1

Wednesday, January 29, 2025 2:30 PM

1. 1D, Mono-energetic, Purely-Absorbing Transport Derivations: The 1D Cartesian transport equation with no scattering and a constant source is

$$\mu \frac{d\psi}{dx} + \Sigma_t \psi = Q \tag{1}$$

and has the general solution of

$$\psi(x) = \psi_i e^{-\frac{\Sigma_t}{\mu}(x-x_i)} + \frac{Q}{\Sigma_t}\left(1 - e^{-\frac{\Sigma_t}{\mu}(x-x_i)}\right) \tag{2}$$

(a) In your own words, define each term in this equation and solution:

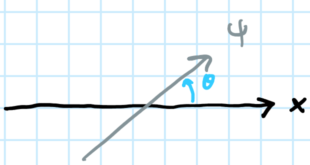
- μ:
- x:
- ψ:
- Σ_t:
- Q:
- ψ_i:
- x_i:

(b) Derive the solution for the average angular flux over a given distance in the x-direction, $\langle\psi\rangle = \frac{1}{x_e - x_i}\int_{x_i}^{x_e} \psi(x)dx$, and put it in the form of: $\langle\psi\rangle = A + B\left[\psi(x_e) - \psi(x_i)\right]$, where A and B are constants. Hint: using a coefficient, $\tau = \frac{\Sigma_t(x_e - x_i)}{\mu}$, and the definition of the exiting angular flux, $\psi(x_e)$, will help.

(c) Is your derivation valid for neutrons traveling in both the positive x direction, where $x_e > x_i$ and $\mu > 0$, and the negative x direction, where $x_e < x_i$ and $\mu < 0$?

a) $\mu = \cos\theta$

where θ is the angle between a flux and the 1D direction we are solving.



b) x is the direction for which we are solving the flux. The material is homogenous in all directions perpendicular to $\vec{x}$.

c) ψ is the angular flux in units of something like $\left[\frac{\text{particles}}{\text{cm}^2 \cdot \text{s} \cdot \text{eV} \cdot \text{sr}}\right]$

So it is the amount of particles traveling

So it is the amount of particles traveling through a unit area per unit time in a specific direction and energy.

d) Σ_t is the total macroscopic cross section, which is the avg. # of reactions per particle per unit track length, where a reaction removes the particle from $\psi(r, \Omega, E, t)$.

e) Q is the source of neutrons per unit volume in the energy and direction of ψ

f) ψ_i is the flux at x_i, which is the inlet, $\psi_i = \psi(x_i)$ in the direction and energy of the flux. (Boundary condition)

g) x_i is the location of the inlet

//

5)

$$\langle\psi\rangle \cdot (x_e - x_i) = \int_{x_i}^{x_e} \psi(x)\,dx$$

$$= \int_{x_i}^{x_e} \psi_i e^{-\frac{\Sigma_t}{\mu}(x-x_i)}\,dx + \frac{Q}{\Sigma_t}\int_{x_i}^{x_e}\left(1 - e^{-\frac{\Sigma_t}{\mu}(x-x_i)}\right)dx$$

$$= \frac{Q}{\Sigma_t}\int_{x_i}^{x_e} dx + \left(\psi_i - \frac{Q}{\Sigma_t}\right)\underbrace{\int_{x_i}^{x_e} e^{-\frac{\Sigma_t}{\mu}(x-x_i)}\,dx}$$

$u = x - x_i \qquad du = dx$

$u_i = 0 \qquad u_e = x_e - x_i$

$$\int_0^{u_e} e^{-\frac{\Sigma_t}{\mu}u}\,du = \left[-\frac{\mu}{\Sigma_t} e^{-\frac{\Sigma_t}{\mu}u}\right]_{u=0}^{u=x_e-x_i}$$

$$= -\frac{\mu}{\Sigma_t}\left(e^{-\frac{\Sigma_t}{\mu}(x_e-x_i)} - 1\right)$$

$$= \frac{Q}{\Sigma_t}(x_e - x_i) + \left(\psi_i - \frac{Q}{\Sigma_t}\right)\frac{\mu}{\Sigma_t}\left(1 - e^{-\tau}\right)$$

$$\langle\psi\rangle = \frac{Q}{\varepsilon_t} + \frac{\mu}{\varepsilon_t}\left(\psi_i - \frac{Q}{\varepsilon_t}\right)\cdot\left(\frac{1}{x_e - x_i}\right)\left(1 - e^{-\tau}\right)$$

Let $\psi_e = \psi(x_e) = \psi_i e^{-\frac{\varepsilon_t}{\mu}(x_e - x_i)} + \frac{Q}{\varepsilon_t}\left(1 - e^{-\frac{\varepsilon_t}{\mu}(x_e - x_i)}\right)$

$$\psi_e = \frac{Q}{\varepsilon_t} + \left[\psi_i - \frac{Q}{\varepsilon_t}\right]e^{-\tau}$$

$$\left(\psi_i - \frac{Q}{\varepsilon_t}\right)\left(1 - e^{-\tau}\right)$$

$$= \psi_i - \psi_i e^{-\tau} - \frac{Q}{\varepsilon_t} + \frac{Q}{\varepsilon_t}e^{-\tau}$$

$$= \psi_i - \left[\frac{Q}{\varepsilon_t} + \left(\psi_i - \frac{Q}{\varepsilon_t}\right)e^{-\tau}\right]$$

$$= \psi_i - \psi_e$$

$$\langle\psi\rangle = \frac{Q}{\varepsilon_t} + \frac{\mu}{\varepsilon_t(x_e - x_i)}\left(\psi_i - \psi_e\right)$$

$$A = \frac{Q}{\varepsilon_t} \qquad B = \frac{\mu}{\varepsilon_t(x_i - x_e)}$$

$$\langle\psi\rangle = A + B\left(\psi_e - \psi_i\right)$$

c) Yes, the derivation is agnostic of sign.
You just have to be careful with the sign.