Standard Halo Model

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We consider a (truncated) Standard Halo Model (SHM) with the following parameters:

- Earth velocity, $\mathbf{v}_\mathrm{e}$ - the mean velocity of the DM halo with respect to the Earth's rest frame;
- Velocity dispersion, σ_v - velocity dispersion of the DM particles in the halo;
- Escape velocity, v_esc - escape velocity of the halo measured in the Galactic rest frame.

The SHM velocity distribution has the following functional form in the Earth's rest frame:

$$f(\mathbf{v}) = \frac{1}{(2\pi\sigma_v^2)^{3/2} N_\mathrm{esc}} \exp\left(-\frac{(\mathbf{v}+\mathbf{v}_\mathrm{e})^2}{2\sigma_v^2}\right) \Theta(v_\mathrm{esc} - |\mathbf{v}+\mathbf{v}_\mathrm{e}|)\,, \tag{1}$$

with

$$N_\mathrm{esc} = \mathrm{erf}\left(\frac{v_\mathrm{esc}}{\sqrt{2}\sigma_v}\right) - \sqrt{\frac{2}{\pi}}\frac{v_\mathrm{esc}}{\sigma_v}\exp\left(-\frac{v_\mathrm{esc}^2}{2\sigma_v^2}\right)\,, \tag{2}$$

and

$$\Theta(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x > 0\,. \end{cases} \tag{3}$$

The speed distribution is then given by:

$$f(v) = \frac{v}{\sqrt{2\pi}\sigma_v v_\mathrm{e} N_\mathrm{esc}} \begin{cases} 0 & \text{if } v > v_\mathrm{esc} + v_\mathrm{e}, \\ \exp[-\frac{(v-v_\mathrm{e})^2}{2\sigma_v^2}] - \exp[-\frac{(v+v_\mathrm{e})^2}{2\sigma_v^2}] & \text{if } v < v_\mathrm{esc} - v_\mathrm{e}, \\ \exp[-\frac{(v-v_\mathrm{e})^2}{2\sigma_v^2}] - \exp[-\frac{v_\mathrm{esc}^2}{2\sigma_v^2}] & \text{otherwise.} \end{cases} \tag{4}$$

This gives, for the velocity integral:

$$\eta(v_\mathrm{min}) = \frac{1}{2v_\mathrm{e}N_\mathrm{esc}} \begin{cases} 0 & \text{if } v_\mathrm{min} > v_\mathrm{esc} + v_\mathrm{e}, \\ \mathrm{erf}[\alpha_+] - \mathrm{erf}[\alpha_-] - 2\frac{v_\mathrm{e}}{\sigma_v}\sqrt{\frac{2}{\pi}}\exp[-\alpha_\mathrm{esc}^2] & \text{if } v_\mathrm{min} < v_\mathrm{esc} - v_\mathrm{e}, \\ \mathrm{erf}[\alpha_\mathrm{esc}] - \mathrm{erf}[\alpha_-] - \frac{v_\mathrm{esc}+v_\mathrm{e}-v_\mathrm{min}}{\sigma_v}\sqrt{\frac{2}{\pi}}\exp[-\alpha_\mathrm{esc}^2] & \text{otherwise.} \end{cases} \tag{5}$$

We have defined:

$$\begin{aligned} \alpha_{\pm}(v_{\rm min}) &= \frac{v_{\rm min} \pm v_{\rm e}}{\sqrt{2}\sigma_v} \,, \\ \alpha_{\rm esc} &= \frac{v_{\rm esc}}{\sqrt{2}\sigma_v} \,. \end{aligned} \tag{6}$$

This can also be written as

$$\eta(v_{\rm min}) = \frac{1}{2 v_{\rm e} N_{\rm esc}} \left({\rm erf}[\tilde{\alpha}_+] - {\rm erf}[\tilde{\alpha}_-] - \frac{2}{\sqrt{\pi}} (\tilde{\alpha}_+ - \tilde{\alpha}_-) \exp[-\alpha_{\rm esc}^2] \right) \,, \tag{7}$$

where

$$\tilde{\alpha}_{\pm}(v_{\rm min}) = \min\left(\alpha_{\pm}, \alpha_{\rm esc}\right) \,. \tag{8}$$