ASTR21200

Observational Techniques in Astrophysics
Lecture 6

Bradford Benson

Lab-1: Overview

5 Rooftop Observing

Weather permitting (we will discuss more in class) with your TAs (Dillon and Rohan) on one of two designated nights on the 5th floor deck of the Eckhardt Research Center (ERC 501)[5]. With them you are going to use a small telescope on the roof, find an object with it, and take an image.

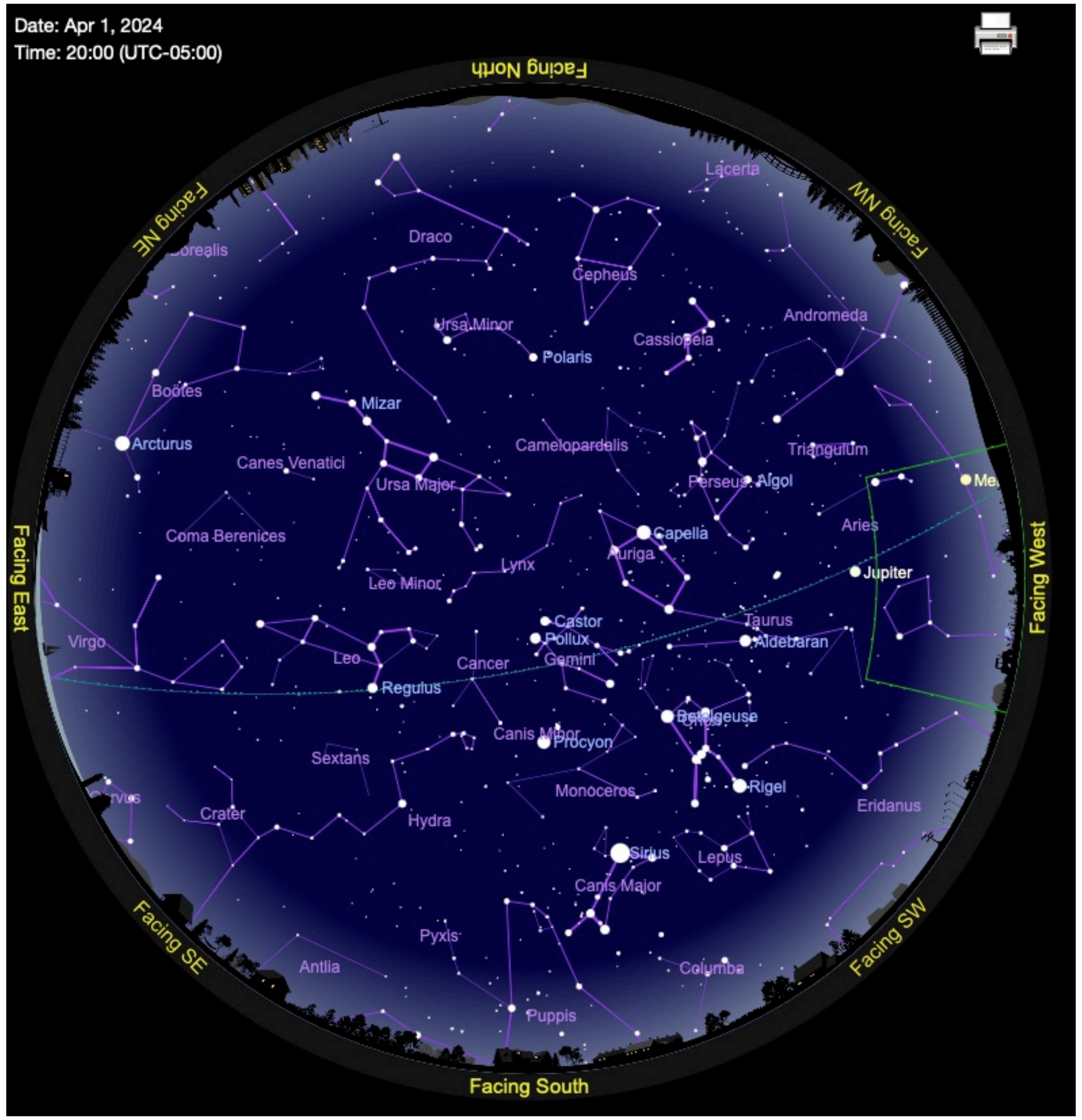
1. *Pick an object:*. See Figure 1 for possible options. Feel free to update the star chart for your particular time and day. Early in the night, you can see some potential options, such as Jupiter in the East. And bright stars like Arcturus, Sirius, Betelgeuse, and the North Star (Polaris).
2. *Find your object:* Use the telescope and try to find your object with the telescope. Are there any physical obstructions preventing you from seeing the object? What are they? If yes, you might want to pick a different object.
3. *Take an image:* Take an image of your object with the camera on the telescope. **Save the image and include it in your Lab Report.**

For your lab report, you will want to:

1. Describe the object you choose, i.e., What is it? Is it a planet, a galaxy, a nebula, a star, etc.?
2. Were there any physical obstructions preventing you from seeing the object? What are they?
3. Were you able to see your object? Why or why not?
4. Save the image and include it in your Lab Report.

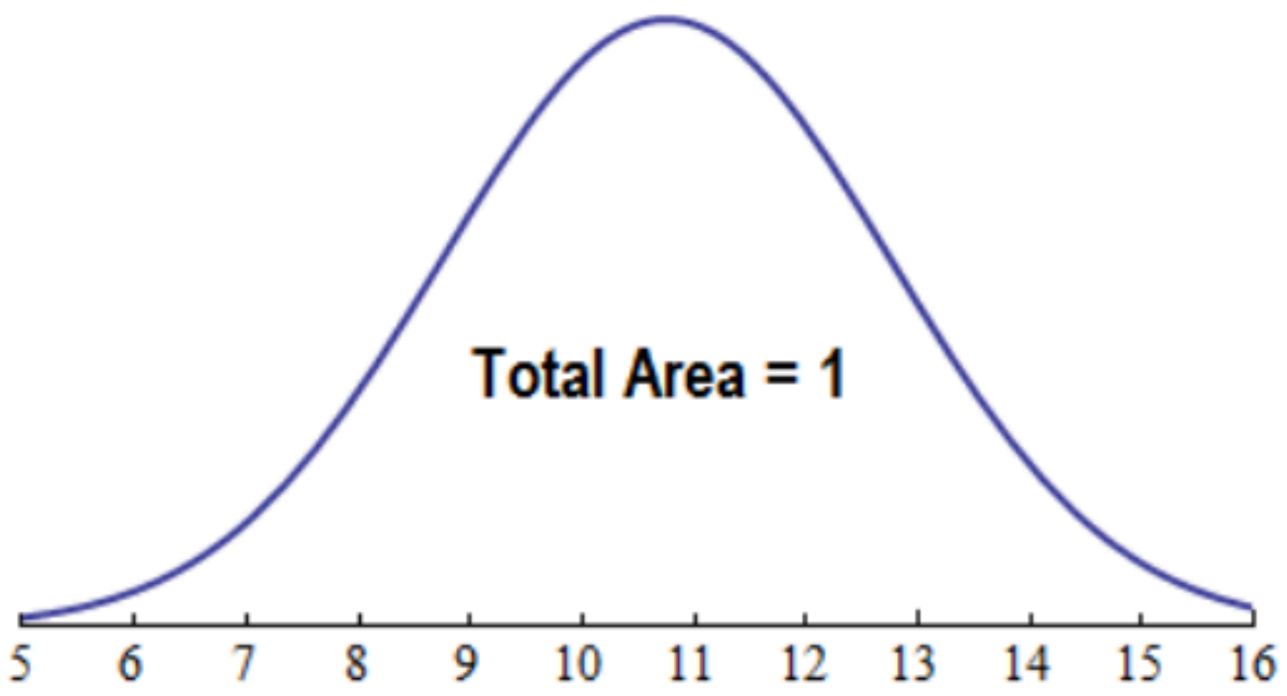
ERC Rooftop observing

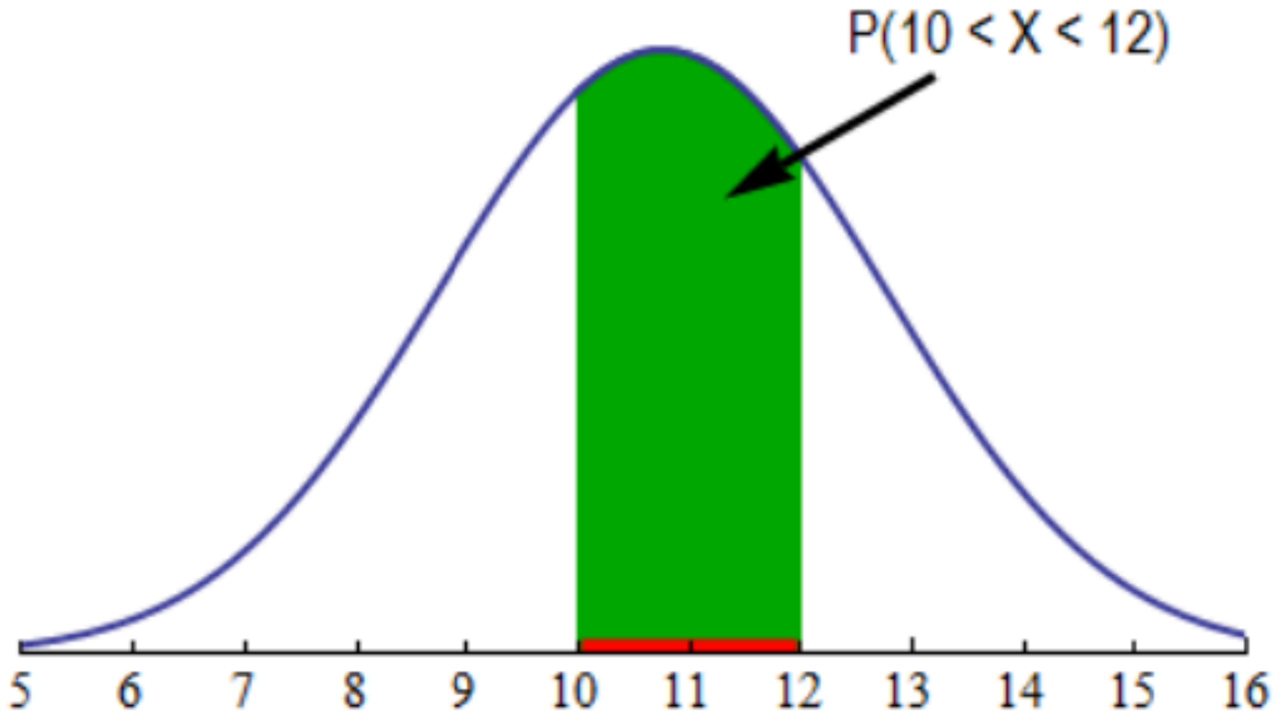
- Rohan and Dillon will lead
- Tuesday April 9 - 9pm ERC 501 deck
- Next time TBD (probably early next week)

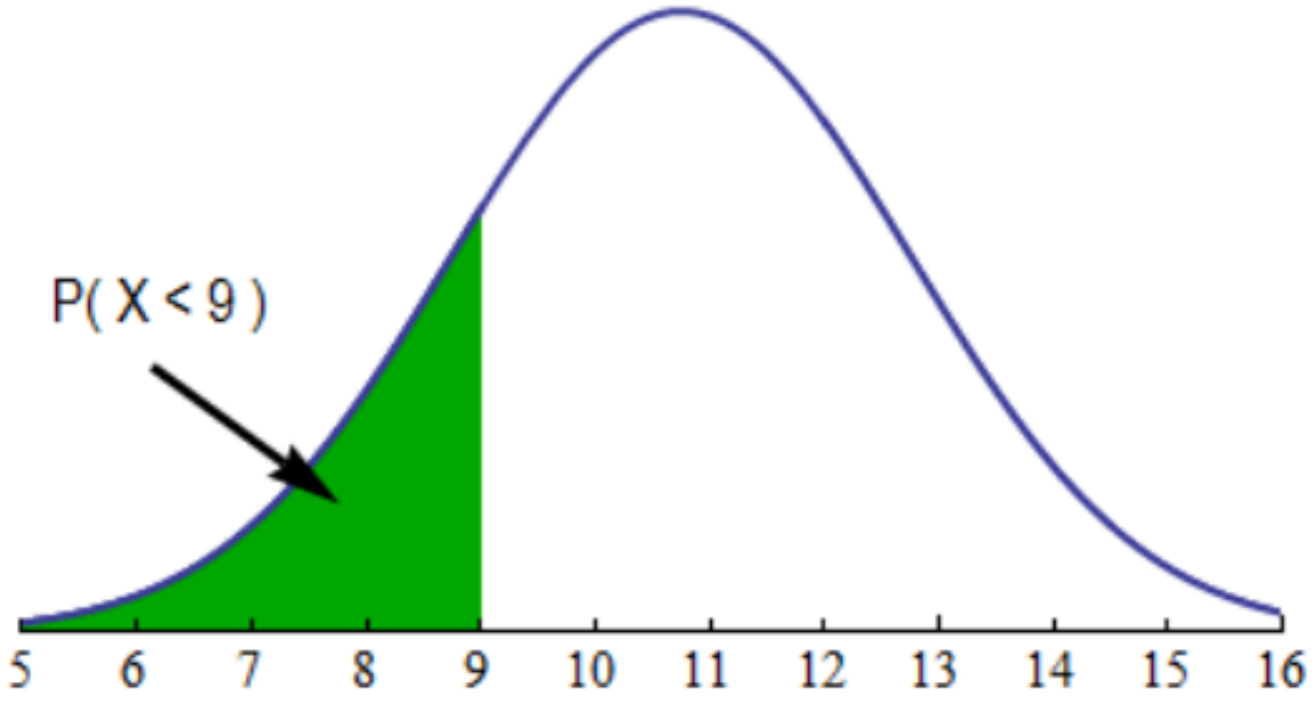


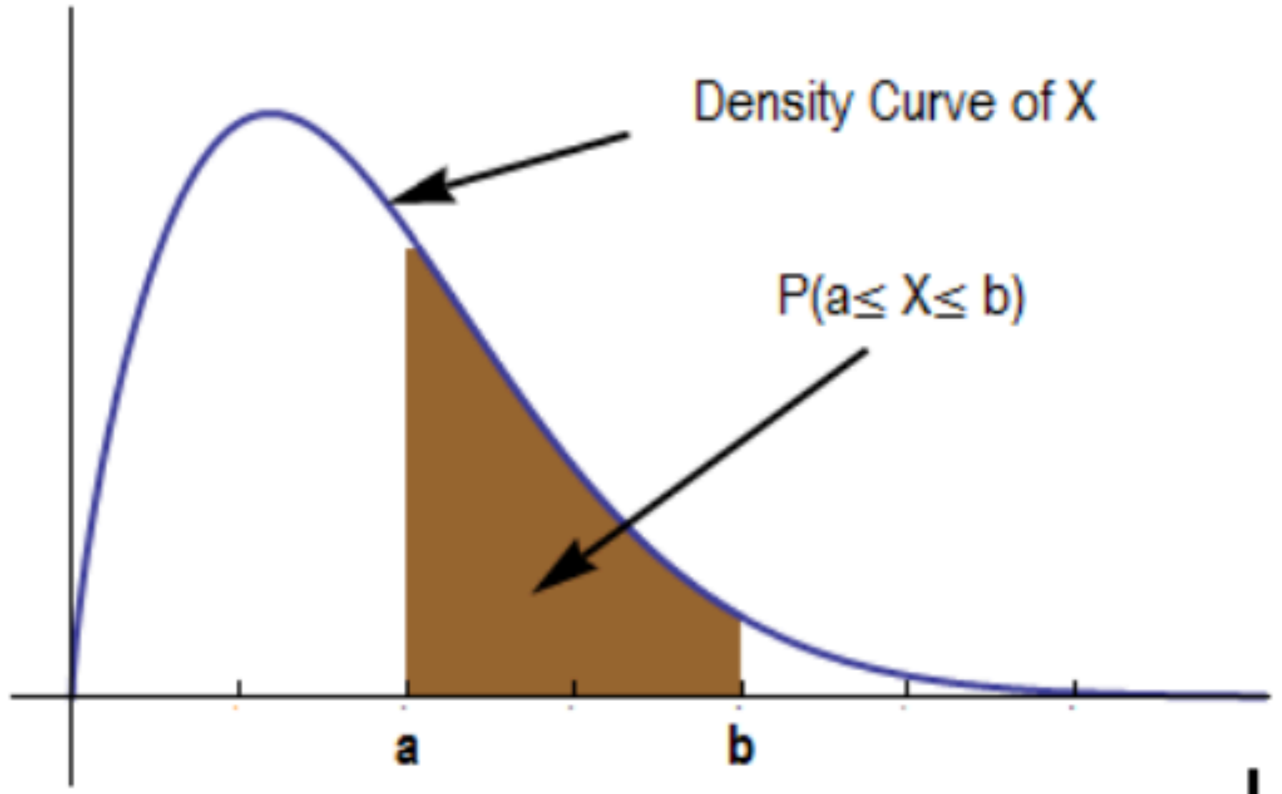
Probability Distributions

- Probability Distribution: Describes the expected (or measured) distribution of measurements.
- Can integrate a probability distribution over range of values to find a probability to be in that range









Lumen Learning

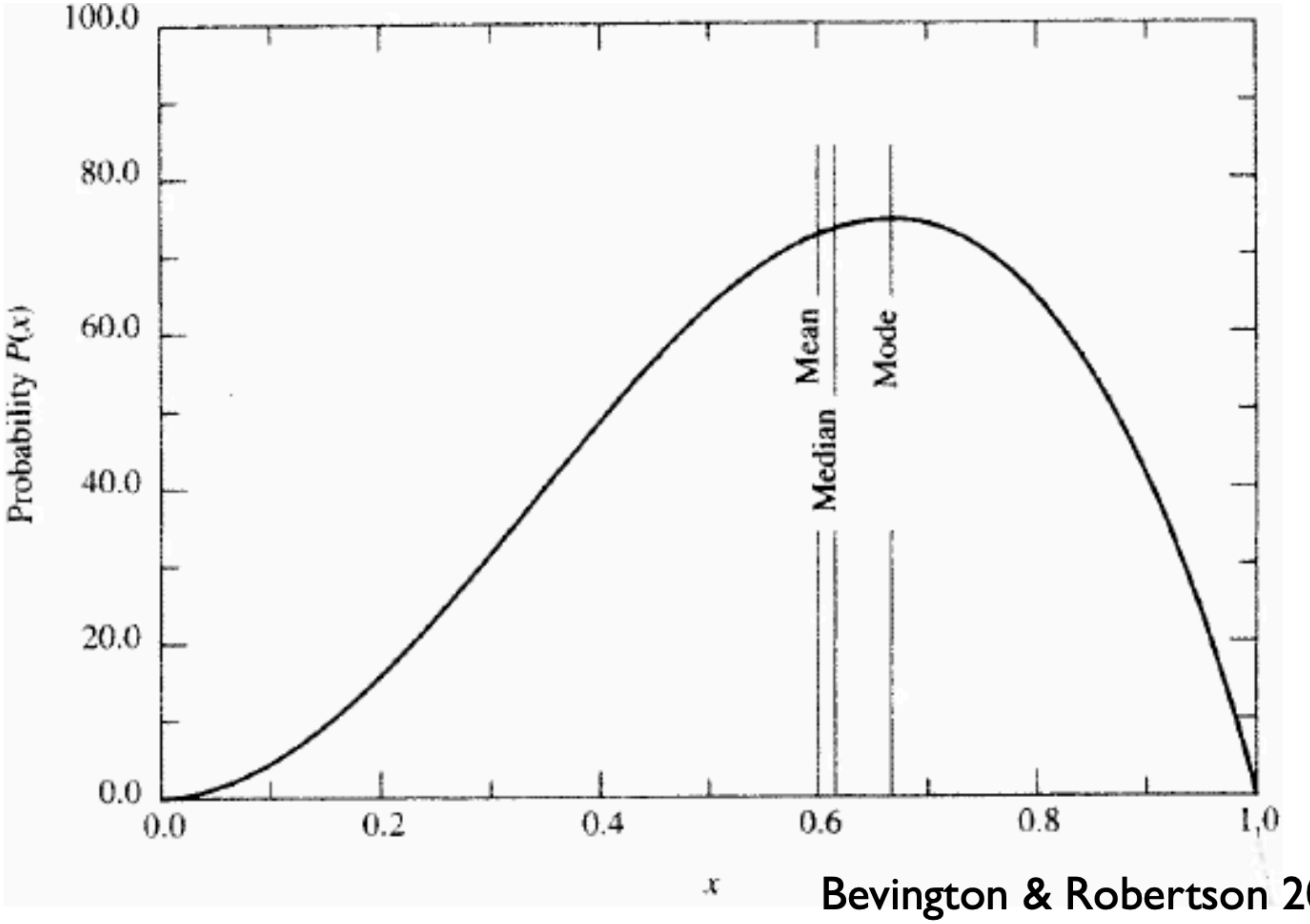
Summary Statistics

- **Mean**: The “average”value, in the limit of N measurements:

$$\bar{x} = \frac{1}{N}\sum_i x_i$$

- **Median**: 50th percentile of distribution, i.e., 50% of the measurements are larger (or smaller) than that value
- **Mode**: The most “common”, or “likely” measurement value

- All three are useful, but will depend on the problem, and possibly the underlying probability distribution being measured



Bevington & Robertson 2003

Deviation, Variance, Standard Deviation

- **Deviation**: of one measurement from the average

$$d_i = x_i - \mu$$

- **Sample variance**: Average of the squares of the deviations.
  - Sample variance can also be estimated from a sample population (i.e., a sample of measurements), using mean as an estimate of the average.

$$\sigma^2 = \frac{1}{N} \sum_i (x_i - \mu)^2$$

$$= \frac{1}{N-1} \sum_i (x_i - \bar{x})^2$$

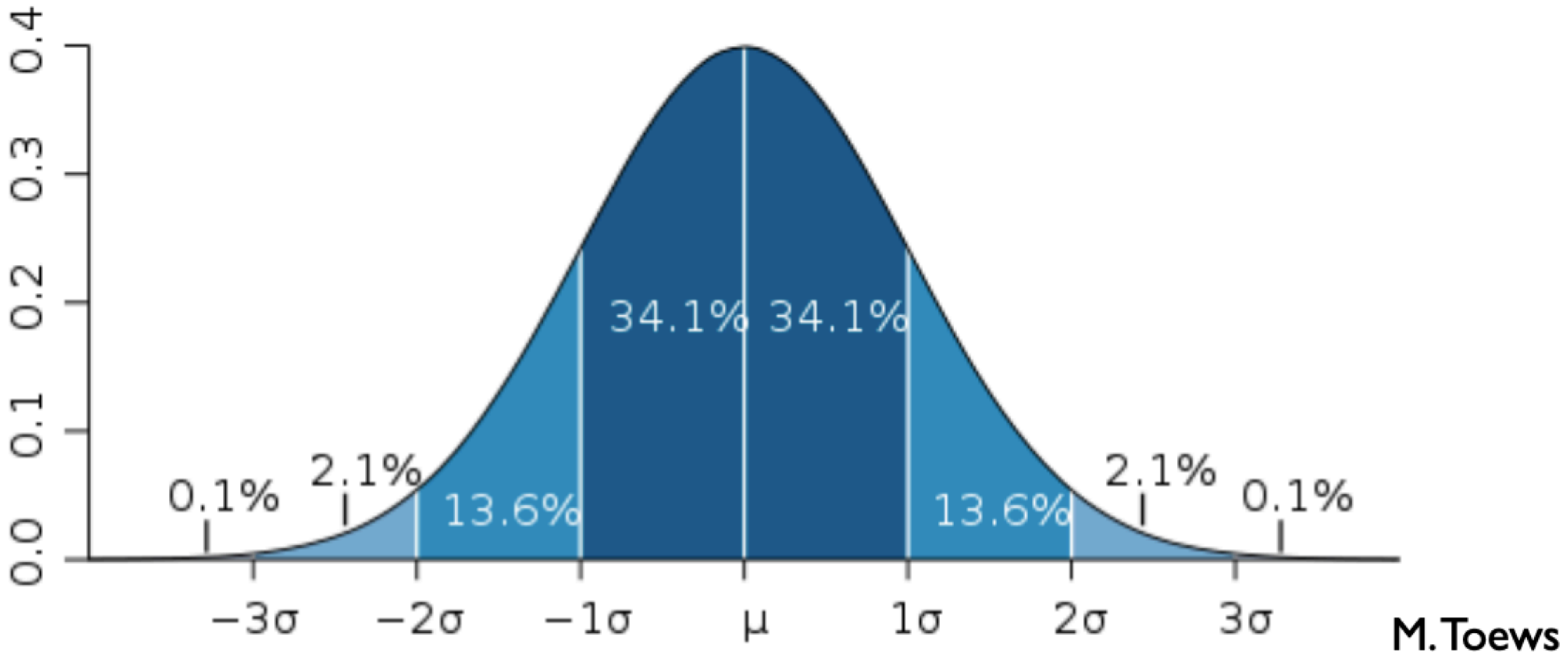
- **Standard Deviation**: The square root of the variance (i.e., σ), or the "typical" deviation around the mean.

Common Probability Distributions

- Three common probability distributions:
  - **Poisson distribution:** Counting experiments for discrete events (e.g., photon counts, N_{counts})
    - **Standard-Deviation: σ = sqrt(N_{counts})**
  - **Binomial distribution:** For experiments with only a small number of possible final states (e.g., coin tosses)
  - **Gaussian (Normal) distribution:** Limiting case of binomial and poisson distributions, for large number of events / measurements

Gaussian Distribution: Example

- Relation between the probability of occurrence and the number of standard deviations away from the mean

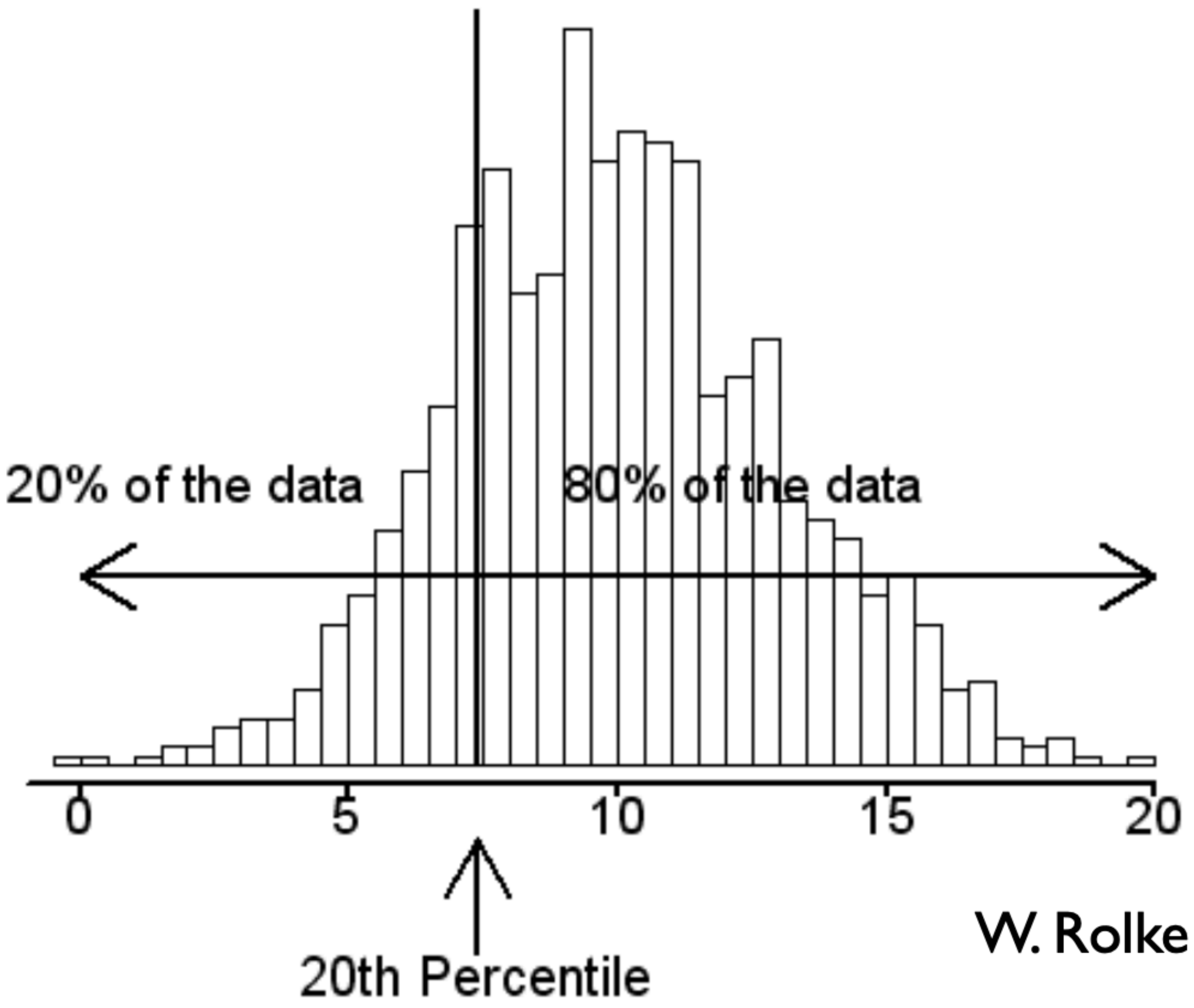


measurements should fall:
- within 1σ of the mean 68.3% of the time
- within 2σ of the mean 95.4% of the time
- within 3σ of the mean 99.73% of the time

Non-Gaussian Distributions

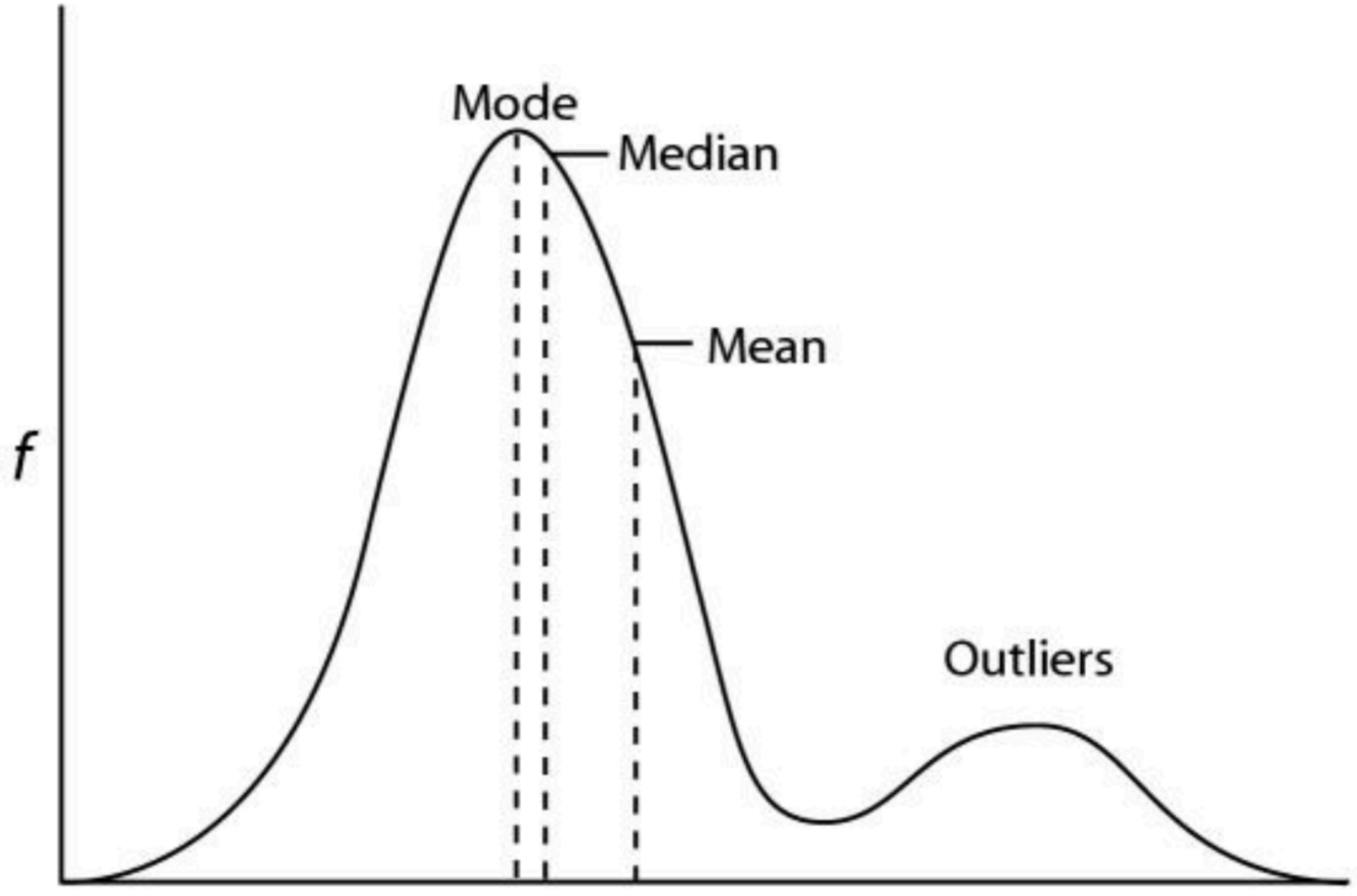
- What if your distribution is non-Gaussian?
- Have to decide on a case-by-case basis
- Percentiles can always sort your data, quote values that are above a certain percentage of the population, e.g.,
    - **Median**: 50th percentile
- Can quote measurement and uncertainty with percentiles
    - e.g., mean and range 68% confidence region

$$99.123^{+0.005}_{-0.004}$$



Outliers

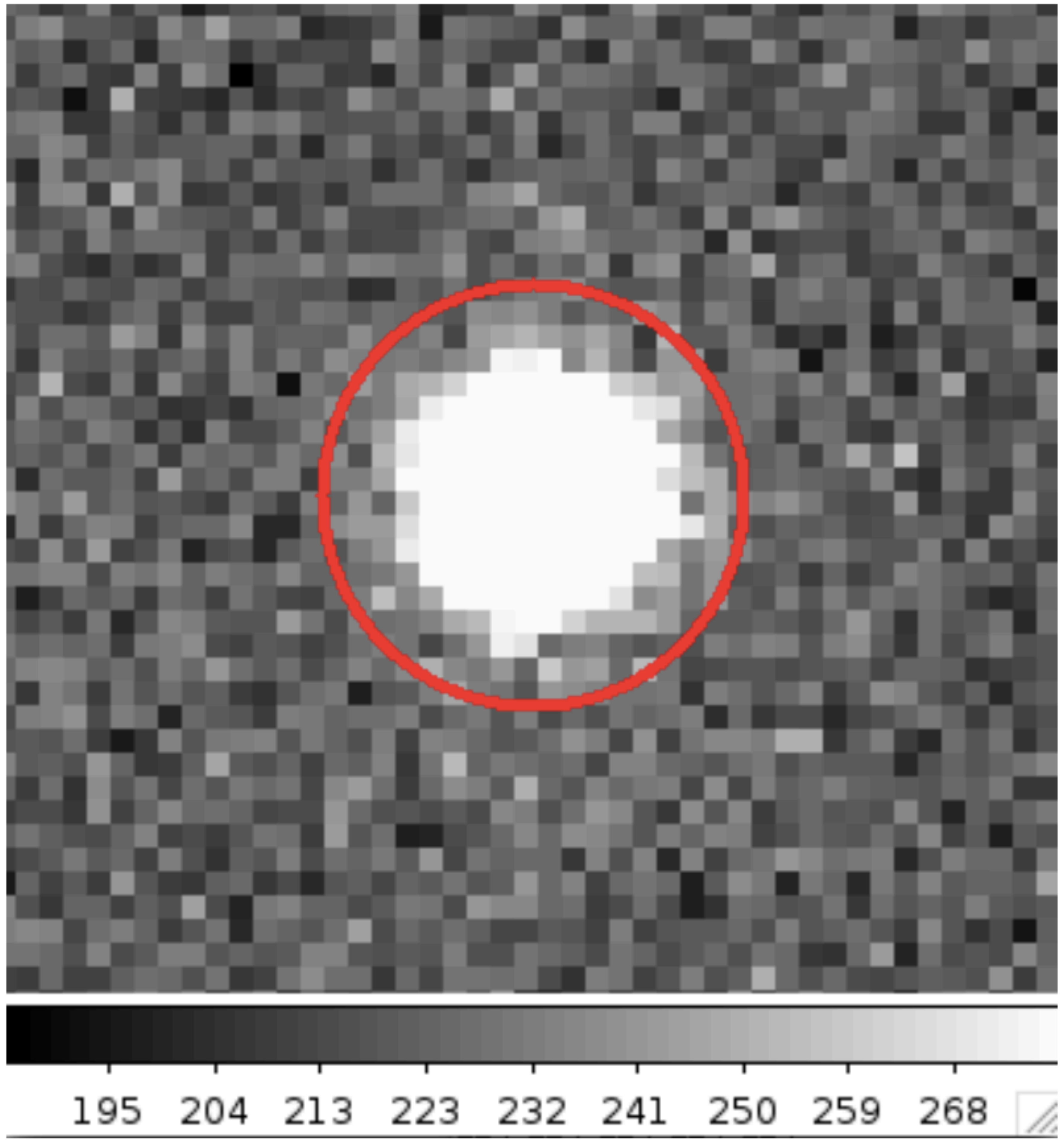
- For Gaussian distribution: Median = Mean
- What if distribution is “almost” normal, but has a few outliers? (e.g., cosmic rays on the CCD)
  - **Mean**: Significantly affected by outliers
  - **Median**: More robust to (a small number of) outliers
- Sometimes its ok to remove gross outliers (e.g., “sigma-clipping”), but need to make sure not to bias your results.



Hedges & Shah 2003

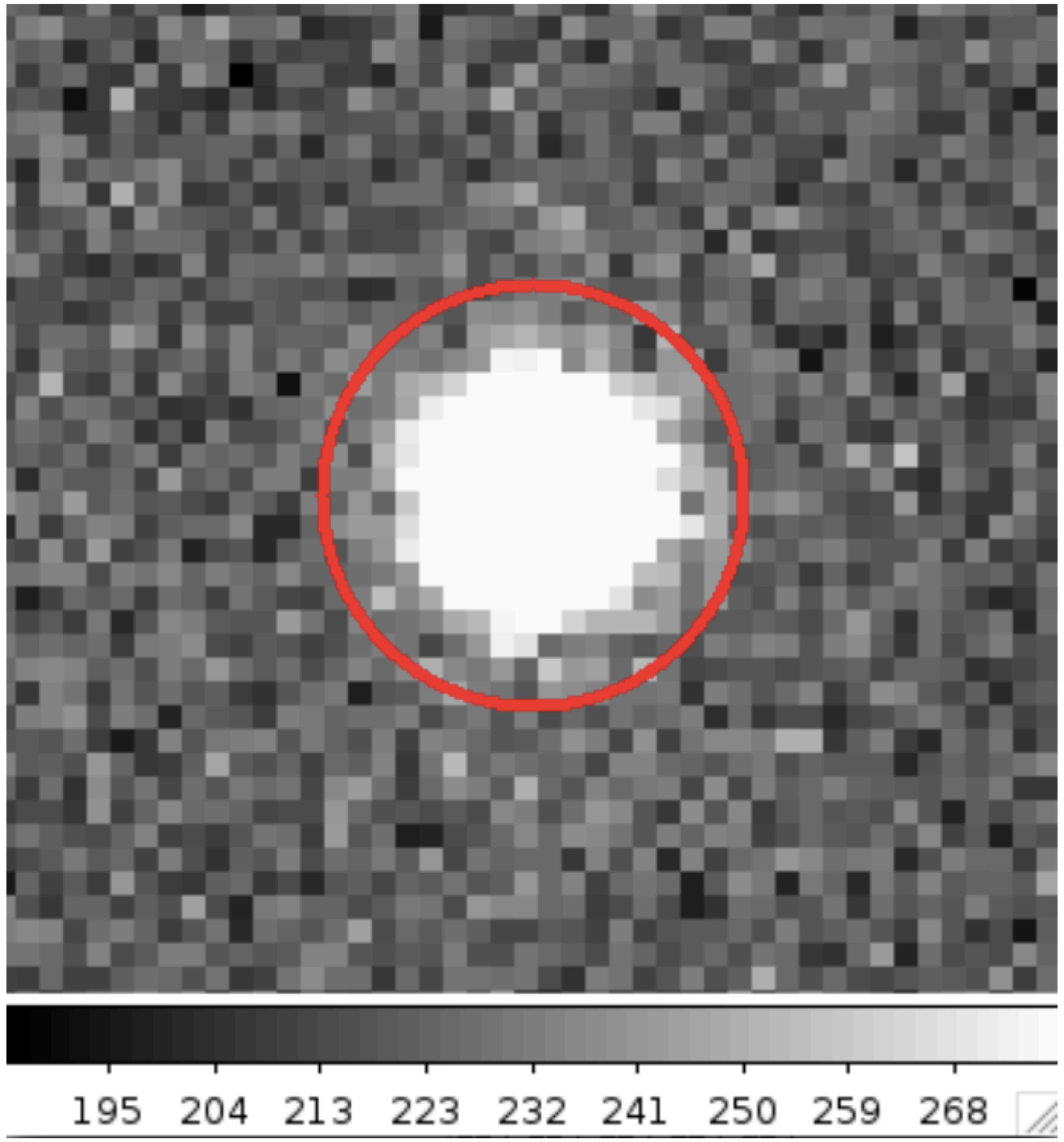
Signal: Flux from an Object

- Flux measured in an aperture
    - (**Total Electrons**) =
        - (**Electrons from Object**, i.e., the Signal) + (**Electrons from Backgrounds**, i.e., atmosphere signal, dark current, etc.)
    - (Signal) = (Total Electrons) - (Background Electrons)
    - $N_{Object} = N_{total} - N_{background}$
- **Note:** From the image alone, we cant really tell which electrons are from the "object" (aka, signal), and which are from backgrounds (aka, noise).



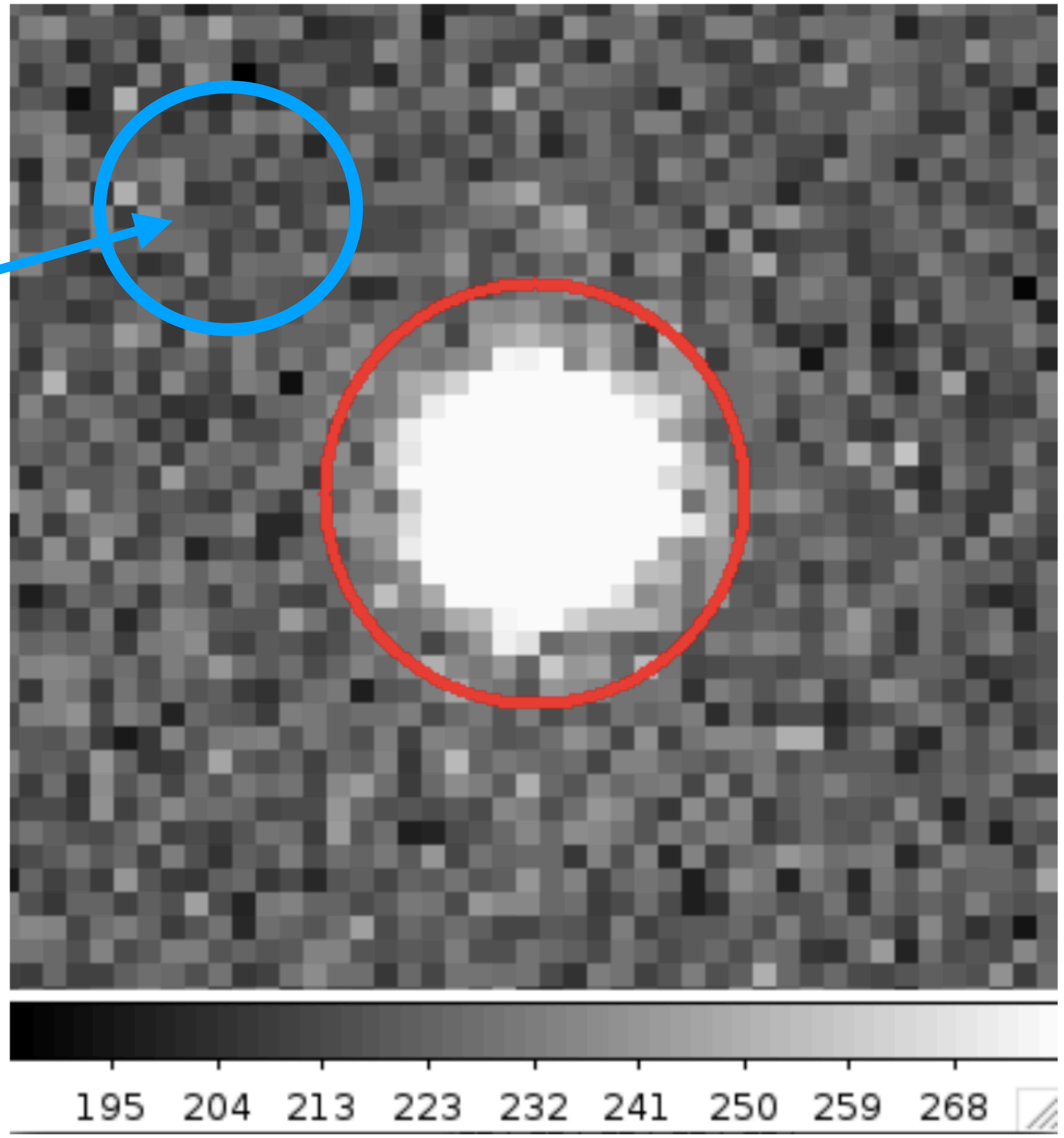
Signal: Flux from an Object

- There is always “noise” in a CCD image
  - Even in a “0-sec” “bias frame”, there will be additional readout noise
- For each pixel, we can imagine that the measurement is drawn from a distribution of **mean (*N*)** and **width (σ)**
- Even our estimates of the “background electrons” will have some uncertainty, so the distribution width above (σ) will be due to some combination in variance of signal, noise, background, etc.



Signal: Flux from an Object

- For example:
  - In “empty” regions of the image, we can measure the noise as the standard deviation of the pixel values.
  - The accuracy with which we measure that standard deviation will improve, the more “background” regions we can average over.

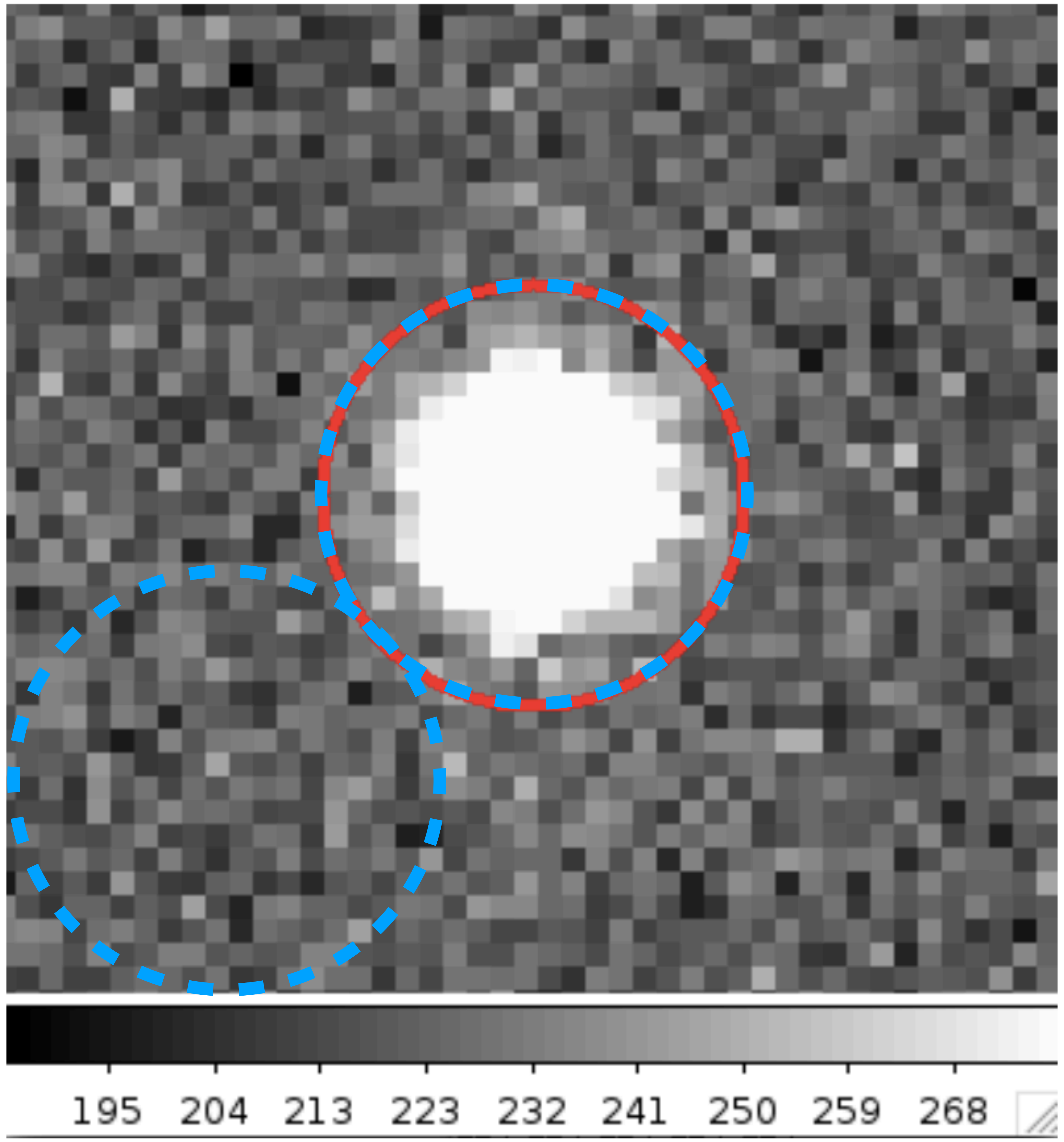


What causes noise?

- Shot noise from the source
- Sky noise
- Dark current noise
- Readout noise

$$\sigma_{\text{object}} = \sqrt{N_{\text{object}}} = \sqrt{S_{\text{object}} \times t}$$

$$\sigma_{\text{sky}} = \sqrt{N_{\text{sky}}} = \sqrt{s_{\text{sky}} \times n_{\text{pix}} \times t}$$



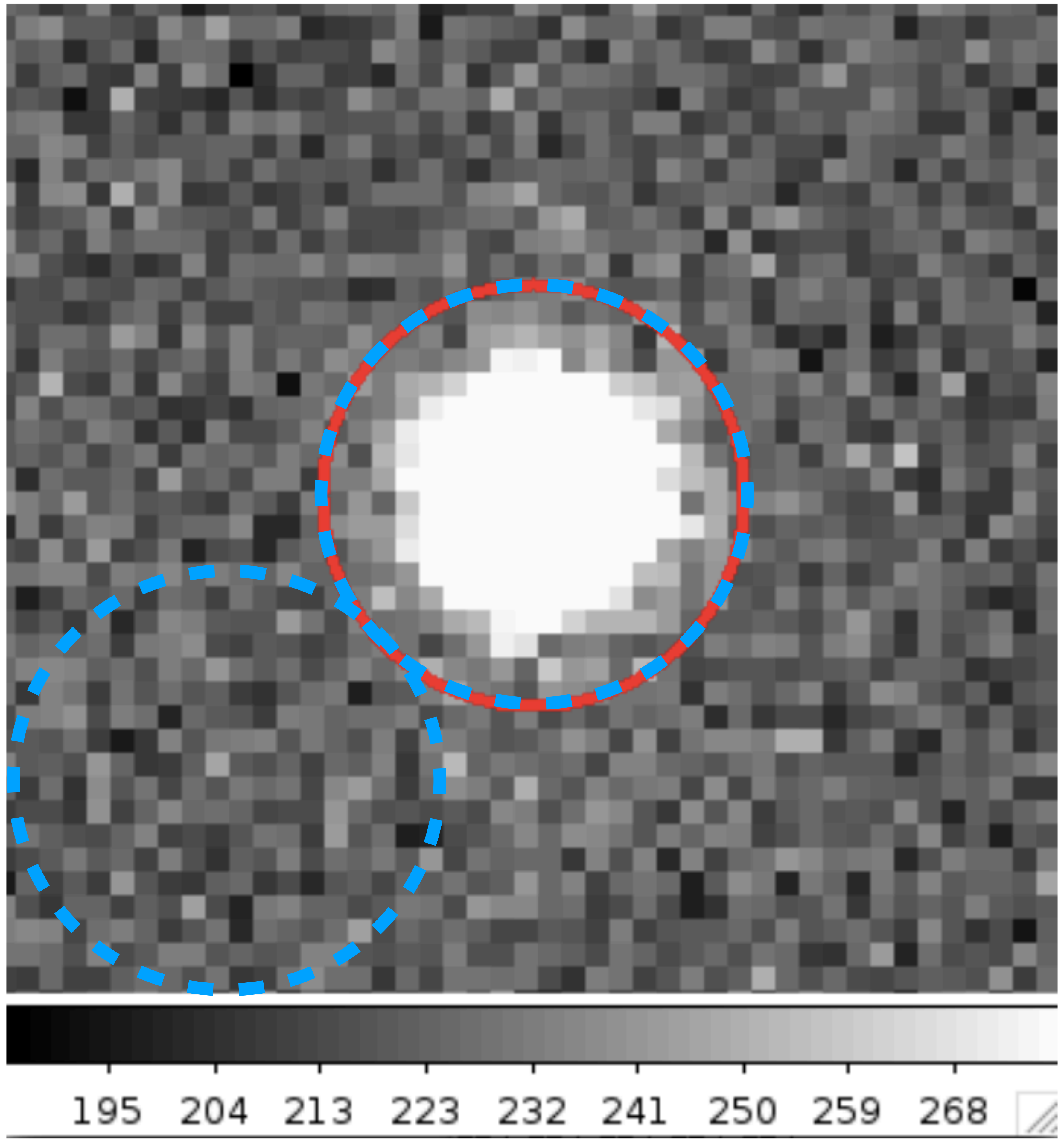
Quadrature Sum, and Signal-to-Noise

- If noise contributions are independent of each other, they add quadratically:

$$\sigma_{\rm total} = \sqrt{\sum_{i \in \text{noise terms}} \sigma_i^2}$$

- From "empty" image region, the total noise will be sum of background contributions

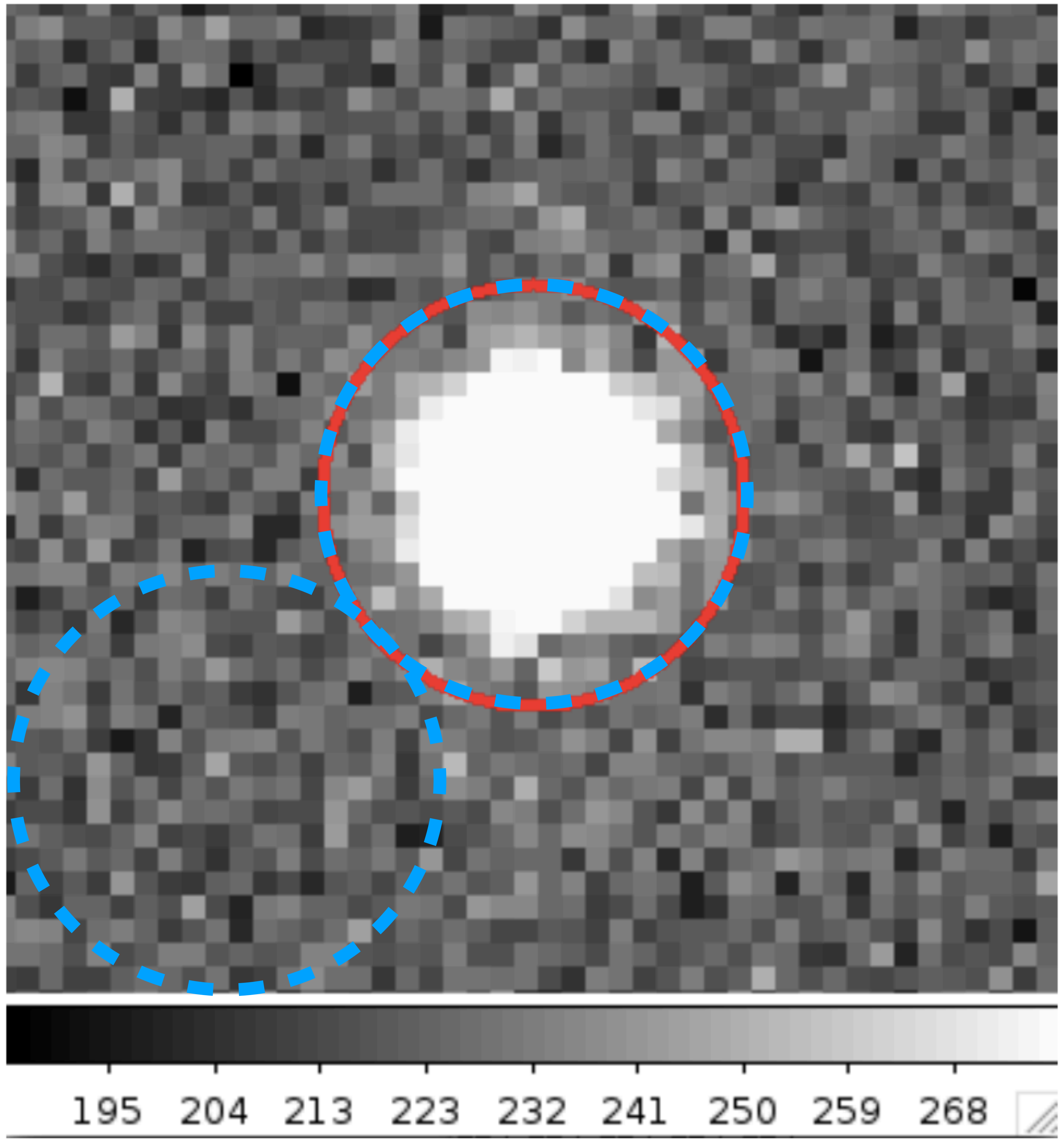
$$\sigma_{\rm bkg} = \sqrt{\sigma_{\rm sky}^2 + \sigma_{\rm dk}^2 + \sigma_{\rm ro}^2}$$



Quadrature Sum, and Signal-to-Noise

- However, for an object, the measurement uncertainty on the flux comes from the background + the shot noise of the object

$$\sigma_{\rm total} = \sqrt{\sigma^2_{\rm object} + \sigma^2_{\rm bkg}}$$
$$= \sqrt{N_{\rm object} + \sigma^2_{\rm bkg}}$$



- Given knowledge of a system, we can predict/ define a signal-to-noise ratio (SNR) as:

$$SNR = \frac{N_{\rm object}}{\sqrt{\sum_{\rm noise} \sigma_i^2}}$$

CCD Equation

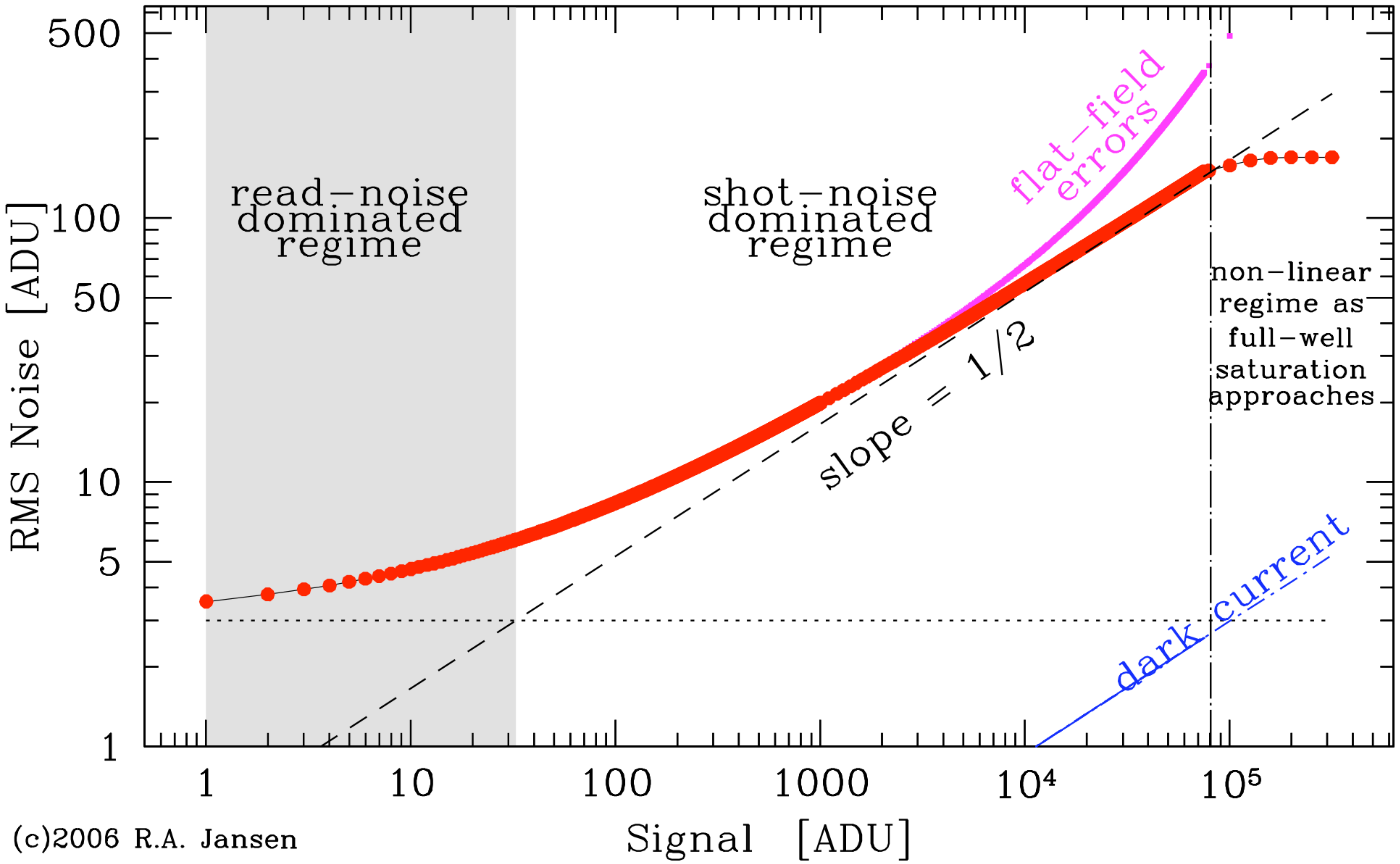
- Read noise follows a Gaussian distribution
- Shot noise (from source, sky, etc.) follows a Poisson distribution

$$N = \sqrt{S_\star + S_S + t \cdot dc + \mathcal{R}^2}$$

N	$S_\star$	S_S	$t \cdot dc$	$\mathcal{R}^2$
Total counts per pixel, electrons	Astronomical Source	Sky background	Dark current	Read noise

CCD Equation

$$\frac{S}{N} = \frac{S_\star}{\sqrt{S_\star + n_{\text{pix}} \cdot \left(1 + \frac{n_{\text{pix}}}{n_{\text{sky}}}\right) \cdot \left(S_S + t \cdot dc + \mathcal{R}^2 + \mathcal{G}^2 \sigma_f^2\right)}}$$



CCD Equation

- In general, do not want to be limited by dark current or readout noise!
- Two limiting cases for the Signal-to-Noise Ratio (SNR) from an object:
  - **1) Very bright object: $N_{\mathrm{object}} >> N_{\mathrm{other}}$**

$$SNR = \frac{N_{\mathrm{object}}}{\sqrt{N_{\mathrm{object}}}} = \frac{s_{\mathrm{object}} \times t}{\sqrt{s_{\mathrm{object}} \times t}}$$

$$\propto \sqrt{t}$$

  - **2) Very faint object: $N_{\mathrm{sky}} >> N_{\mathrm{other}}$**

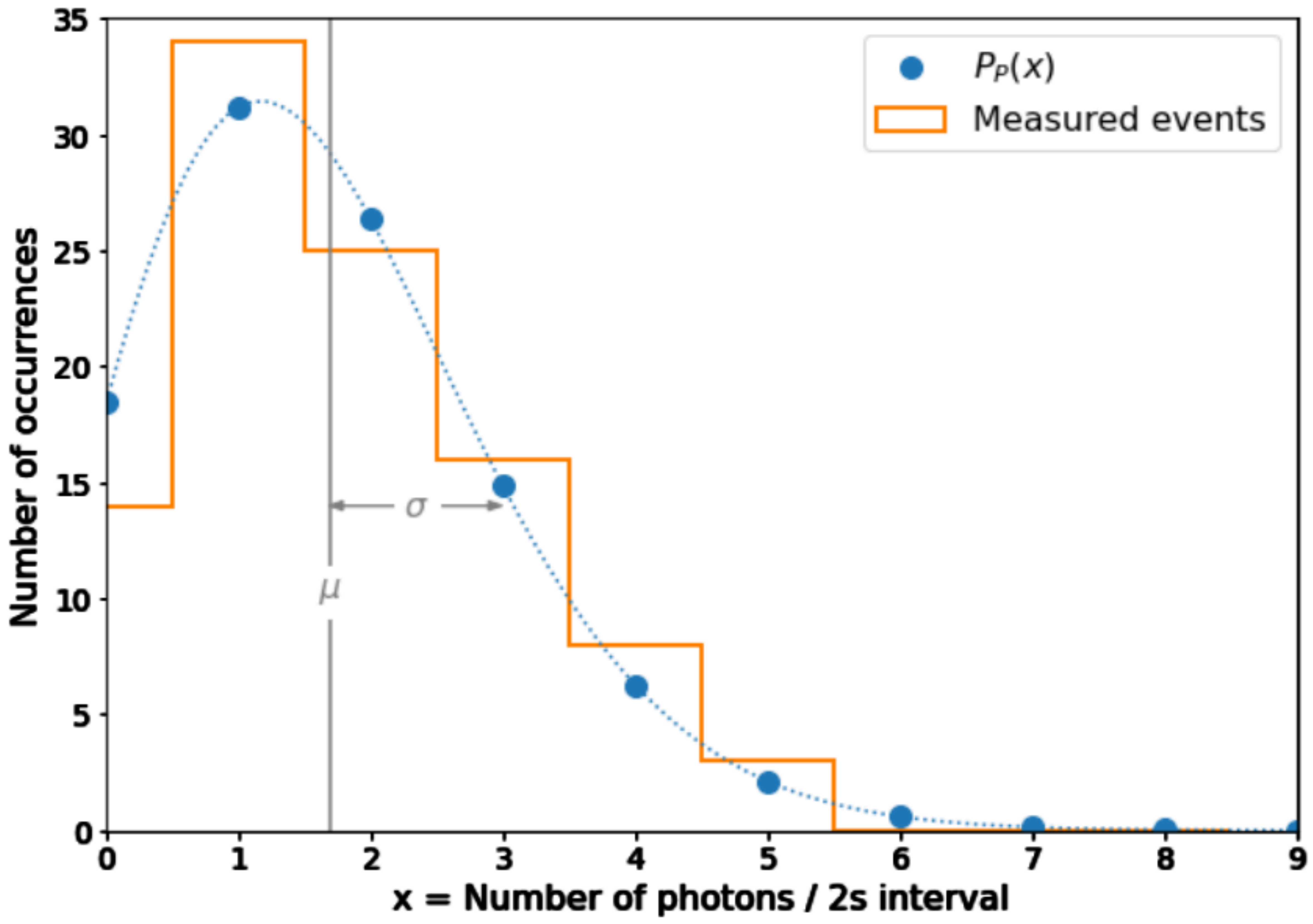
$$SNR = \frac{N_{\mathrm{object}}}{\sqrt{N_{\mathrm{sky}}}} = \frac{s_{\mathrm{object}} \times t}{\sqrt{s_{\mathrm{sky}} \times n_{\mathrm{pix}} \times t}}$$

$$\propto \sqrt{t}$$

Extras

Poisson Distribution: Example

- A detector measures the number of gamma-ray photons per 2-second intervals, making 100 measurements



$$P_{\mathrm{P}}(x|\mu) = \frac{\mu^x}{x!} e^{-\mu}$$

measured mean:

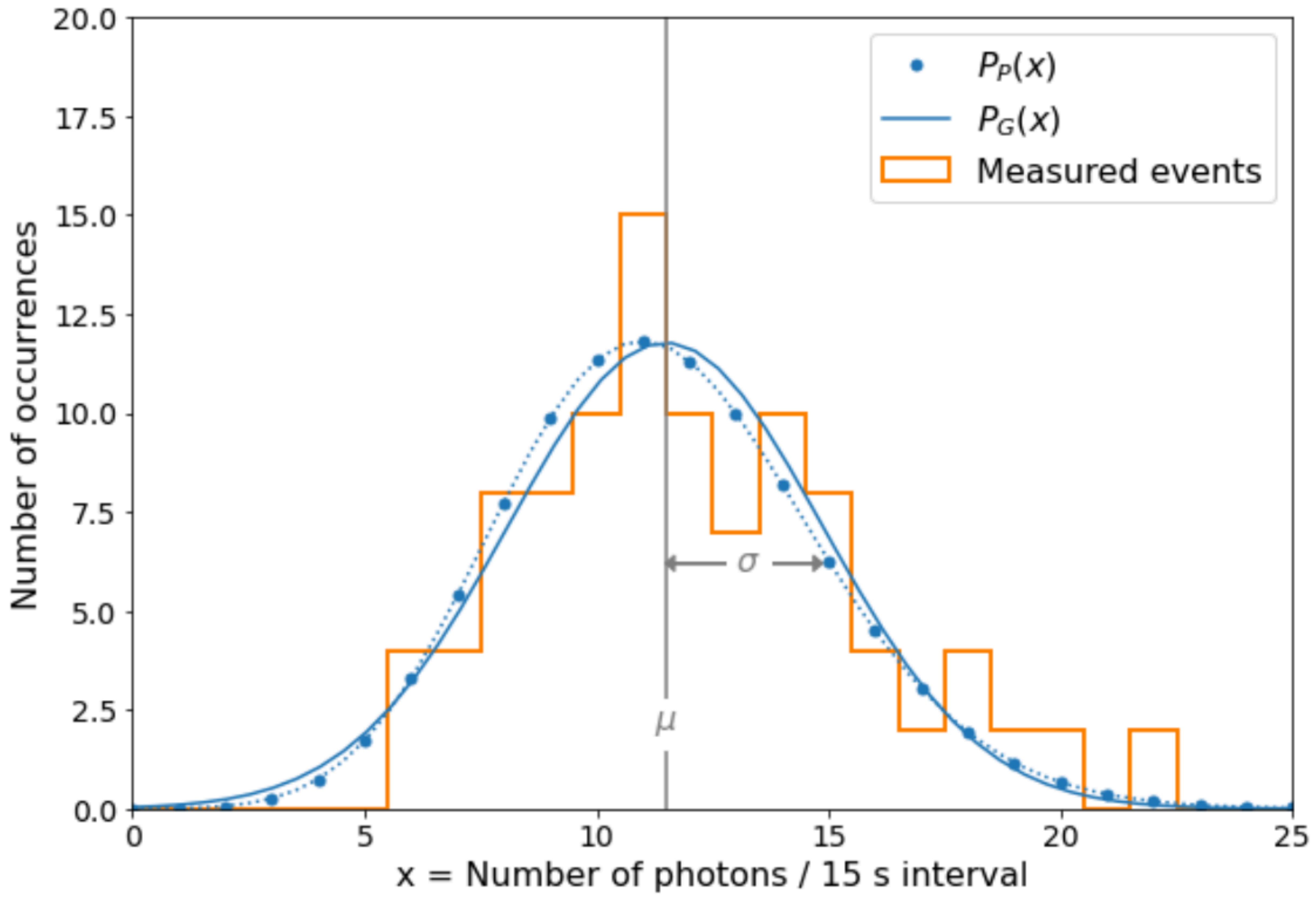
$$\bar{x} = 1.69$$

blue points:

$$P_{\mathrm{P}}(x|1.69)$$

Gaussian Distribution: Example

- A detector measures the number of gamma-ray photons per 15-second intervals, making 60 measurements



$$P_{\mathrm{G}}(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

measured mean:

$$\bar{x} = 11.48$$

blue points:

$$P_{\mathrm{P}}(x|11.48)$$

blue curve:

$$P_{\mathrm{G}}(x|\bar{x},\sqrt{\bar{x}})$$

Data is starting to be fit well by a Gaussian distribution

Last Homework Problem

6. **Photometry**: Observing on Magellan ($D = 6.5$m) with the Megacam CCD imager[1], the sky background in r-band varies between a magnitude of 21 to 20 per square arcsec during a new and full moon. Assume the following, the r-band is centered at λ=600 nm with a bandwidth $\Delta\lambda$=200 nm. The r-band magnitude, M_R, can be written as $M_R = -2.5\log(f_R/f_0)$ where $f_0 = 3000$ Jy. The pixel scale is 0.08-arcsec per pixel, and the quantum efficiency of the detectors is about 0.8 at this wavelength. Note that 1 Jy = 1×10^{-26} W m^{-2} Hz^{-1}.

   (a) Whats the ratio of photo-electrons per pixel that you expect to see for the new vs full moon?

   Using equation 2, we know the flux ratio between two sources is $10^{-\Delta m/2.5} = 10^{-0.4} \sim 0.4$

   (b) Integrate for 100-sec. What is the average number of photo-electrons that you see per pixel during the new moon? You can either calculate this yourself (i.e., from first principles, given the above information), or use whatever information that you can find on the Magellan website (but cite your sources).

   Starting from the equation $M_R = -2.5\log(f_R/f_0)$, we can calculate f_R for the magnitude of a new moon, or $M_R = 21$ per square arcsec. Solving for $f_R = (3000\text{Jy})10^{-M_R/2.5} = 1.19\times10^{-5}$ Jy arcsec^{-2} $= 1.19\times10^{-31}$ W m^{-2} Hz^{-1} arcsec^{-2}

   The units of Jy is W m^{-2} Hz^{-1}, so to convert to a flux, we need to multiply by a throughput ($A\Omega$), a bandwidth ($\Delta\nu$), and an efficiency, η. The area of the telescope is $A = \pi(6.5\text{m}/2)^2$, the solid angle (Ω) is $(0.08)^2$ arcsec^2, the bandwidth in frequency can be calculated from $d\nu = d\lambda(c/\lambda^2) \sim 1.48\times10^14$ Hz. Multiplying these factors, we find that the flux is $f_R(A\Omega)(\Delta\nu) = 3.7\times10^{-18}$ W = 3.7 aW (or attowatts).

   To convert from a a power to a number of photons, we need to multiply by the integration time (Δt) to convert W to Joules, and then also divide by the average energy per photon ($h\nu_{avg}$), to convert Joules to the number of photons. Each absorbed phtoton should also create one electron count in the CCD (times an efficiency factor, which we've already corrected for above). So putting it all together, the equation for number of electrons measured by the CCD should be:

$$N_{electrons} = f_R\frac{(A\Omega)(\Delta\nu)(\eta)(\Delta t)}{h\nu_{avg}} = (3.7\times10^{-18}\text{W})(0.8)(180\text{sec})/(3.3\times10^{-19}\text{J}) = 1634 \quad (5)$$

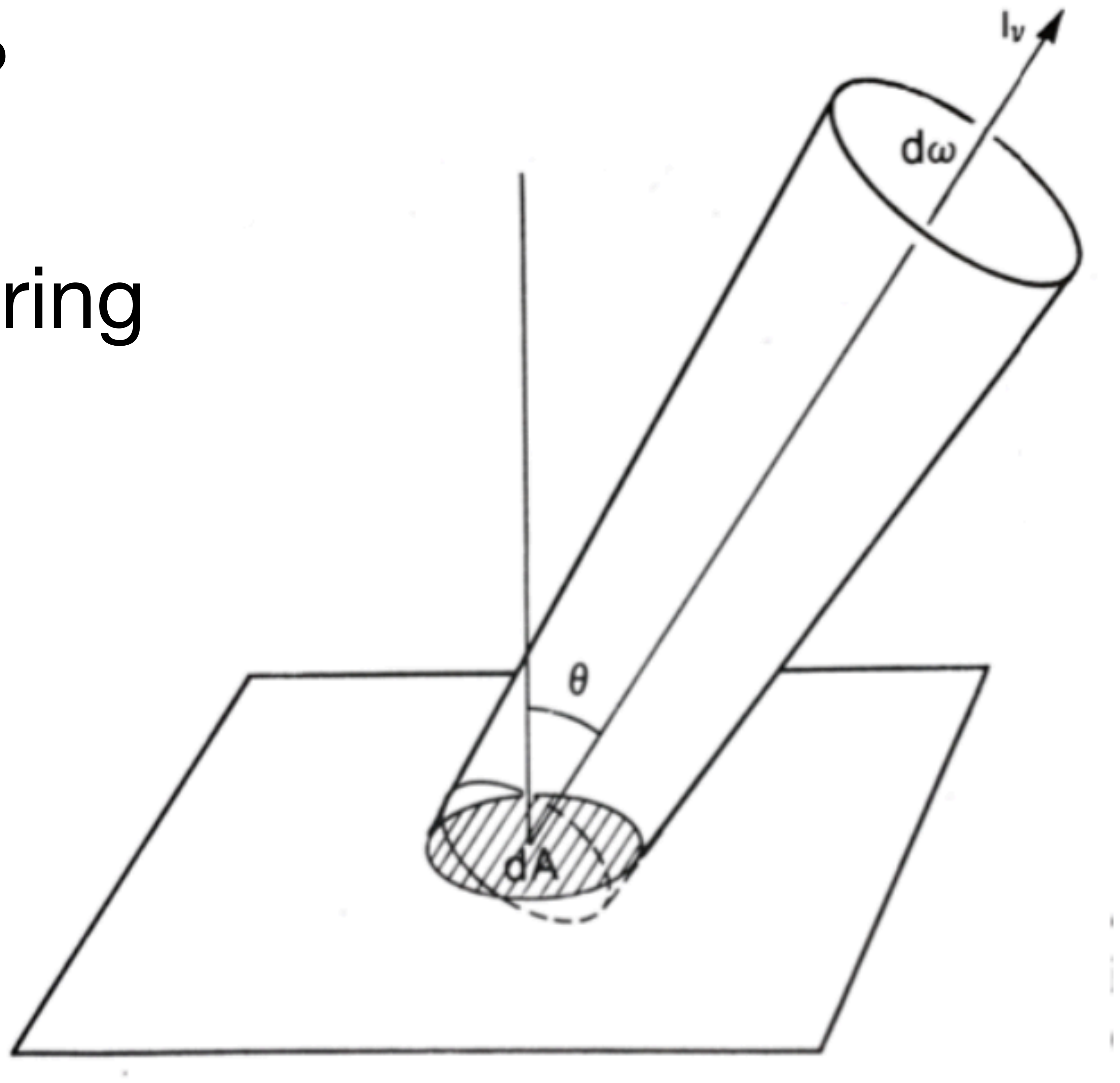
[1] https://www.lco.cl/magellan-instruments/

Flux and Intensity

- In Astronomy, we often characterize the flux from, or intensity of, an object, but what do we mean by that?
- Amount of energy (dE_ν) passing through an area, *dA*, within solid angle $d\Omega$, in frequency range $[\nu, \nu + d\nu]$, during time *dt* is:

$$dE_\nu = I_\nu \, dA \cos\theta \, d\omega \, dt \, d\nu$$

- Where:
  - $dA d\Omega$ could be something like the size (and effective) collecting area of your detector
  - **I_ν: Specific Intensity**
    - Units of J / [s m^2 Hz steradian]
    - An intrinsic property of the object (i.e., it should not depend on the observer or the measurement)



Karttunen et al.

Weighted Mean

- **Deviation**: of one measurement from the average

$$d_i = x_i - \mu$$

- **Weighted Mean**: Average of the squares of the deviations.

$$\sigma^2 = \frac{1}{N}\sum_i (x_i - \mu)^2$$

$$= \frac{1}{N-1}\sum_i (x_i - \bar{x})^2$$

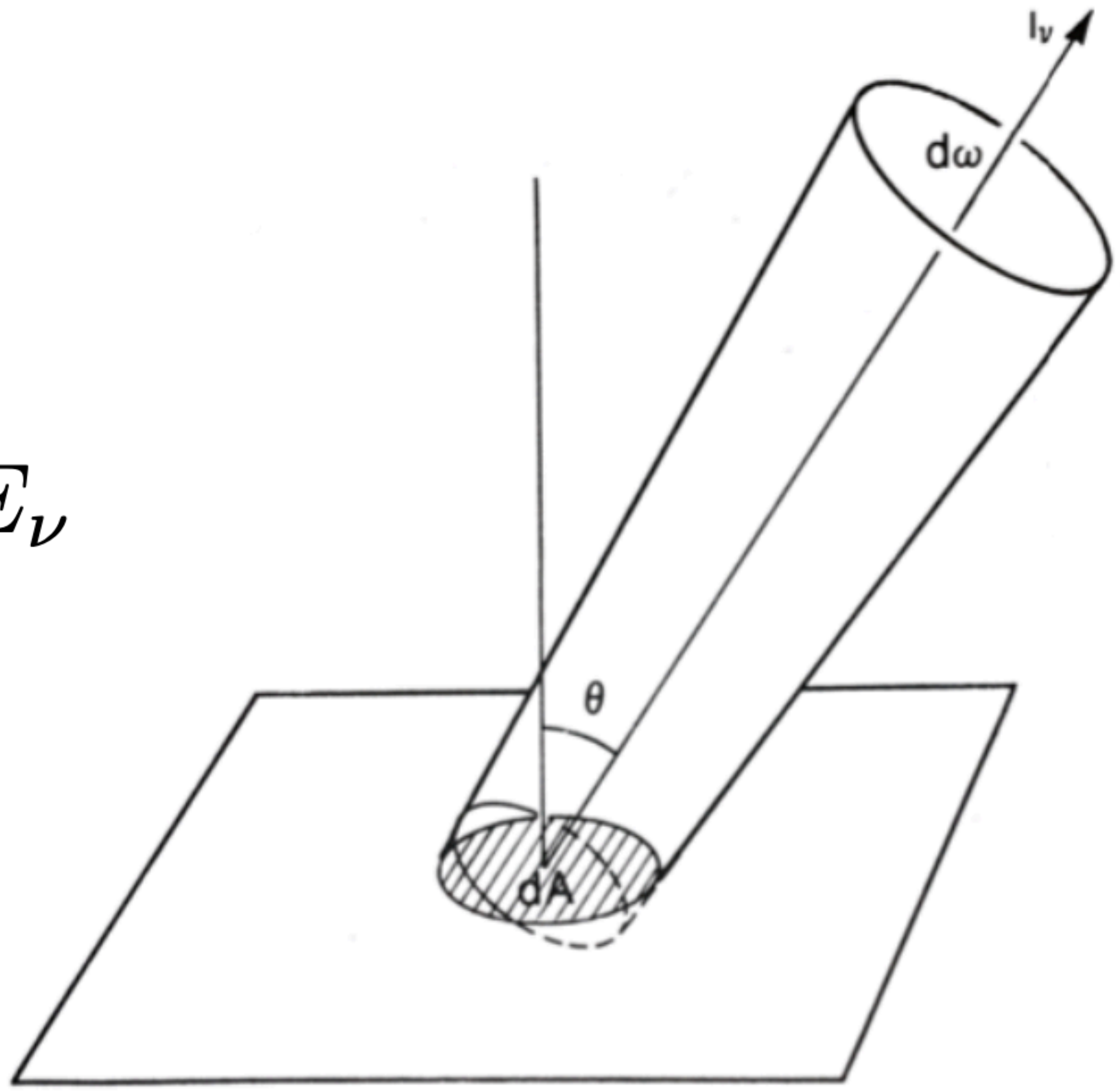
- **Gauss**: The square root of the variance (i.e., σ), or the “typical” deviation around the mean.

Flux and Intensity

- We measure Flux by integrating the Specific Intensity over solid angle

$$dE_\nu = I_\nu \, dA \cos\theta \, d\omega \, dt \, d\nu$$

$$f_\nu = \int_\Omega d\omega \cos\theta \; I_\nu$$
$$= \frac{1}{dA \; dt \; d\nu} \int_\Omega dE_\nu$$



Karttunen et al.

- **Spectral Flux Density, f_ν:**
  - Energy per area, per time, per frequency interval
  - We usually observe f_ν (or f_ν integrated over the frequency band of our detector)
  - Depends on the distance between the source and the observer.