1

Monday, February 10, 2025 1:46 PM

1. Infinite homogeneous medium: There is a material that is infinitely large in space ($\frac{d\psi}{dx} = 0$) with an isotropic source ($Q^0 = 1$). For this problem, you don't need a space-angle dependent transport code to solve it, but by solving it with your code, with a spatial mesh, you can verify it works for even the simplest of problems by comparing with the analytic solution and see how quickly it solves problems for each scattering cross section. Note that some problems may not have a solution.
   - Use reflecting boundaries on each size
   - Use a problem size of 100cm with ten 1cm mesh cells.
   - Use the minimum number of angular unknowns P_1 (aka diffusion) and S_2 (Gauss Legnedre with 2 unknowns)
   - If you're feeling ambitious, try it with more anglular degrees of freedom and see how the time-to-solution changes.

Steady State Analytical Solution

$$\frac{d\psi}{dx} + \Sigma_t \psi(x) = Q$$

$$\psi(x) = \frac{Q}{\Sigma_t}$$

$$\Sigma_t \psi(x) = Q + \phi \Sigma_s$$

$$\psi(x) = \frac{1}{\Sigma_t - \Sigma_s} Q$$

//

Discrete ordinances

m directions

$$\phi = \int_{-1}^{1} \psi(\mu)\, d\mu \approx \sum_{m=0}^{m} \psi(\mu_m) w_m$$

where $\sum_{m}^{m} w_m = 2$

Weights and angles are determined by the quadrature set

```
number_of_directions = 2
angles, weights = np.polynomial.legendre.leggauss(number_of_directions)
print(angles)
print(weights)
```

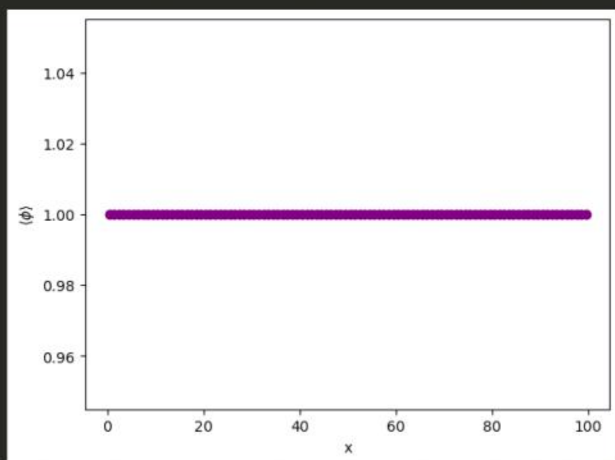
✓ 0.0s Open 'weights' in Data Wrangler

```
[-0.57735027  0.57735027]
[1. 1.]
```

```python
# Problem 1:
number_of_directions = 2
angles, weights = np.polynomial.legendre.leggauss(number_of_directions)
DiscreteOrdinances(
    angles=list(angles),
    weights=list(weights),
    left_boundary="reflecting",
    right_boundary="reflecting",
    volumetric_source=1,
    sigma_t=1,
    sigma_s=0,
    title_start="test",
    x_end=100,
    n_surfaces= 101
).angular_flux();
```

✓ 0.1s

```
test: Converged after 0 iterations
```



This matches with the simple analytic solution

$$\frac{Q}{\Sigma_t} = \frac{1}{1} = 1$$

(Tested for other cases too)

2

Tuesday, February 11, 2025 9:51 PM

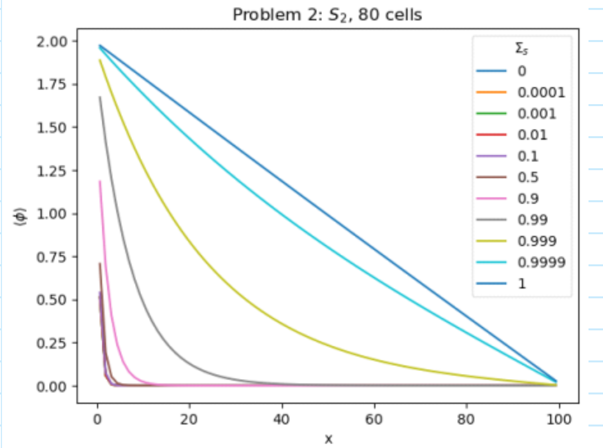
2. Source-Free 1D transport: There is a single material that has a finite size (100 cm) with no source ($Q^0 = 0$) and an isotropic incident angular flux on the left-side, $\psi(0, \mu > 0) = 1$, and a vacuum boundary on the right: $\psi(100, \mu < 0) = 0$. For each scattering cross section, try to solve the problem by increasing the number of space and direction degrees of freedom. All problems will have a solution, but not all methods will be able to solve them.

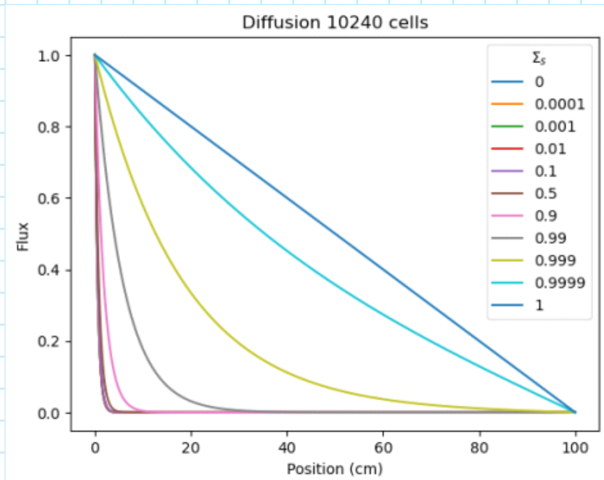
   - For the spatial mesh, begin with 10 mesh cells and increase by factors of four: 40, 160, 640, 2560, and 10240.

- For discrete ordinates (S_n), increase the S_n order using a Gauss-Lengendre quadrature for n=2, 4, 8, 16, 32, and 64 directions.
- For spherical harmonics (P_n), increase the P_n order for n=1, 3, 5, 7, 9

Sn code: https://github.com/bradenpecora/ME388F/blob/main/HW3/discrete_ordinates.ipynb

Diffusion code: https://github.com/bradenpecora/ME388F/blob/main/HW3/diffusion.ipynb





Specifying the boundary condition with angular flux for diffusion is hard… I just chose 1
The shape is the same.

$$-\frac{d}{dx}\left[D\frac{d\phi}{dx}\right] + \Sigma_a \phi = Q = 0$$

In the limit that

$$\Sigma_s \to 1, \quad \Sigma_a \to 0$$

$$\frac{d^2\phi}{dx^2} = 0$$

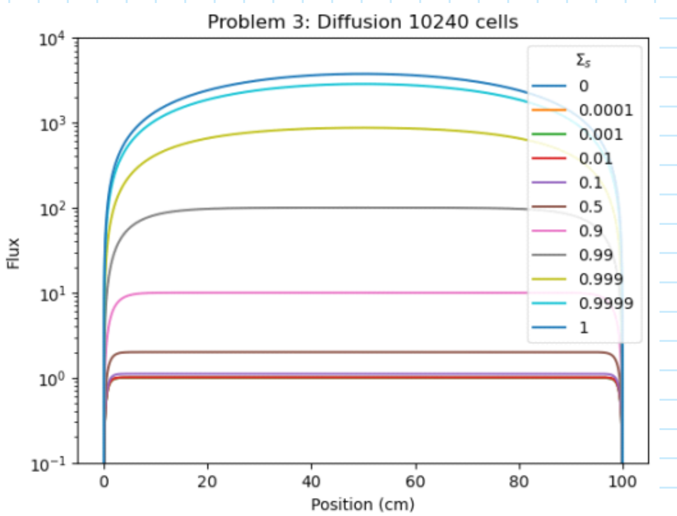
$\therefore$ Linear

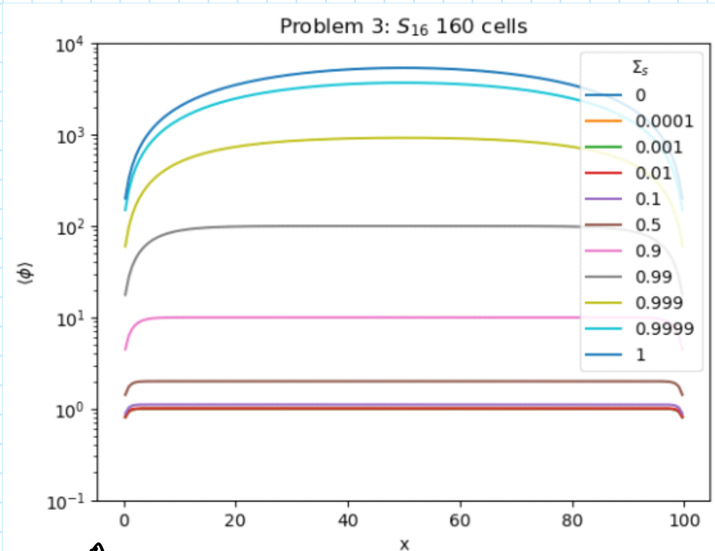
It is correct that the problem becomes linear as $\Sigma_s \to 1$

3

Tuesday, February 11, 2025 11:12 PM

3. Finite source-driven transport: There is a single material that has a finite size (100 cm) with an isotropic source ($Q^0 = 1$). All problems will have a solution, but not all methods will be able to solve them.
   - For the spatial mesh, begin with 10 mesh cells and increase by factors of four: 40, 160, 640, 2560, and 10240.
   - For discrete ordinates (S_n), increase the S_n order using a Gauss-Lengendre quadrature for n=2, 4, 8, 16, 32, and 64 directions.
   - For spherical harmonics (P_n), increase the P_n order for n=1, 3, 5, 7, 9





Plotting scalar flux averaged at cells, so we don't see the flux go to 0 at our surfaces

Thought provoking analyses

Thursday, February 13, 2025 5:04 PM

1. For the infinite homogeneous problem, which problem(s) was/were unsolvable and why? Why did this become solvable for problems 2 and 3?

Diffusion for Sigma scatter = 1 was literally unsolvable as it resulted in a singular matrix.
The analytical solution to the problem is infinity.

The solution for discrete ordinates didn't converge after 10,000 iterations for Sigma_s = 0.9999 and =1,
But the answer approaches the actual answer (except, of course, when the solution is infinity).

When sigma_s =1 and you have reflective boundary conditions, neutrons can't be absorbed or leave, so you can't solve a steady state problem as neutrons are constantly being produced but can't leave the system.

For problems 2 and 3, we don't have reflective boundary conditions, so neutrons can leave the system.

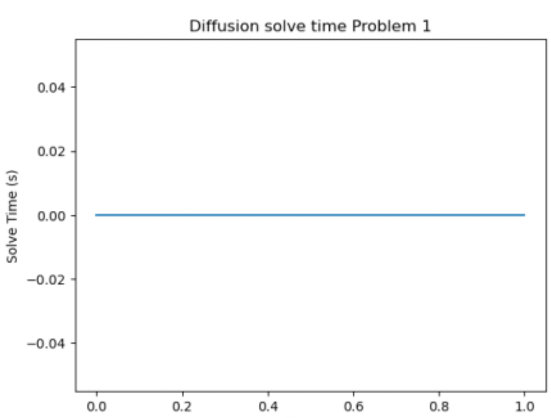
2. For the other infinite homogeneous problems:
   (a) Did your code have a spatially-flat solution?
   (b) Was it analytically correct?
   (c) How did the time to solution compare as a function of scattering for your solution method? Why do you think that is?

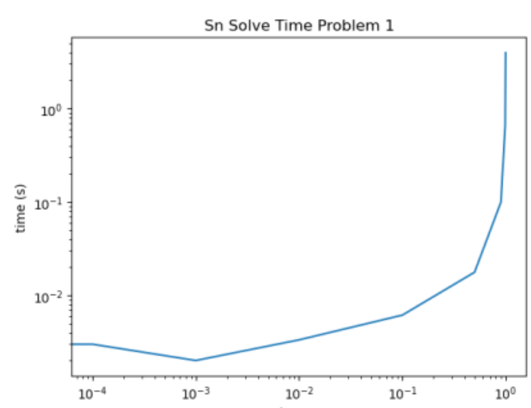
a) Yes, I had a spatially flat solution.

b) The analytical solution is angular flux = source / sigma_absorption. I got this answer.

c) For diffusion, the time to solution didn't change with time, there is no iteration for my implementation.

   For discrete ordinates, the relation is a power or exponential fit or something. As you have more scattering, you need more and more flux iteration to get the correct answer, so the problem takes longer. As Sigma_s approaches 1, the problem can't converge so the time goes to infinity.



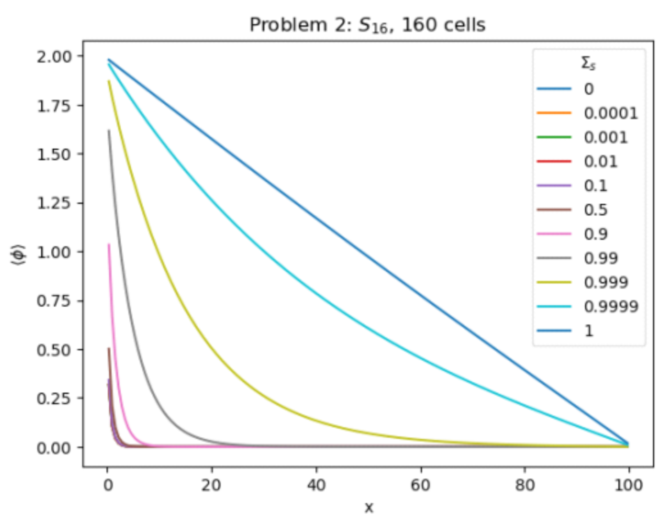


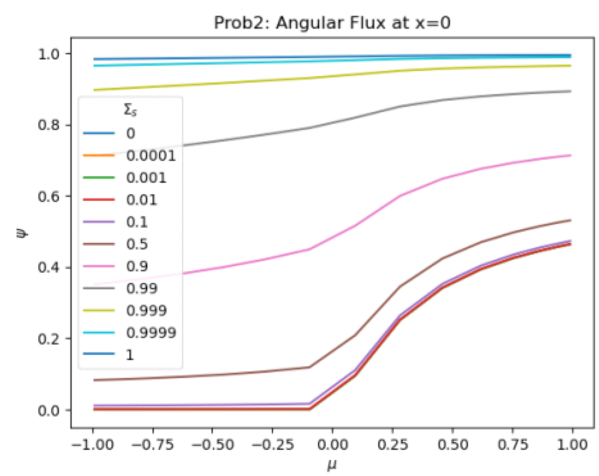
3. For problem 2, which method(s) could not solve which problem(s) and why?

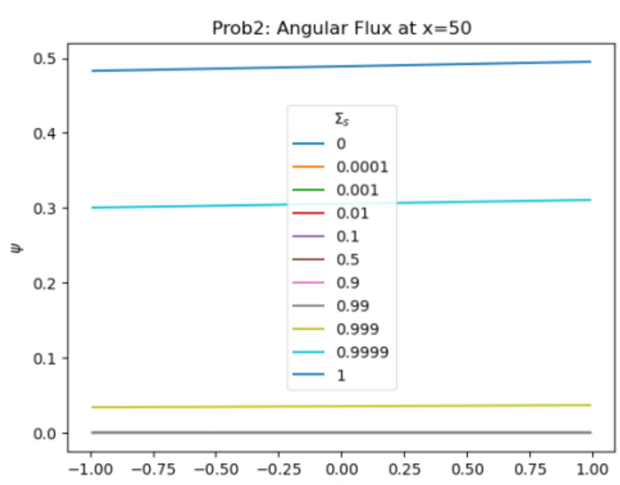
I was able to solve all diffusion problems, even for the largest mesh. We are only using first order spherical harmonics, so the matrix isn't as big (the matrix scales with every two increases in order since we can simplify).

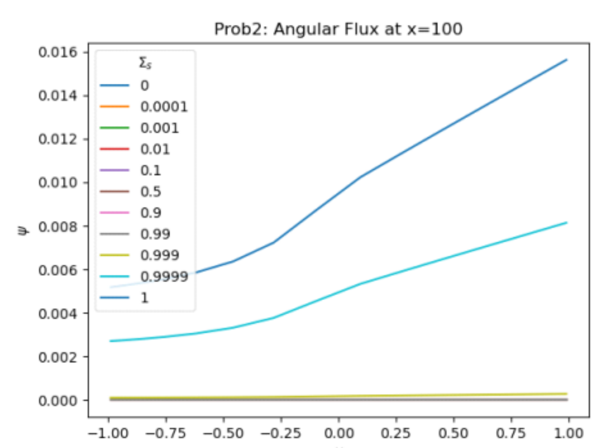
I didn't run out of memory for S64 with 10240 cells, but it takes a very long time to solve... I could save memory by iterating between angles at the expense of computational cost. The size of the matrix is number of directions

4. For problems 2 and 3: for the highest angular and spatial solutions you could produce, plot the angular flux on the left ($x = 0$), in the center ($x = 50$), and on the right side ($x = 100$).
   (a) How do the shapes compare for each scattering cross section and problems? Do the methods seem to agree?
   (b) Is the flux approximately linearly-anisotropic for any/all of the scattering cross sections?





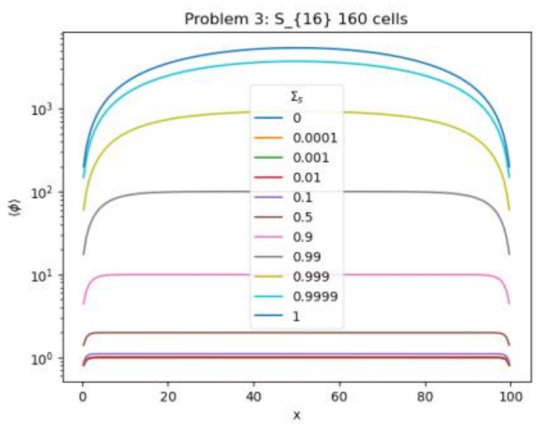


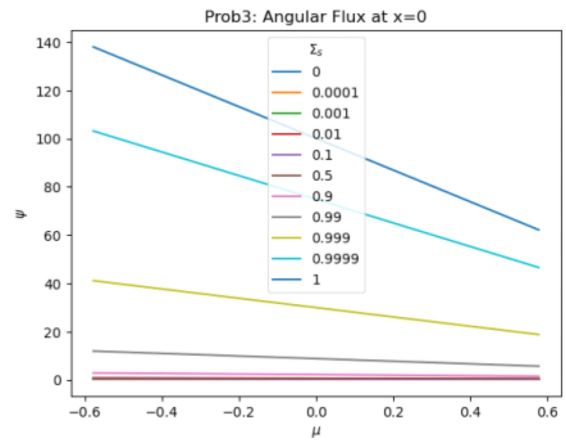


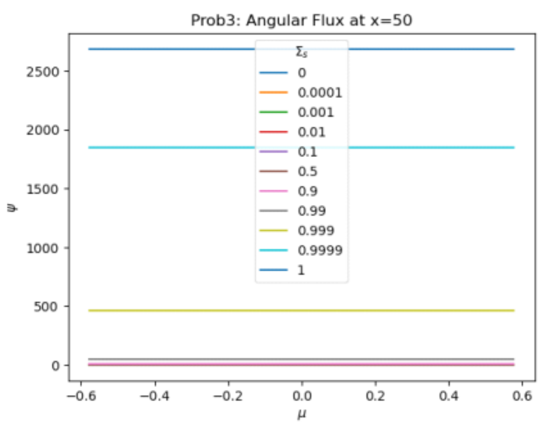
For problem 2, we have an incoming rightward flux at x=0
The flux gets pretty isotropic at x=0 with increasing cross section since there is more scattering causing

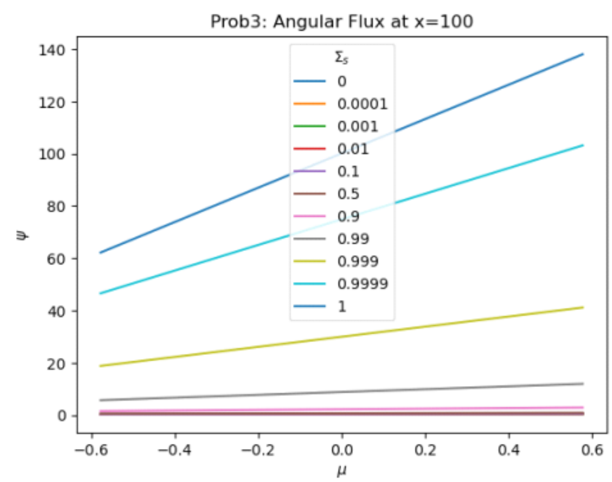
flux to go leftward
The flux gets kind of linearly anisotropic at the right side x =100 as scattering increases
The average flux increases with sigma scatter (makes sense)

I can't really compare to diffusion since I don't have angular flux. The scalar fluxes have the same shape though.





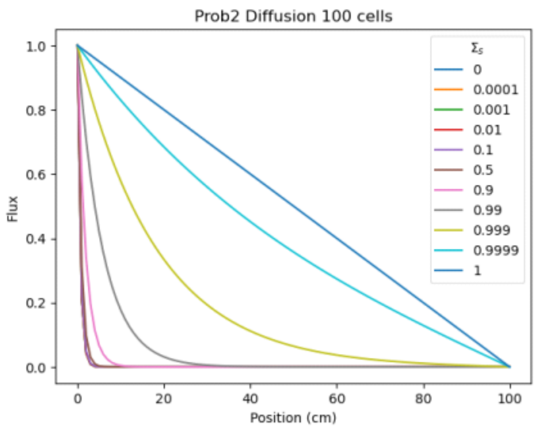


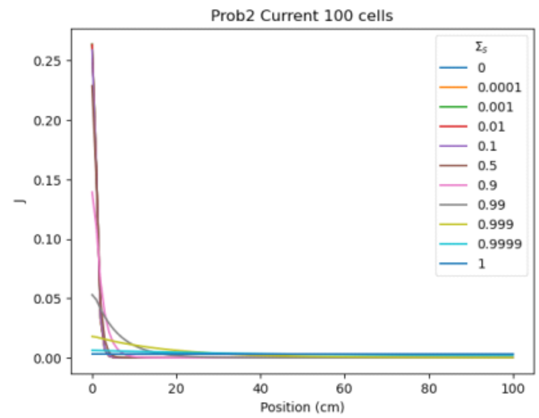


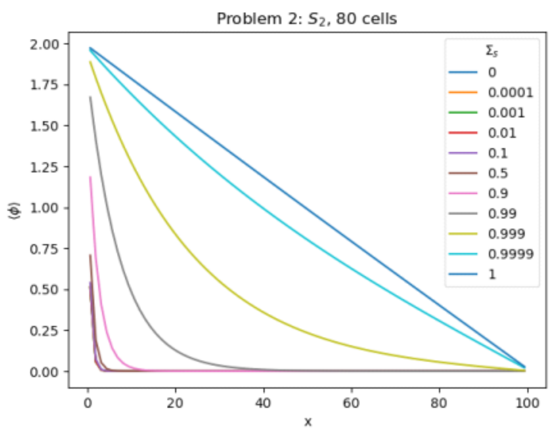
For problem 3 (volumetric source, vacuum boundary conditions)

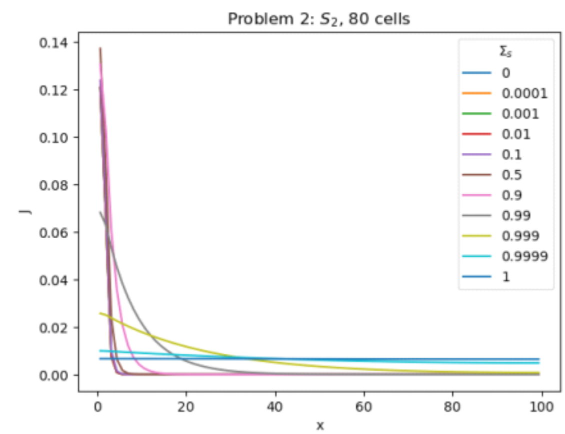
The fluxes are significantly more linearly anisotropic. The slope and average value increase with scattering cross section.

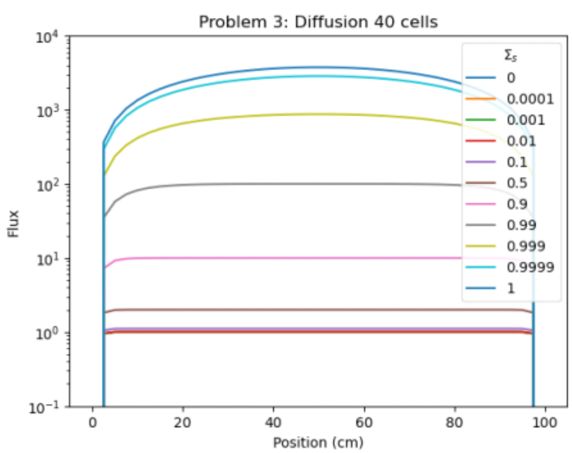
5. For problems 2 and 3: plot the spatial distribution of the scalar flux and current (ϕ^1) for a sample of cases. What did you learn?

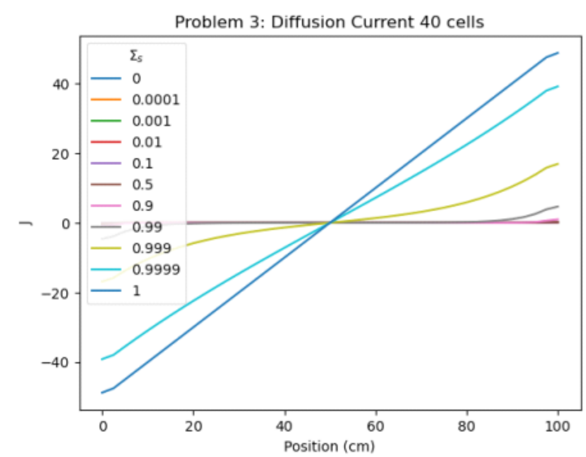

Prob2 Diffusion 100 cells
Flux
Position (cm)
Σ_s
0
0.0001
0.001
0.01
0.1
0.5
0.9
0.99
0.999
0.9999
1

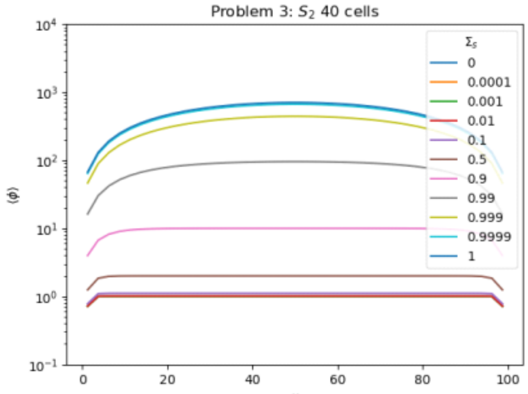

Prob2 Current 100 cells
J
Position (cm)
Σ_s
0
0.0001
0.001
0.01
0.1
0.5
0.9
0.99
0.999
0.9999
1

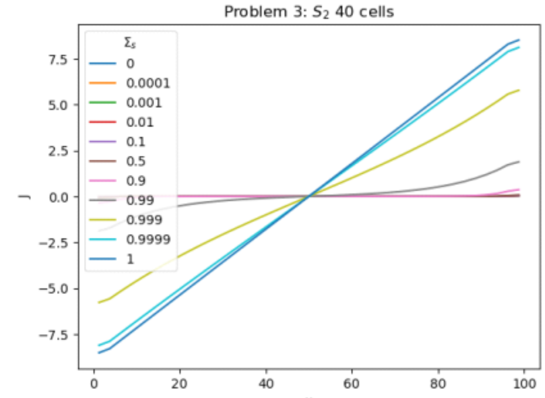

Problem 2: S_2, 80 cells
$\langle\phi\rangle$
x
Σ_s
0
0.0001
0.001
0.01
0.1
0.5
0.9
0.99
0.999
0.9999
1

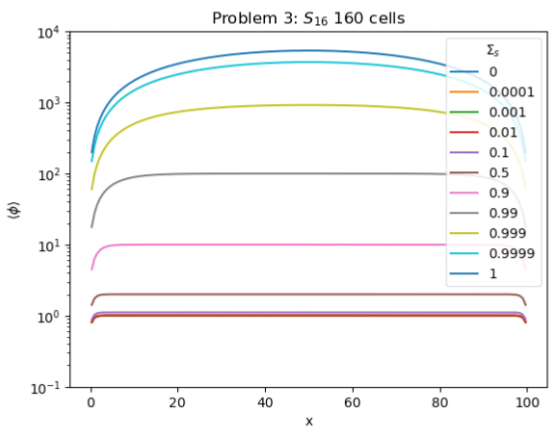

Problem 2: S_2, 80 cells
J
x
Σ_s
0
0.0001
0.001
0.01
0.1
0.5
0.9
0.99
0.999
0.9999
1

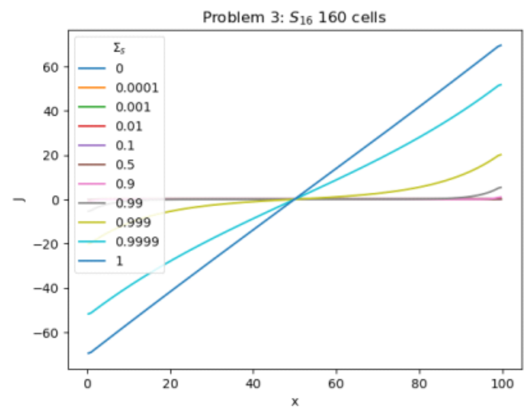

Problem 3: Diffusion 40 cells
Flux
Position (cm)
Σ_s
0
0.0001
0.001
0.01
0.1
0.5
0.9
0.99
0.999
0.9999
1


Problem 3: Diffusion Current 40 cells
J
Position (cm)
Σ_s
0
0.0001
0.001
0.01
0.1
0.5
0.9
0.99
0.999
0.9999
1


Problem 3: S_2 40 cells
$\langle\phi\rangle$
x
Σ_s
0
0.0001
0.001
0.01
0.1
0.5
0.9
0.99
0.999
0.9999
1


Problem 3: S_2 40 cells
J
x
Σ_s
0
0.0001
0.001
0.01
0.1
0.5
0.9
0.99
0.999
0.9999
1





Problem 2 diffusion, didn't really know the boundary condition since we are given angular flux as a boundary and not scalar flux

Problem 3: For high scattering cross sections, Sn doesn't get the answer. I needed more cells and angles. This could be a false convergence thing.

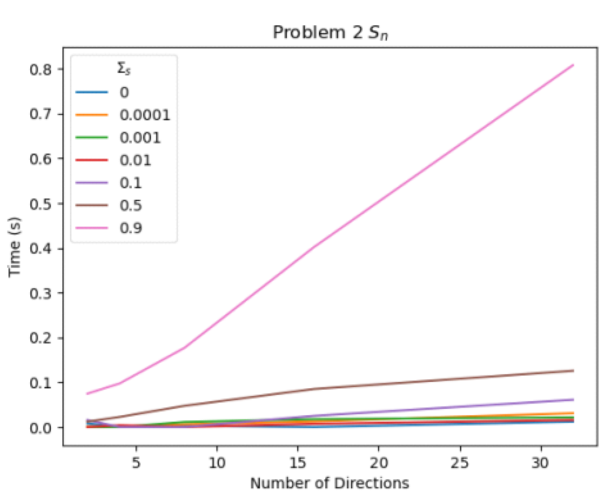
They mostly look the same. Changing mesh size or number of ordinates doesn't really change the answer that much.

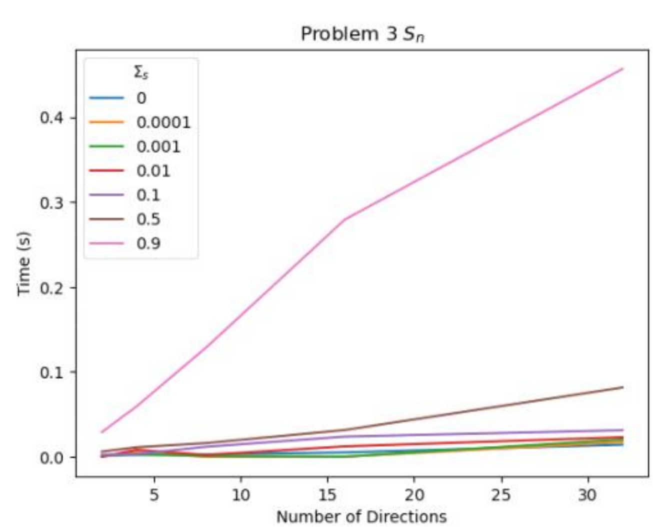
6. For problem 3, which method(s) could not solve which problem(s) and why?

There is the same memory constraint before and the problems end up taking a while.

I could solve diffusion with 10240 cells. I stopped at S16 160. There may be a memory constraint.

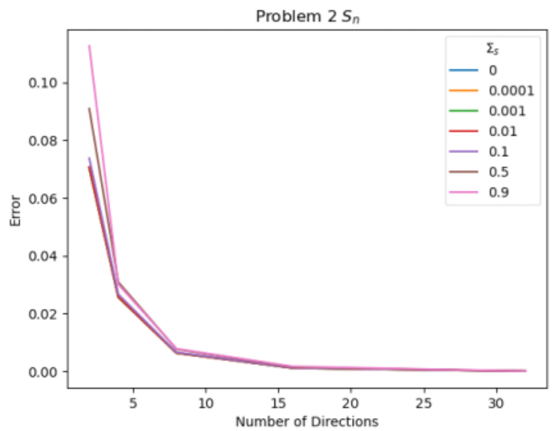
7. For a given mesh in problem 2 and 3, plot the time to solution as a function of the S_n order and P_n order.

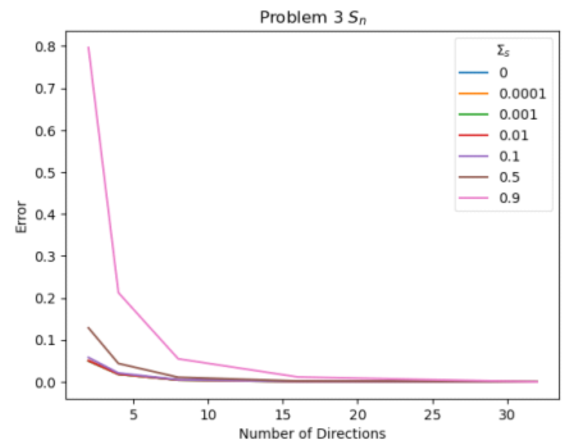




Time to solution is linearly related to the order of Sn.

8. For problems 2 and 3: If the highest order you could solve (in S_n and P_n) for the largest mesh you could solve with were considered *truth*, plot the error in the scalar flux for a given angular quadrature compared to the *true* solution. As the angular order increases, does the error smoothly approach the *true* solution or does it bounce around unpredictably? Is that true for both S_n and P_n





Error decreases exponentially with number of direction (order) for both problem 2 and 3.
The decrease in error is pretty smooth and predictable.