

# Project - Conceptual Brief (draft)

## Context and Motivation

In Swiss table tennis, each competitive player is assigned a rating. This rating can be interpreted as a player's current performance level.

- A player's Elo rating reflects their skill level, with a few wins and losses over time.
- The difference between two players reflects how well one player performs against the other.
- When a player wins or loses a match, their Elo rating is updated.

In this sense, the Elo rating is a trajectory that is so ordered by points evolve over time as matches are played.

While the Elo update rule is deterministic once a match is played, it is uncertain. Players face different opponents, and competition can lead to very different ranking trajectories.

As a competitive player, I am not interested in predicting my future Elo rating. Instead, I want to understand the outcomes of matches. How much could my Elo realistically increase or decrease?

## Core Idea of the Project

The key idea of the project is to simulate multiple possible Elo rating trajectories over a fixed period of time.

Instead of asking:

"What will my Elo be next month?"

the project asks:

"If this month were replayed many times under random conditions, what would we observe?"

To answer this question, the project simulates this month thousands of times, each time with different initial Elo ratings and final results than a single predicted value.

## What Is Being Simulated

The project simulates the evolution of the Elo rating over a month, starting from the beginning of the month as a probabilistic experiment of events.

At a high level, each simulated month follows these steps:

1. A monthly competition is defined consisting of league matches.
2. For each match, participants are selected from institutional rosters or tournament pools.

3. Probabilities of winning go to the match.
4. A win or loss is randomly drawn based on this probability.
5. Elo points are updated using the official Swiss Elo update rule.
6. The entire process is repeated many times. Monte Carlo simulation distribution of elo points is then obtained.

The goal is not to predict a single future Elo value, but plausible realizations of the same competitive month.

All detailed modeling assumptions (competition form and parameter values) are specified except for hypotheses.

### Why Monte Carlo Simulation Is Used

A single month of competition is subject to substantial fluctuations. Opponents vary in strength, and small differences in opponents' elo values can lead to large differences in results.

Monte Carlo simulation addresses this uncertainty by each time with different random draws from the distribution of elo values rather than a single point estimate.

This distributional perspective allows us to quantify:

- expected Elo change,
- downside and upside risk,
- and the overall variability of ranking outcomes.

### Role of Win Probability Modeling

A key component of the simulation is the assignment of win probabilities.

The project deliberately distinguishes between:

- the simulation of match outcomes
- the update of Elo points follows the official Swiss Elo update rule.

Two alternative models are used to generate match outcomes:

1. Official-based probabilities are directly from the Swiss Elo update rule.
2. Machine-learning-based probabilities are empirically from historical data involving many players.

Importantly, these two models' outcomes are generated independently. All Elo point updates are based on the same underlying principle that ranking evolution remains realistic and comparable.

### What Is Compared Across Simulations

The simulation is run separately under each probability constant.

The comparison focuses on:

- how distribution of math probabilities, Elo outcomes
- whether dispersion, skewness, or tail risk differ
- and how sensitive ranking uncertainty is to parameters

The project does not aim to declare one probability in a strict sense. Instead, it evaluates how different modeling profiles for ranking evolution.

### Output of the Project

The final outputs of the simulation are:

- distributions of Elo values,
- summary statistics (mean, quantiles),
- and visual comparisons between scenarios.

These outputs form the basis for interpretation.

### Transition to Modeling Choices & Hypotheses

The remainder of the project formalizes the elements of the model in part:

- the exact structure of a competitive month,
- opponent sampling rules,
- win probability models,
- and the Elo update mechanism.

The code implementation is expected to follow these additional assumptions.

## Research Question

How uncertainty is reflected in the Elo distribution of a Swiss table tennis month of competition, and how sensitivity is modeled (official vs. Edged probability distributions) under different alternative hypotheses.

## Modeling Choices and Hypotheses

### Scope and Objective of the Model

The objective of the model is not to predict a distribution of performance in a Swiss table tennis month. The model therefore focuses on capturing uncertainty for the analysis.

The analysis is restricted to one type of model to allow the model to remain interpretable while capturing evolution.

## Structural Assumptions on Competition Form

A month of competition is modeled as a fixed and static period to approximate a realistic competitive workload rate.

The player is assumed to participate in:

- two league, eenaccohu nctbehisse stting doon in du anbt oahds
- one national tournament

To ensure a consistent number of matches across a hypothetical baseline, it guarantees a fixed workload. The national tournament is assumed to break up the month into two groups of 10 days each, with a break in placement between the two tournaments.

This structure effectively reduces the standard Swiss tournament structure to 10 days, which reduces the number of matches while maintaining the intensity of competition over a month while remaining consistent with Monte Carlo simulations. The implications of this

## Match Outcome Assumptions

Each match outcome is independent of previous matches.

- Match scores, set differences, and margin of victory are independent of previous matches.
- The effect of these omitted variables is implicitly captured by the random draw of opponents.

Match outcomes are independent of previous matches, meaning that any dependence between matches operates only within the same day.

## Opponent Sampling Assumptions

Opponents faced by the player during the month are drawn from a pool of eligible opponents for the national tournament, reflecting the institutional constraints of the tournament.

League opponents (based on the database), rostered by team.

For league play, the opponents are drawn from a pool of fixed at the beginning of the league. For each opponent faced by the player is modelled as a fraction of the total opponents in the league.

To reflect selection patterns in team matches, the fraction of opponents faced by the player is assumed to be proportional to the fraction of matches played by that player divided by the total number of matches played by all players.

Tournament opponents (A/B pool with no rematches).

For the national tournament, eligible opponents are drawn from a pool of fixed size. The database restricts opponents to those who have not yet participated in tournaments whose results have been published. This ensures that the analysis remains competitive.

In the baseline model, opponents are sampled under minimal tournament realism without regard to the location of an opponent has been faced in the tournament, tournament simulation. This approximation captures repeated matchups, while remaining computational efficient.

## Win Probability Models

To simulate match outcomes within the Monte Carlo probability that the focal player wins a given match, a probability model-based Elo-based Bradley-Terry-Luce model. The purpose of this comparison is not to but to assess how sensitive ranking uncertainty is to

### Elo-based Probability Model

In the first approach, win probabilities are computed based on Elo rating difference. Under this formulation, the probability of a win is given by

$$\frac{1}{1 + e^{(E_{\text{focal}} - E_{\text{opponent}})/100}}$$

where  $E_{\text{focal}}$  and  $E_{\text{opponent}}$  denote the Elo ratings of the two players.

This formulation is directly embedded in the Swiss system. It encodes the assumption that the Elo rating fully specifies the outcome of a match.

### Machine Learning Probability Model

The second approach replaces the Elo-based probability estimation from historical match data.

Rather than imposing a predefined functional relationship between probabilities, the machine learning model infers the relationship from matches. Conceptually, the model is trained to estimate the probability of a win based on information about the two players.

To avoid overfitting and to ensure generalization, the model is trained on a dataset comprising many more transfers than only matches played by the focal player. The player within the Monte Carlo simulation.

### Minimal Feature Specification

In line with the objective of maintaining a simple machine learning model, the feature set

- Elo rating of the focal player

- Elo rating of the opponent

No additional contextual variables (such as recent included in the baseline specification. This design remains directly comparable to the Elo system. The same information is also available in the Swiss ranking.

### Model Choice and Interpretation

A logistic regression model is used as the baseline due to its well-known flexibility and transparency:

- Logistic regression is a well-known model.
- Its structure closely mirrors the logistic function estimated from data rather than fixed a priori.
- The resulting probability function can be interpreted using the Elo formula.

Importantly, the machine learning model is not used for construction. It simply provides a mapping between observed frequencies in the data.

### Role in the Monte Carlo Simulation

Both probability models are embedded separately to structural assumptions, opponent sampling procedure differences observed in the Elo system. The main difference is in how the Elo system handles draws rather than wins.

### Interpretation Scope

Differences between the two simulations are mainly present in uncertainty probability estimation, not as definitive comparison highlights how the Elo system is too in probability modeling.

## Elo Update Mechanism

After each simulated match, the player's Elo rating is updated according to the Elo update rule. For a given match, the Elo change is calculated as:

$$\text{Elo} \times (\text{result} / \text{expected})^{1/4},$$

where:

- $K = 15$  is the Elo adjustment factor,
- result is 1 in case of a win and 0 in case of a loss,
- expected is the probability of winning according to the Swiss ranking.

The win probability is computed using the fixed Elo ratings of the opponents.

$$= \frac{1}{1 + 10^{\frac{\text{opp play}}{200}}}.$$

If the playema pahesi dat es gi the month, the total by the sum of individual match updates:

$$E \mid_{m \in \Theta_n} f_h = 1$$

The player's Elo rating at the end of the month is

$E \mid_e \rho \neq E \mid_s \rho_a +_r t E \mid_m \rho_n, t h$

where legend notes the Elo rating at the beginning of

In the Monte Carlo simulation, match-based probability model or the random probability model described always followed Swiss, English and German simulated random system.

## Monte Carlo Assumptions

Uncertainty in ranking methods net zero ICUs and identifying issues in using

- The same month of competition is simulated repeatedly
  - Each simulation uses independent random draws
  - The collection of final Elo ratings across simulated outcomes.

The number of simulations is chosen sufficiently large to obtain stable properties such as quantiles and variance.

## Simplifications and Excluded Factors

Several variables factors are intentionally excluded from

- Physical condition, fatigue, and injuries
  - Psychological effects
  - Strategic behavior and learning during tournaments
  - Long-term form dynamics

These exclusions are motivated by data limitations and interpretable. Their potential impact is discussed.

## Interpretation Framework

The resulting Elo distribution is more concentrated than random noise.

Differences between distributions obtained under uncertainty than purely random noise.

## Methodology (Conceptual Algorithm)

This section describes the simulation procedure for Elo ratings.

The methodology translates the modeling choices without introducing additional specific assumptions or implications.

### Step 1: Initialization

At the beginning of each simulation run:

- A focal player is defined with an initial Elo rating.
- This Elo rating remains fixed throughout the reference rating for all probability and Elo calculations.
- All model parameters (competition structure, opponents) are fixed and identical across simulation runs.

As a result, all variability in simulated outcomes is due to match outcomes.

### Step 2: Definition of the Competitive Month

Each simulation run represents one stylized month.

The competitive month consists of a fixed sequence of:

- two league encounters, each composed of three matches.
- one national tournament with a fixed total number of matches.

The tournament structure is designed to guarantee a minimum number of matches per team. It includes institutional constraints such as the absence of ties.

The player is assumed to participate in all scheduled matches.

### Step 3: Opponent Selection

For each match in the simulated month, an opponent is selected.

- League opponents are drawn from the rosters. Selection probabilities reflect observed participation rates.
- Tournament opponents are sampled from the available pool, subject to a maximum constraint within the tournament.

Opponent Elo ratings are treated as known and fixed.

### Step 4: Win Probability Assignment

Once an opponent is selected, a probability of win is computed. Two alternative probability models are considered:

- Elo-based probabilities using the official Swiss Elo rating.
- Match-based probabilities from historical matches using the same Elo inputs.

In both cases, win probabilities depend on the fixed Elo rating.

#### StepMatch Outcome Simulation

Given the assigned win probability:

- The match outcome is simulated as a binary random variable.
- Outcomes are generated using independent random assigned probabilities.

This step captures the intrinsic randomness of competition.

#### StepElo Update Calculation

After each simulated match:

- An Elo change is computed using the official Swiss Elo rating formula.
- The expected score used in the update formula is the player's Elo rating and the opponent's Elo rating.

Although Elo changes are cumulative, they do not affect subsequent probabilities within the same month.

The total Elo change over the month is obtained.

#### StepEndMonth Elo Recording

At the end of the simulated month:

- The player's final Elo rating is computed as the mean of all simulated outcomes.
- This final Elo value represents one possible realization of the model.

#### StepMonte Carlo Repetition

Steps are repeated a large number of times.

Each repetition represents an alternative but plausible scenario differing only through random opponent selection and Elo rating.

This procedure generates a distribution of final Elo ratings.

#### StepAggregation and Analysis

From the simulated distribution:

- summary statistics (mean, variance, quantiles) are computed
- distributional properties such as dispersion are assessed
- results are compared across probability models

No individual simulation run is interpreted as a single outcome, but rather as a distribution used for interpretation.

### Methodological Clarification

The simulation does not validate the model's assumptions, it only provides a way to answer the following question:

Given a fixed competitive month and fixed opponents, what is the expected Elo outcome?

### Why This Methodology Is Consistent

- It respects the institutional rule that Elo ratings are additive
- It ensures full comparability across probability distributions
- It provides a clean additive structure for interpretation

## Expected Results and Interpretation

This section outlines the qualitative patterns and properties of the Elo distribution, derived directly from the structure of the model and the assumptions described in the previous sections.

The purpose of this section is not to anticipate specific results, but rather to understand through which uncertainty arises and to define interpretation.

### Expected Shape of the Elo Distribution

Given that the monthly Elo change is modeled as a normal distribution, the resulting distribution is expected to be approximately unimodal, centered around a mean close to the expected value.

- approximately unimodal,
- centered around a mean close to the expected value,
- with dispersion increasing as the number of matches increases.

However, because individual match outcomes are binomial, the resulting distribution is expected to be skewed. The probability of winning a match against a stronger opponent is lower than 50%, while the probability of winning a match against a weaker opponent is higher than 50%.

## Expected Role of Match Volume and Component

The simulated month includes a relatively large number of matches in a tournament.

As a result:

- the variance of the monthly Elo change is explained by the variance of a single match outcome,
- extreme positive or negative outcomes become partially moderate.

The tournament component, which introduces a bias, is expected to contribute disproportionately to the downside risk relative to league play alone.

## Expected Impact of Fixed Elo Within the League

Because the Elo rating is held fixed throughout the month:

- win probabilities do not adapt to interim success rates,
- early wins do not mechanically increase the probability of future wins.

This absence of feedback implies that the total Elo rating is the sum of independent realizations. Consequently, the distribution reflects uncertainty rather than dynamic momentum effects.

This design choice is expected to yield cleaner insights into psychological dynamics.

## Expected Comparison Between Probabilistic Models

When comparing simulation-based probability models with Elo-based probability model, several qualitative differences are expected:

If both models produce similar average Elo growth, the Elo-based probability model is expected to have more predictable properties:

- dispersion may differ if one model assigns more weight to certain segments,
- tail risk may increase if the machine learning model present in the Elo formula,
- skewness may differ if one model has asymmetric chances against specific opponent segments.

Importantly, any such difference does not necessarily mean that one model is superior to the other. It is important to consider the context and the specific goals of the analysis.

## Expected Sensitivity to Opponent Sampling

Because opponents are sampled probabilistically most often, the outcome is expected to be sensitive to the realization.

Simulations in which the player faces an unusual distribution of opponents will populate the lower or upper tails of the distribution. Ranking uncertainty arises at the extremes of the distribution due to limited competition exposure.

## Interpretation Framework

The simulated Elo distribution is large enough to be useful in financial contexts.

In this interpretation:

- the mean Elo change represents expected performance
- the dispersion captures ranking volatility
- the lower tail reflects downside risk
- the upper tail reflects upside potential

The comparison across different distributions is intended to test the sensitivity of the model to modeling assumptions, rather than providing a

## Role of This Section

This section establishes the context in which the empirical results are evaluated. Deviations between observed results and the model provide informative signals about the interaction between stochastic variability.