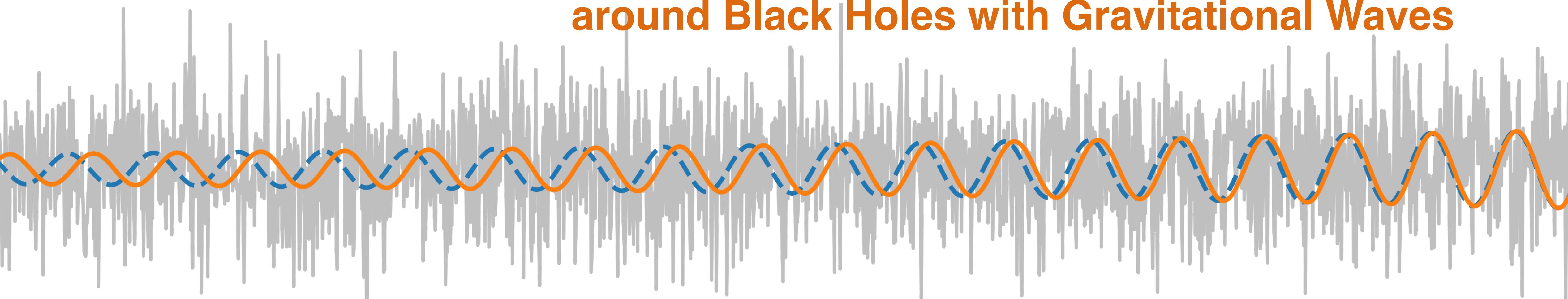


# Detecting, Discovering and Measuring Dark Matter around Black Holes with Gravitational Waves



 [kavanagh@ifca.unican.es](mailto:kavanagh@ifca.unican.es)

 @BradleyKavanagh

Bradley J Kavanagh  
Instituto de Física de Cantabria  
(CSIC-Universidad de Cantabria)

25th November 2021 - IFT Seminar, Madrid

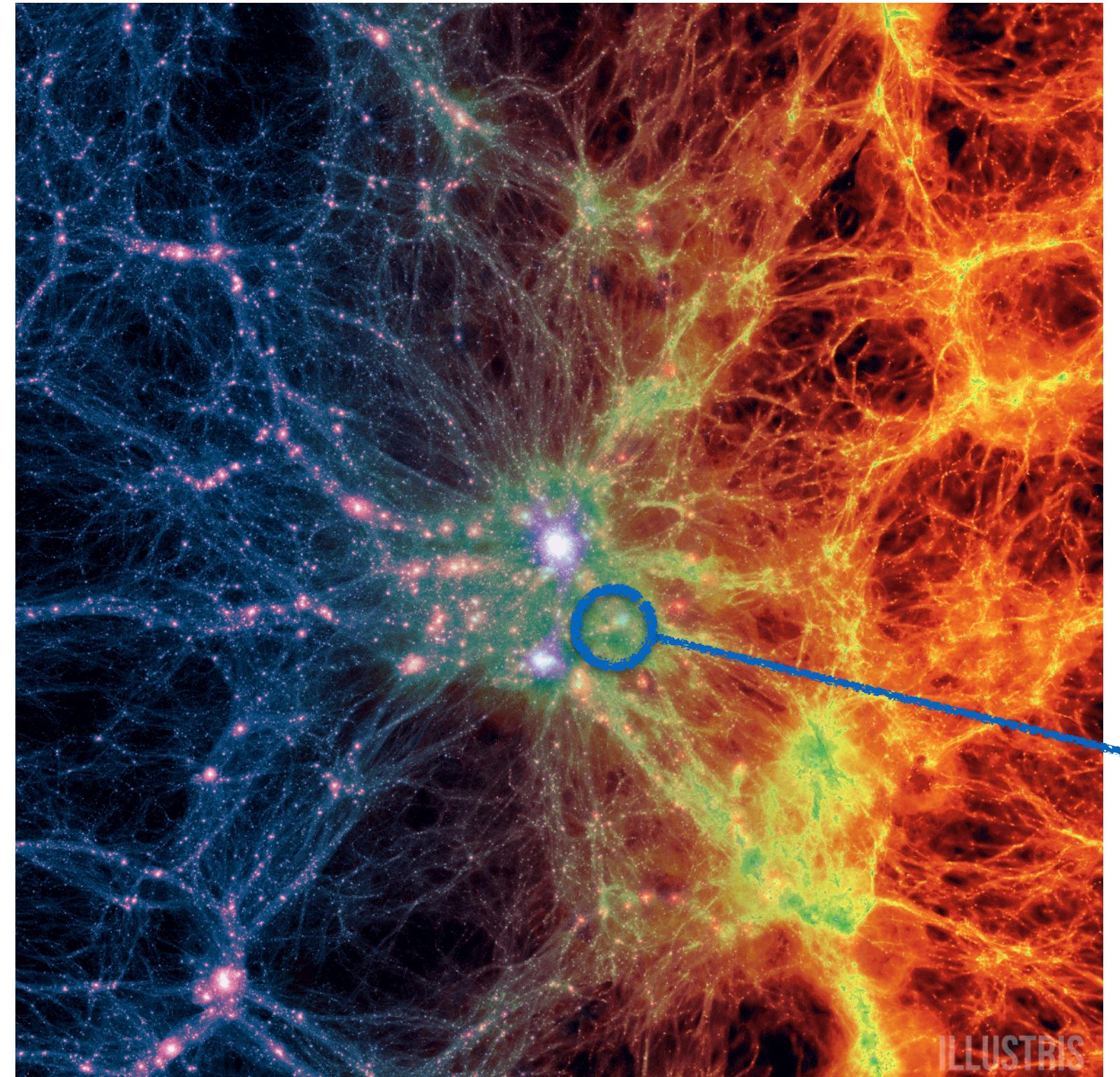


EXCELENCIA  
MARÍA  
DE MAEZTU



**CSIC** UC  
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS  
UNIVERSIDAD  
DE CANTABRIA

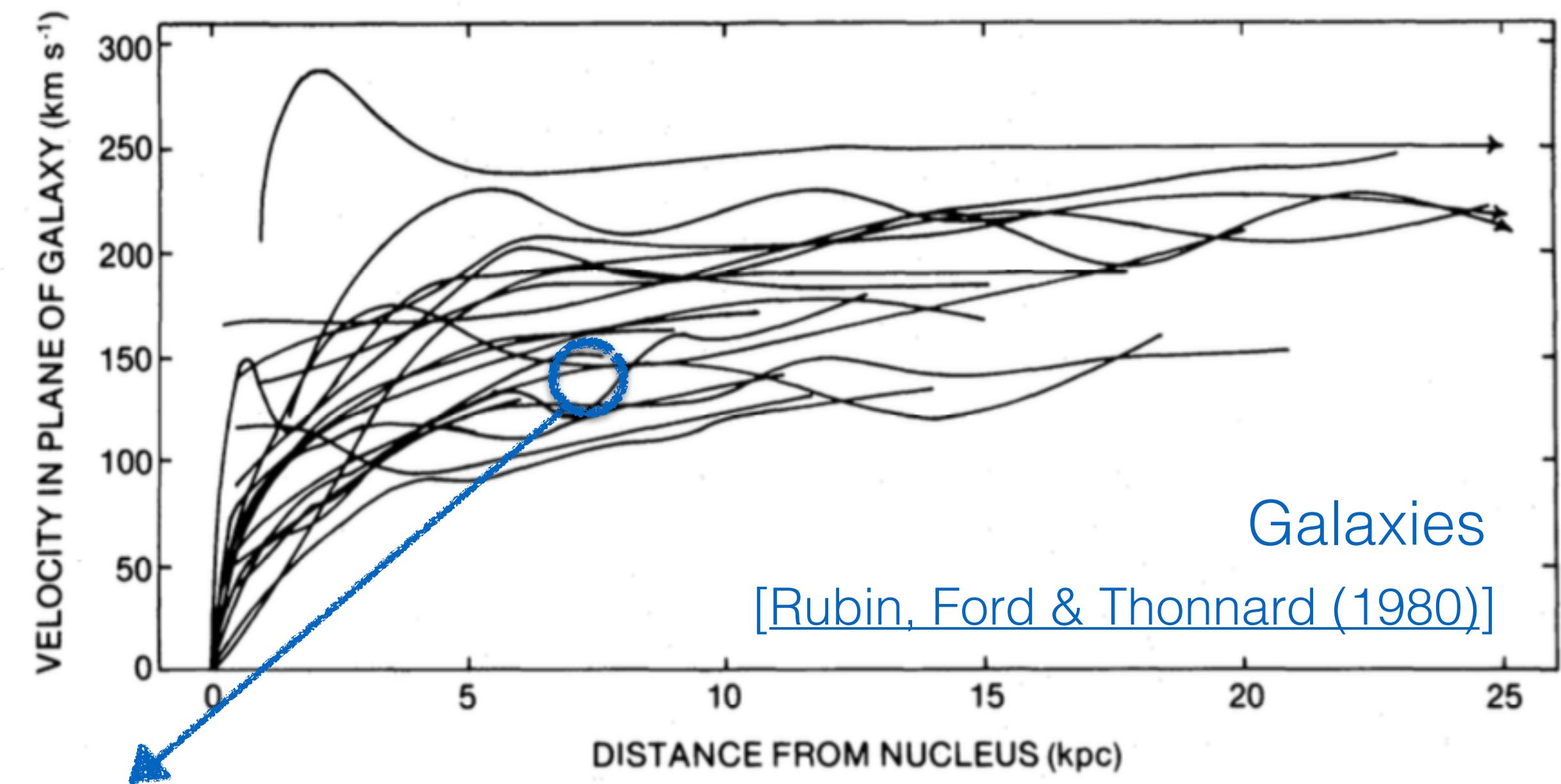
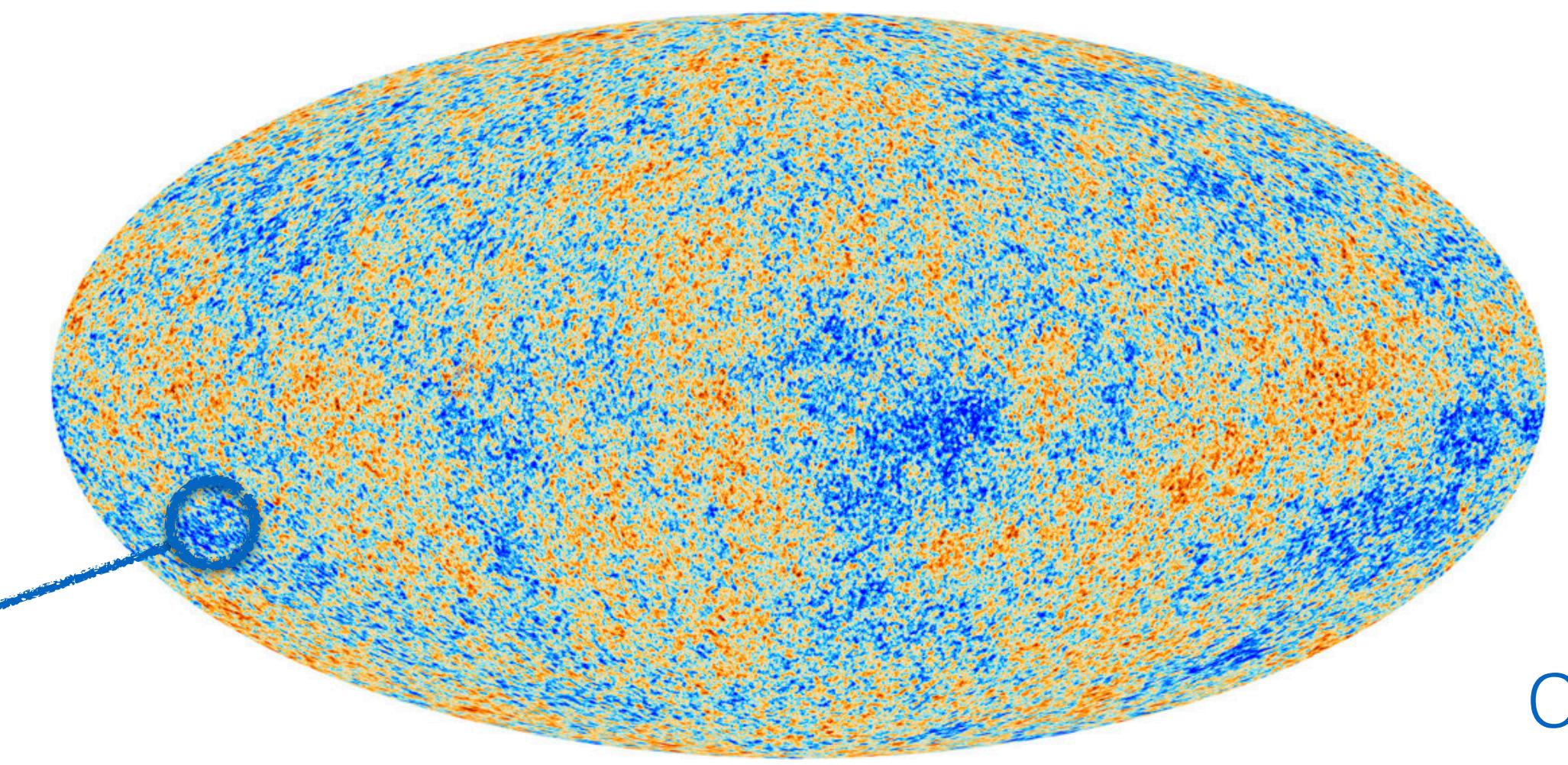
# Dark Matter on all scales



Galaxy clusters

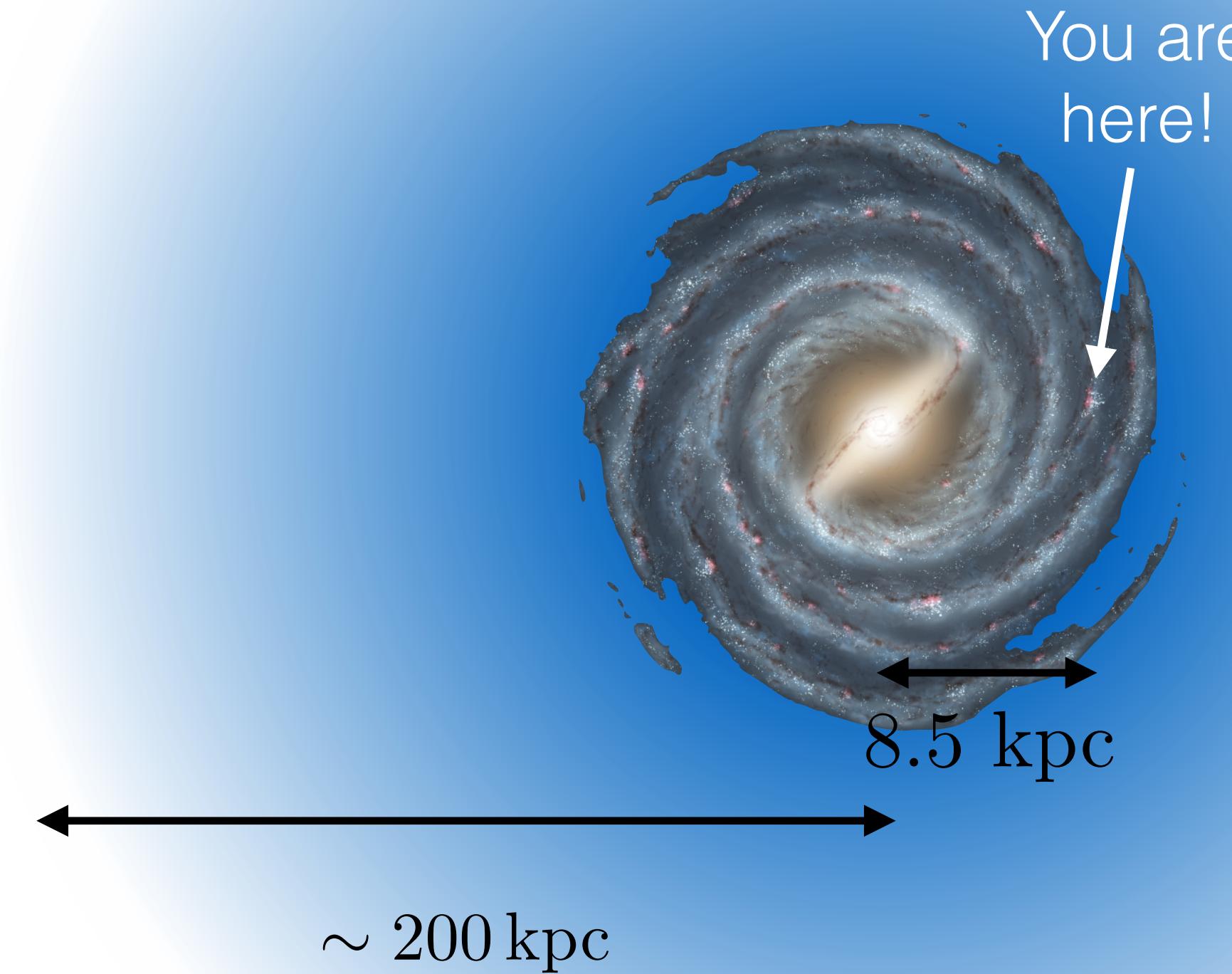
[Illustris, [1405.2921](#)]

[[astro-ph/0006397](#)]



# Dark Matter at Earth

**NOT TO SCALE**



Global and local estimates of  
DM at Solar radius give:

$$\begin{aligned}\rho_\chi &\sim (0.2 - 0.8) \text{ GeV cm}^{-3} \\ &\sim (0.005 - 0.02) M_\odot \text{ pc}^{-3}\end{aligned}$$

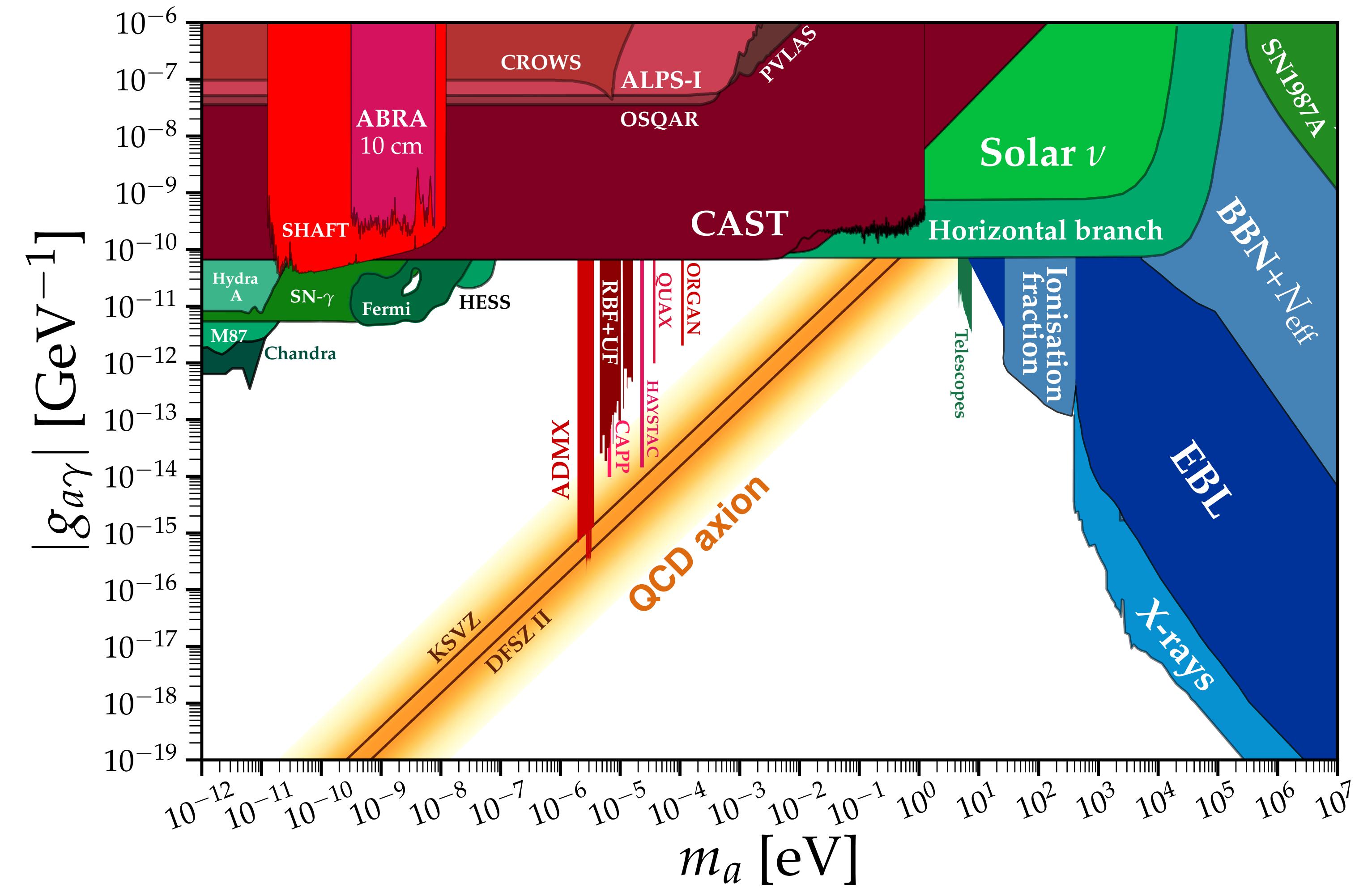
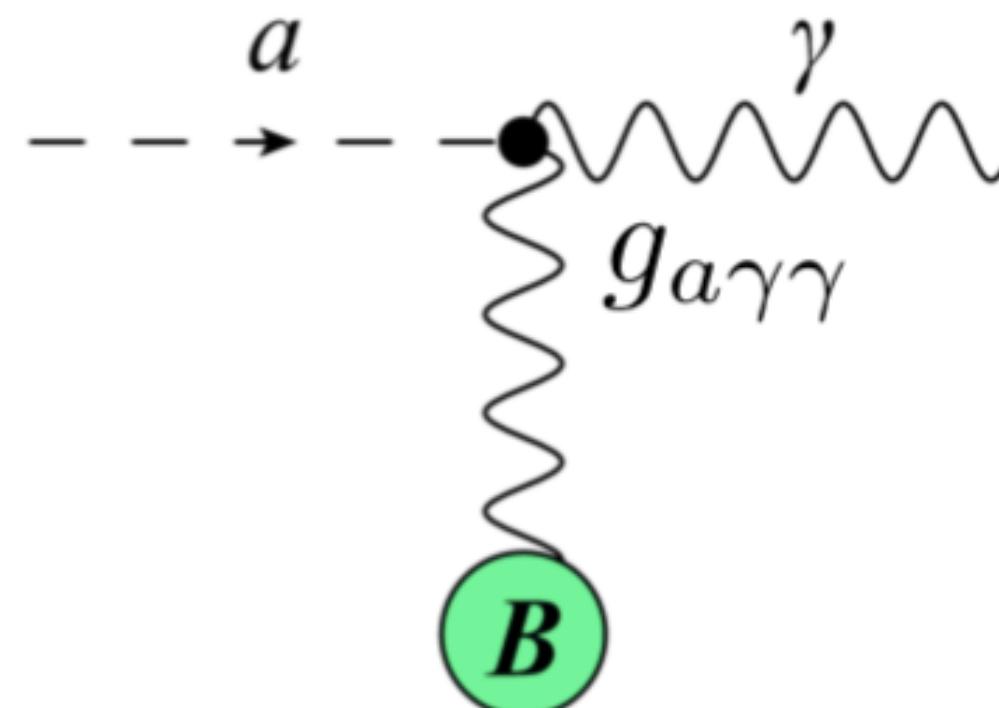
E.g. Iocco et al. [1502.03821],  
Garbari et al. [1206.0015], Read [1404.1938]

# Axion Dark Matter

[1510.07633, 2003.01100]

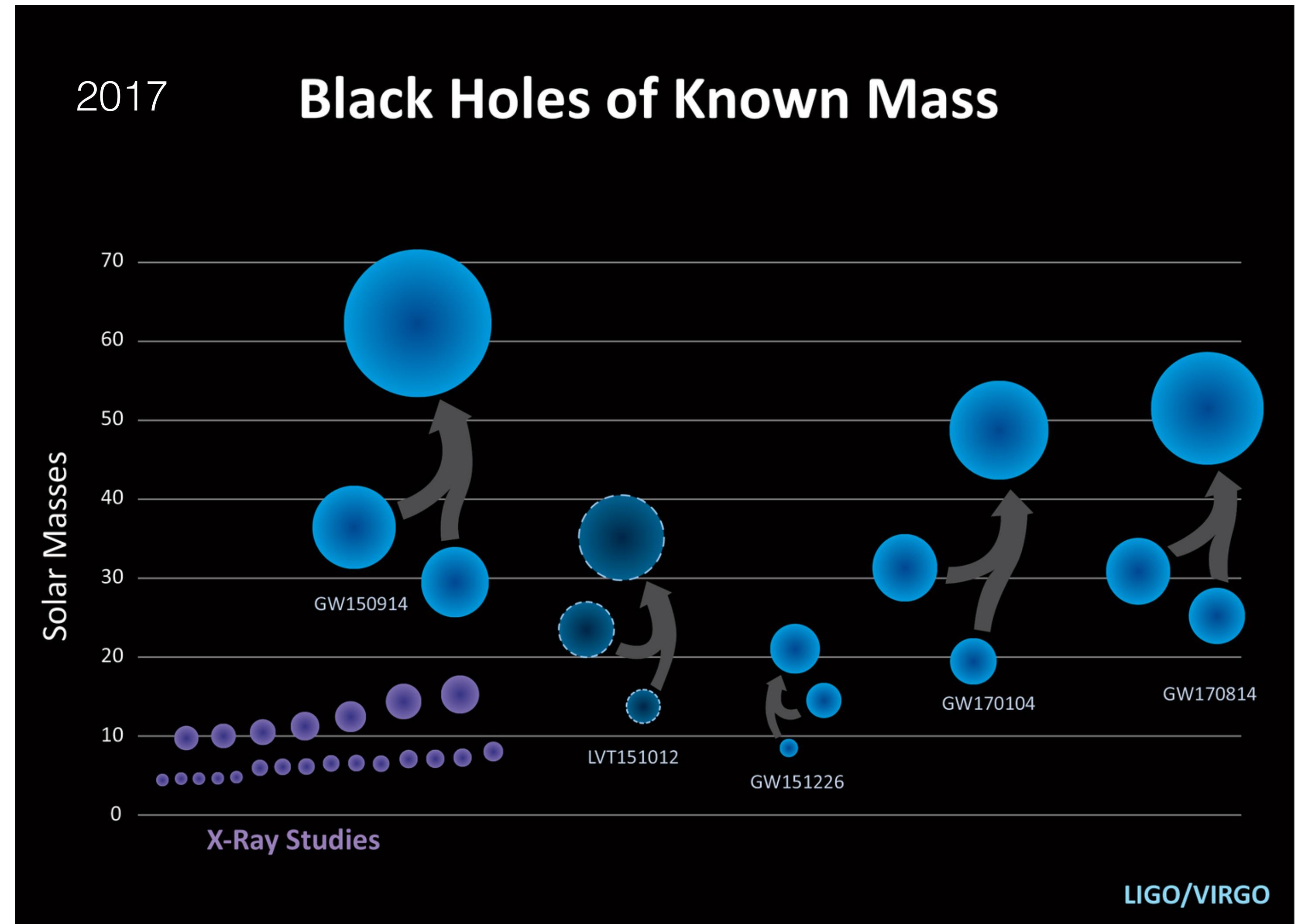
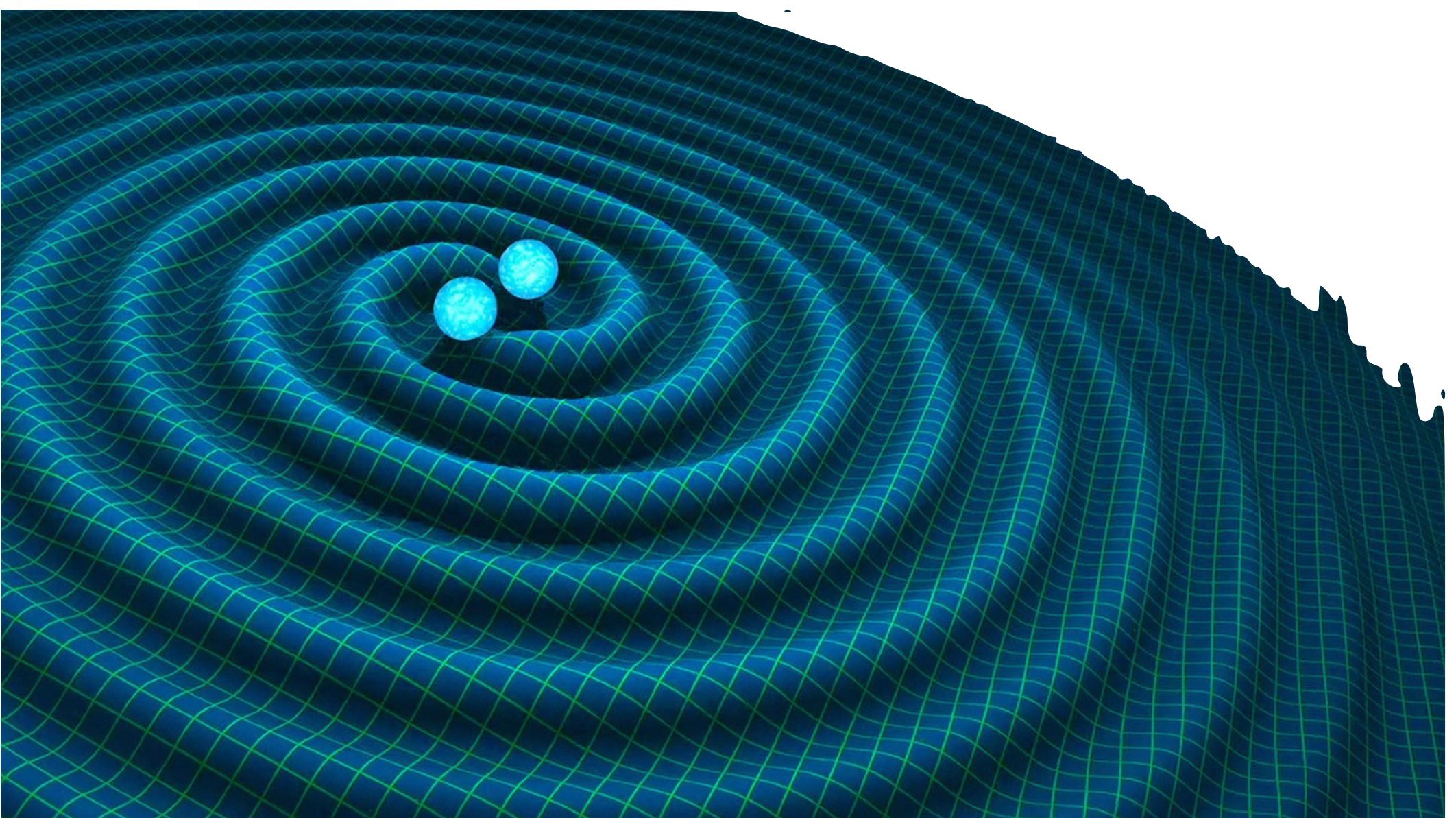
Dark Matter could be in the form of light pseudo scalar ‘axions’, which may convert to photons (and vice versa) in an external magnetic field:

$$\begin{aligned}\mathcal{L} &\supset -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} \\ &= -\frac{1}{4} g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B}\end{aligned}$$



# Gravitational Waves

R. HURT / CALTECH-JPL /  
HANDOUT/ ESA

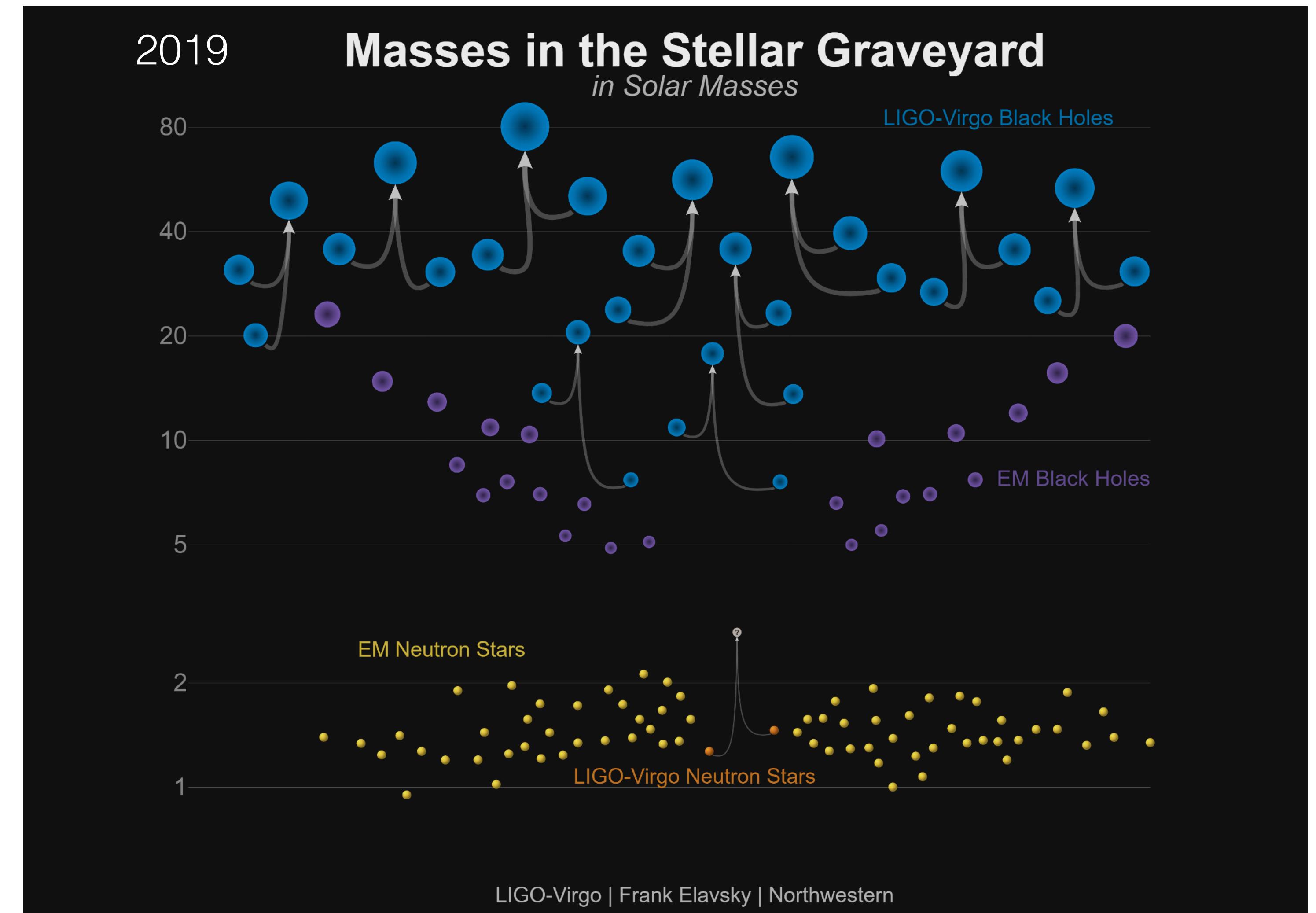
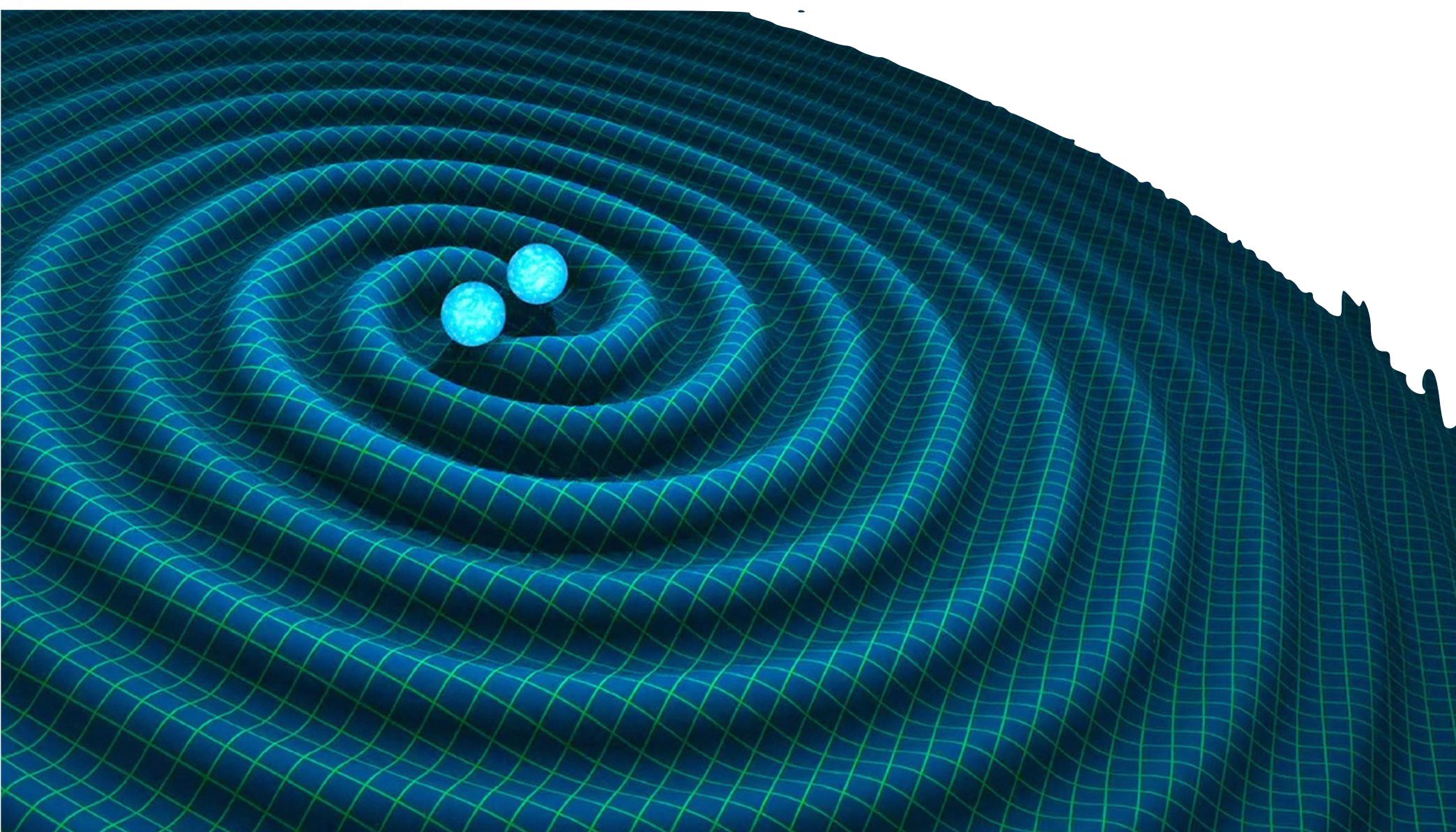


Credit: LIGO/Caltech/Sonoma State (Aurore

LIGO/Virgo/Northwestern Univ.  
(Frank Elavsky, Aaron Geller)

# Gravitational Waves

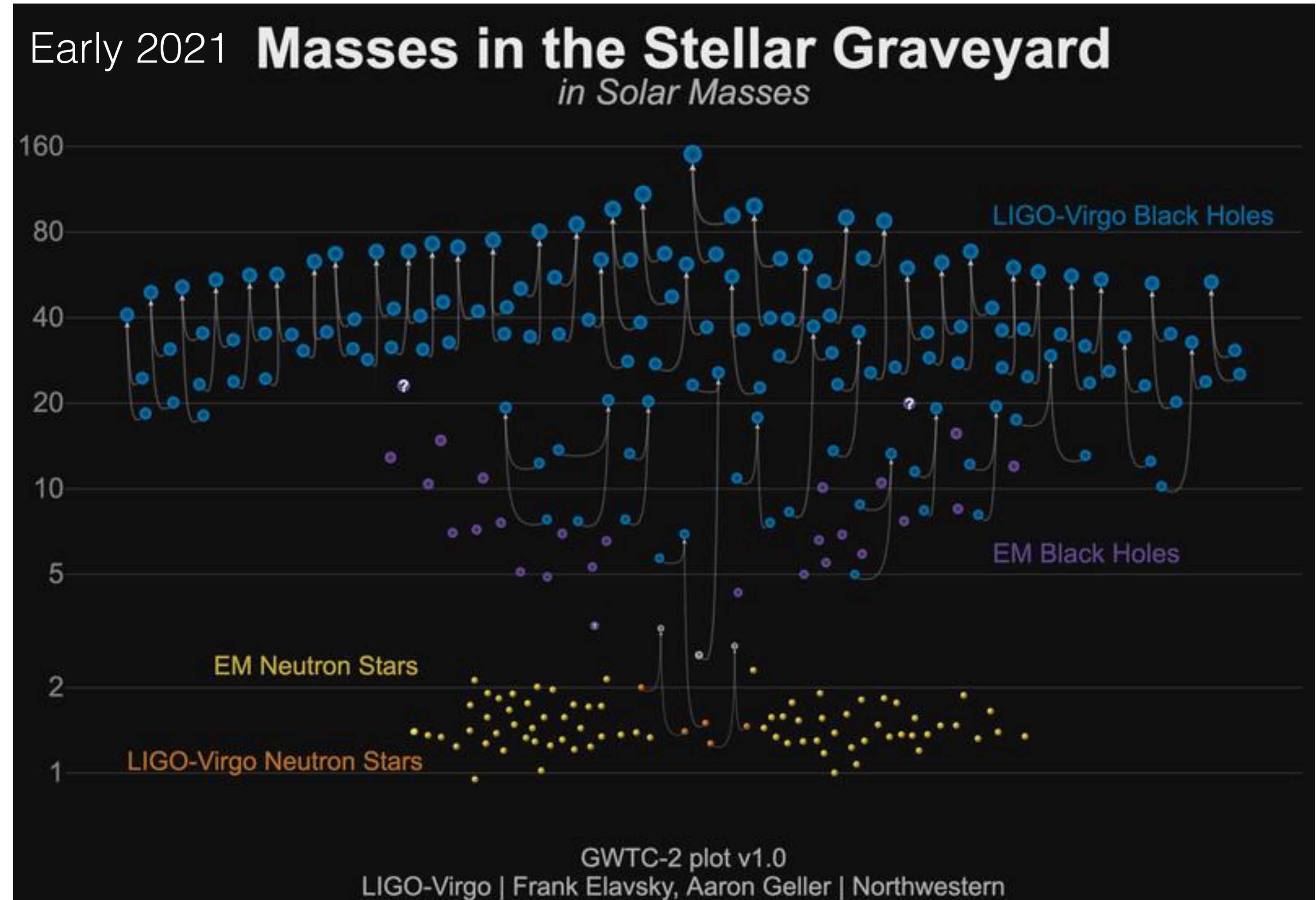
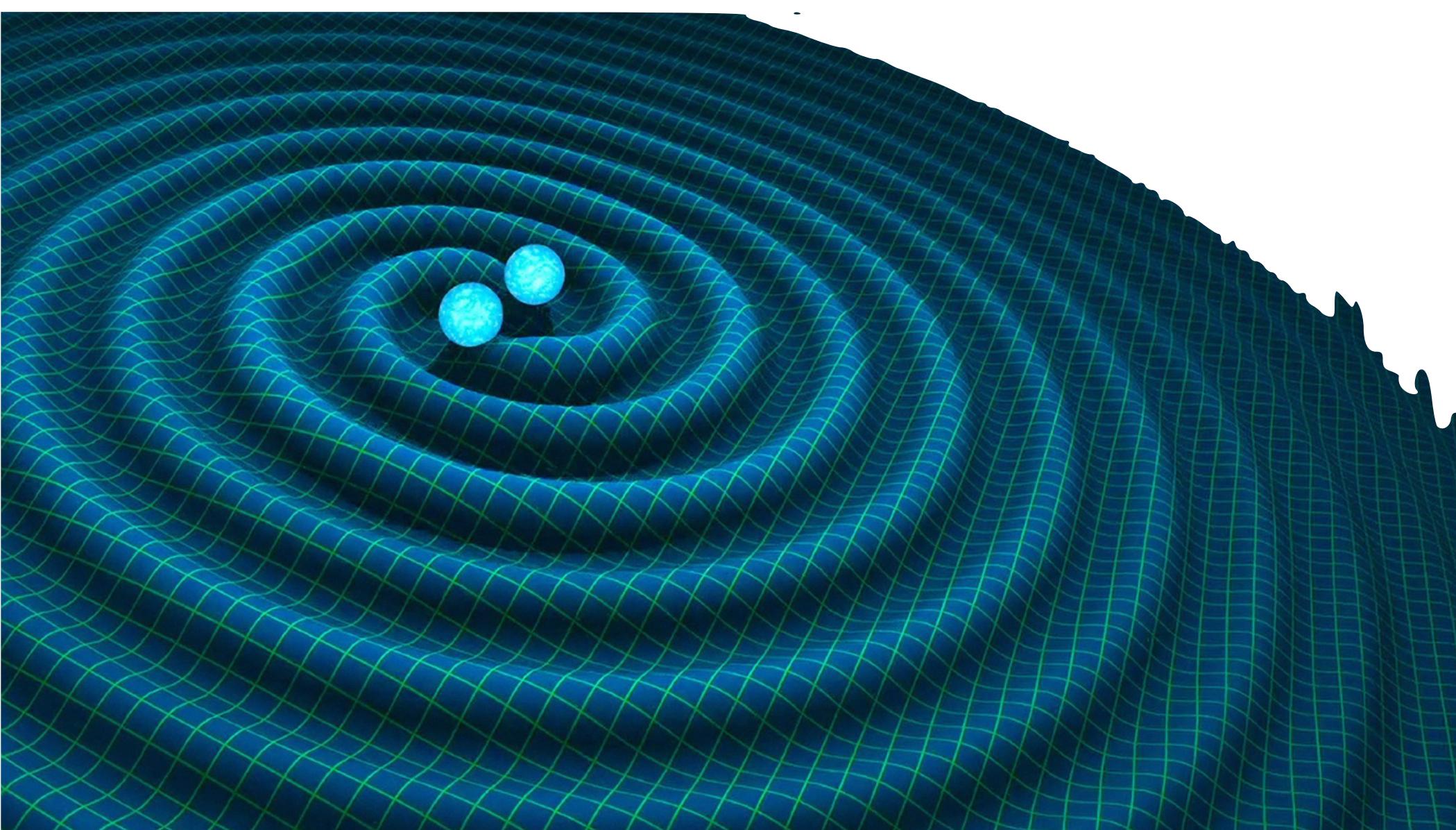
R. HURT / CALTECH-JPL /  
HANDOUT/ ESA



LIGO/Virgo/Northwestern Univ.  
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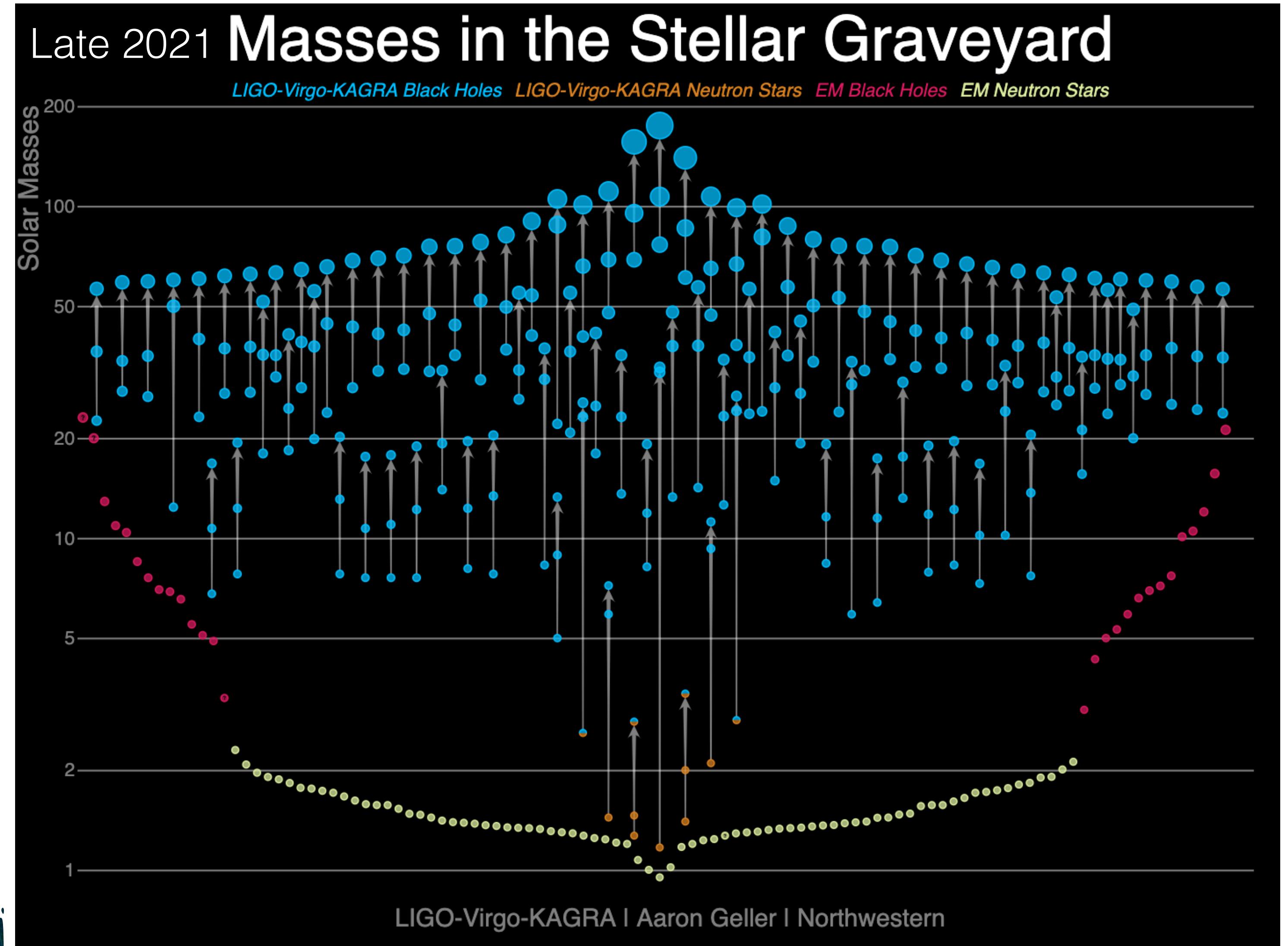
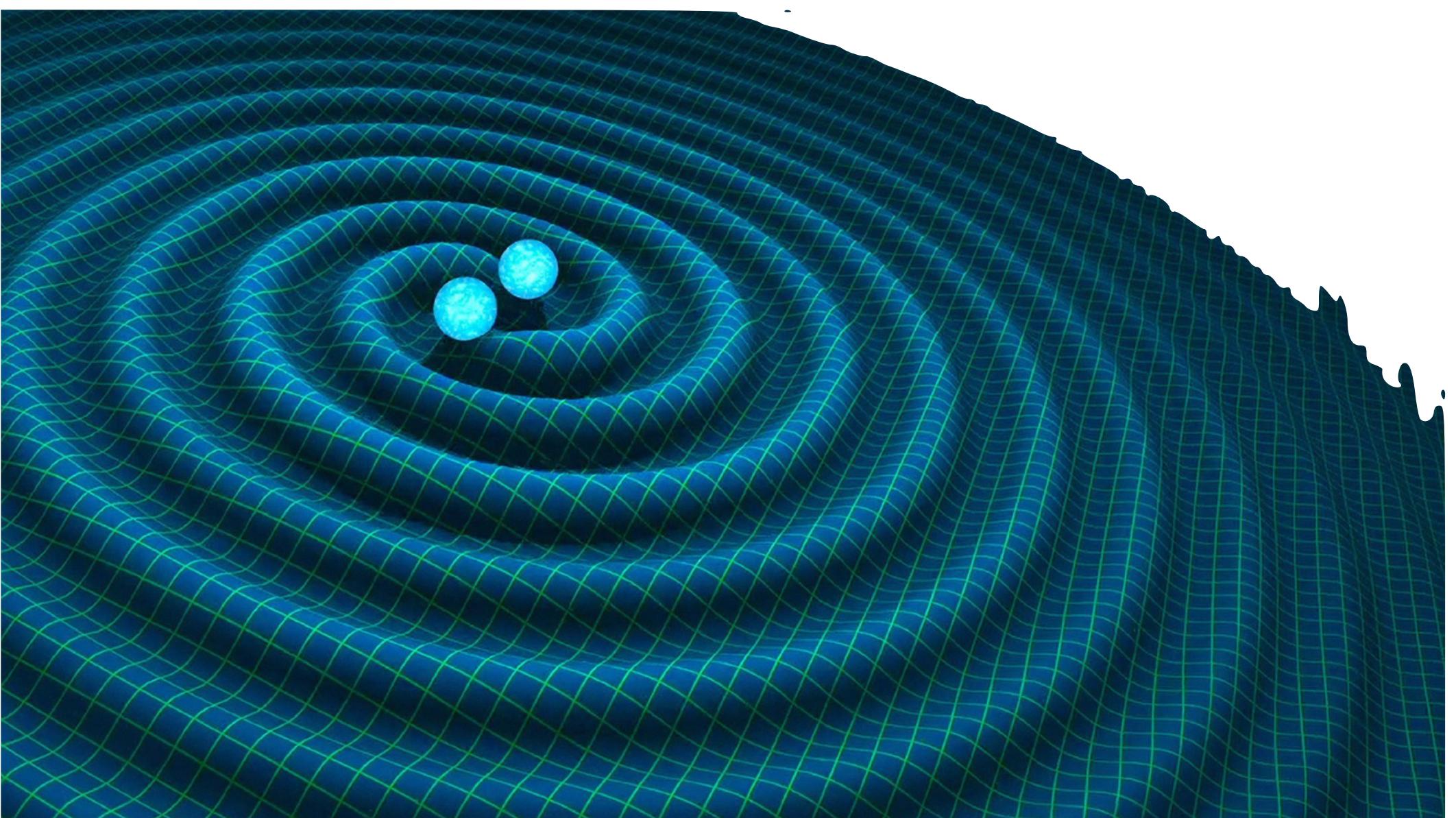
R. HURT / CALTECH-JPL /  
HANDOUT/ ESA



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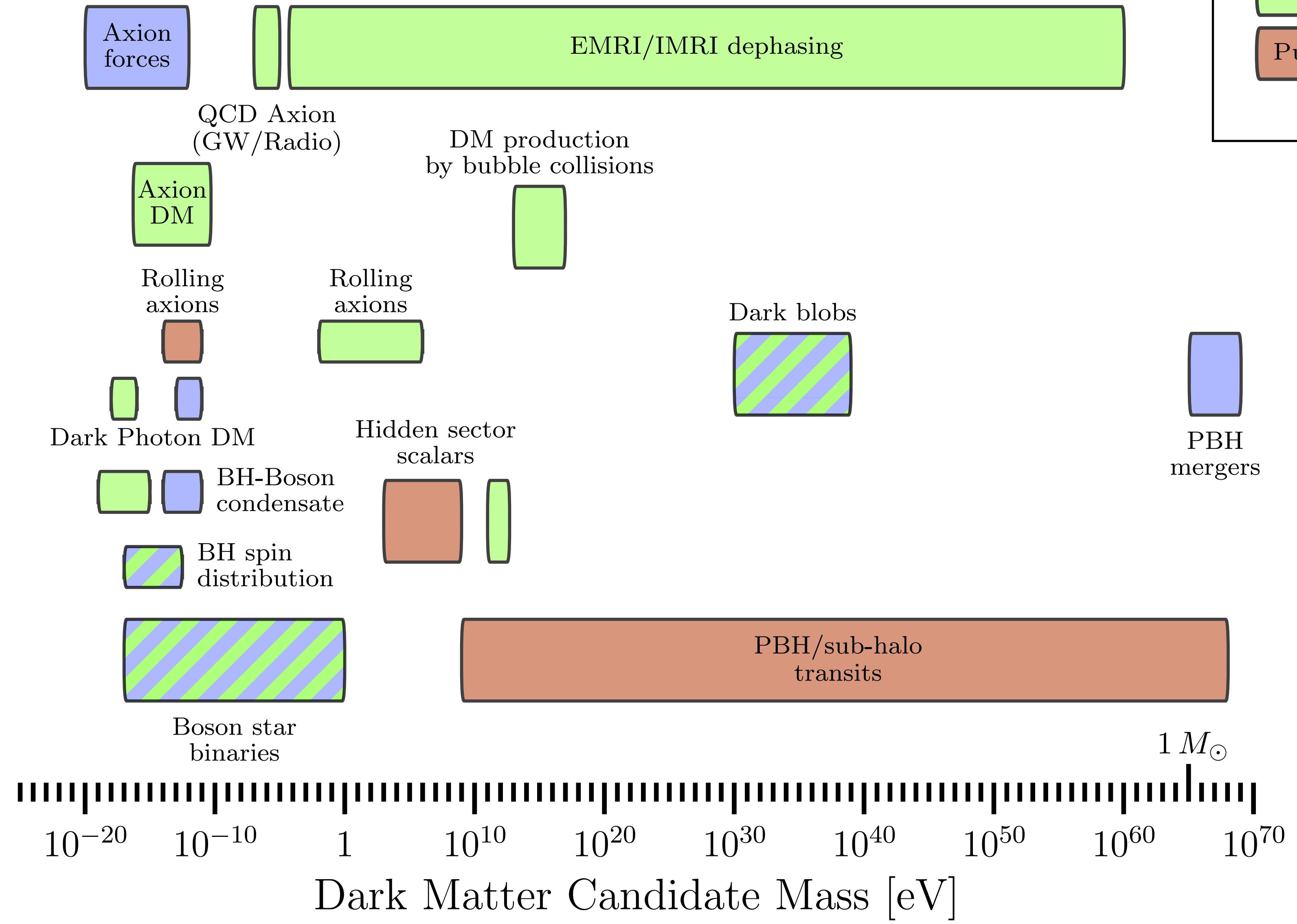
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HANDOUT/ ESA

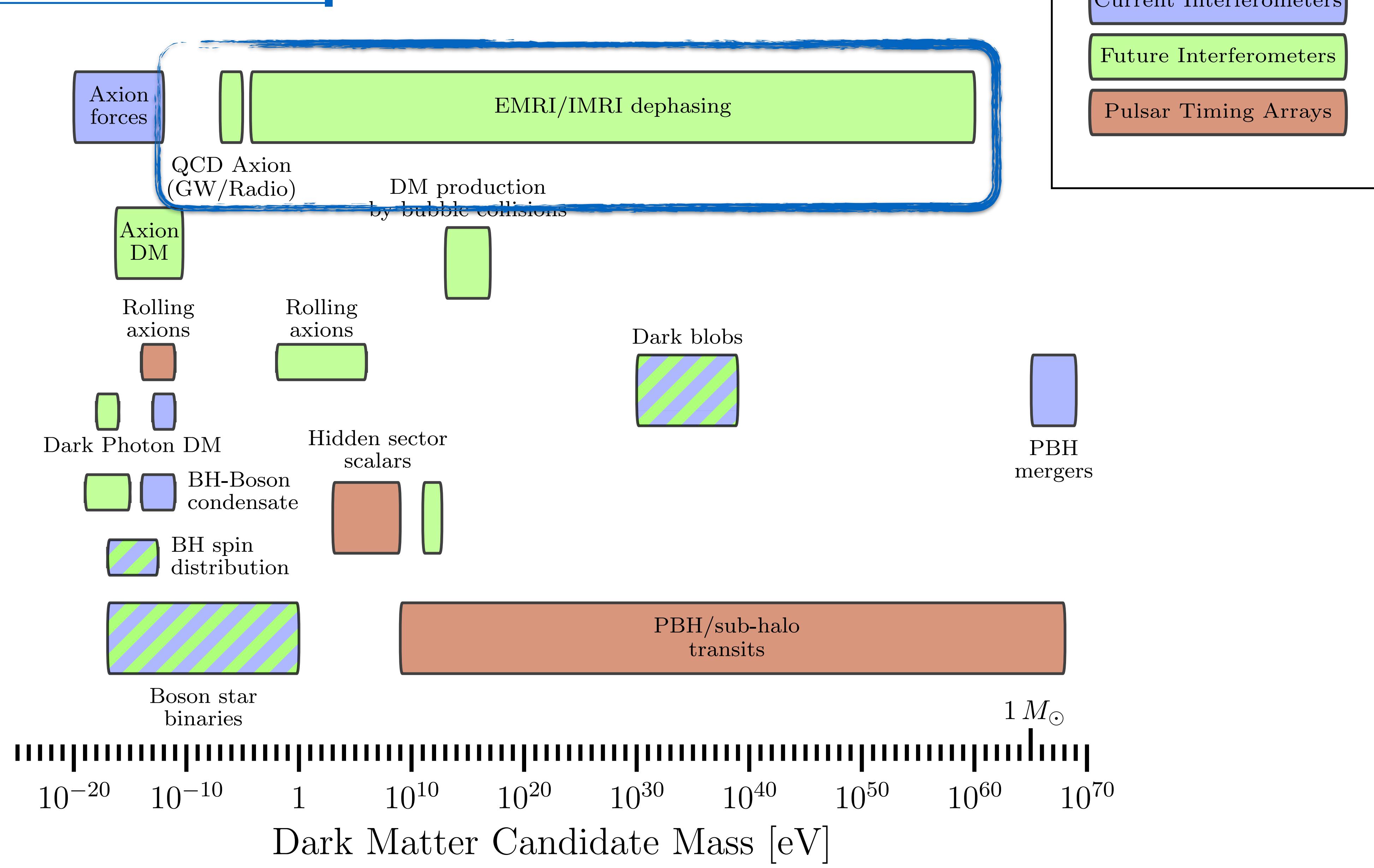


LIGO/Virgo/Northwestern Univ.  
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# GW probes of DM



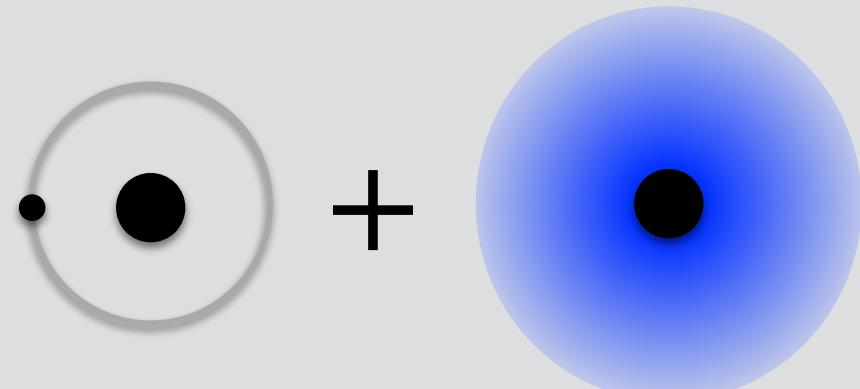
# GW probes of DM



# Overview

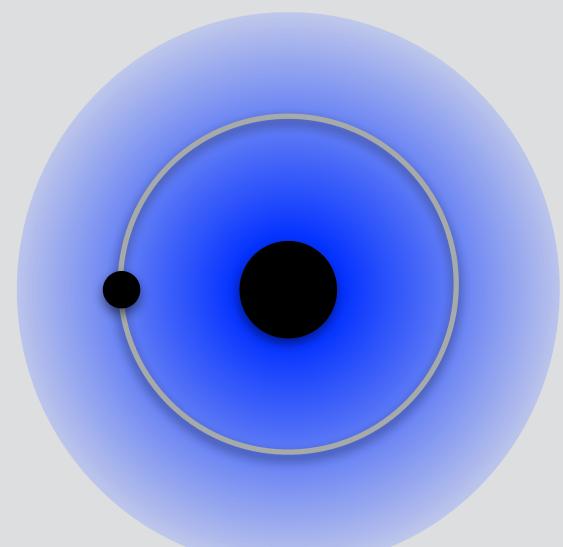
## Intermediate Mass-Ratio Inspirals (IMRIs) and Dark Matter spikes

[Eda et al, [1301.5971](#), and others]



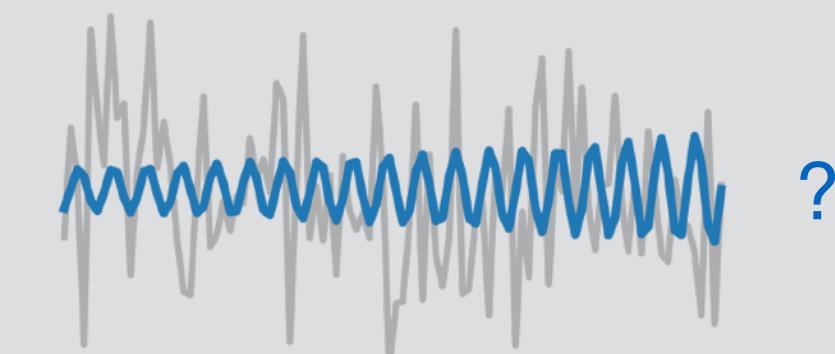
## Dark Matter ‘de-phasing’ revisited

[BJK, Nichols, Gaggero, Bertone, [2002.12811](#)]



## Measuring Dark Matter around Black Holes

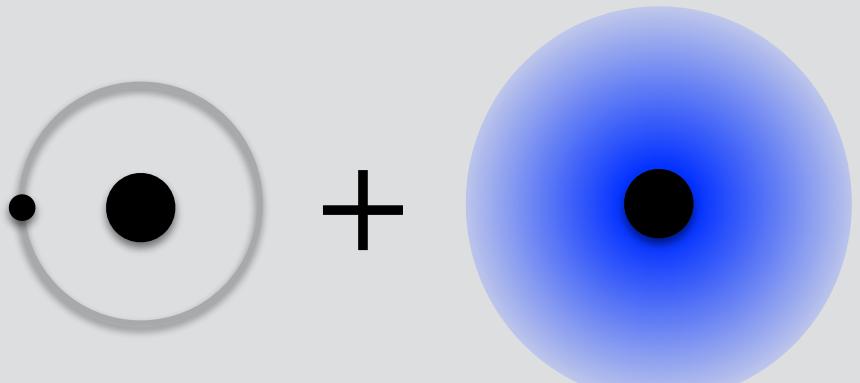
[Coogan, Bertone, Gaggero, BJK & Nichols, [2108.04154](#)]



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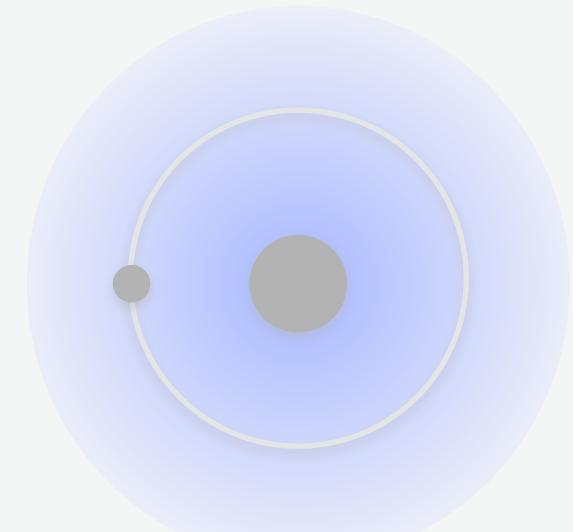
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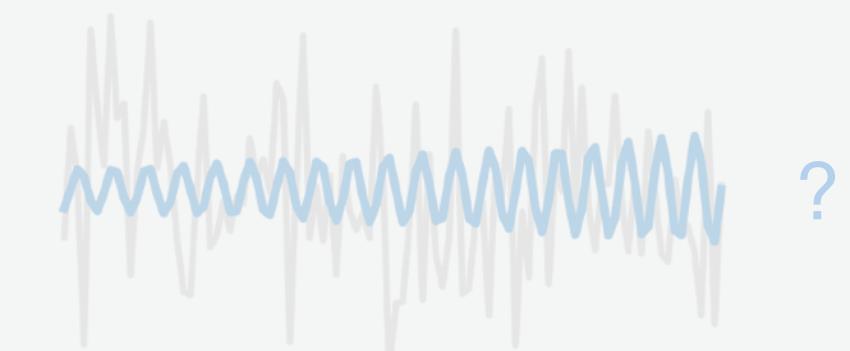
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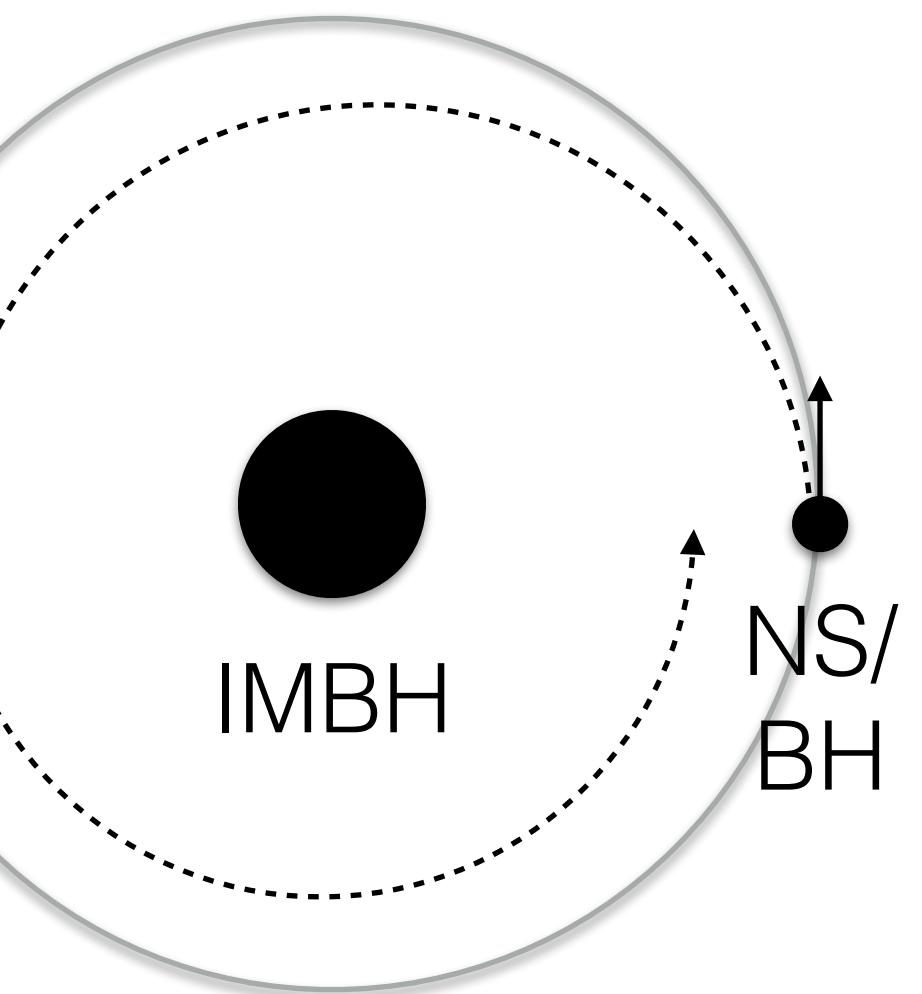
# Intermediate Mass Ratio Inspiral (IMRI)

Stellar mass compact object (NS/BH) inspirals towards intermediate mass black hole (IMBH)

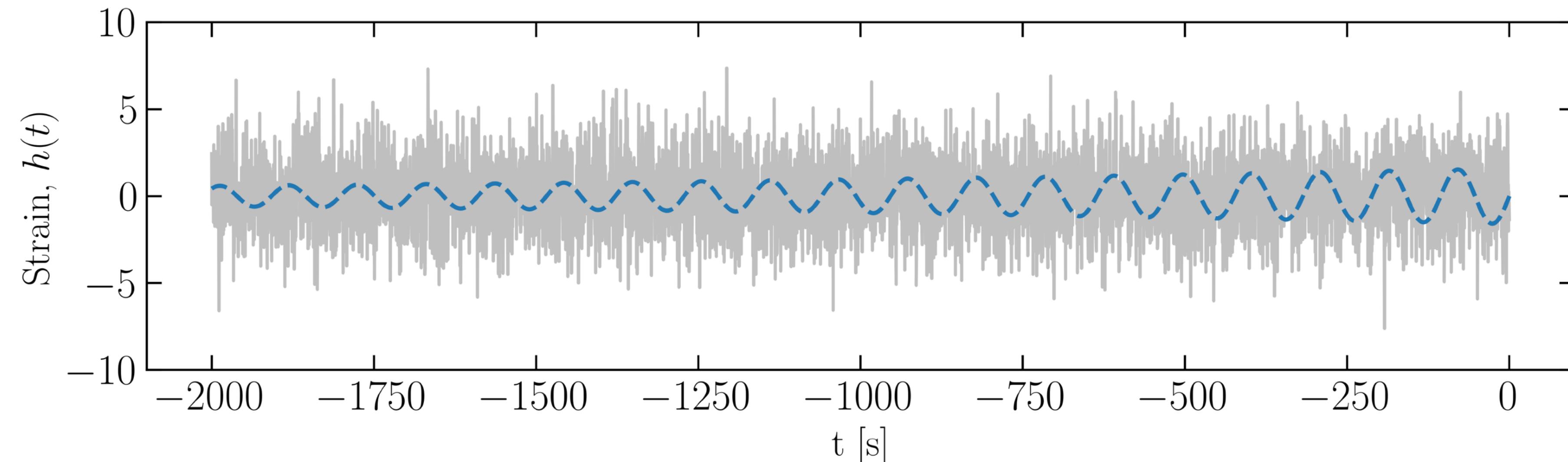
$$M_{\text{IMBH}} \sim 10^3 - 10^5 M_\odot$$

GW emission causes long, slow inspiral:

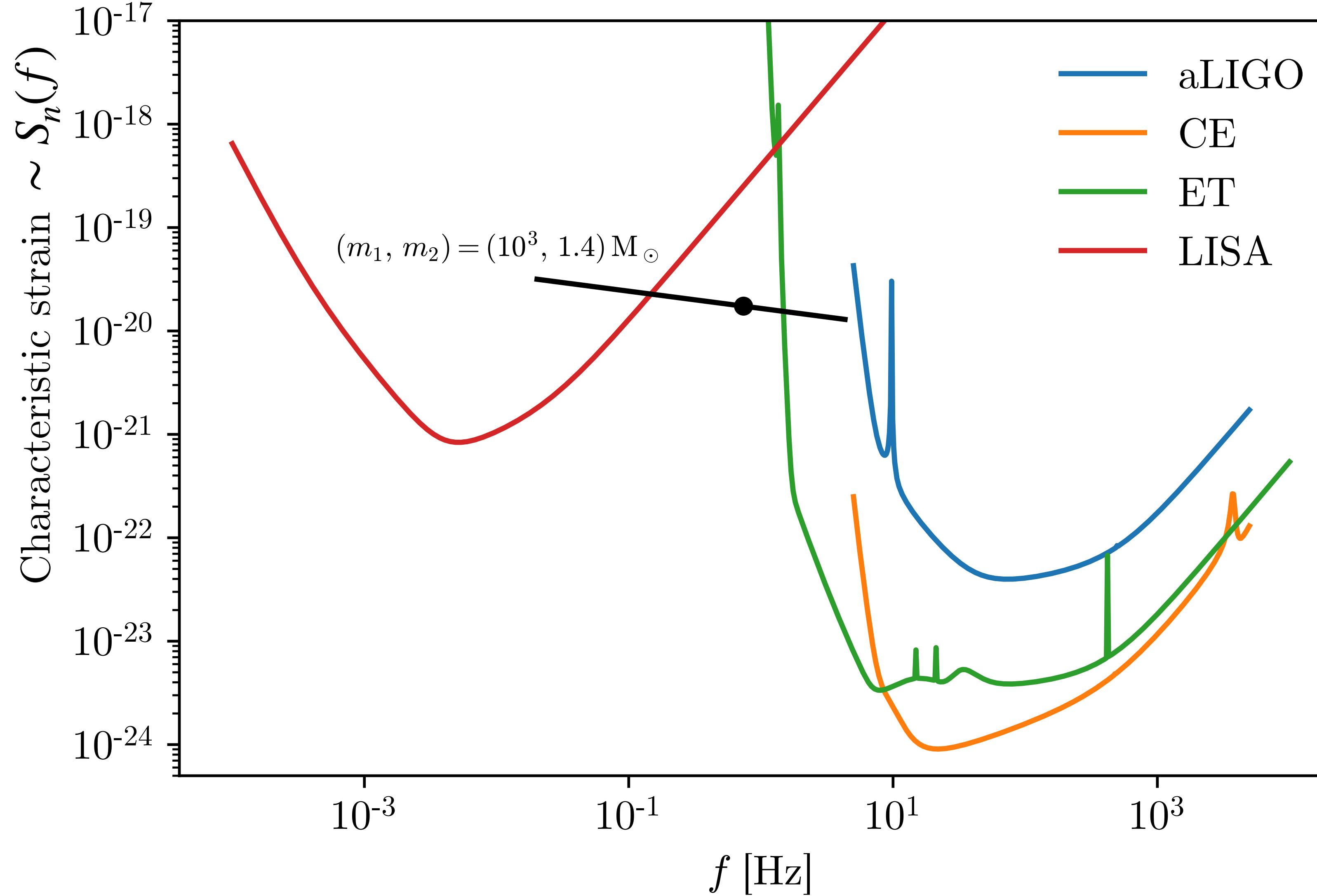
$$\dot{E}_{\text{GW}} \approx \frac{32G^4}{5c^5} \frac{M_{\text{IMBH}}^3 M_{\text{NS}}^2}{r^5} \propto (f_{\text{GW}})^{10/3}$$



$$M_{\text{NS/BH}} \sim 1 M_\odot$$



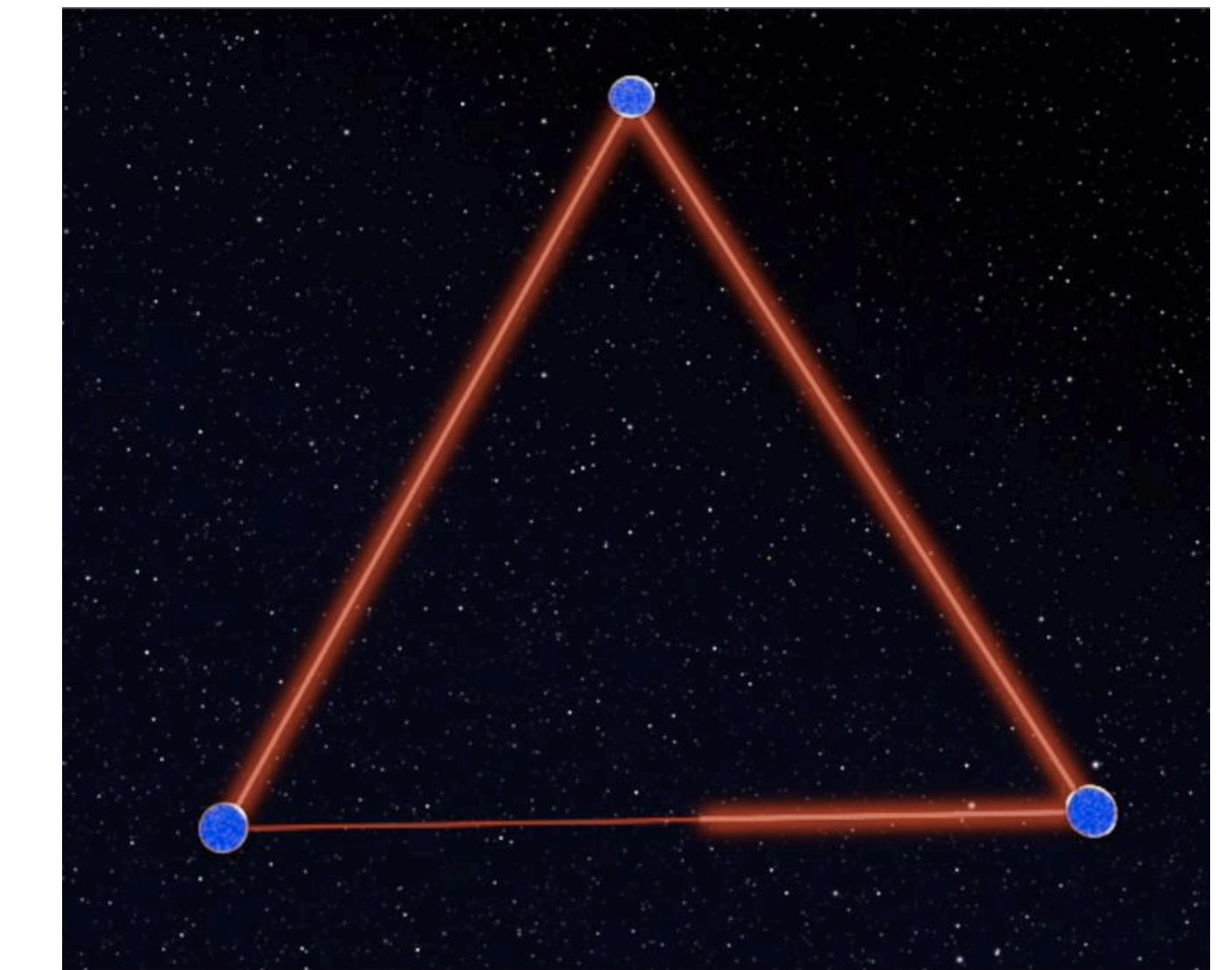
# LISA: GWs in Space



**Laser Interferometer Space Antenna**  
(planned for the 2030s)

[1702.00786]

© AEI / MM / exozet

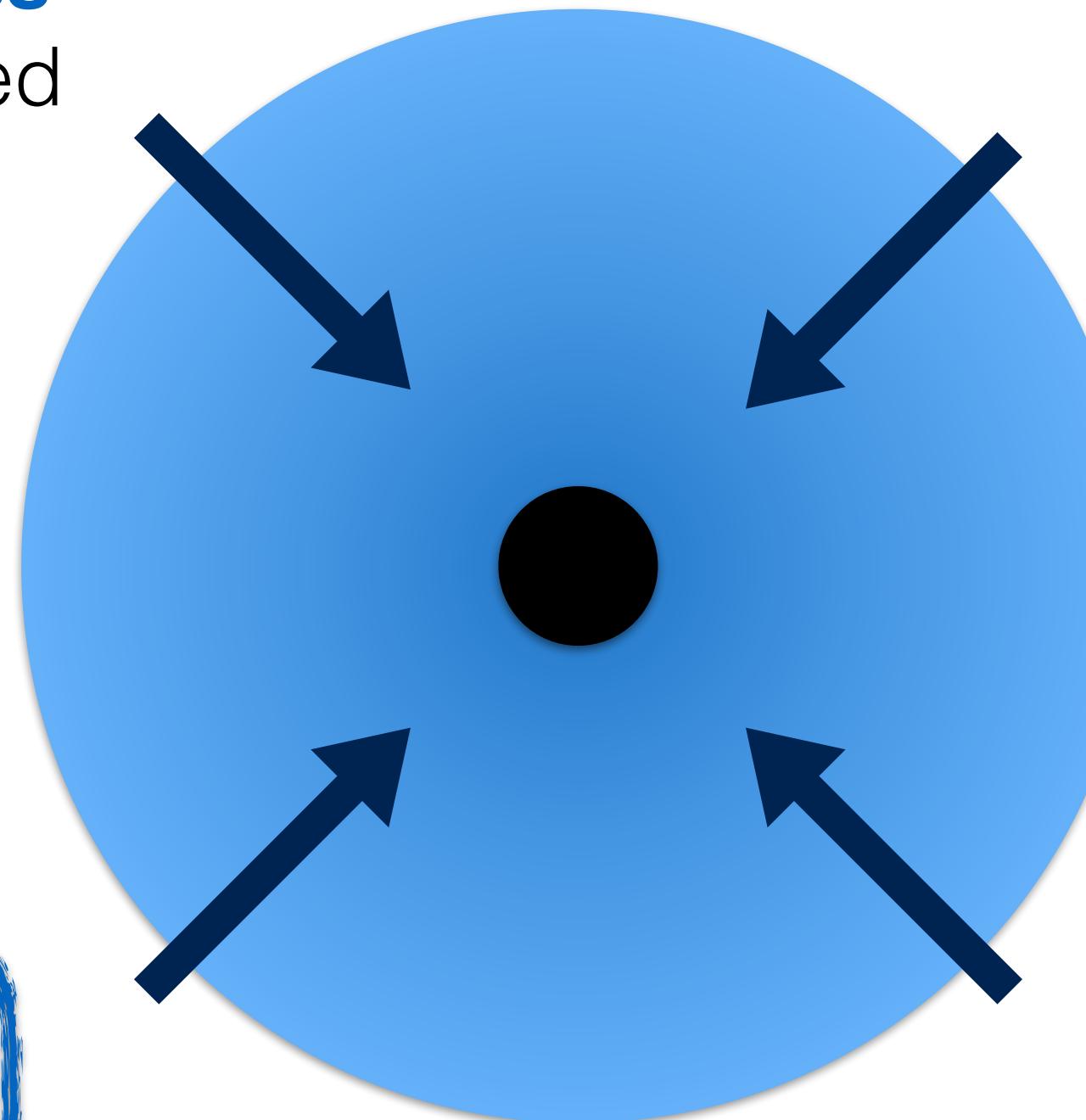


LISA could detect  
~ 3 - 10 IMRIs per year

[1711.00483]

# Dark Matter Spikes

Consider now a cold **DM ‘spike’** or **‘dress’** around the central BH (not to be confused with ultralight boson clouds).



Study the following benchmarks:

$$m_1 = 10^3 M_\odot$$

$$m_2 = 1 M_\odot$$

$$\rho_{\text{DM}} = \rho_6 \left( \frac{10^{-6} \text{ pc}}{r} \right)^{\gamma_{\text{sp}}}$$

## Astrophysical scenario

$$\gamma_{\text{sp}} = 7/3 \approx 2.3333\dots$$

$$\rho_6 \approx 5.45 \times 10^{15} M_\odot \text{ pc}^{-3}$$

...depending on a number of environmental factors...

[[astro-ph/9906391](#), [astro-ph/0509565](#),  
[1305.2619](#), ...]

## PBH scenario

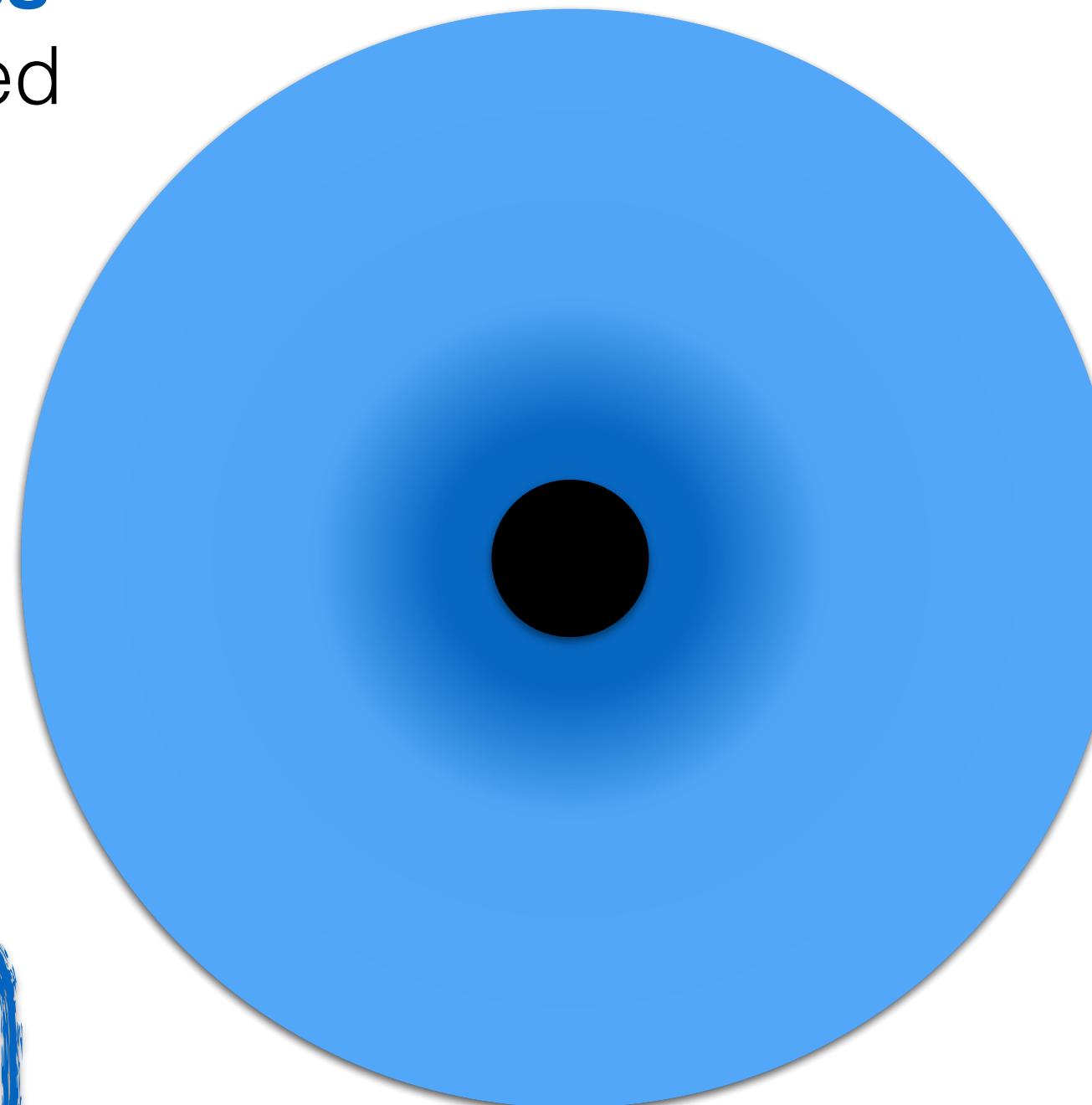
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[[Bertschinger \(1985\)](#), [astro-ph/0608642](#),  
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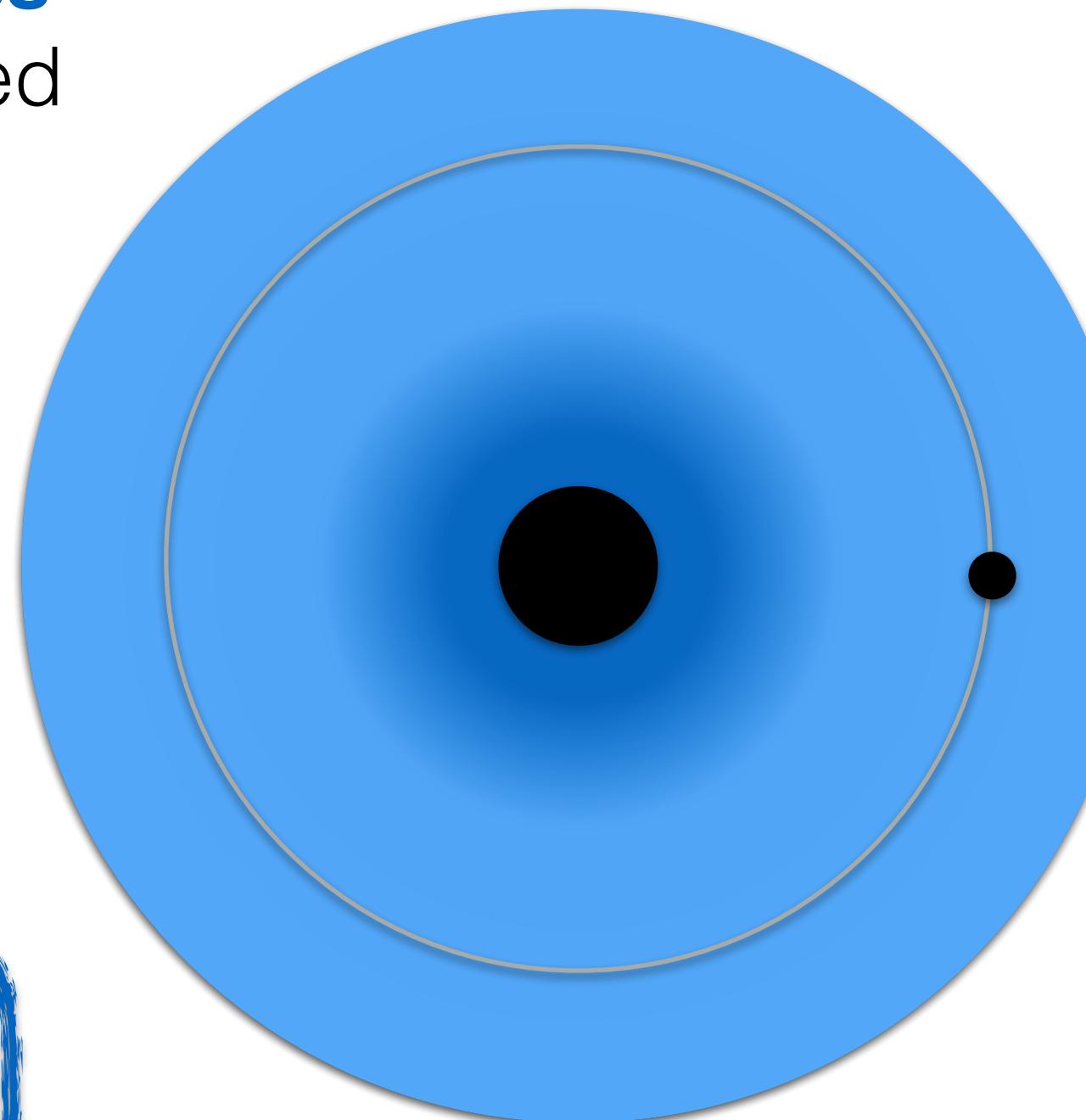
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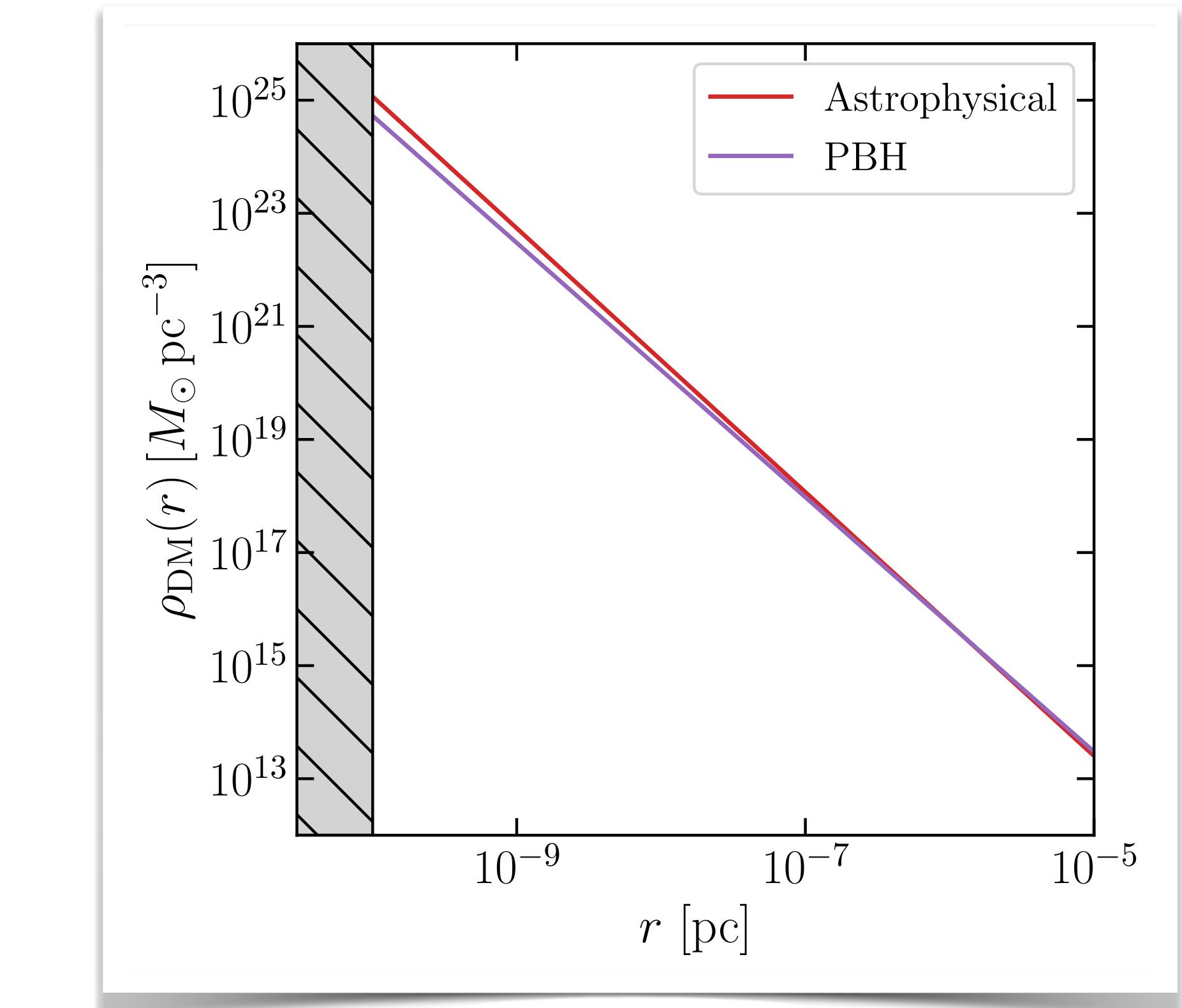
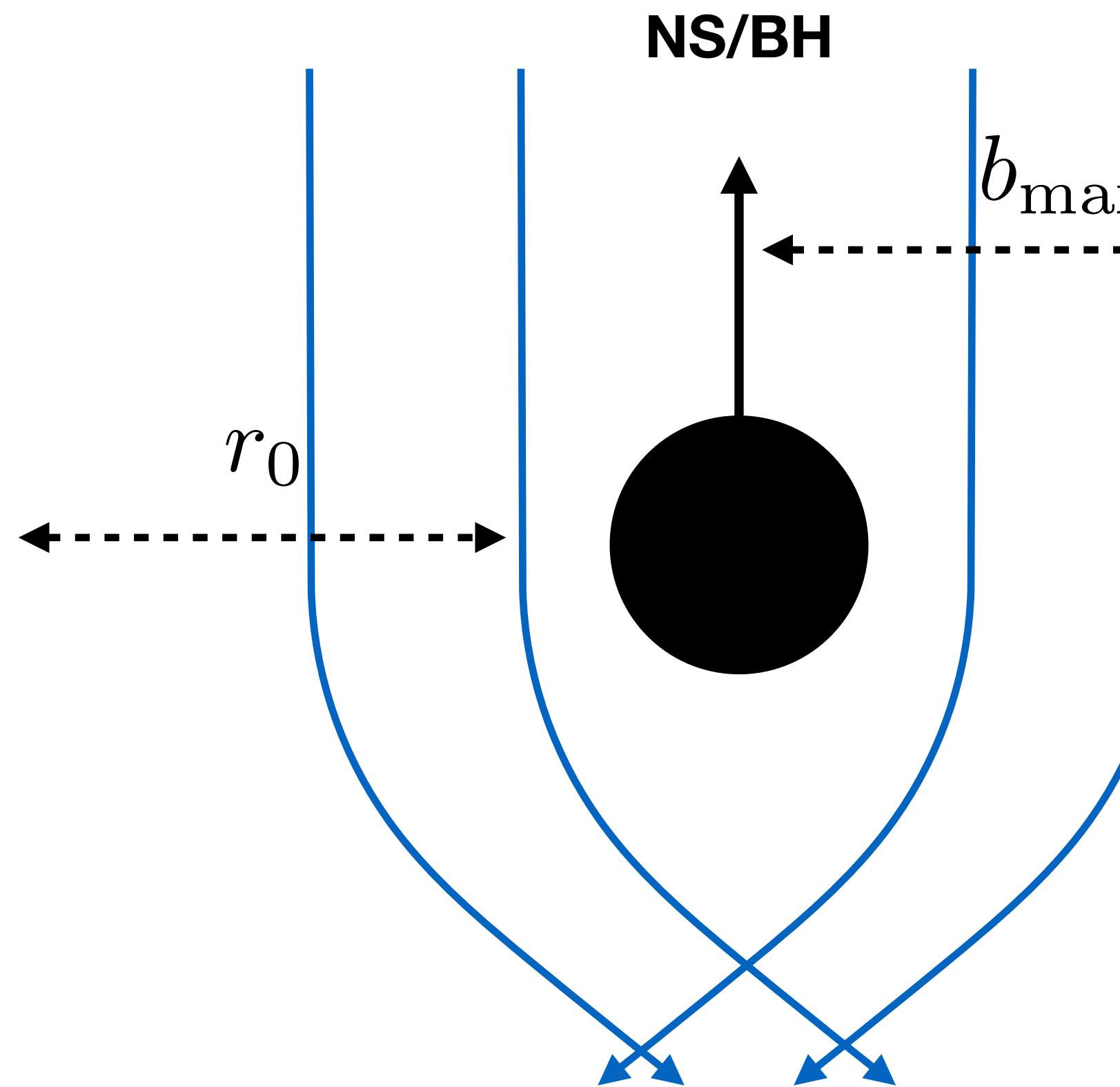
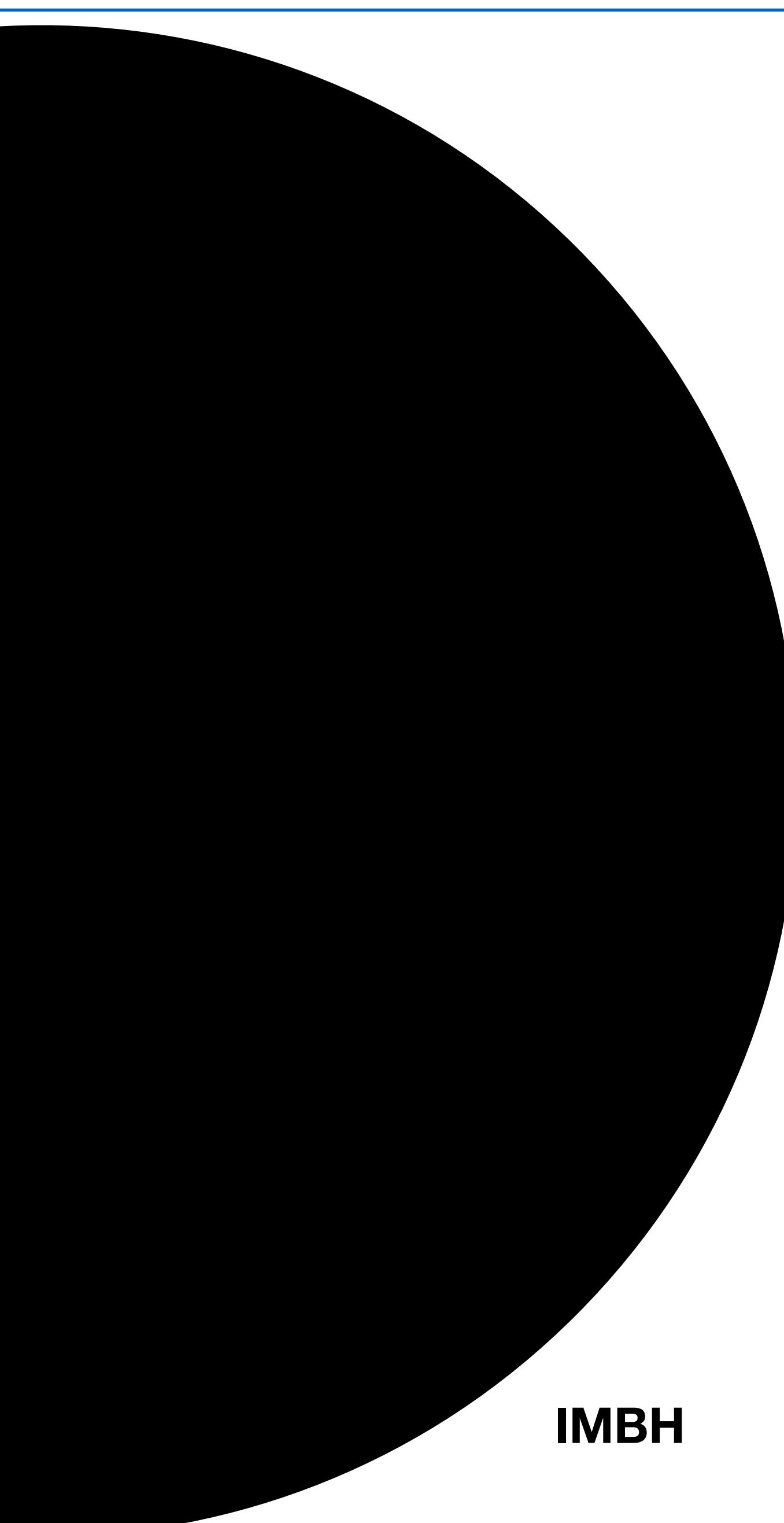
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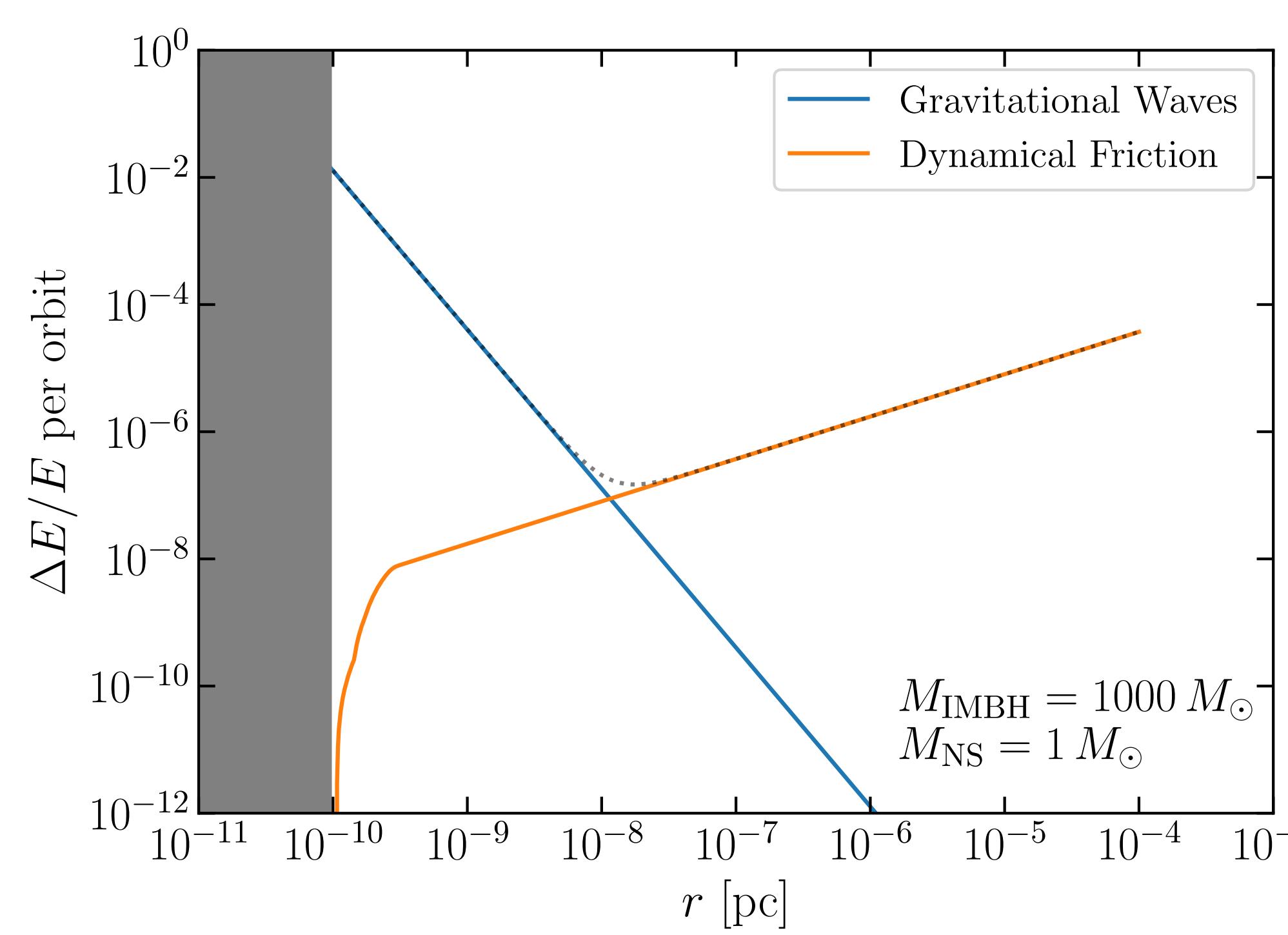
# Dynamical Friction

[Chandrasekhar, 1943]

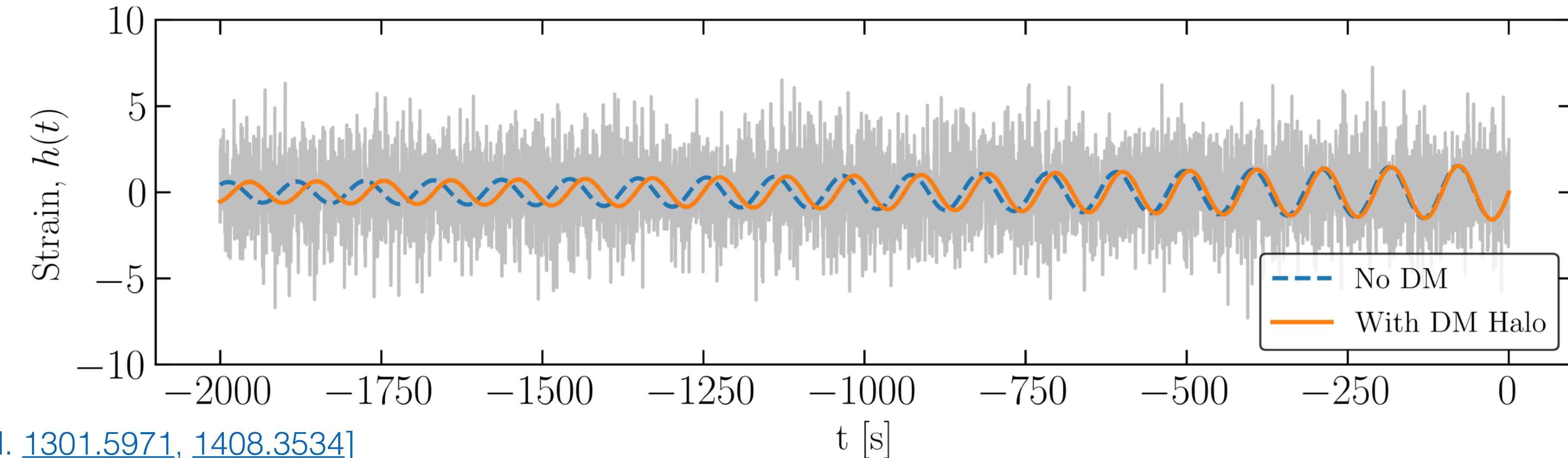
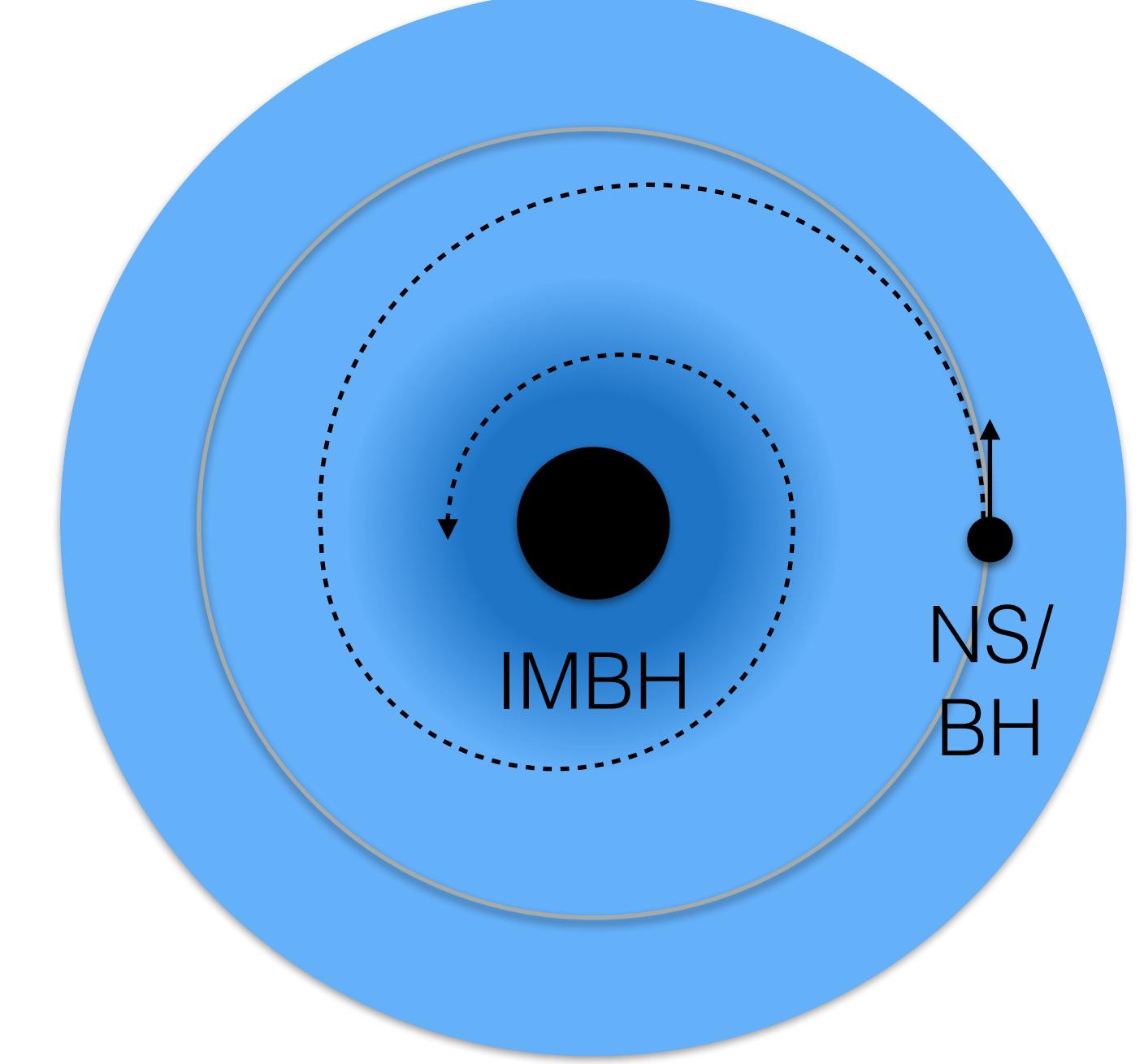
$$\dot{E}_{\text{DF}} \sim \frac{4\pi G^2 M_{\text{NS}}^2 \xi(v) \rho_{\text{DM}}(r)}{v_{\text{NS}}} \ln \Lambda \propto (f_{\text{GW}})^{\frac{2}{3}\gamma-3}$$



# IMRI + Dark Matter



$$-\dot{E}_{\text{orb}} = \dot{E}_{\text{GW}} + \dot{E}_{\text{DF}}$$

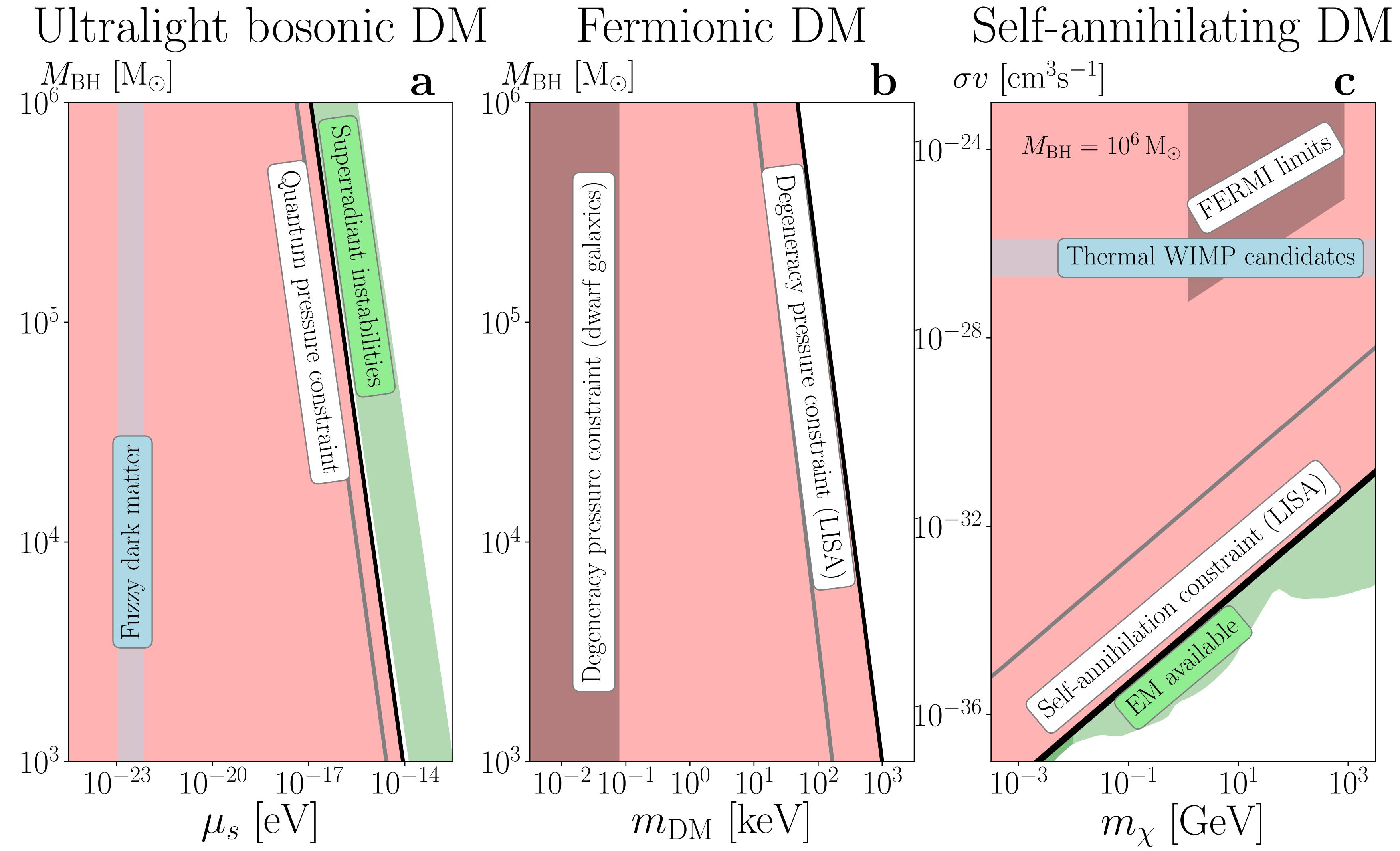


[See e.g. Eda et al. [1301.5971](#), [1408.3534](#)]

# Nature of Dark Matter

Red regions would be ruled out by observation of a DM spike!

[1906.11845]

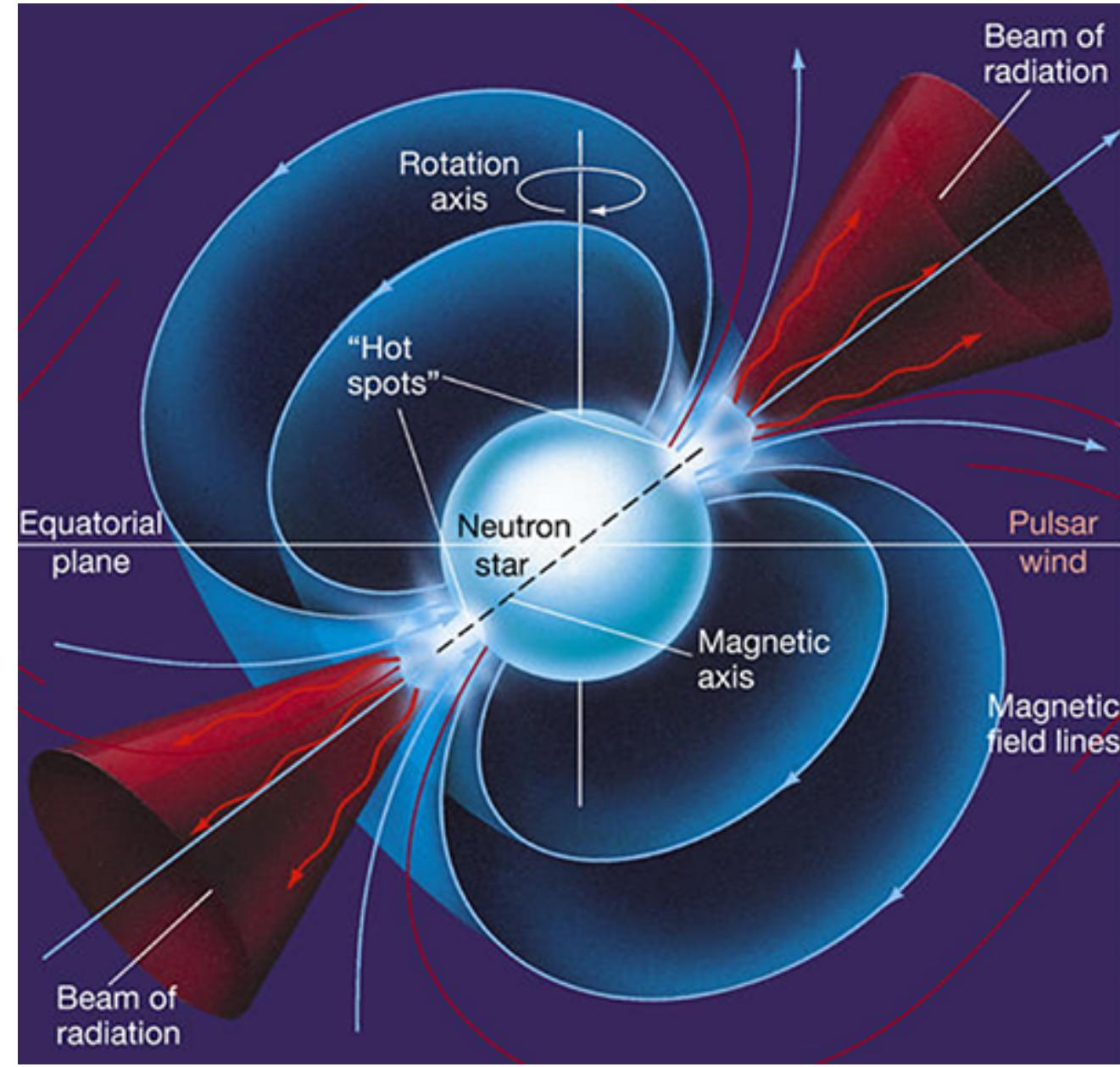


[See also Bertone, Coogan, Gaggero, **BJK** & Weniger, [1905.01238](#)]

# Multimessenger QCD axions

[Edwards, Chianese, **BJK**, Nissanke & Weniger, [1905.04686](#)]

© 2005 Pearson Prentice Hall, Inc

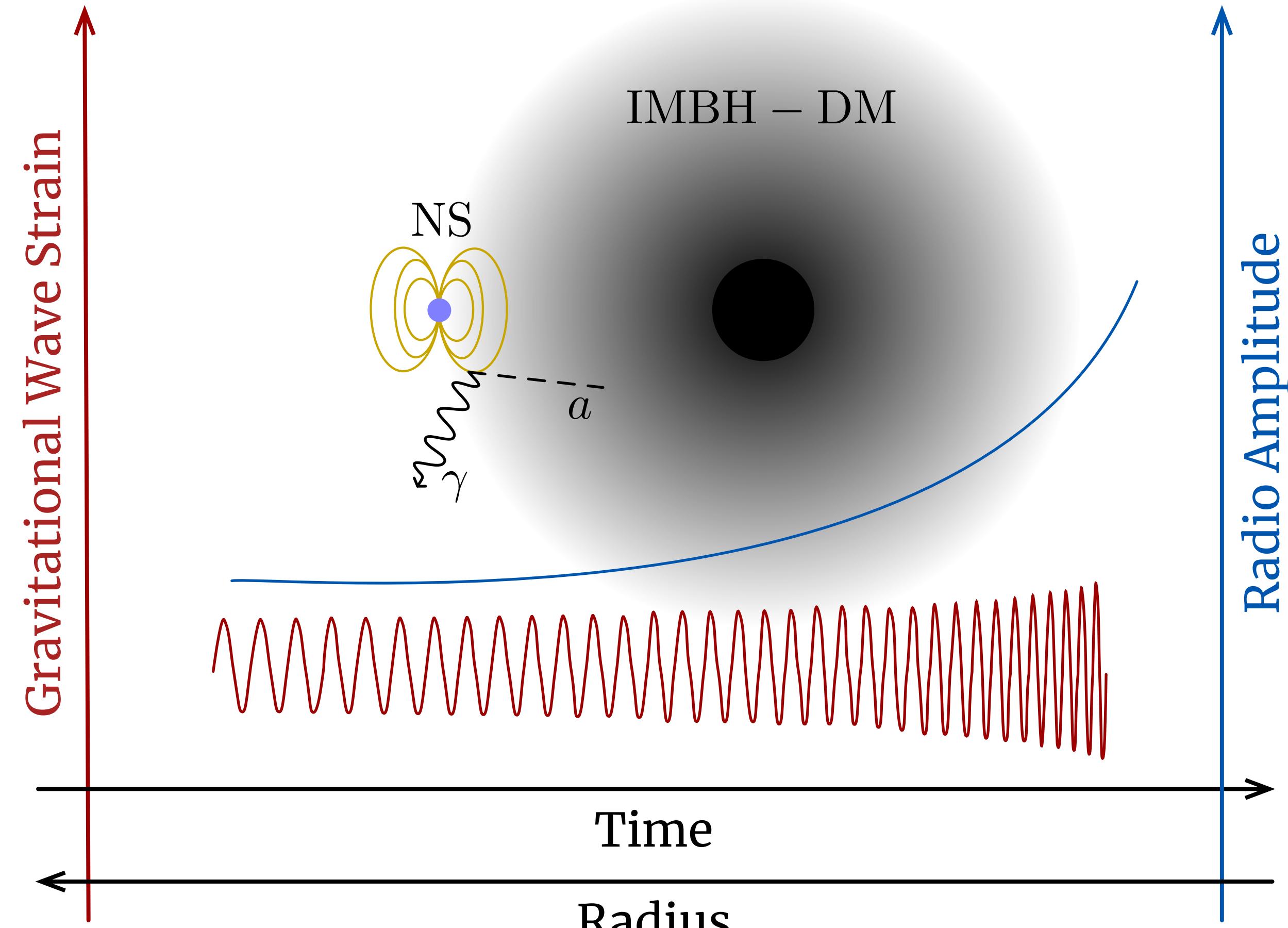


Old neutron stars can have extremely high magnetic fields:  $B_0 = 10^{12} - 10^{15} \text{ G}$

Surrounded by a dense plasma which allows 'resonant' conversion when axion mass matches plasma mass:

$$\omega_p(B_0, P) = m_a/2\pi$$

[\[1803.08230\]](#), [\[1804.03145\]](#),  
[\[1811.01020\]](#), [\[1910.11907\]](#)



Future radio observations should be able to probe QCD axion DM in the range  $10^{-7} - 10^{-5} \text{ eV}$ , while LISA would constrain the DM density close to the IMBH!

# 'De-phasing' signal

Consider our astrophysical benchmark system, starting at some initial separation:

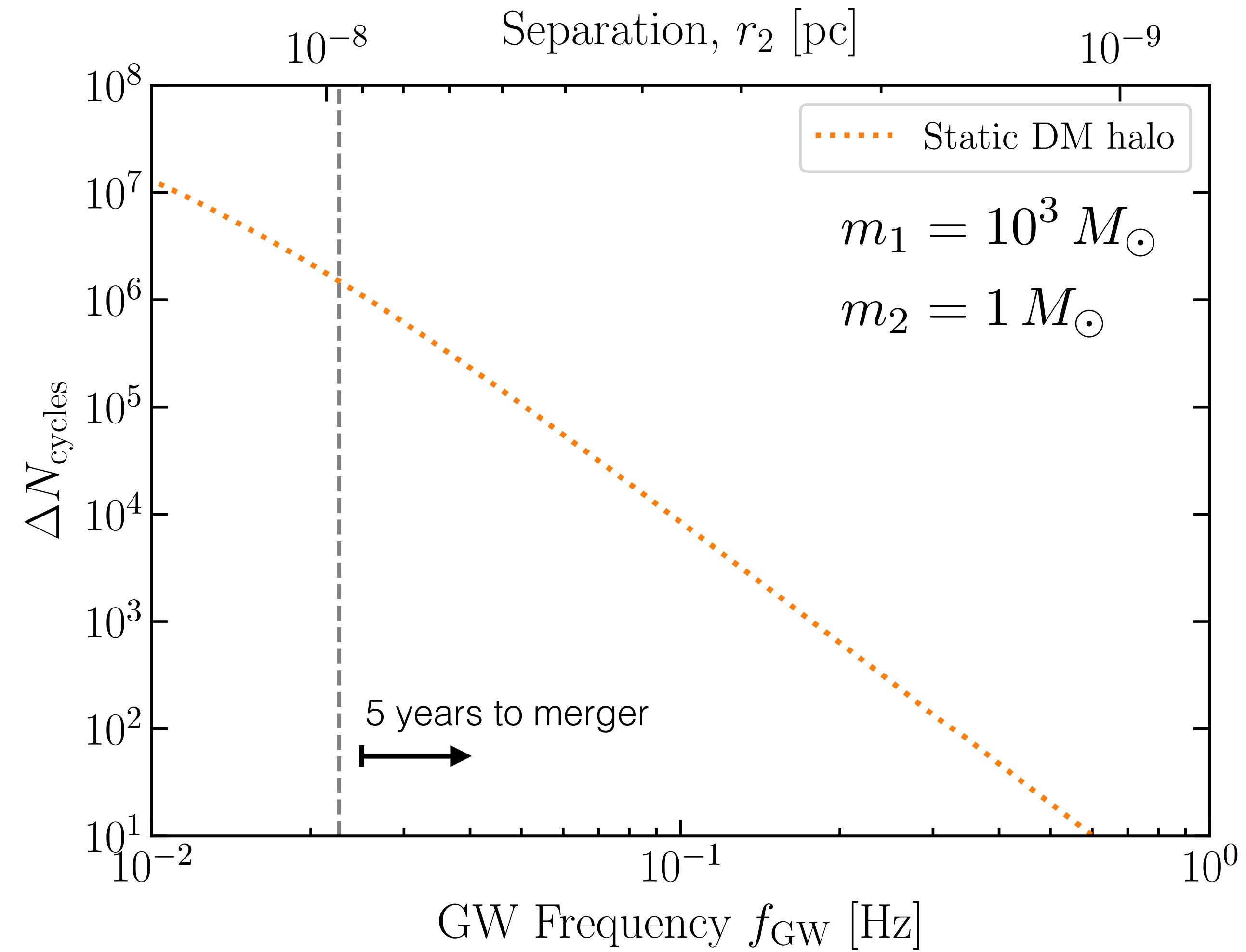
$$r_{\text{ini}} \sim 10^{-8} \text{ pc}$$



$$t_{\text{merge}}^{\text{vacuum}} \sim 5 \text{ yr}$$

$$N_{\text{cycles}}^{\text{vacuum}} \sim 6 \times 10^6$$

How does DM affect the number of cycles?



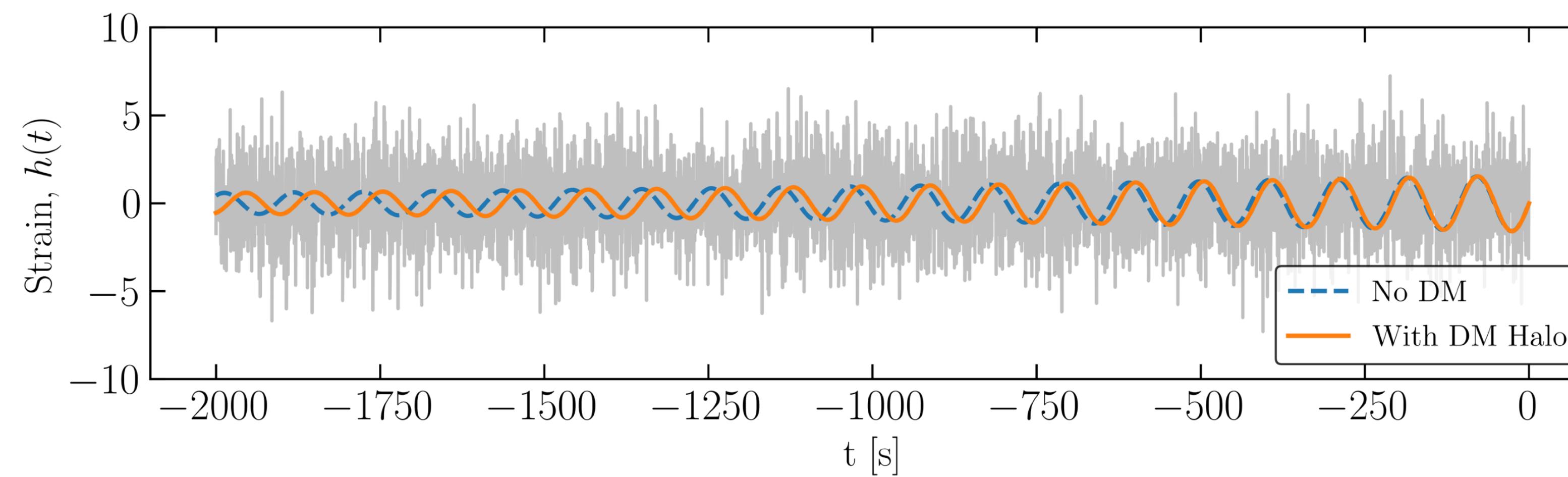
[Eda et al. [1301.5971](#), [1408.3534](#);  
see also [1302.2646](#), [1404.7140](#), [1404.7149](#)]

But, we need to model this signal very carefully...

**INTERMISSION**

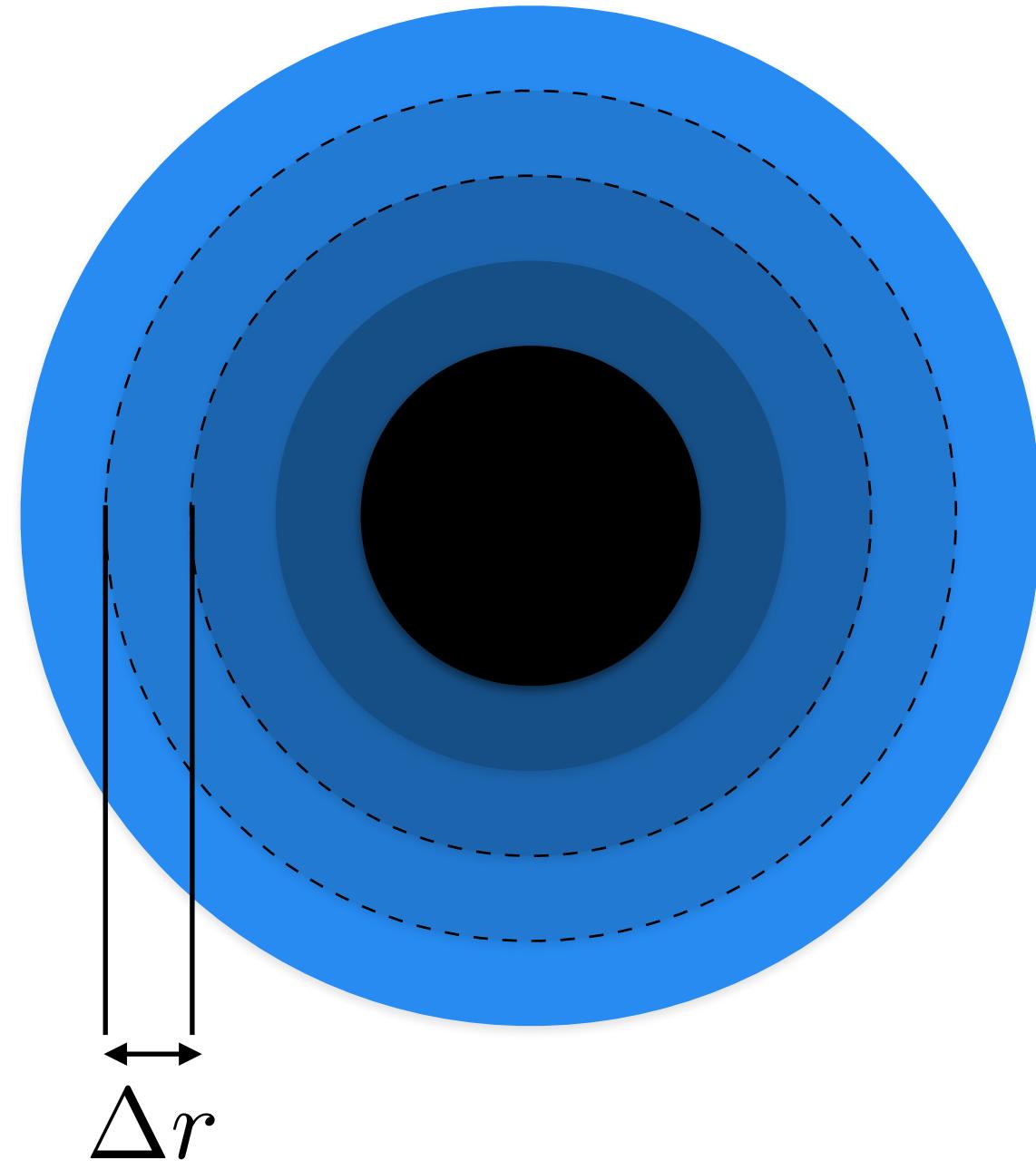
# INTERMISSION



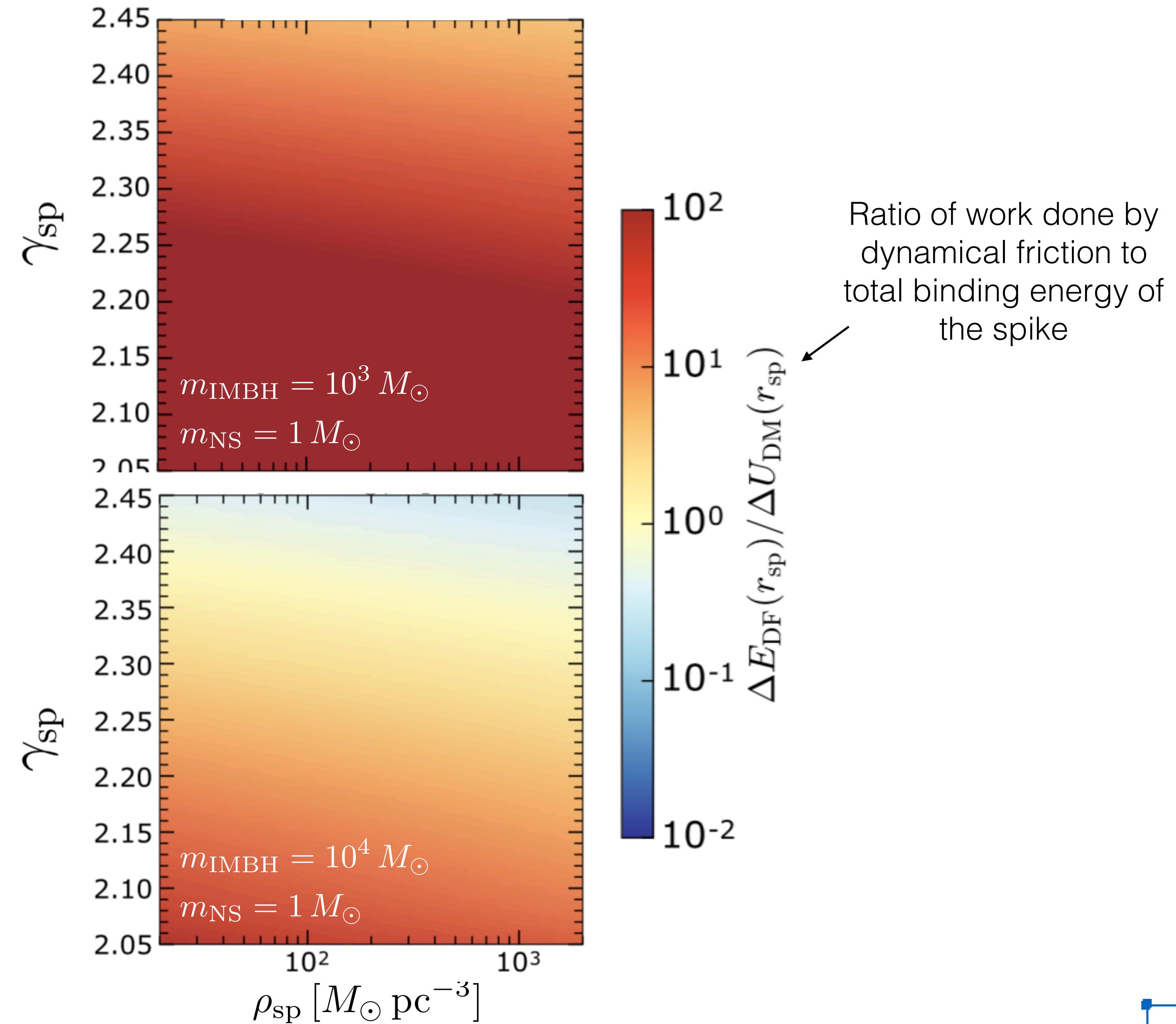


# Energy Budget

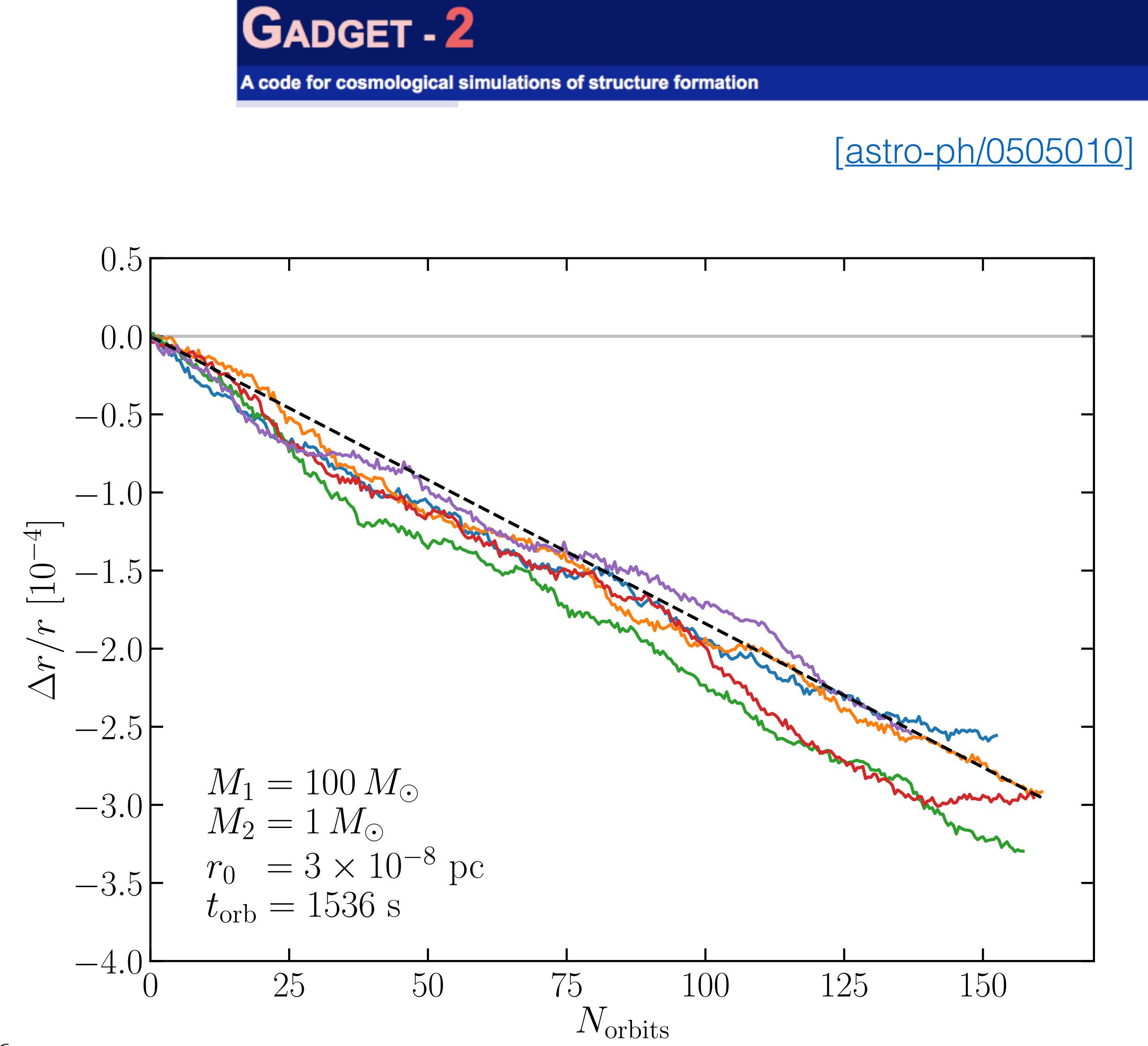
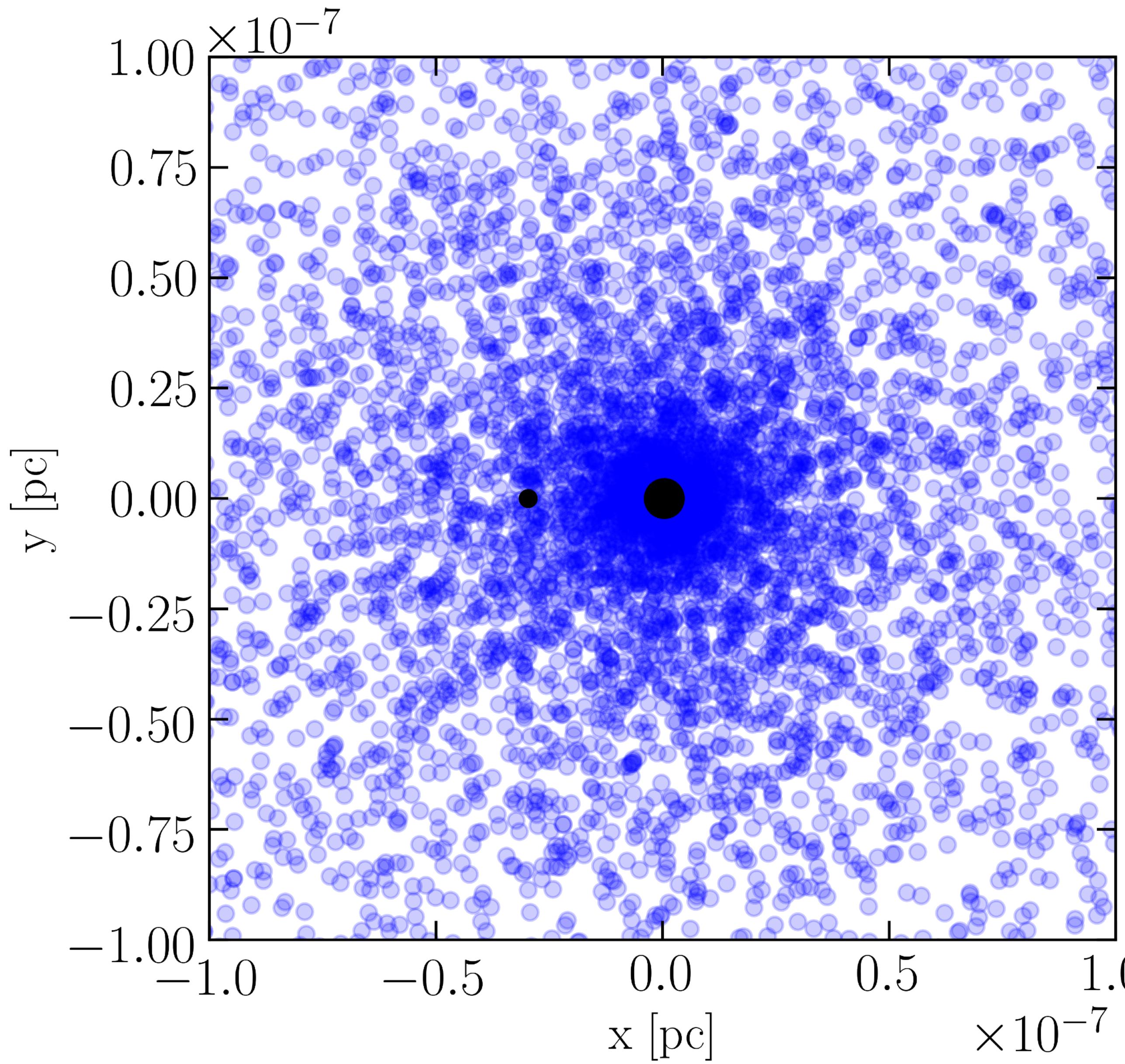
Q: How much energy is *available* for dynamical friction?



A: Binding energy of DM  
 $\Delta U_{\text{DM}}$  over radius  $\Delta r$



# N-body Simulations (?)



[BJK, Nichols, Gaggero, Bertone, 2002.12811]

# Halo Feedback

Follow semi-analytically the phase space distribution of DM:

$$f = \frac{dN}{d^3\mathbf{r} d^3\mathbf{v}} \equiv f(\mathcal{E})$$

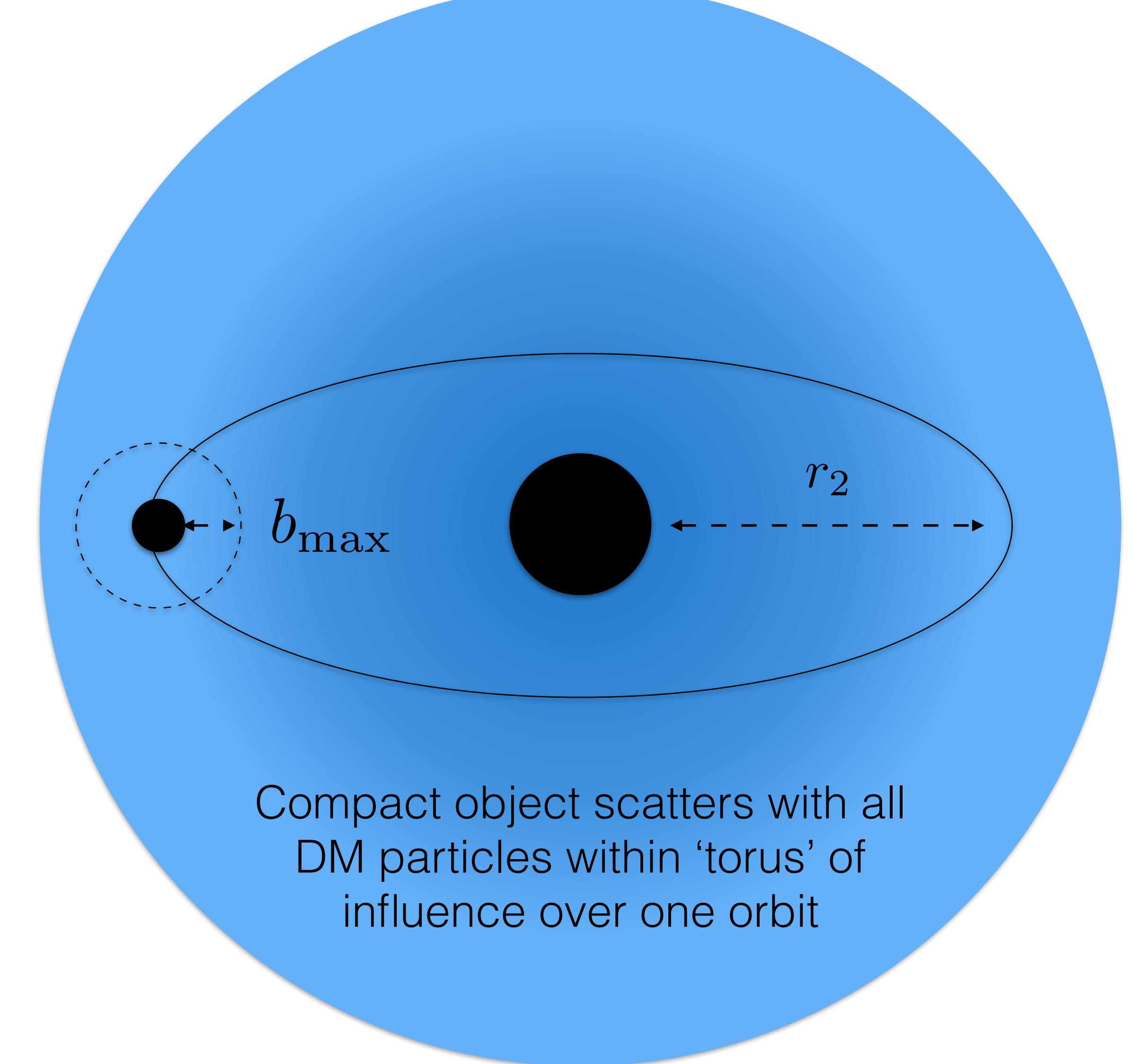
$$\mathcal{E} = \Psi(r) - \frac{1}{2}v^2$$

Each particle receives a ‘kick’ through gravitational scattering

$$\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$$

Reconstruct density from distribution function:

$$\rho(r) = \int d^3\mathbf{v} f(\mathcal{E})$$



[**BJK**, Nichols, Gaggero, Bertone, [2002.12811](#)]

[Code available online:  
[github.com/bradkav/HaloFeedback](https://github.com/bradkav/HaloFeedback)]

# Self-consistent evolution

---

Assuming everything evolves slowly compared to the orbital period:

$$T_{\text{orb}} \frac{df(\mathcal{E})}{dt} = -p_{\mathcal{E}} f(\mathcal{E}) + \int \left( \frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}} \right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E}-\Delta\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

$P_{\mathcal{E}}(\Delta\mathcal{E})$  - probability for a particle with energy  $\mathcal{E}$  to scatter and receive a 'kick'  $\Delta\mathcal{E}$

$$p_{\mathcal{E}} = \int P_{\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

- total probability for a particle with energy  $\mathcal{E}$  to scatter

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Particles scattering from  
 $\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$

Particles scattering from  
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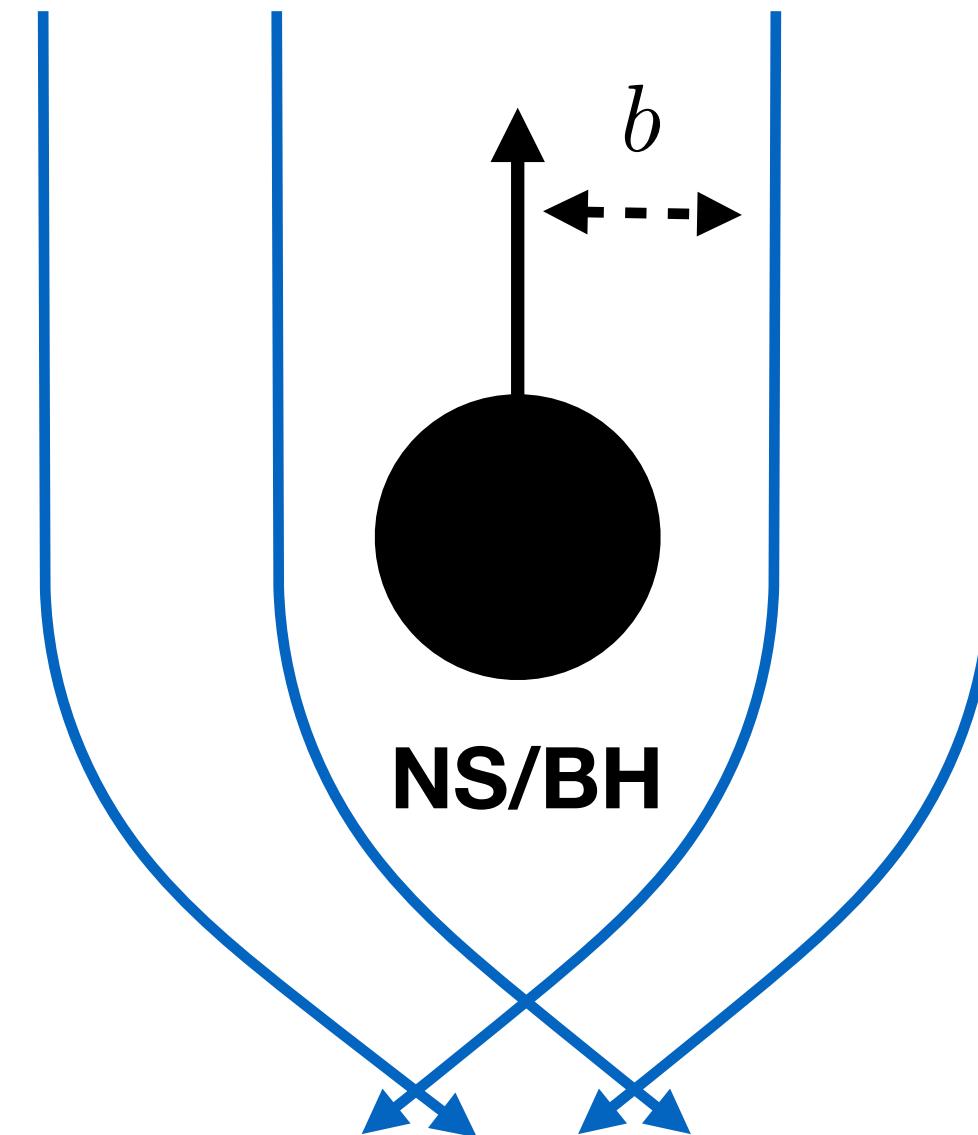
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# Scattering probability

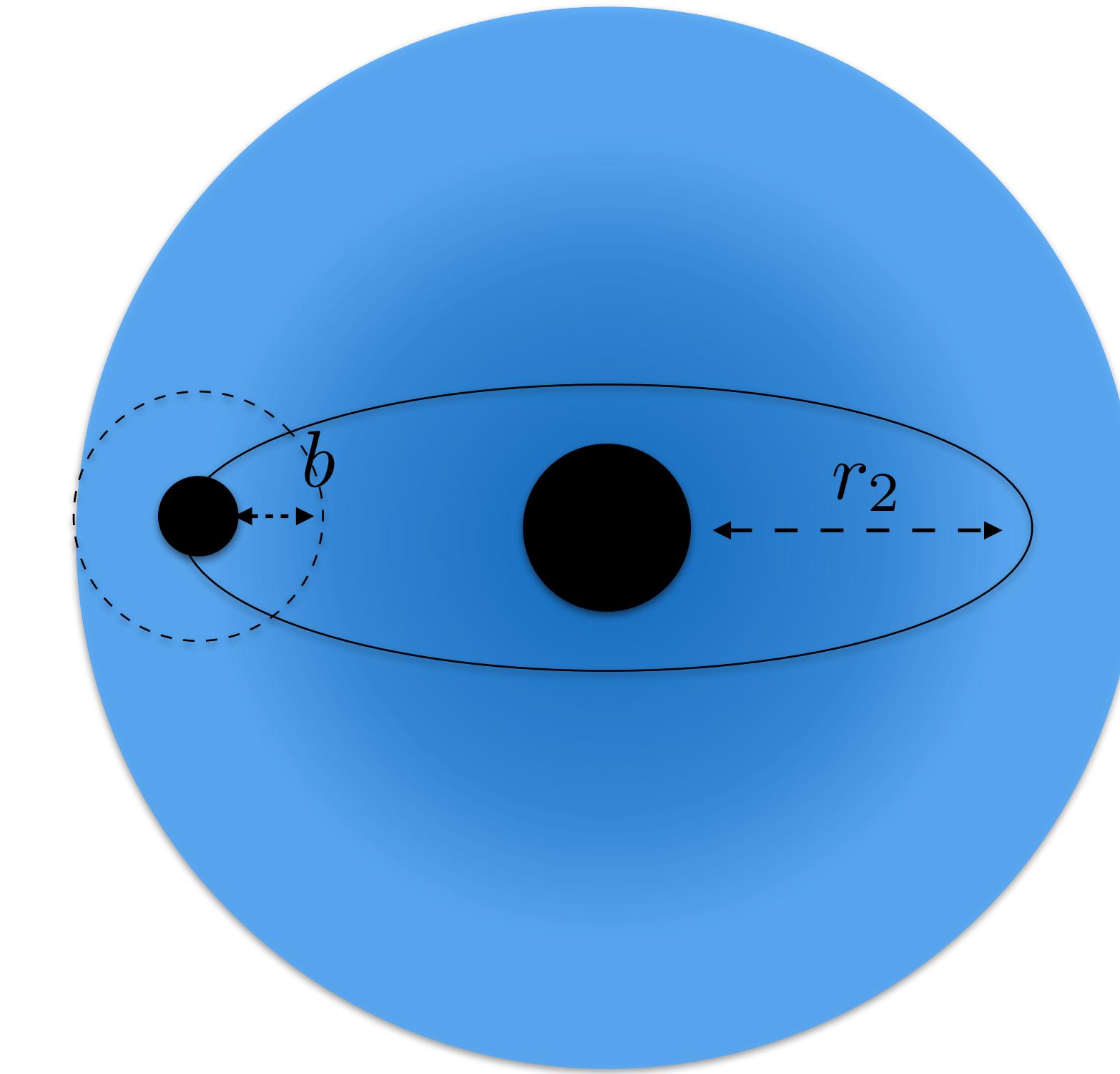
Two body scattering problem relates energy exchange to impact parameter:

$$\Delta\mathcal{E}(b) = -2v_0^2 \left[ 1 + \frac{b^2 v_0^4}{G^2 m_2^2} \right]^{-1}$$



Scattering probability becomes a *geometric* problem:

$$P_{\mathcal{E}}(\Delta\mathcal{E}) \propto P(b|\mathcal{E})$$



Code available online: [github.com/bradkav/HaloFeedback](https://github.com/bradkav/HaloFeedback)  
(See also <https://github.com/DMGW-Goethe/imripy>)

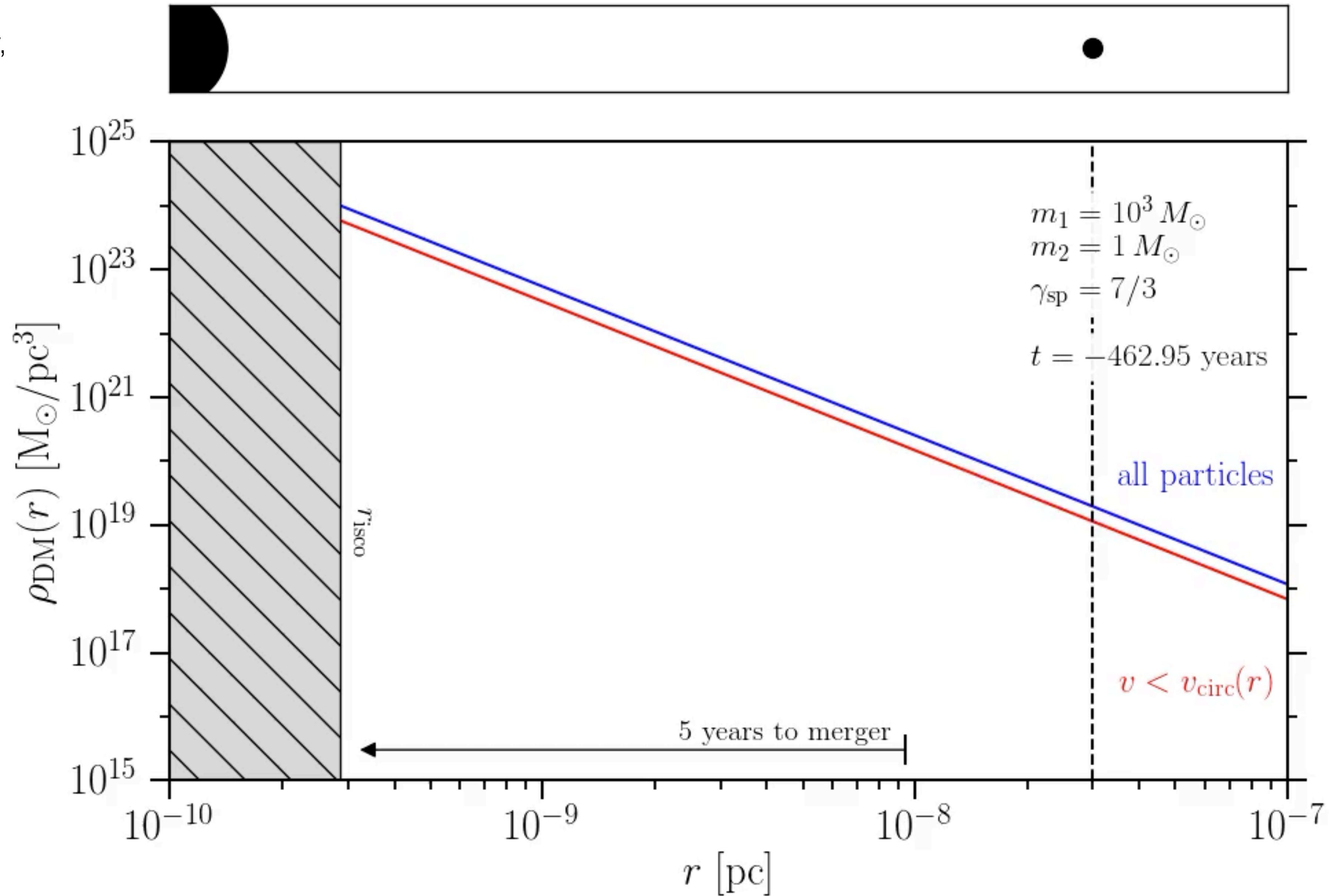
# Full evolution of the system

Newtonian motion of the binary,  
Taking into account:

- GW emission
- Dynamical Friction
- DM Halo Feedback

Density of the DM spike is  
depleted (and replenished...)

This is one of the reasons we  
want to look at IMRIs/EMRIs...



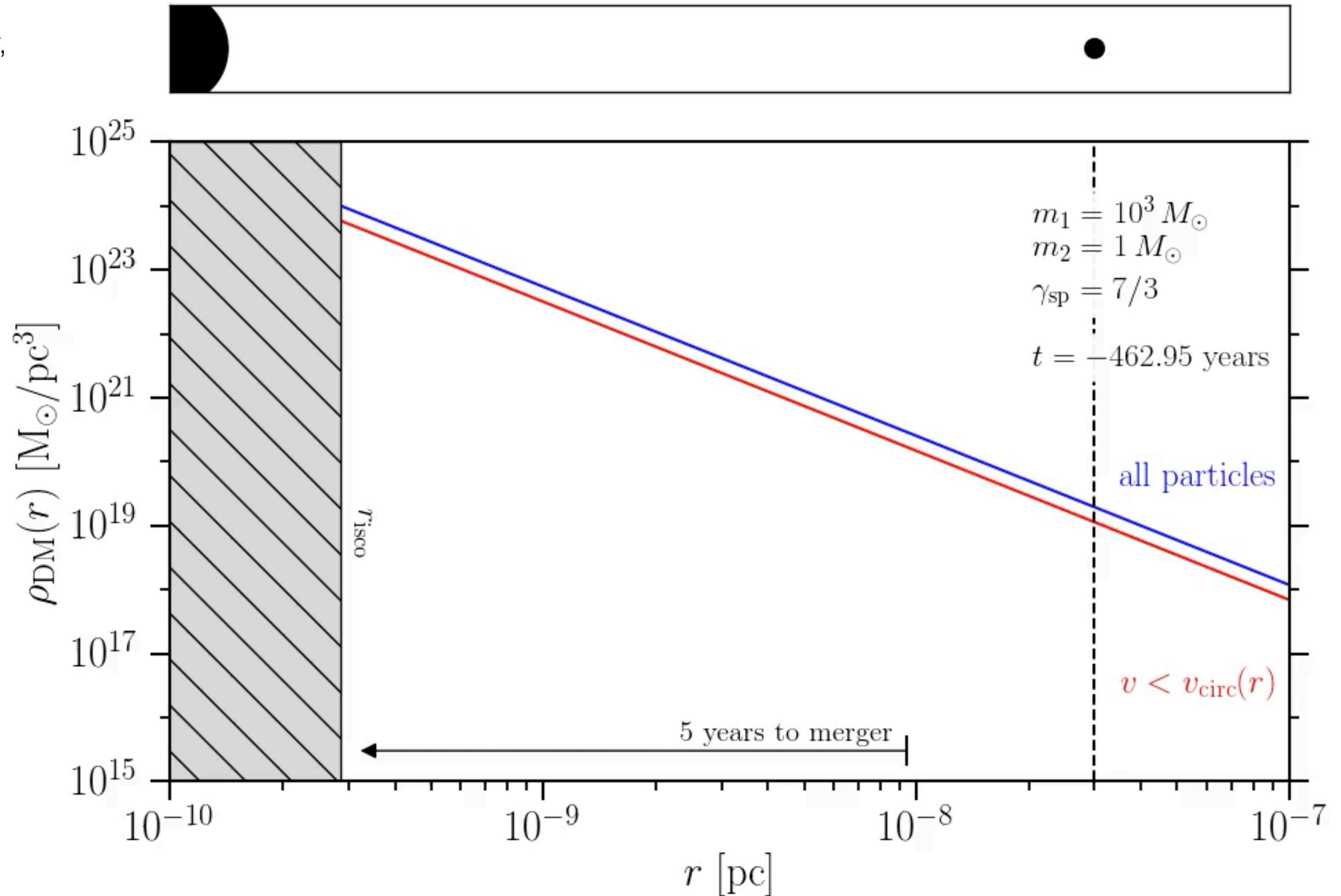
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# Self-consistent dephasing

[BJK, Nichols, Gaggero, Bertone, [2002.12811](#)]

Consider our astro benchmark system,  
starting at some initial separation:

$$r_{\text{ini}} \sim 10^{-8} \text{ pc}$$

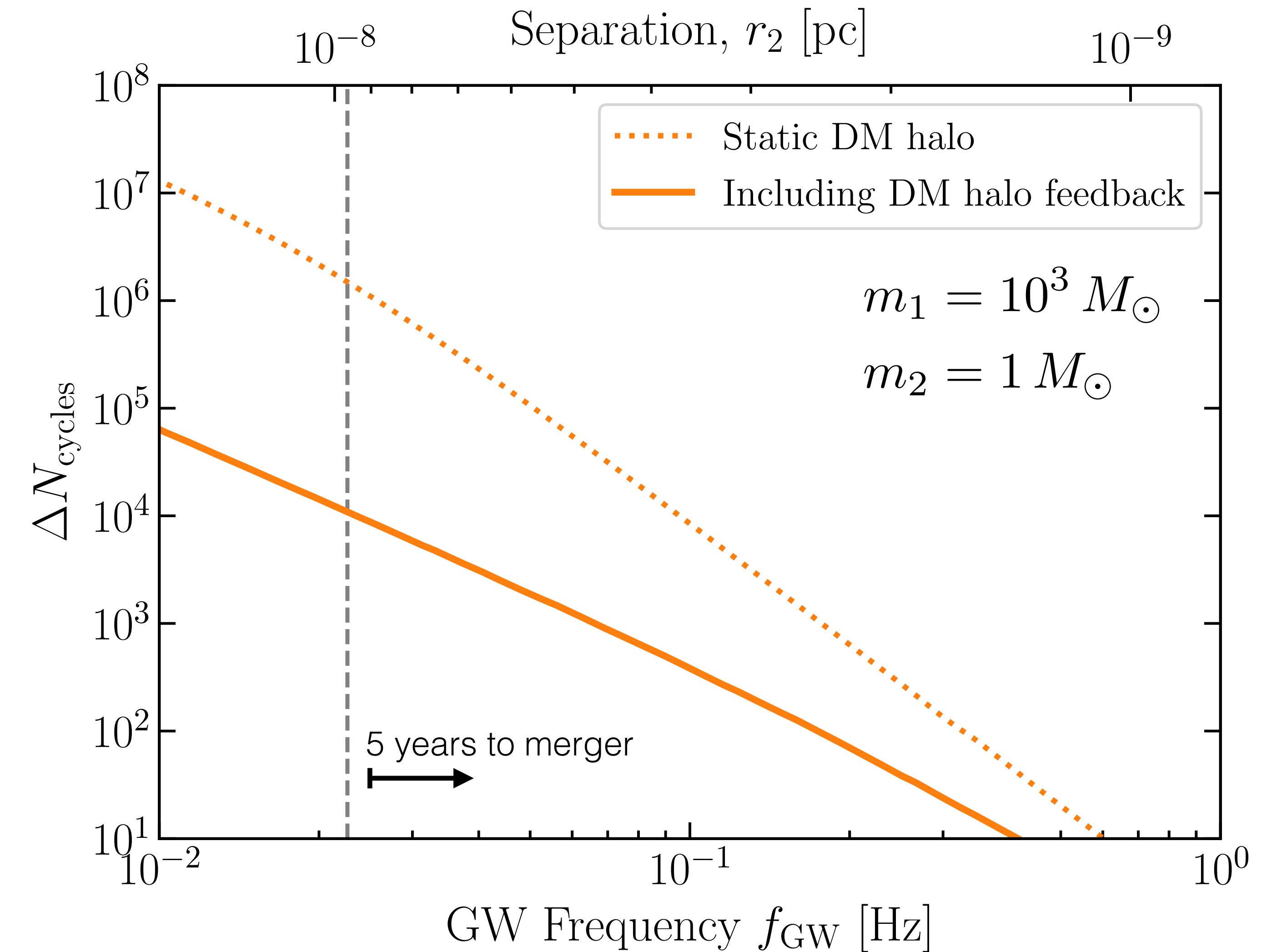
$$t_{\text{merge}}^{\text{vacuum}} \sim 5 \text{ yr}$$

$$N_{\text{cycles}}^{\text{vacuum}} \sim 6 \times 10^6$$

**DM dephasing**

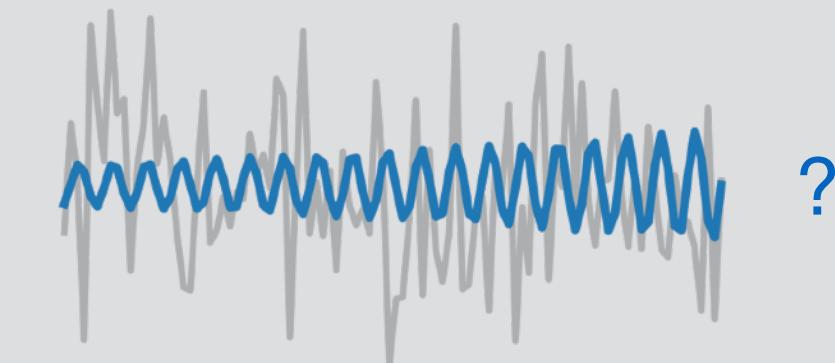
$$\Delta N_{\text{cycles}} \sim \mathcal{O}(10^4) \text{ cycles} \sim \% \text{ level}$$

Change in the number of GW cycles to merger,  
starting at some initial frequency/separation:



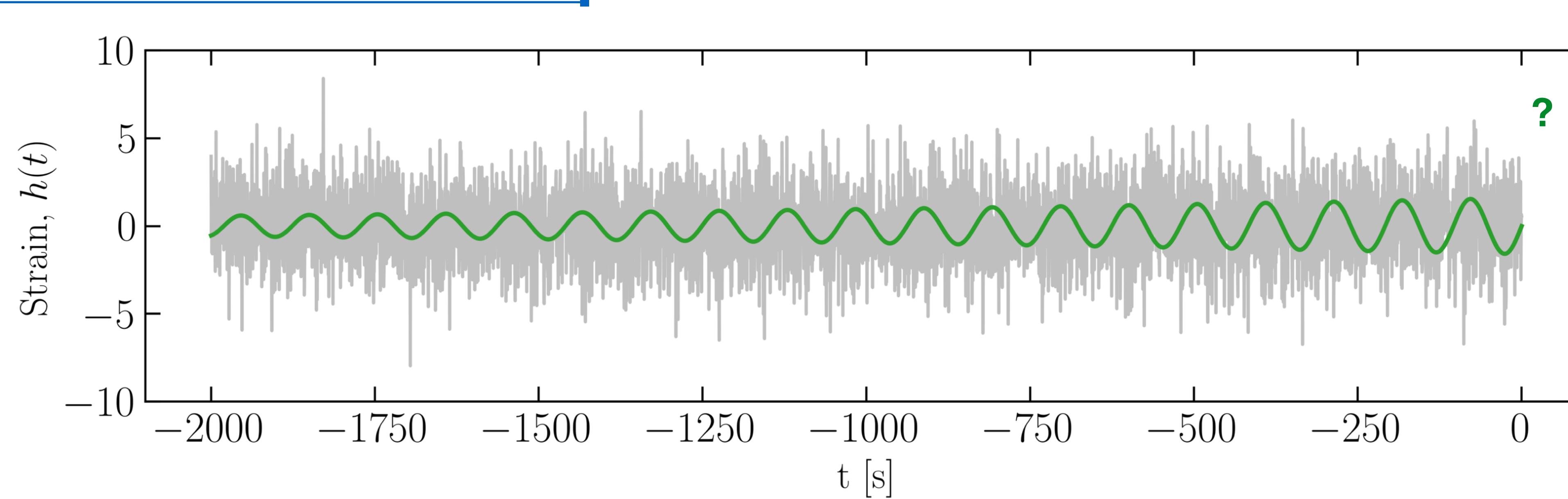
## ***Measuring Dark Matter around Black Holes***

[Coogan, Bertone, Gaggero, **BJK** & Nichols, [2108.04154](#)]



# A more realistic scenario

[Coogan, Bertone, Gaggero, **BJK** & Nichols, [2108.04154](#)]



Want to address questions of:

- **Detectability** - is the event loud enough to detect?
- **Discoverability** - can we tell it apart from a *GR-in-vacuum* waveform?
- **Measurability** - can we pin down the properties of the system (*especially the DM*)?

# Detectability

A signal may be **detectable** with LISA using matched filtering with a signal-to-noise ratio (SNR)  $\gtrsim 15\dots$

[\[1905.11998\]](#)

Match between waveforms  $a$  and  $b$   
defined as:

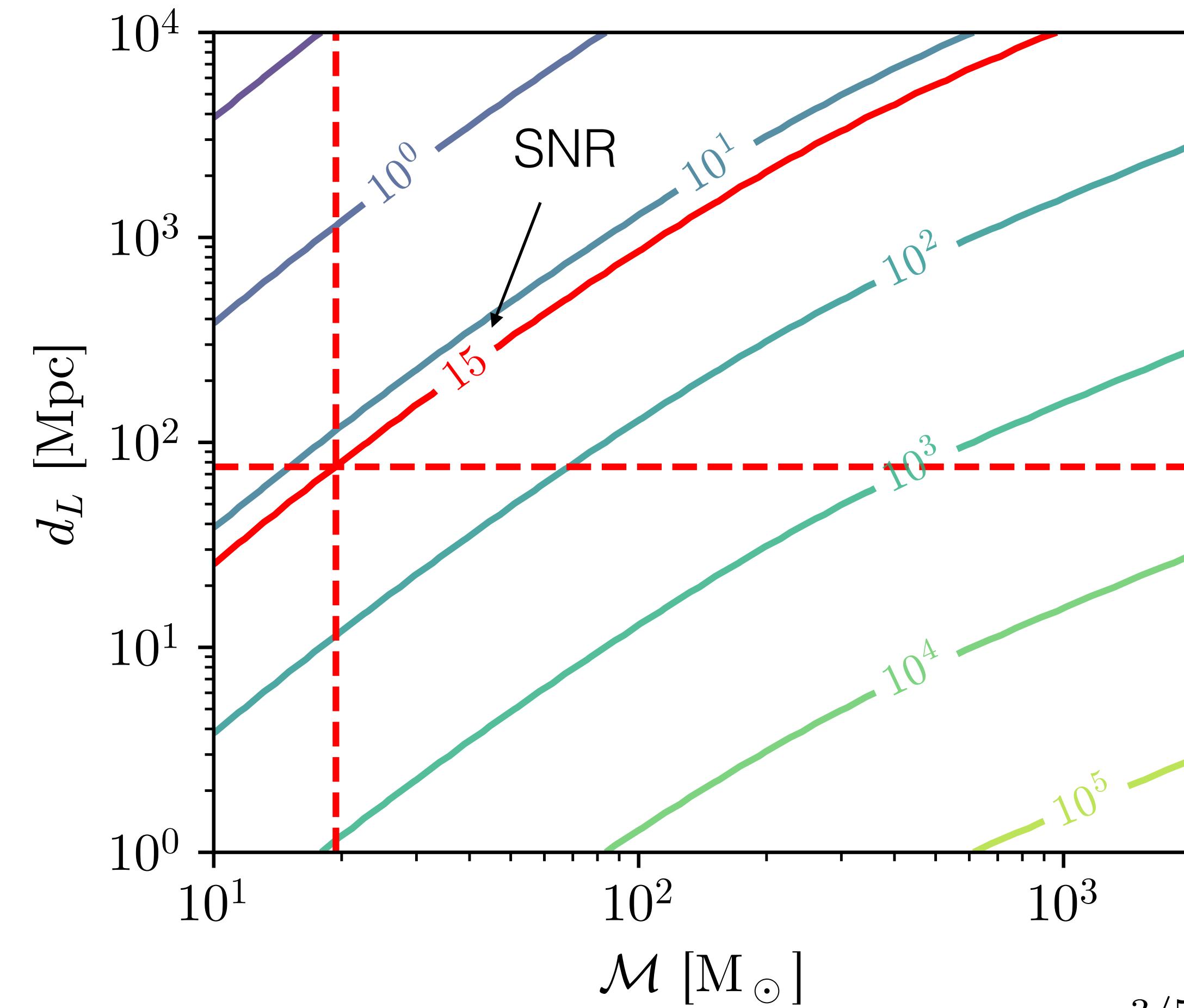
$$\langle a | b \rangle = 4 \operatorname{Re} \int_0^\infty df \frac{\tilde{a}(f)^* \tilde{b}(f)}{S_n(f)}$$

LISA noise curve

Optimal SNR for waveform  $s$  is then:

$$\text{SNR}(s) = \sqrt{\langle s | s \rangle}$$

NB: Presence of the dark dress  
does not substantially affect SNR

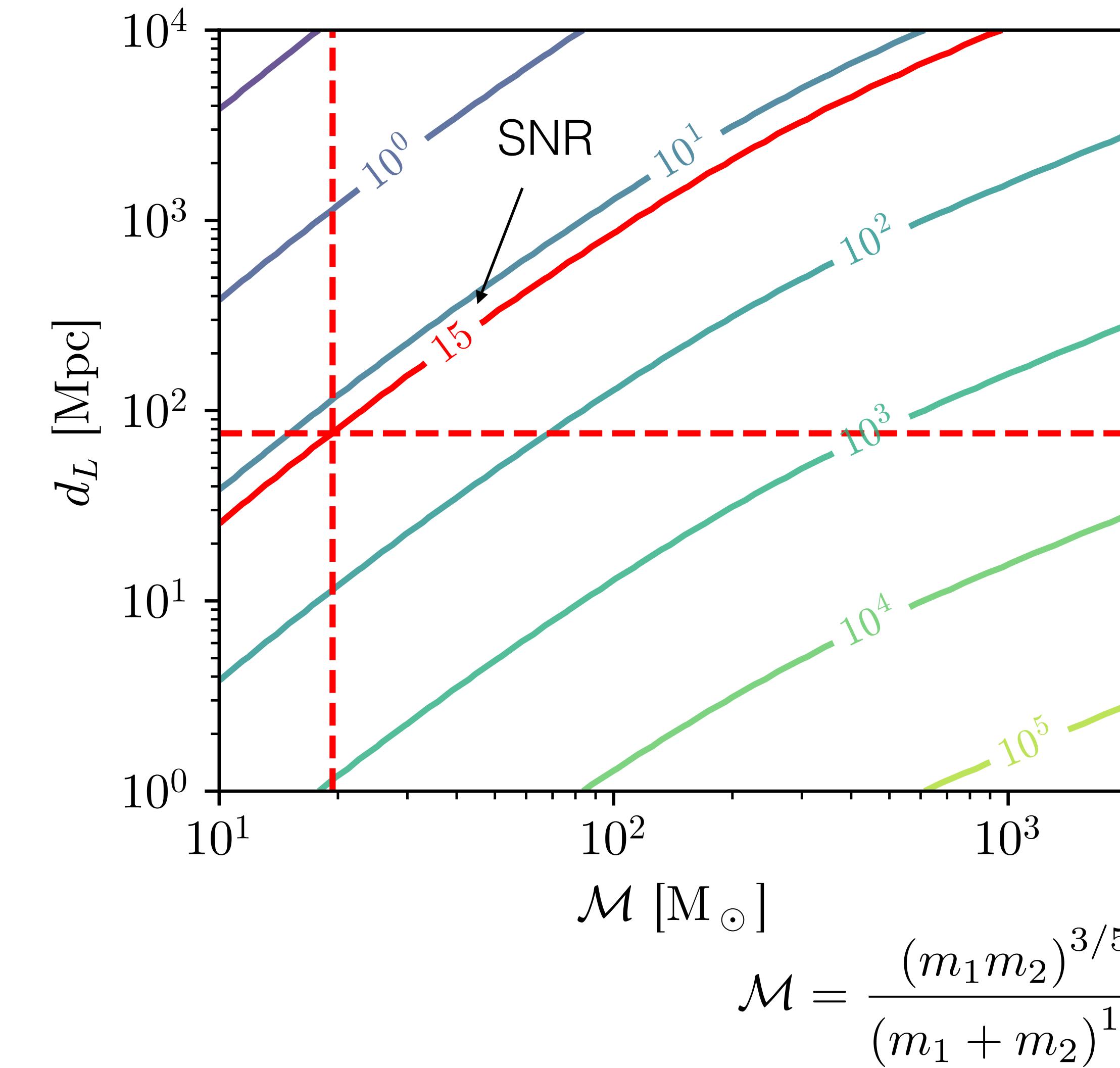
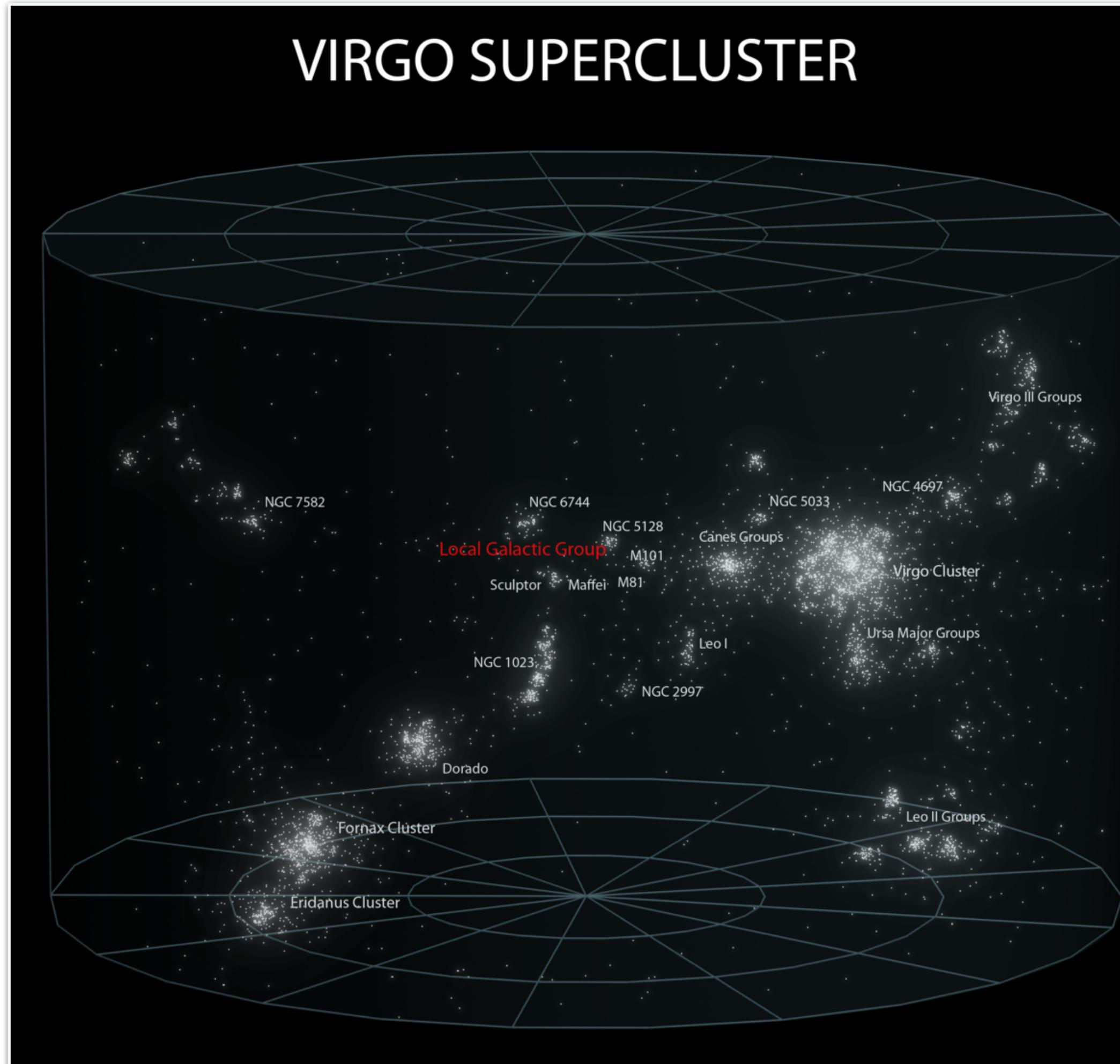


$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

# Detectability

A signal may be **detectable** with LISA using matched filtering with a signal-to-noise ratio (SNR)  $\gtrsim 15\dots$

[\[1905.11998\]](#)

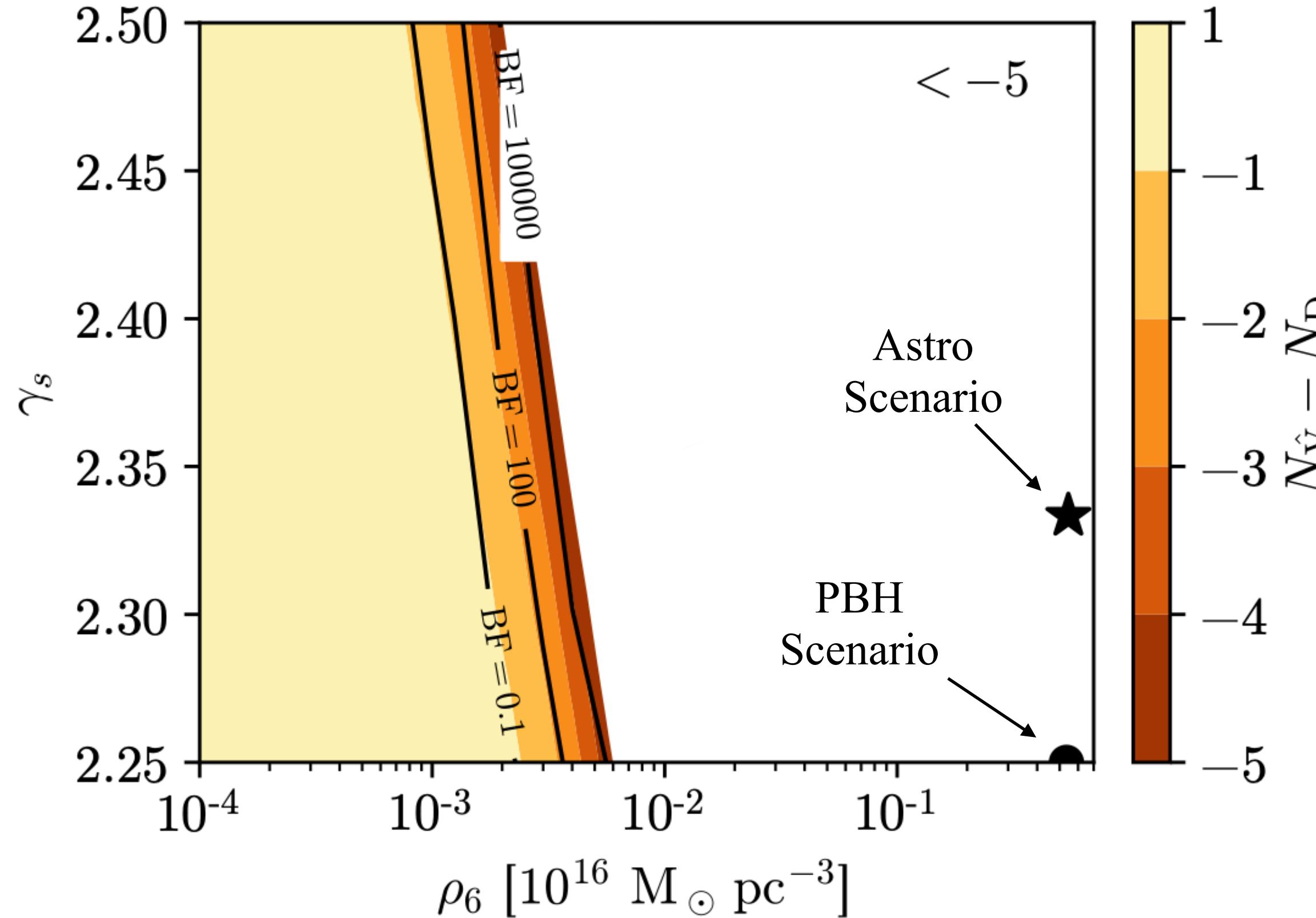


# Discoverability

$$q = m_2/m_1$$

We'll call a DM spike **discoverable** if it can be distinguished from a GR-in-vacuum system.

Given the data  $d$ , compare Bayesian evidence  $p(d)$  for **V**acuum and **D**ressed systems:



$$\theta_V = \{\mathcal{M}\} \quad \text{vs.} \quad \theta_D = \{\gamma_{\text{sp}}, \rho_6, \mathcal{M}, \log_{10} q\}$$

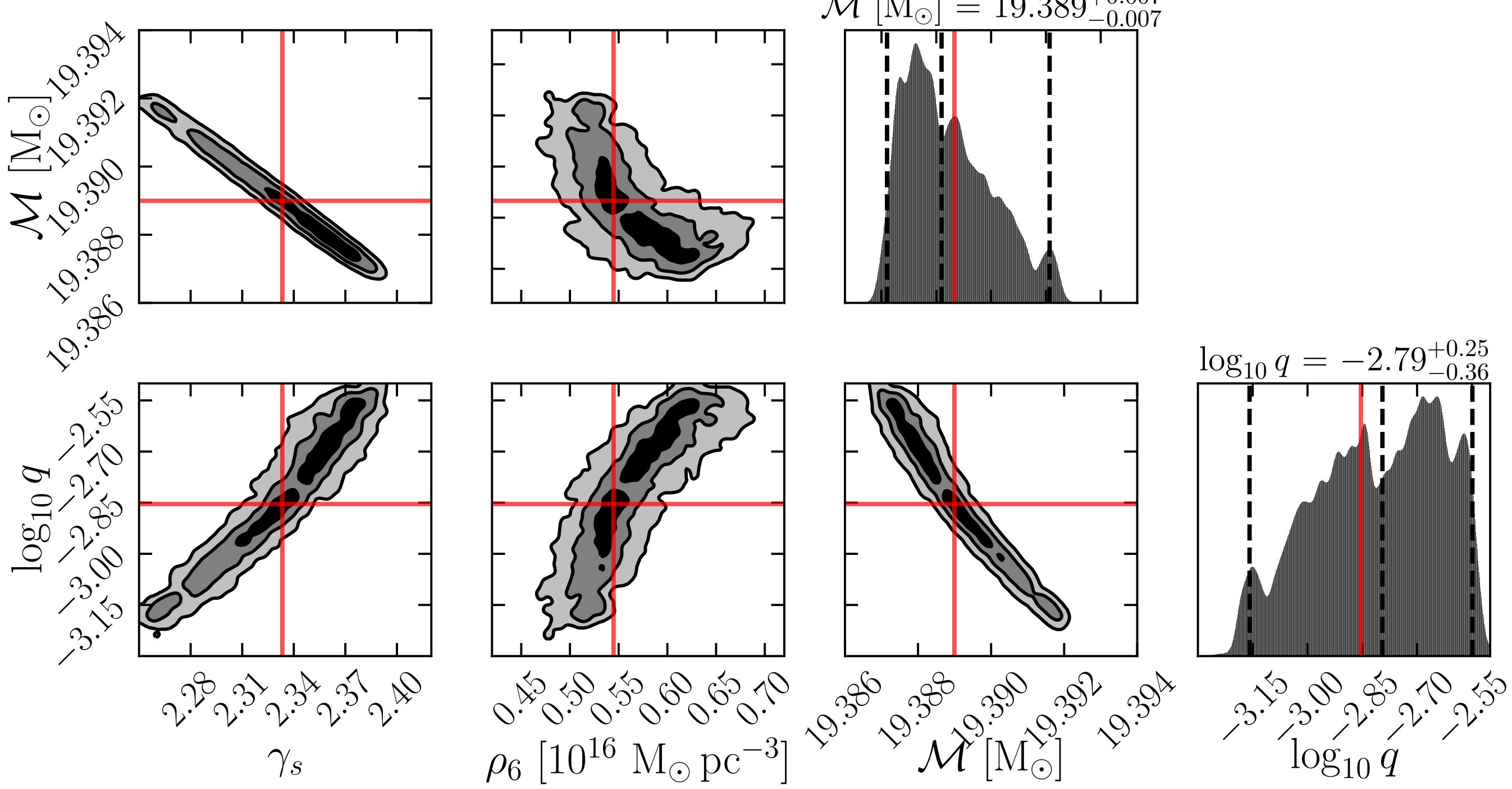
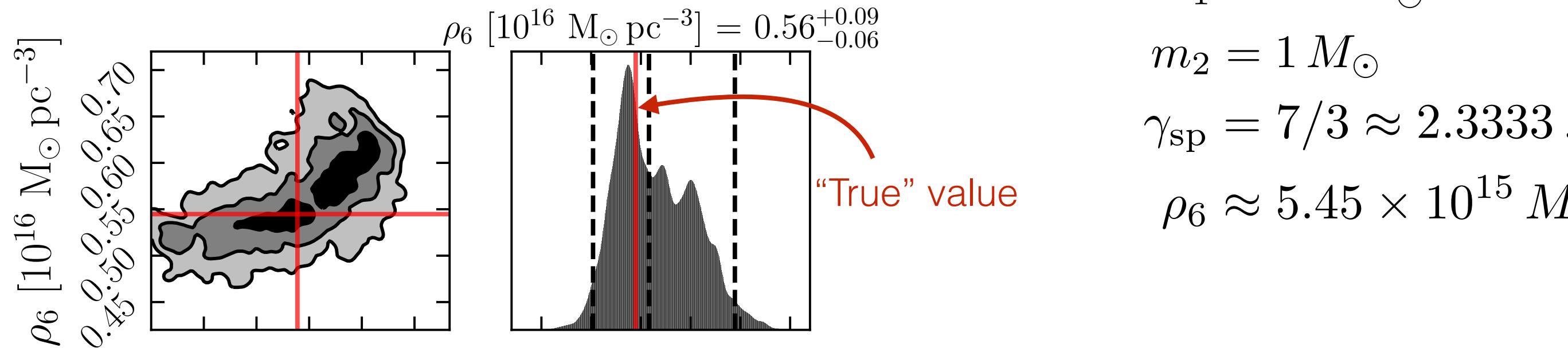
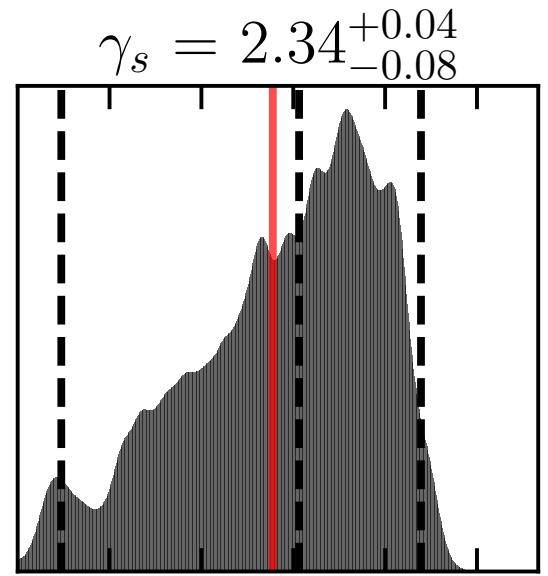
(maximising over extrinsic variables  
 $\theta_{\text{ext}} \equiv \{D_L, \phi_c, \tilde{t}_c\}$ )

$$p(d) = \int d\theta \mathcal{L}(\theta) p(\theta)$$

$$\text{BF}(d) \equiv \frac{p(d|\text{D})}{p(d|\text{V})}$$

Use an approximate waveform parametrisation in terms of  $\theta_D$

[Code available online:  
<https://github.com/adam-coogan/pydd>]



# Measurability

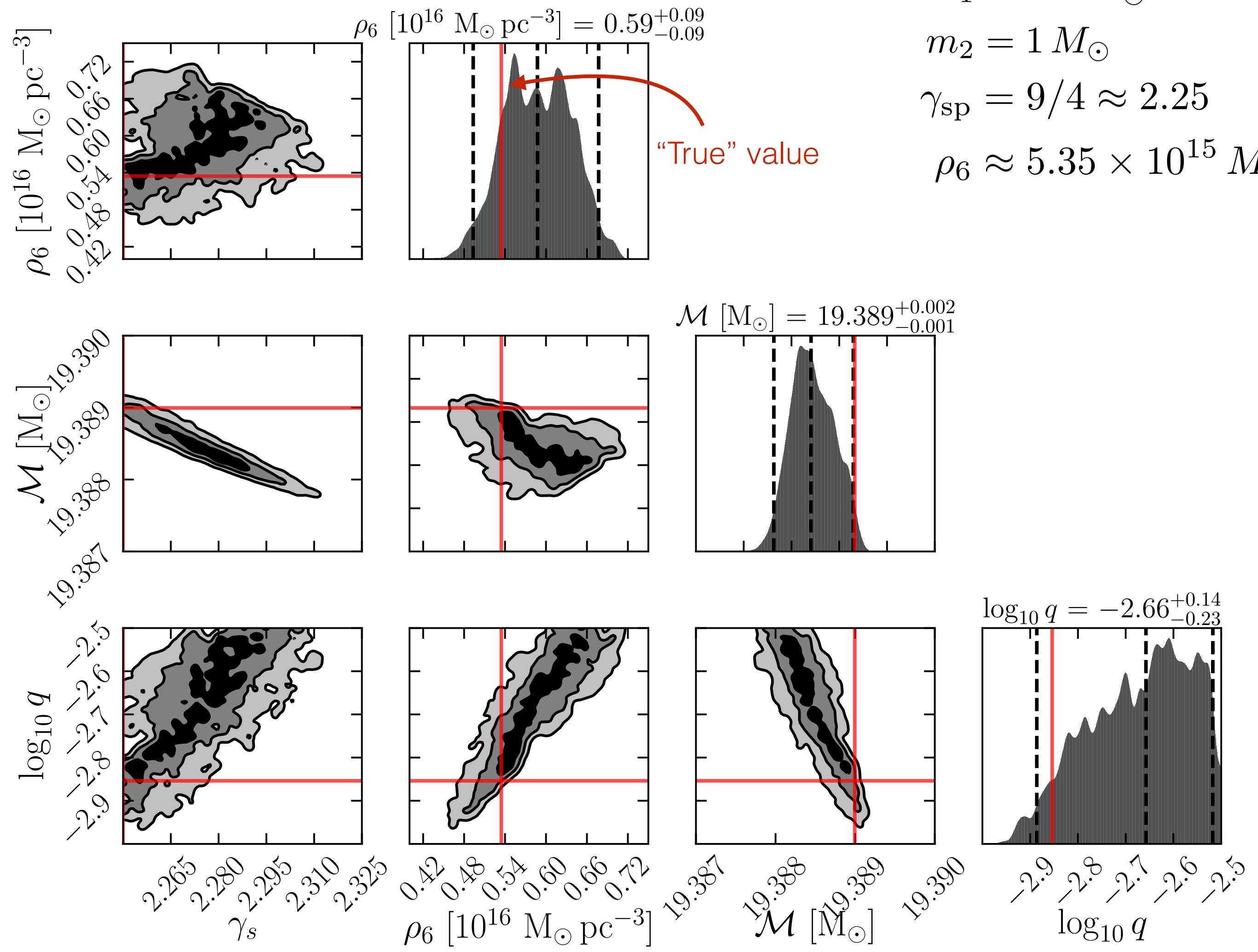
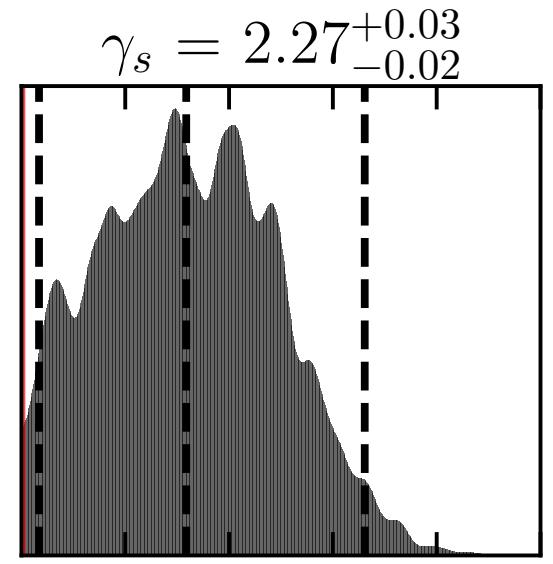
## Astrophysical scenario

$$m_1 = 10^3 M_\odot$$

$$m_2 = 1 M_\odot$$

$$\gamma_{\text{sp}} = 7/3 \approx 2.3333\dots$$

$$\rho_6 \approx 5.45 \times 10^{15} M_\odot \text{ pc}^{-3}$$



# Measurability

## PBH scenario

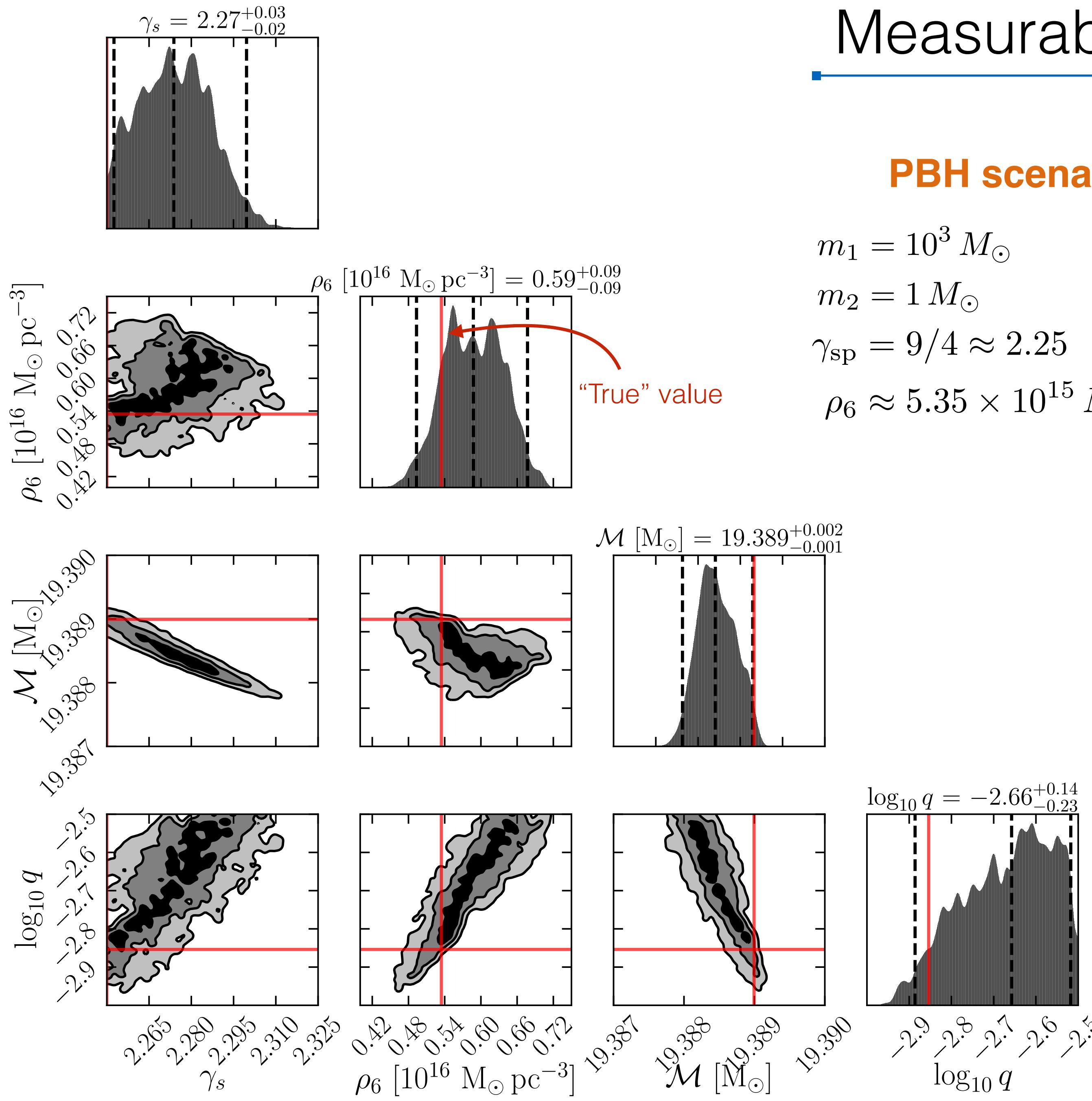
$$m_1 = 10^3 M_\odot$$

$$m_2 = 1 M_\odot$$

$$\gamma_{\text{sp}} = 9/4 \approx 2.25$$

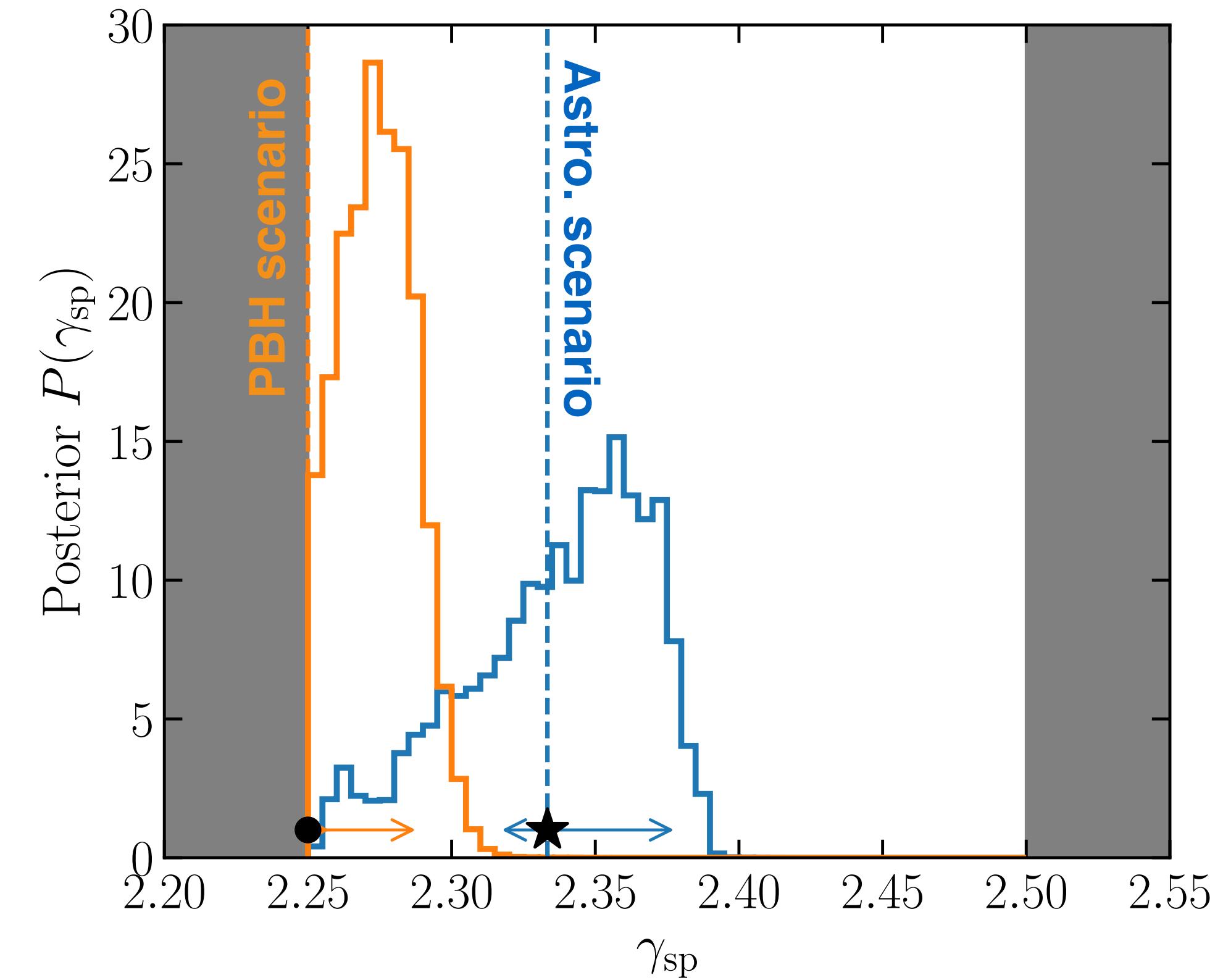
$$\rho_6 \approx 5.35 \times 10^{15} M_\odot \text{ pc}^{-3}$$

# Measurability

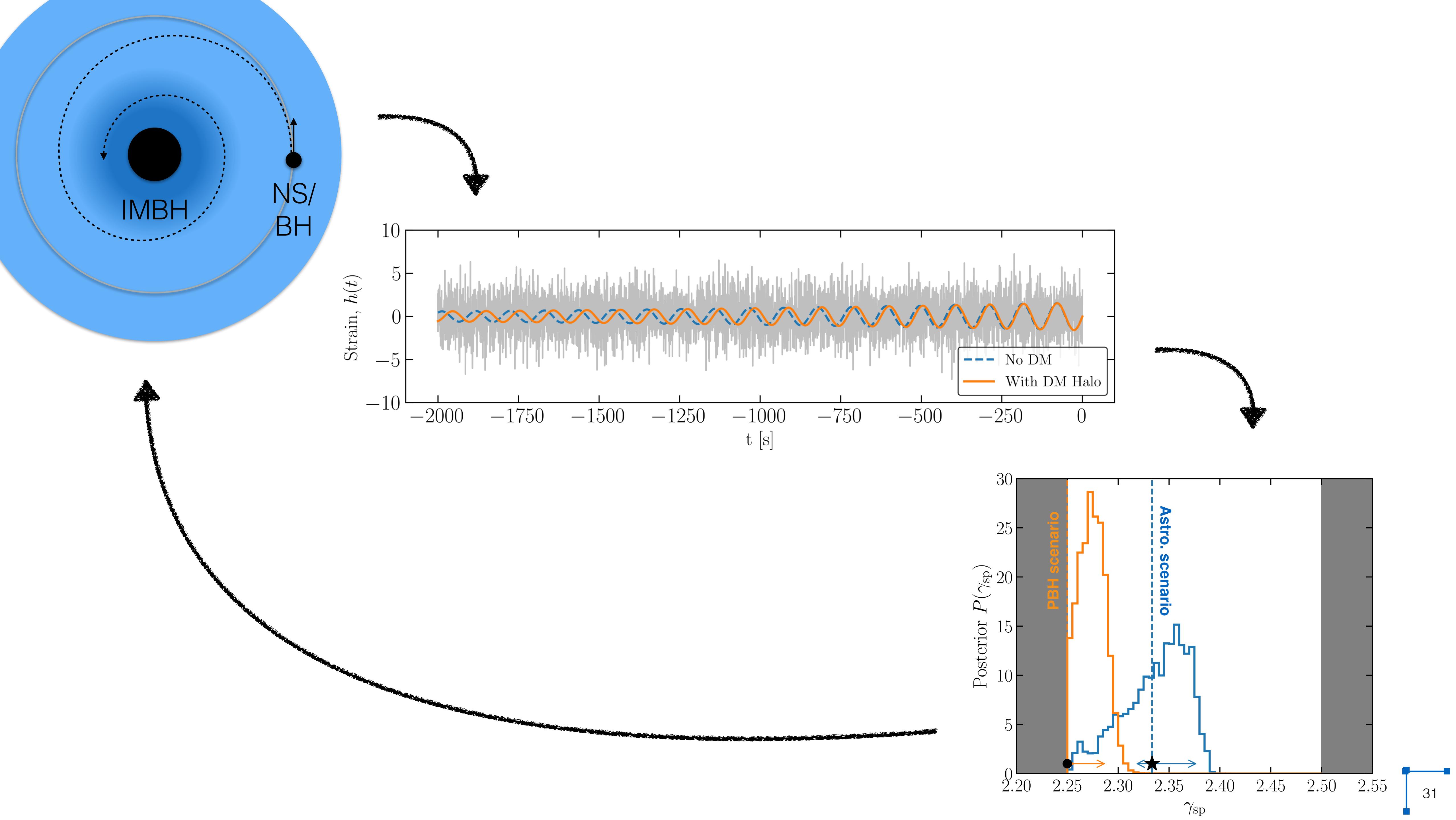


## PBH scenario

$$\begin{aligned} m_1 &= 10^3 M_\odot \\ m_2 &= 1 M_\odot \\ \gamma_{\text{sp}} &= 9/4 \approx 2.25 \\ \rho_6 &\approx 5.35 \times 10^{15} M_\odot \text{ pc}^{-3} \end{aligned}$$



We may be able to distinguish different *shapes* of spike  
 → Different DM models and formation mechanisms!



# Plans for the future



Gianfranco Bertone  
(GRAPPA, Amsterdam)



Pippa Cole  
(GRAPPA, Amsterdam)



Adam Coogan  
(Mila, Montreal)



Jose Maria Diego  
(IFCA, Santander)



Daniele Gaggero  
(IFT, Madrid)



Pratibha Jangra  
(IFCA, Santander)



David Nichols  
(U. Virginia)



Francesca Scarcella  
(IFT, Madrid)

## Improved modelling

- Injection and evolution of angular momentum in the spike
- Orbital eccentricity
- Post-Newtonian corrections
- Better N-body approaches [[AMUSE?](#)]

## Detection methods

- Producing template banks for LISA searches
- *Surrogate models for waveform generation*
- Incoherent searches for continuous GWs
- ‘General’ de-phased waveform templates [[2004.06729](#)]

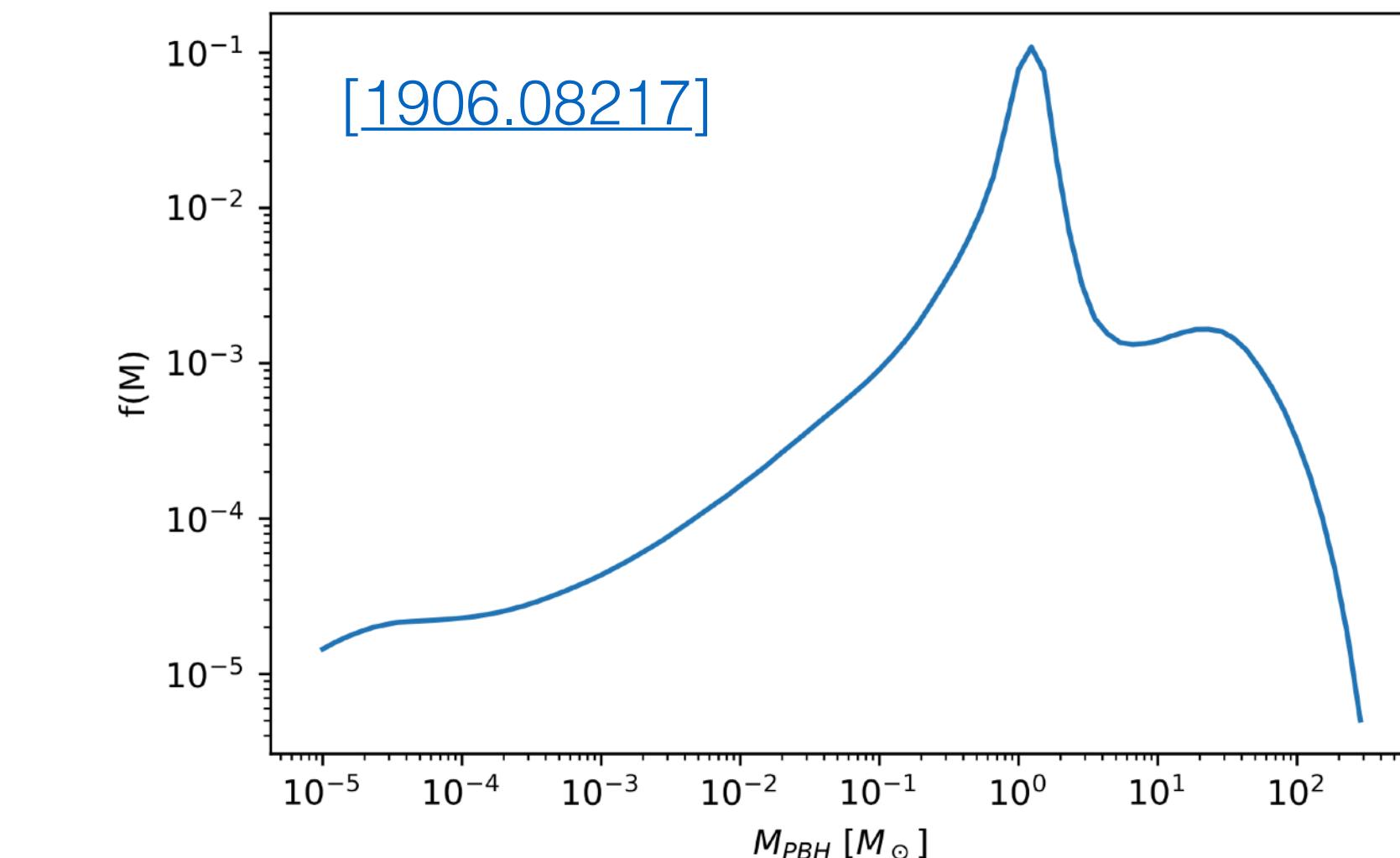
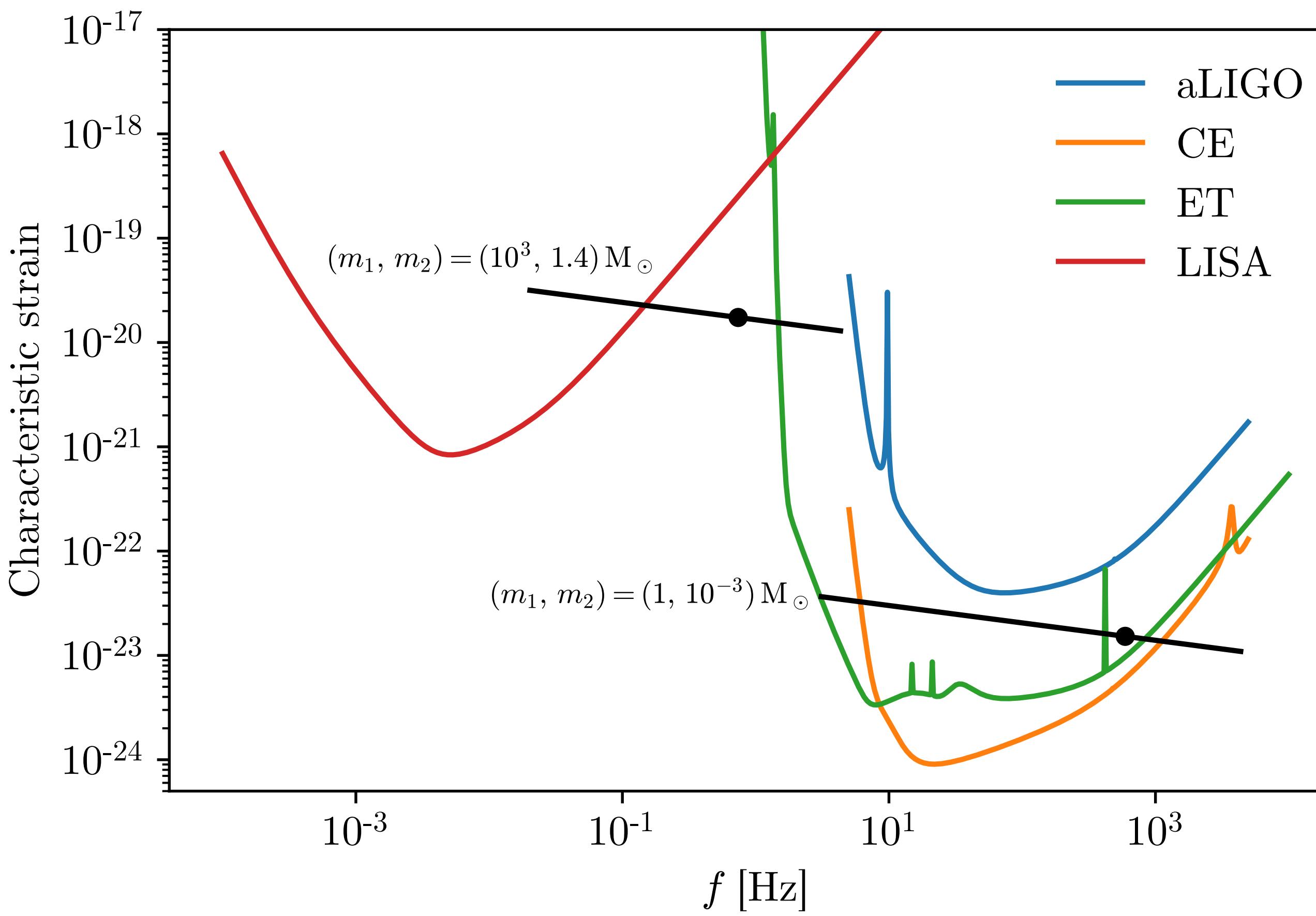
## Detection prospects

- How many IMRI systems form? How many with BH/NSs?
- *How many systems have a (surviving) spike?*
- Comparison with dephasing due to baryons, or due to ultralight bosons (gravitational atoms)
- *What about ground-based detectors? Low mass PBHs?*

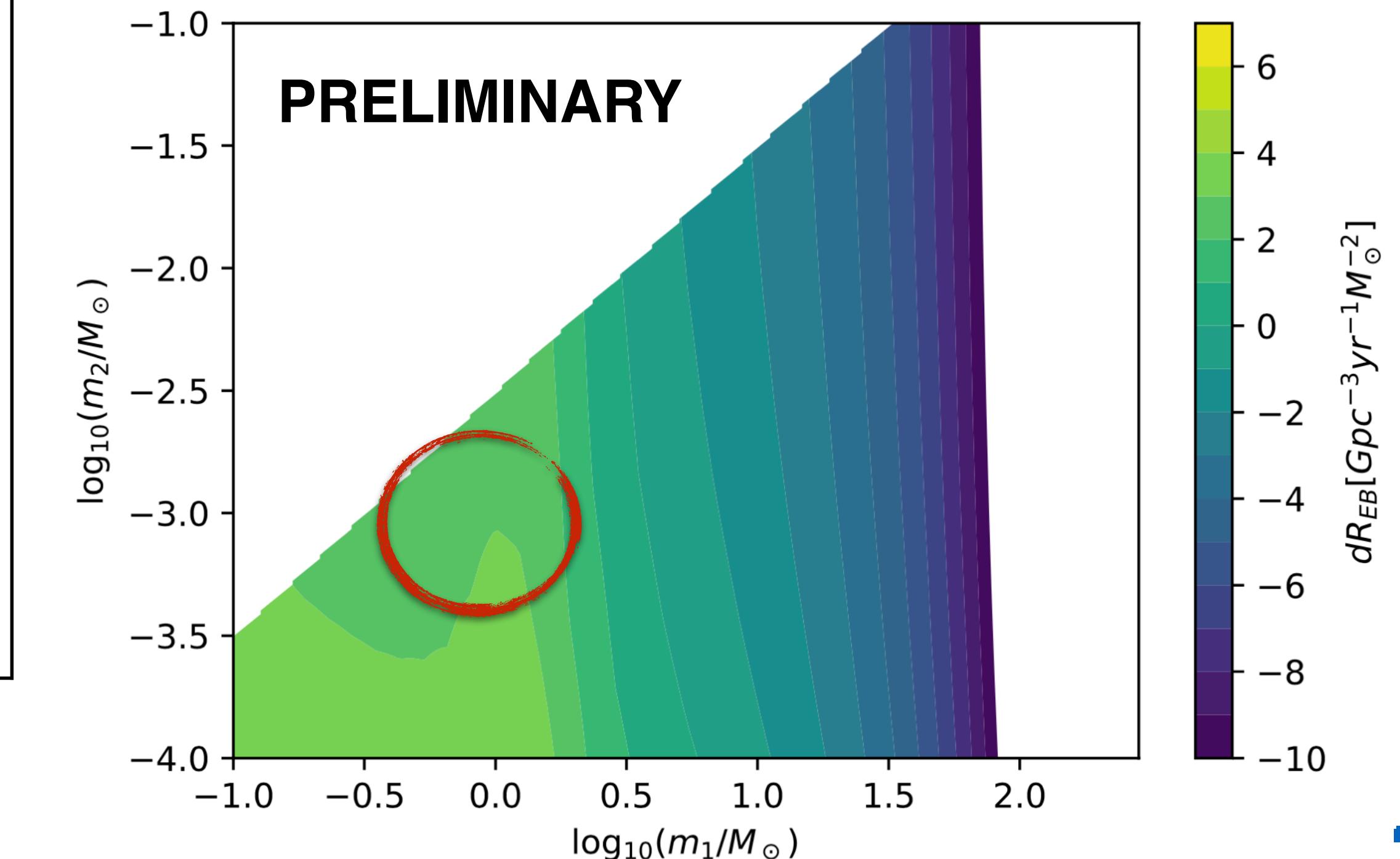
...and others...

# Ground-based telescopes

Low mass **PBH binaries** could be detected with ground based detectors such as LIGO or Einstein Telescope (ET)



(b)  $k_p = 5 \times 10^5 \text{ Mpc}^{-1}$ ,  $f_{PBH} = 0.085$ .



# Conclusions

Dark Matter ‘de-phasing’ is an extremely promising GW signature, which needs to be **modelled carefully**

[**BJK**, Nichols, Gaggero, Bertone, [2002.12811](#)]

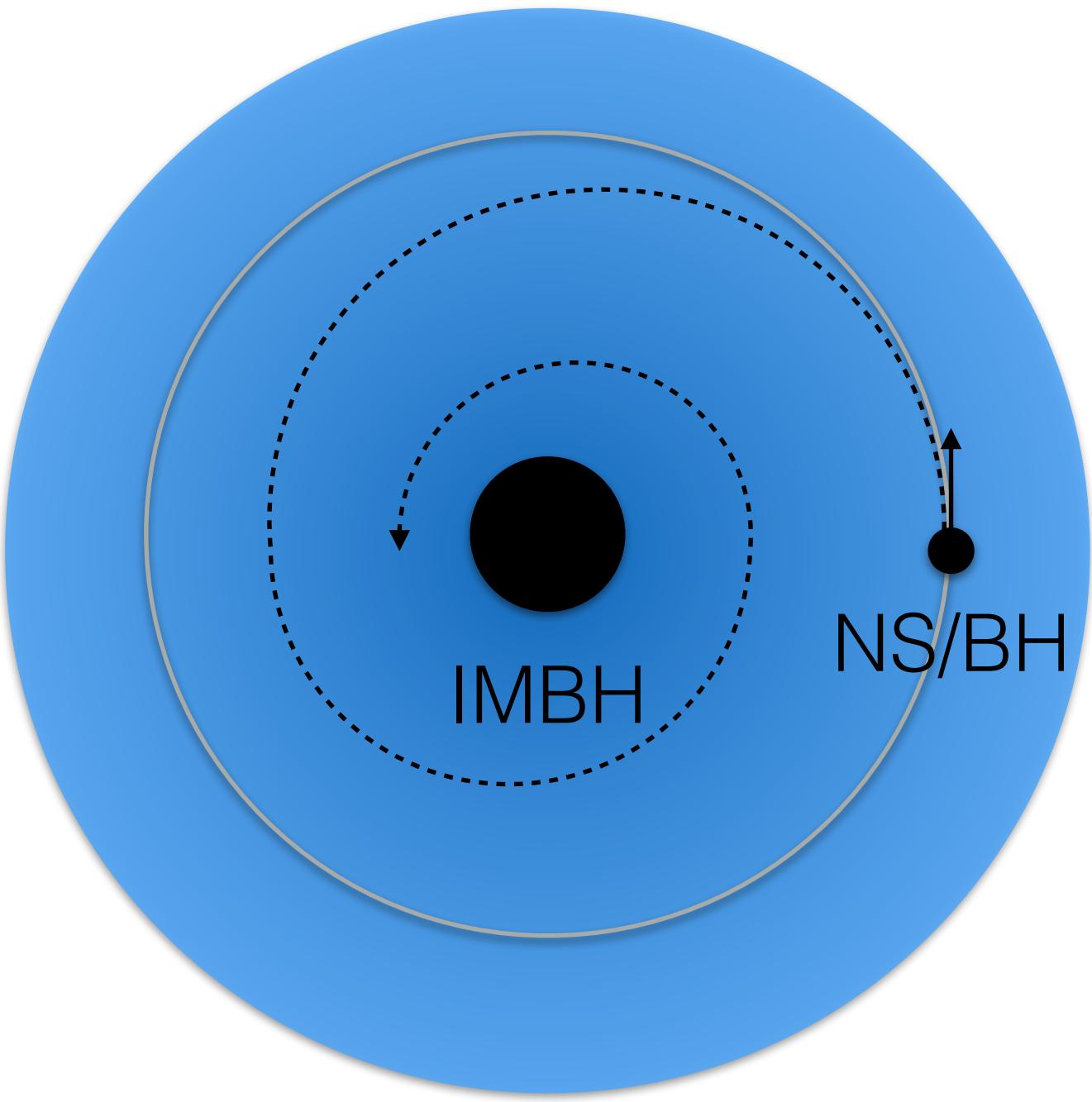
With LISA, such systems should be **detectable, discoverable** against vacuum-only systems, and the properties **measurable**.

[Coogan, Bertone, Gaggero, **BJK** & Nichols, [2108.04154](#)]

These signals could probe the **nature of Dark Matter** and pave the way towards a **multi-messenger detection** of Dark Matter

[Edwards, Chianese, **BJK**, Nissanke & Weniger, [1905.04686](#)]

**There are lots of open questions remaining,  
but they’re well worth answering!**



# Conclusions

Dark Matter ‘de-phasing’ is an extremely promising GW signature, which needs to be **modelled carefully**

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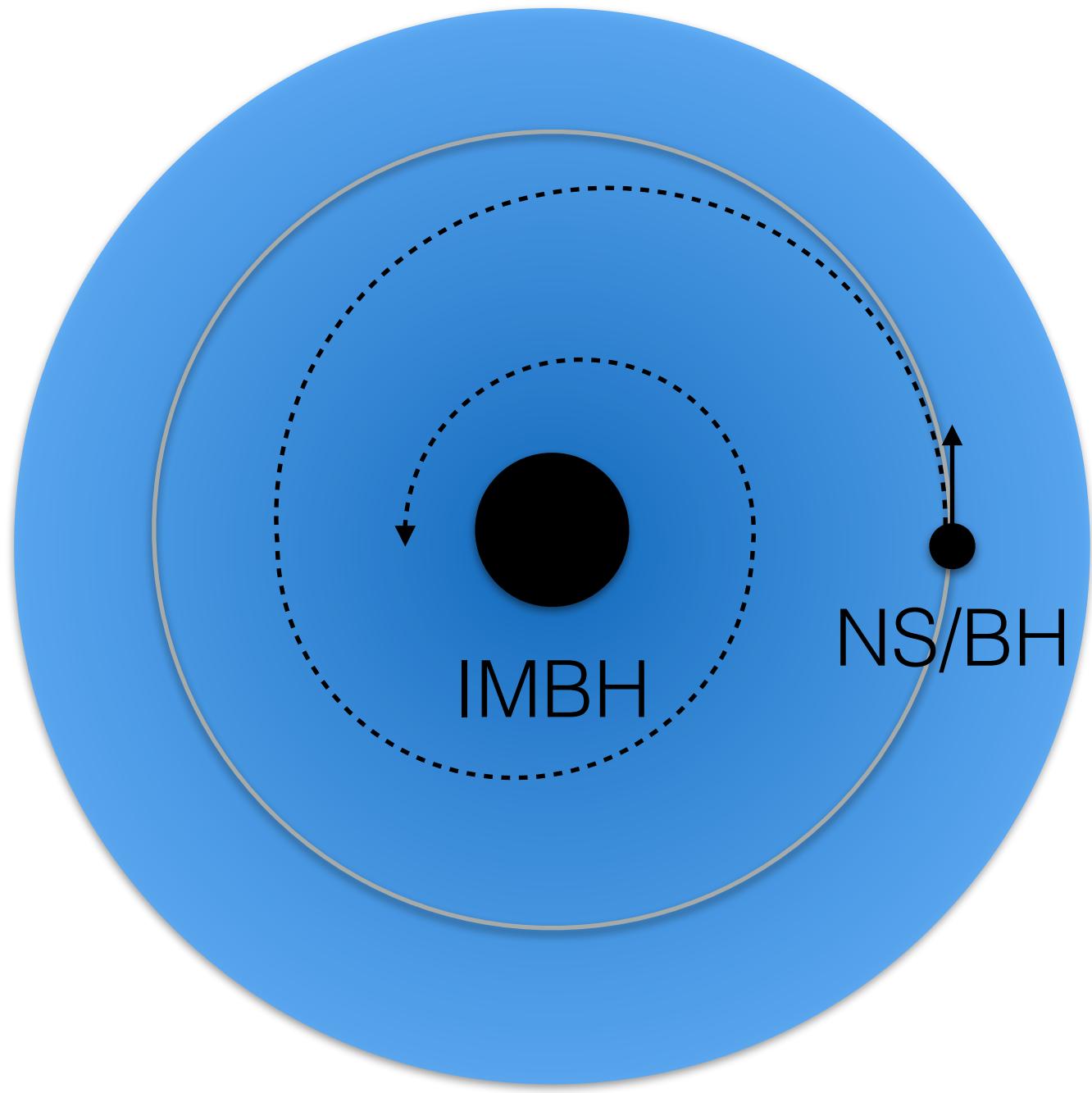
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**There are lots of open questions remaining,  
but they’re well worth answering!**



**Thank you!**

# Backup Slides

# Dark Matter ‘spikes’ (1)

Depending on the formation mechanism of the IMBH,  
expect an over-density of DM:

For BH forming in an NFW halo,  
from adiabatic growth expect:

$$\gamma_{\text{sp}} = 7/3 \approx 2.333$$

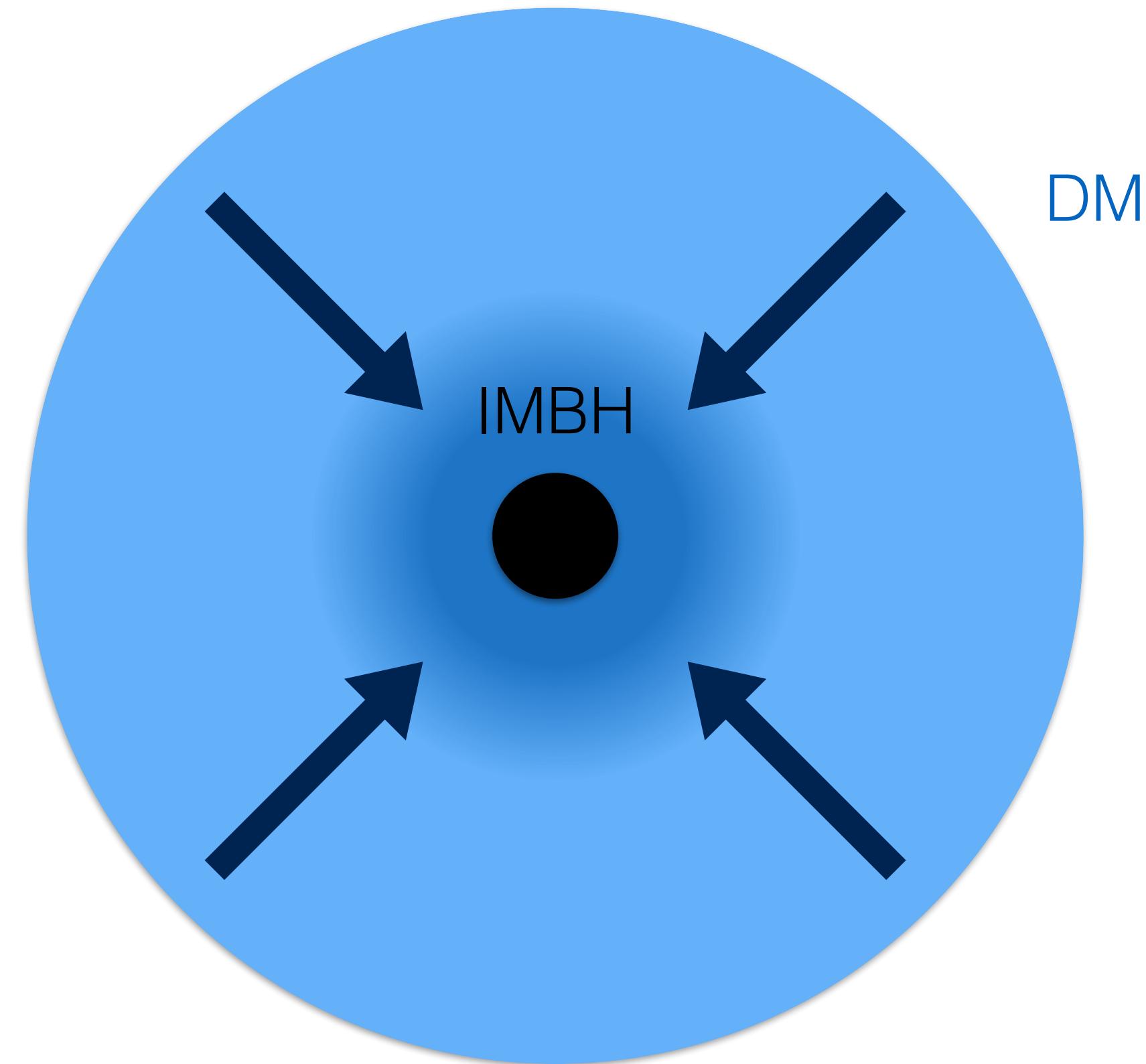
For  $10^5$  Solar mass IMBH, forming at  
 $z \sim 20$ , get typical values:

$$\rho_6 \approx 5.45 \times 10^{15} M_\odot \text{pc}^{-3}$$

$$r_{\text{sp}} \approx 0.5 \text{ pc}$$

Density can reach  $\rho \gtrsim 10^{24} M_\odot \text{pc}^{-3}$   
( $\sim 10^{24}$  times larger than local density)

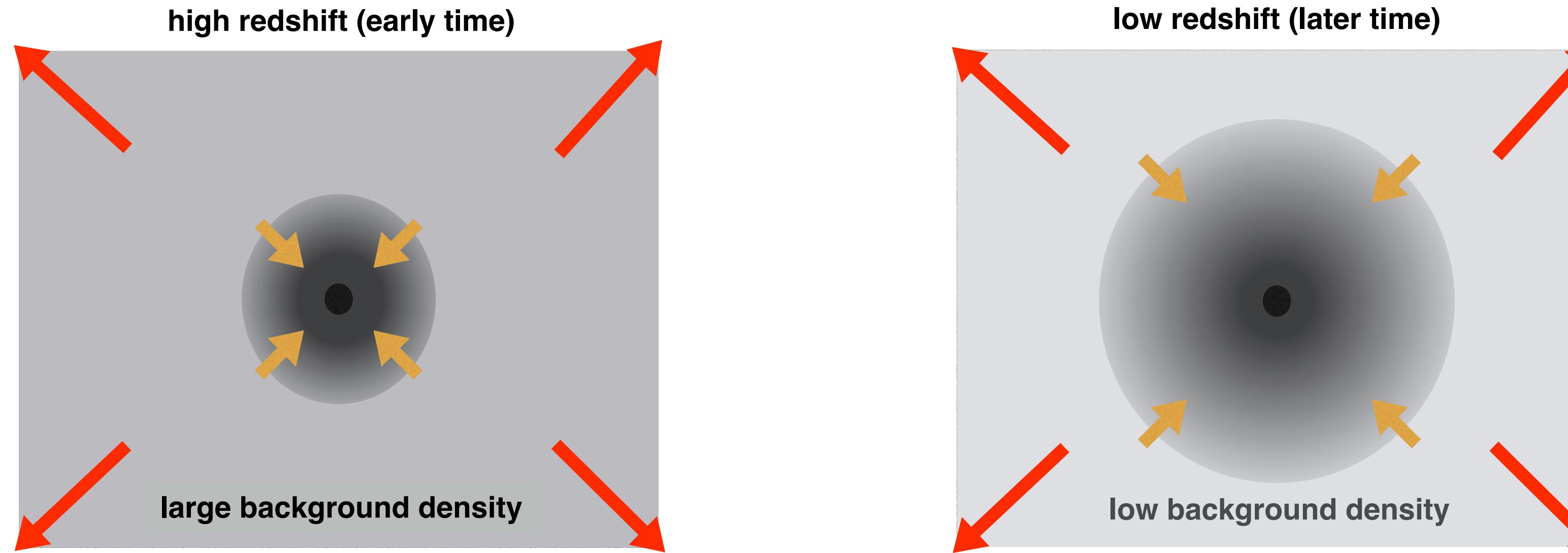
$$\rho_{\text{DM}} = \rho_6 \left( \frac{10^{-6} \text{ pc}}{r} \right)^{\gamma_{\text{sp}}}$$



[[astro-ph/9906391](#), [astro-ph/0501555](#), [astro-ph/0501625](#), [astro-ph/0509565](#), [0902.3665](#), [1305.2619](#)]

# Dark Matter 'spikes' (2)

*Primordial black holes seed the formation of 'local' DM halos:*



$$R_{\text{tr}}(z) = 0.0063 \left( \frac{M_{\text{PBH}}}{M_{\odot}} \right) \left( \frac{1 + z_{\text{eq}}}{1 + z} \right) \text{ pc}$$

$$\rho(r) \propto r^{-9/4}$$

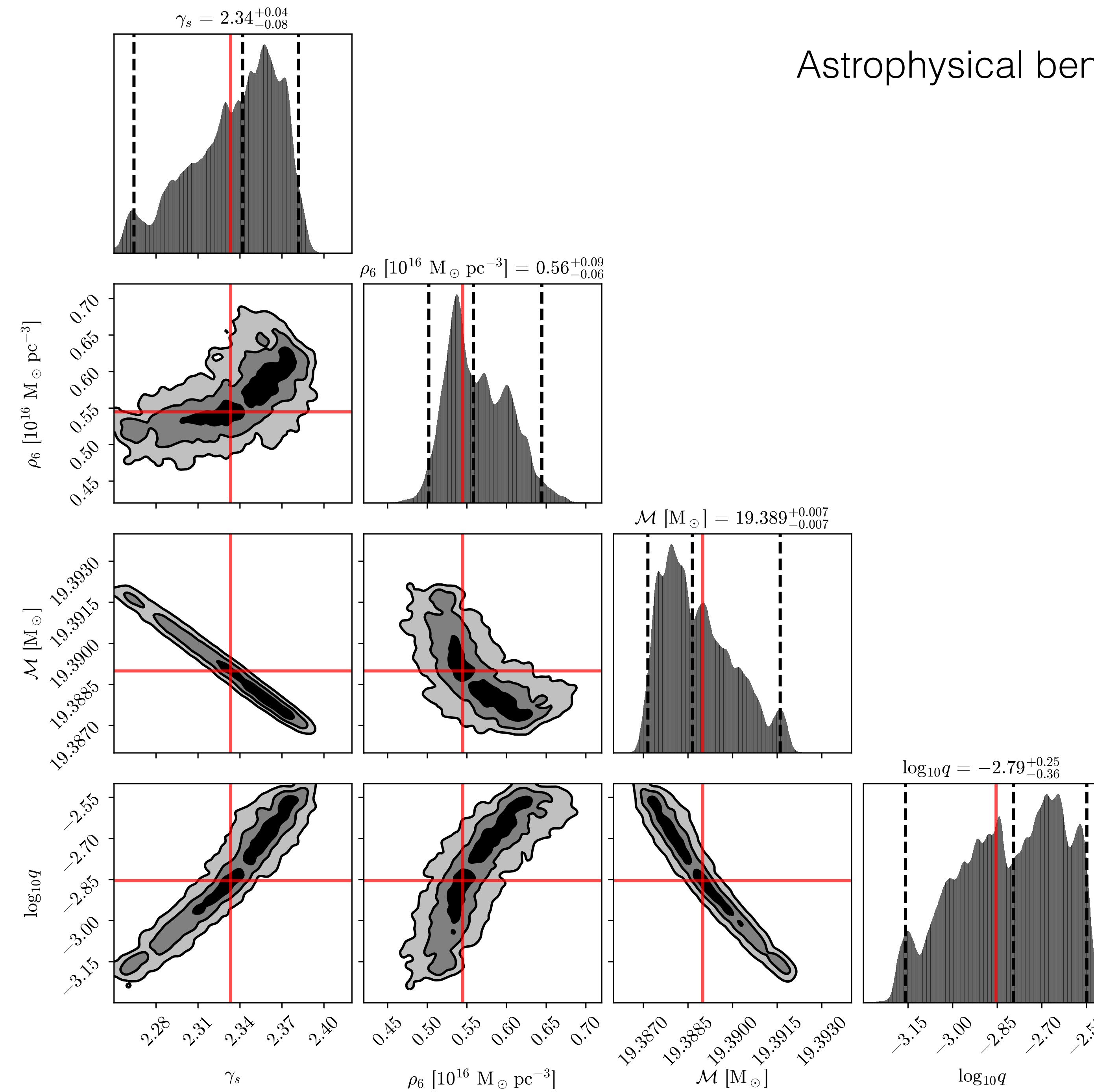
By matter-radiation equality,  $M_{\text{halo}} \sim M_{\text{PBH}}$

shamelessly ripped off from Daniele Gaggero]

[[Bertschinger \(1985\)](#)]  
[[0706.0864](#), [1901.08528](#)]

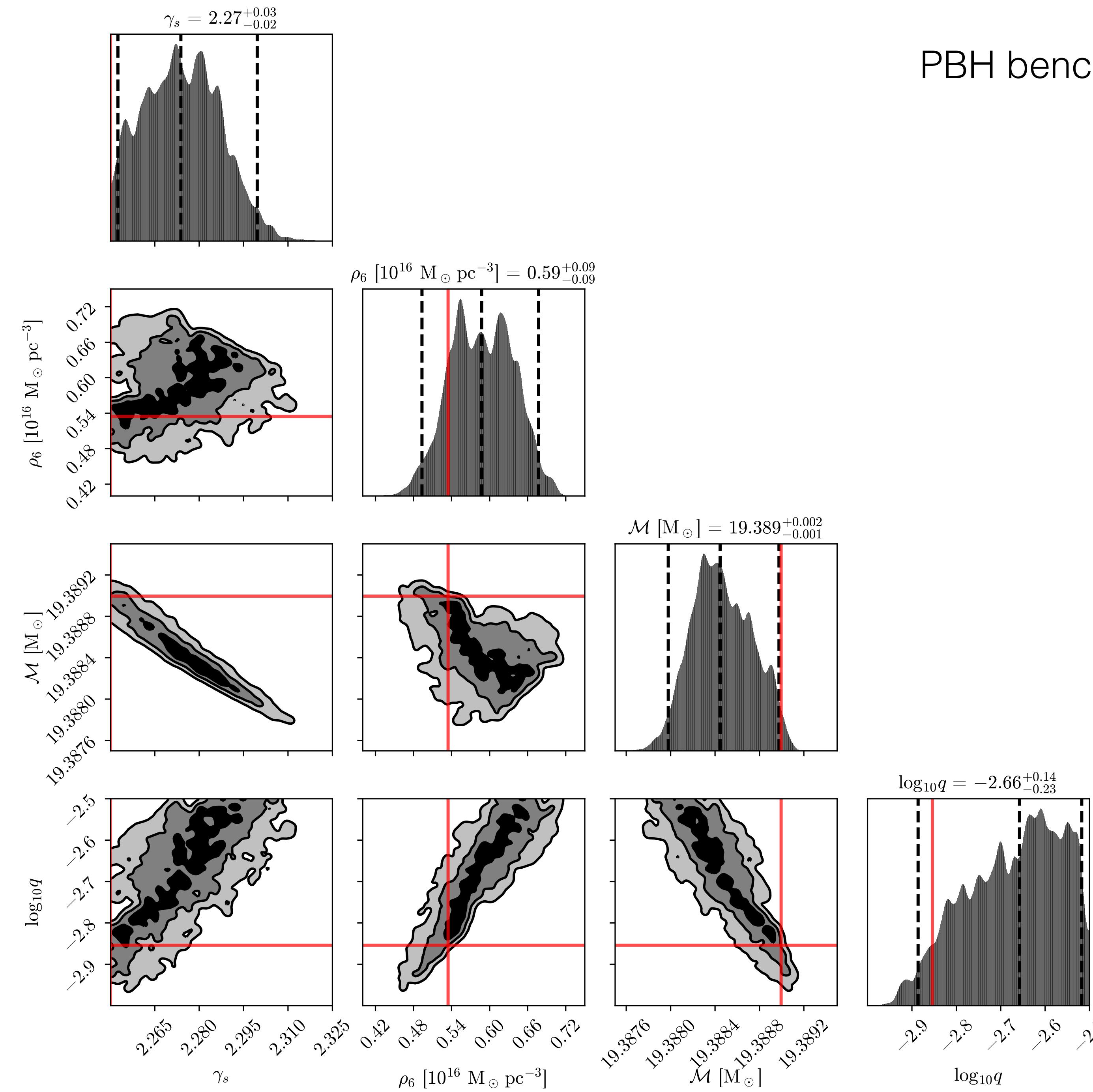
# Measurability

Astrophysical benchmark

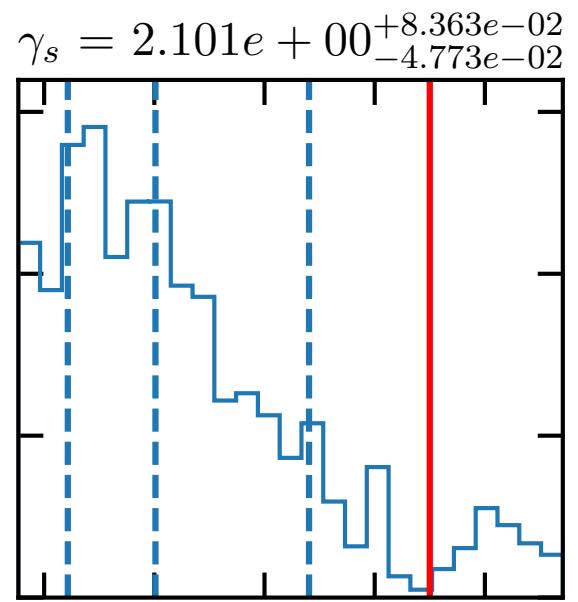


# Measurability

PBH benchmark

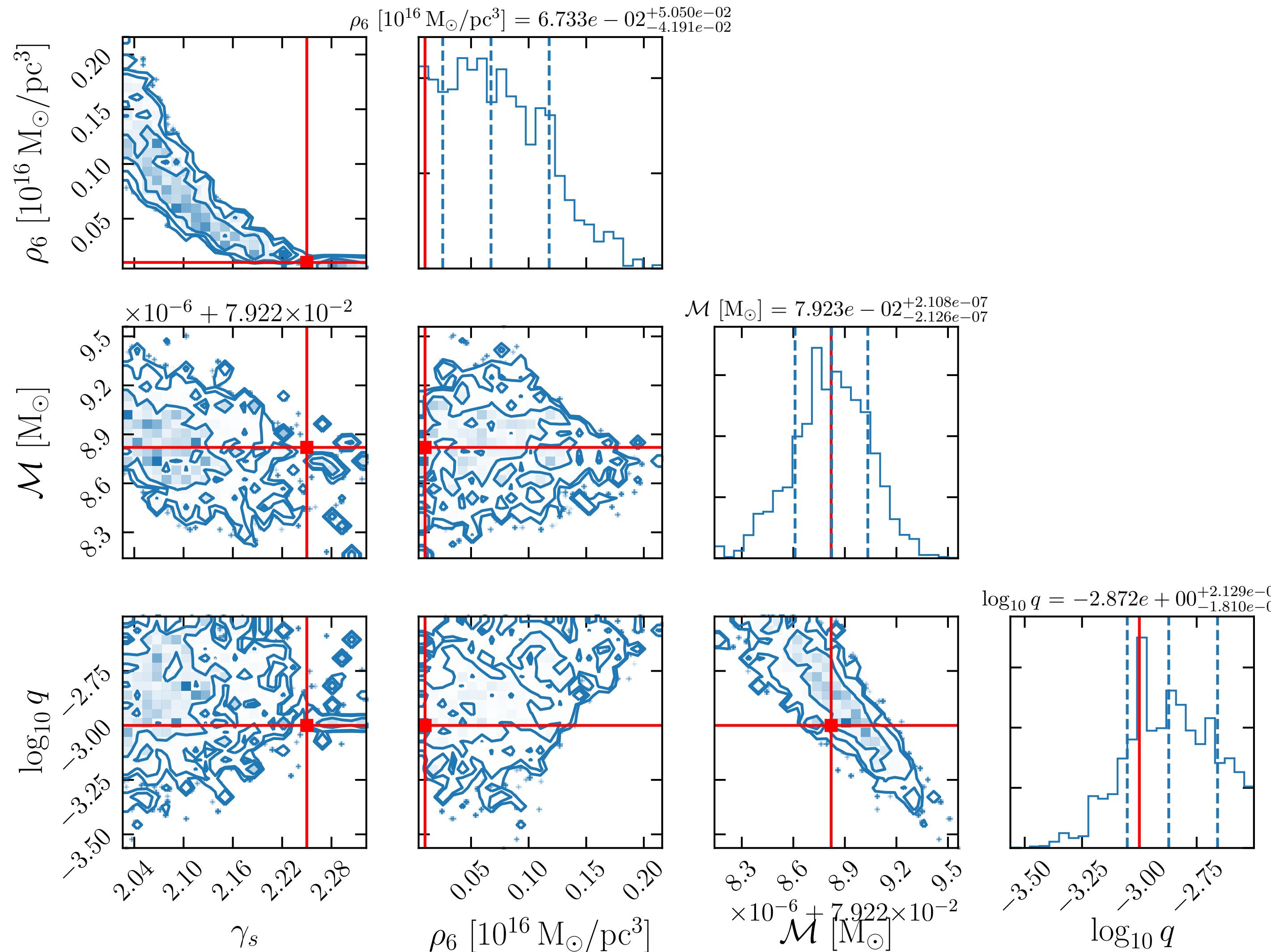


# Low mass PBHs



$m_1 = 5 M_\odot$ ;  $m_2 = 0.005 M_\odot$ ; aLIGO

**PRELIMINARY**

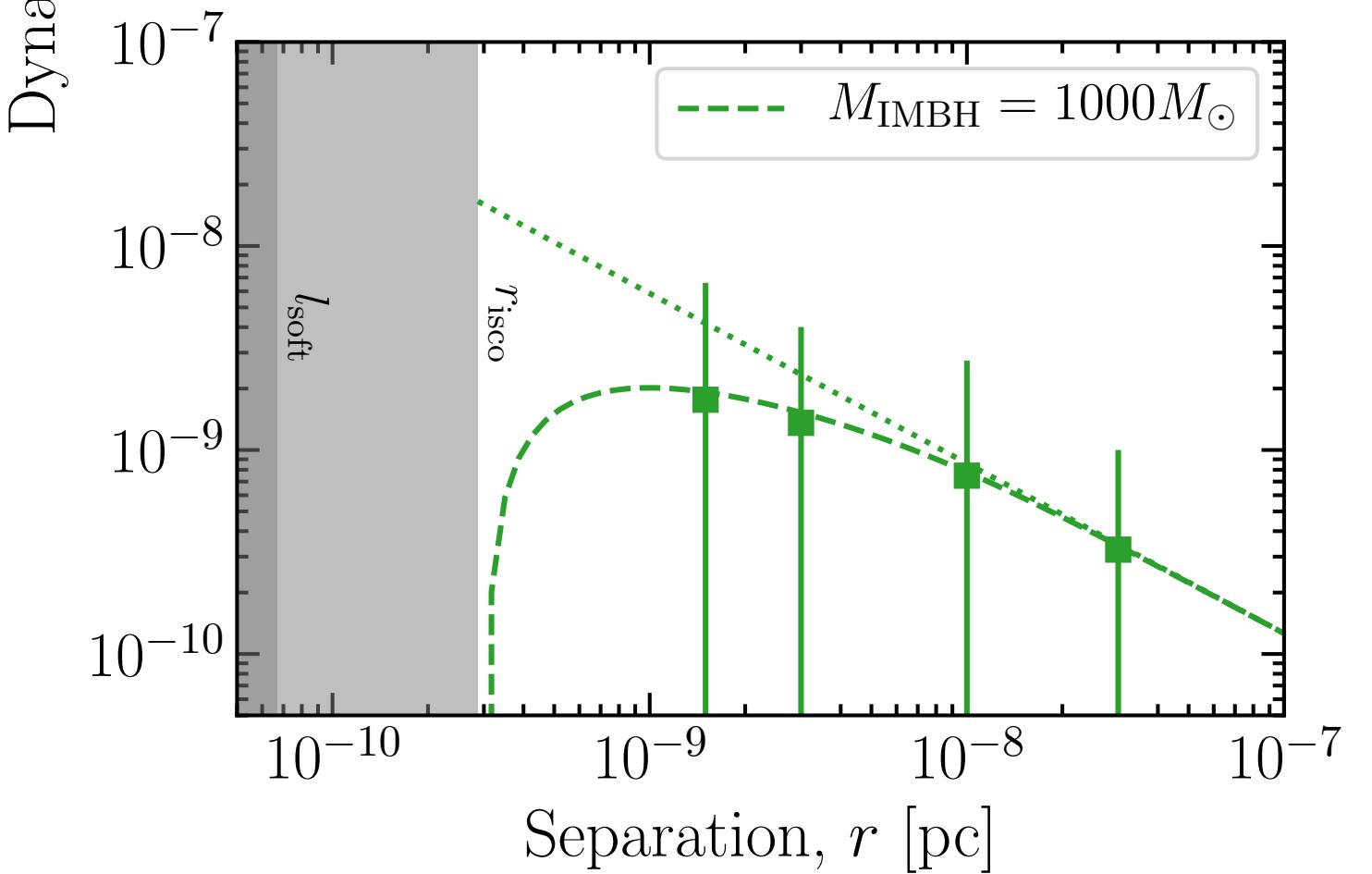
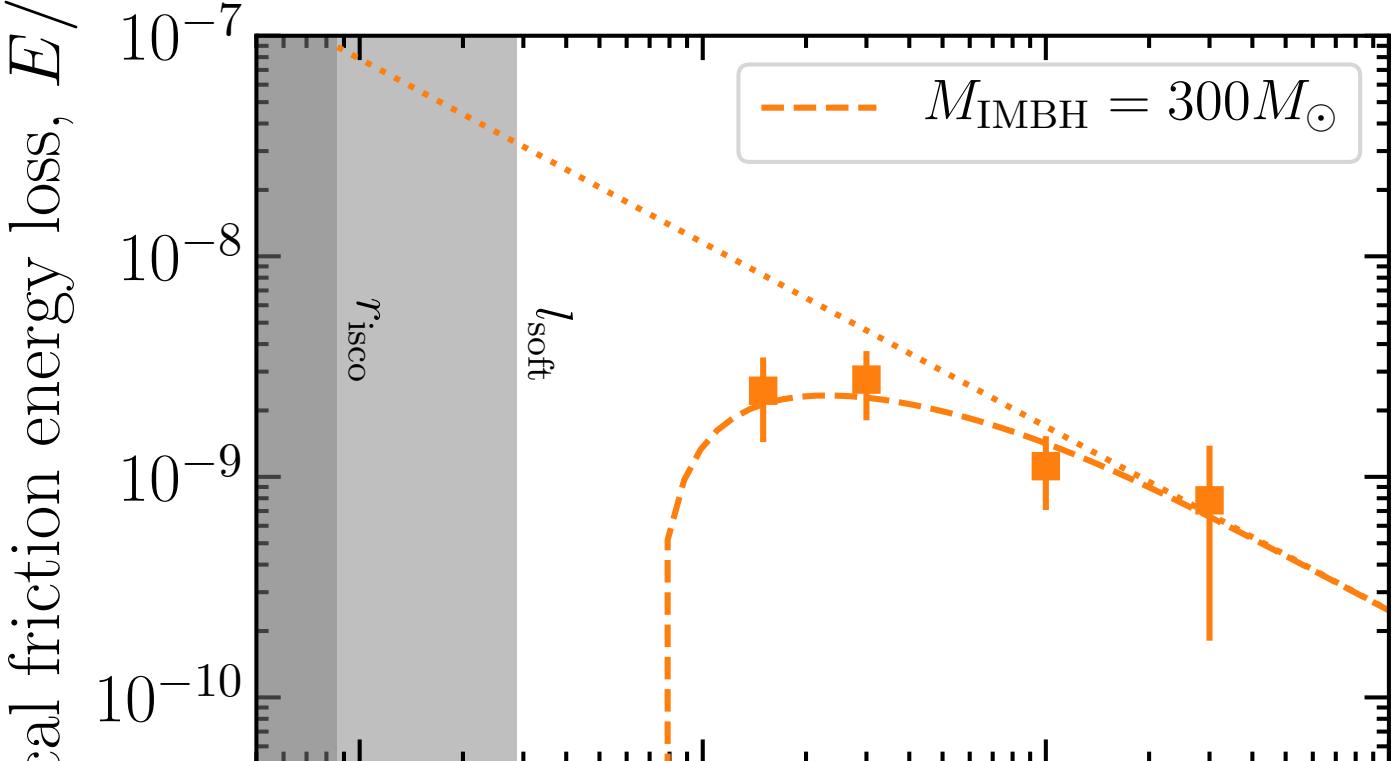
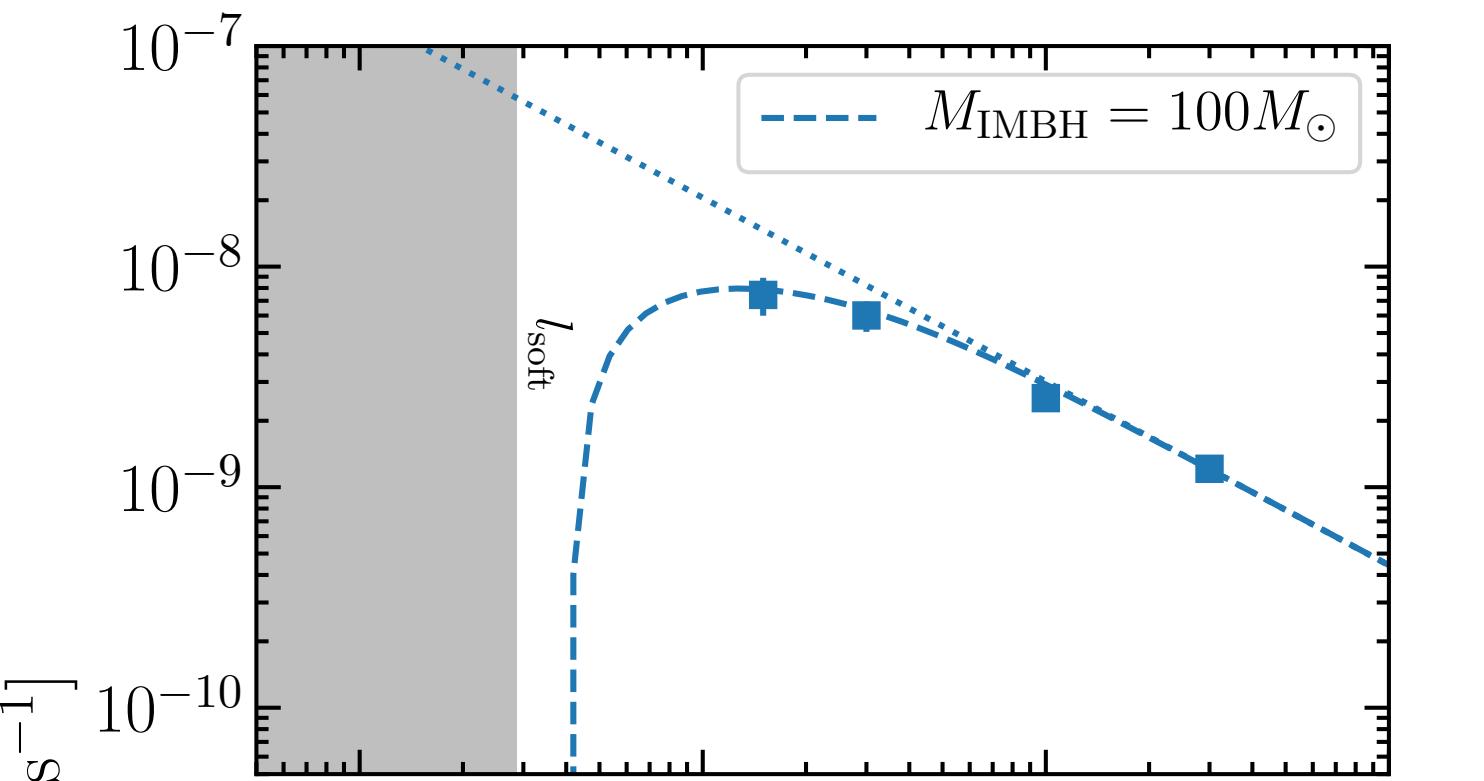


# N-body results

Dependence of dynamical friction force on mass and separation matches expectations

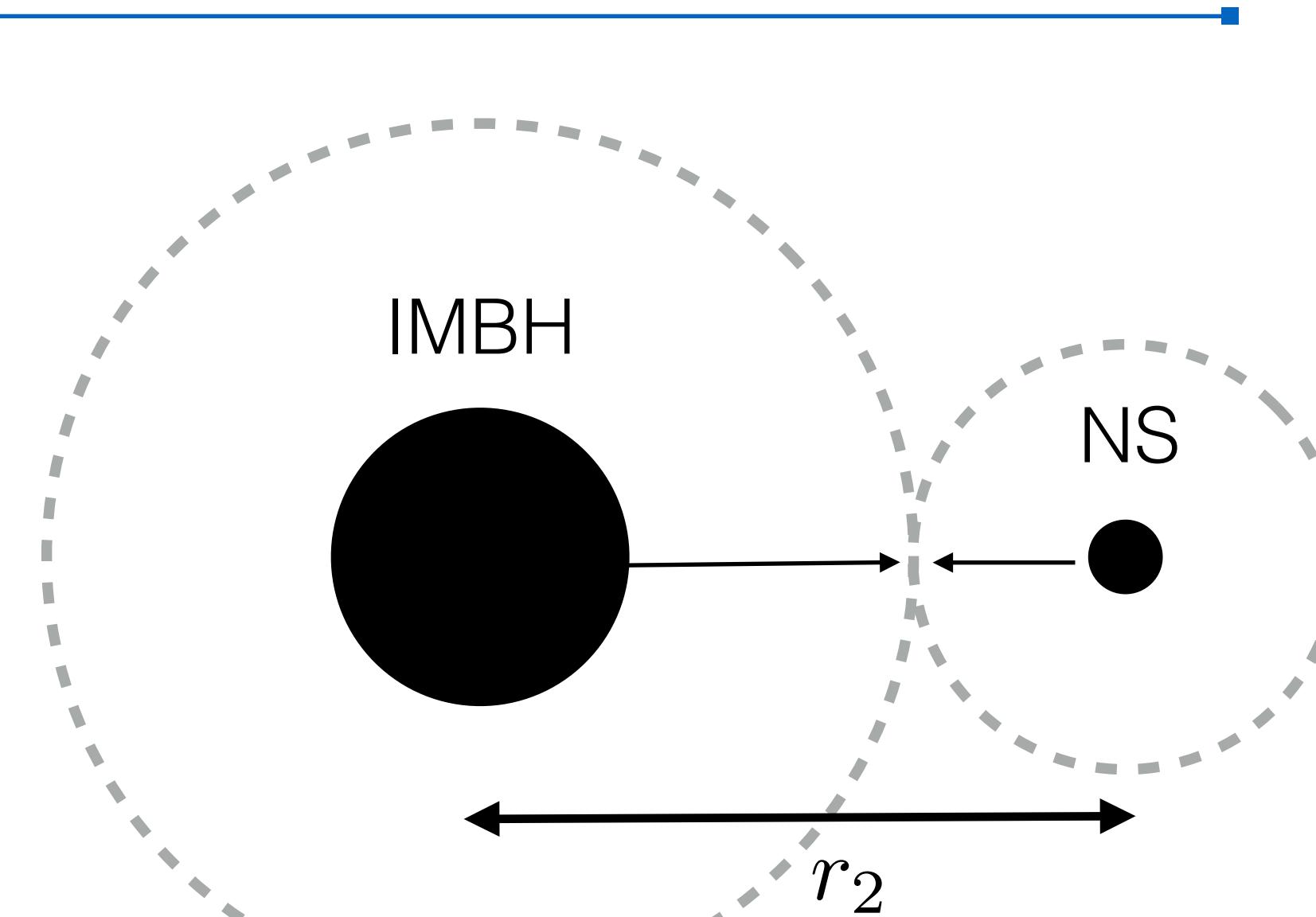
Dynamical friction traces local DM density (to better than 1%)

Drop off in DF force at small separations due to softening of simulations



# N-body results

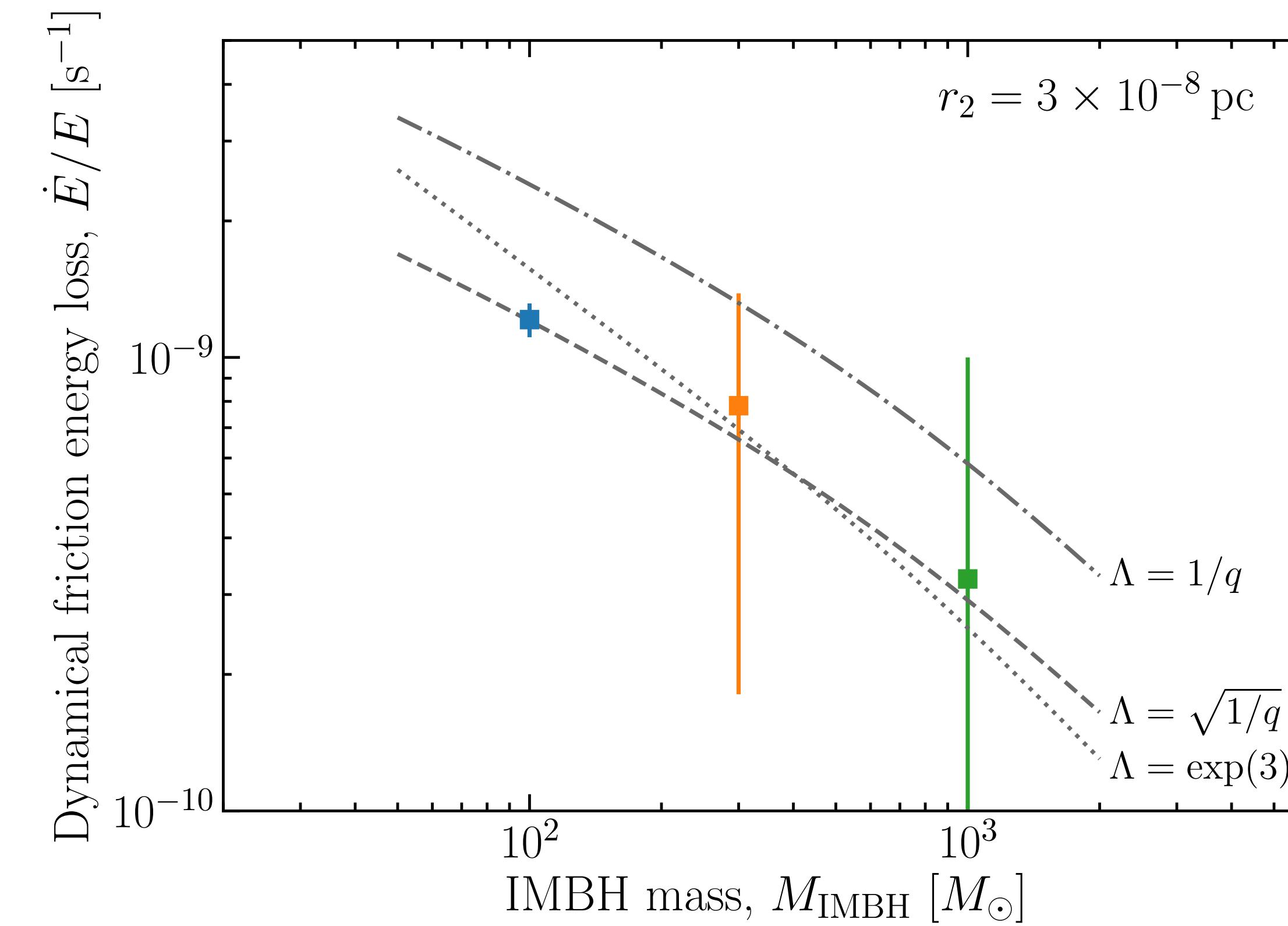
$$q \equiv m_{\text{NS}}/m_{\text{IMBH}} \ll 1$$



$$\begin{aligned}\Lambda &= b_{\max} \frac{v_0^2}{G m_{\text{NS}}} \\ &= \frac{b_{\max}}{q r_2} \\ &= 1/\sqrt{q}\end{aligned}$$

Allows us to calibrate the maximum impact parameter; tells us which particles scatter with the NS.

$$b_{\max} = \sqrt{q} r_2 \sim 3\% r_2$$

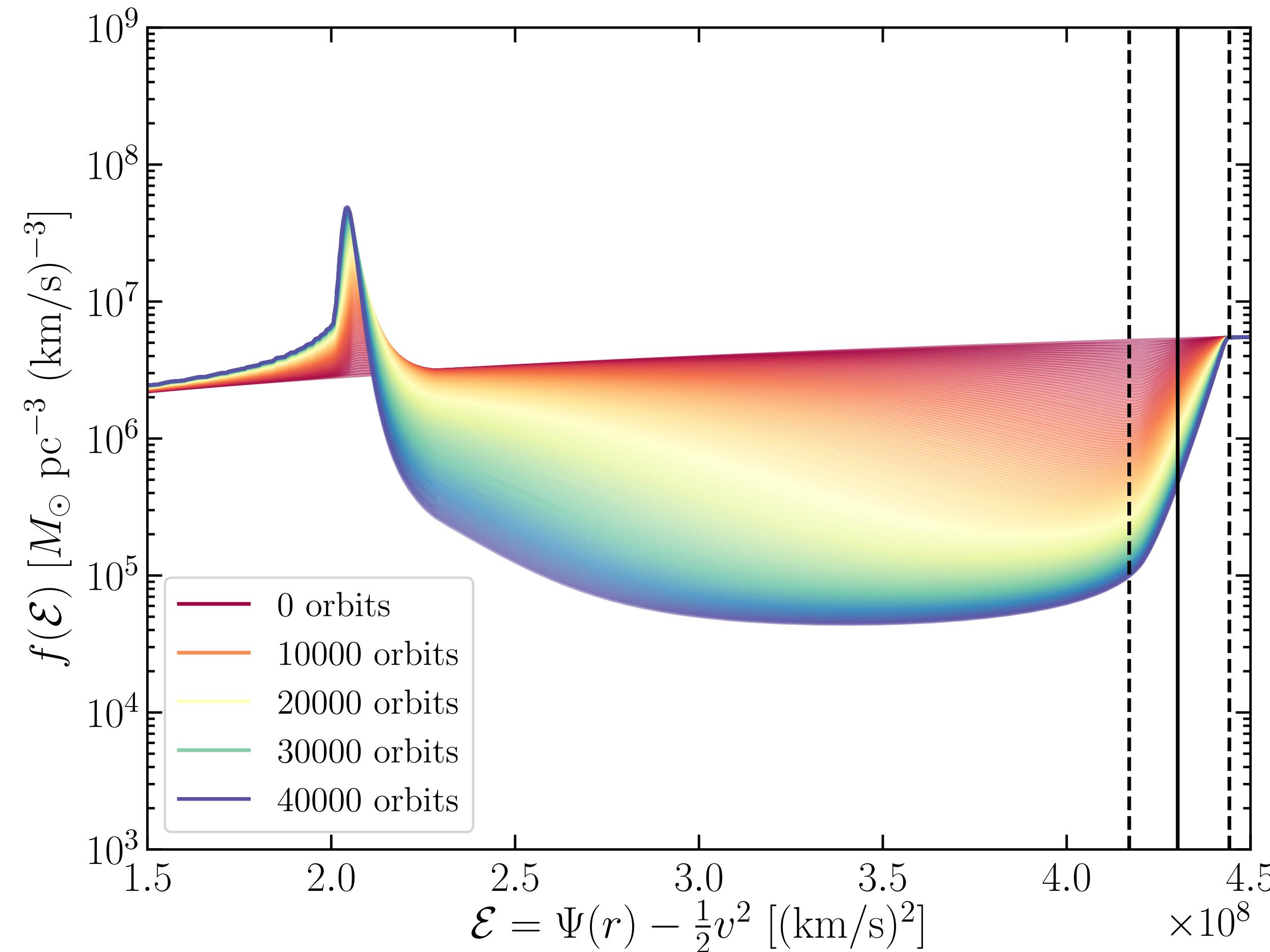


# Assumptions

---

- Spherical symmetry and isotropy of the DM halo
- DM particles only scatter within an impact parameter
$$b < b_{\max} = \Lambda \times G_N M_{\text{NS}} / v_{\text{NS}}^2$$
- DM distribution is ‘locally’ uniform
$$b_{\max} \ll r_0$$
- Halo ‘relaxation’ is instantaneous
- Orbital properties evolve slowly compared to the orbital period

# Distribution function



Self-consistently reconstruct density from distribution function:

$$\rho(r) = 4\pi \int_0^{v_{\max}(r)} v^2 f(\mathcal{E}) dv$$

# Numbers of cycles

$$m_1 = 10^3 M_\odot, N_{\text{cycles}} = 5.71 \times 10^6 \text{ in vacuum}$$

	$\gamma_{\text{sp}} = 1.5$	$\gamma_{\text{sp}} = 2.2$	$\gamma_{\text{sp}} = 2.3$	$\gamma_{\text{sp}} = 2.\bar{3}$
Static	< 1	$2.4 \times 10^4$	$1.6 \times 10^5$	$2.9 \times 10^5$
Dynamic	< 1	$2.7 \times 10^2$	$1.9 \times 10^3$	$3.5 \times 10^3$

$$m_1 = 10^4 M_\odot, N_{\text{cycles}} = 3.20 \times 10^6 \text{ in vacuum}$$

	$\gamma_{\text{sp}} = 1.5$	$\gamma_{\text{sp}} = 2.2$	$\gamma_{\text{sp}} = 2.3$	$\gamma_{\text{sp}} = 2.\bar{3}$
Static	< 1	$1.4 \times 10^3$	$8.7 \times 10^3$	$1.6 \times 10^4$
Dynamic	< 1	$6.2 \times 10^2$	$4.0 \times 10^3$	$7.4 \times 10^3$

TABLE I. **Change in the number of cycles  $\Delta N_{\text{cycles}}$  during the inspiral.** Change in the total number of GW cycles due to dynamical friction, starting 5 years from the merger.

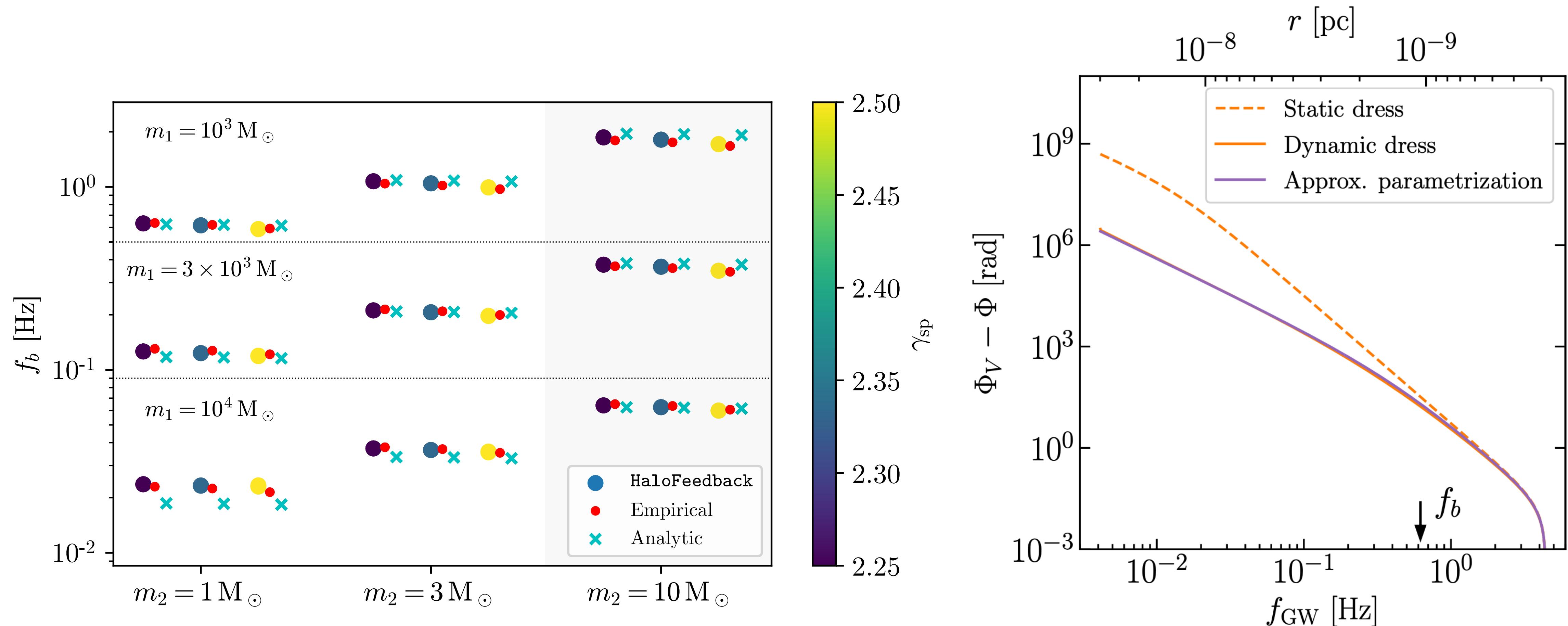
# Phase parametrisation

$$\vartheta = \frac{5}{2\gamma_e}, \quad \lambda = \frac{11 - 2(\gamma_{\text{sp}} + \gamma_e)}{3}, \quad \eta = \frac{5 + 2\gamma_e}{2(8 - \gamma_{\text{sp}})} \left( \frac{f_{\text{eq}}}{f_b} \right)^{\frac{11 - 2\gamma_{\text{sp}}}{3}}$$

$$\hat{\Phi}(f) \equiv \Phi^V(f) \\ \times \left\{ 1 - \eta y^{-\lambda} \left[ 1 - {}_2 F_1 \left( 1, \vartheta, 1 + \vartheta, -y^{-\frac{5}{3\vartheta}} \right) \right] \right\}$$

$$f_b = \beta \left( \frac{m_1}{1000 M_\odot} \right)^{-\alpha_1} \left( \frac{m_2}{M_\odot} \right)^{\alpha_2} \left[ 1 + \zeta \log \frac{\gamma_{\text{sp}}}{\gamma_r} \right], \quad (35)$$

where  $\alpha_1 = 1.4412$ ,  $\alpha_2 = 0.4511$ ,  $\beta = 0.8163 \text{ Hz}$ ,  $\zeta = -0.4971$  and  $\gamma_r = 1.4396$ .



# Axions and neutron stars

Produce a photon with axion energy  $m_a \sim 10^{-6} \text{ eV} \sim 240 \text{ MHz}$



Conversion happens at a radius  $r_c$ , with probability:  $p_{a\gamma} \propto \frac{g_{a\gamma\gamma}^2 B(r_c)^2}{2v_c}$

Radiated power is given by:  $\frac{d\mathcal{P}}{d\Omega} \sim 2 \times p_{a\gamma} \rho_{\text{DM}}(r_c) v_c r_c^2$

Probe axions in the mass range

$$m_a \sim 10^{-7} \text{ eV} \quad \text{up to} \quad m_a \sim 10^{-5} \text{ eV}$$

Frequency range of  
radio telescopes

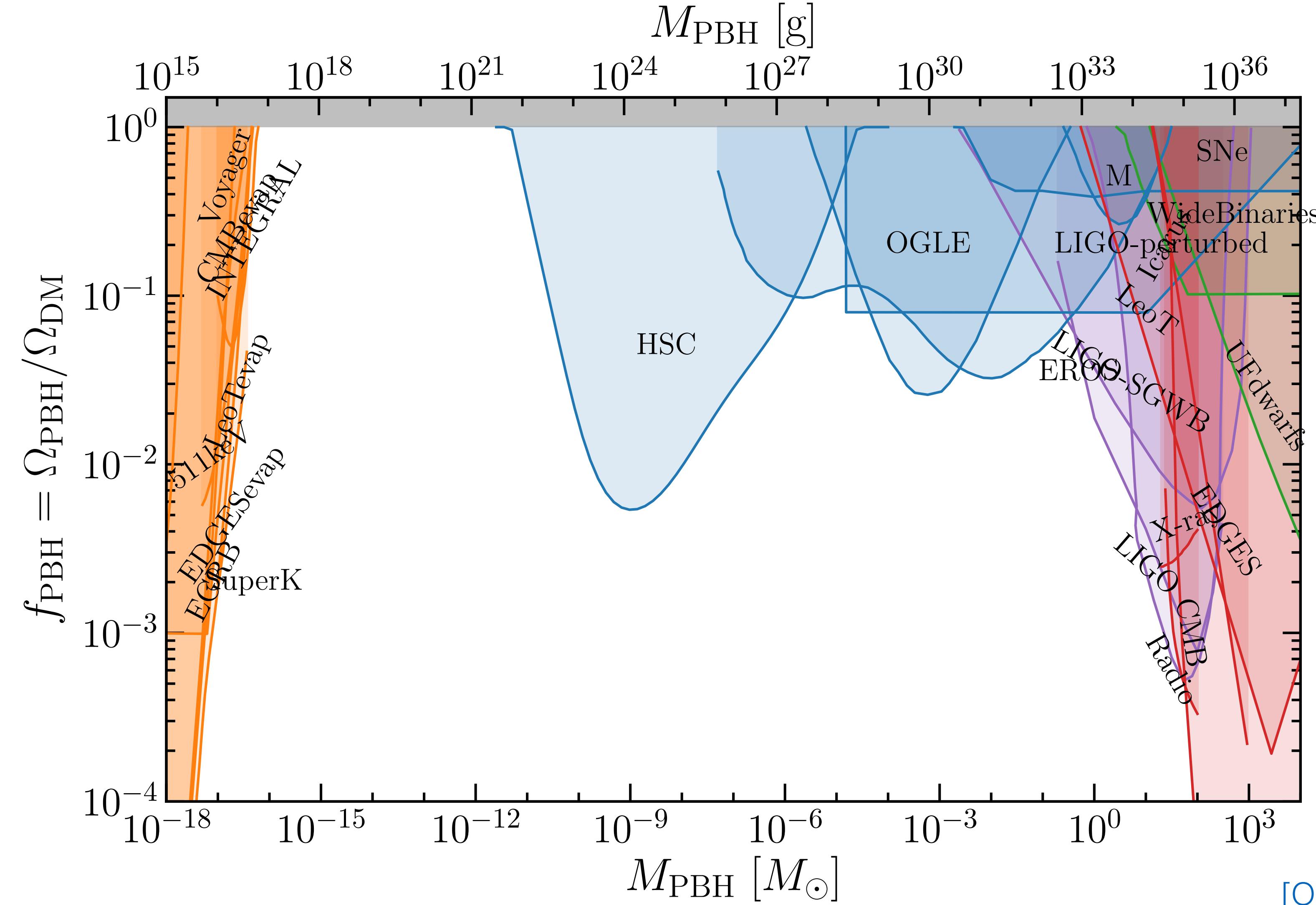
Require conversion  
*outside NS*

[[1803.08230](#), [1804.03145](#), [1811.01020](#), [1910.11907](#)]

# PBH Constraints

[Green & **BJK**, 2007.10722]

[Code online: [github.com/bradkav/PBHbounds](https://github.com/bradkav/PBHbounds)]

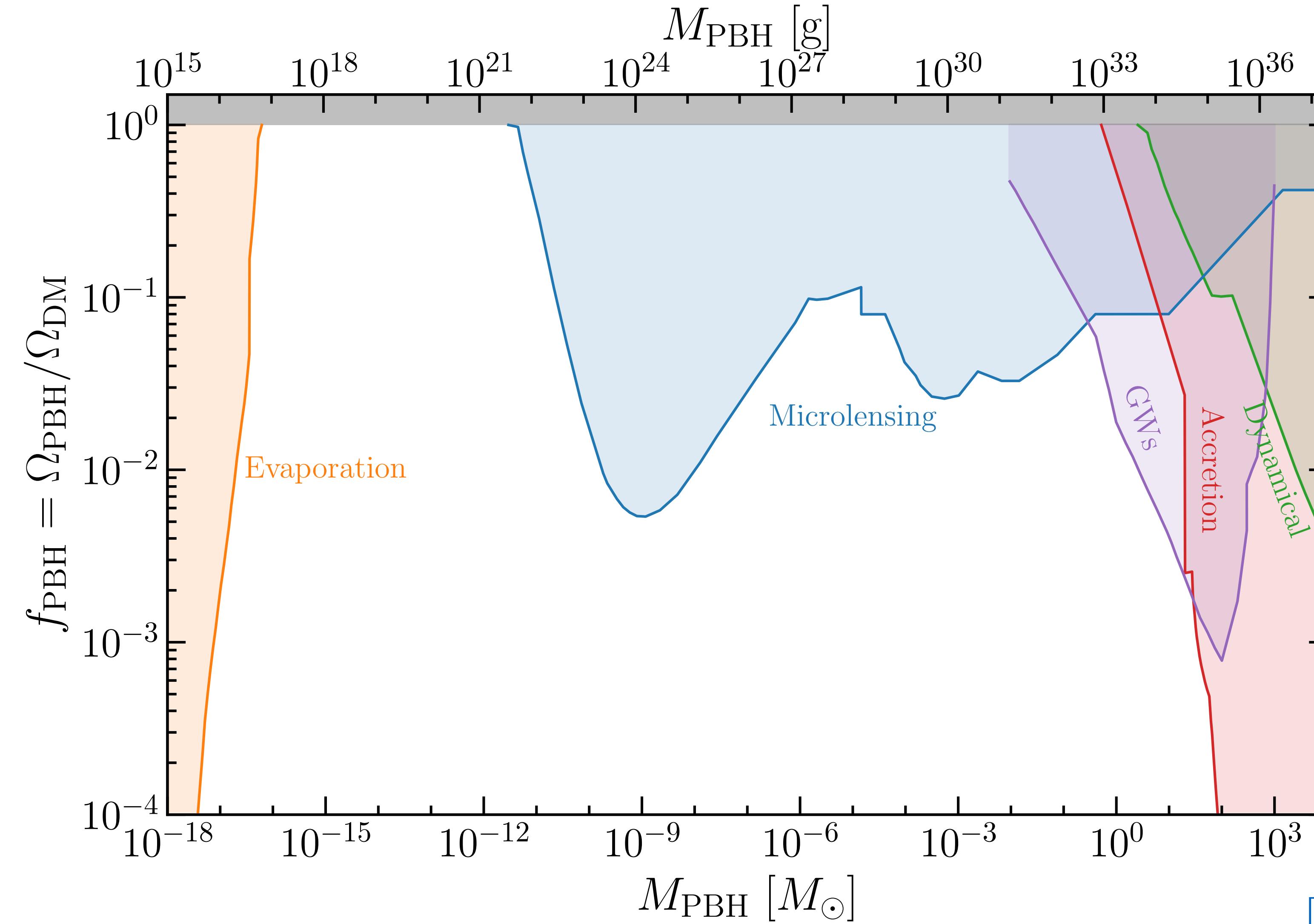


[Other reviews: [1801.05235](#),  
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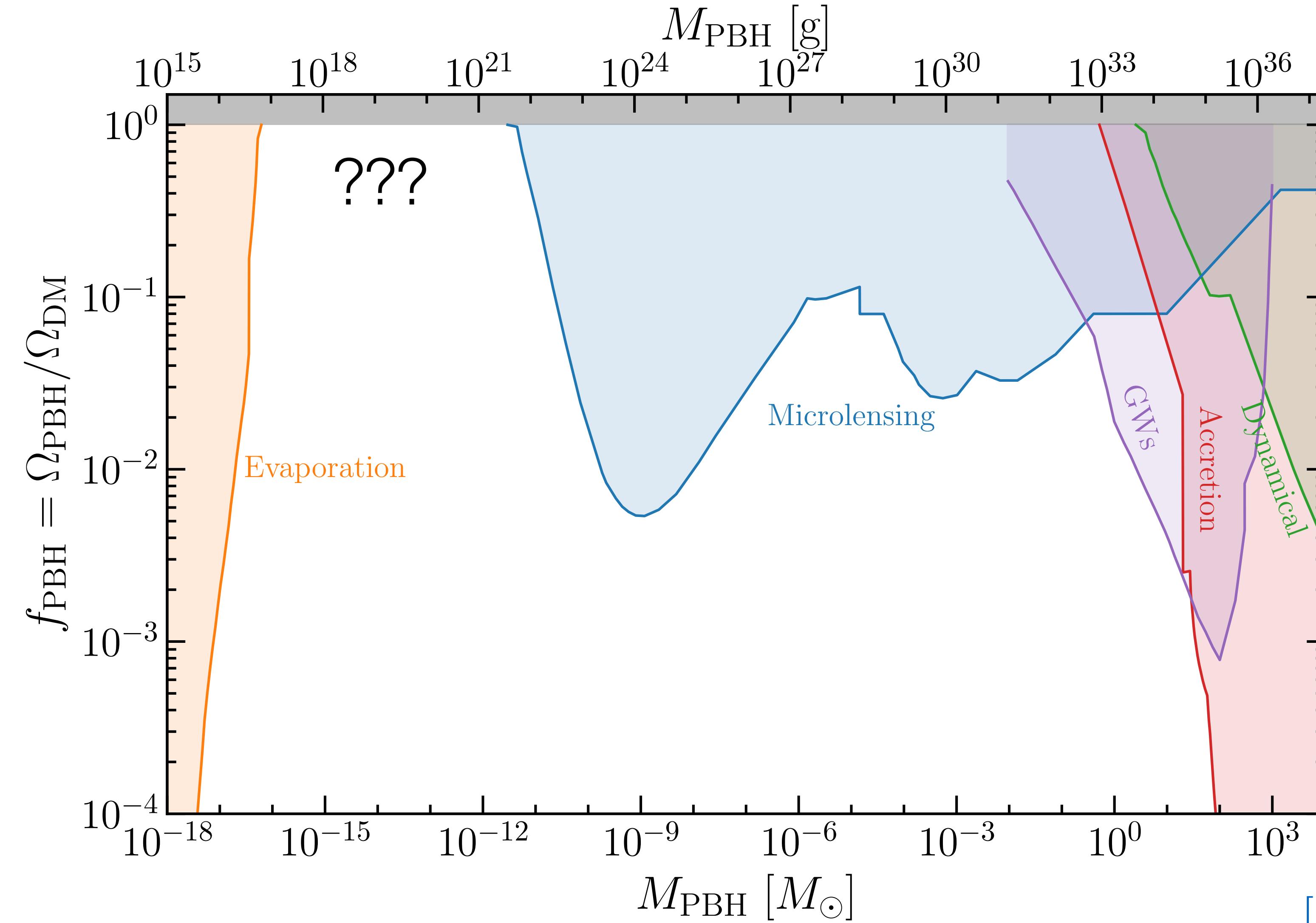


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[2002.12778](https://arxiv.org/abs/2002.12778), [2006.02838](https://arxiv.org/abs/2006.02838)]

# QCD Axion Reach

SKA should be able to probe QCD axion DM in the range  $10^{-7} - 10^{-5}$  eV.

