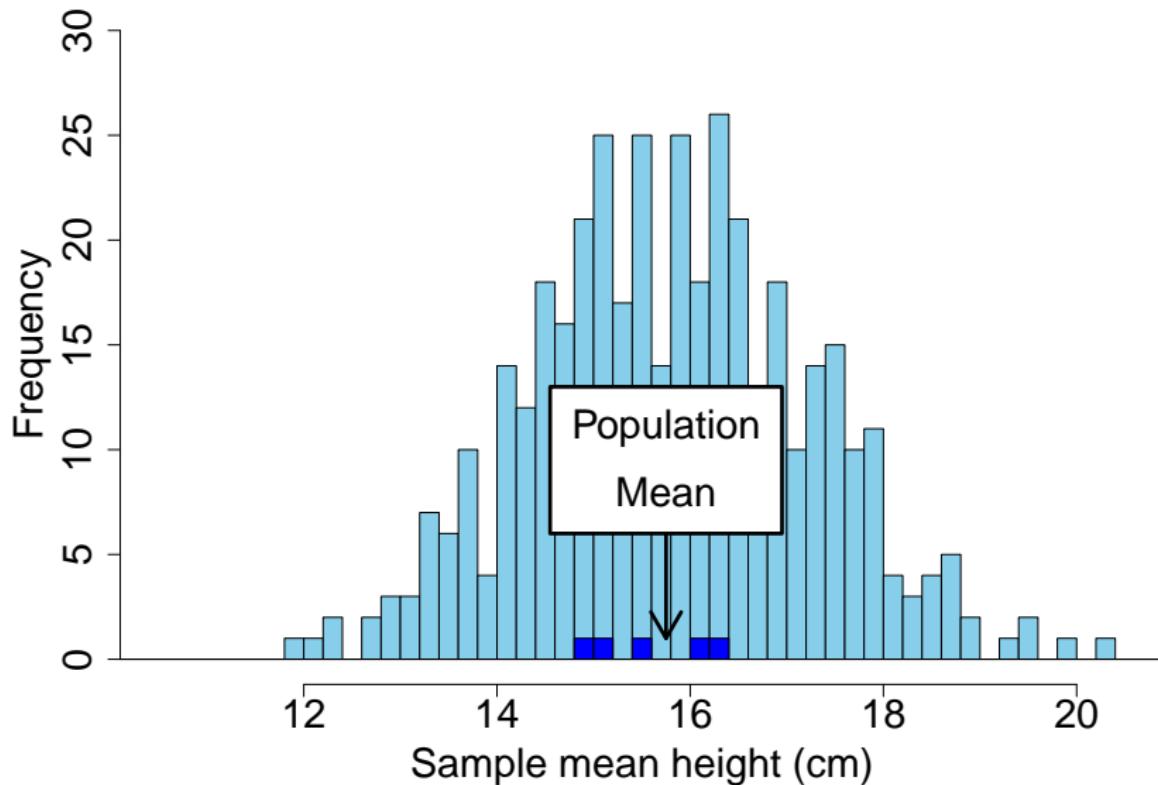
- ▶ What is our expectation?
- ▶ What is our uncertainty?

What is the **distribution** of
the *sample mean* (\bar{x}) around
the *population mean* (μ_x)?

The standard error: Sample means and population mean



The standard error: Sample means and population mean



The standard error: Sample means and population mean

- ▶ What is the **distribution** of the *sample mean* (\bar{x}) around the *population mean* (μ_x)?
- ▶ **Standard error:** The standard deviation of sample means around the population mean

The standard error: estimation

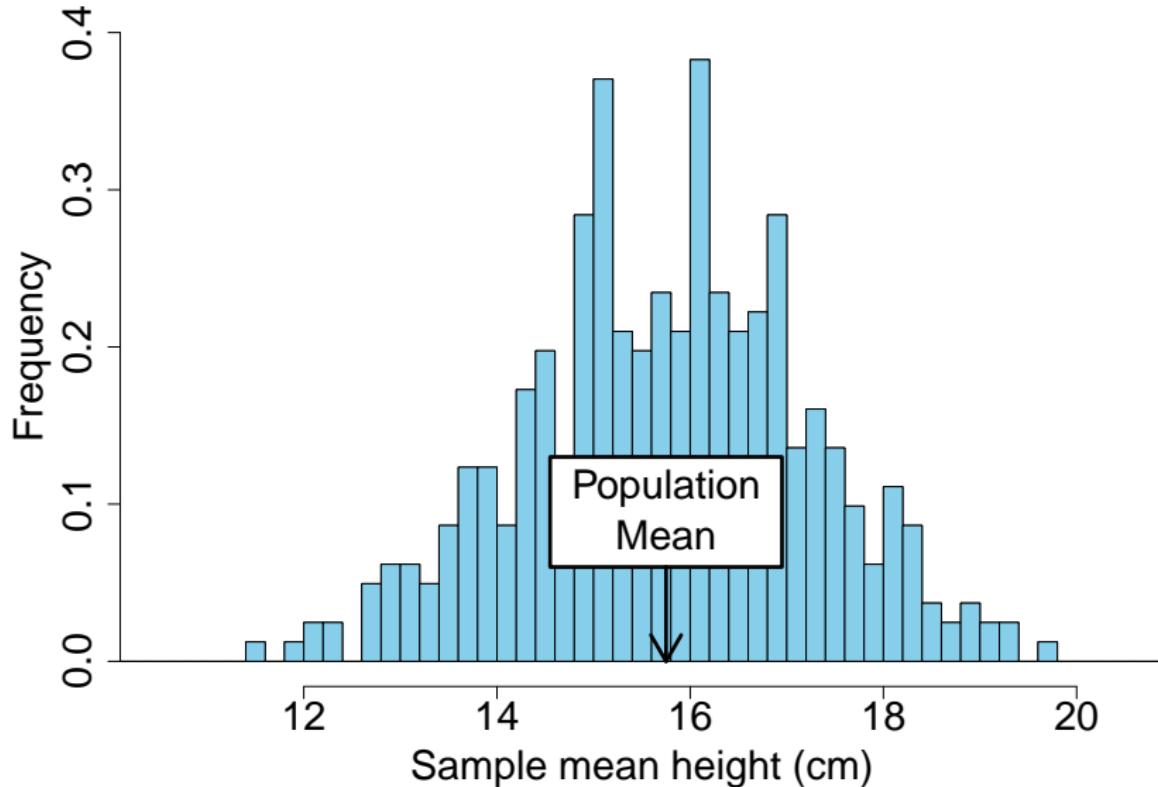
$$SE = \frac{\text{Sample standard deviation}}{\sqrt{\text{Sample size}}}$$

The standard error: estimation

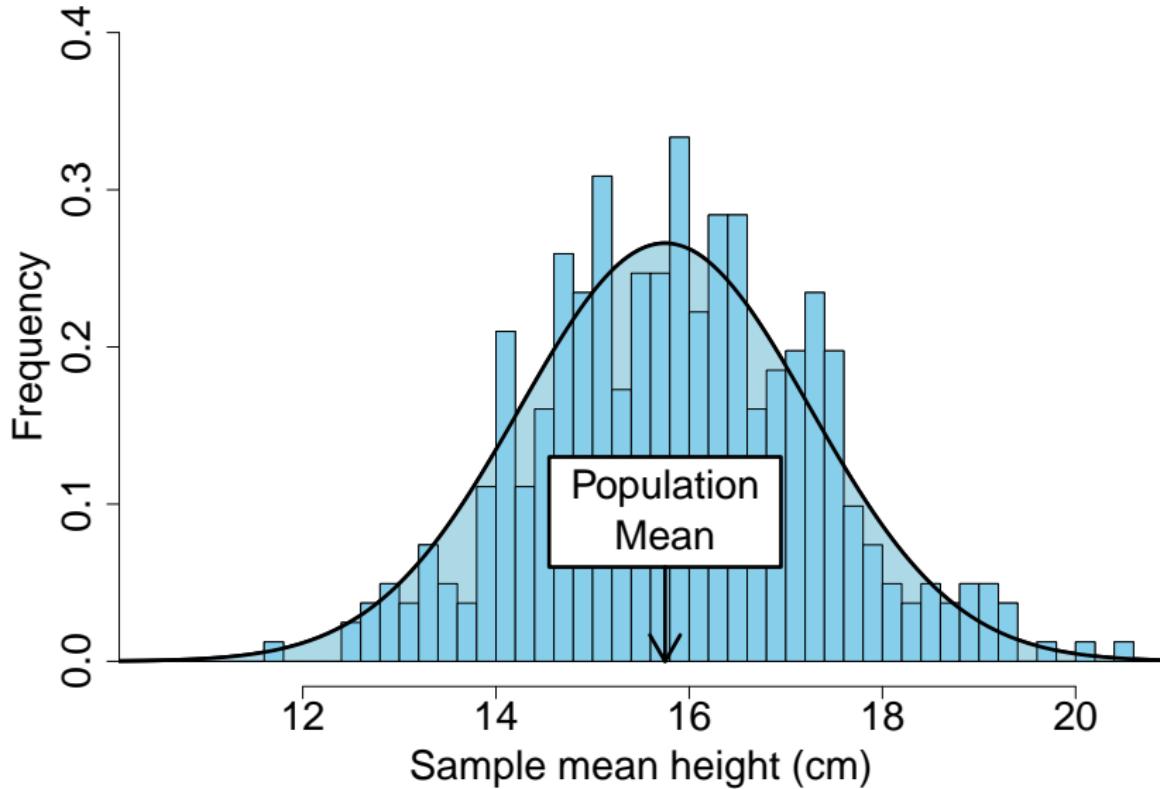
$$SE = \frac{\text{Sample standard deviation}}{\sqrt{\text{Sample size}}}$$

$$SE = \frac{s}{\sqrt{N}}$$

The standard error



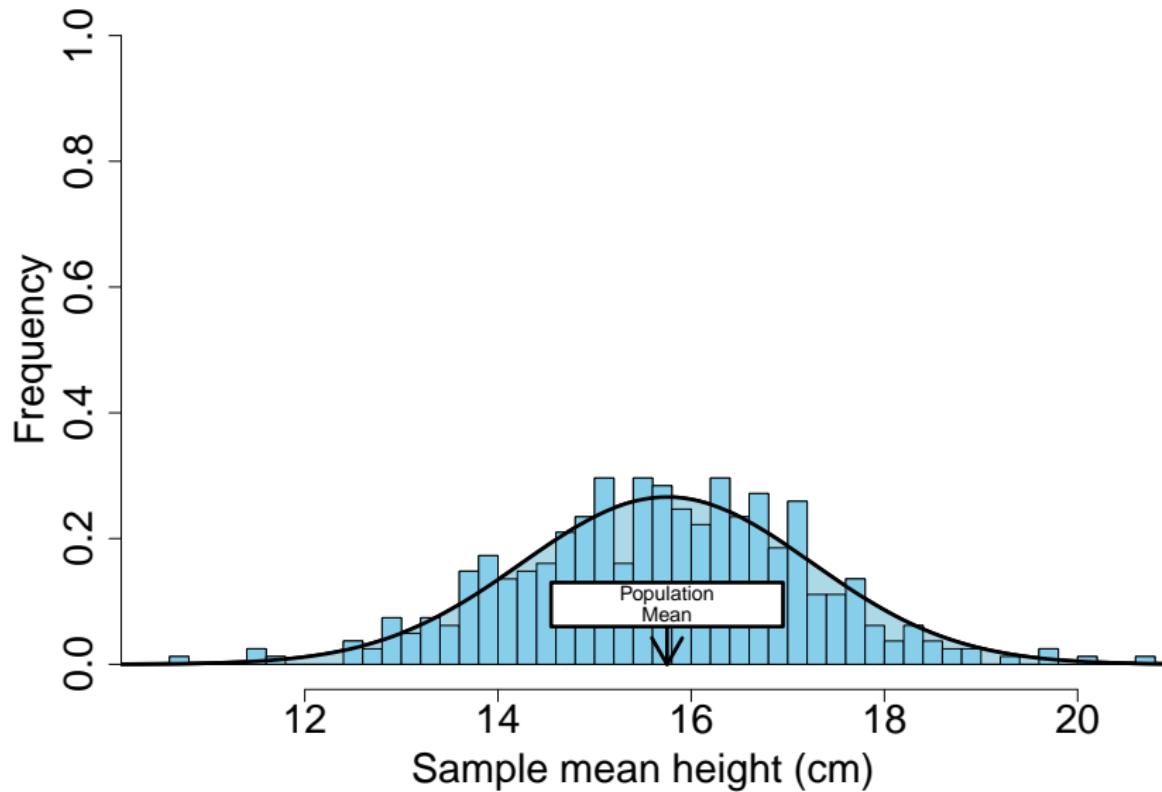
The standard error



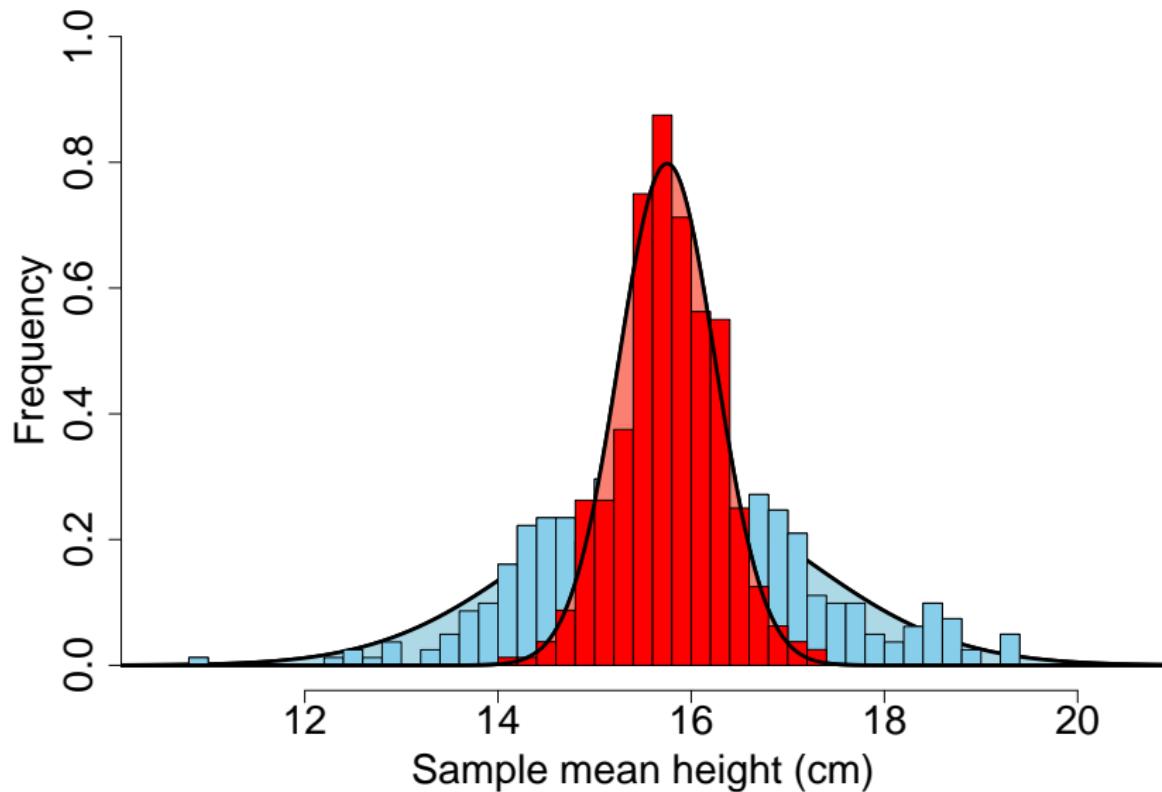
The standard error: Sample means and population mean

- ▶ **Standard error:** The standard deviation of sample means around the population mean
- ▶ Standard error measures the uncertainty of the sample mean

The standard error



The standard error



The standard error in jamovi

The screenshot shows the jamovi interface with the 'Descriptives' module open. On the left is the 'Descriptives' panel, and on the right is the 'Results' panel.

Descriptives Panel:

- Variables:** Soil organic carbon (g C / kg soil) is selected.
- Split by:** Soil type is selected.
- Descriptives:** Variables across columns is selected.
- Statistics:** Sample Size, Central Tendency, Percentile Values, Dispersion, Distribution, Mean Dispersion, and Outliers are selected.
- Sample Size:** N and Missing are checked.
- Central Tendency:** Mean, Median, Mode, and Sum are checked.
- Percentile Values:** Cut points for equal groups and Percentiles (25, 50, 75) are selected.
- Dispersion:** Std. deviation, Variance, Range, Std. error of Mean, and Confidence interval for Mean are selected.
- Distribution:** Minimum, Maximum, Skewness, Kurtosis, Shapiro-Wilk, and Most extreme values are selected.
- Mean Dispersion:** Std. error of Mean and Confidence interval for Mean are selected.
- Outliers:** Most extreme values are selected.

Results Panel:

Descriptives

	Soil organic carbon (g C / kg soil)
N	34
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