

Project - Conceptual Brief (draft)

Context and Motivation

In Swiss table tennis, each competitive player is assigned an Elo rating. This rating can be thought of as a player's current

- A player with a high rating is considered a player with fewer losses.
- The difference between two players reflects how well one performs against the other.
- When a player wins or loses a match, their Elo rating is updated.

In this sense, a player's Elo rating evolves over time as matches are played.

While the Elo update rule is deterministic once a match outcome is known, the outcome of a match is uncertain. Players face different opponents, match scheduling can lead to very different ranking trajectories.

As a competitive player, I am not interested in predicting my future Elo rating. Instead, I want to understand the range of possible outcomes. How much could my Elo realistically increase or decrease?

Core Idea of the Project

The key idea of the project is to simulate a player's Elo rating over a fixed trajectory.

Instead of asking:

"What will my Elo be next month?"

the project asks:

"If this month were replayed many times under realistic conditions, what would we observe?"

To answer this question, a simulation of a month is run thousands of times, each time with different starting conditions. The final Elo rating is then averaged to provide a predicted value.

What Is Being Simulated

The project simulates the evolution of a player's Elo rating over a fixed trajectory, treating this month as a probabilistic sequence of events.

At a high level, each simulated month follows the following structure:

1. A monthly competition is defined, consisting of league matches and tournaments (rosters or tournament pools).
2. For each match, a probabilistic outcome is determined based on the players' current Elo ratings and other factors.

3. A probability of a swing in the match.

4. A win or loss is randomly drawn based on this probability.

5. Elo points are updated using the official Swiss Elo update.

6. The entire month is simulated using the Monte Carlo simulation distribution of Elo points.

The goal is not to predict a single future Elo value, but to generate plausible realizations of the same competitive month.

All detailed modeling assumptions (competition format and parameter values) are specified in the model description.

Why Monte Carlo Simulation Is Used

A single month of competition is subject to substantial uncertainty. Opponents vary in strength, and small differences in performance can lead to large changes in ranking.

Monte Carlo simulation addresses this uncertainty by simulating each time with different random draws of performance values, rather than a single point estimate.

This distributional perspective allows us to quantify:

- expected Elo change,
- downside and upside risk,
- and the overall variability of ranking outcomes.

Role of Win Probability Modeling

A key component of the simulation is the assignment of win probabilities to each match.

The project deliberately distinguishes between:

- the simulation of match outcomes,
- the update of Elo points, which follows the official Swiss Elo update.

Two alternative models are used to generate match win probabilities:

1. Official Elo probabilities derived directly from the Swiss Elo ratings.
2. Machine-learning probabilities estimated empirically from historical data involving many players.

Importantly, these two models generate different distributions of outcomes. All Elo point updates are computed using the same simulation, ensuring that the ranking evolution remains realistic and comparable.

What Is Compared Across Simulations

The simulation is run separately under each probability model, and the results are compared.

The comparison focuses on:

- how the distribution of monthly Elo outcomes
- whether dispersion, skewness, or tail risk differ
- and how sensitive ranking uncertainty is to parameter choices

The project does not aim to declare one probability distribution in any sense. Instead, it evaluates how different modeling profiles for ranking evolution.

Output of the Project

The final outputs of the simulation are:

- distribution of monthly Elo values,
- summary statistics (mean, quantiles),
- and visual comparisons between scenarios.

These outputs form the basis for interpretation.

Transition to Modeling Choices & Hypotheses

The remainder of the project formalizes the elements of the model. In particular, the modeling choices and hypotheses are:

- the exact structure of a competitive month,
- opponent sampling rules,
- win probability models,
- and the Elo update mechanism.

The code implementation is expected to follow the additional assumptions.

Research Question

How uncertain is the distribution of a Swiss table tennis player's monthly Elo rating, and how sensitive is this uncertainty to modeling choices? (official Elo probabilities are used as a baseline for comparison)

Modeling Choices and Hypotheses

Scope and Objective of the Model

The objective of the model is not to predict a specific distribution of monthly Elo ratings for a Swiss table tennis player. The model is intended to explore the uncertainty and variability inherent in the ranking process.

The analysis is structured to ensure that the model remains interpretable while capturing the complexity of the ranking evolution.

Structural Assumptions on Competition Format

A month of competition is modeled as a fixed and static workload to approximate a realistic competitive workload rate.

The player is assumed to participate in:

- two league, tiered competition, consisting of 6 home and 6 away matches
- one national tournament

To ensure a consistent number of matches across a hypothetical player, it is assumed that a fixed workload tournament is assumed to be evenly divided among the group of players. This ensures that the number of matches is consistent across all players.

This structure reflects the standard Swiss tournament system, which reduces the number of matches played by each player, while maintaining the intensity of competition over a month while running Monte Carlo simulations. The implications of this structure are discussed in the next section.

Match Outcome Assumptions

Each match outcome is modeled as a random variable.

- Match scores, set differences, and margin of victory
- The effect of these omitted variables is implicit in the model.

Match outcomes are assumed to be independent of each other, meaning that any dependence between matches operates only over time.

Opponent Sampling Assumptions

Opponents faced by the player during the month are sampled from the database for the national tournament, reflecting the institutional structure of the tournament.

League opponents (based on the league table), roster

For league play, the opponent is sampled from the league table, assuming a fixed number of opponents at the beginning of the league. For each of the opponents faced by the player is modeled as a random variable.

To reflect selection patterns in team matches, the number of matches played by that player is assumed to be proportional to the number of matches played by that player divided by the total number of matches played by all players.

Tournament opponents (A/B pool with no rematches)

For the national tournament, eligible opponents are sampled from the database, restricted to those players who are eligible to play in the tournament. The player participates only in tournaments whose results are included in the database, and it ensures that the analysis remains at the player level.

In the baseline model, opponents are sampled under minimal tournament realism with the assumption that after an opponent has been faced in the tournament, tournament simulation. This approximation captures repeated matchups, while remaining computationally

Win Probability Models

To simulate match outcomes within the Monte Carlo probability that the focal player wins a given match, a probability model-based chess model is used in the baseline model. The purpose of this comparison is not to but to assess how sensitive ranking uncertainty is

Elo-Based Probability Model

In the first approach, win probabilities are computed from a formulation. Under this model, the probability of a difference between their Elo ratings and is given

$$p_{ij} = \frac{1}{1 + 10^{\frac{r_j - r_i}{200}}}$$

where r_i and r_j denote the Elo ratings of players i and j .

This formulation is directly embedded in the Swiss tournament theoretical. It assumes that the Elo rating fully determines match outcomes are probabilistically determined by rating

Machine Learning Probability Model

The second approach replaces the fixed Elo probability estimate learned from historical match data.

Rather than imposing a predefined functional relationship between win probabilities, the machine learning model infers from historical matches. Conceptually, the model characterizes the relationship between features and outcomes to estimate the probability

To avoid overfitting and to ensure generalization, the model is trained on a dataset comprising many players rather than only matches played by the focal player. The model is used within the Monte Carlo simulation.

Minimal Feature Specification

In line with the objective of minimizing model complexity, the machine learning model uses a set of features:

- Elo rating of the focal player

- Elo rating of the opponent

No additional contextual variables (such as recent performance) are included in the baseline specification. This design remains directly comparable to the standard Elo system.

Model Choice and Interpretation

A logistic regression model is used as the baseline, chosen for its flexibility and transparency:

- Logistic regression allows for flexible modeling.
- Its structure closely mirrors the logistic formula, with coefficients estimated from data rather than fixed a priori.
- The resulting probability function can be interpreted as an updated version of the Elo formula.

Importantly, the machine learning model is not a black box; its structure is transparent and interpretable, allowing us to understand how it relates to the observed frequencies in the data.

Role in the Monte Carlo Simulation

Both probability models are embedded within a larger framework that accounts for structural assumptions, opponent sampling procedures, and the differences observed in the data. The model is designed to capture the underlying patterns in the data, rather than to simply describe the observed differences.

Interpretation Scope

Differences between the two simulations are not interpreted as definitive conclusions about the underlying process, but rather as a probabilistic estimate, not as a definitive comparison. The primary goal is to provide a probabilistic estimate of the underlying process.

Elo Update Mechanism

After each simulated match, the player's Elo rating is updated according to the following rule. For a given match, the Elo change is calculated as:

$$\Delta E_i = K \times (result_i - E_i)$$

where:

- K is the Elo adjustment factor,
- $result_i$ is 1 in case of a win and 0 in case of a loss,
- E_i is the player's current Elo rating.

The win probability is calculated using the fixed Elo ratings of both players.

$$= \frac{1}{1 + 10^{\frac{\text{opp player} - \text{player}}{200}}}$$

Importantly, within a given month, the player's Elo rating at the beginning of the month

If the player plays n matches in the month, the total update is the sum of individual match updates:

$$E_{\text{month}} - E_{\text{start}} = 1$$

The player's Elo rating at the end of the month is

$$E_{\text{end}} = E_{\text{start}} + E_{\text{month}}$$

where E_{start} denotes the Elo rating at the beginning of the month.

In the Monte Carlo simulation, match-based probabilities are used to approximate the continuous probability model or the continuous probability model described above. This is always followed by a simulation of the match outcome, which is then used to update the Elo rating. This is consistent with the Elo system.

Monte Carlo Assumptions

Uncertainty in ranking is modeled using Monte Carlo simulation.

- The same month of competition is simulated repeatedly.
- Each simulation uses independent random draws.
- The collection of final Elo ratings across simulations is used to estimate the distribution of outcomes.

The number of simulations is chosen sufficiently large to estimate the desired properties such as quantiles and variance.

Simplifications and Excluded Factors

Several factors are intentionally excluded from the simulation.

- Physical condition, fatigue, and injuries
- Psychological effects
- Strategic behavior and learning during tournaments
- Long-term form dynamics

These exclusions are motivated by data limitations and interpretability. Their potential impact is discussed in the appendix.

Interpretation Framework

The resulting Elo distributions are not directly interpretable in financial modeling.

Differences between distributions obtained under uncertainty are greater than purely random noise.

Methodology (Conceptual Algorithm)

This section describes the simulation procedure for Elo ratings.

The methodology translates the modeling choices without introducing additional assumptions or implementation details.

Step 1: Initialization

At the beginning of each simulation run:

- A focal player is defined with an initial Elo rating.
- This Elo rating remains fixed throughout the simulation as a reference rating for all probability and Elo updates.
- All model parameters (competition structure, opponent selection, etc.) are fixed and identical across simulation runs.

As a result, all variability in simulated outcomes is attributed to random noise and match outcomes.

Step 2: Definition of the Competitive Month

Each simulation run represents one stylized month of competition.

The competitive month consists of a fixed sequence of events:

- two league encounters, each composed of three matches.
- one national tournament with a fixed total number of matches.

The tournament structure is designed to guarantee realism and includes institutional constraints such as the absence of international matches.

The player is assumed to participate in all scheduled matches.

Step 3: Opponent Selection

For each match in the simulated month, an opponent is selected.

- League matches: opponents are drawn from the rosters of competing teams, with selection probabilities reflecting observed participation rates.
- Tournament matches: opponents are sampled from the pool of eligible players, subject to tournament constraints within the tournament structure.

Opponent Elo ratings are treated as known and fixed at the time of selection.

Step 4: Win Probability Assignment

Once an opponent is selected, a probability of win is computed using the official Swiss Elo rating. Two alternative probability models are considered:

- Elo-based probabilities computed using the official Swiss Elo rating.
- Machine-learning probabilities estimated from historical match data using the same Elo inputs.

In both cases, win probabilities depend on the fixed Elo rating.

Step 5: Match Outcome Simulation

Given the assigned win probability:

- The match outcome is simulated as a binary random variable.
- Outcomes are generated using independent random draws from the assigned probabilities.

This step captures the intrinsic randomness of chess.

Step 6: Elo Update Calculation

After each simulated match:

- An Elo change is computed using the official Swiss Elo update formula.
- The expected score used in the update formula is the player's rating and the opponent's Elo rating.

Although Elo changes are small, they accumulate over time, affecting subsequent probabilities within the same month.

The total Elo change over the month is obtained by summing individual updates.

Step 7: End-of-Month Elo Recording

At the end of the simulated month:

- The player's final Elo rating is computed as the initial rating plus the total change.
- This final Elo value represents one possible realization of the rating model.

Step 8: Monte Carlo Repetition

Steps 1-7 are repeated a large number of times.

Each repetition represents an alternative but plausible scenario, differing only through random opponent selection and match outcomes.

This procedure generates a large sample of final Elo ratings.

Step 9: Aggregation and Analysis

From the simulated distribution:

- summary statistics (mean, variance, quantiles) and
- distributional properties such as dispersion and
- results are compared across probability models

No individual simulation run is interpreted as a result used for interpretation.

Methodological Clarification

The simulation does not simulate the process directly following the question:

Given a fixed competitive month and fixed Elo on the first day, what Elo outcomes arise purely from randomness and probability?

Why This Methodology Is Consistent

- It respects the institutional rule that Elo ratings are updated monthly
- It ensures full comparability across probability models
- It provides a clean additive structure for interpretation

Expected Results and Interpretation

This section outlines the qualitative patterns and predictions from the Monte Carlo simulation, taking into account the predictions derived directly from the structure of the model and the assumptions described in the previous sections.

The purpose of this section is not to anticipate the results through which uncertainty arises and to define the expected results.

Expected Shape of the Elo Distribution

Given that the monthly Elo change is modeled as a random variable, the distribution of Elo ratings is expected to be

- approximately unimodal,
- centered around a mean close to the expected Elo rating
- with dispersion increasing as the number of matches increases

However, because individual match outcomes are based on probabilities that differ from 50%, the resulting distribution may exhibit mild skewness, which may arise if the probabilities are not symmetrically distributed around 50%.

Expected Role of Match Volume and Component

The simulated month includes a relatively large number of matches, including a tournament.

As a result:

- the variance of the monthly Elo change is expected to be the sum of the variance of a single match outcome,
- extreme positive or negative outcomes become proportionally more moderate.

The tournament component, which introduces a break in the continuity of play, is expected to contribute disproportionately to the downside risk relative to league play alone.

Expected Impact of Fixed Elo Within the Tournament

Because the Elo rating is held fixed throughout the tournament:

- win probabilities do not adapt to interim success or failure,
- early wins do not mechanically increase the probability of winning the tournament.

This absence of feedback implies that the total Elo change is the sum of independent realizations. Consequently, the distribution of outcomes is expected to be more static, reflecting uncertainty rather than dynamic momentum effects.

This design choice is expected to yield cleaner results by ignoring the psychological dynamics that often influence performance.

Expected Comparison Between Probabilistic Models

When comparing simulated Elo-based probability models with Elo-based probability models, several qualitative differences are expected.

If both models produce similar average Elo change, the model with the more dynamic Elo change is expected to be more accurate. However, the following properties are expected to differ:

- dispersion may differ if one model assigns more weight to individual match outcomes,
- tail risk may increase if the machine learning model is present in the Elo formula,
- skewness may differ if one model uses a non-symmetric probability distribution for match outcomes against specific opponent segments.

Importantly, any such differences are not definitive evidence of predictive superiority. They are merely indicators of model behavior that require further investigation.

Expected Sensitivity to Opponent Sampling

Because opponents are sampled probabilistically, the expected outcome is expected to be sensitive to the realized opponent.

Simulations in which the player faces an unusual opponent will populate the lower or upper tails of the distribution. Ranking uncertainty arises not only from the stochastic nature of competition exposure.

Interpretation Framework

The simulated Elo distribution is a large distribution of outcomes in financial contexts.

In this interpretation:

- the mean Elo change represents expected performance,
- the dispersion captures ranking volatility,
- the lower tail reflects downside risk,
- the upper tail reflects upside potential.

The comparison across information systems involves the use of the model to modeling assumptions, rather than providing a direct comparison.

Role of This Section

This section establishes the conditions under which the empirical results are evaluated. Deviations between observed results and the model provide informative signals about the interaction between stochasticity and the model.