ASTR21200

Observational Techniques in Astrophysics
Lecture 6

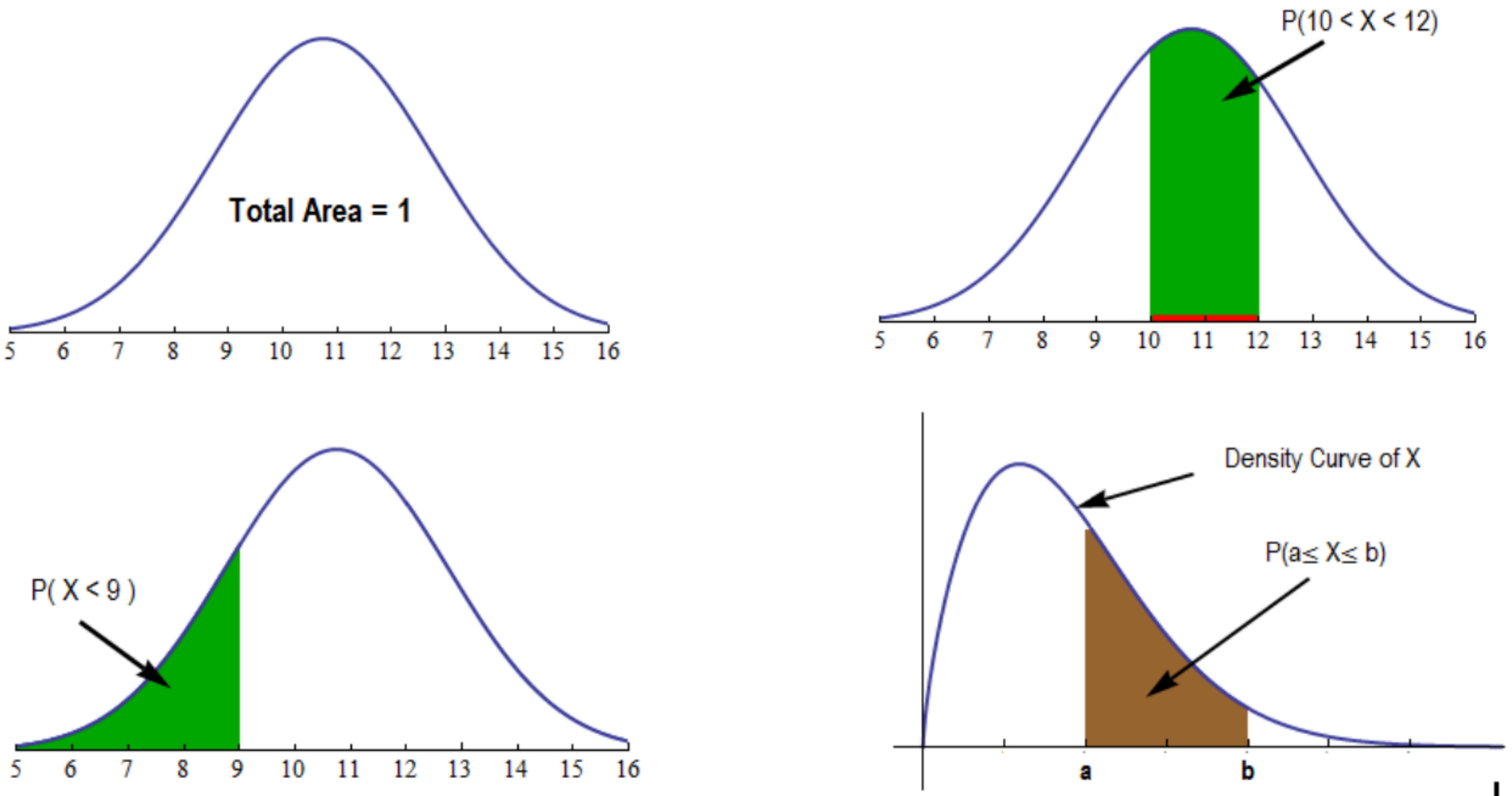
Bradford Benson

For Next ~1-Week

- For Lab-2, we will start using Stone Edge Observatory (SEO) to make new observations of objects
- SEO Tutorials:
  - Over the next week (Jan-24 to Jan-30), Al Harper will be available to run SEO tutorials upon request.
  - Because of the time difference observing nights will start at 9pm central
    - If interested, please Slack "daharper" (Al Harper) to request a night.
    - Al will post an announcement in the class Slack, if others want to join.
    - Use the class zoom at: https://uchicago.zoom.us/j/94887021971?pwd=QjJRWGhCZ0tJbjFVYzY2T2ZQVmZWdz09

Probability Distributions

- Probability Distribution: Describes the expected (or measured) distribution of measurements.
- Can integrate a probability distribution over range of values to find a probability to be in that range



Lumen Learning

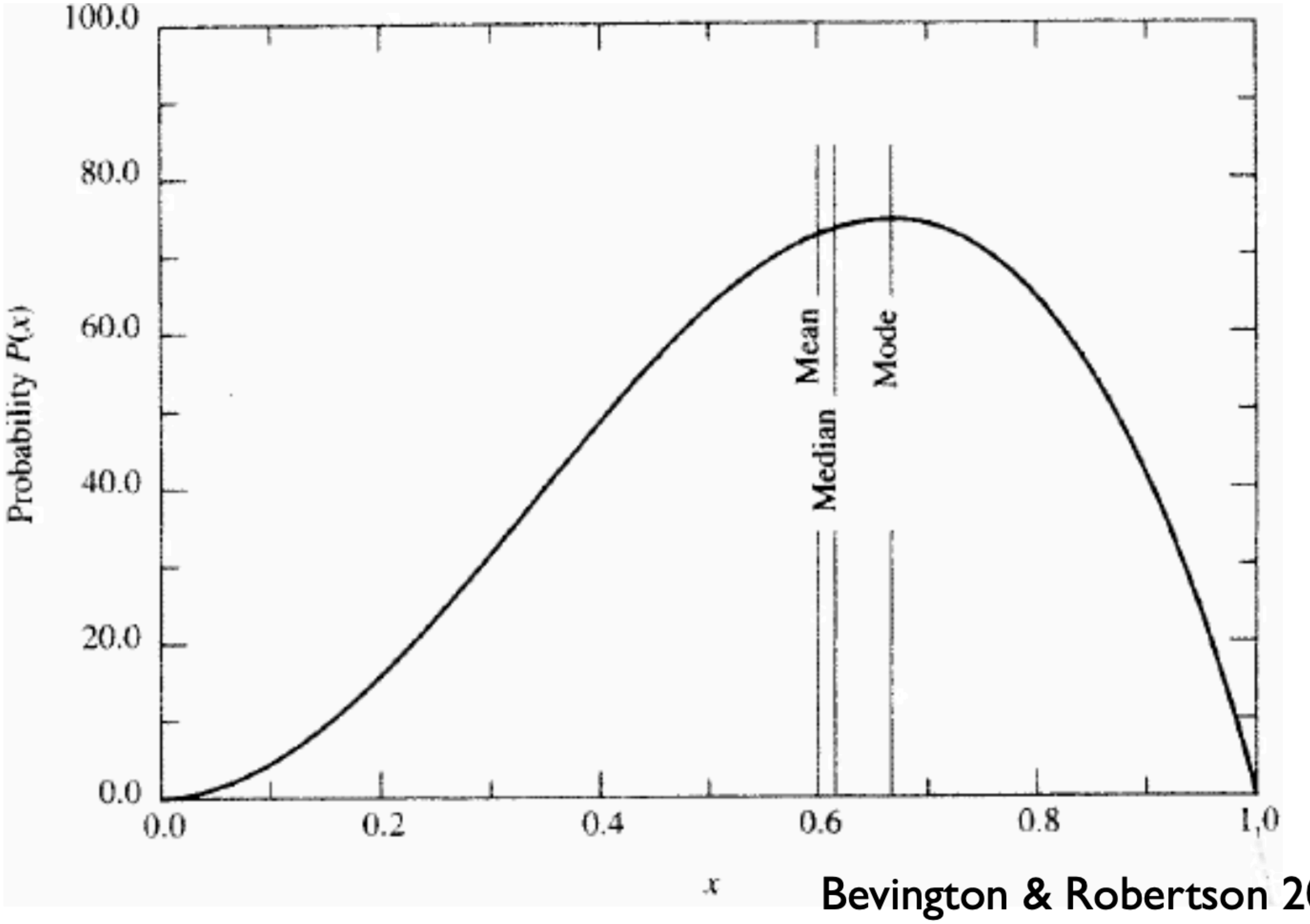
Summary Statistics

- **Mean**: The “average”value, in the limit of N measurements:

$$\bar{x} = \frac{1}{N}\sum_i x_i$$

- **Median**: 50th percentile of distribution, i.e., 50% of the measurements are larger (or smaller) than that value
- **Mode**: The most “common” or “likely” measurement value

- All three are useful, but will depend on the problem, and possibly the underlying probability distribution being measured



Bevington & Robertson 2003

Deviation, Variance, Standard Deviation

- **Deviation**: of one measurement from the average

$$d_i = x_i - \mu$$

- **Sample variance**: Average of the squares of the deviations.
  - Sample variance can also be estimated from a sample population (i.e., a sample of measurements)

$$\sigma^2 = \frac{1}{N}\sum_i (x_i - \mu)^2$$

$$= \frac{1}{N-1}\sum_i (x_i - \bar{x})^2$$

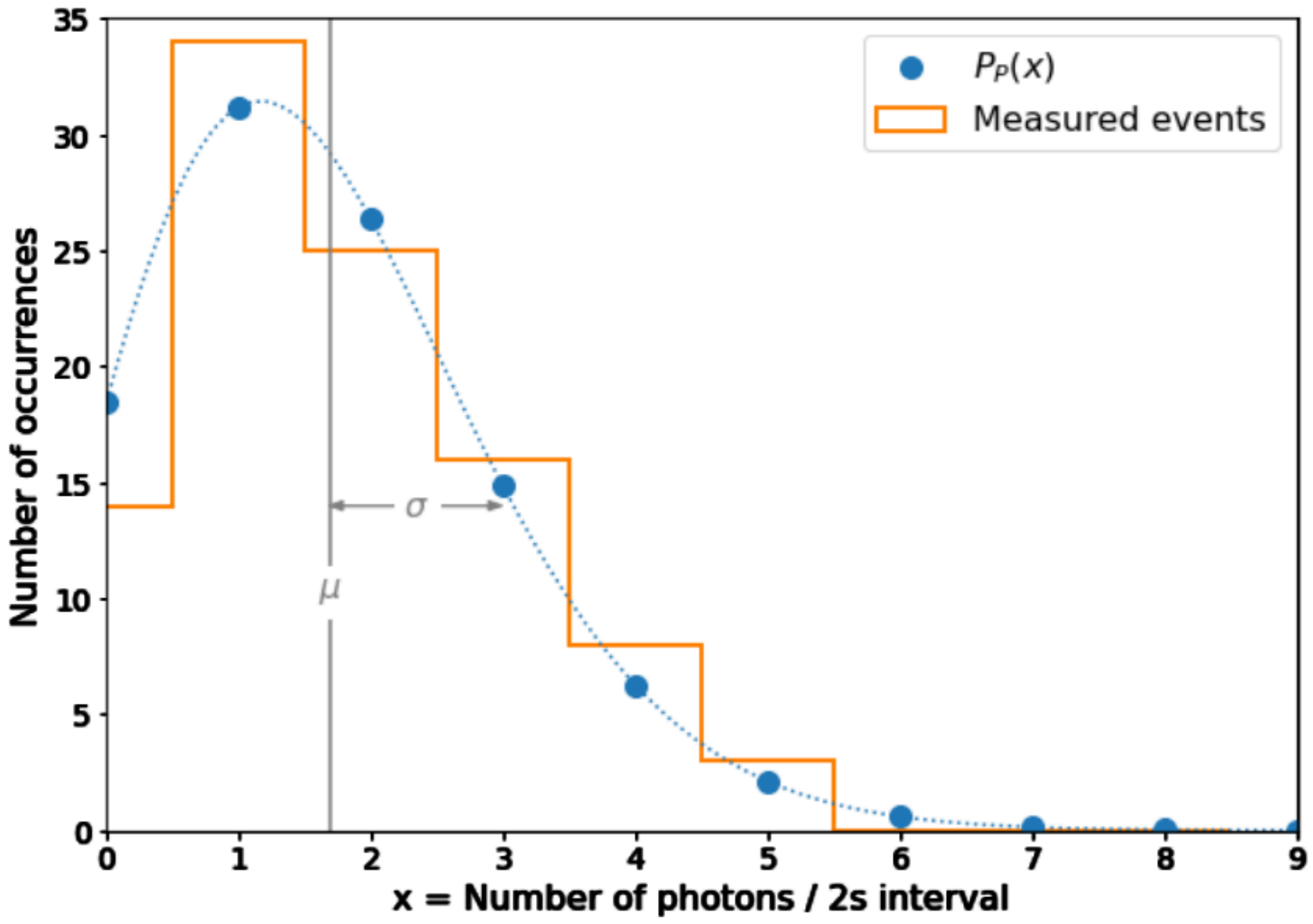
- **Standard Deviation**: The square root of the variance (i.e., σ), or the "typical" deviation around the mean.

Common Probability Distributions

- Three common probability distributions:
  - **Poisson distribution:** Counting experiments for discrete events (e.g., photon counts, N_{counts})
    - Standard-Deviation: $\sigma = \mathrm{sqrt}(N_{counts})$
  - **Binomial distribution:** For experiments with only a small number of possible final states (e.g., coin tosses)
  - **Gaussian (Normal) distribution:** Limiting case of binomial and poisson distributions, for large number of events / measurements

Poisson Distribution: Example

- A detector measures the number of gamma-ray photons per 2-second intervals, making 100 measurements



$$P_{\rm P}(x|\mu) = \frac{\mu^x}{x!}e^{-\mu}$$

measured mean:

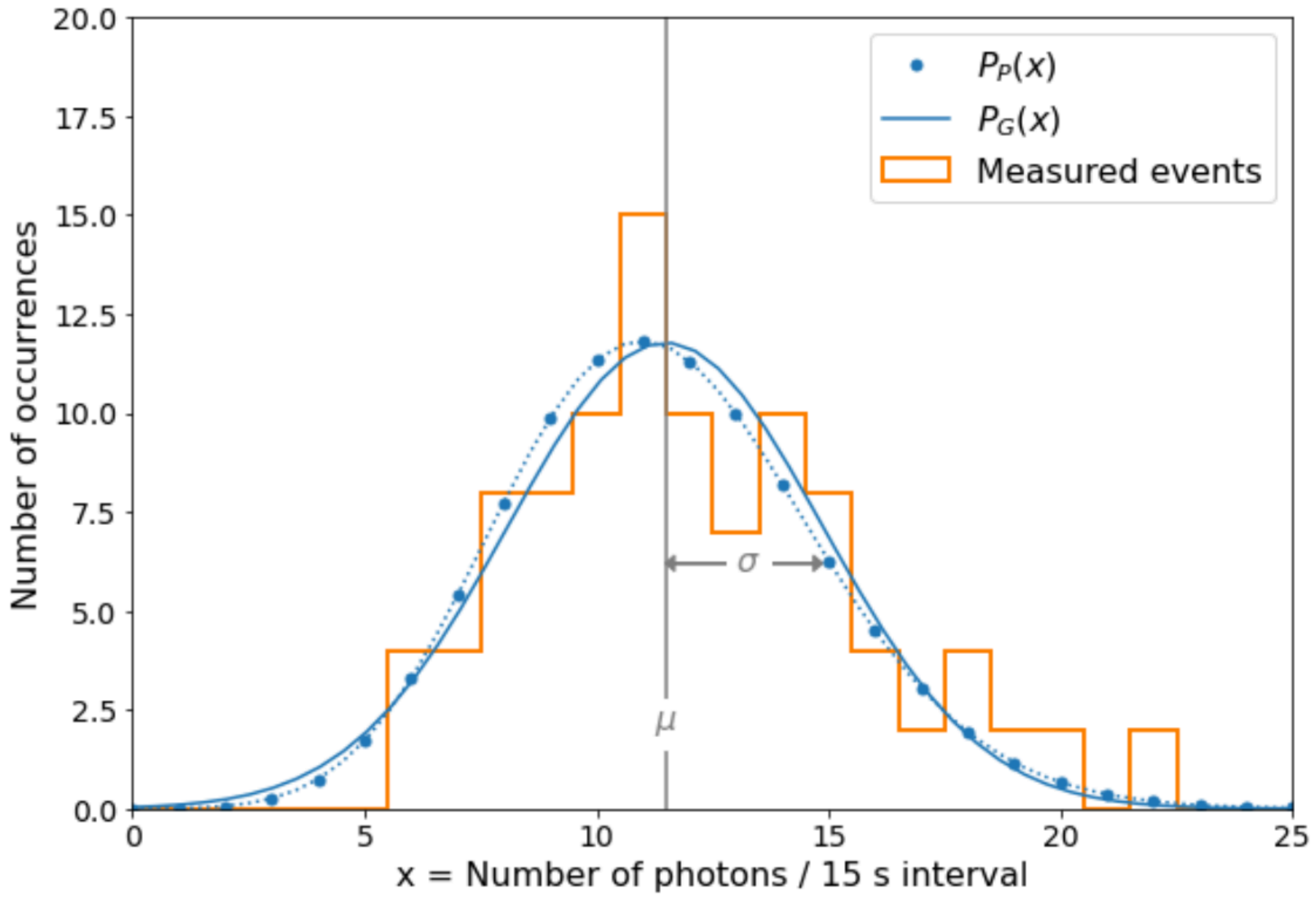
$$\bar{x} = 1.69$$

blue points:

$$P_{\rm P}(x|1.69)$$

Gaussian Distribution: Example

- A detector measures the number of gamma-ray photons per 15-second intervals, making 60 measurements



$$P_{\mathrm{G}}(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

measured mean:

$$\bar{x} = 11.48$$

blue points:

$$P_{\mathrm{P}}(x|11.48)$$

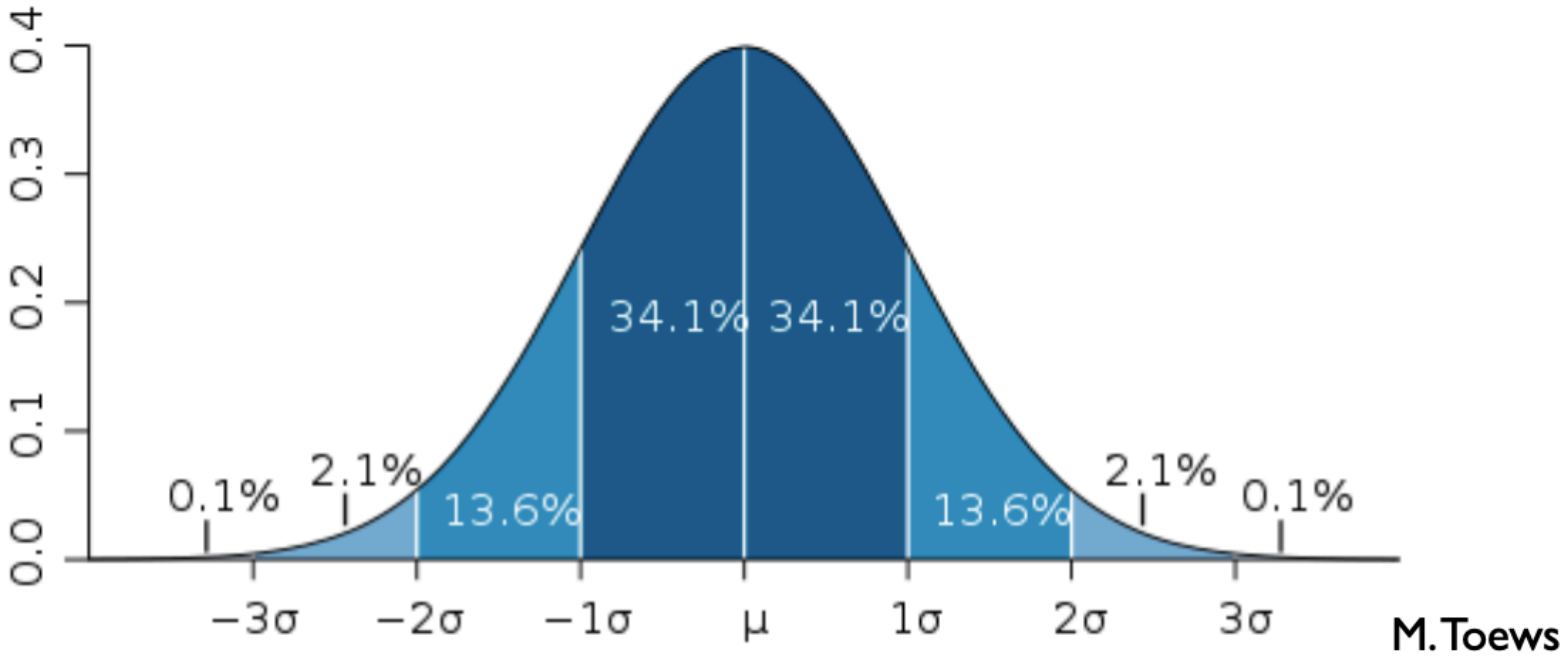
blue curve:

$$P_{\mathrm{G}}(x|\bar{x},\sqrt{\bar{x}})$$

Data is starting to be fit well by a Gaussian distribution

Gaussian Distribution: Example

- Relation between the probability of occurrence and the number of standard deviations away from the mean



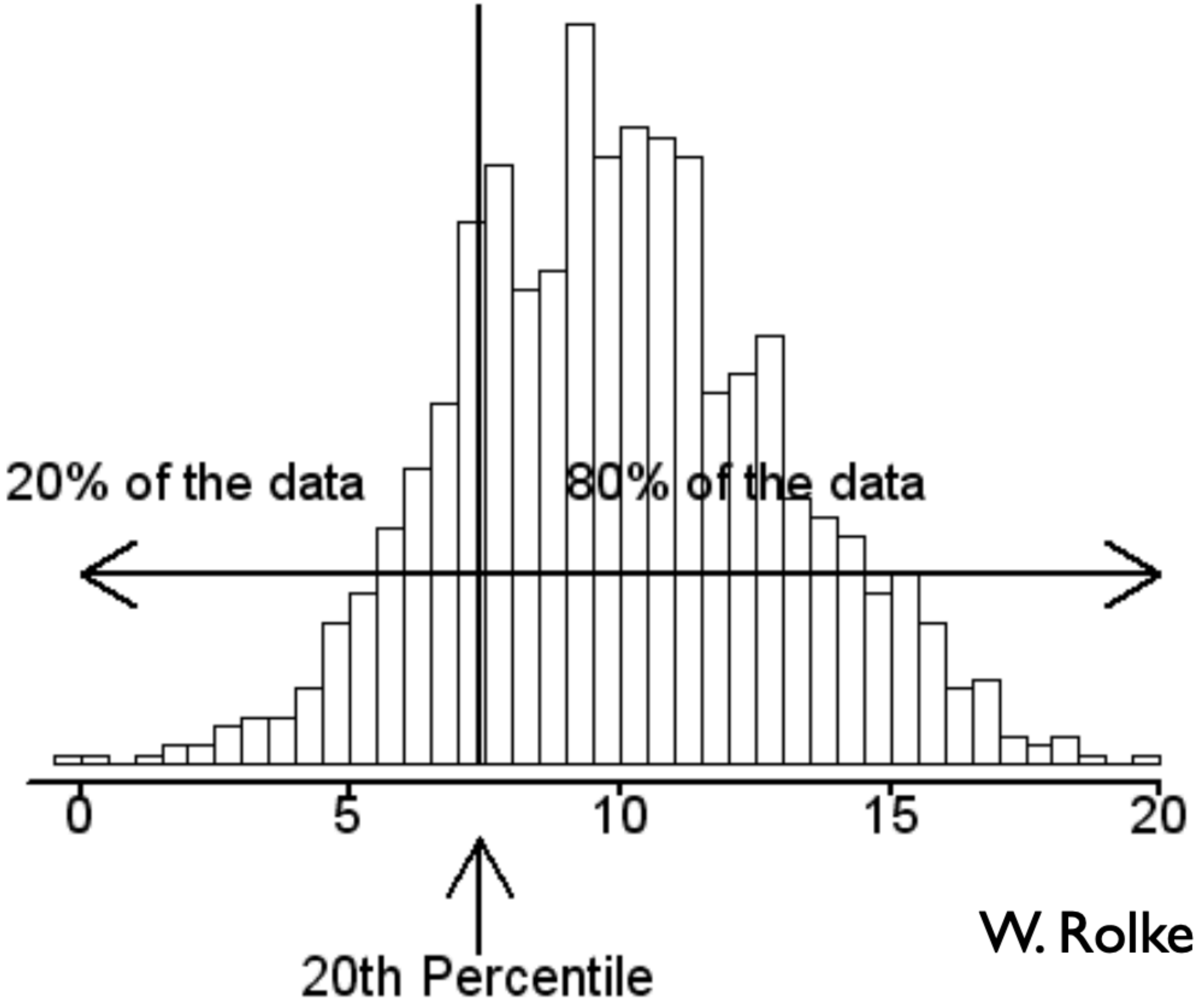
measurements should fall:

- within 1σ of the mean 68.3% of the time
- within 2σ of the mean 95.4% of the time
- within 3σ of the mean 99.73% of the time

Non-Gaussian Distributions

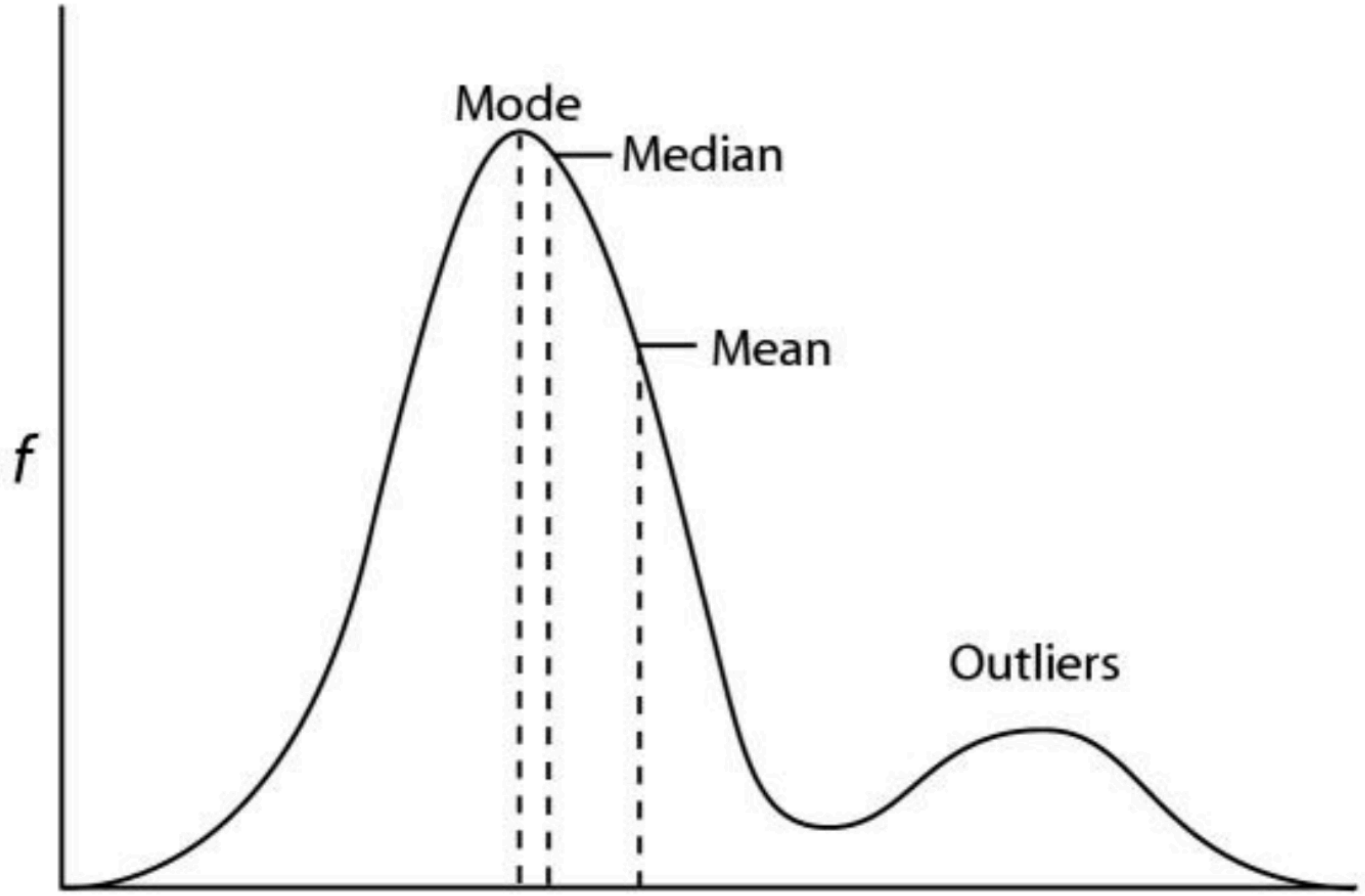
- What if your distribution is non-Gaussian?
- Have to decide on a case-by-case basis
- Percentiles can always sort your data, quote values that are above a certain percentage of the population, e.g.,
    - **Median**: 50th percentile
- Can quote measurement and uncertainty with percentiles
    - e.g., mean and range 68% confidence region

$$99.123^{+0.005}_{-0.004}$$



Outliers

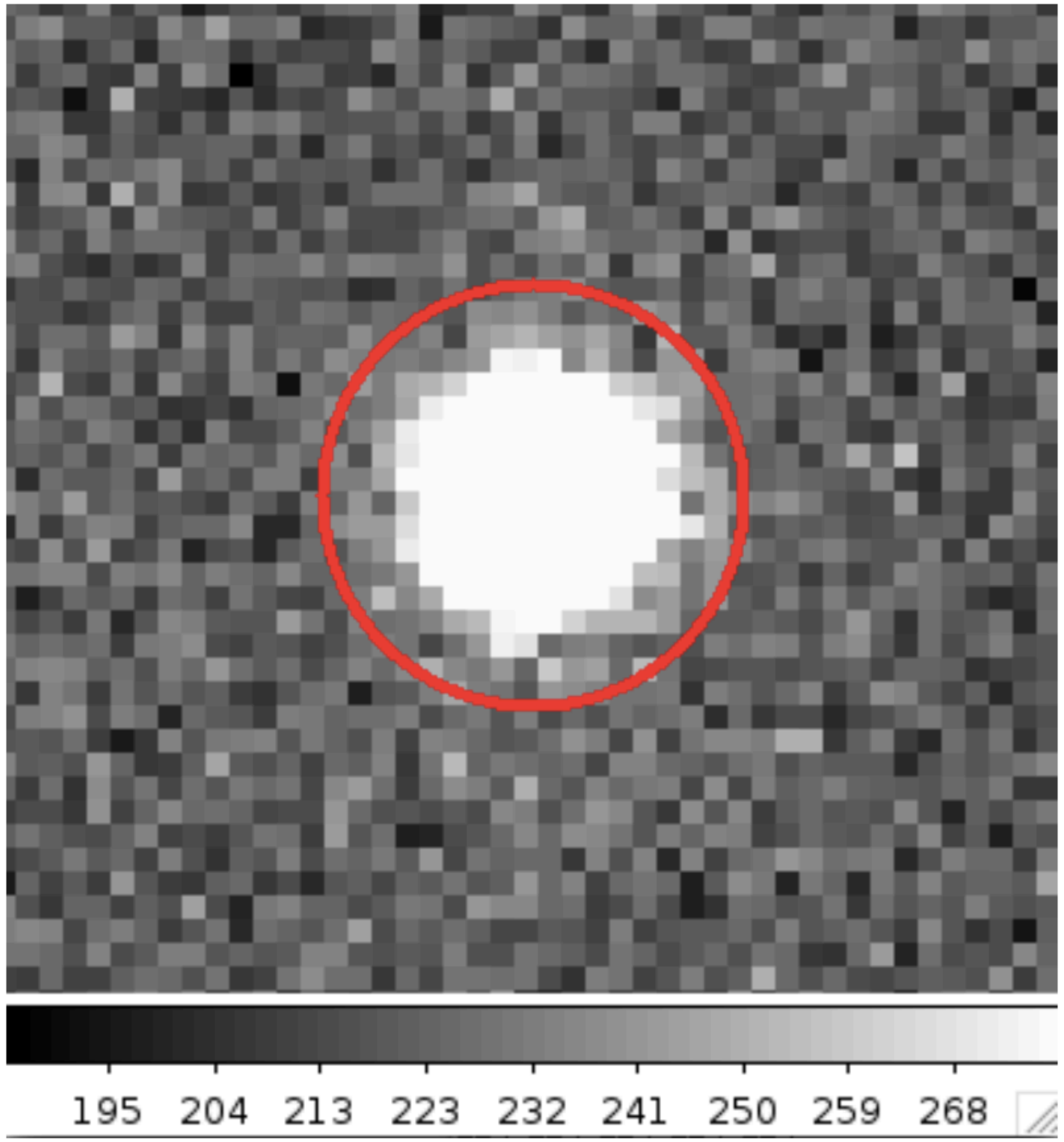
- For Gaussian distribution: Median = Mean
- What if distribution is "almost" normal, but has a few outliers? (e.g., cosmic rays on the CCD)
  - **Mean**: Significantly affected by outliers
  - **Median**: More robust to (a small number of) outliers
- Sometimes its ok to remove gross outliers (e.g., "sigma-clipping"), but need to make sure not to bias your results.



Hedges & Shah 2003

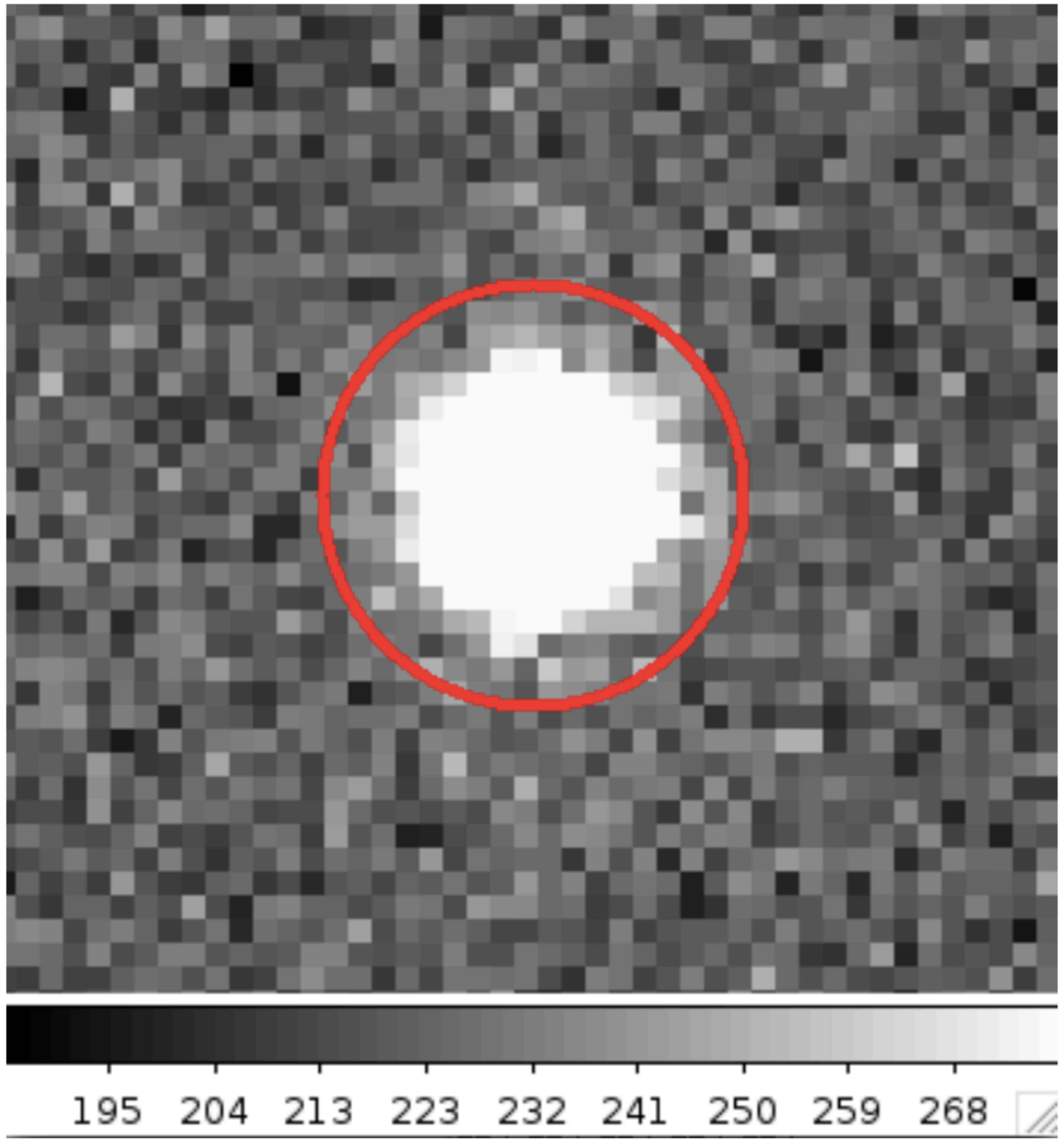
Signal: Flux from an Object

- Flux measured in an aperture
    - (Total Electrons) =
        - (Electrons from Object, i.e., the Signal) + (Electrons from backgrounds, i.e., atmosphere signal, dark current, etc.)
    - (Signal) = (Total Electrons) - (Background Electrons)
    - $N_{Object} = N_{total} - N_{background}$
- Note: From the image alone, we cant really tell which electrons are from the "object" (aka, signal), and which are from backgrounds (aka, noise).



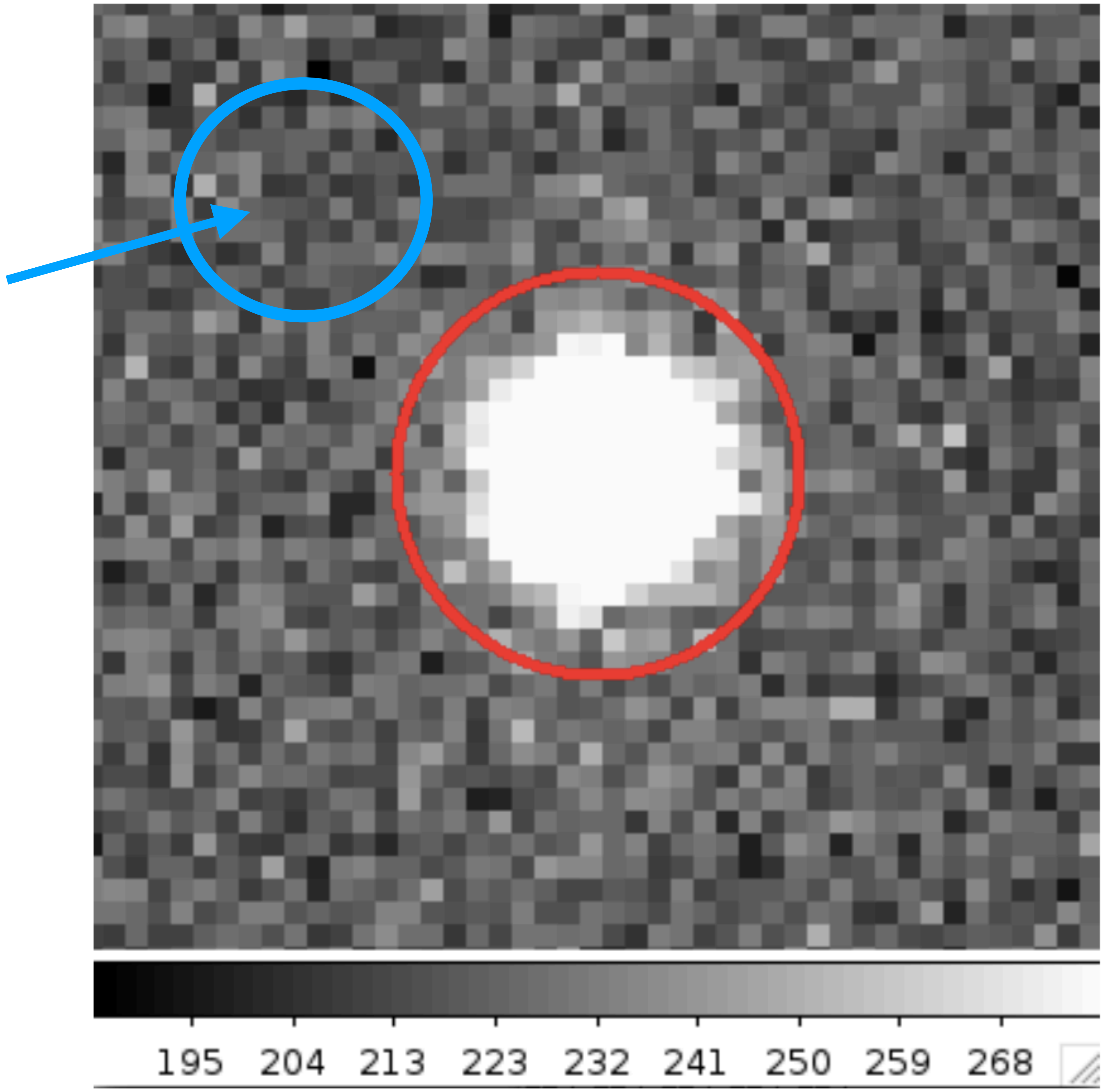
Signal: Flux from an Object

- There is always “noise” in a CCD image
  - Even in a “0-sec” “bias frame”, there will be additional readout noise
- For each pixel, we can imagine that the measurement is drawn from a distribution of mean *N* and width (σ)
- Even our estimates of the “background electrons” will have some uncertainty, so the distribution width above (σ) will be due to some combination in variance of signal, noise, background, etc.



Signal: Flux from an Object

- For example:
  - In "empty" regions of the image, we can measure the noise as the standard deviation of the pixel values.
  - The accuracy with which we measure that standard deviation will improve, the more "background" regions we can average over.

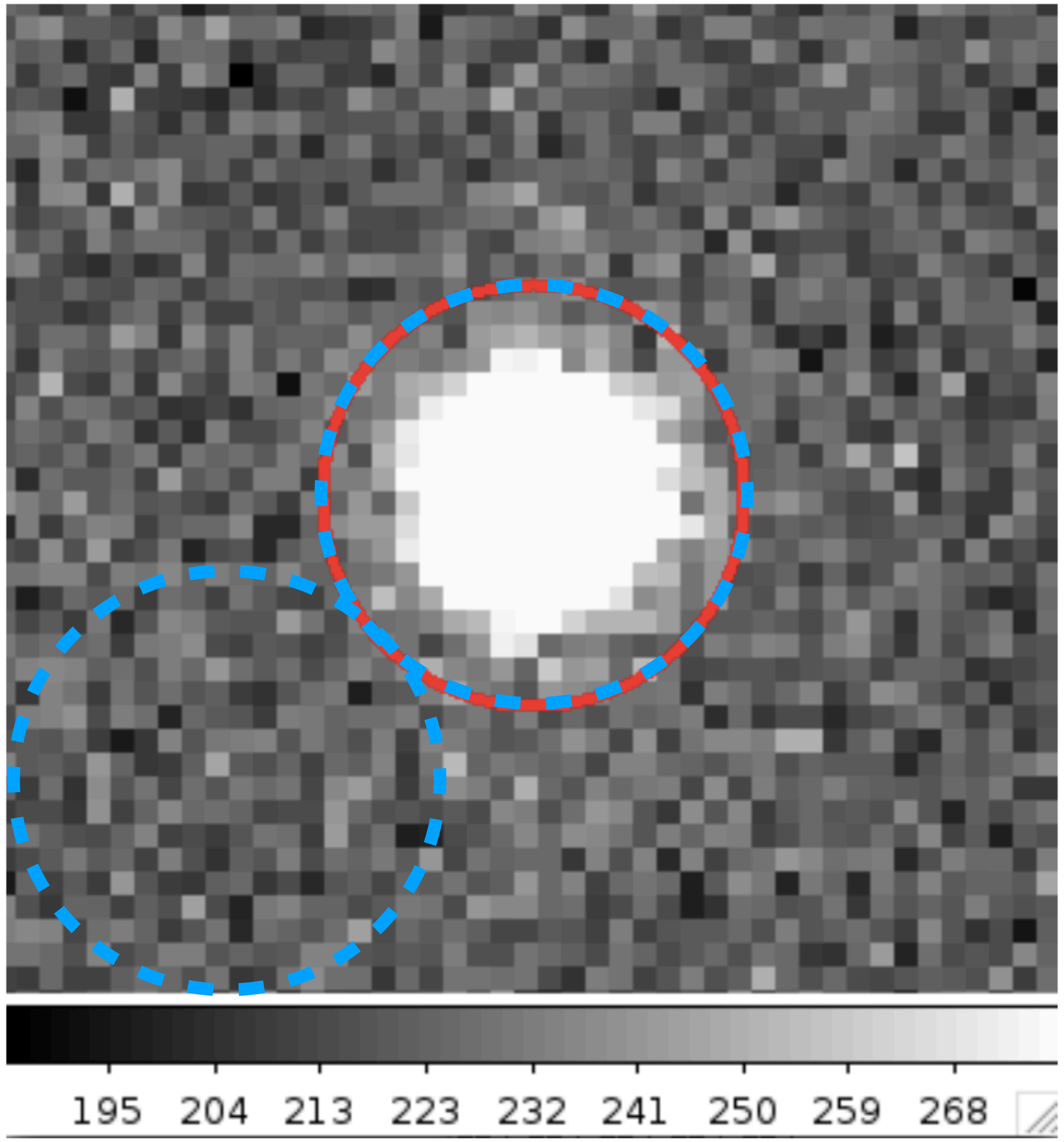


What causes noise?

- Shot noise from the source
- Sky noise
- Dark current noise
- Readout noise

$$\sigma_{\text{object}} = \sqrt{N_{\text{object}}} = \sqrt{S_{\text{object}} \times t}$$

$$\sigma_{\text{sky}} = \sqrt{N_{\text{sky}}} = \sqrt{s_{\text{sky}} \times n_{\text{pix}} \times t}$$



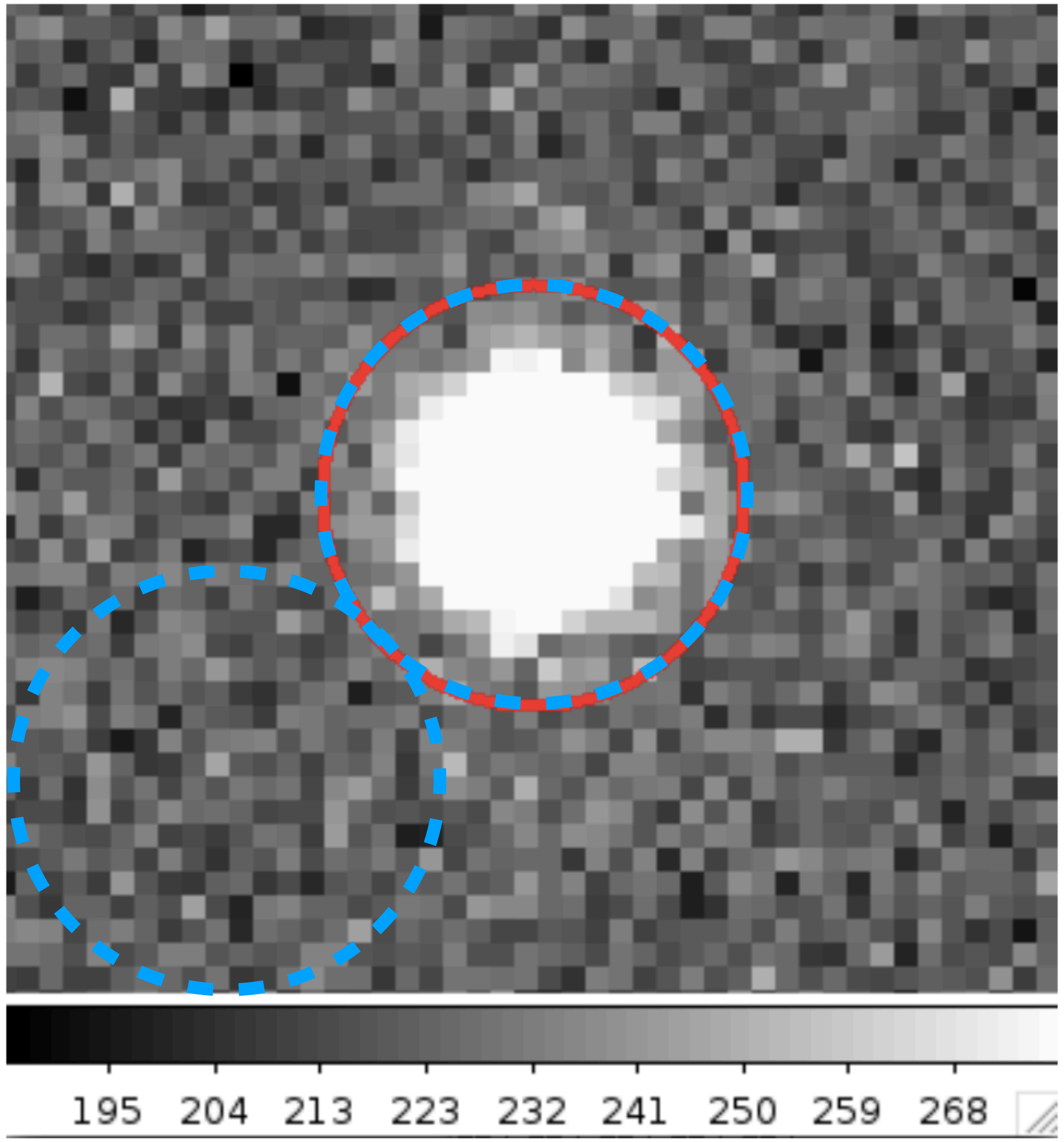
Quadrature Sum, and Signal-to-Noise

- If noise contributions are independent of each other, they add quadratically:

$$\sigma_{\rm total} = \sqrt{\sum_{i \,\in\, \rm noise\ terms} \sigma_i^2}$$

- From "empty" image region, the total noise will be sum of background contributions

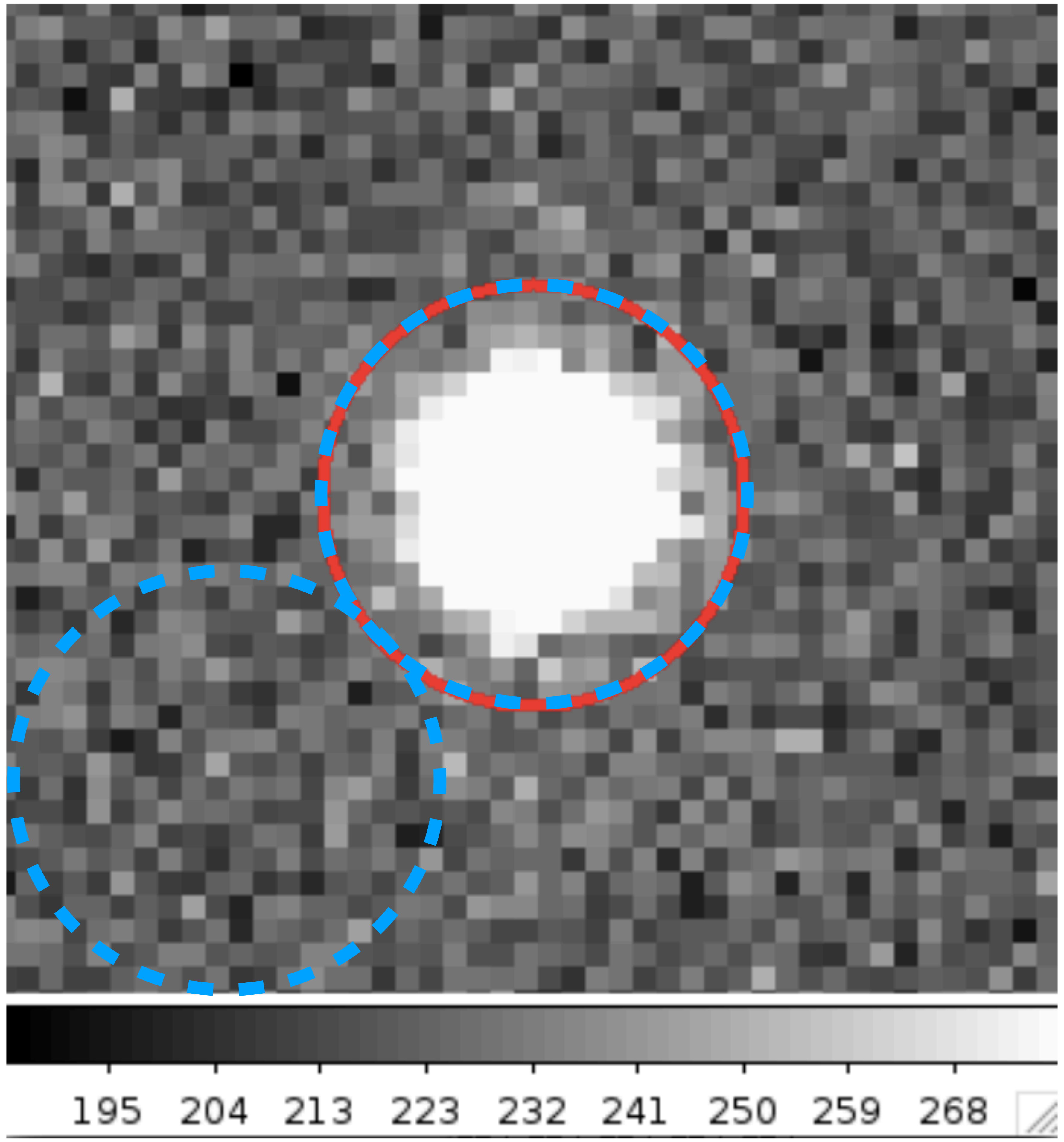
$$\sigma_{\rm bkg} = \sqrt{\sigma_{\rm sky}^2 + \sigma_{\rm dk}^2 + \sigma_{\rm ro}^2}$$



Quadrature Sum, and Signal-to-Noise

- However, for an object, the measurement uncertainty on the flux comes from the background + the shot noise of the object

$$\sigma_{\rm total} = \sqrt{\sigma^2_{\rm object} + \sigma^2_{\rm bkg}}$$
$$= \sqrt{N_{\rm object} + \sigma^2_{\rm bkg}}$$



- Given knowledge of a system, we can predict/ define a signal-to-noise ratio (SNR) as:

$$SNR = \frac{N_{\rm object}}{\sqrt{\sum_{\rm noise} \sigma_i^2}}$$

CCD Equation

- Read noise follows a Gaussian distribution
- Shot noise (from source, sky, etc.) follows a Poisson distribution

$$N = \sqrt{S_\star + S_S + t \cdot dc + \mathcal{R}^2}$$

Total counts per pixel, electrons

Astronomical Source

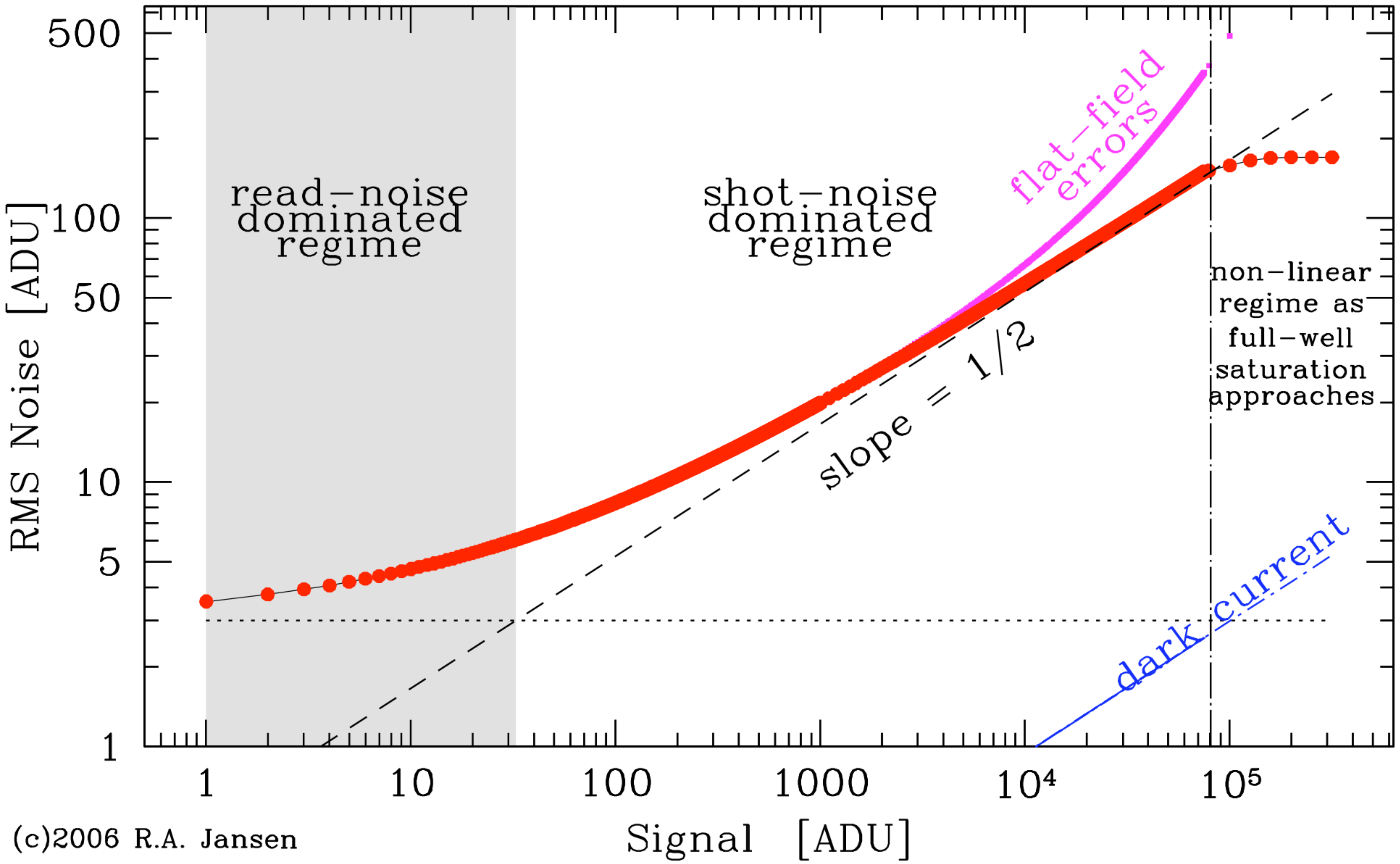
Sky background

Dark current

Read noise

CCD Equation

$$\frac{S}{N} = \frac{S_\star}{\sqrt{S_\star + n_{\text{pix}} \cdot \left(1 + \frac{n_{\text{pix}}}{n_{\text{sky}}}\right) \cdot \left(S_S + t \cdot dc + \mathcal{R}^2 + \mathcal{G}^2 \sigma_f^2\right)}}$$



CCD Equation

- In general, do not want to be limited by dark current or readout noise!
- Two limiting cases:
  - **1) Very bright object:** $N_{object} >> N_{other}$

$$SNR = \frac{N_{\mathrm{object}}}{\sqrt{N_{\mathrm{object}}}} = \frac{s_{\mathrm{object}} \times t}{\sqrt{s_{\mathrm{object}} \times t}}$$

$$\propto \sqrt{t}$$

  - **2) Very faint object:** $N_{sky} >> N_{other}$

$$SNR = \frac{N_{\mathrm{object}}}{\sqrt{N_{\mathrm{sky}}}} = \frac{s_{\mathrm{object}} \times t}{\sqrt{s_{\mathrm{sky}} \times n_{\mathrm{pix}} \times t}}$$

$$\propto \sqrt{t}$$

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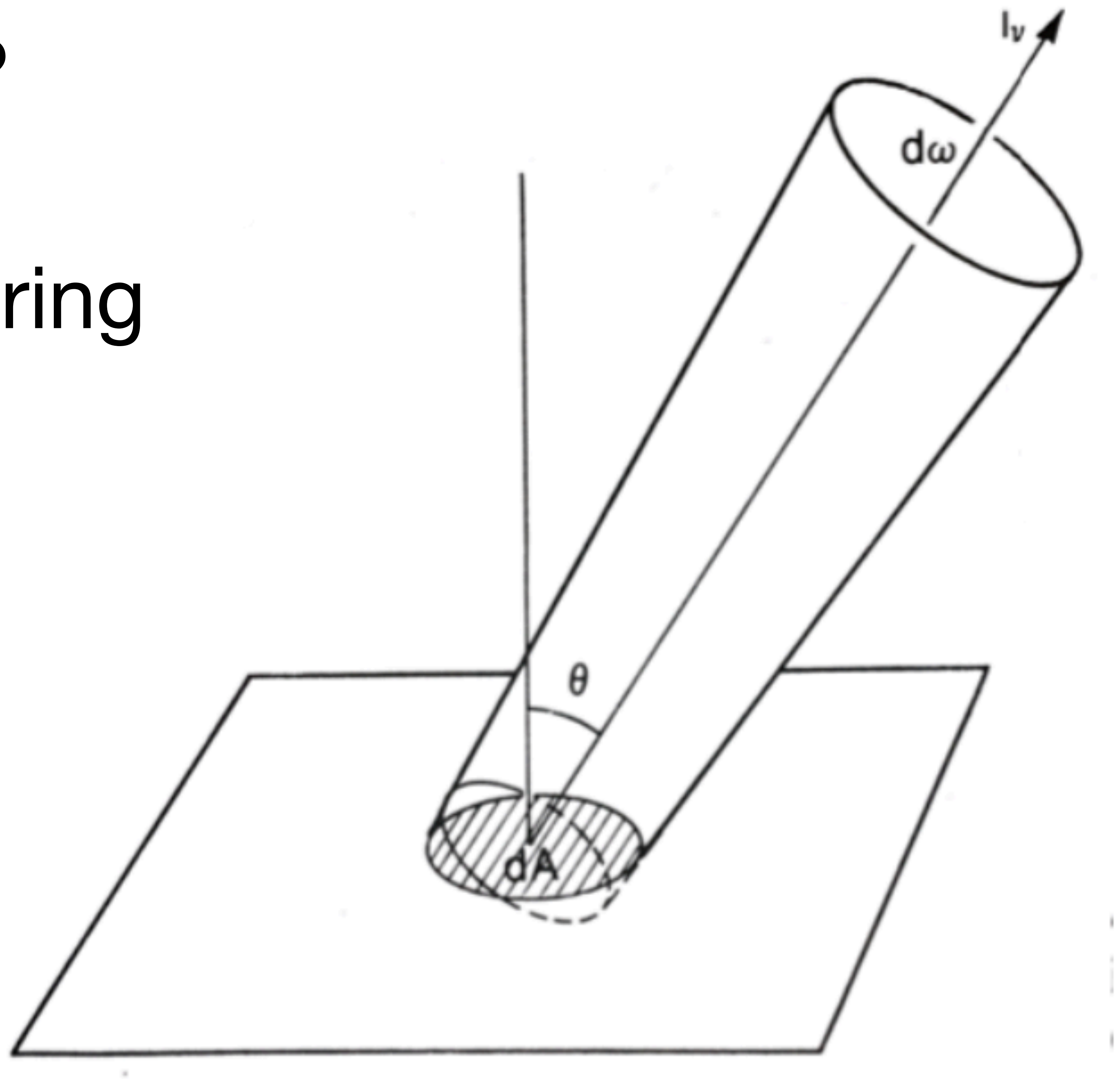
END

Flux and Intensity

- In Astronomy, we often characterize the flux from, or intensity of, an object, but what do we mean by that?
- Amount of energy (dE_ν) passing through an area, *dA*, within solid angle *dΩ*, in frequency range [*ν*,*ν*+*dν*], during time *dt* is:

$$dE_\nu = I_\nu \, dA \cos\theta \, d\omega \, dt \, d\nu$$

- Where:
  - dAdΩ could be something like the size (and effective) collecting area of your detector
  - **I_ν: Specific Intensity**
    - Units of J / [s m^2 Hz steradian]
    - An intrinsic property of the object (i.e., it should not depend on the observer or the measurement)



Karttunen et al.

Weighted Mean

- **Deviation**: of one measurement from the average

$$d_i = x_i - \mu$$

- **Weighted Mean**: Average of the squares of the deviations.

$$\sigma^2 = \frac{1}{N}\sum_i (x_i - \mu)^2$$

$$= \frac{1}{N-1}\sum_i (x_i - \bar{x})^2$$

- **Gauss**: The square root of the variance (i.e., σ), or the "typical" deviation around the mean.

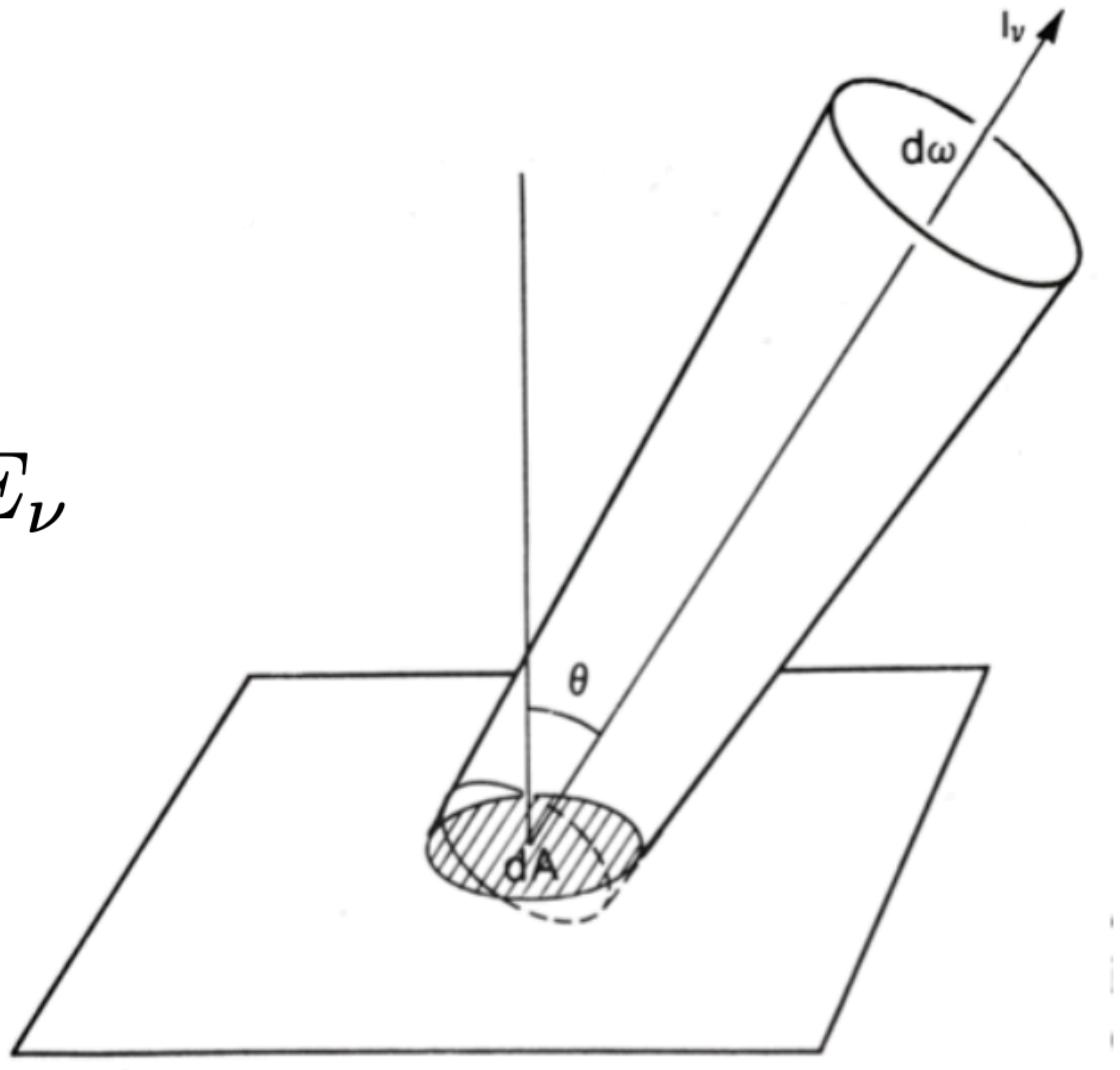
Flux and Intensity

- We measure Flux by integrating the Specific Intensity over solid angle

$$dE_\nu = I_\nu \, dA \cos\theta \, d\omega \, dt \, d\nu$$

$$f_\nu = \int_\Omega d\omega \cos\theta \; I_\nu$$
$$= \frac{1}{dA \; dt \; d\nu} \int_\Omega dE_\nu$$

- **Spectral Flux Density, *f_ν*:**
    - Energy per area, per time, per frequency interval
    - We usually observe f_ν (or f_ν integrated over the frequency band of our detector)
    - Depends on the distance between the source and the observer.



Karttunen et al.