Causal Inference

Prediction and Regression

Brad Hackinen

Agenda:

- Intro to Regression
- Regression for Causal Inference
- The Table 2 Fallacy

Intro to Regression

Motivating Example: Estimating the Effect of Superhost Status

r/airbnb_hosts • 1y ago
CryptographerMurky12

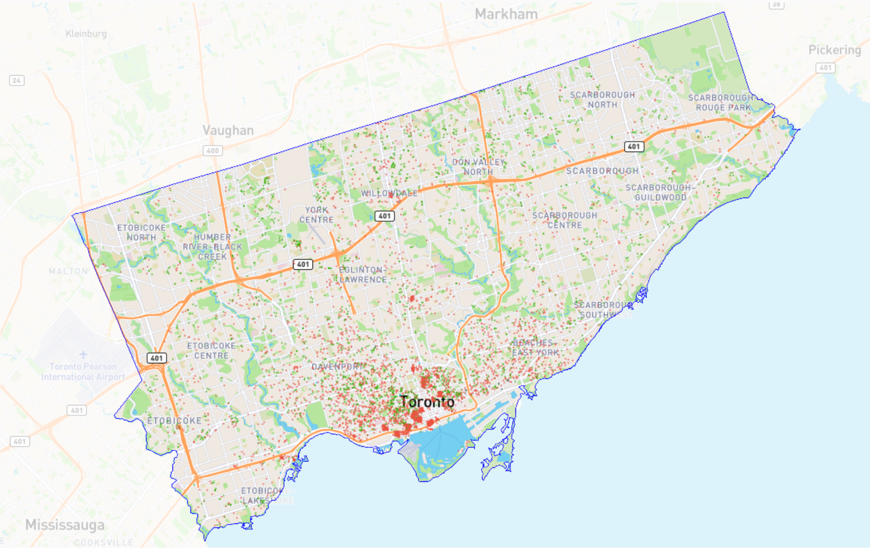
Just became a Superhost – does it actually make a difference?

Discussion

I've just become a Superhost and was wondering if anyone who's been one for a while has actually seen any noticeable benefits. Did you experience an increase in bookings or any other perks? Curious to hear your thoughts!

Research Question: Does Superhost status increase revenue for Airbnb hosts?

Data: 6000+ listings in Toronto scraped by InsideAirbnb



Practical Questions:

- What do we need to control for?
- What data is available?
- Are there any variables in the data that we should *not* control for?
- What estimation method should we use?
- How do we interpret the results?

Q: Have you ever run a linear regression? How would you use a regression to estimate the effect of Superhost status on revenue?

Regression Overview

Regression is a flexible tool for analyzing data in which we summarize statistical relationships in a dataset by “fitting” a function to match the data as closely as possible, and then examine the properties of the fitted function.

Regression Functions

In general, we have N observations $i = 1, 2, 3, \dots, N$, and for each observation:

- A "***dependent variable***" denoted Y_i
- One or more "***independent variables***" denoted $X_{1i}, X_{2i}, X_{3i}, \dots$

The fitted regression function $\hat{f}$ produces a predicted value for the dependent variable $\hat{Y}_i$ as a function of the independent variables:

$$\hat{Y}_i = \hat{f}(X_{1i}, X_{2i}, X_{3i}, \dots)$$

Regression Equations

The function we are fitting to the data is often described by a ***regression equation***.

A regression equation describes how the independent variables are used to predict the dependent variable. It usually includes:

- ***Parameters*** that are selected when fitting the regression function
- ***Error terms*** that describe random variables that are:
  - Not observable in the data
  - Assumed to explain the difference between actual and predicted values.

Example 1 (Simple linear regression):

$$Y_i = \alpha + \beta X_i + \varepsilon_i$$

- Y_i is the dependent variable
- X_i is a single independent variable
- α and β are regression parameters
- We call α the ***intercept term***
- ε_i is an error term

Example 2 (Multiple linear regression):

$$Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + \varepsilon_i$$

- Y_i is the output of the function
- $X_{1i}, X_{2i}, X_{3i}, \ldots, X_{ki}$ are the covariates
- $\alpha, \beta_1, \beta_2, \beta_3, \ldots, \beta_k$ are regression parameters
- ε_i is an error term

How do we “fit” the regression?

The most common approach is to use ***Ordinary Least Squares (OLS)*** .

Ordinary Least Squares (OLS)

Define the ***residual*** e_i for each observation i as the difference between the observed Y_i and output of the regression equation:

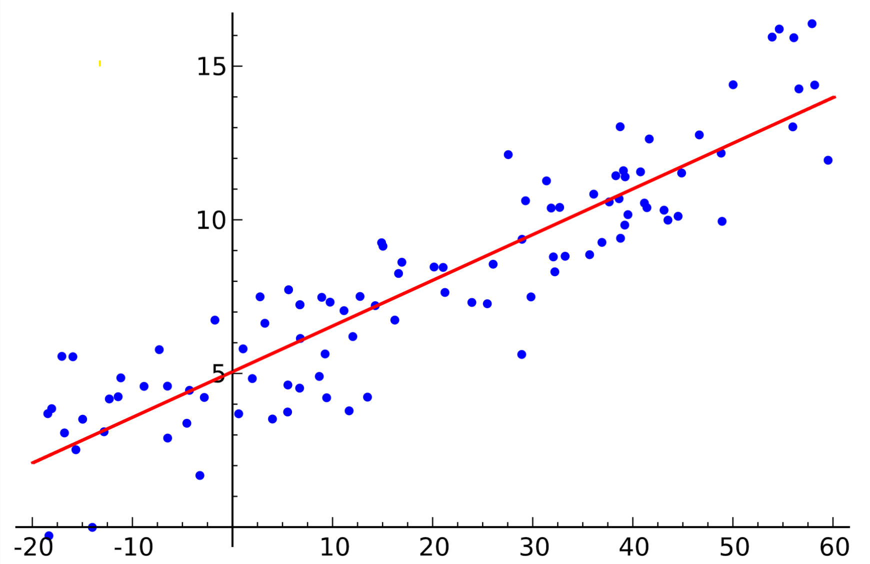
$$e_i = Y_i - \alpha - \beta_1 X_{1i} - \beta_2 X_{2i} - \beta_3 X_{3i} - \cdots - \beta_k X_{ki}$$

Then the OLS ***parameter estimates*** are the set of parameters that achieve the lowest *Sum of Squared Residuals (SSR)*:

$$SSR = \sum_{i=1}^{N} e_i^2$$

We label the parameter estimates with "hats" to indicate they are specific estimated quantities: $\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \ldots \hat{\beta}_k$.

Visualizing Simple Linear Regression



Linear Dependence

The OLS estimator only has a unique solution if the independent variables are *linearly independent*.

Definition: Vectors $X_0, X_1, X_2, \ldots, X_k$ are called ***linearly dependent*** or ***collinear*** if any variable X_j can be expressed as a weighed sum of the other variables.

If you try to run a regression that contains any collinear variables, you will get an error.

Example: *BirthYear*, *Year*, and *Age* (as of December 31st), are collinear since:

$$Age_i = Year_i - BirthYear_i$$

Conditional Expectation Functions

OLS estimates a linear approximation of a ***conditional expectation function***.

$$E[Y_i|X_{1i}, X_{2i}, X_{3i}, \ldots, X_{ki}] \approx \hat{\alpha} + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i} + \cdots + \hat{\beta}_k X_{ki}$$

It can be shown that (in the limit of infinite sample size), OLS provides the lowest mean square error out of any linear estimator.

Nonlinearity

The OLS prediction may not be very accurate if the true conditional expectation function is *nonlinear*. This is an important limitation of OLS.

However, many functions that appear to be nonlinear can be represented as a linear combination of appropriately transformed independent variables.

Example: The following regression estimates a conditional expectation function that is quadratic in X_i but linear in X_i^2:

$$Y_i = \alpha + \beta_1 X_i + \beta_2 X_i^2 + \varepsilon_i$$

Non-linear regression

Regression does not need to be linear. There are many different approaches that allow the output of the regression to be a non-linear function of the inputs. Popular approaches include:

- **Non-parametric methods** that estimate smooth curves using local observations
- **Generalized Linear Models** feed the output of an "inner" linear function through a non-linear function to predict the output:
    - **Logistic regression** produces values in the range $(0, 1)$. Often used to predict probabilities with binary data.
    - **Probit regression** is similar to logistic regression with non-linear function that is based on the normal distribution.
    - **Poisson Regression** produces predictions in the range $[0, \infty)$. Used to analyze count data, or other non-negative variables.

Regression in R

`RegressionInR.Rmd`

Goal:

- A quick tour of how to run regressions in R and interpret the output.

Regression for Causal Inference

The key question for causal inference is: *What does OLS estimate?*

...in a simple linear regression?

$$Y_i = \alpha + \delta D_i + \varepsilon_i$$

...in a multiple linear regression with controls?

$$Y_i = \alpha + \delta D_i + \beta X_i + \varepsilon_i$$

R Demo

Let's look at a simple example in R.

`RegressionForCI.Rmd`

Goal:

- Try a simple regression
- Try multiple regression with controls

What do they estimate?

Linear Regression without Controls

First, let's consider a simple linear regression:

$$Y_i = \alpha + \delta D_i + \varepsilon_i$$

Recall that the fitted OLS parameters provide the *Best Linear Prediction* of the outcome conditional on treatment.

$$E[Y_i|D_i] \approx \hat{\alpha} + \hat{\delta} D_i$$

This linear prediction can be written separately for treated and untreated units to find the predicted potential outcomes:

$$E[Y_i|D_i = 1] \approx \hat{\alpha} + \hat{\delta}$$
$$E[Y_i|D_i = 0] \approx \hat{\alpha}$$

Therefore,

$$\hat{\delta} \approx E[Y_i|D_i = 1] - E[Y_i|D_i = 0]$$

Look familiar?

In fact, in a simple linear regression of an outcome Y_i on a binary treatment indicator D_i like this:

$$Y_i = \alpha + \delta D_i + \varepsilon_i$$

The parameter estimate $\hat{\delta}$ is mathematically equivalent to measuring the difference-in-means between the treated and untreated groups.

Interpreting OLS Regression with Controls

If we include an additional covariate X in the regression:

$$Y_i = \alpha + \delta D_i + \beta X_i + \varepsilon_i$$

Then $\hat{\delta}$ measures the average difference between two conditional estimates:

$$E[Y_i|D_i = 1] \approx \hat{\alpha} + \hat{\delta} + \beta X_i$$

$$E[Y_i|D_i = 0] \approx \hat{\alpha} + \beta X_i$$

However! These two estimates assume that β is the same for treated and untreated units. This may not be realistic!

OLS and Heterogeneous Treatment Effects

The common regression equation

$$Y_i = \alpha + \delta D_i + \beta X_i + \varepsilon_i$$

assumes that β is constant across treatment groups.

This is equivalent to assuming that the average treatment effect is constant across units ($ATT = ATE = ATU$).

If $ATT \neq ATU$, then $\hat{\delta}$ will not estimate the ATT,ATU, or ATE (though it will be somewhere between the ATT and ATU).

A more flexible specification

One way to account for heterogeneous treatment effects is to modify our regression as follows:

1. Compute the mean $\overline{X}$ within our target group (e.g. the treated units for the ATT, or the whole sample for the ATE)
2. Replace X with $\widetilde{X}_i = X - \overline{X}$ in our regression equation
3. Add an interaction term $D_i\widetilde{X}$ to allow for varying slopes in each treatment group.

$$Y_i = \alpha + \delta D_i + \beta_1 \widetilde{X}_i + \beta_2 D_i \widetilde{X}_i + \varepsilon_i$$

G-Computation

Another alternative is to use G-computation to estimate the treatment effect.

1. Fit a model to that allows for different relationships between X and Y in each treatment group by including an *interaction* between D and X.
2. Use the model to predict potential outcomes for all units
3. Estimate the ATE/ATT/ATU as the average difference between predicted $\hat{Y}(1)$ and $\hat{Y}(1)$ in the selected group.

Another Problem: The Table 2 Fallacy

People have a habit of interpreting the coefficients on control variables in a regression as if they were causal effects. This is a mistake!

Let's look at an example in R.

`Table2Fallacy.Rmd`

The Linearity Assumption

The key difference between OLS and the other estimators we have discussed is that OLS imposes the assumption that all the statistical relationships are linear.

- If true, OLS will perform better than any alternative estimator
- If there are important non-linear relationships, then OLS may be biased (we are only controlling for a *linear approximation* of the confounding variables)
- The linearity assumption is particularly important when we have units outside the common support. When we are *extrapolating* based on this linearity assumption.