Table of Contents


```
clear all;
close all;
```

ELEC 4700 Assignment 2: Finite Difference Method

ELEC 4700 Assignment - 1 Braden Bale (101072763)

Question 1(a)

The matrix form of the Finite Difference Method is used solve for the electrostatic potential of a recetangular region with insolating sides. Below some varibles for simulation are set up, with F and G being used for the matrices.

```
L = 3;
W = 2;

dx = 0.25;
dy = 0.25;
nx = L/dx;
ny = W/dy;

V0 = 1;

F = zeros(nx*ny, 1);
G = sparse(nx*ny, nx*ny);
```

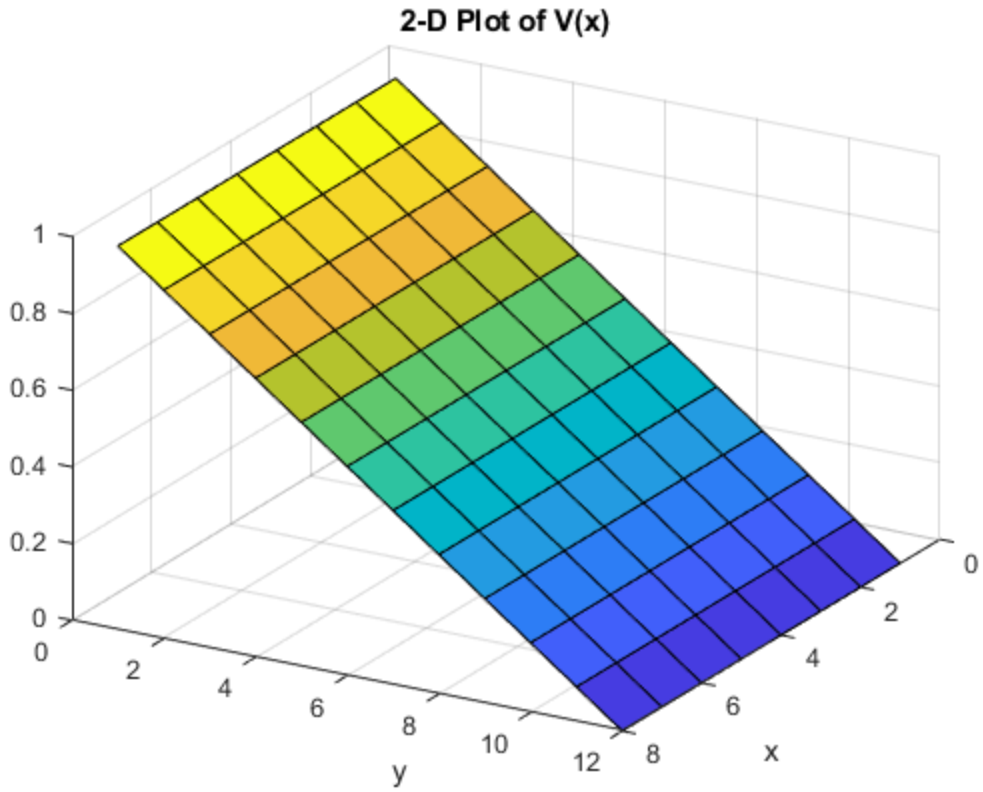
Below the Finite Difference Method is used to set up the G and F matrices .

```
for i = 1:(nx)
    for j = 1:(ny)
        n = (j + (i-1).*ny);
        if i == 1
            G(n, n) = 1;
            F(n) = V0;
        elseif i == nx
            G(n, n) = 1;
            F(n) = 0;
        elseif j == 1
```

```
            G(n, n) = -1/dy^2 + (1/dx^2)*-2;
            G(n, ((j-1) + (i-1).*ny)) = 1/(dx^2);
            G(n, ((j+1) + (i-1).*ny)) = 1/(dx^2);
            G(n, (j + (i).*ny)) = 1/(dy^2);
        elseif j == ny
            G(n, n) = -1/dy^2 + (1/dx^2)*-2;
            G(n, ((j-1) + (i-1).*ny)) = 1/(dx^2);
            G(n, ((j+1) + (i-1).*ny)) = 1/(dx^2);
            G(n, (j + (i-2).*ny)) = 1/(dy^2);
        else
            G(n, n) = (1/dx^2 + 1/dy^2)*-2;
            G(n, ((j-1) + (i-1).*ny)) = 1/(dx^2);
            G(n, ((j+1) + (i-1).*ny)) = 1/(dx^2);
            G(n, (j + (i-2).*ny)) = 1/(dy^2);
            G(n, (j + (i).*ny)) = 1/(dy^2);
        end
    end
end

V = G\F;

V = reshape(V,[],nx)';
```

Finally, in Figure 1 below a 2-D plot of V(x) is shown.

```
figure(1);
surf(V);
title('2-D Plot of V(x)');
xlabel('x');
ylabel('y');
view(120, 30);
```



Question 1(b)

When comparing the analytical solution to the solution retrieved from using the Finite Difference Method the analytical solution could not be found. This code is commented out as it does not correctly complete the question.

Below the matrices G anf F are set up similarily to Question 1(a).

```
% for i = 1:(nx)
%     for j = 1:(ny)
%         n = (j + (i-1).*ny);
%         if i == 1
%             G(n, n) = 1;
%             F(n) = V0;
%         elseif i == nx
%             G(n, n) = 1;
%             F(n) = V0;
%         elseif j == 1
%             G(n, n) = 1;
%             F(n) = 0;
%         elseif j == ny
%             G(n, n) = 1;
%             F(n) = 0;
%         else
%             G(n, n) = (1/dx^2 + 1/dy^2)*-2;
%             G(n, ((j-1) + (i-1).*ny)) = 1/(dx^2);
```

```
%             G(n, ((j+1) + (i-1).*ny)) = 1/(dx^2);
%             G(n, (j + (i-2).*ny)) = 1/(dy^2);
%             G(n, (j + (i).*ny)) = 1/(dy^2);
%         end
%     end
% end
```

The analytical solution was attempted by building a matrices which was the same size as V using xp and xp below and repeating them for the length of nx and ny.

```
% VA = zeros(nx, ny);
%
% max = 1000;
%
% a = W;
% b = L;
%
% xp = linspace(-L, L, ny);
% yp = linspace(0, W, nx);
%
%
%
% for i = 1: nx
%    x(i, :) = xp;
% end
%
% for j = 1: ny
%    y(:, j) = yp;
% end
%
```

Below the for loop to caluculate the analytical solution is shown.

```
% for k = 1: max
%     n = k*2 -1;
%     VA = VA + (1/n)*((cosh((n*pi.*x)./a))./(cosh((n.*pi.*b)./
a))).*(sin((n.*pi.*y)./a));
% end
%
% VA = VA.*4.*1./pi;
%
%
% V = G\F;
%
% V = reshape(V,[],nx)';
%
% figure(1);
% surf(V);
% xlabel('x');
% ylabel('y');
%
% figure(2);
% surf(VA');
% xlabel('x');
```

```
% ylabel('y');
```

Question 2

From Question 1, conductivity of a surface could be added to the area. Two boxes were added to create a bottle-neck to understand conductivity's effect on the system. The conductivity in the boxes is set to 10^-2 and outside the boxes is set to 1. For this question a conductivity map, voltage map, electric field matrix and current density matrix were set up and shown.

```
rho_out_of_box = 1;
rho_in_box = 10^-2;
```

Below the boxees for the conductivity map are set up.

```
cMap = ones(nx, ny).*rho_out_of_box;
for i = 1: nx
    for j = 1 : ny
        if ((i > ((0.5*nx)- (0.1*nx))) && (i <((0.5*nx)+(0.1*nx))) &&
 (j > ((0.5*ny)+(0.25*ny))))
            cMap(i, j) = rho_in_box;
        elseif ((i > ((0.5*nx)- (0.1*nx))) && (i <
 ((0.5*nx)+(0.1*nx))) && (j < ((0.5*ny)-(0.25*ny))))
            cMap(i, j) = rho_in_box;
        end
    end
end

G2 = sparse(nx*ny);
B = zeros(1,nx*ny);
```

The conductivity map for the Question is set up using the Finite Difference Method.

```
for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        if i == 1
            G2(n, n) = 1;
            B(n) = 1;
        elseif i == nx
            G2(n, n) = 1;
            B(n) = 0;
        elseif j == 1
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nyp = j+1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

            G2(n,n) = -(rxp+rxm+ryp);
            G2(n,nxp) = rxp;
            G2(n,nxm) = rxm;
```

```
            G2(n,nyp) = ryp;
        elseif j ==  ny
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nym = j-1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            rym = (cMap(i,j) + cMap(i,j-1))/2.0;

            G2(n,n) = -(rxm+rxp+rym);
            G2(n,nxm) = rxm;
            G2(n,nxp) = rxp;
            G2(n,nym) = rym;
        else
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nym = j-1 + (i-1)*ny;
            nyp = j+1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            rym = (cMap(i,j) + cMap(i,j-1))/2.0;
            ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

            G2(n,n) = -(rxm+rxp+rym+ryp);
            G2(n,nxm) = rxm;
            G2(n,nxp) = rxp;
            G2(n,nym) = rym;
            G2(n,nyp) = ryp;
        end

    end
end
```

Then the voltage map is set up below.

```
V2 = G2\B';

Vmap = zeros(nx,ny);
for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        Vmap(i, j) = V2(n);
    end
end
```

Finally, with the voltage map found the electric field can be found.

```
for i = 1:nx
    for j = 1:ny
        if i == 1
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i, j));
        elseif i == nx
```

```
            Ex(i, j) = (Vmap(i, j) - Vmap(i - 1, j));
        else
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i - 1, j)) * 0.5;
        end
        if j == 1
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j));
        elseif j == ny
            Ey(i, j) = (Vmap(i, j) - Vmap(i, j - 1));
        else
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j - 1)) * 0.5;
        end
    end
end

Ex = -Ex;
Ey = -Ey;

Jx = cMap.*Ex;
Jy = cMap.*Ey;
```

Figure 2 below shows the conductiviy map.

```
figure(2)
surf(cMap);
xlabel('x');
ylabel('y');
title('Conductivity Map');
```

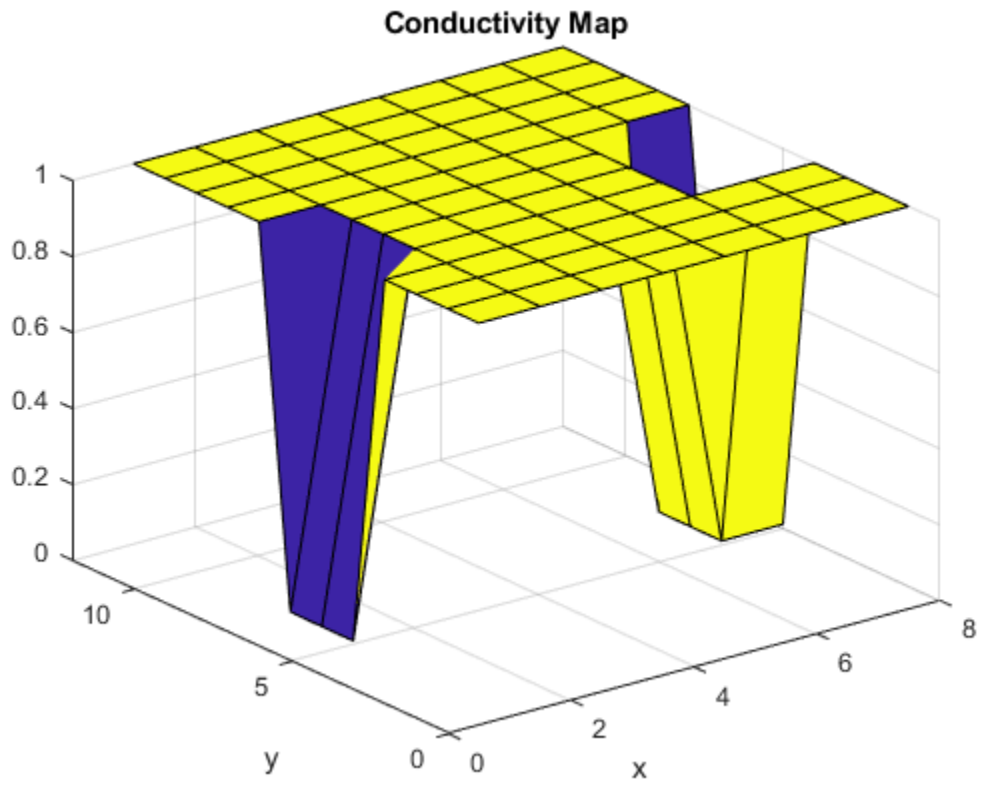


Figure 3 below shows the potential map.

```
figure(3)
surf(Vmap);
xlabel('x');
ylabel('y');
title('Potential Map');
```

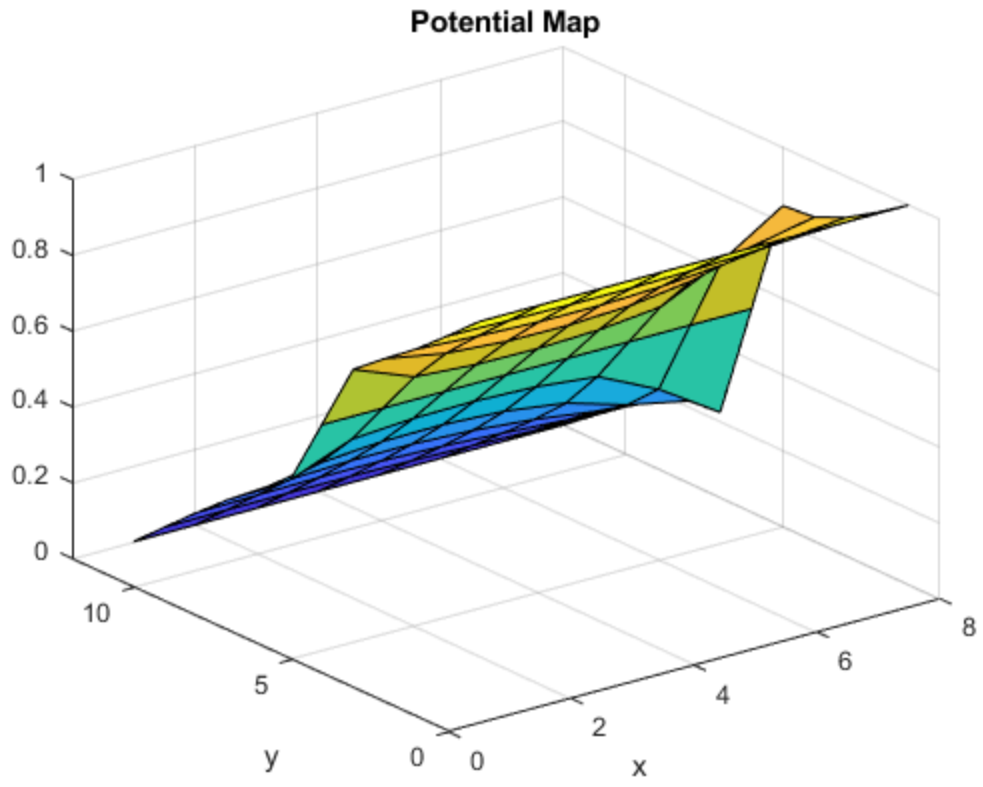


Figure 4 below shows the electric field.

```
figure(4)
quiver(Ex', Ey',  1);
xlabel('x');
ylabel('y');
title('Electric Field');
```

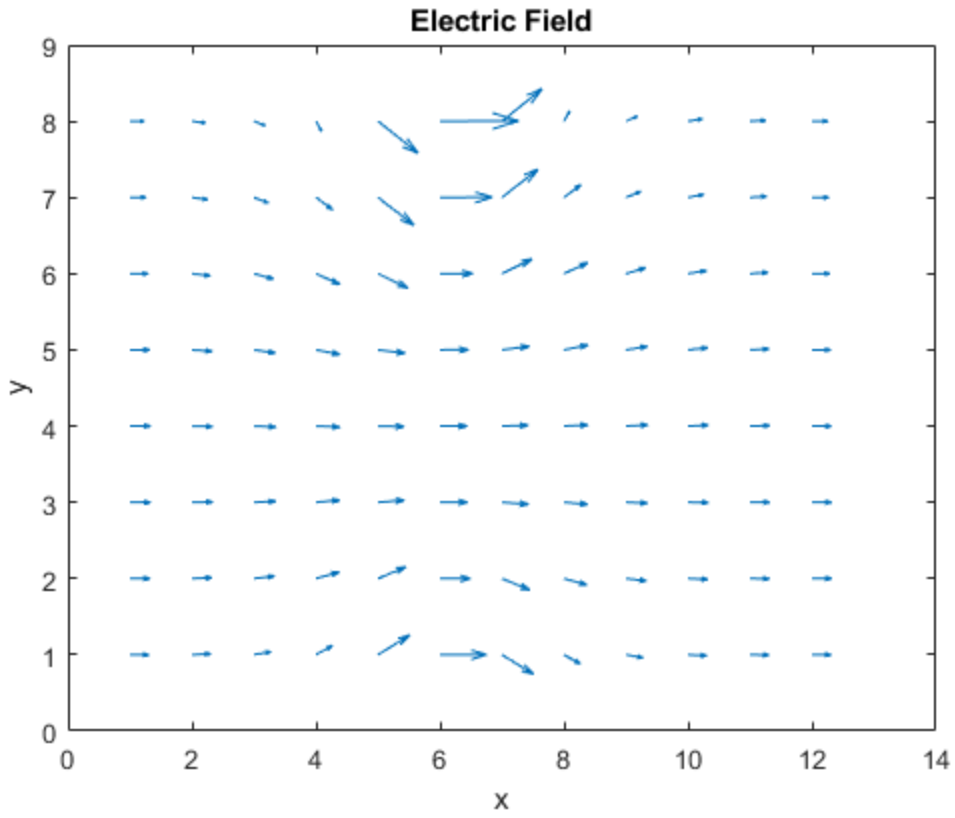
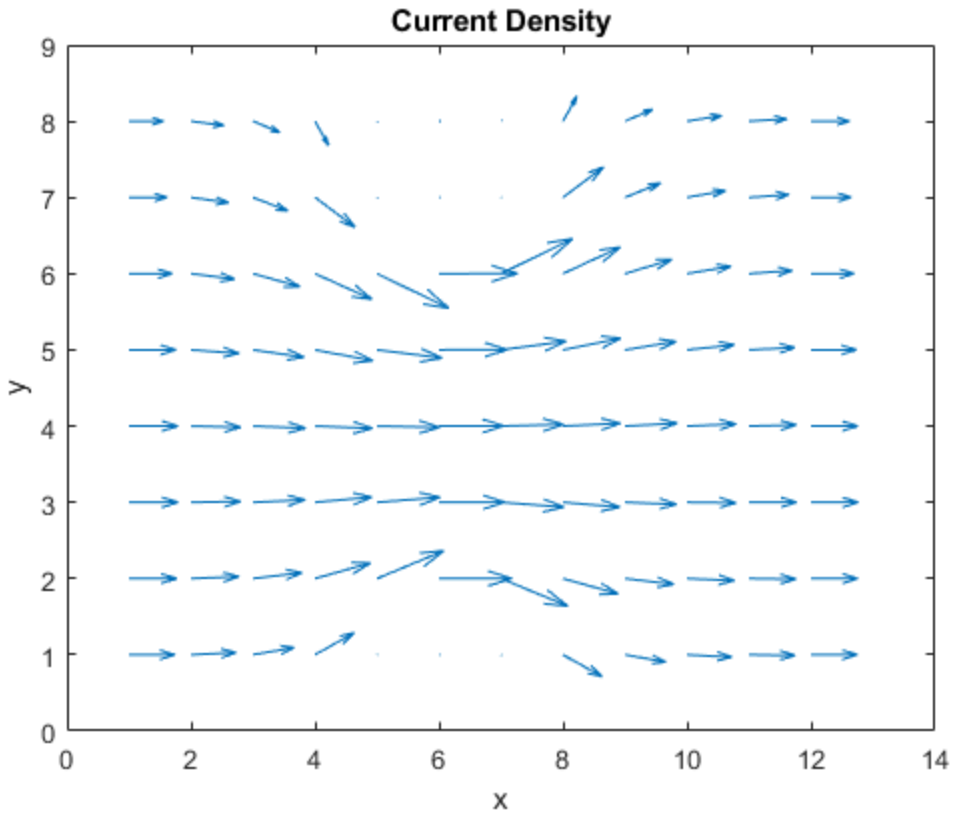


Figure 5 below shows the current density.

```
figure(5)
quiver(Jx', Jy', 1);
xlabel('x');
ylabel('y');
title('Current Density');
```



From this section onward small changes are made to the code to demonstrate different tests to the system.

```
Jx = 0;
Jy = 0;
Ex = 0;
Ey = 0;
Vmap = 0;
cMap = 0;
V2 = 0;

G2 = sparse(nx*ny);
B = zeros(1,nx*ny);
```

First, a new current density is set. The variable dx and dy are changed from 0.25 to 0.10.

```
dx = 0.10;
dy = 0.10;
nx = L/dx;
ny = W/dy;

cMap = ones(nx, ny).*rho_out_of_box;
for i = 1: nx
    for j = 1 : ny
        if ((i > ((0.5*nx)- (0.1*nx))) && (i <((0.5*nx)+(0.1*nx))) &&
 (j > ((0.5*ny)+(0.3*ny))))
            cMap(i, j) = rho_in_box;
```

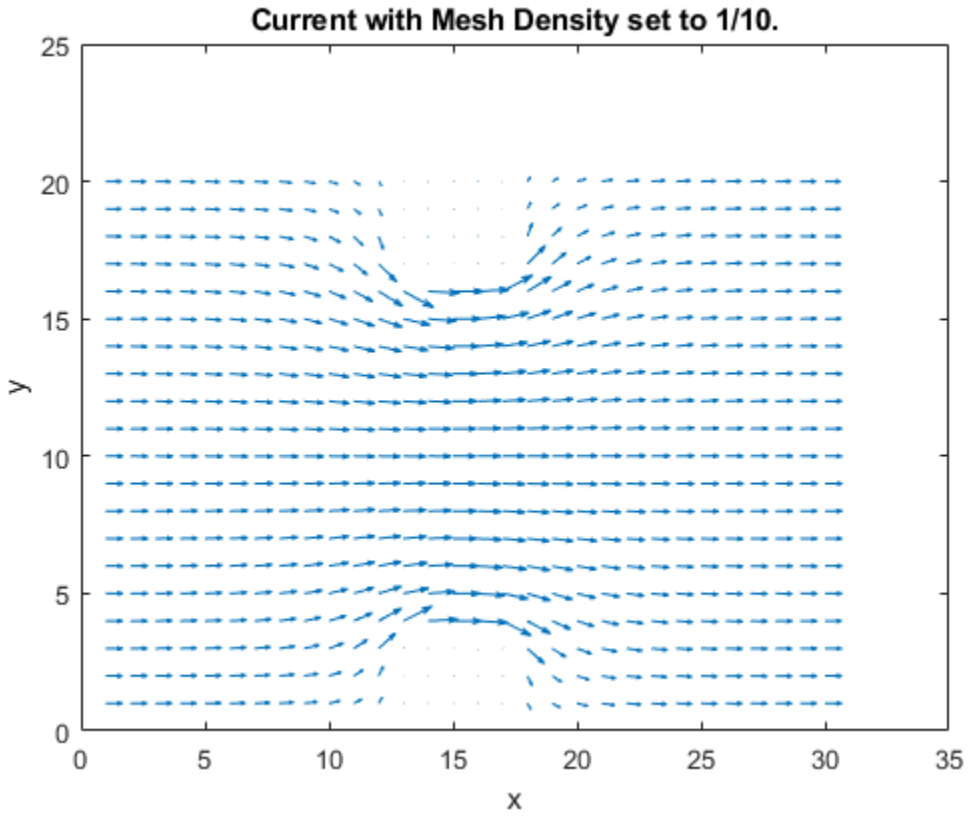
```
        elseif ((i > ((0.5*nx)- (0.1*nx))) && (i <
 ((0.5*nx)+(0.1*nx))) && (j < ((0.5*ny)-(0.3*ny))))
            cMap(i, j) = rho_in_box;
        end
    end
end

for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        if i == 1
            G2(n, n) = 1;
            B(n) = 1;
        elseif i == nx
            G2(n, n) = 1;
            B(n) = 0;
        elseif j == 1
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nyp = j+1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

            G2(n,n) = -(rxp+rxm+ryp);
            G2(n,nxp) = rxp;
            G2(n,nxm) = rxm;
            G2(n,nyp) = ryp;
        elseif j ==  ny
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nym = j-1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            rym = (cMap(i,j) + cMap(i,j-1))/2.0;

            G2(n,n) = -(rxm+rxp+rym);
            G2(n,nxm) = rxm;
            G2(n,nxp) = rxp;
            G2(n,nym) = rym;
        else
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nym = j-1 + (i-1)*ny;
            nyp = j+1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            rym = (cMap(i,j) + cMap(i,j-1))/2.0;
            ryp = (cMap(i,j) + cMap(i,j+1))/2.0;
```

```
            G2(n,n) = -(rxm+rxp+rym+ryp);
            G2(n,nxm) = rxm;
            G2(n,nxp) = rxp;
            G2(n,nym) = rym;
            G2(n,nyp) = ryp;
        end

    end
end



V2 = G2\B';

Vmap = zeros(nx,ny);
for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        Vmap(i, j) = V2(n);
    end
end

for i = 1:nx
    for j = 1:ny
        if i == 1
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i, j));
        elseif i == nx
            Ex(i, j) = (Vmap(i, j) - Vmap(i - 1, j));
        else
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i - 1, j)) * 0.5;
        end
        if j == 1
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j));
        elseif j == ny
            Ey(i, j) = (Vmap(i, j) - Vmap(i, j - 1));
        else
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j - 1)) * 0.5;
        end
    end
end

Ex = -Ex;
Ey = -Ey;

Jx = cMap.*Ex;
Jy = cMap.*Ey;
```

Figure 6 is shown below.

```
figure(6)
quiver(Jx', Jy', 1);
xlabel('x');
ylabel('y');
```

```
title('Current with Mesh Density set to 1/10.');
```



Next, the size of the bottleneck is varied.

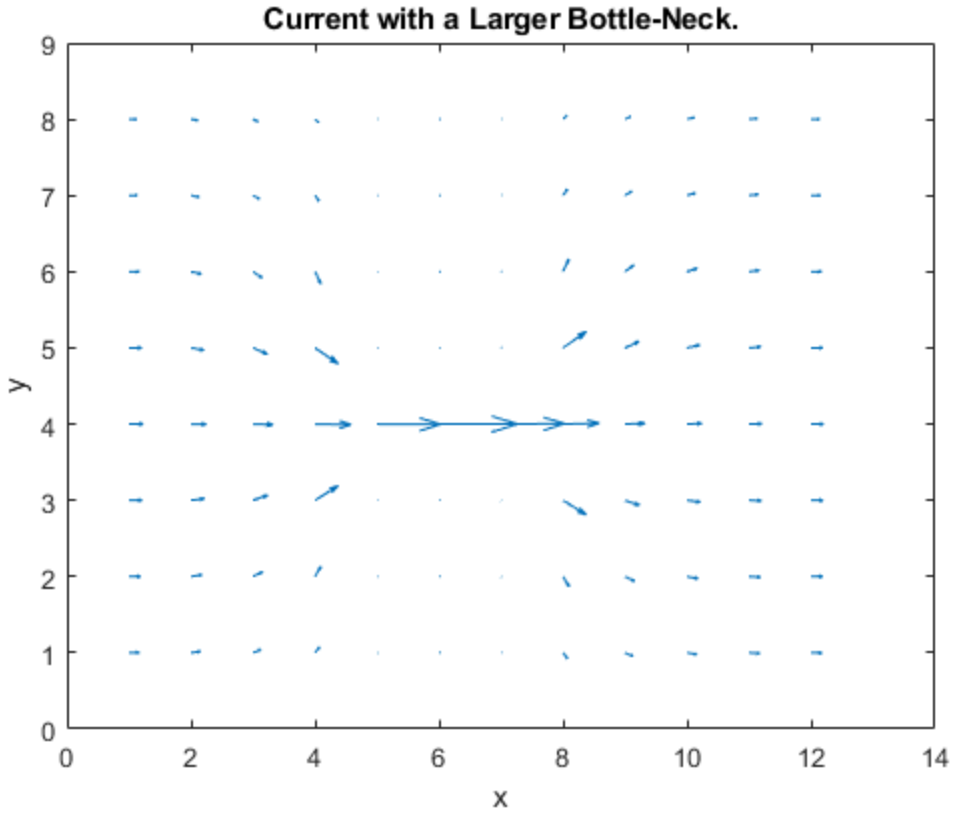
```
Jx = 0;
Jy = 0;
Ex = 0;
Ey = 0;
Vmap = 0;
V2 = 0;

dx = 0.25;
dy = 0.25;
nx = L/dx;
ny = W/dy;

G2 = sparse(nx*ny);
B = zeros(1,nx*ny);

cMap = ones(nx, ny).*rho_out_of_box;
for i = 1: nx
    for j = 1 : ny
        if ((i > ((0.5*nx)- (0.1*nx))) && (i <((0.5*nx)+(0.1*nx))) &&
 (j > ((0.5*ny)+(0.1*ny))))
            cMap(i, j) = rho_in_box;
        elseif ((i > ((0.5*nx)- (0.1*nx))) && (i <
 ((0.5*nx)+(0.1*nx))) && (j < ((0.5*ny)-(0.1*ny))))
```

```
            cMap(i, j) = rho_in_box;
        end
    end
end

for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        if i == 1
            G2(n, n) = 1;
            B(n) = 1;
        elseif i == nx
            G2(n, n) = 1;
            B(n) = 0;
        elseif j == 1
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nyp = j+1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

            G2(n,n) = -(rxp+rxm+ryp);
            G2(n,nxp) = rxp;
            G2(n,nxm) = rxm;
            G2(n,nyp) = ryp;
        elseif j ==  ny
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nym = j-1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            rym = (cMap(i,j) + cMap(i,j-1))/2.0;

            G2(n,n) = -(rxm+rxp+rym);
            G2(n,nxm) = rxm;
            G2(n,nxp) = rxp;
            G2(n,nym) = rym;
        else
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nym = j-1 + (i-1)*ny;
            nyp = j+1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            rym = (cMap(i,j) + cMap(i,j-1))/2.0;
            ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

            G2(n,n) = -(rxm+rxp+rym+ryp);
            G2(n,nxm) = rxm;
```

```
            G2(n,nxp) = rxp;
            G2(n,nym) = rym;
            G2(n,nyp) = ryp;
        end

    end
end

V2 = G2\B';

Vmap = zeros(nx,ny);

for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        Vmap(i, j) = V2(n);
    end
end

for i = 1:nx
    for j = 1:ny
        if i == 1
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i, j));
        elseif i == nx
            Ex(i, j) = (Vmap(i, j) - Vmap(i - 1, j));
        else
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i - 1, j)) * 0.5;
        end
        if j == 1
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j));
        elseif j == ny
            Ey(i, j) = (Vmap(i, j) - Vmap(i, j - 1));
        else
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j - 1)) * 0.5;
        end
    end
end

Ex = -Ex;
Ey = -Ey;

Jx = cMap.*Ex;
Jy = cMap.*Ey;
```

Figure 7 is shown below.

```
figure(7)
quiver(Jx', Jy', 1);
xlabel('x');
ylabel('y');
title('Current with a Larger Bottle-Neck.');
```



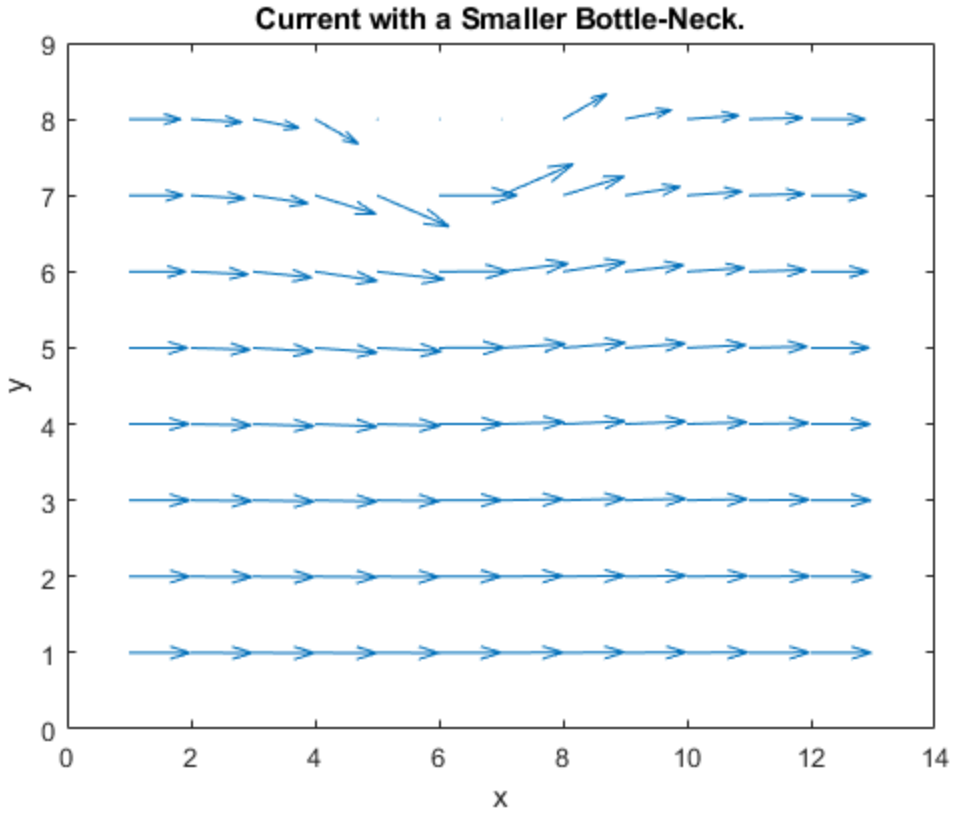

```
Jx = 0;
Jy = 0;
Ex = 0;
Ey = 0;
Vmap = 0;
cMap = 0;
V2 = 0;

G2 = sparse(nx*ny);
B = zeros(1,nx*ny);

cMap = ones(nx, ny).*rho_out_of_box;
for i = 1: nx
    for j = 1 : ny
        if ((i > ((0.5*nx)- (0.1*nx))) && (i <((0.5*nx)+(0.1*nx))) &&
 (j > ((0.5*ny)+(0.4*ny))))
            cMap(i, j) = rho_in_box;
        elseif ((i > ((0.5*nx)- (0.1*nx))) && (i <
 ((0.5*nx)+(0.1*nx))) && (j < ((0.5*ny)-(0.4*ny))))
            cMap(i, j) = rho_in_box;
        end
    end
end

for i = 1:nx
    for j = 1:ny
```

```
            n = j + (i - 1) * ny;

            if i == 1
                G2(n, n) = 1;
                B(n) = 1;
            elseif i == nx
                G2(n, n) = 1;
                B(n) = 0;
            elseif j == 1
                nxm = j + (i-2)*ny;
                nxp = j + (i)*ny;
                nyp = j+1 + (i-1)*ny;

                rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
                rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
                ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

                G2(n,n) = -(rxp+rxm+ryp);
                G2(n,nxp) = rxp;
                G2(n,nxm) = rxm;
                G2(n,nyp) = ryp;
            elseif j ==  ny
                nxm = j + (i-2)*ny;
                nxp = j + (i)*ny;
                nym = j-1 + (i-1)*ny;

                rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
                rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
                rym = (cMap(i,j) + cMap(i,j-1))/2.0;

                G2(n,n) = -(rxm+rxp+rym);
                G2(n,nxm) = rxm;
                G2(n,nxp) = rxp;
                G2(n,nym) = rym;
            else
                nxm = j + (i-2)*ny;
                nxp = j + (i)*ny;
                nym = j-1 + (i-1)*ny;
                nyp = j+1 + (i-1)*ny;

                rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
                rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
                rym = (cMap(i,j) + cMap(i,j-1))/2.0;
                ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

                G2(n,n) = -(rxm+rxp+rym+ryp);
                G2(n,nxm) = rxm;
                G2(n,nxp) = rxp;
                G2(n,nym) = rym;
                G2(n,nyp) = ryp;
            end

        end
    end
```

```
V2 = G2\B';

Vmap = zeros(nx,ny);
for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        Vmap(i, j) = V2(n);
    end
end

for i = 1:nx
    for j = 1:ny
        if i == 1
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i, j));
        elseif i == nx
            Ex(i, j) = (Vmap(i, j) - Vmap(i - 1, j));
        else
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i - 1, j)) * 0.5;
        end
        if j == 1
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j));
        elseif j == ny
            Ey(i, j) = (Vmap(i, j) - Vmap(i, j - 1));
        else
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j - 1)) * 0.5;
        end
    end
end

Ex = -Ex;
Ey = -Ey;

Jx = cMap.*Ex;
Jy = cMap.*Ey;
```

Figure 8 is shown below.

```
figure(8)
quiver(Jx', Jy', 1);
xlabel('x');
ylabel('y');
title('Current with a Smaller Bottle-Neck.');
```



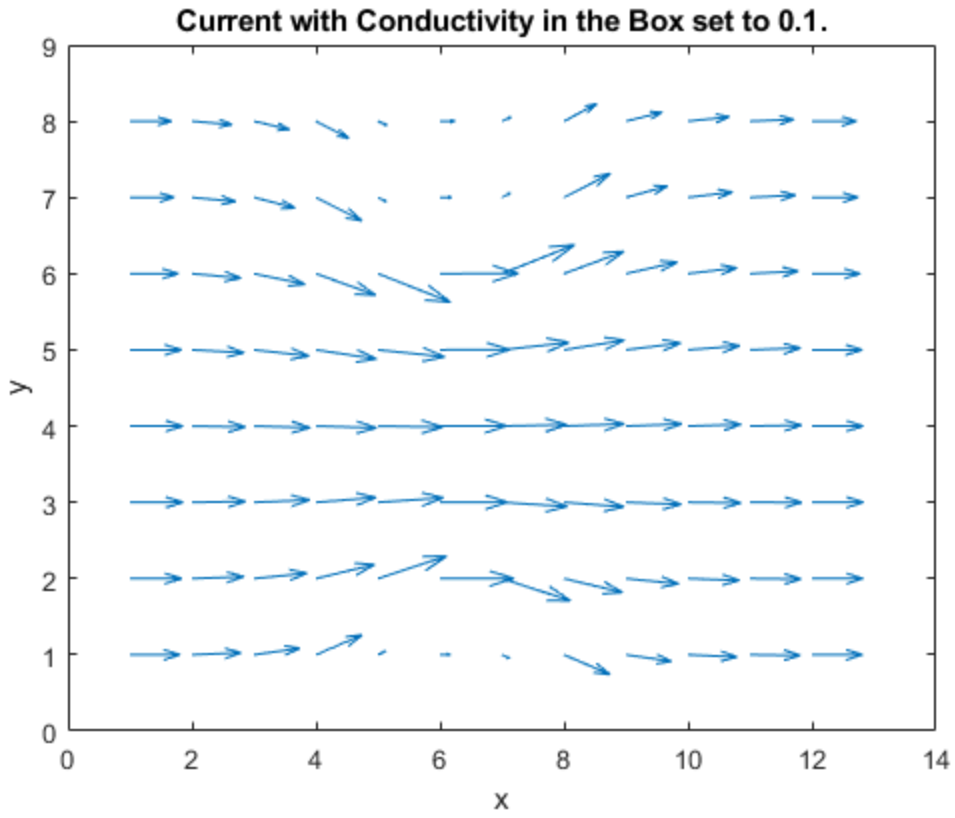

```
% Then, the conductivity in the box was varied.

Jx = 0;
Jy = 0;
Ex = 0;
Ey = 0;
Vmap = 0;
cMap = 0;
V2 = 0;

G2 = sparse(nx*ny);
B = zeros(1,nx*ny);

rho_in_box = 0.1;

cMap = ones(nx, ny).*rho_out_of_box;
for i = 1: nx
    for j = 1 : ny
        if ((i > ((0.5*nx)- (0.1*nx))) && (i <((0.5*nx)+(0.1*nx))) &&
 (j > ((0.5*ny)+(0.25*ny))))
            cMap(i, j) = rho_in_box;
        elseif ((i > ((0.5*nx)- (0.1*nx))) && (i <
 ((0.5*nx)+(0.1*nx))) && (j < ((0.5*ny)-(0.25*ny))))
            cMap(i, j) = rho_in_box;
        end
    end
```

```
end

for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        if i == 1
            G2(n, n) = 1;
            B(n) = 1;
        elseif i == nx
            G2(n, n) = 1;
            B(n) = 0;
        elseif j == 1
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nyp = j+1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

            G2(n,n) = -(rxp+rxm+ryp);
            G2(n,nxp) = rxp;
            G2(n,nxm) = rxm;
            G2(n,nyp) = ryp;
        elseif j ==  ny
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nym = j-1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            rym = (cMap(i,j) + cMap(i,j-1))/2.0;

            G2(n,n) = -(rxm+rxp+rym);
            G2(n,nxm) = rxm;
            G2(n,nxp) = rxp;
            G2(n,nym) = rym;
        else
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nym = j-1 + (i-1)*ny;
            nyp = j+1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            rym = (cMap(i,j) + cMap(i,j-1))/2.0;
            ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

            G2(n,n) = -(rxm+rxp+rym+ryp);
            G2(n,nxm) = rxm;
            G2(n,nxp) = rxp;
            G2(n,nym) = rym;
            G2(n,nyp) = ryp;
```

```
        end

    end
end

V2 = G2\B';

Vmap = zeros(nx,ny);
for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        Vmap(i, j) = V2(n);
    end
end

for i = 1:nx
    for j = 1:ny
        if i == 1
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i, j));
        elseif i == nx
            Ex(i, j) = (Vmap(i, j) - Vmap(i - 1, j));
        else
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i - 1, j)) * 0.5;
        end
        if j == 1
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j));
        elseif j == ny
            Ey(i, j) = (Vmap(i, j) - Vmap(i, j - 1));
        else
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j - 1)) * 0.5;
        end
    end
end



Ex = -Ex;
Ey = -Ey;

Jx = cMap.*Ex;
Jy = cMap.*Ey;
```

Figure 9 is shown below.

```
figure(9)
quiver(Jx', Jy', 1);
xlabel('x');
ylabel('y');
title('Current with Conductivity in the Box set to 0.1.');
```



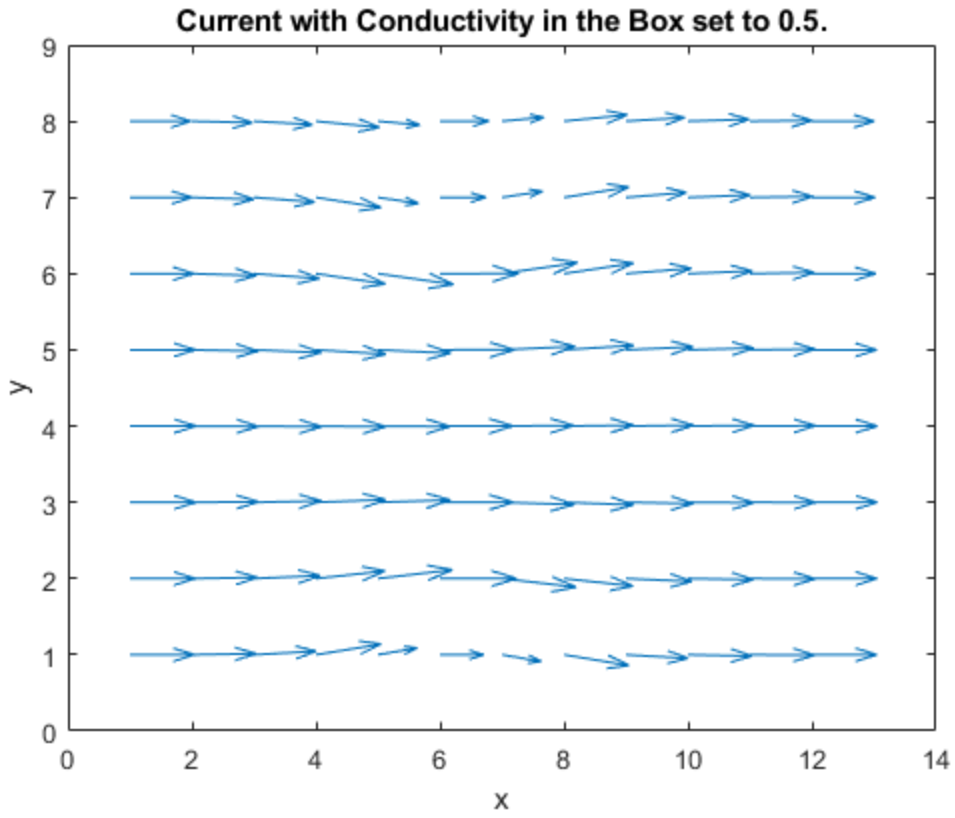

```
Jx = 0;
Jy = 0;
Ex = 0;
Ey = 0;
Vmap = 0;
cMap = 0;
V2 = 0;

G2 = sparse(nx*ny);
B = zeros(1,nx*ny);

rho_in_box = 0.5;

cMap = ones(nx, ny).*rho_out_of_box;
for i = 1: nx
    for j = 1 : ny
        if ((i > ((0.5*nx)- (0.1*nx))) && (i <((0.5*nx)+(0.1*nx))) &&
 (j > ((0.5*ny)+(0.25*ny))))
            cMap(i, j) = rho_in_box;
        elseif ((i > ((0.5*nx)- (0.1*nx))) && (i <
 ((0.5*nx)+(0.1*nx))) && (j < ((0.5*ny)-(0.25*ny))))
            cMap(i, j) = rho_in_box;
        end
    end
end
```

```
for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        if i == 1
            G2(n, n) = 1;
            B(n) = 1;
        elseif i == nx
            G2(n, n) = 1;
            B(n) = 0;
        elseif j == 1
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nyp = j+1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

            G2(n,n) = -(rxp+rxm+ryp);
            G2(n,nxp) = rxp;
            G2(n,nxm) = rxm;
            G2(n,nyp) = ryp;
        elseif j ==  ny
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nym = j-1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            rym = (cMap(i,j) + cMap(i,j-1))/2.0;

            G2(n,n) = -(rxm+rxp+rym);
            G2(n,nxm) = rxm;
            G2(n,nxp) = rxp;
            G2(n,nym) = rym;
        else
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nym = j-1 + (i-1)*ny;
            nyp = j+1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            rym = (cMap(i,j) + cMap(i,j-1))/2.0;
            ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

            G2(n,n) = -(rxm+rxp+rym+ryp);
            G2(n,nxm) = rxm;
            G2(n,nxp) = rxp;
            G2(n,nym) = rym;
            G2(n,nyp) = ryp;
        end
```

```
    end
end

V2 = G2\B';

Vmap = zeros(nx,ny);
for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        Vmap(i, j) = V2(n);
    end
end

for i = 1:nx
    for j = 1:ny
        if i == 1
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i, j));
        elseif i == nx
            Ex(i, j) = (Vmap(i, j) - Vmap(i - 1, j));
        else
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i - 1, j)) * 0.5;
        end
        if j == 1
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j));
        elseif j == ny
            Ey(i, j) = (Vmap(i, j) - Vmap(i, j - 1));
        else
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j - 1)) * 0.5;
        end
    end
end

Ex = -Ex;
Ey = -Ey;

Jx = cMap.*Ex;
Jy = cMap.*Ey;
```

Figure 10 is shown below.

```
figure(10)
quiver(Jx', Jy', 1);
xlabel('x');
ylabel('y');
title('Current with Conductivity in the Box set to 0.5.');
```



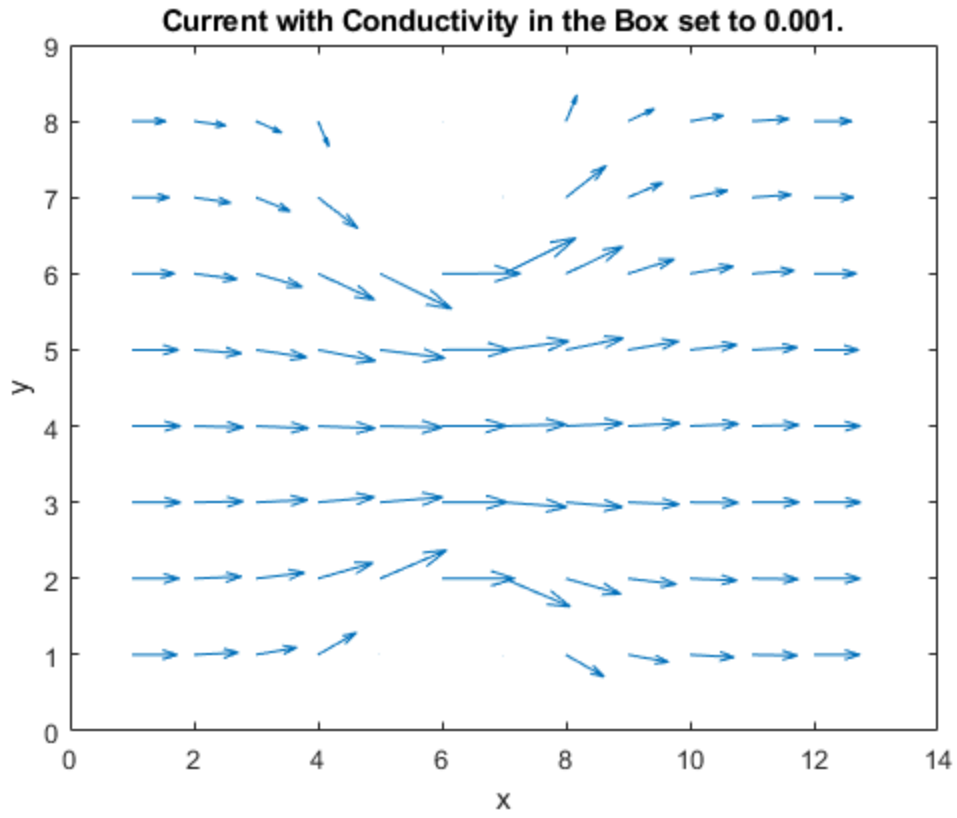

```
Jx = 0;
Jy = 0;
Ex = 0;
Ey = 0;
Vmap = 0;
cMap = 0;
V2 = 0;

G2 = sparse(nx*ny);
B = zeros(1,nx*ny);

rho_in_box = 0.001;

cMap = ones(nx, ny).*rho_out_of_box;
for i = 1: nx
    for j = 1 : ny
        if ((i > ((0.5*nx)- (0.1*nx))) && (i <((0.5*nx)+(0.1*nx))) &&
 (j > ((0.5*ny)+(0.25*ny))))
            cMap(i, j) = rho_in_box;
        elseif ((i > ((0.5*nx)- (0.1*nx))) && (i <
 ((0.5*nx)+(0.1*nx))) && (j < ((0.5*ny)-(0.25*ny))))
            cMap(i, j) = rho_in_box;
        end
    end
end
```

```
for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        if i == 1
            G2(n, n) = 1;
            B(n) = 1;
        elseif i == nx
            G2(n, n) = 1;
            B(n) = 0;
        elseif j == 1
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nyp = j+1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

            G2(n,n) = -(rxp+rxm+ryp);
            G2(n,nxp) = rxp;
            G2(n,nxm) = rxm;
            G2(n,nyp) = ryp;
        elseif j ==  ny
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nym = j-1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            rym = (cMap(i,j) + cMap(i,j-1))/2.0;

            G2(n,n) = -(rxm+rxp+rym);
            G2(n,nxm) = rxm;
            G2(n,nxp) = rxp;
            G2(n,nym) = rym;
        else
            nxm = j + (i-2)*ny;
            nxp = j + (i)*ny;
            nym = j-1 + (i-1)*ny;
            nyp = j+1 + (i-1)*ny;

            rxm = (cMap(i,j) + cMap(i-1,j))/2.0;
            rxp = (cMap(i,j) + cMap(i+1,j))/2.0;
            rym = (cMap(i,j) + cMap(i,j-1))/2.0;
            ryp = (cMap(i,j) + cMap(i,j+1))/2.0;

            G2(n,n) = -(rxm+rxp+rym+ryp);
            G2(n,nxm) = rxm;
            G2(n,nxp) = rxp;
            G2(n,nym) = rym;
            G2(n,nyp) = ryp;
        end
```

```
    end
end

V2 = G2\B';

Vmap = zeros(nx,ny);
for i = 1:nx
    for j = 1:ny
        n = j + (i - 1) * ny;

        Vmap(i, j) = V2(n);
    end
end

for i = 1:nx
    for j = 1:ny
        if i == 1
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i, j));
        elseif i == nx
            Ex(i, j) = (Vmap(i, j) - Vmap(i - 1, j));
        else
            Ex(i, j) = (Vmap(i + 1, j) - Vmap(i - 1, j)) * 0.5;
        end
        if j == 1
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j));
        elseif j == ny
            Ey(i, j) = (Vmap(i, j) - Vmap(i, j - 1));
        else
            Ey(i, j) = (Vmap(i, j + 1) - Vmap(i, j - 1)) * 0.5;
        end
    end
end

Ex = -Ex;
Ey = -Ey;

Jx = cMap.*Ex;
Jy = cMap.*Ey;
```

Figure 11 is shown below.

```
figure(11)
quiver(Jx', Jy', 1);
xlabel('x');
ylabel('y');
title('Current with Conductivity in the Box set to 0.001.');
```



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