ASTR21200: Homework 3 (HW3)

1. **Poisson statistics and Error Propagation:** The Poisson distribution describes the probability to observe x events during a certain measurement interval, given a mean rate μ:

$$P_P(x|\mu) = \frac{\mu^x}{x!}e^{-\mu} \tag{1}$$

   (a) During your first observation, you measured $N = 2,500$ photons from the object (with a negligible sky brightness). What is the uncertainty on this measurement?

   (b) For your second observation, you plan to increase the observing time by a factor of four, such that you would expect $N = 10,000$ photons from the object. Given your measurements from your first observation, what is the probability that you measure more than 10,400 photons from the object? What is the probability that you measure more than 11,000 photons from the object? State your reasoning.

   (c) When you repeat the second observation on the following night, unfortunately the sky brightness is much worse, and contributing $N = 20,000$ photons from sky brightness alone (i.e., this is in addition to the expected $N = 10,000$ photons from the object). By measuring other blank regions of the sky, you are able to estimate and subtract the sky brightness in the region around your object, and measure $N = 10,000$ photons from the object (as you predicted). For this second observation alone, what is the uncertainty on the photon counts from the object from this measurement?

2. **χ^2 and Goodness of Fit:**

One active area of research in cosmology today, is trying to understand the source of the so-called *Hubble Tension.* This generally refers to the current disagreement between local measurements of the expansion rate of the Universe (typically characterized by the *Hubble constant*), and what is predicted by measurements of the cosmic microwave background (CMB) fit a standard cosmological model. To measure the *Hubble constant* in the local universe (sometimes described to be at redshift, $z < 0.1$), we most often use various *standard candles*, or astrophysical sources that have a known luminosity. Since different *standard candles* might have different sources of systematic uncertainties, there have been several techniques developed to test for this, however there can still be differences based on analysis choices by different groups.

Using data from the table below, answer the following questions::

H_0 (km s^{-1} Mpc^{-1})	**Method**	**Reference**
	Local Probes	
		arXiv:2408.11770
73.4 ± 2.1	Cepheid variables	Riess et al., 2024
72.2 ± 2.2	Tip of Red Giant Branch	Riess et al., 2024
72.1 ± 2.2	J-Region Asymptotic Giant Branch	Riess et al., 2024
		arXiv: 2408.06153
70.4 ± 1.9	Cepheid variables	Freedman et al., 2024
68.8 ± 2.2	Tip of Red Giant Branch	Freedman et al., 2024
67.8 ± 2.7	J-Region Asymptotic Giant Branch	Freedman et al., 2024
	Cosmic Microwave Background (CMB)	
67.4 ± 0.5	Planck CMB	arXiv:1807.06209

(a) Using only the local probe measurements from Riess et al. (2024), what is the best-fit Hubble constant and uncertainty?

(b) Using only the local probe measurements from Freedman et al. (2024), what is the best-fit Hubble constant and uncertainty?

(c) What is the offset of the local probe measurements (calculated in parts a & b) compared to the CMB measurement, in units of σ? Where σ is the combined uncertainty, or the expected standard deviations, from the CMB and the local probe measurements? Calculate for both the 1) Riess et al. (2024) and 2) Freedman et al. (2024) best-fit Hubble constant.

(d) Assume that the CMB and Riess et al. (2024) measurement can be fit by a single Hubble constant. What would be the χ^2 of the fit to those two data points? What is the probability to exceed (PTE) this χ^2 value? This relatively low PTE is what would be considered the *Hubble Tension* in the literature.

(e) The Riess et al. (2024) and Freedman et al. (2024) measurements were based on local probes of the Hubble constant that used mostly the same objects and data, i.e., they are not independent measurements and we can't statistically combine them because it would be double counting measurements. However, because these analyses are effectively using the same data, we would expect them to agree at a level much better than their statistical uncertainty. How does the offset between the Riess et al. (2024) and Freedman et al. (2024) compare to their statistical uncertainty? What does this imply about the systematic uncertainty of these measurements, relative to the statistical uncertainty? What might be a more reasonable estimate of the systematic uncertainty for these local probe measurements based on the difference between the Riess and Freedman results?

(f) Repeat the χ^2 and PTE calculation in part *(d)* using this new estimate of the systematic uncertainty.