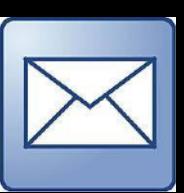


# Black Hole Inspirals: Lessons for Dark Matter



Bradley J. Kavanagh  
(IFCA, UC-CSIC, Santander)

Invisibles Workshop, Bologna  
3rd July 2024



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@BradleyKavanagh



# i F (A)

## Instituto de Física de Cantabria

Keep an eye out for our biannual conference:

*Dark Matter 2025: From the Smallest to the Largest Scales*

...Coming Spring/Summer 2025

[Previous edition: [DM2023](#)]

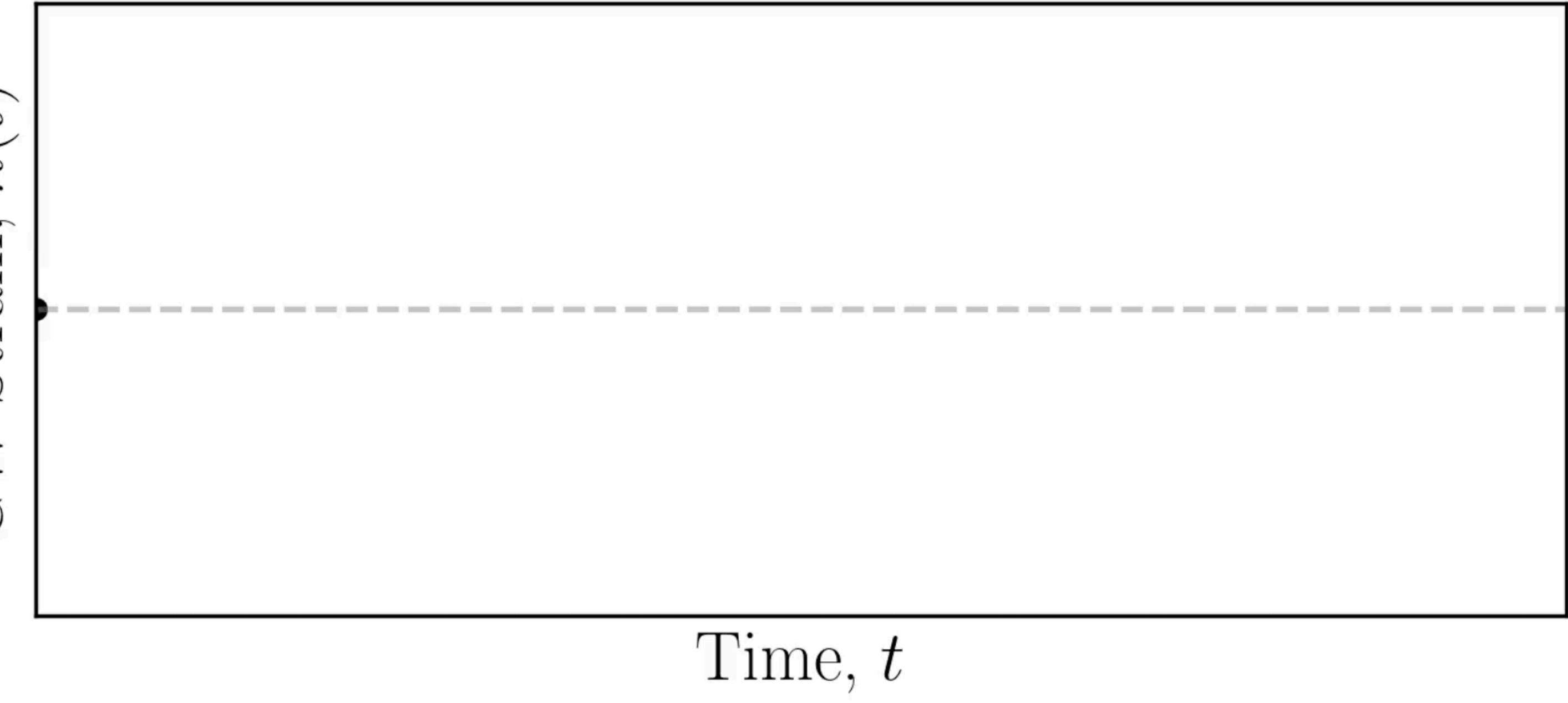


# Gravitational Wave Dephasing

[Animations online]

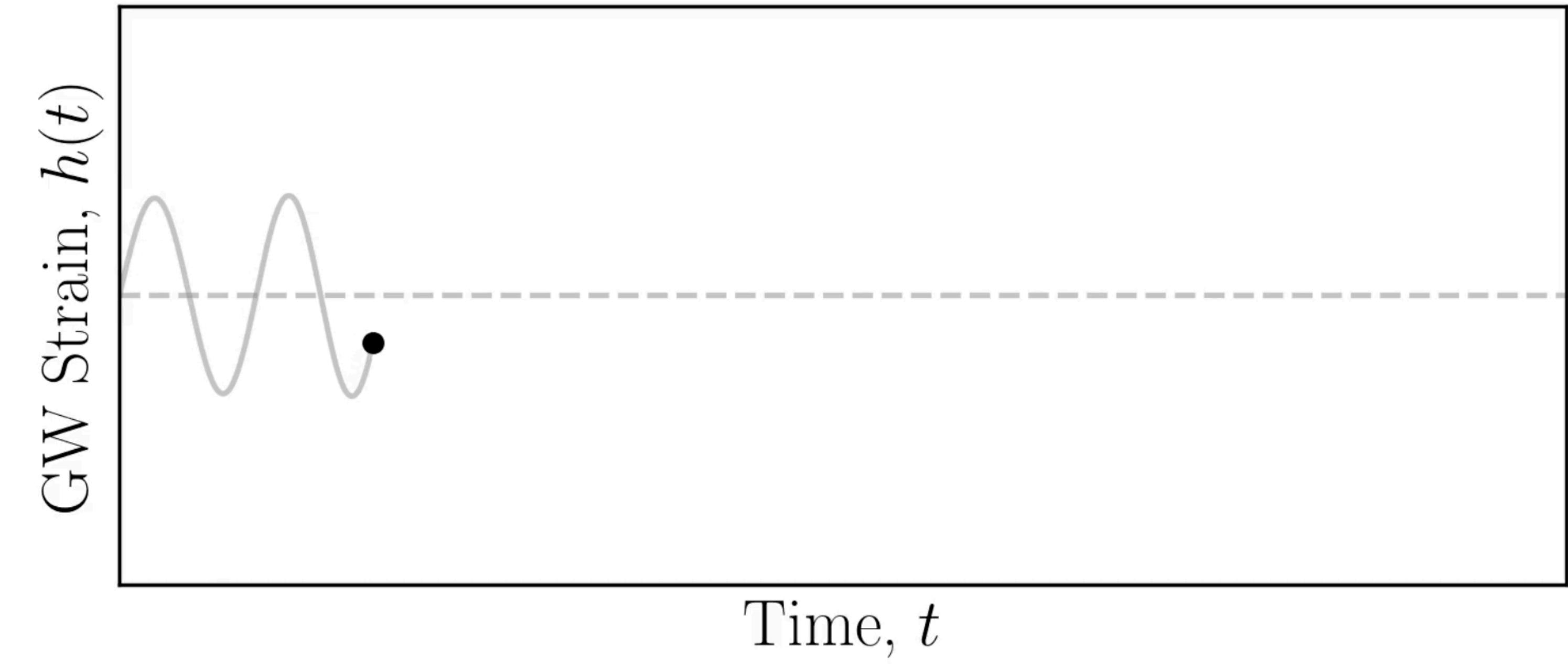
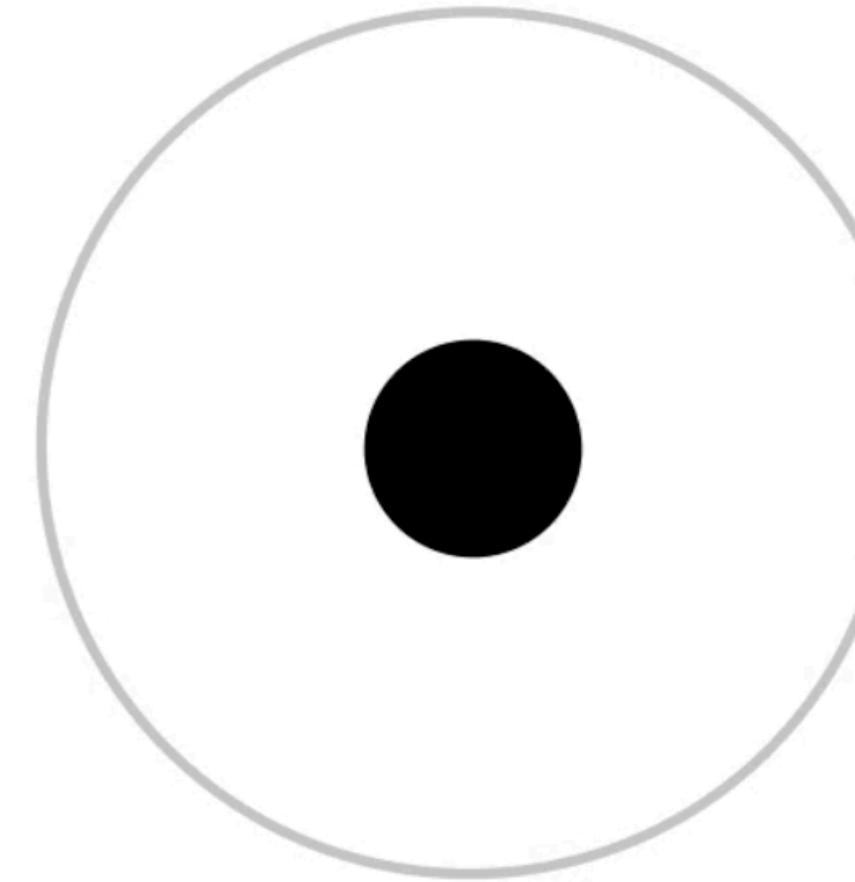


GW Strain,  $h(t)$



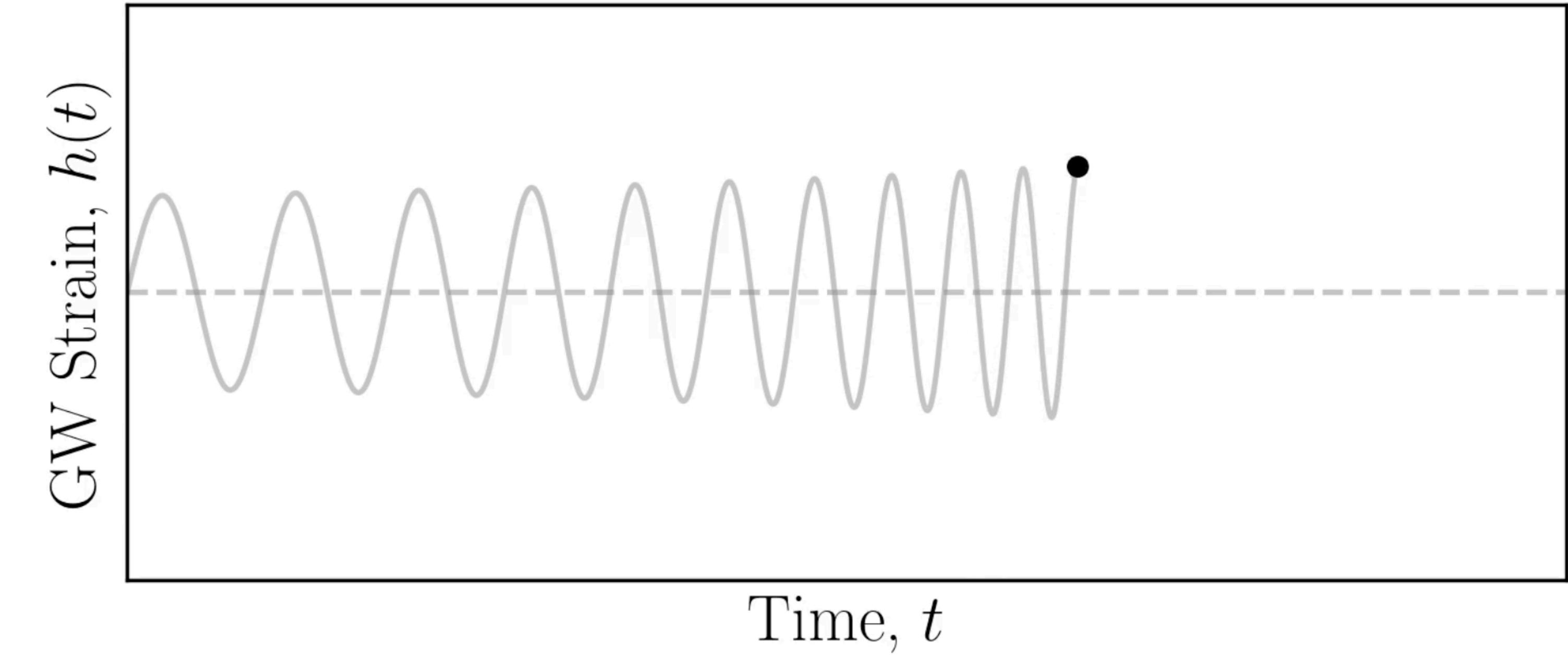
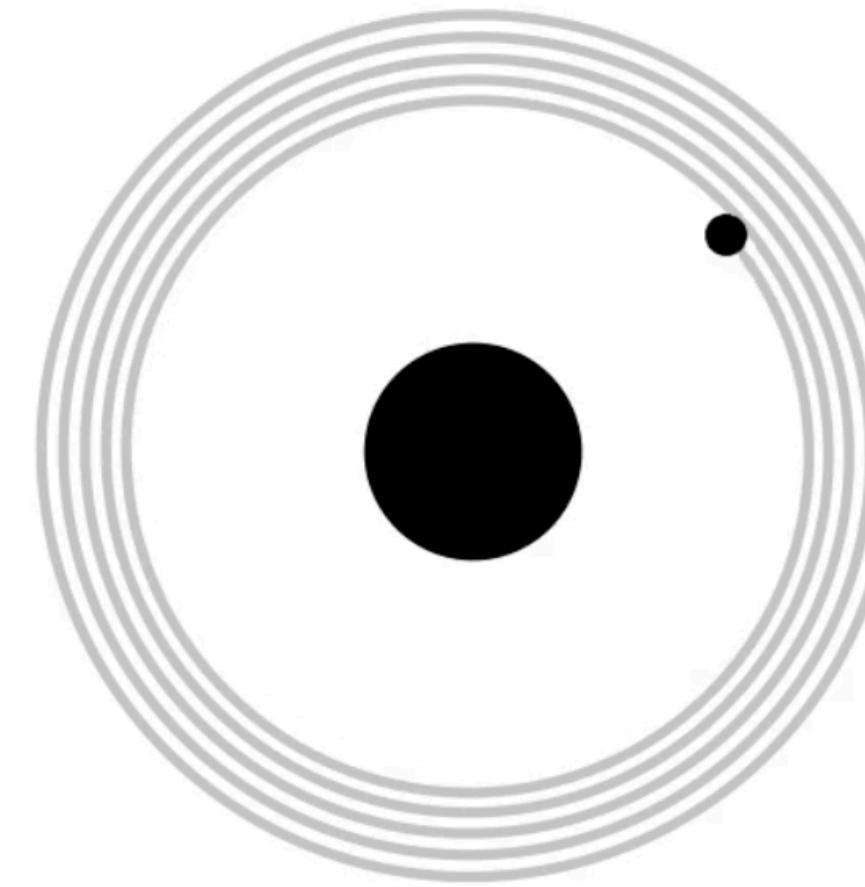
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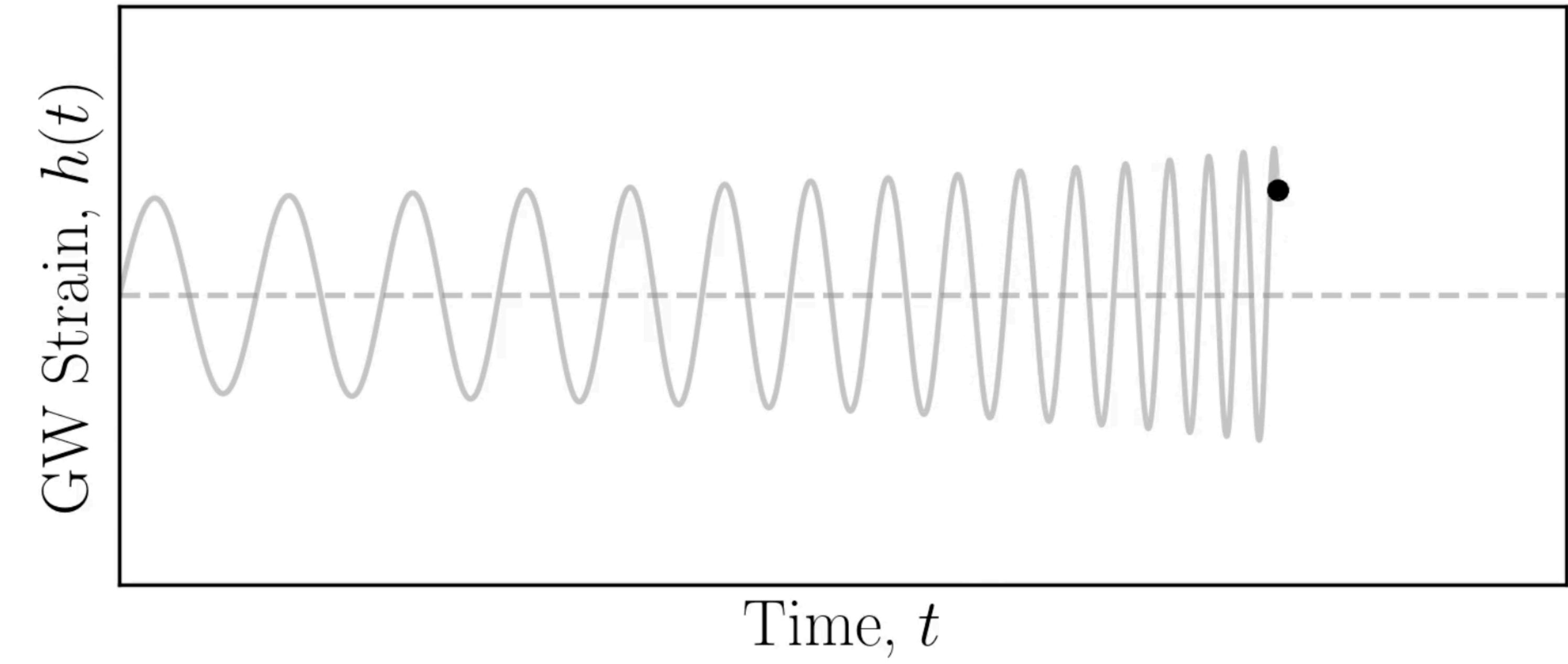
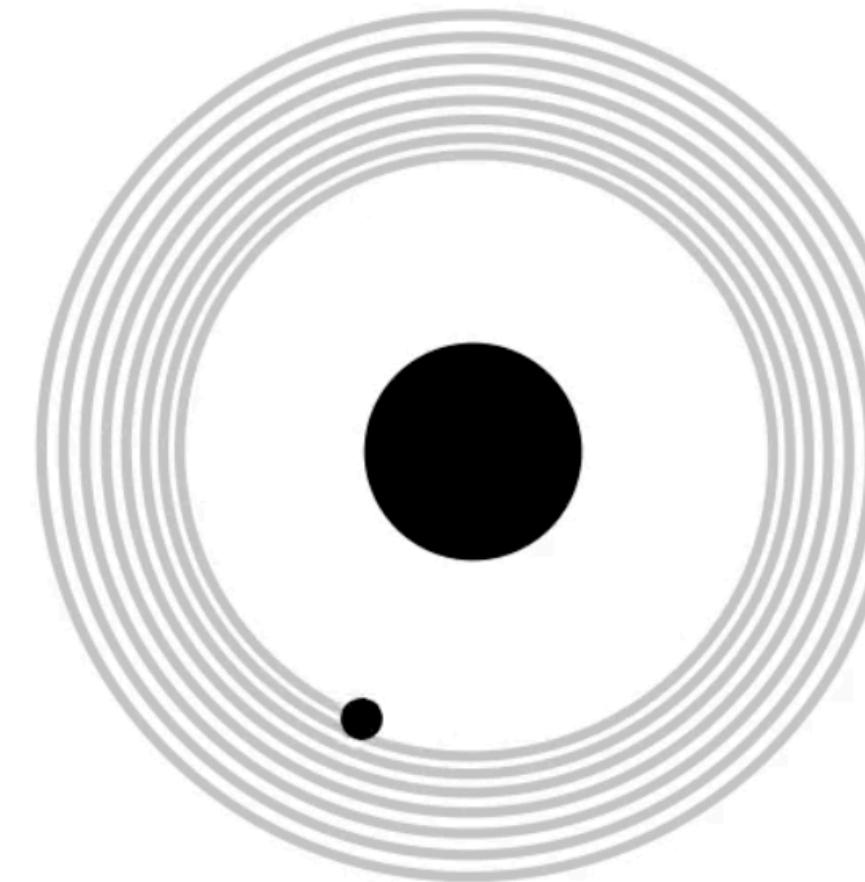
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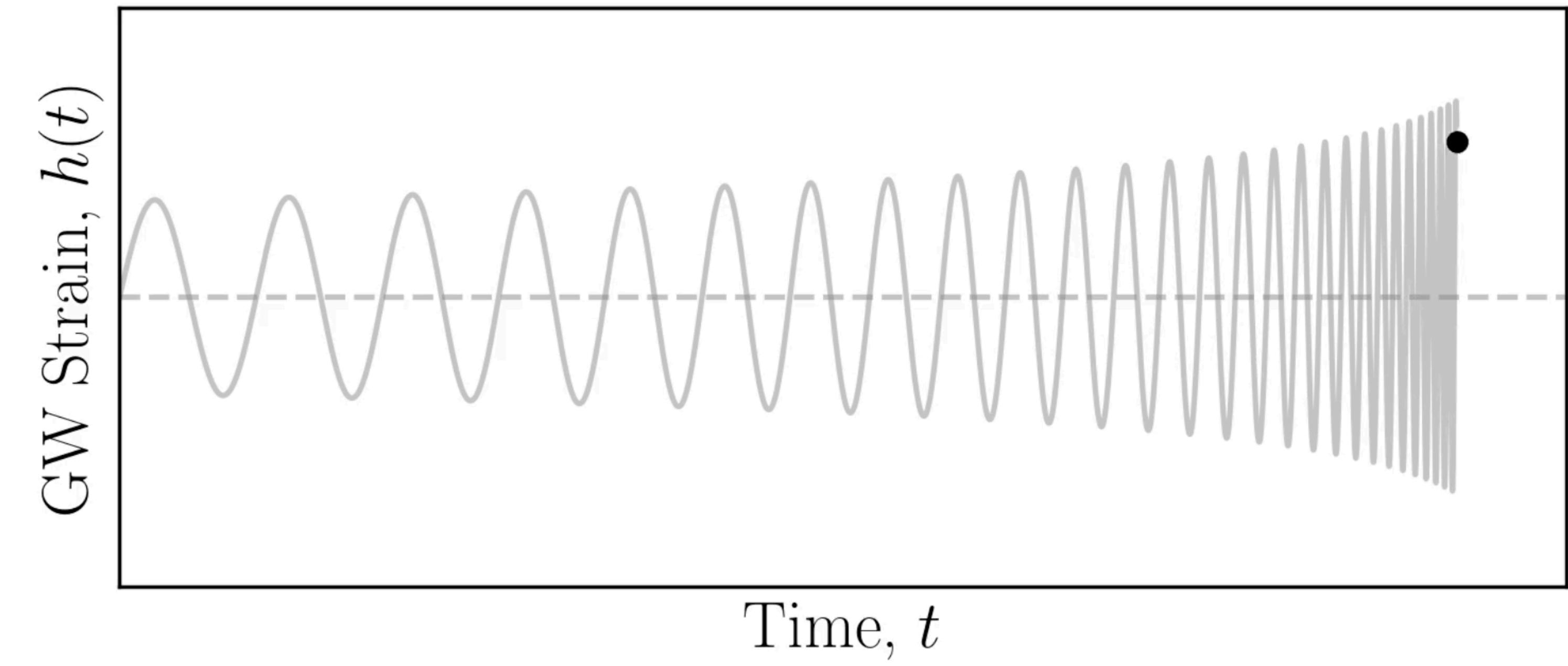
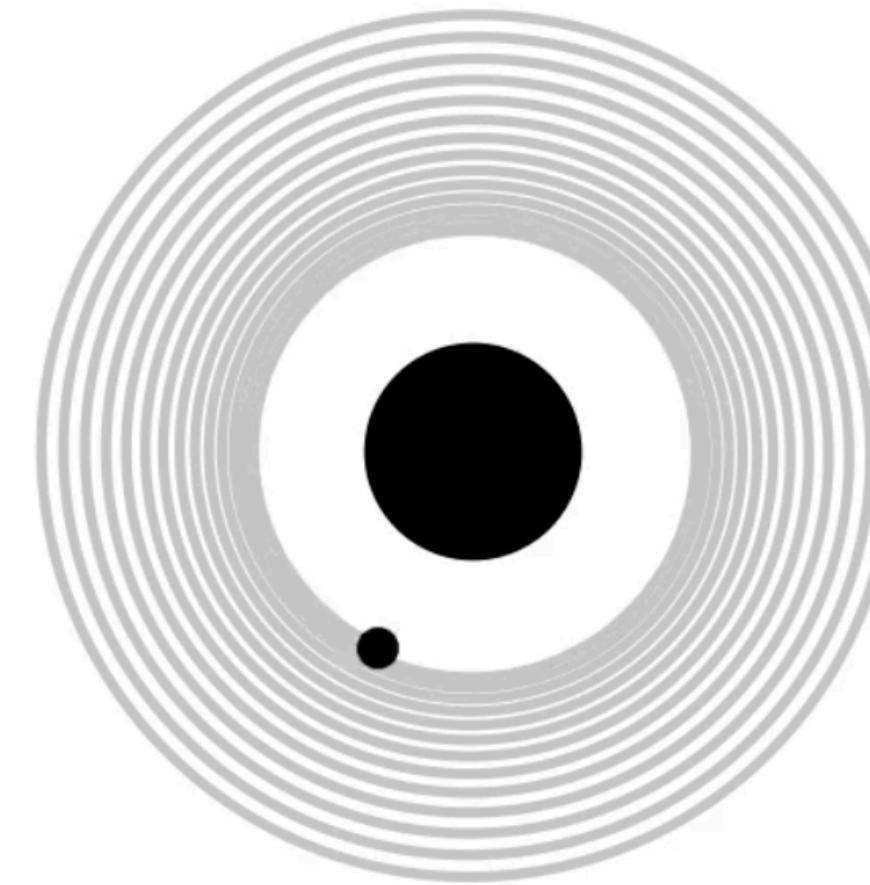
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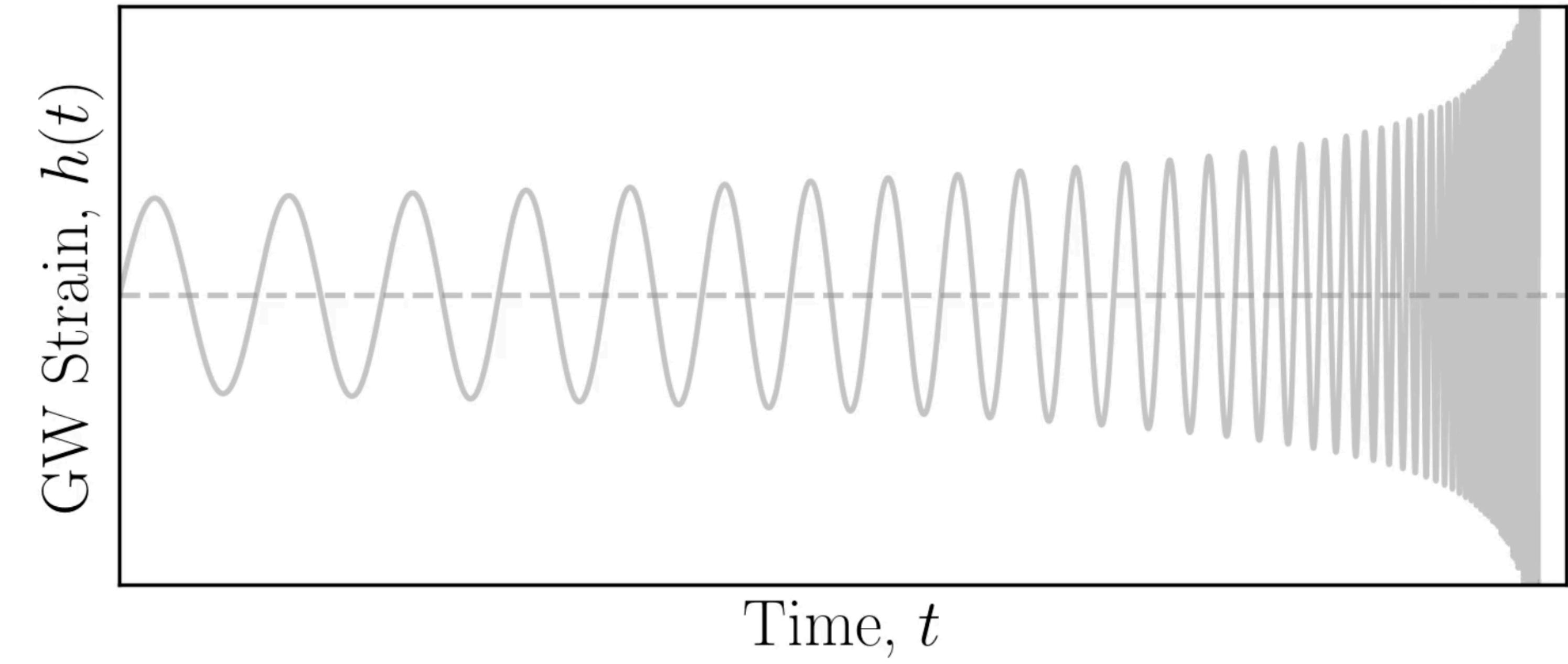
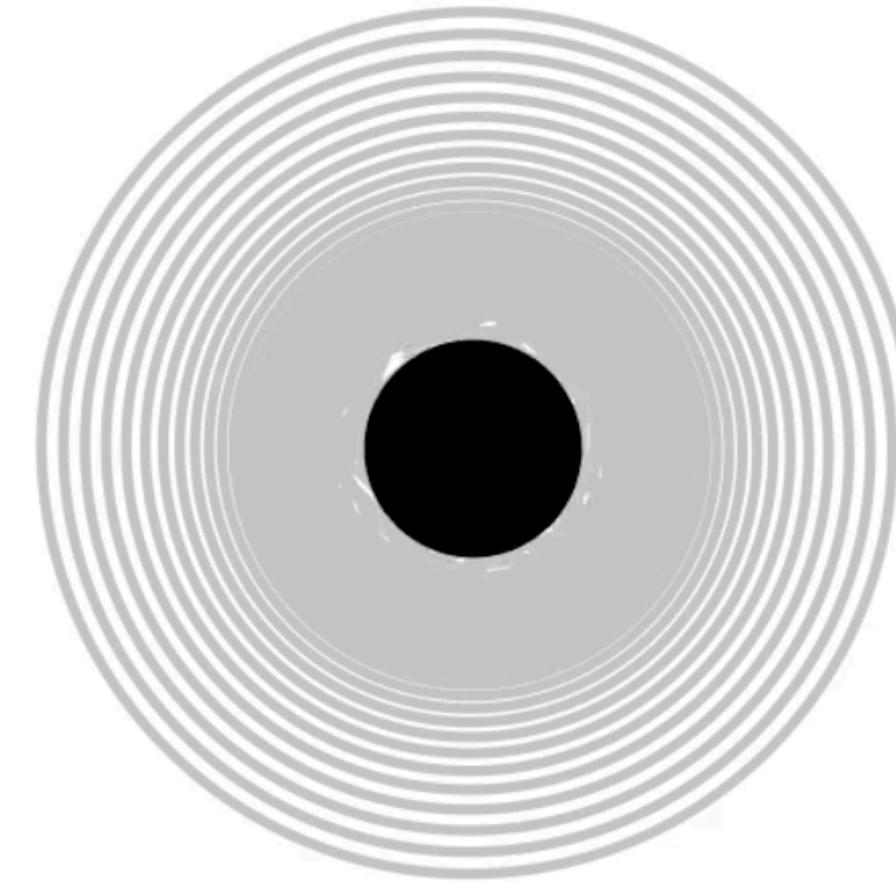
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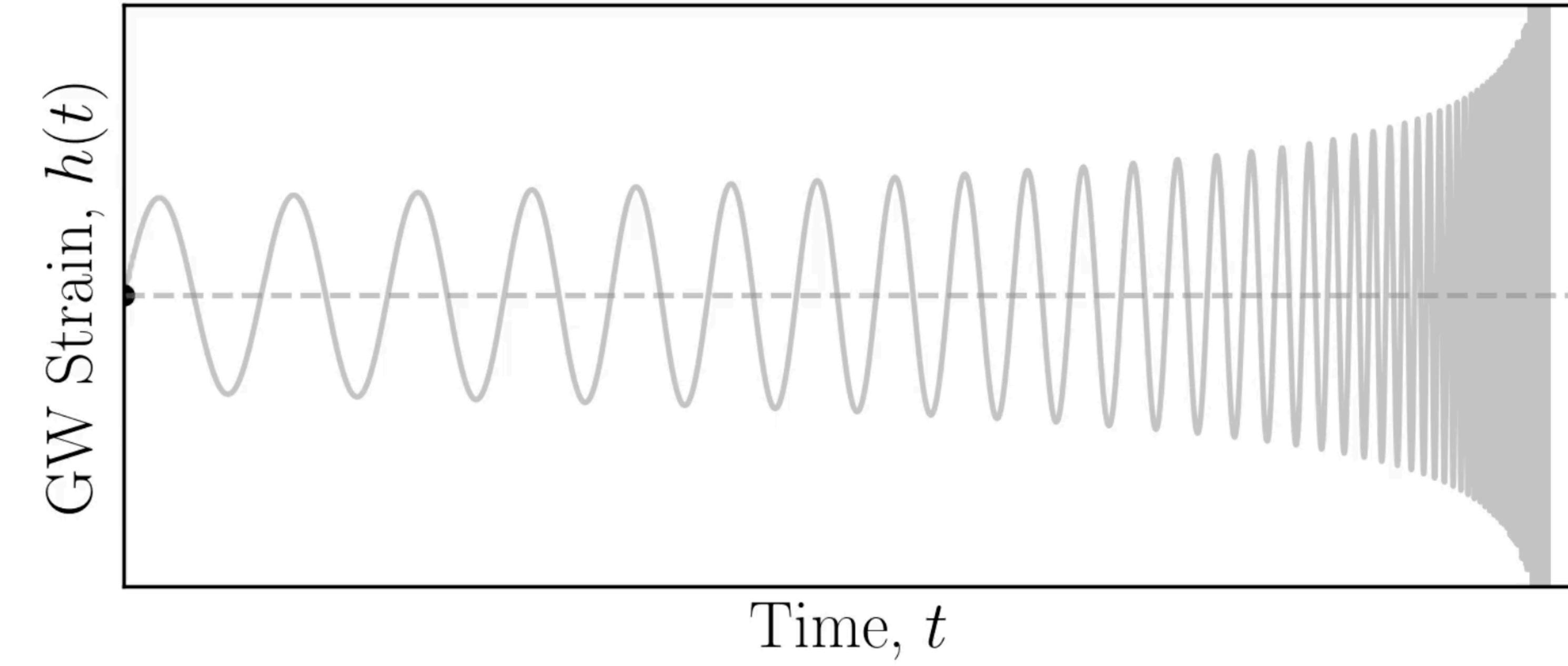
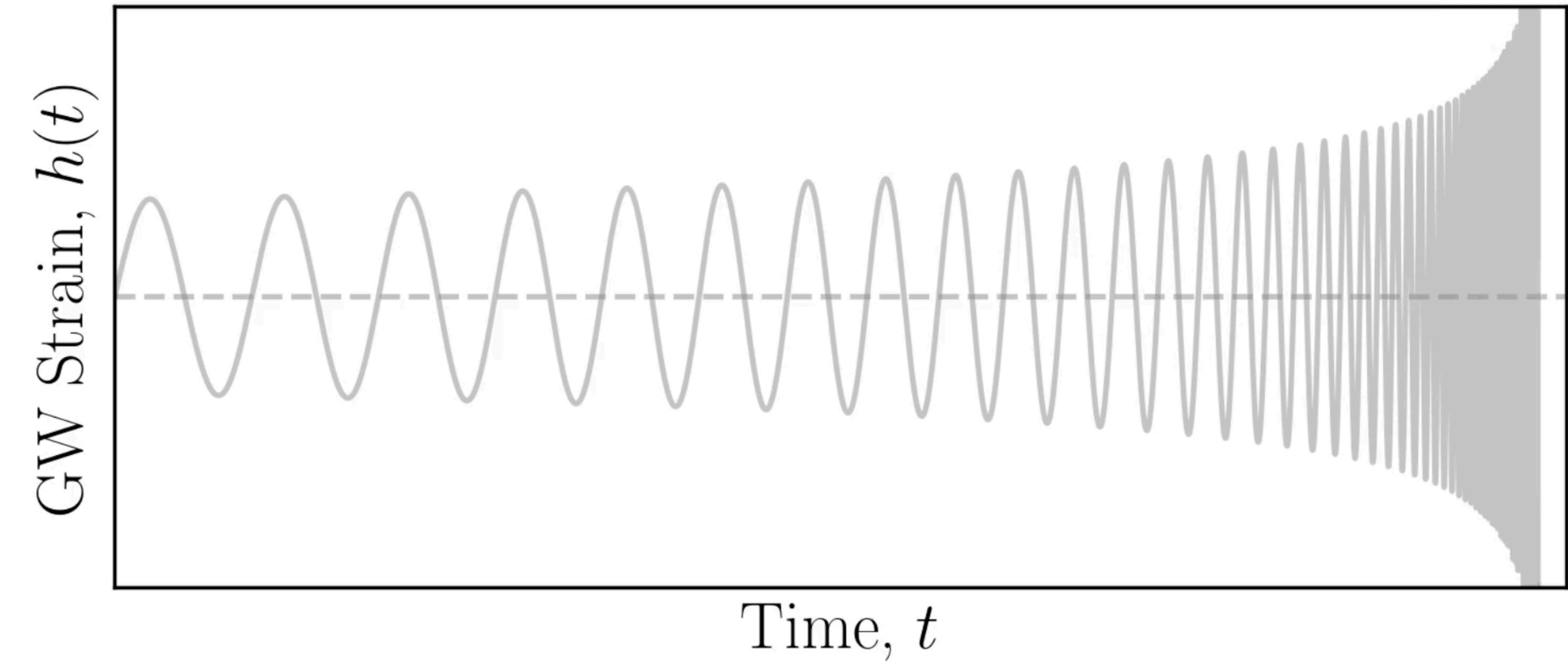
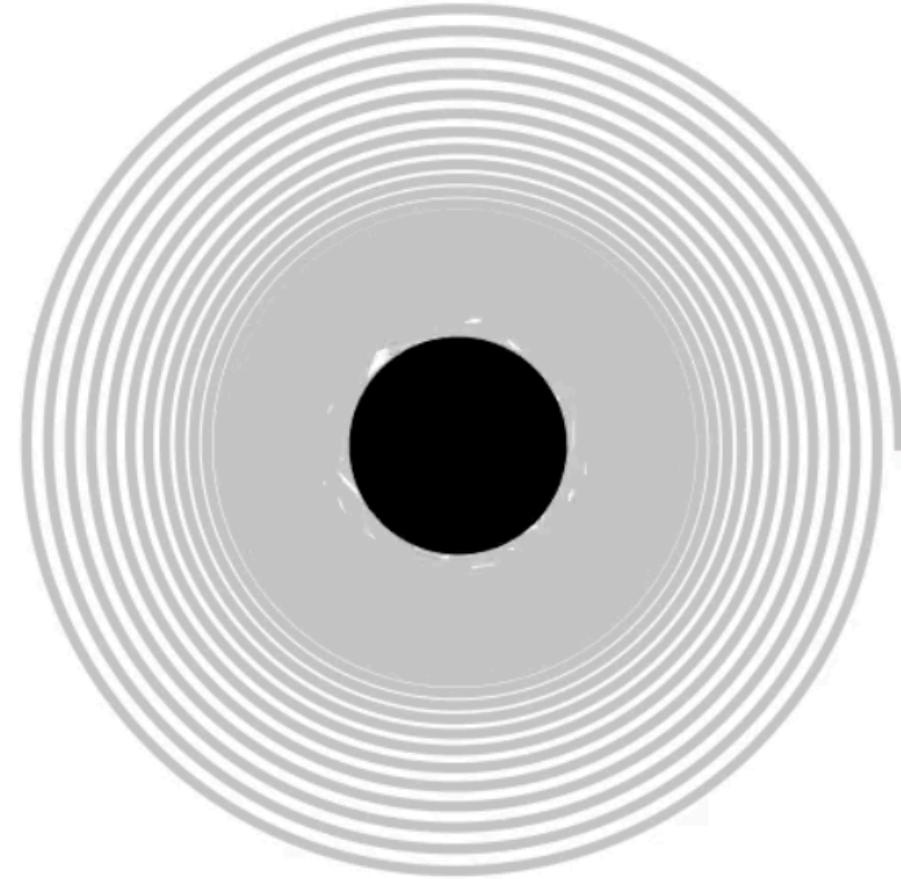
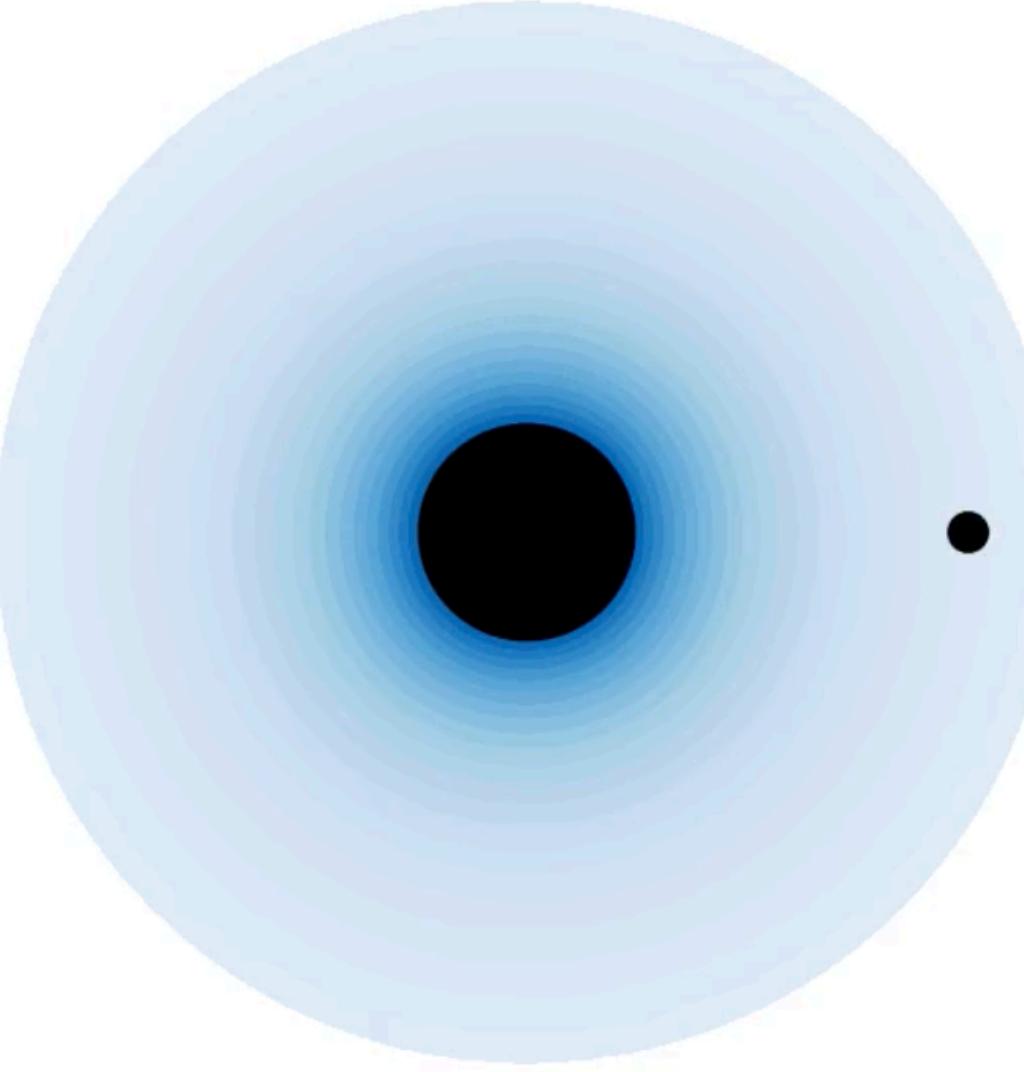
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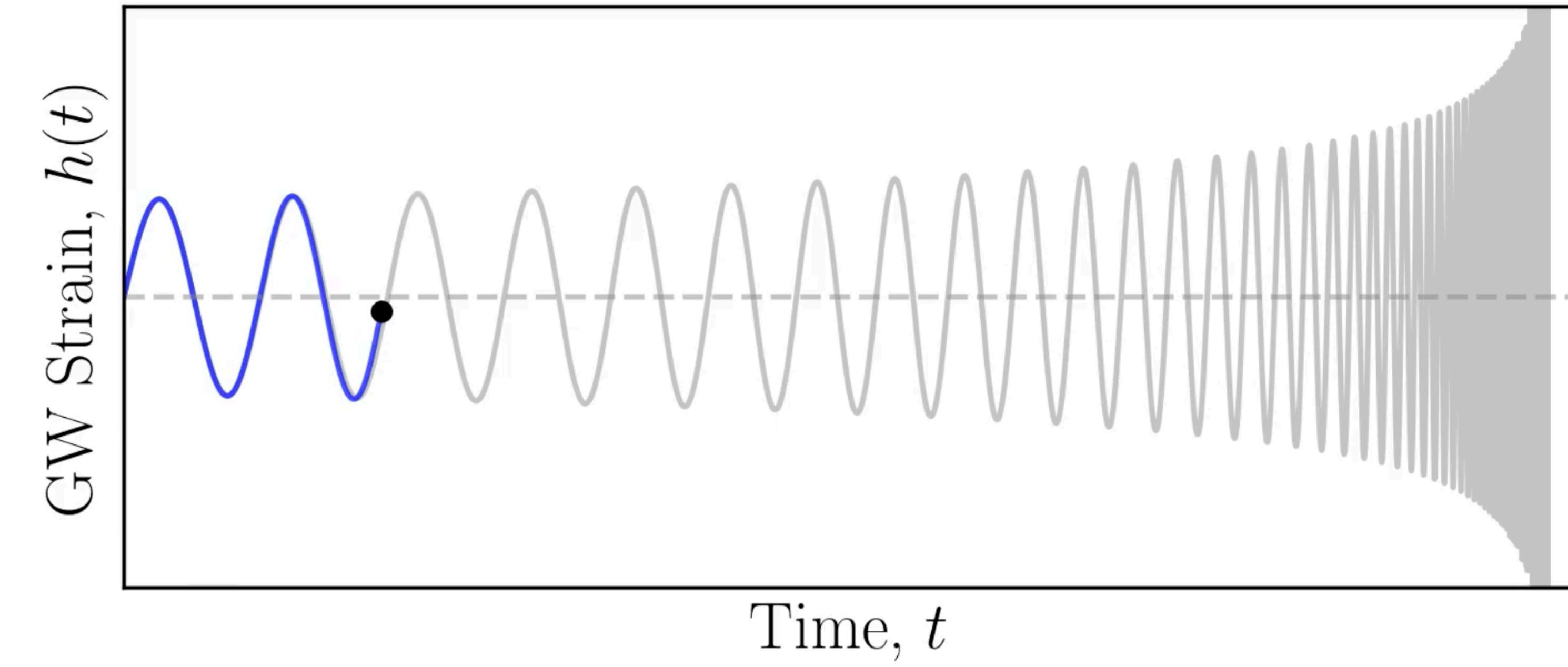
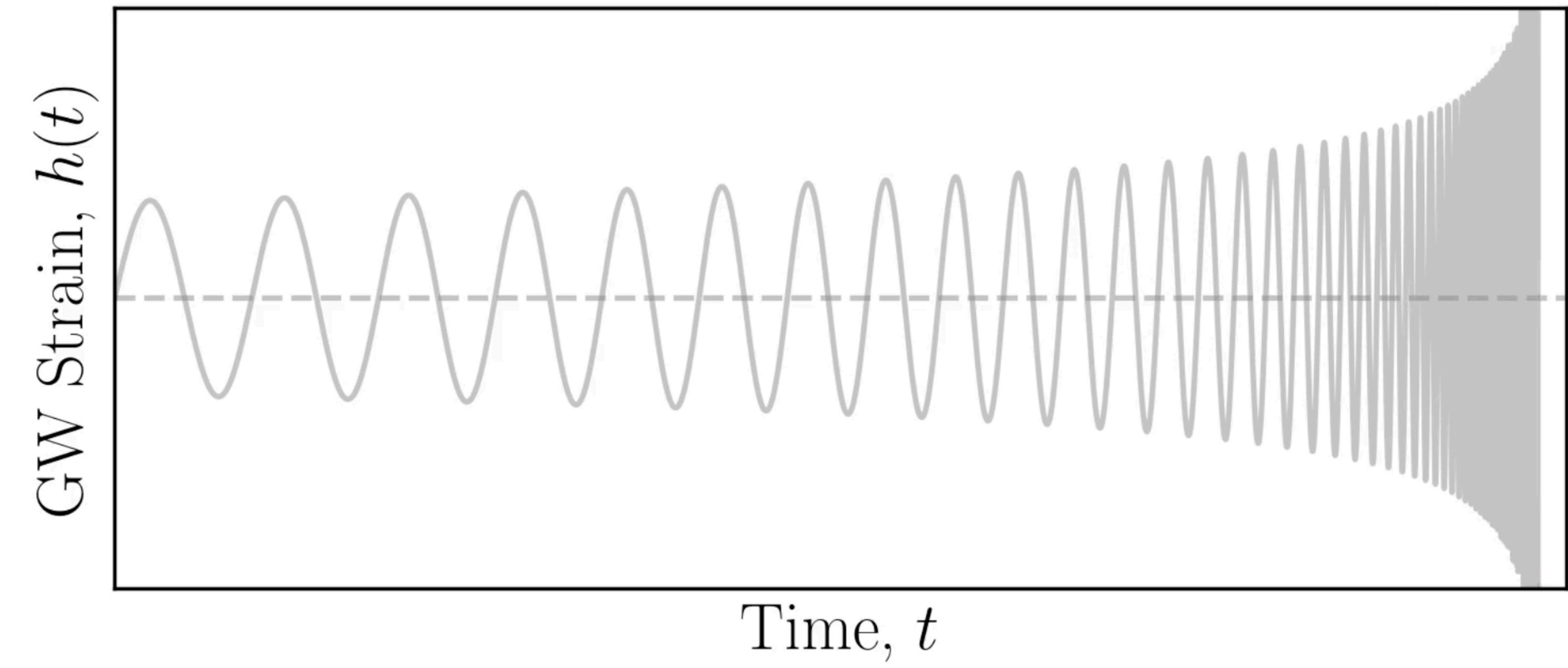
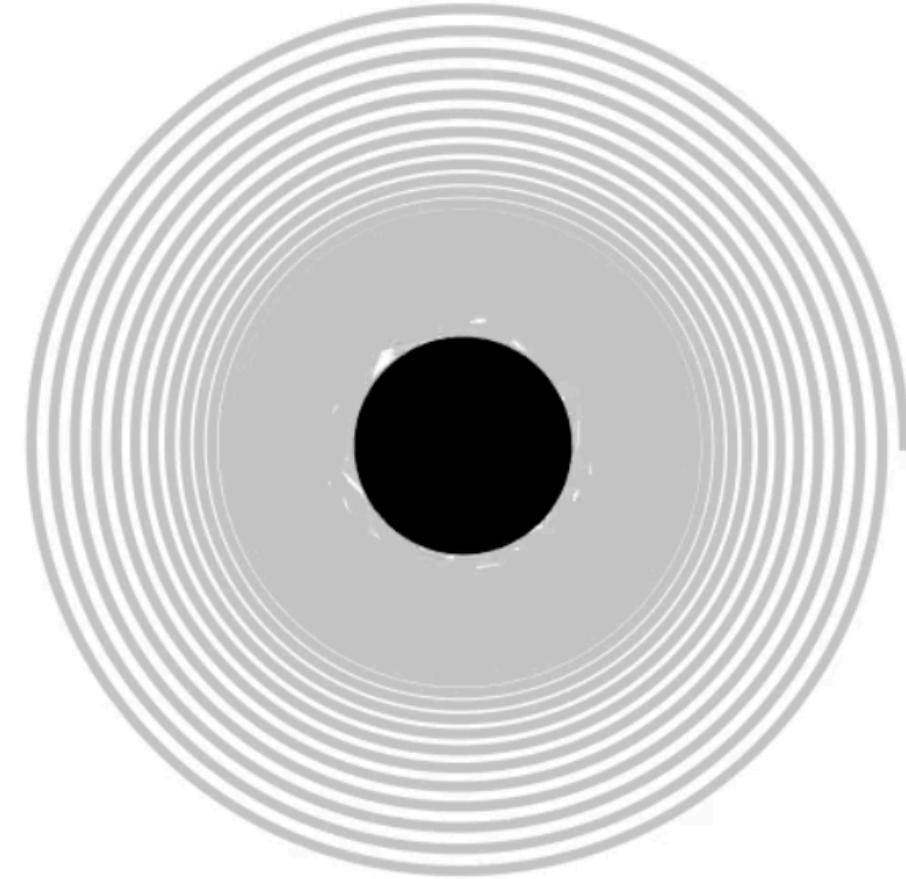
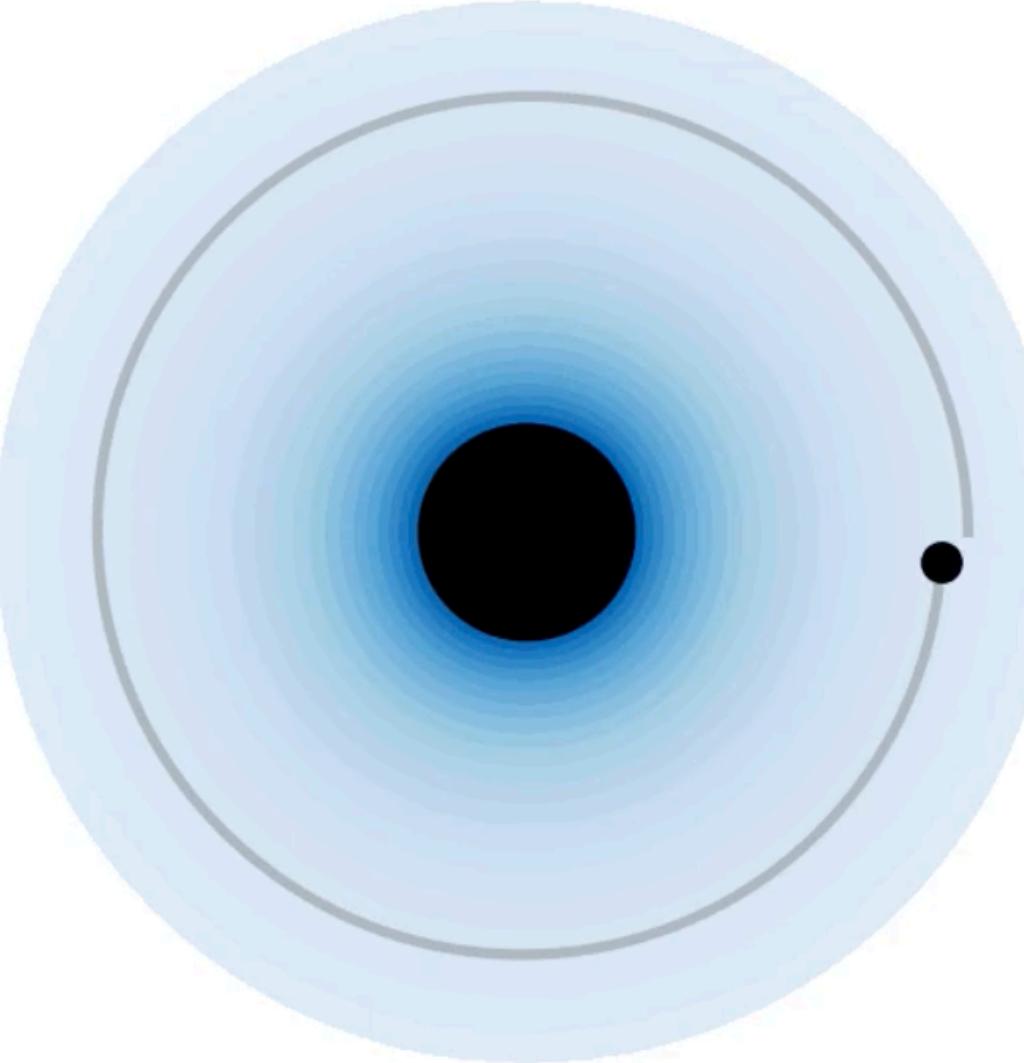
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[Animations online]



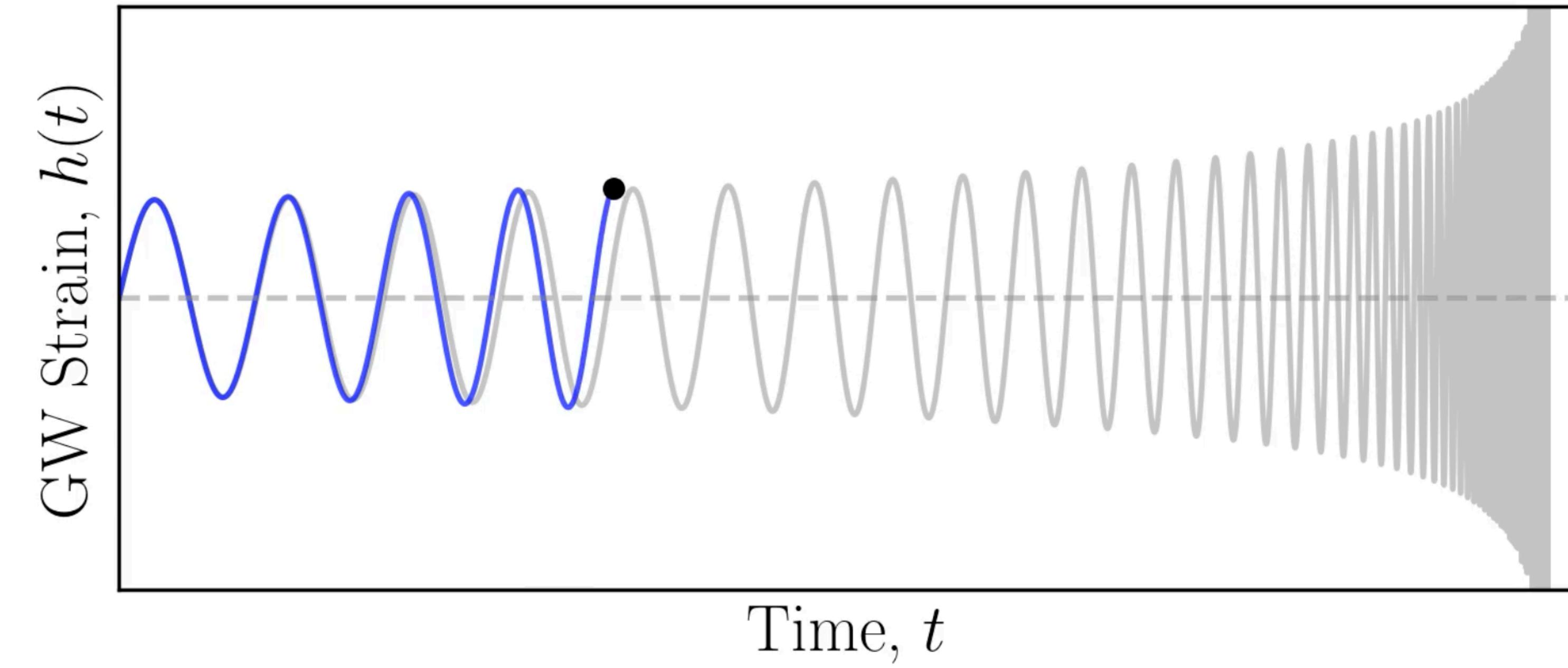
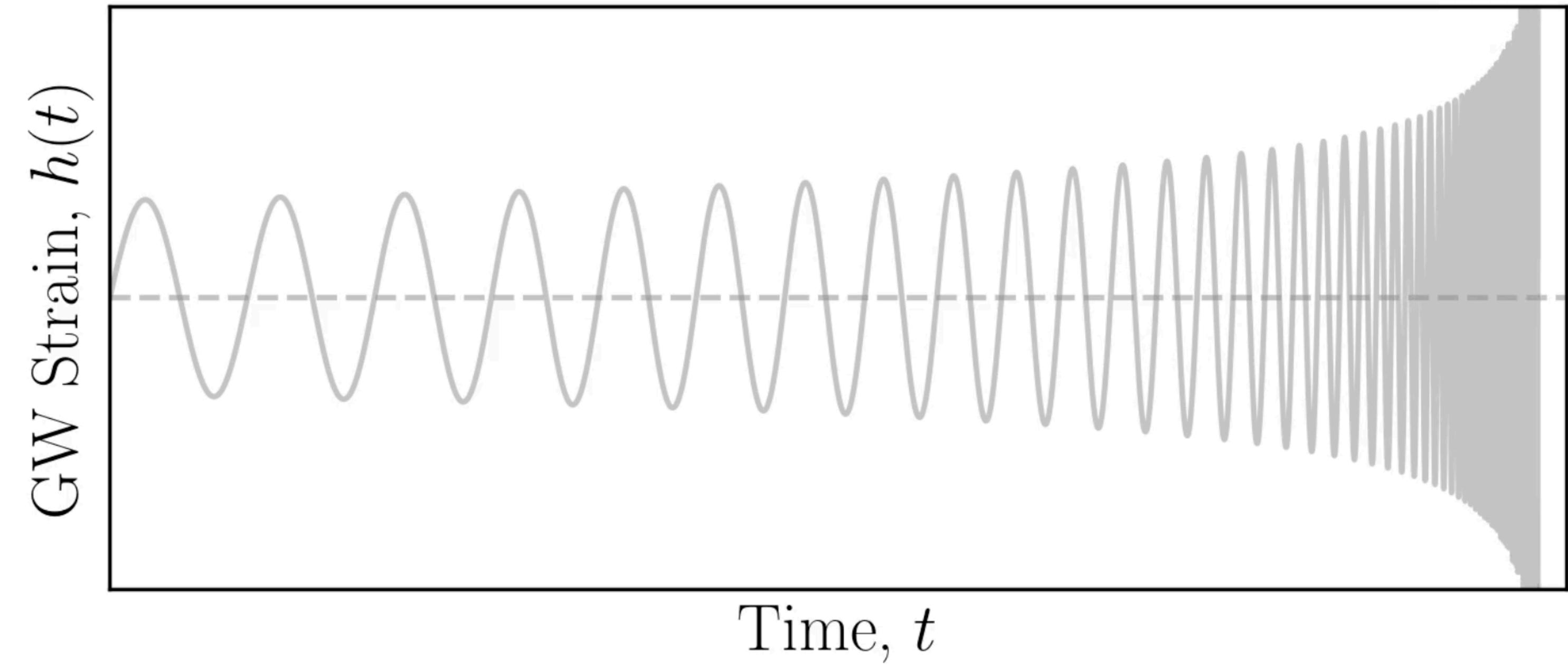
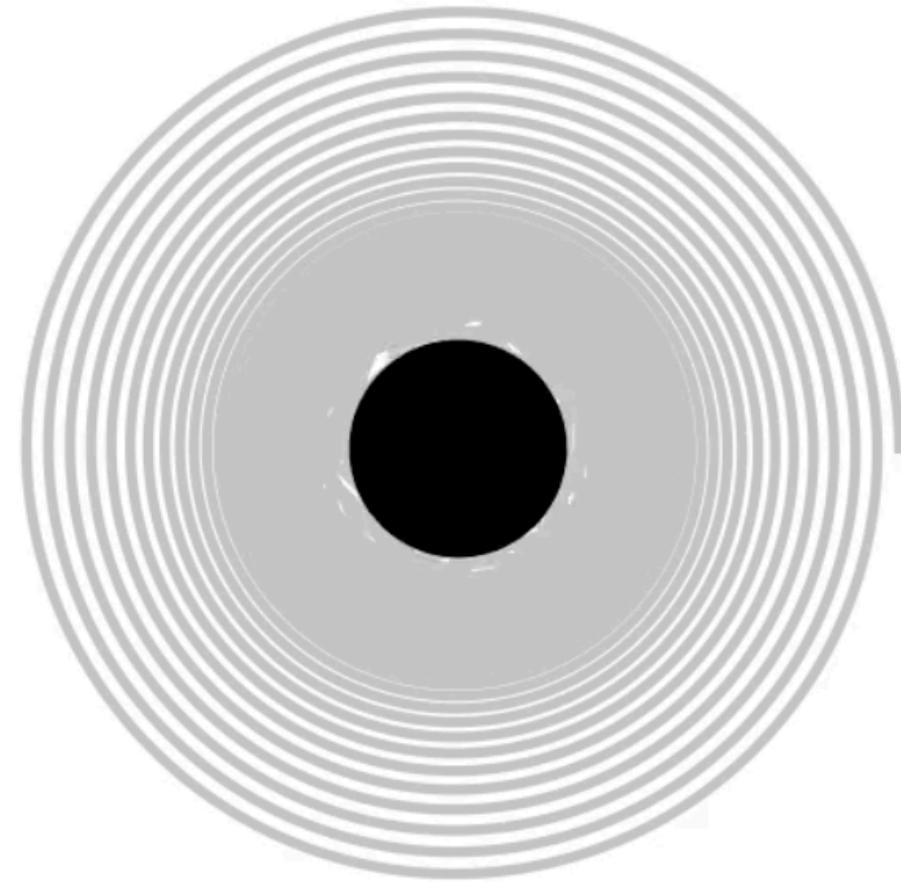
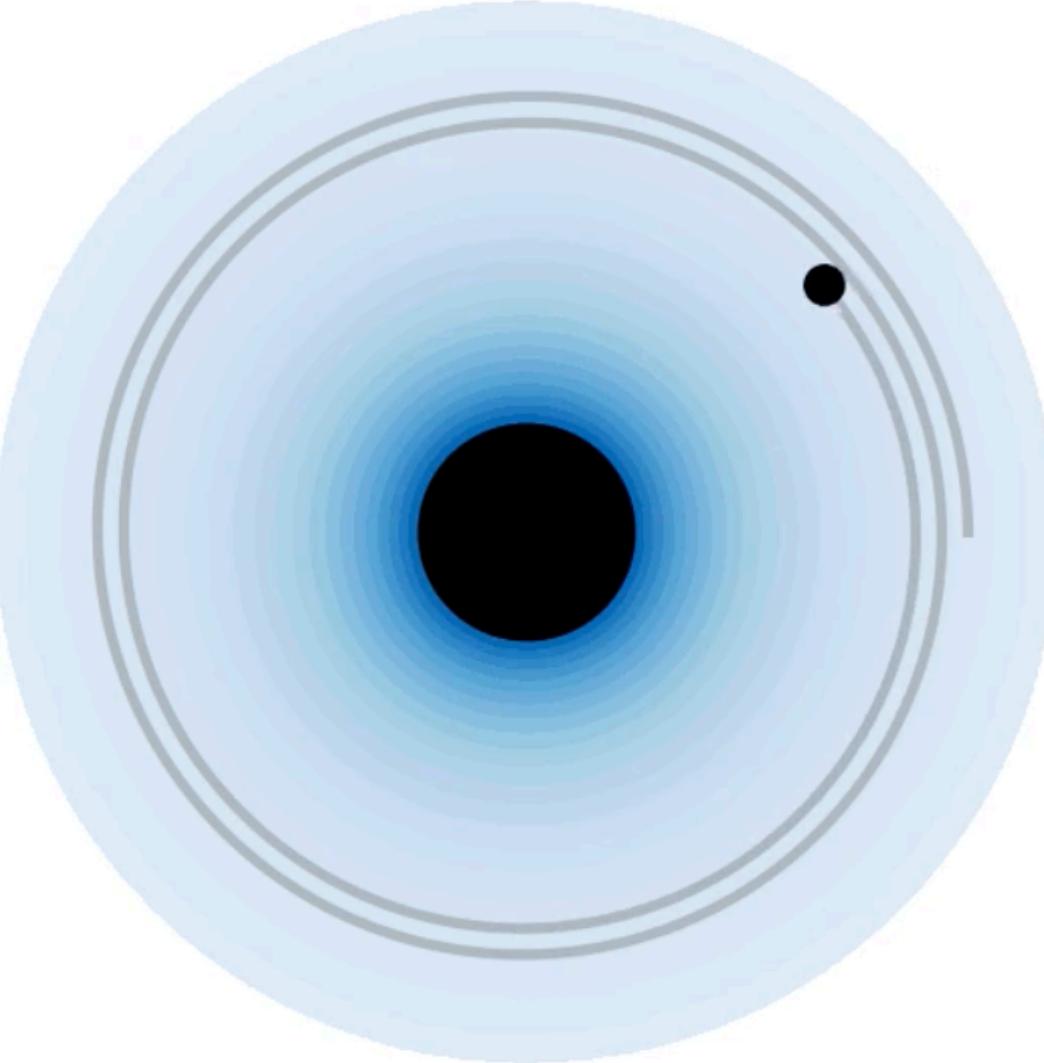
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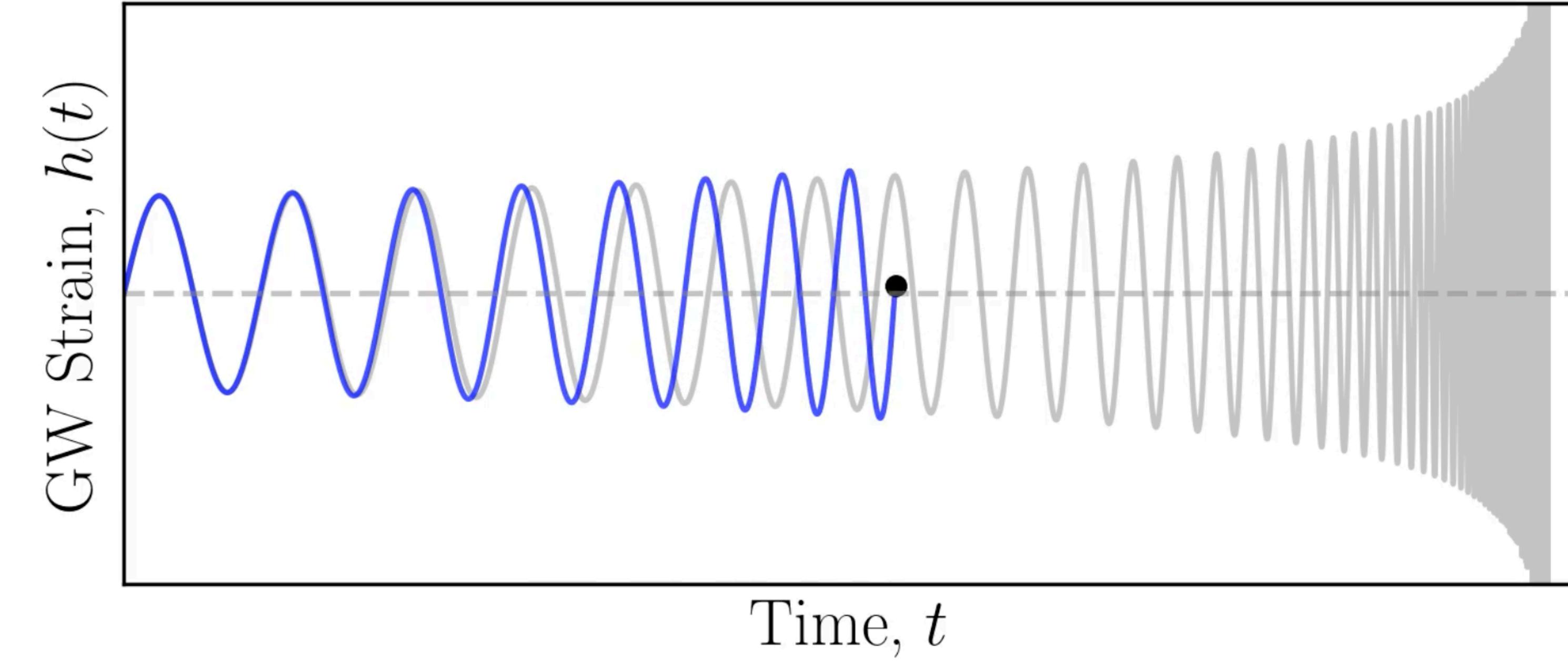
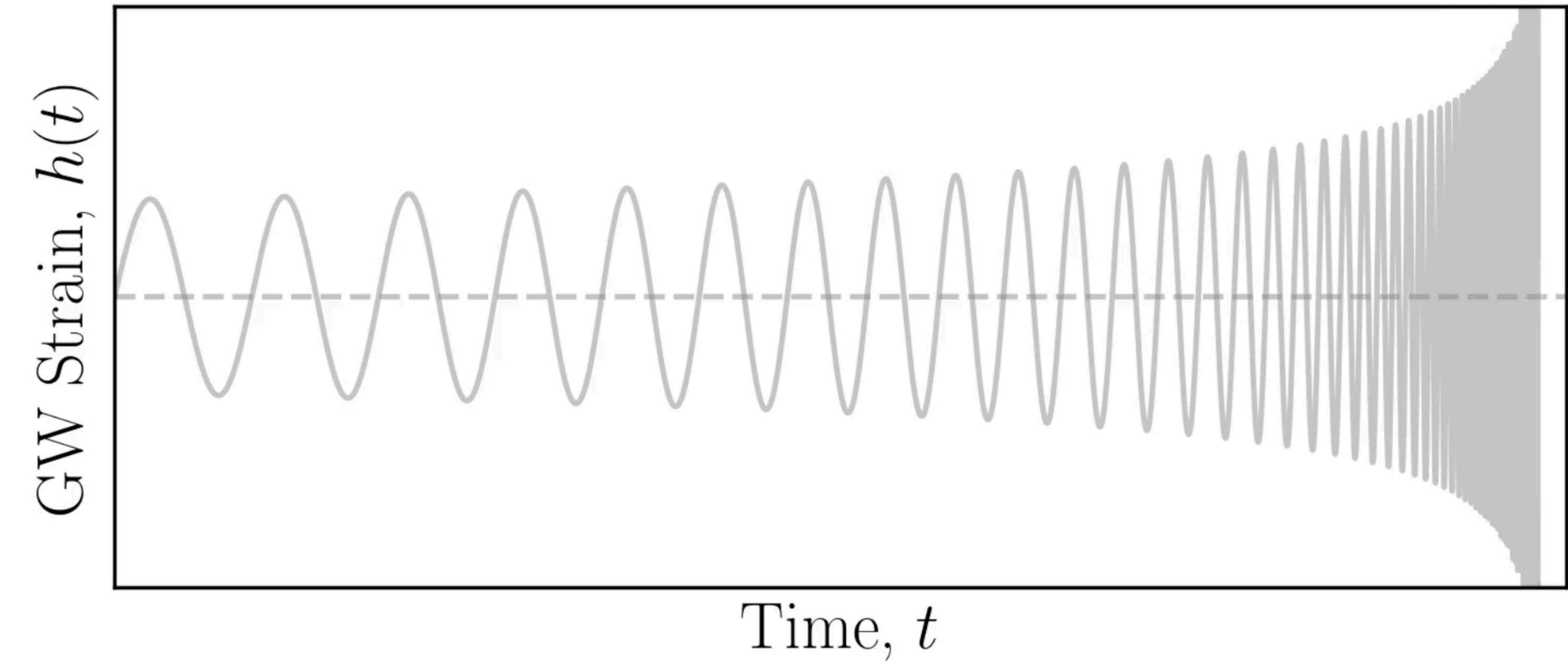
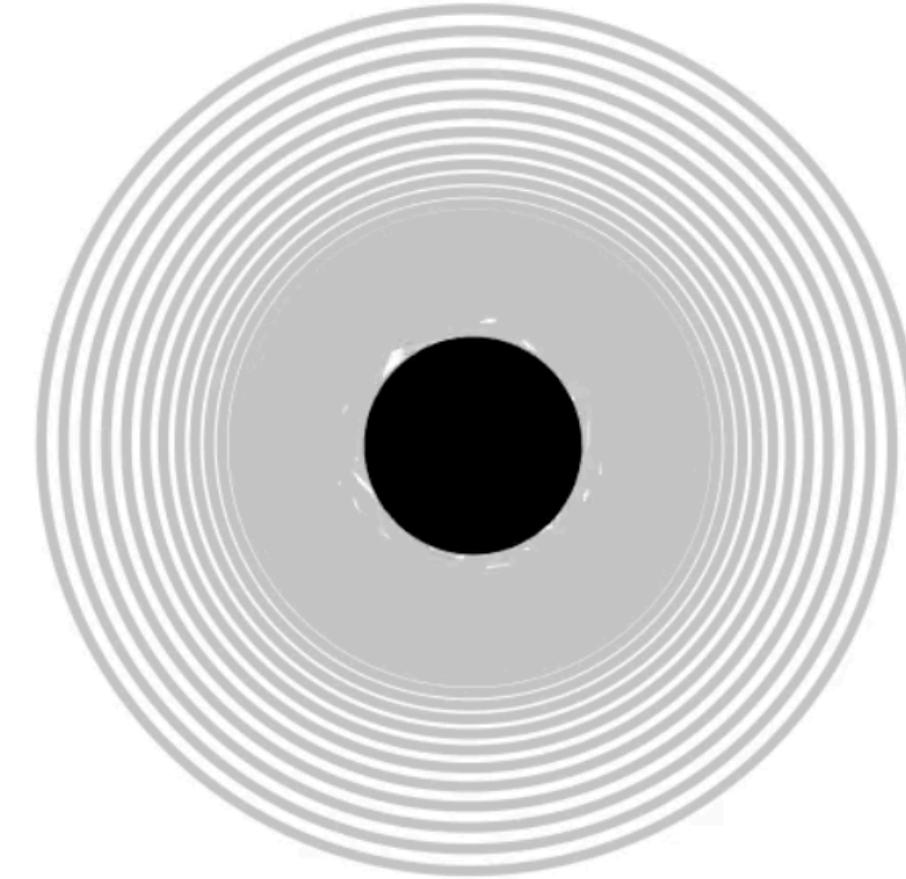
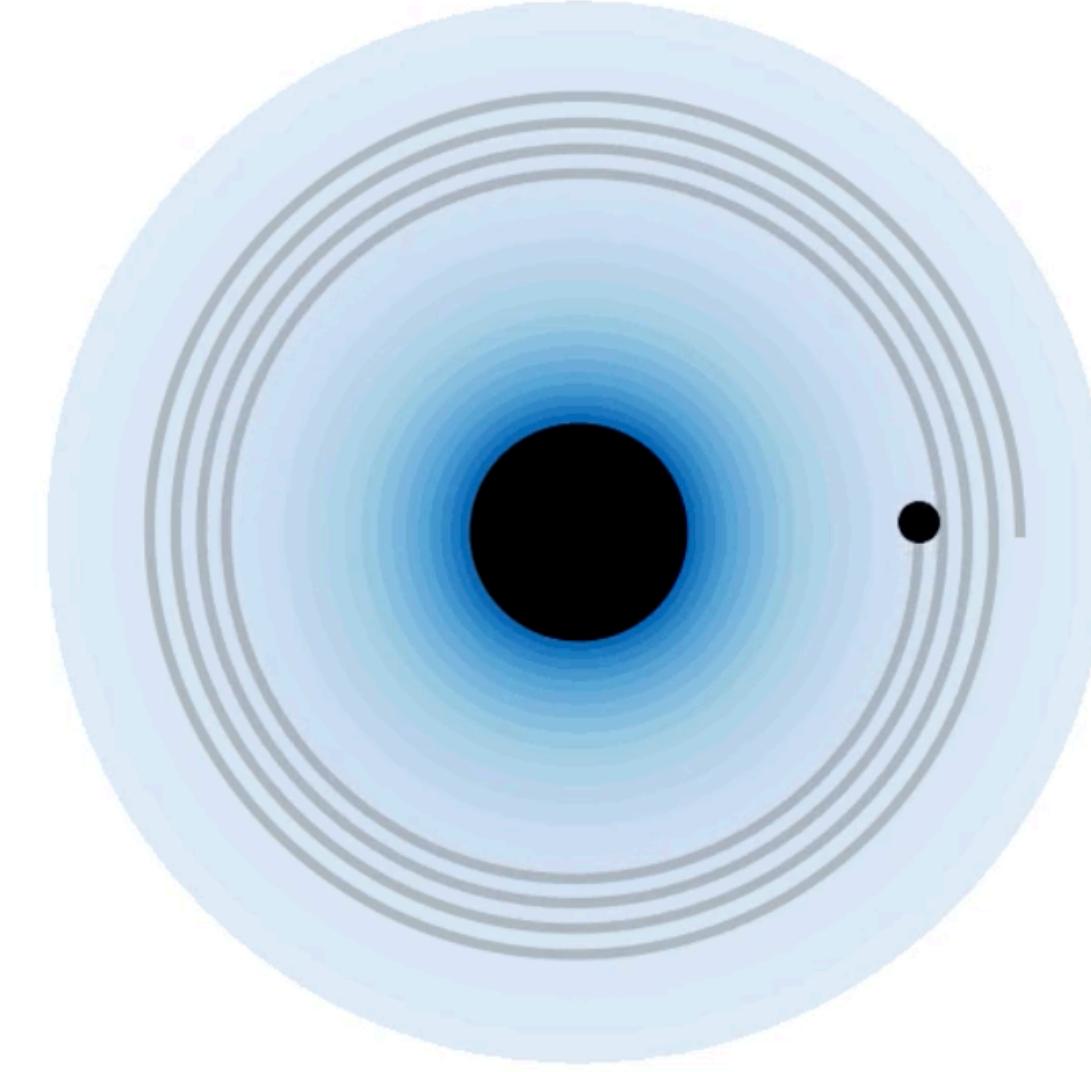
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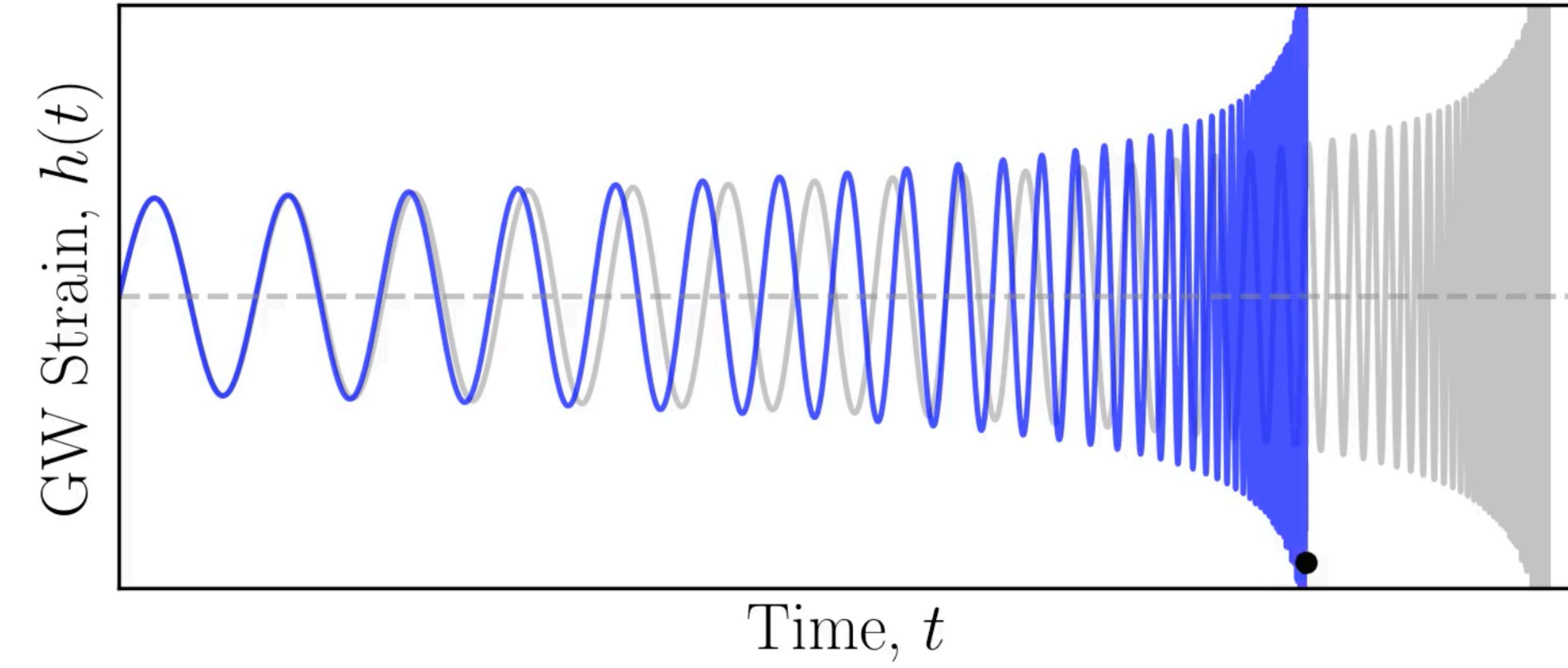
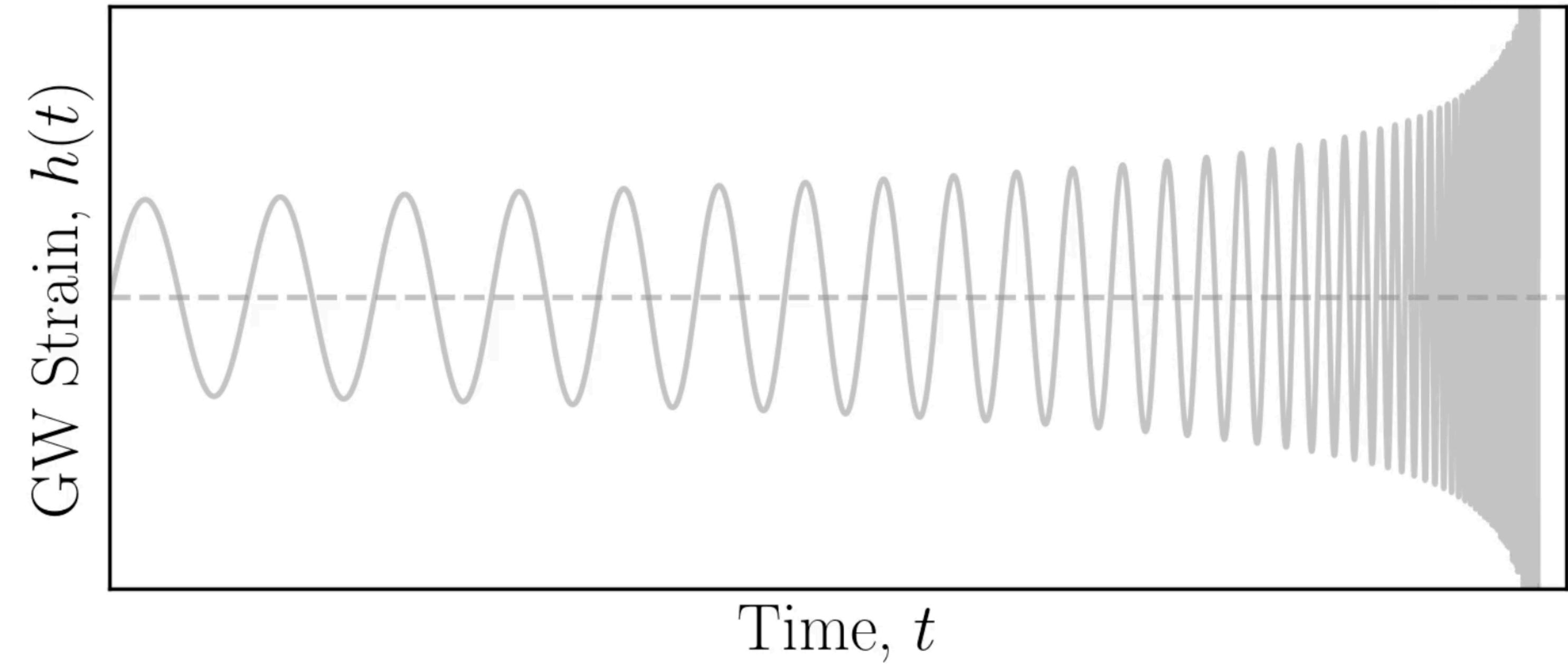
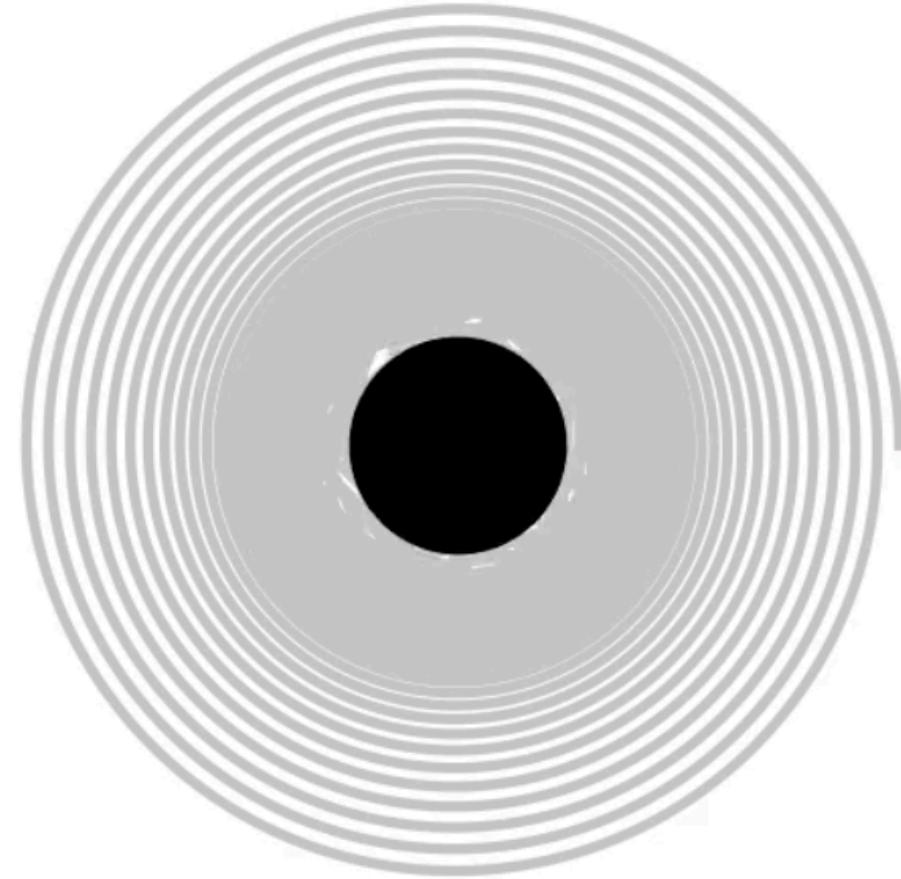
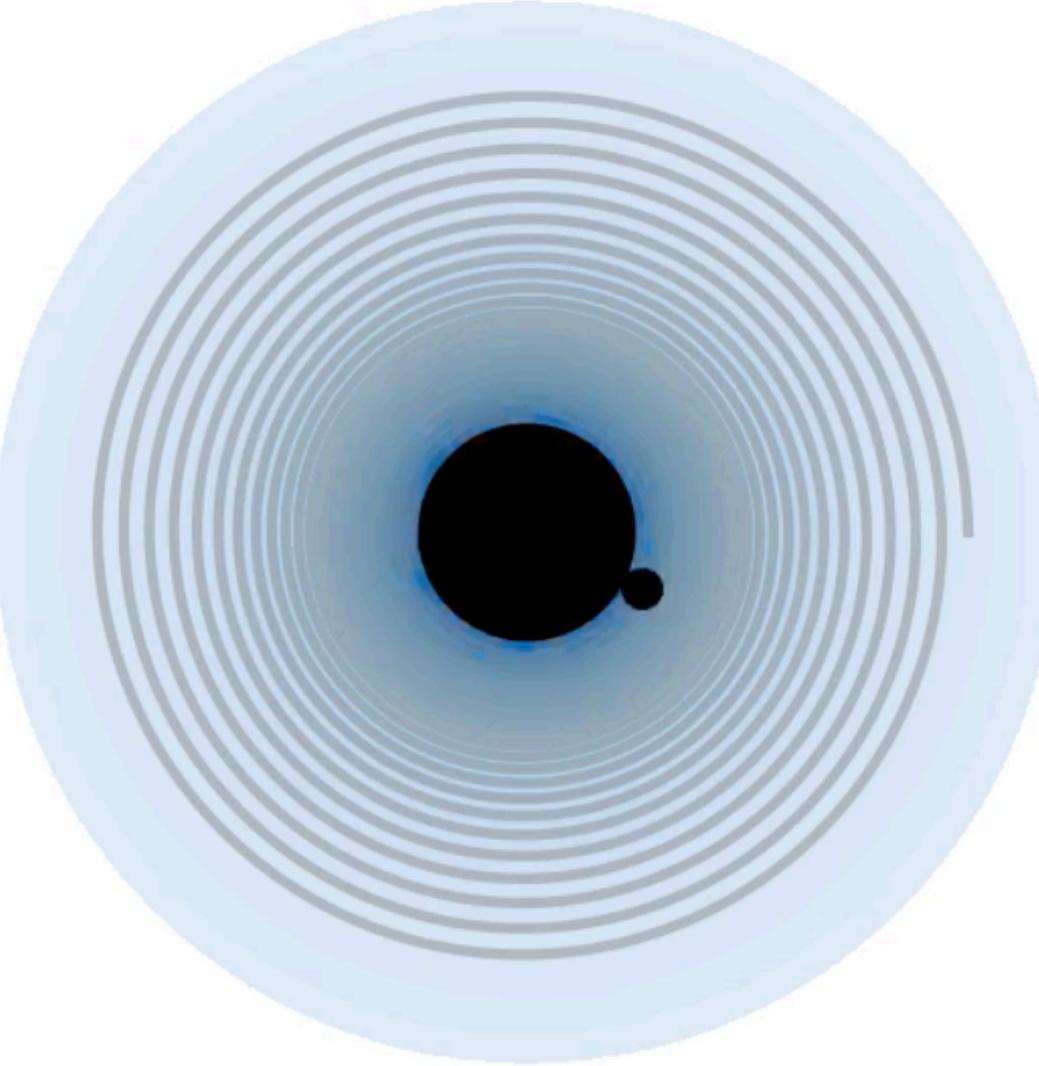
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[Animations online]



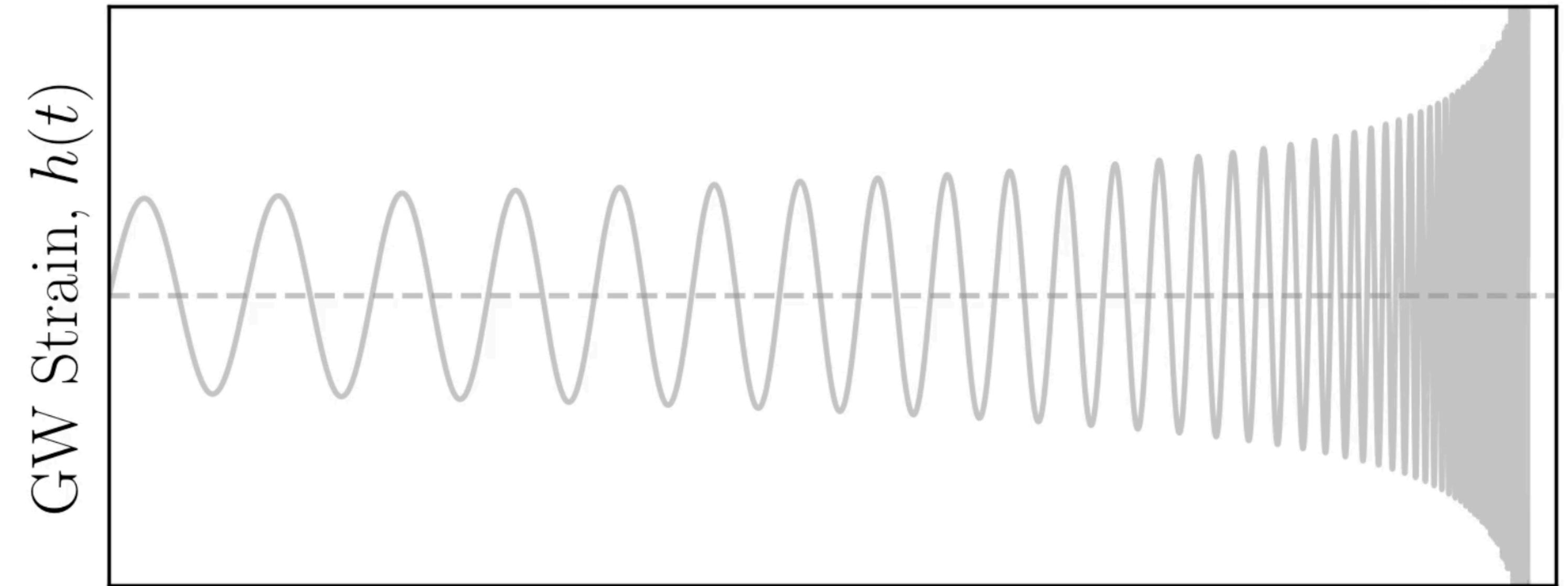
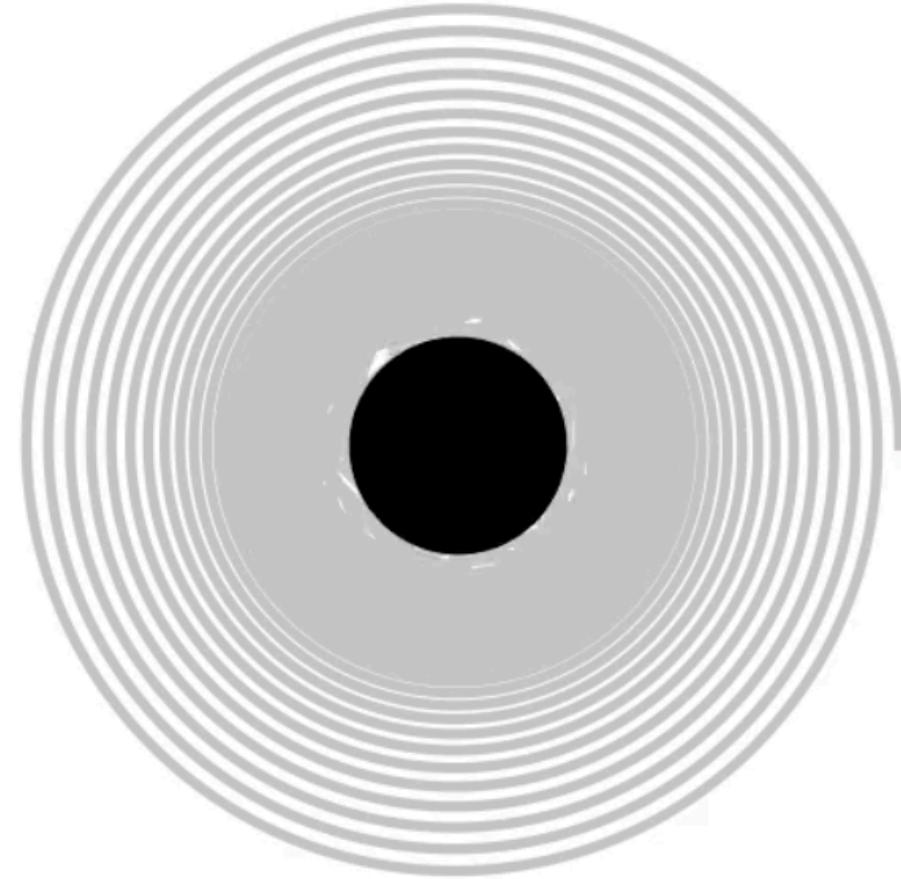
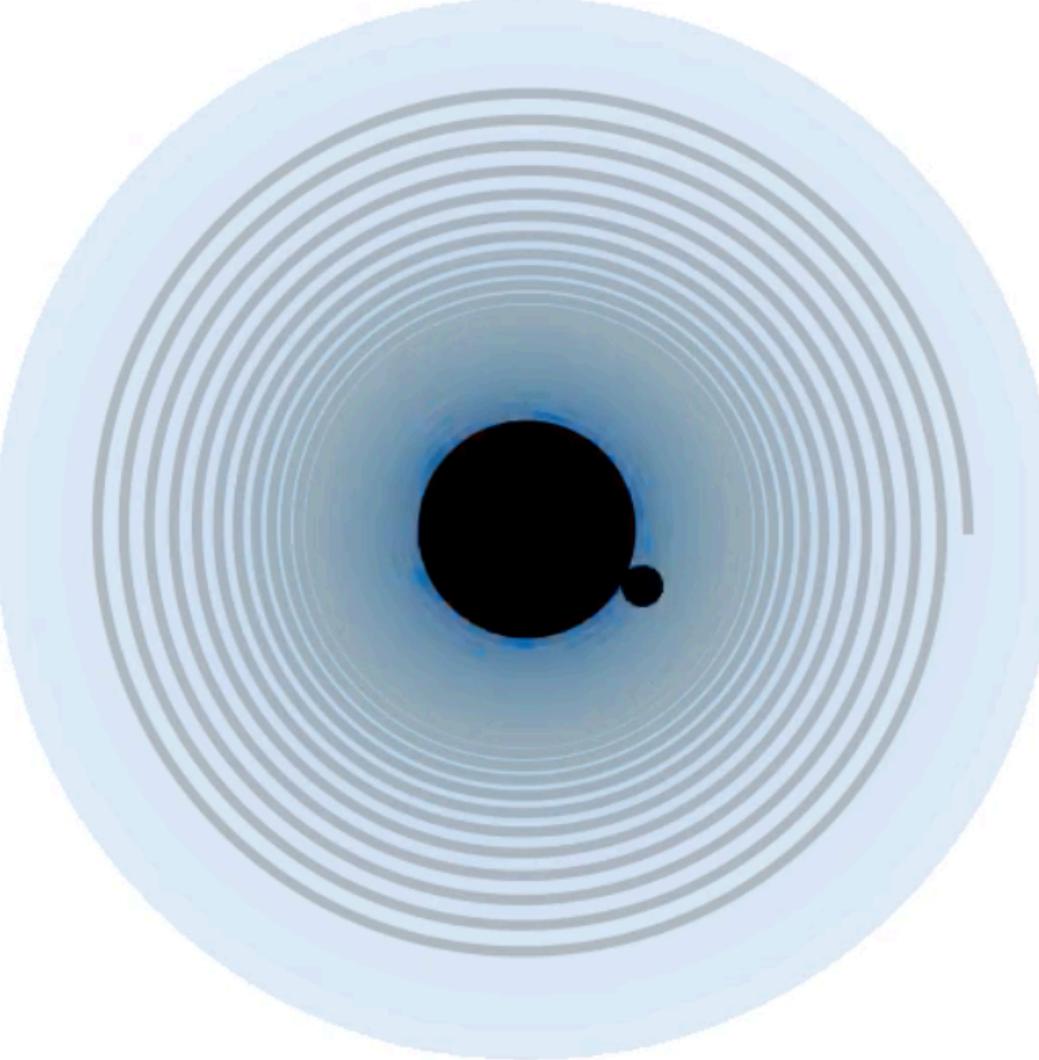
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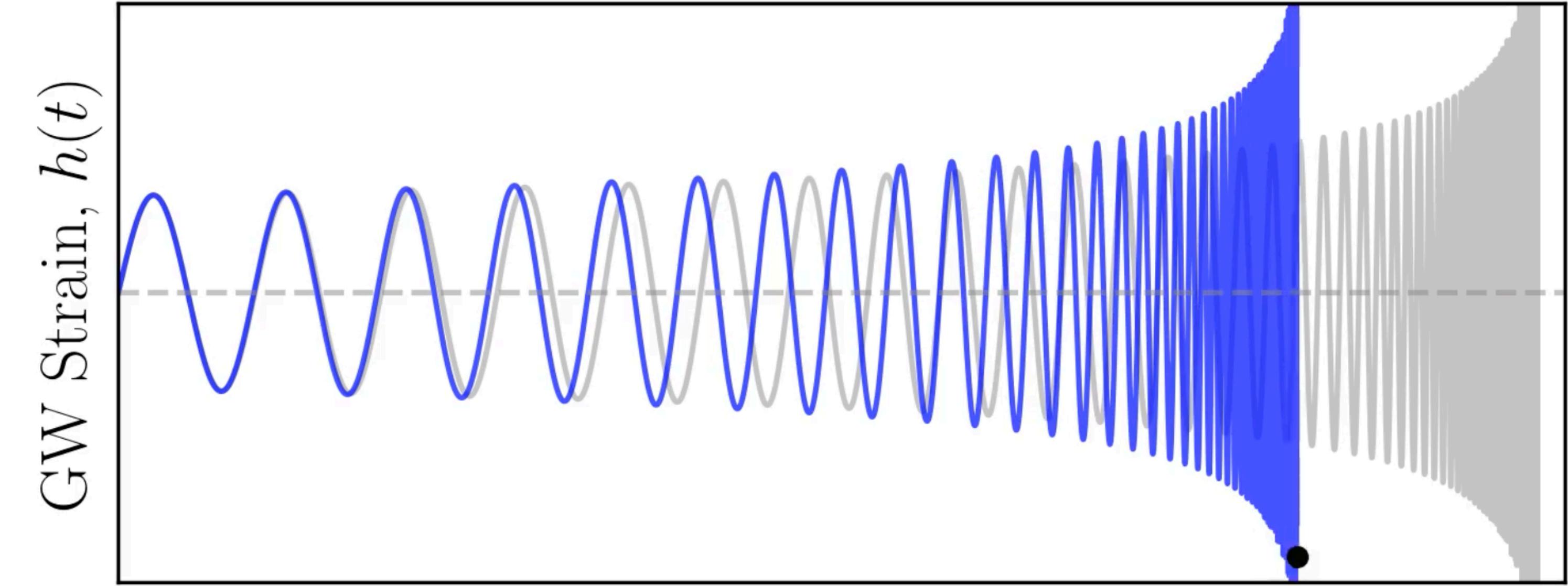
# Gravitational Wave Dephasing

[Animations online]



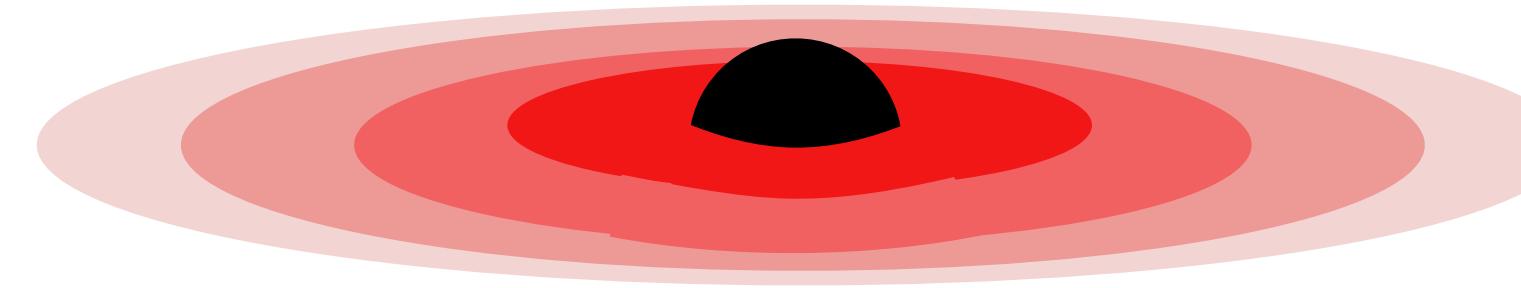
Time,  $t$

**“Dephasing” ( $\Delta N_{\text{GW}}$ )**

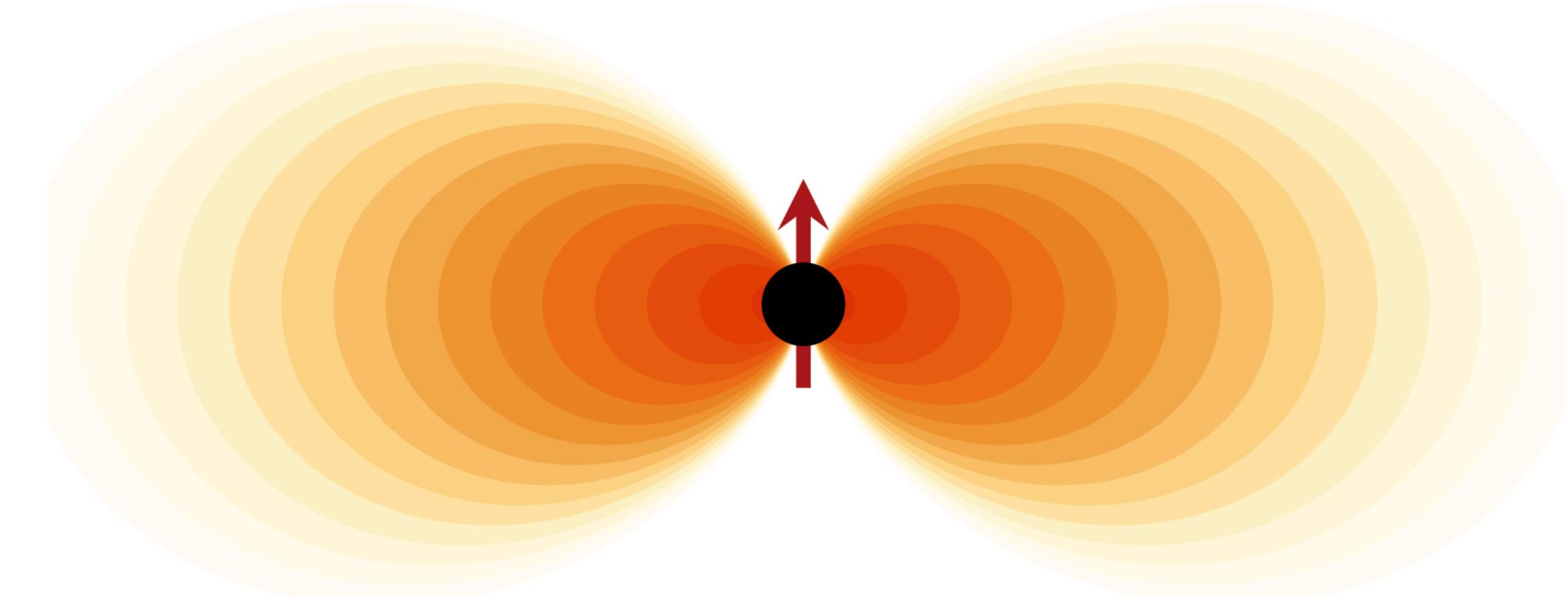


Time,  $t$

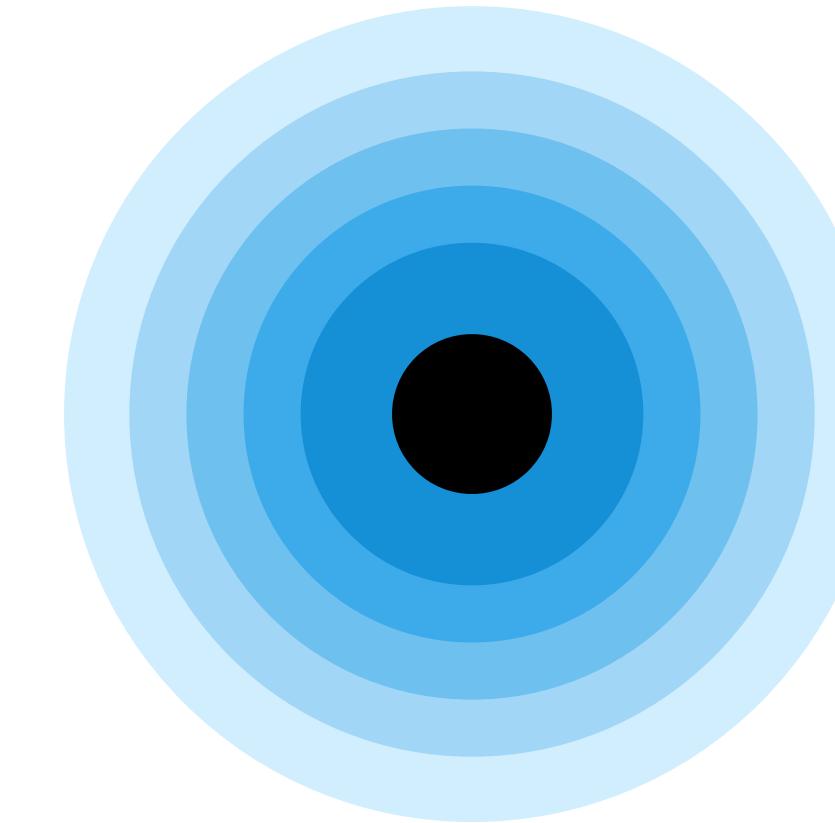
# Black Hole Environments



Baryonic  
**Accretion Disks**

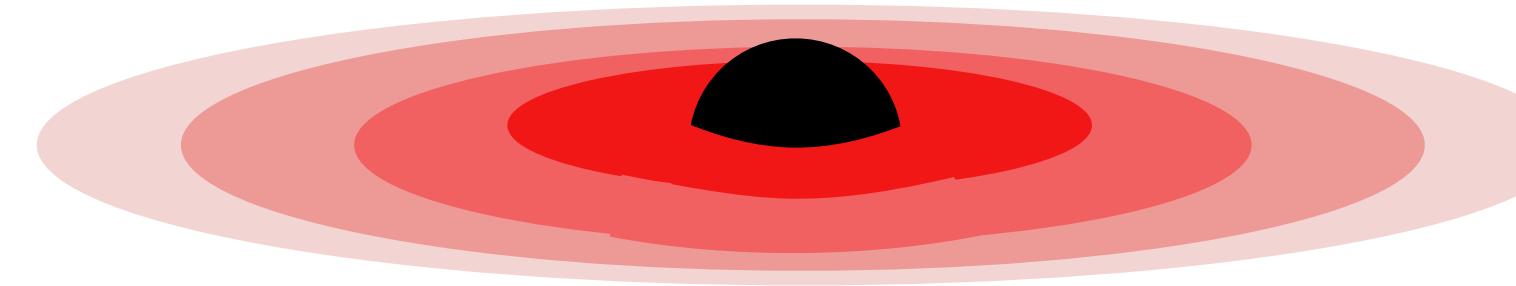


**'Gravitational Atoms'** of  
Ultralight Bosons

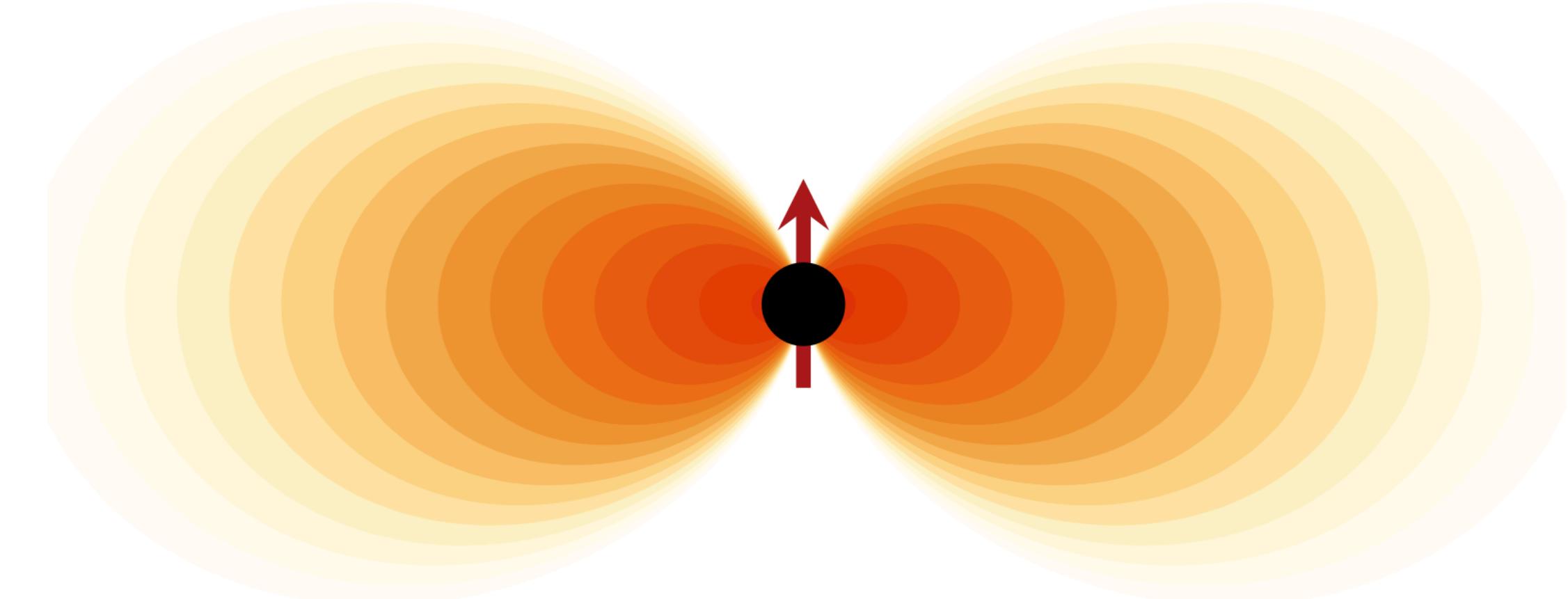


Particle Dark Matter  
**'Spikes'** or **'Dresses'**

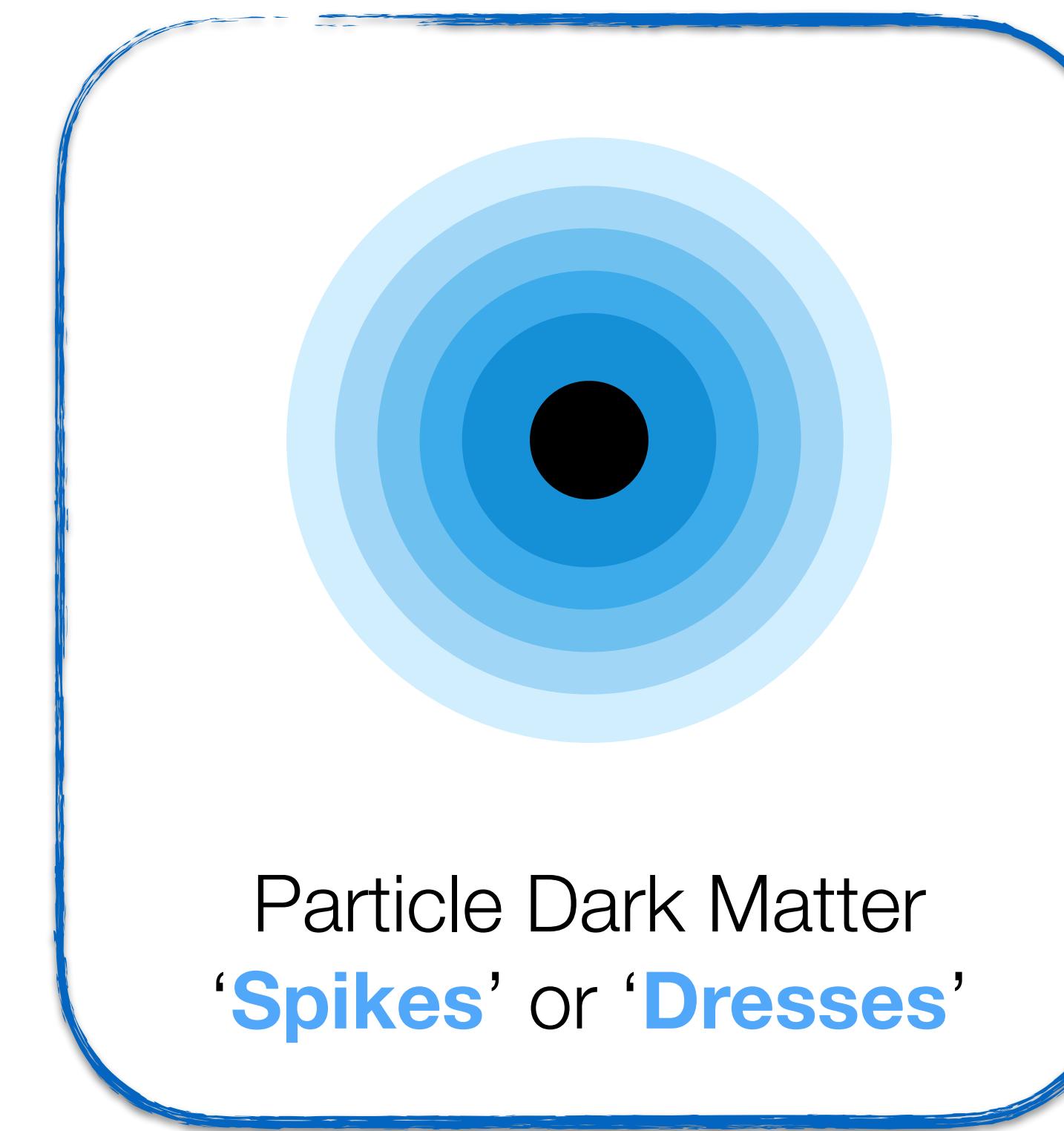
# Black Hole Environments



Baryonic  
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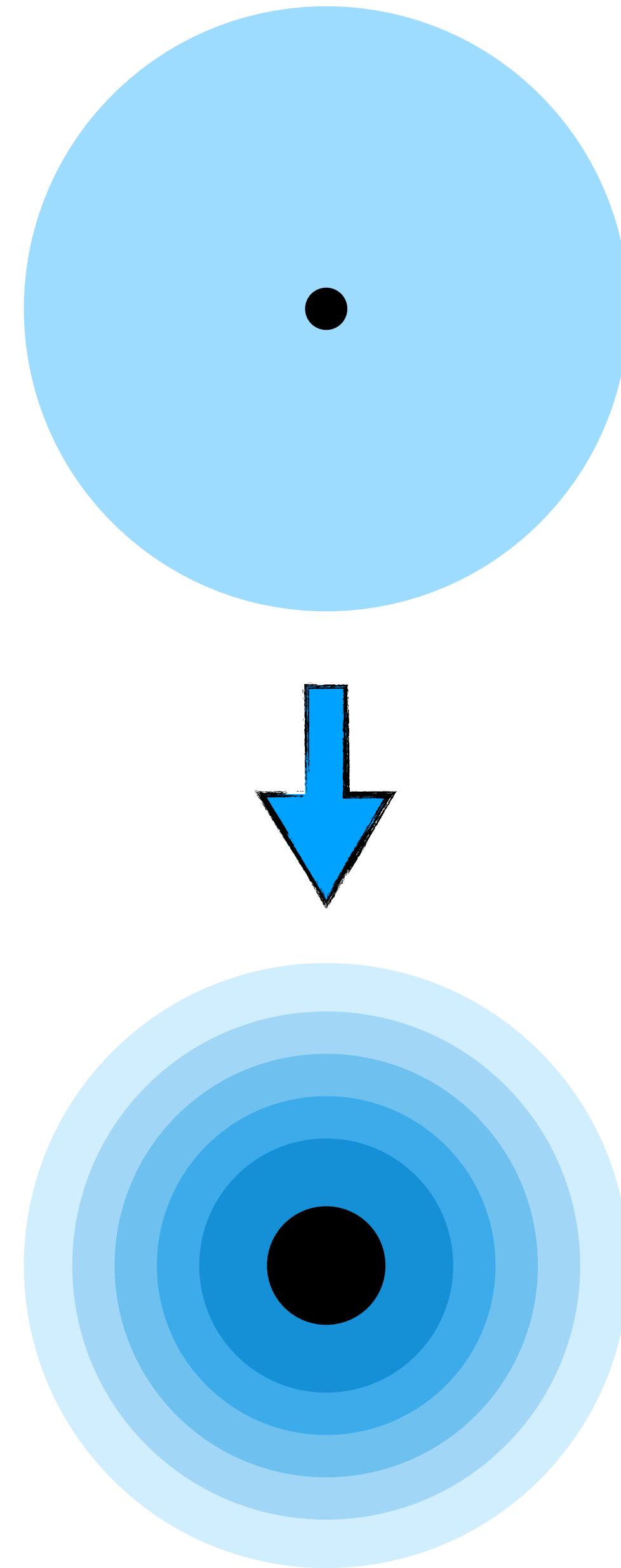


**'Gravitational Atoms'** of  
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Particle Dark Matter  
**'Spikes'** or **'Dresses'**

# Dark Matter Spikes



'**Spikes**' or '**dresses**' of cold, particle-like DM may form around BHs:

Around BHs which form from large density fluctuations in the early Universe (i.e. Primordial Black Holes)

## "PBH scenario"

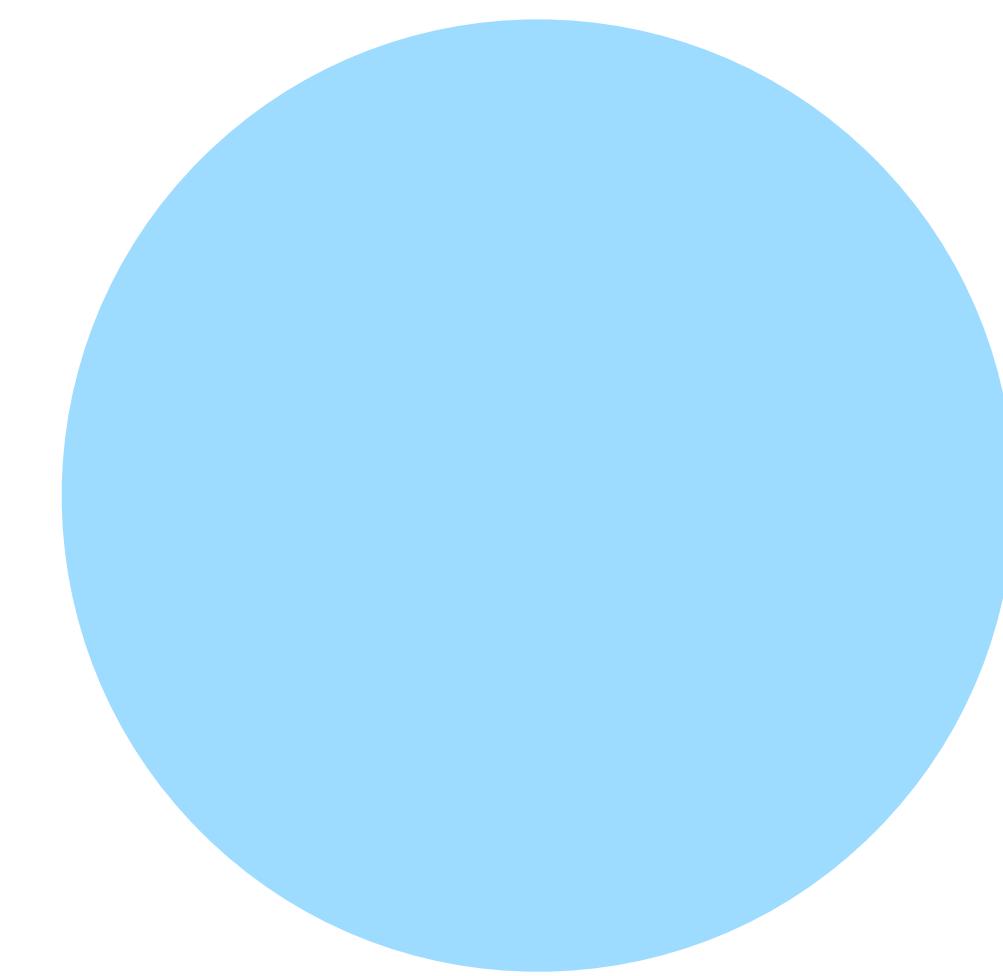
[[Bertschinger \(1985\)](#), [astro-ph/0608642](#),  
[1901.08528](#), ...]

From the slow ('adiabatic') growth of a BH at the centre of a DM halo

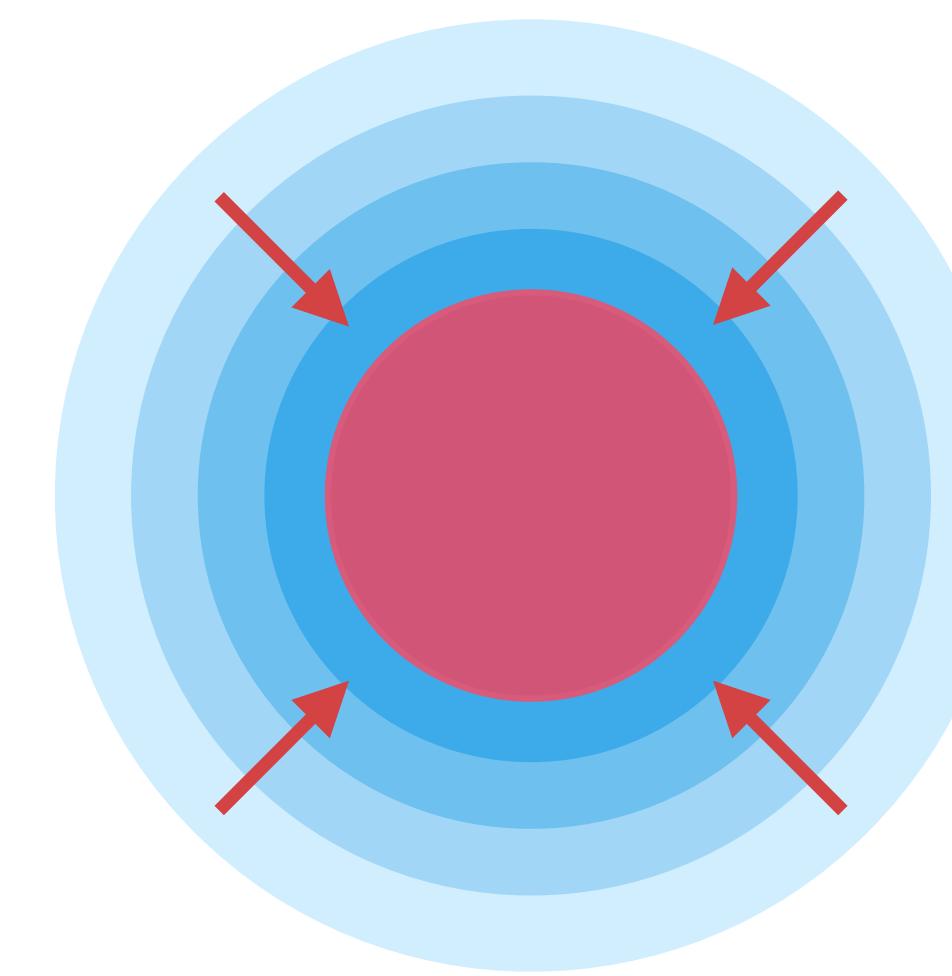
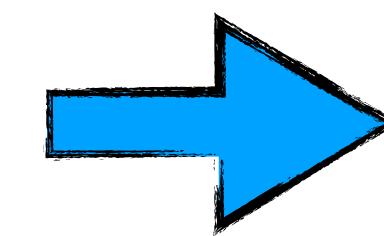
## "Astrophysical scenario"

[[astro-ph/9906391](#), [astro-ph/0509565](#),  
[1305.2619](#), ...]

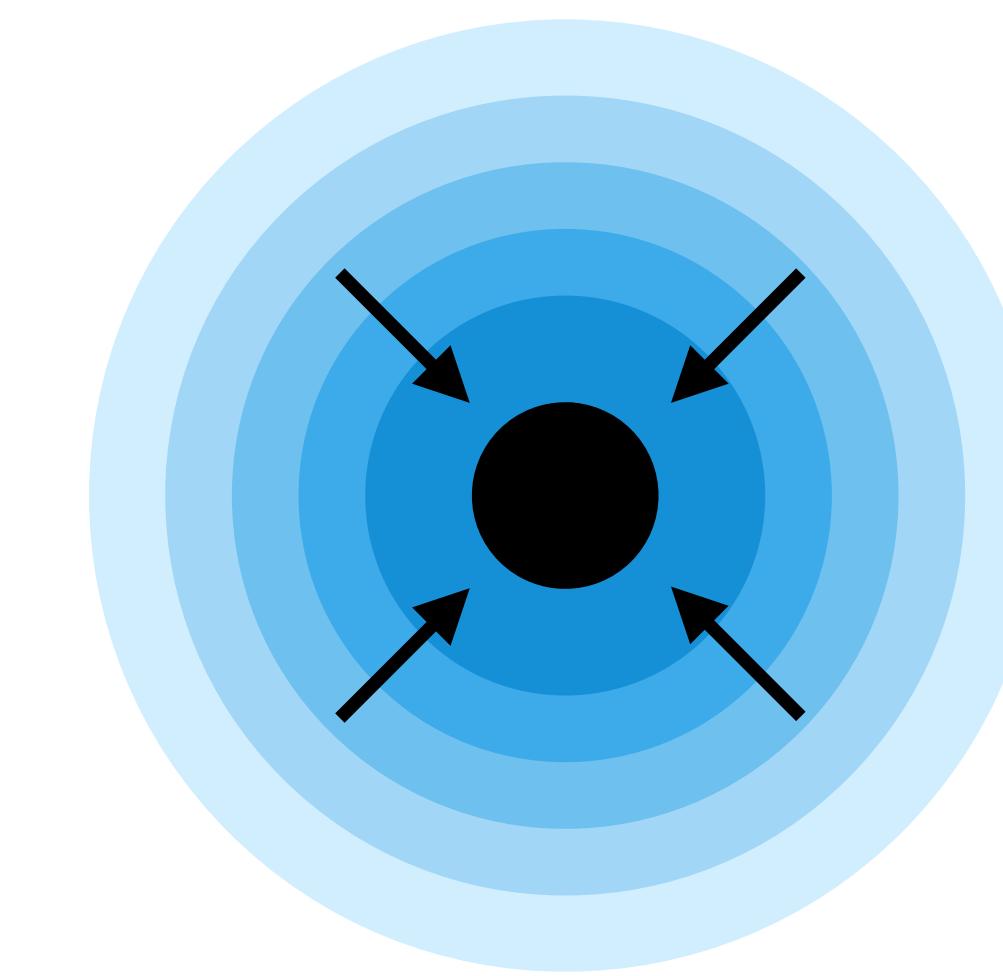
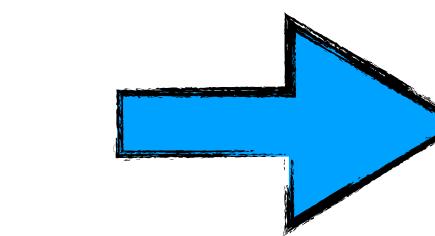
# Dark Matter 'Mounds'



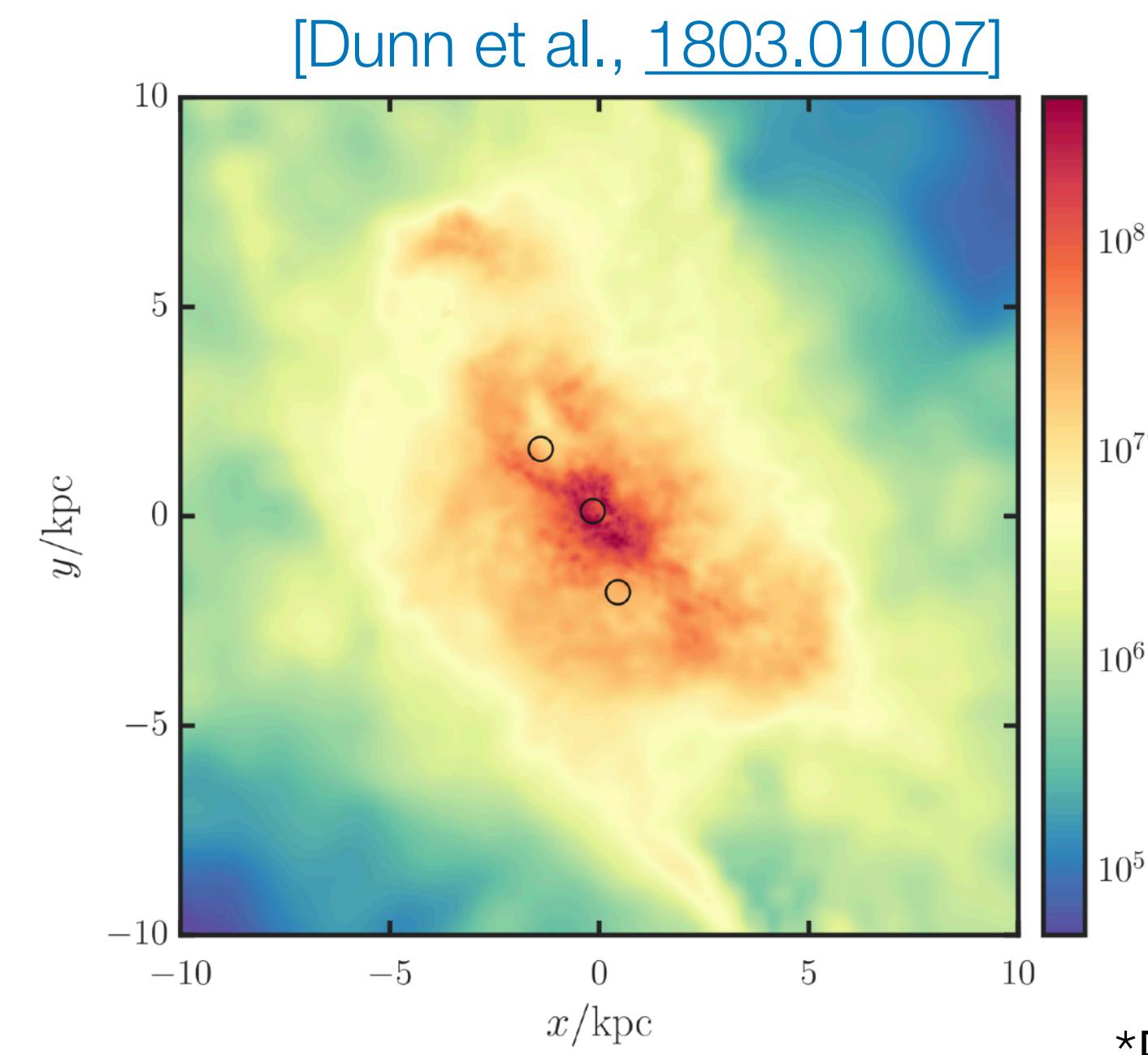
NFW Dark Matter Halo  
at high redshift  $z \gtrsim 15$



**Supermassive 'star'**



**Direct Collapse Black Hole**  
+ Dense **DM Spike/Mound\***

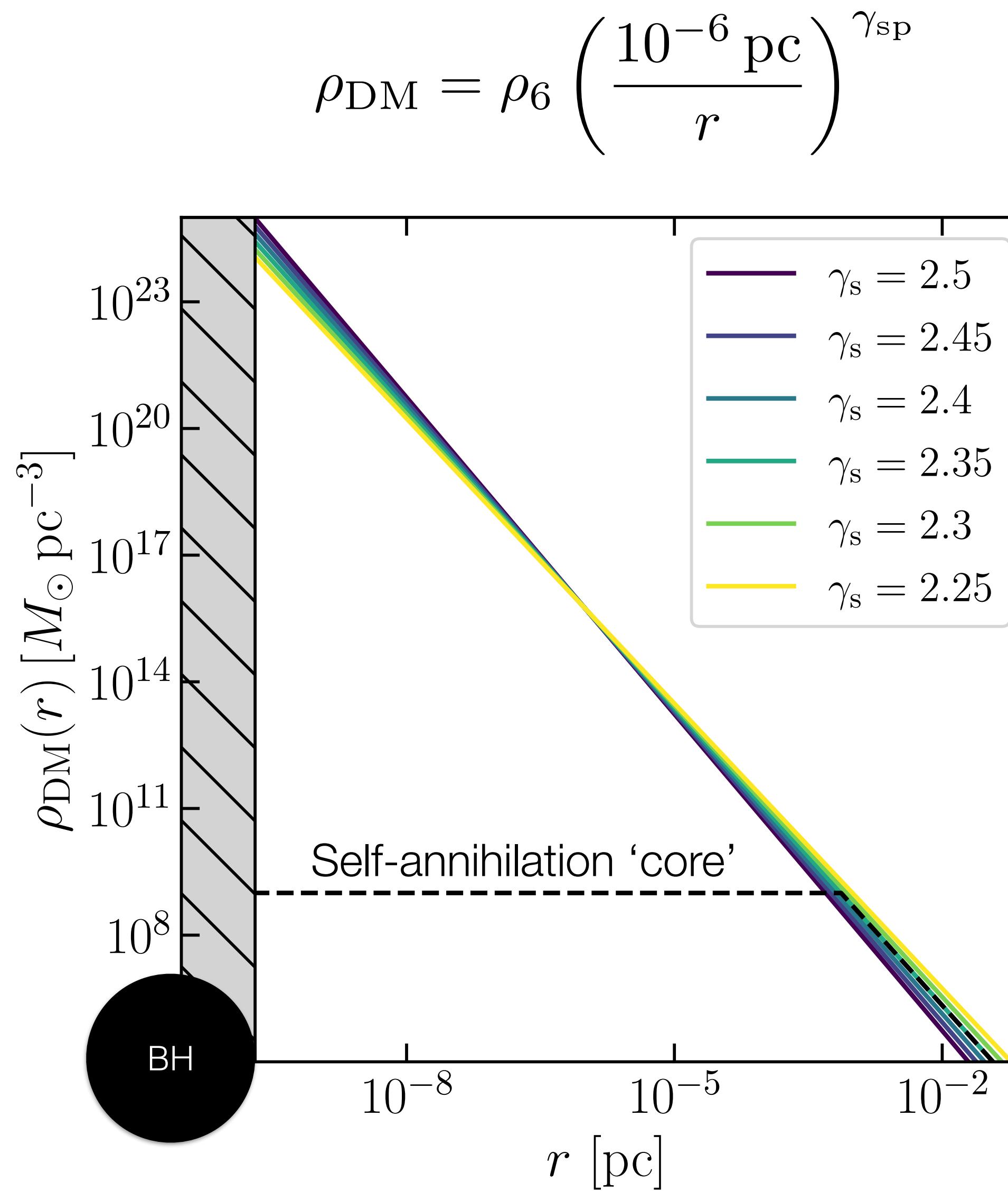


$$m_{\text{DCBH}} \sim 10^3 - 10^5 M_\odot$$

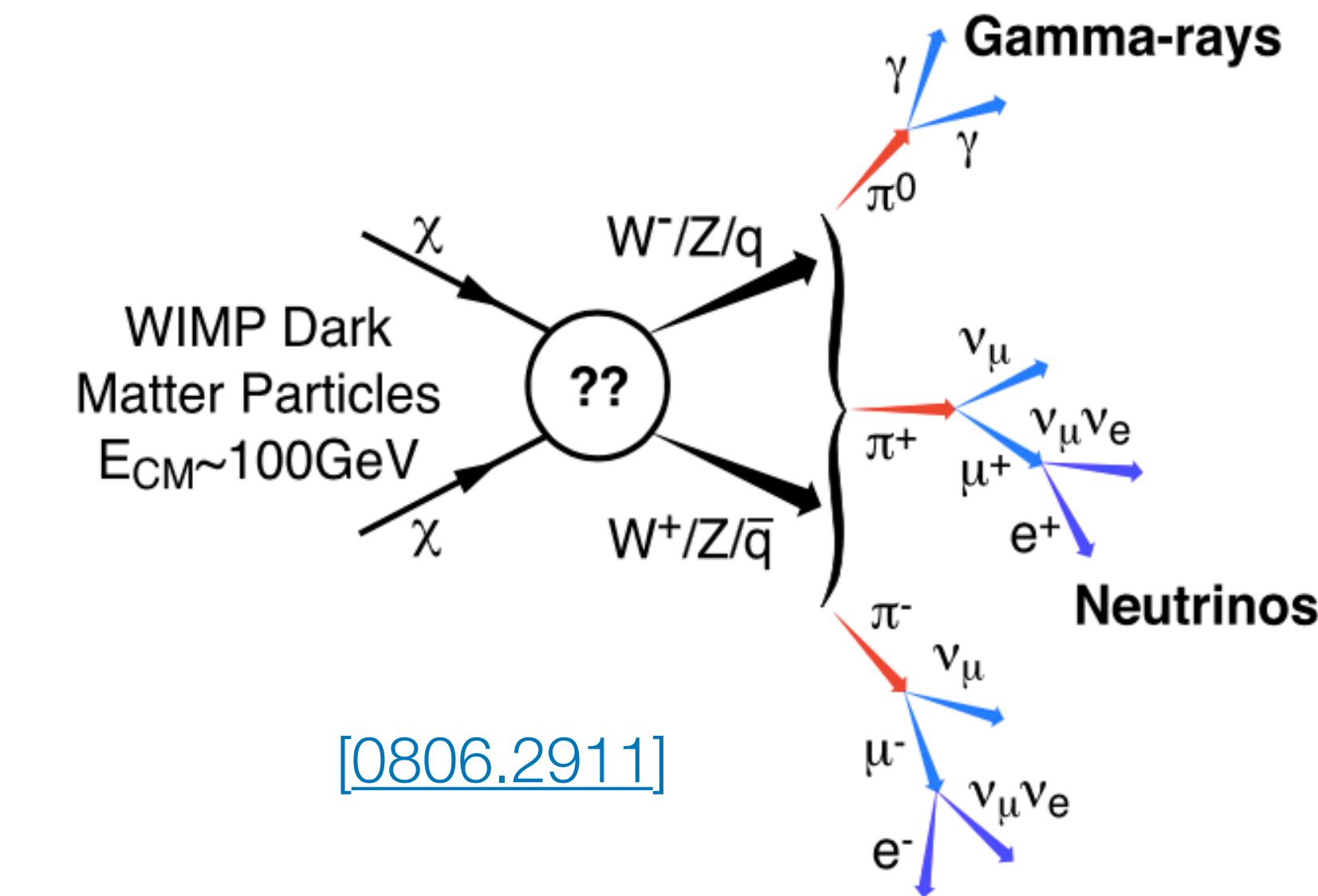
[Bertone et al. (including **BJK**), [2404.08731](#);  
and work in progress with Abram Perez and Pratika Dayal]

\*Precise details of formation may affect slope  
and density of DM very close to BH

# DM annihilation?



$$\rho_{\text{DM, local}} \sim 10^{-2} M_\odot/\text{pc}^3$$



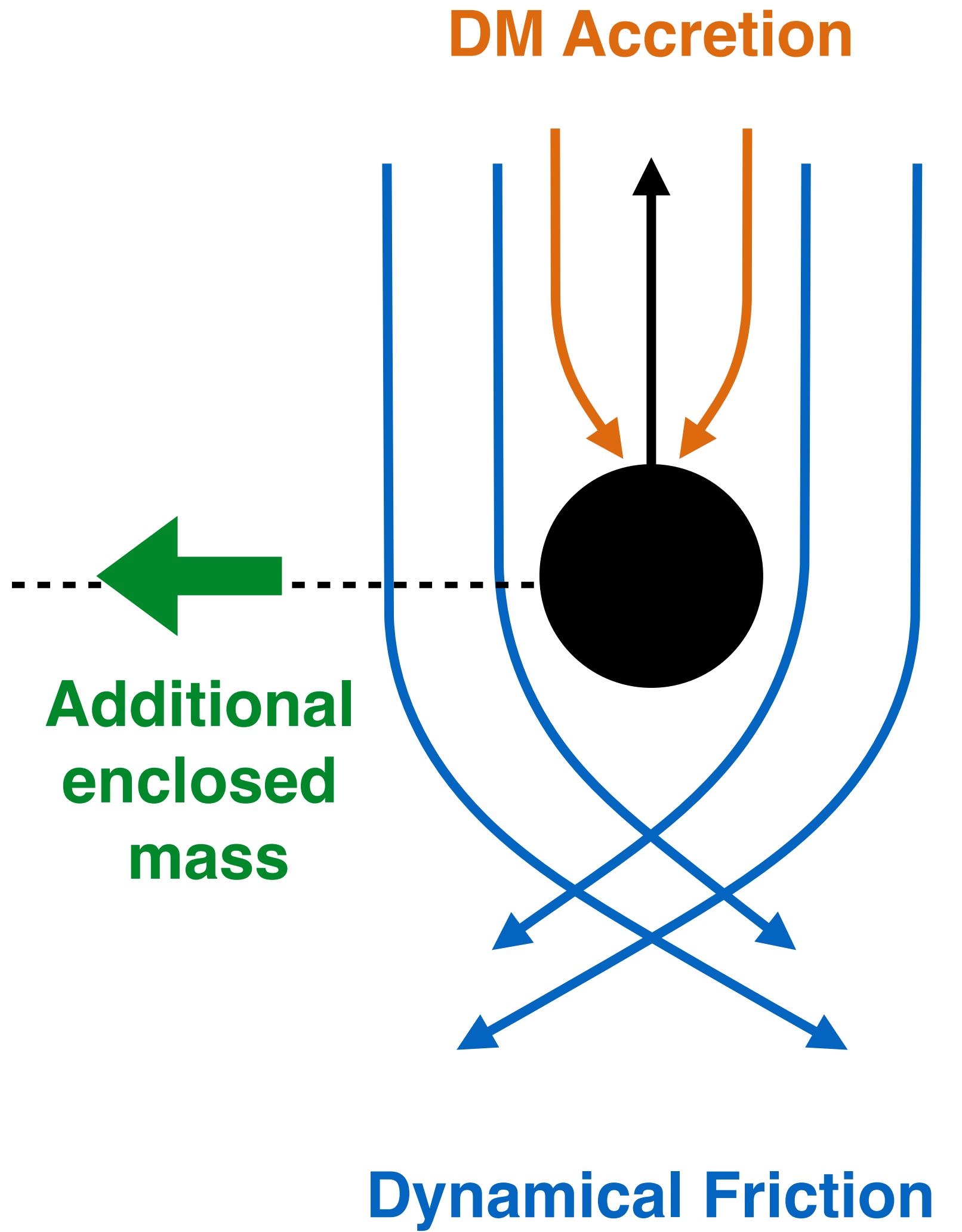
DM self-annihilation can suppress the spike density, but can still lead to large (diffuse and point source) fluxes of gamma-rays and neutrinos

[E.g. Lacroix & Silk, [1712.00452](#), Bertone et al., [1905.01238](#), Freese et al., [2202.01126](#)]

What about **non-annihilating DM**?

# Impact of DM Spikes

$m_1 = 1000 M_\odot$   
 $m_2 = 1 M_\odot$   
 $\gamma_{\text{sp}} = 7/3$   
 $\rho_6 = 5.45 \times 10^{15} M_\odot \text{ pc}^{-3}$



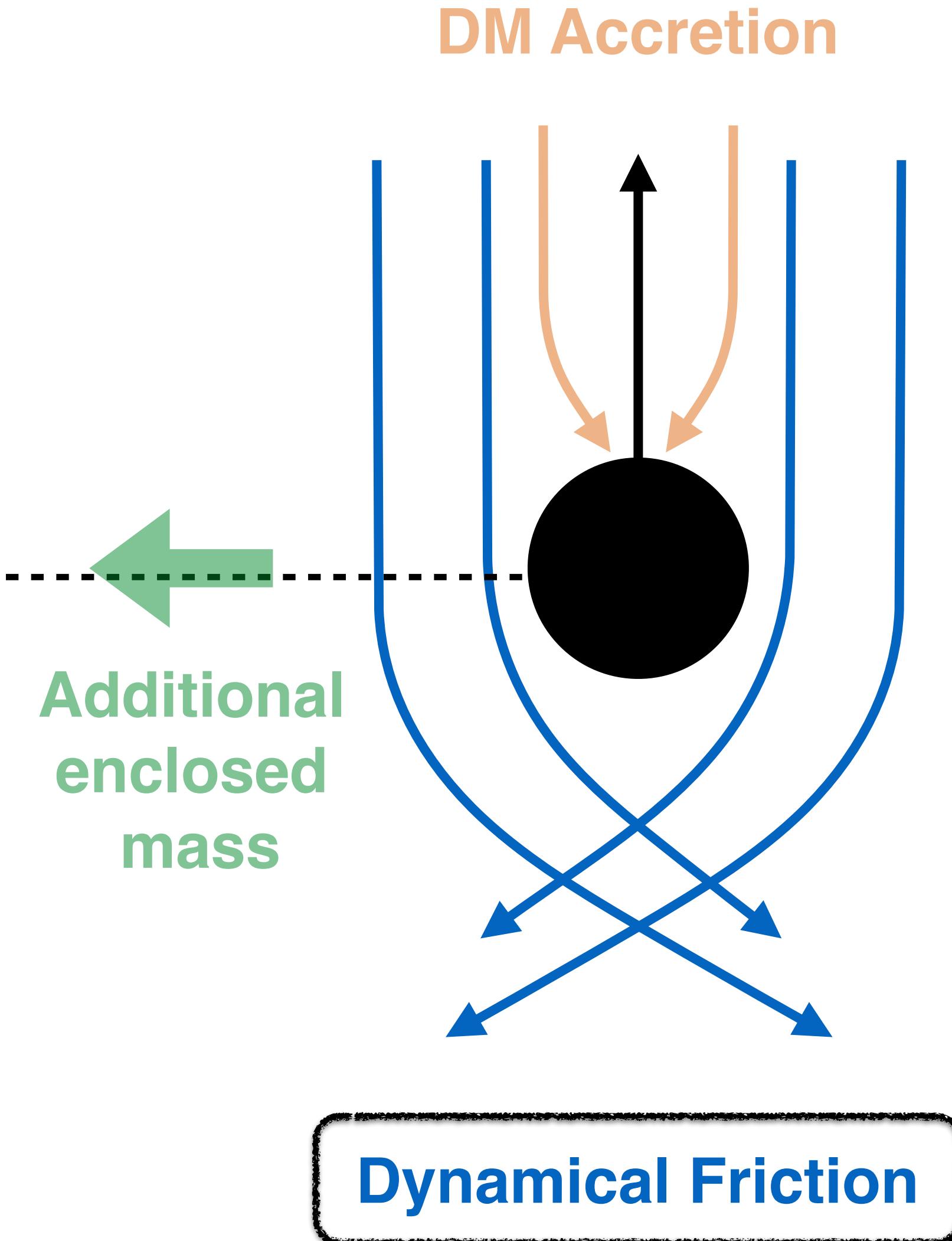
[See e.g. Eda et al. [1301.5971](#), [1408.3534](#),  
Macedo et al., [1302.2646](#); Cardoso & Maselli, [1909.05870](#)]

# Impact of DM Spikes

$$\begin{aligned}m_1 &= 1000 M_{\odot} \\m_2 &= 1 M_{\odot} \\\gamma_{\text{sp}} &= 7/3 \\\rho_6 &= 5.45 \times 10^{15} M_{\odot} \text{ pc}^{-3}\end{aligned}$$

$$\dot{E}_{\text{DF}} \sim \frac{4\pi G^2 m_2^2 \rho_{\text{DM}}(r) \xi(v)}{v} \ln \Lambda$$

[See e.g. Eda et al. [1301.5971](#), [1408.3534](#),  
Macedo et al., [1302.2646](#); Cardoso & Maselli, [1909.05870](#)]



Solve the Newtonian system of a quasi-circular inspiral  $r_2(t)$ :

$$\boxed{\begin{aligned} -\dot{E}_{\text{orb}} &= \dot{E}_{\text{GW}} + \dot{E}_{\text{DF}} \\ E_{\text{orb}} &\approx -\frac{Gm_1m_2}{2r_2} \end{aligned}}$$

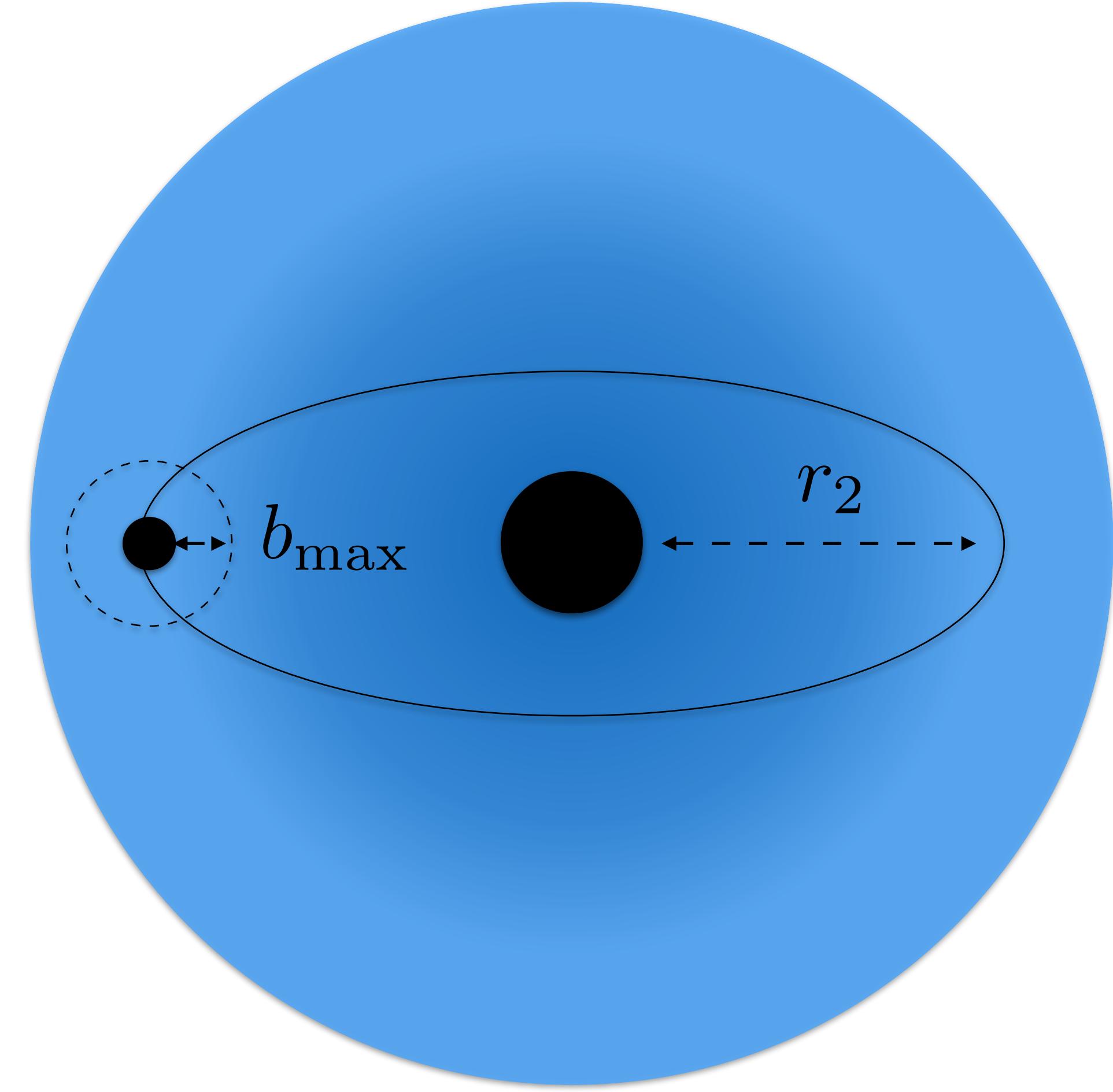
But dynamical friction injects (a lot of) energy into the DM spike!

Each particle passing close to the orbiting BH receives a ‘kick’ through gravitational scattering:

$$\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E} \quad \text{with} \quad \mathcal{E} = \Psi(r) - \frac{1}{2}v^2$$

Simultaneously evolve the DM distribution function:

$$\frac{\partial f(\mathcal{E})}{\partial t}$$

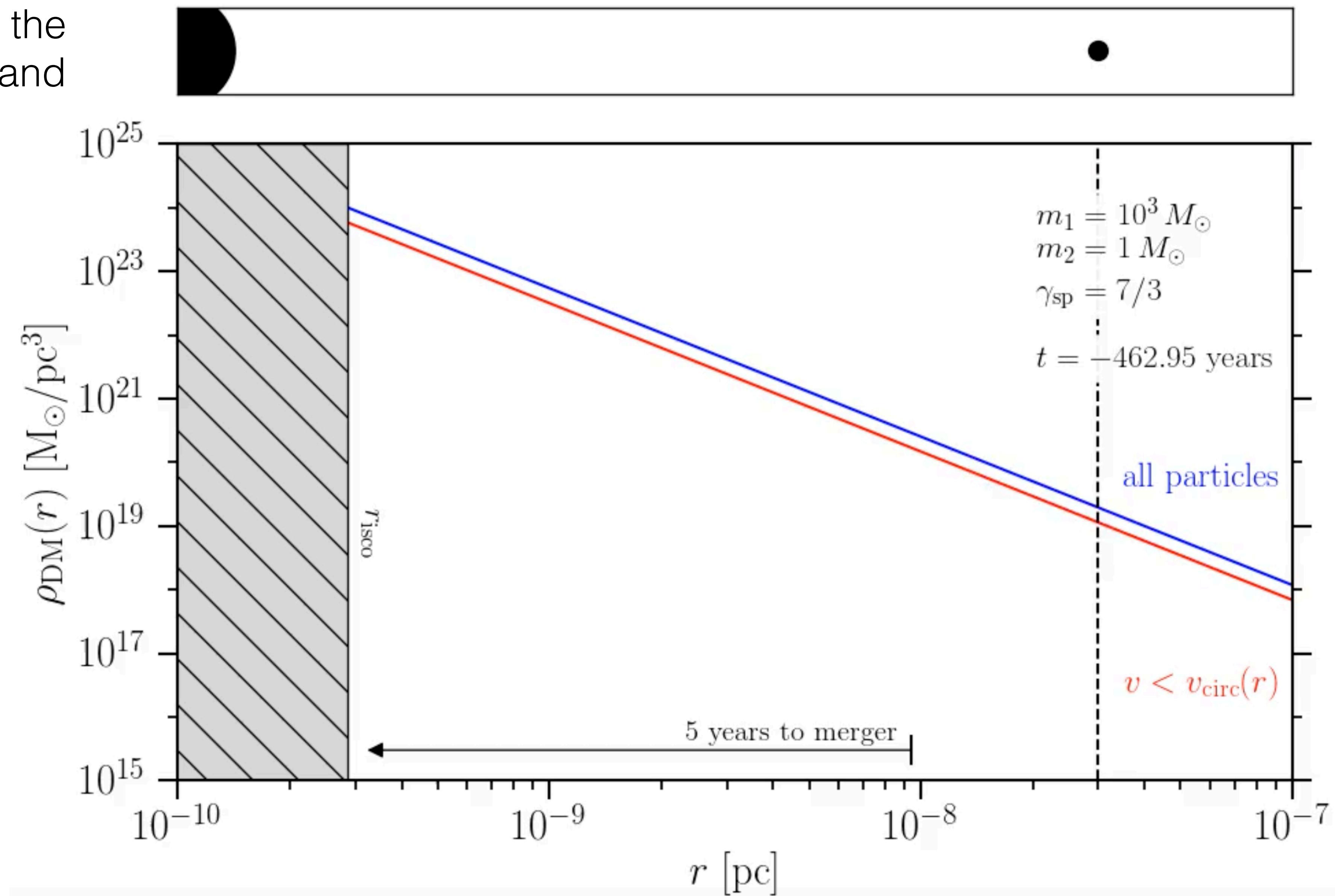


### HaloFeedback

[Code available online:  
[github.com/bradkav/HaloFeedback](https://github.com/bradkav/HaloFeedback)]

# Co-evolution

Find that the density of the DM spike is depleted (and replenished...)



This is one of the reasons we want to look at IMRIs/EMRIs...

[BJK, Nichols, Gaggero, Bertone, [2002.12811](#)]

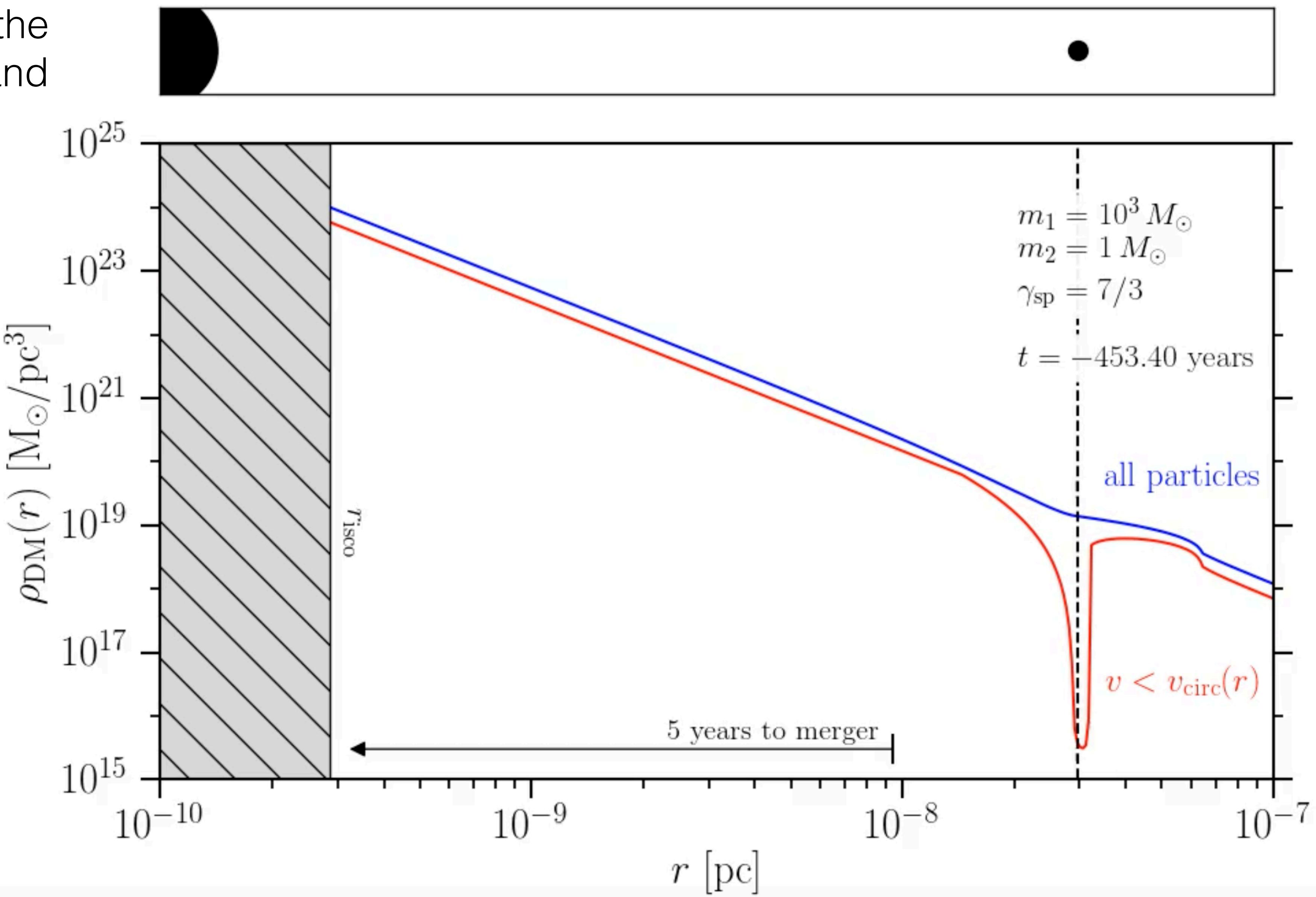
[Movies: [tinyurl.com/GW4DM](#)]

[Code: [github.com/bradkav/HaloFeedback](#)]

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Size of the dephasing effect is **reduced from  $\mathcal{O}(1)$  to  $\mathcal{O}(1\%)$ .**



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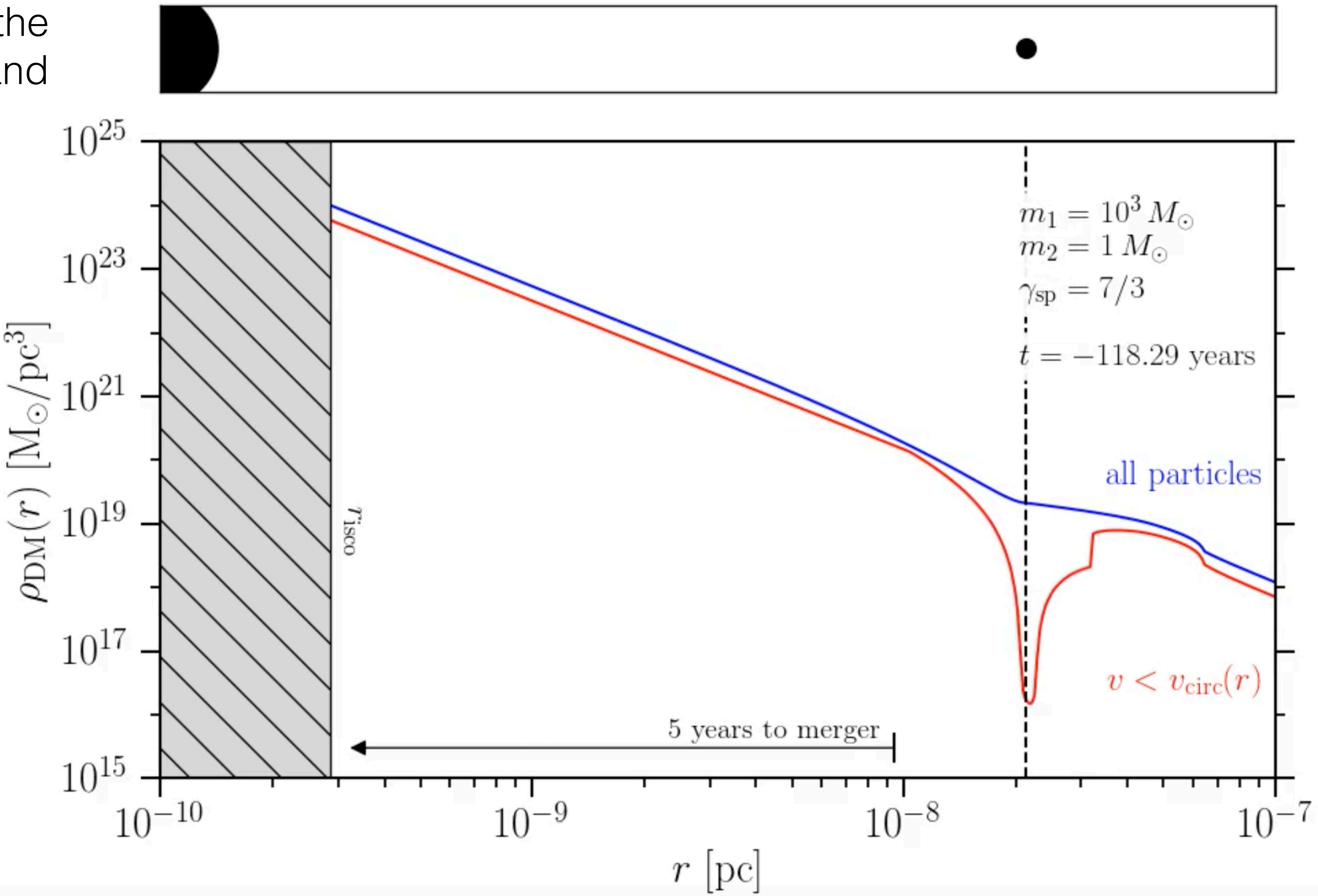
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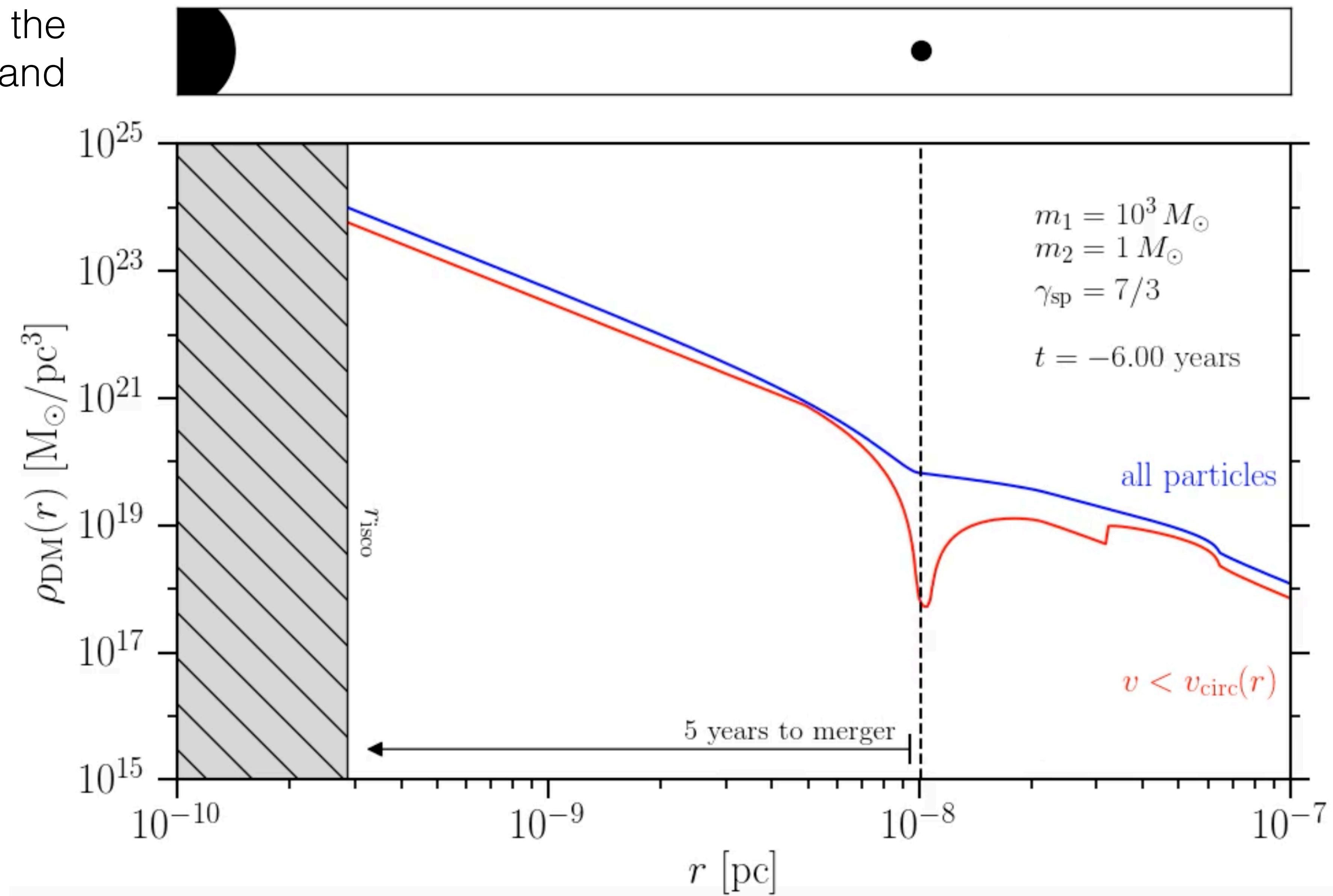
[BJK, Nichols, Gaggero, Bertone, [2002.12811](#)]

[Movies: [tinyurl.com/GW4DM](#)]

[Code: [github.com/bradkav/HaloFeedback](#)]

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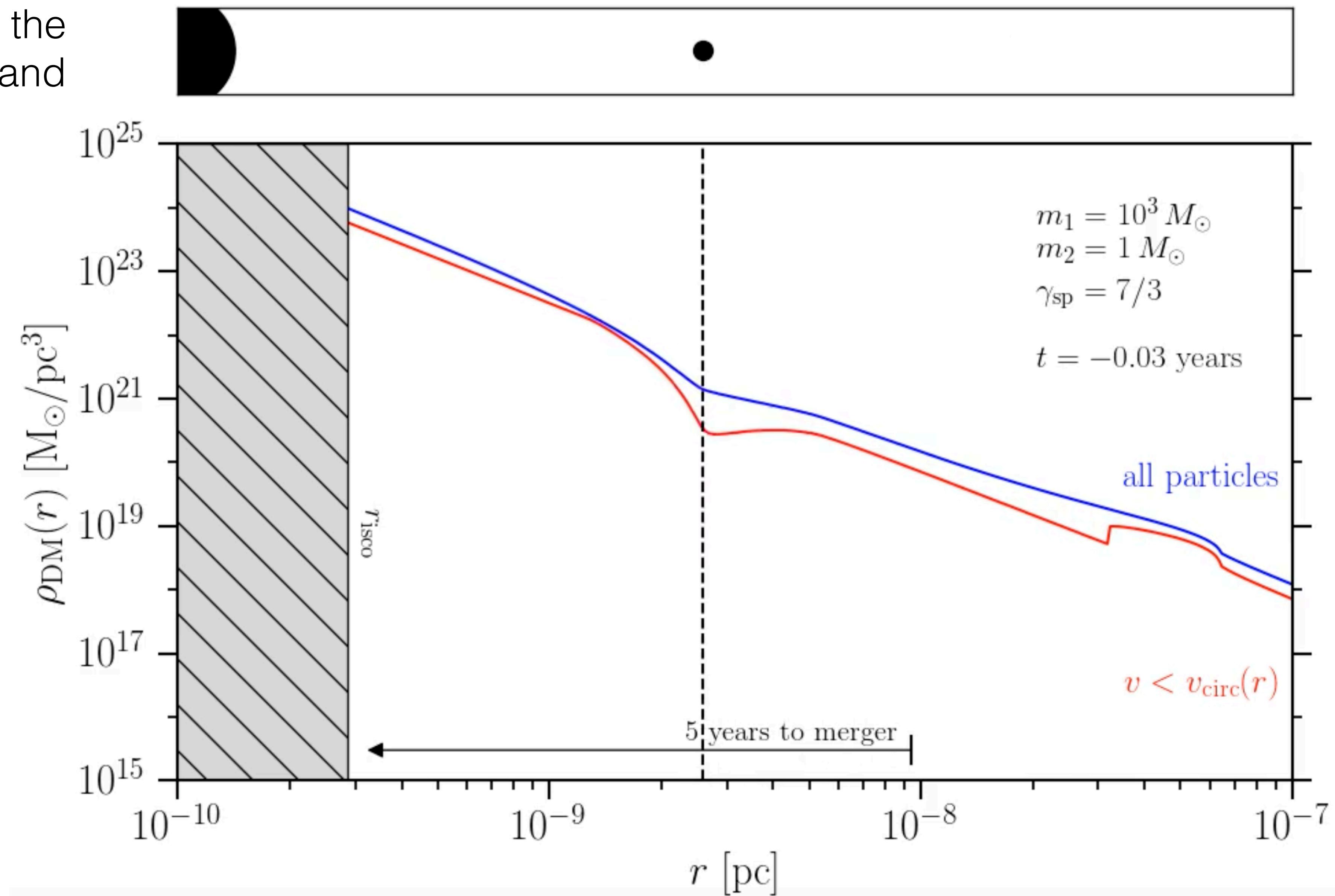
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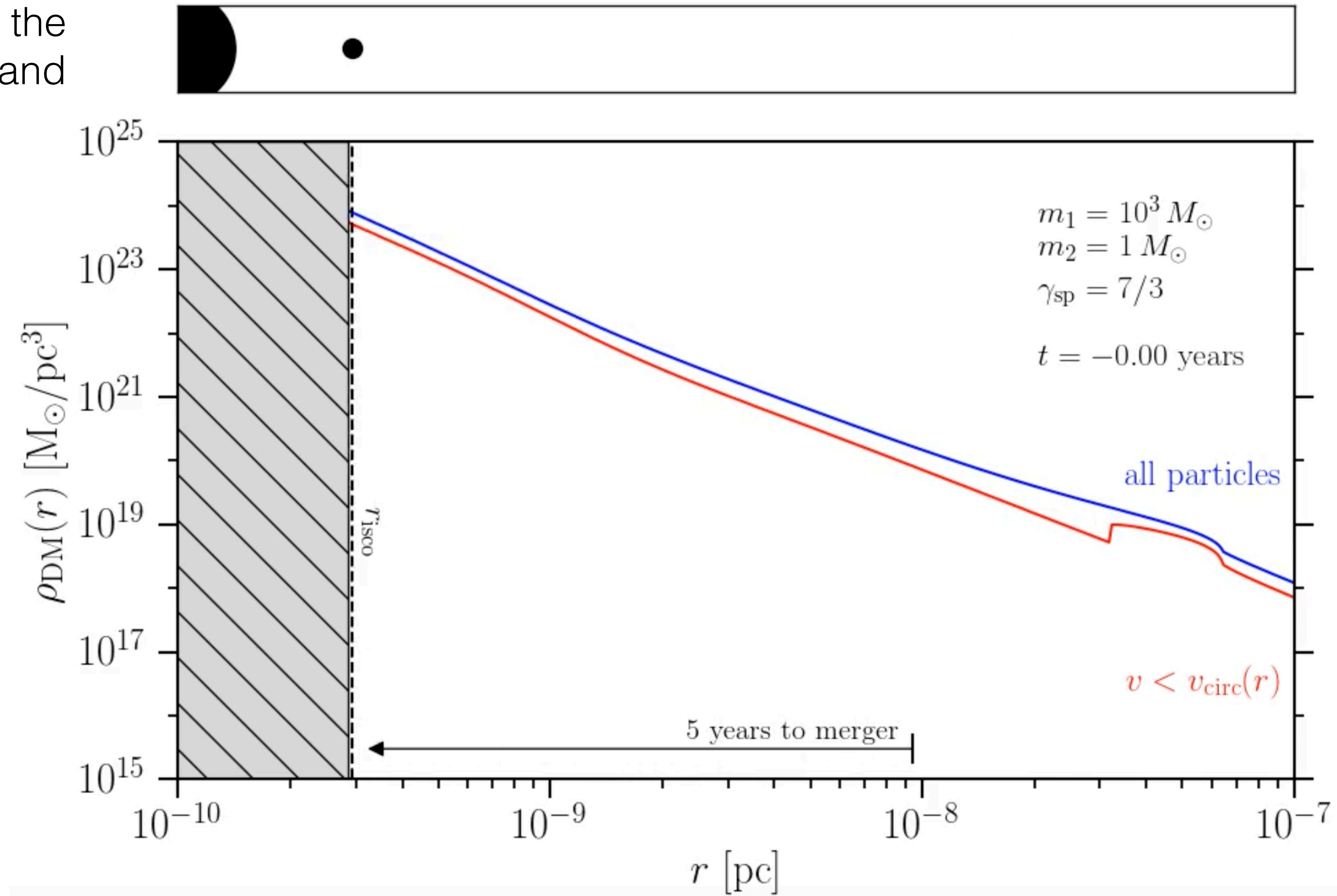
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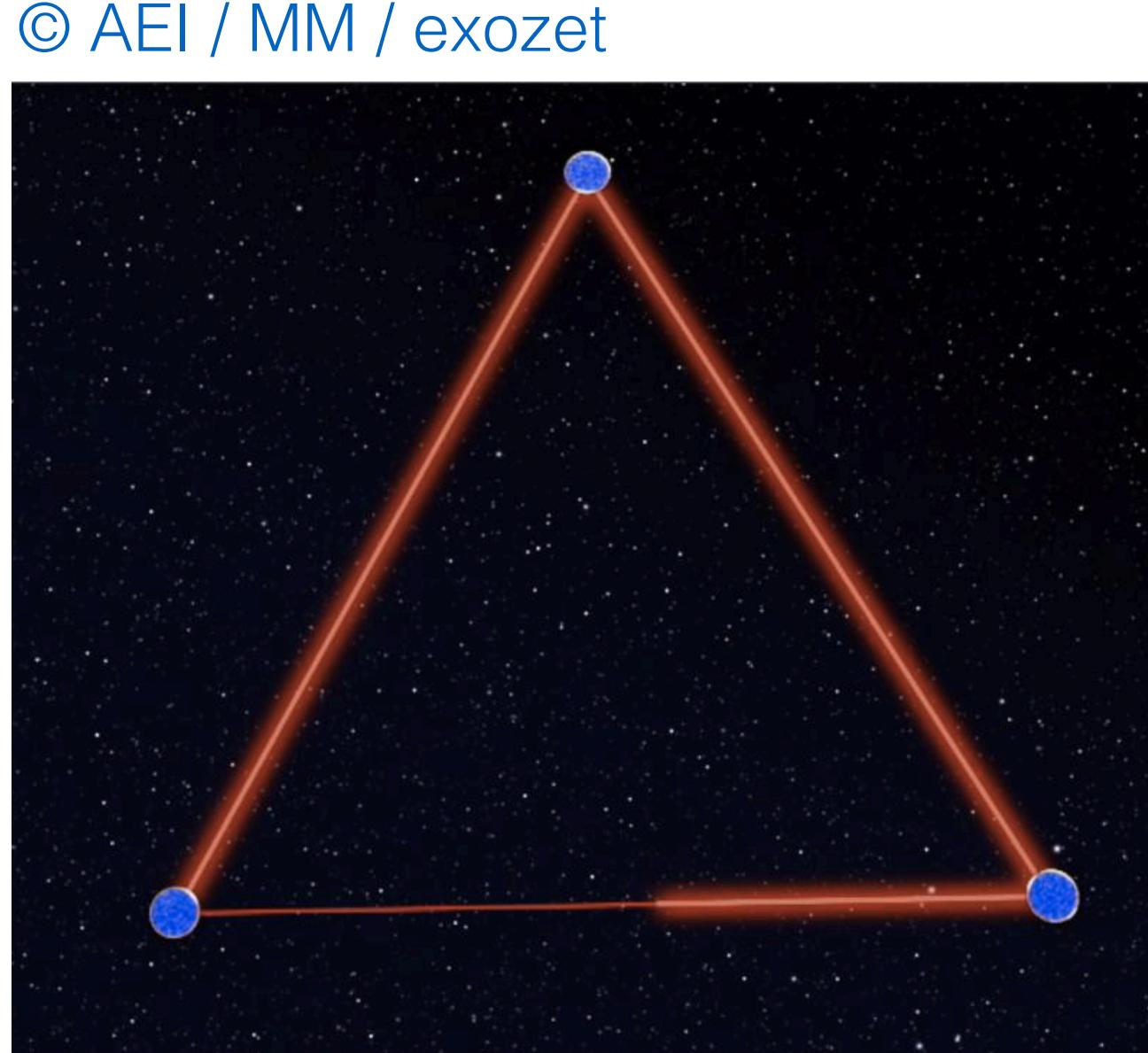
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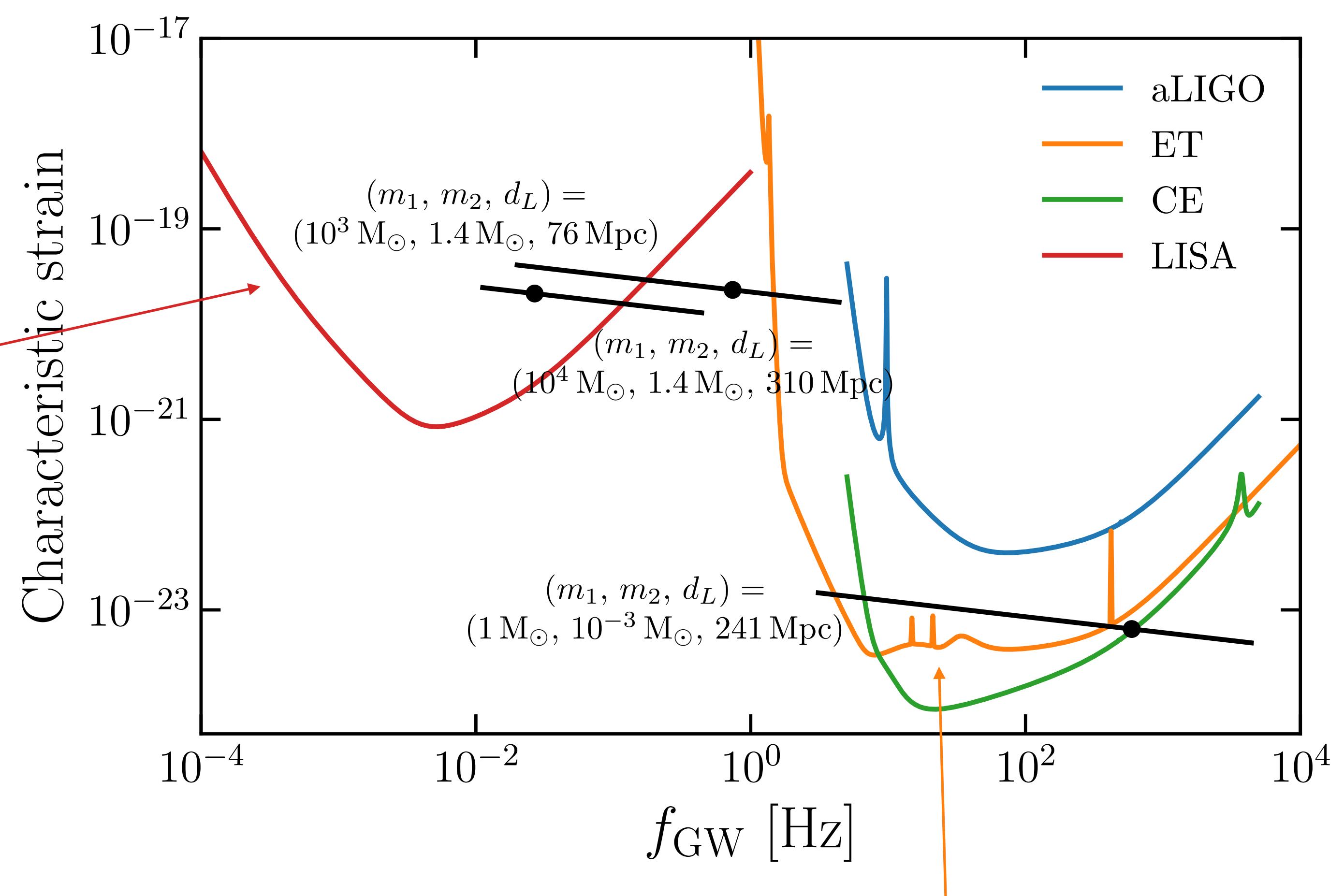
[Code: [github.com/bradkav/HaloFeedback](#)]

# GW Sensitivity



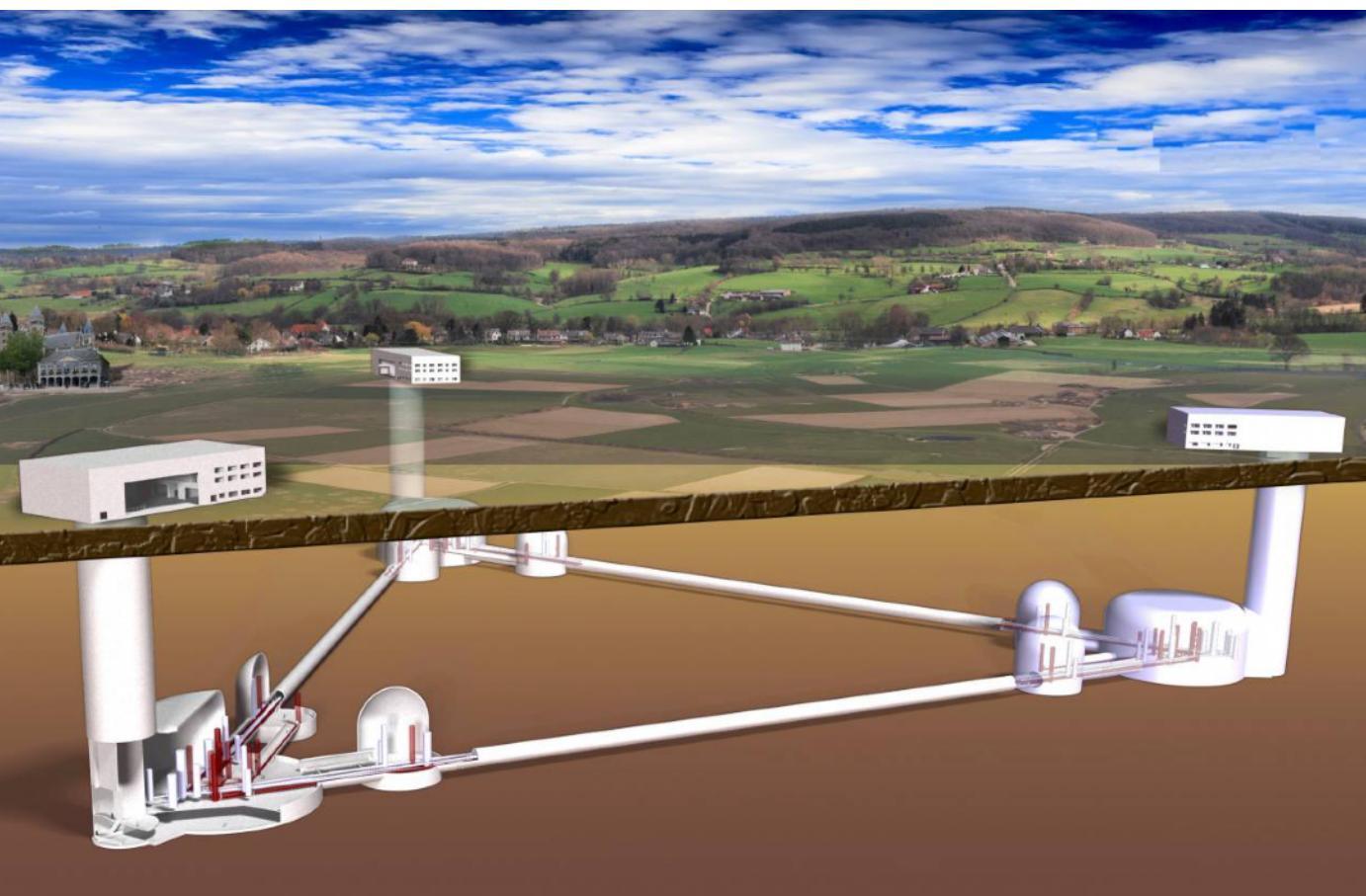
**Laser Interferometer Space Antenna**  
(planned for the 2030s)

[\[1907.06482\]](#)

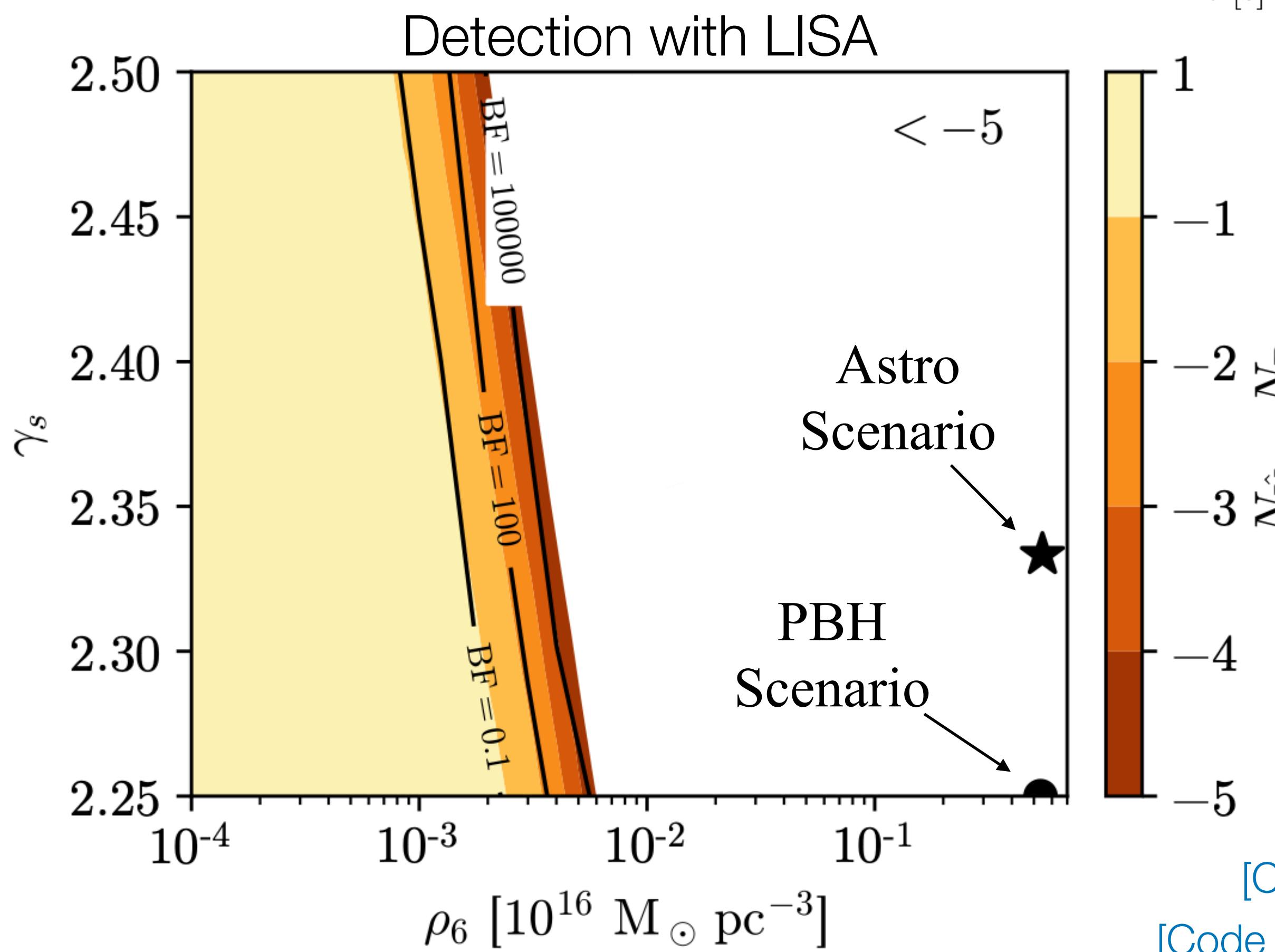
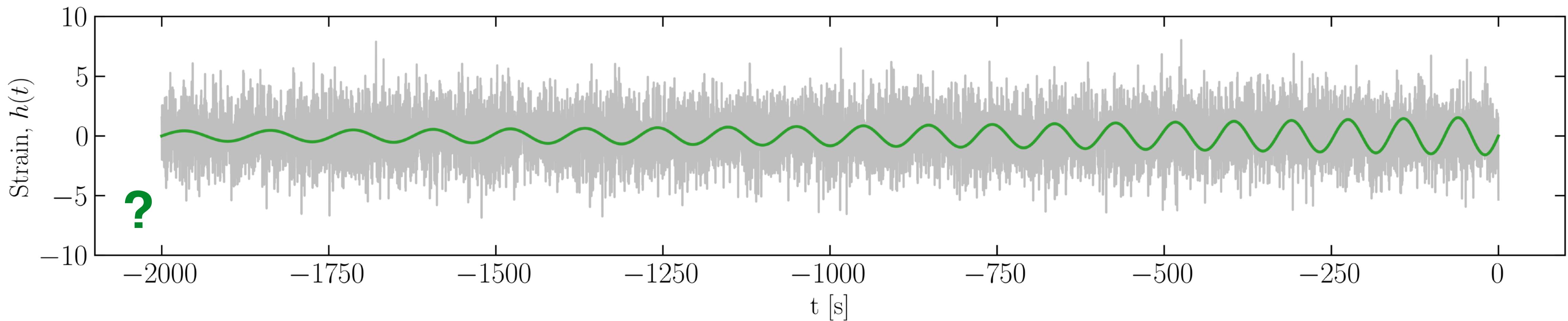


**Einstein Telescope**

[\[1912.02622\]](#)



# Discoverability



Compare **Bayes factor (BF)** for the vacuum case (V) and the DM dressed case (D)

$$\theta_V = \{\mathcal{M}\}$$

vs.

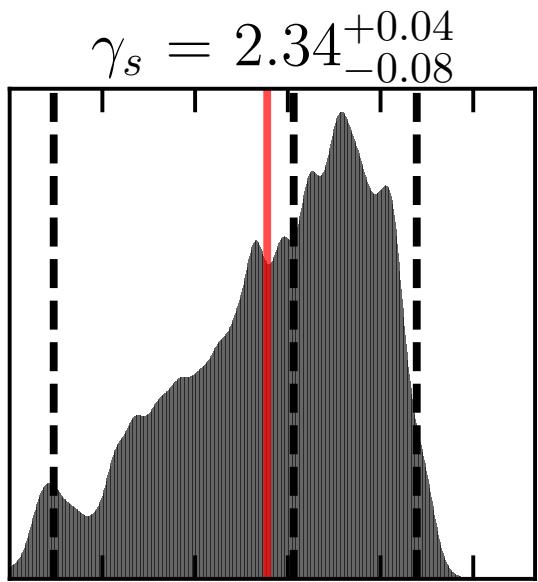
$$\theta_D = \{\gamma_{\text{sp}}, \rho_6, \mathcal{M}, \log_{10} q\}$$

Number of GW cycles of dephasing

[Coogan, Bertone, Gaggero, **BJK** & Nichols, [2108.04154](#)]  
[Code available online: <https://github.com/adam-coogan/pydd>]

# Measurability

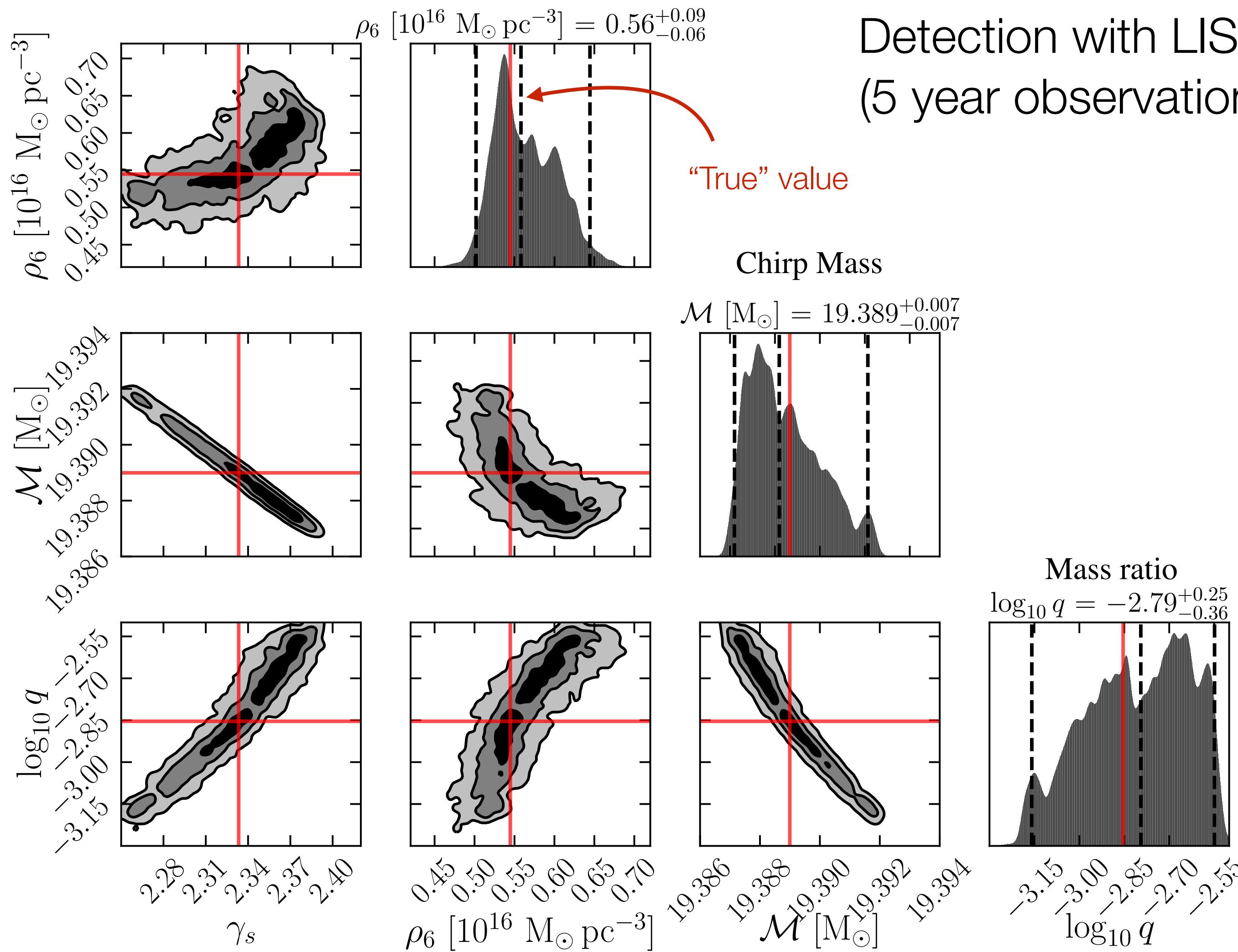
## Astrophysical scenario



$$m_1 = 10^3 M_\odot$$

$$m_2 = 1 M_\odot$$

Detection with LISA  
(5 year observation)



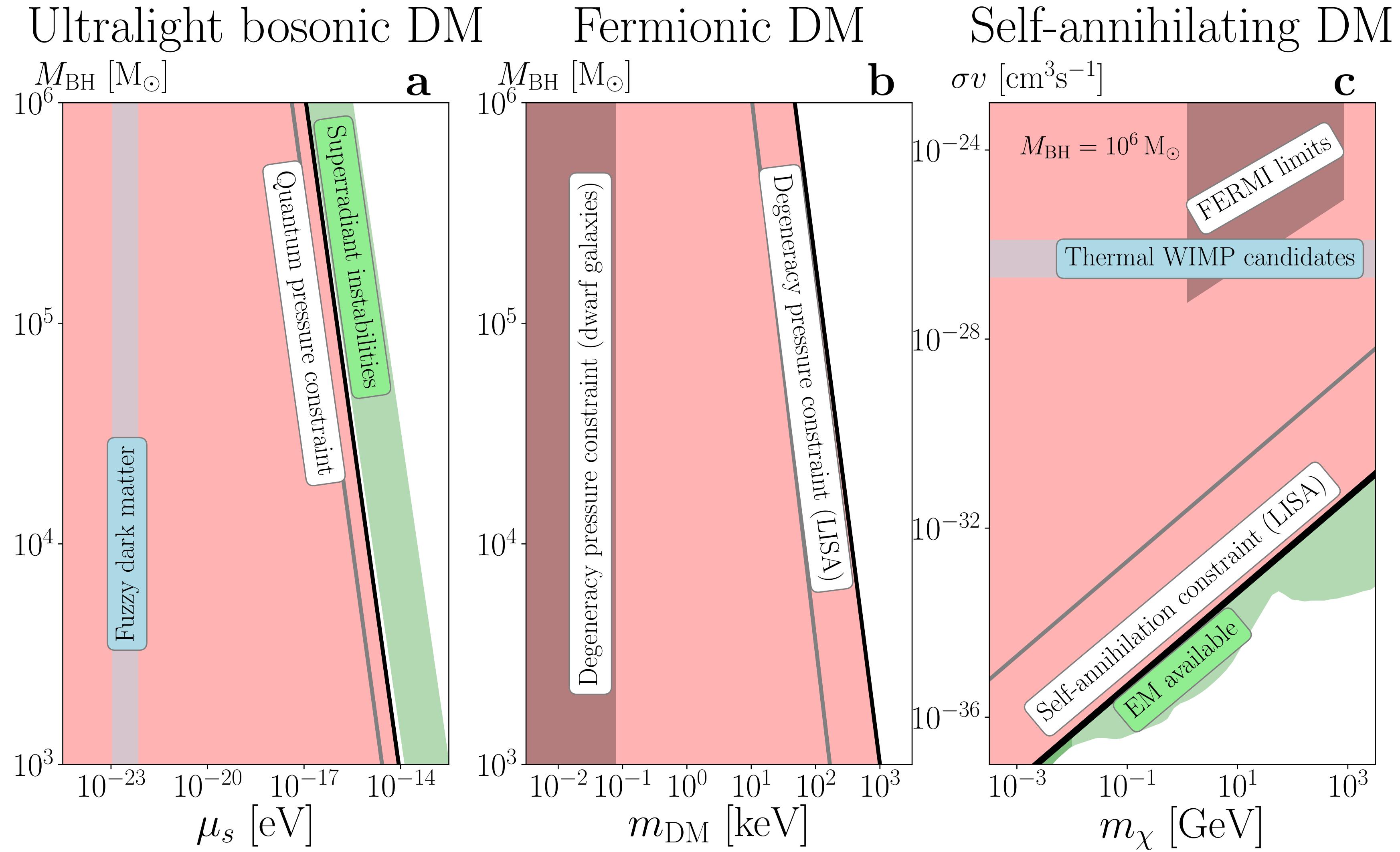
[Coogan, Bertone, Gaggero,  
**BJK** & Nichols, 2108.04154]

[Code: [github.com/adam-coogan/pydd](https://github.com/adam-coogan/pydd)]

[See also Cole, Coogan, **BJK**, Bertone,  
[2207.07576](https://arxiv.org/abs/2207.07576) for the PBH scenario ]

**Red regions** would be ruled out by observation of a DM spike!

[1906.11845]



[See also Bertone, Coogan, Gaggero, **BJK** & Weniger, 1905.01238]

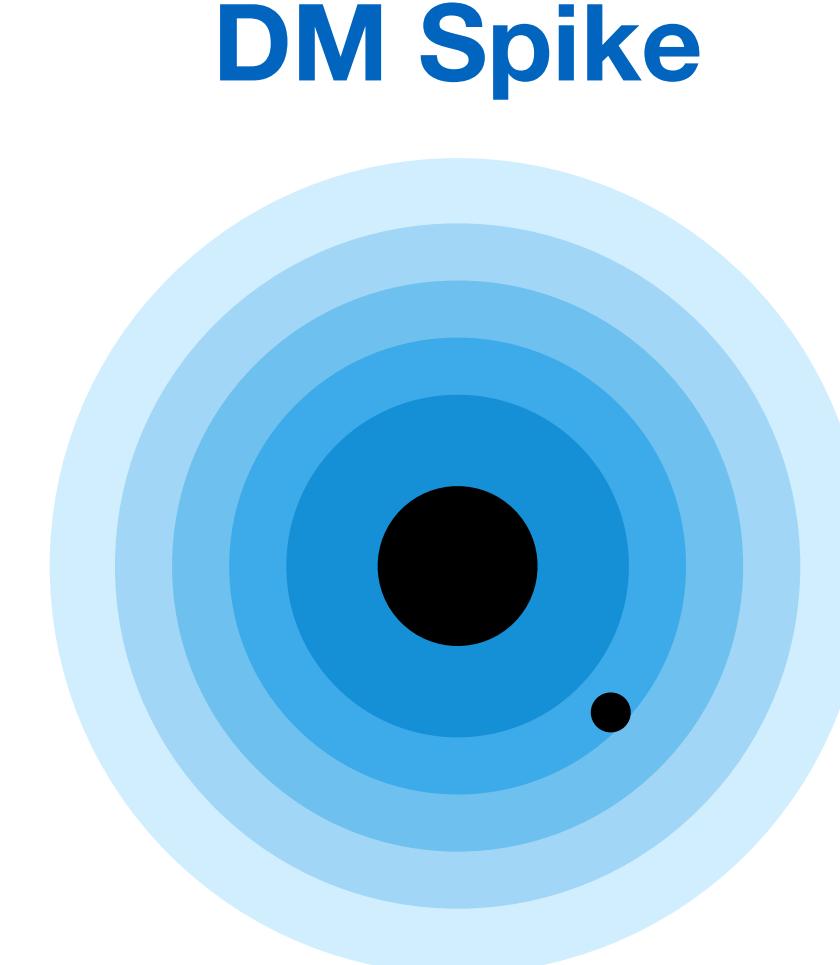
# Environmental confusion?

Generate waveform assuming:

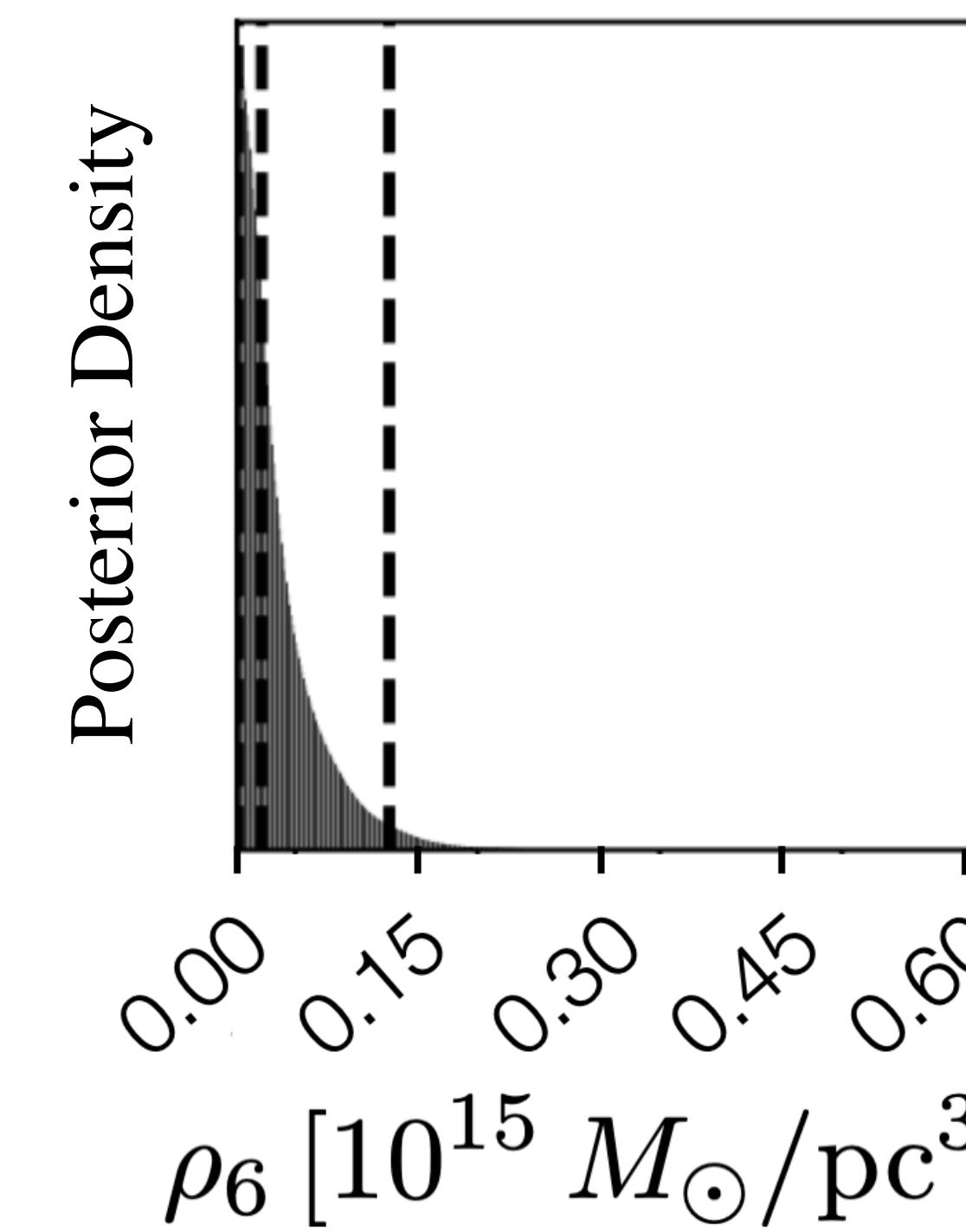


$$\Sigma(r) = \Sigma_0 \left( \frac{r}{r_0} \right)^{-1/2}$$

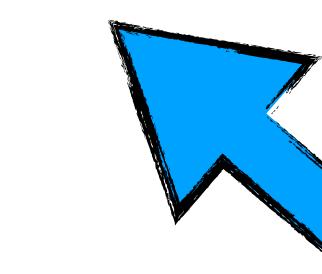
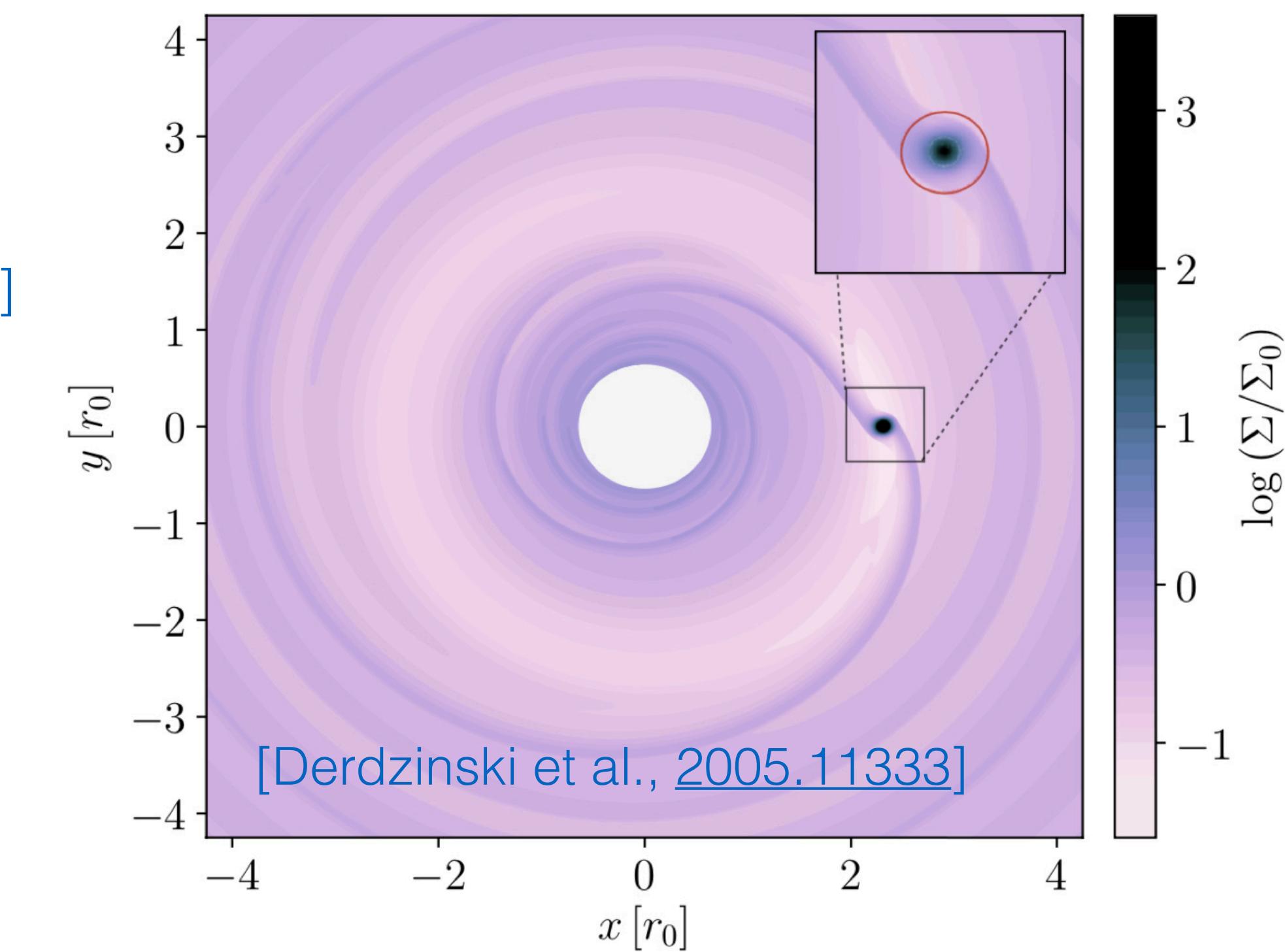
Fit signal assuming:



[Cole et al. (including **BJK**),  
[2211.01362](#), Nature Astronomy]



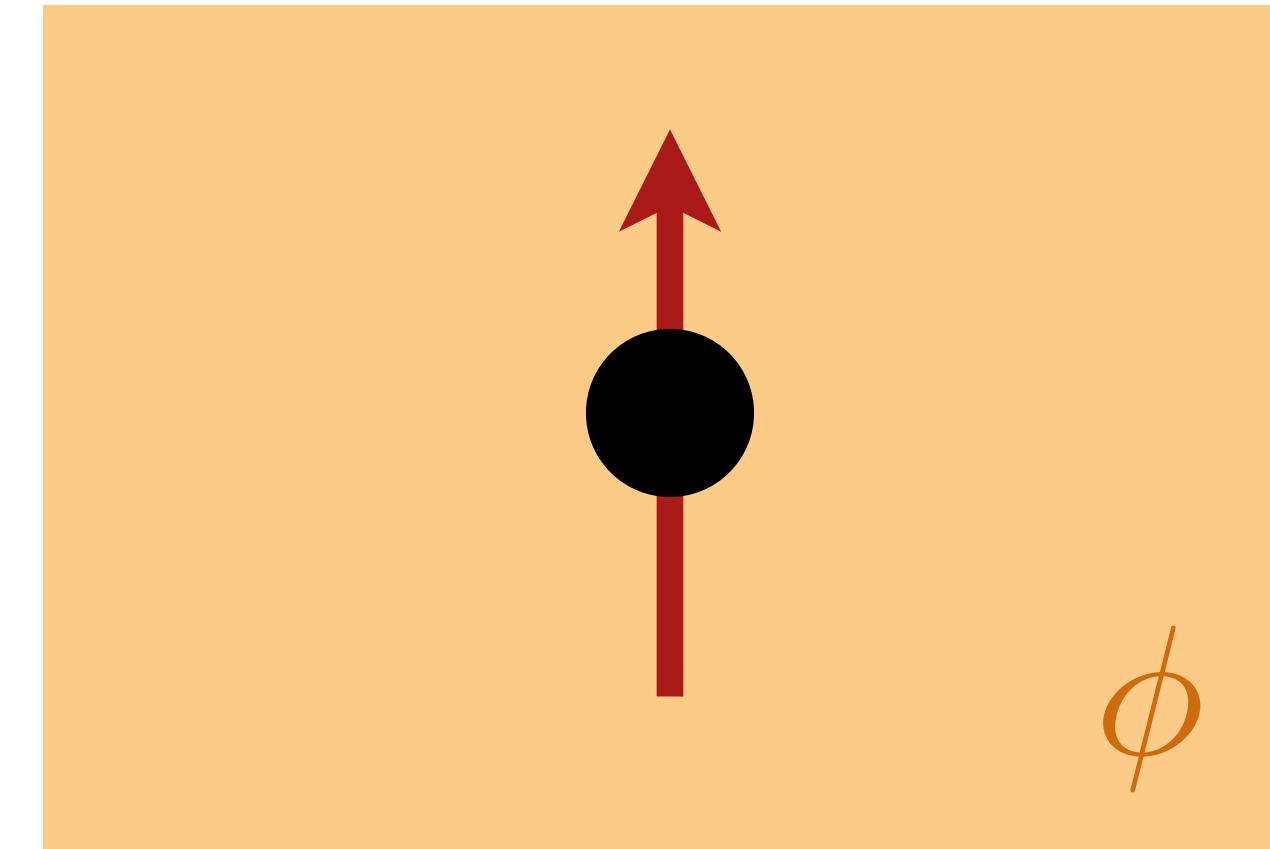
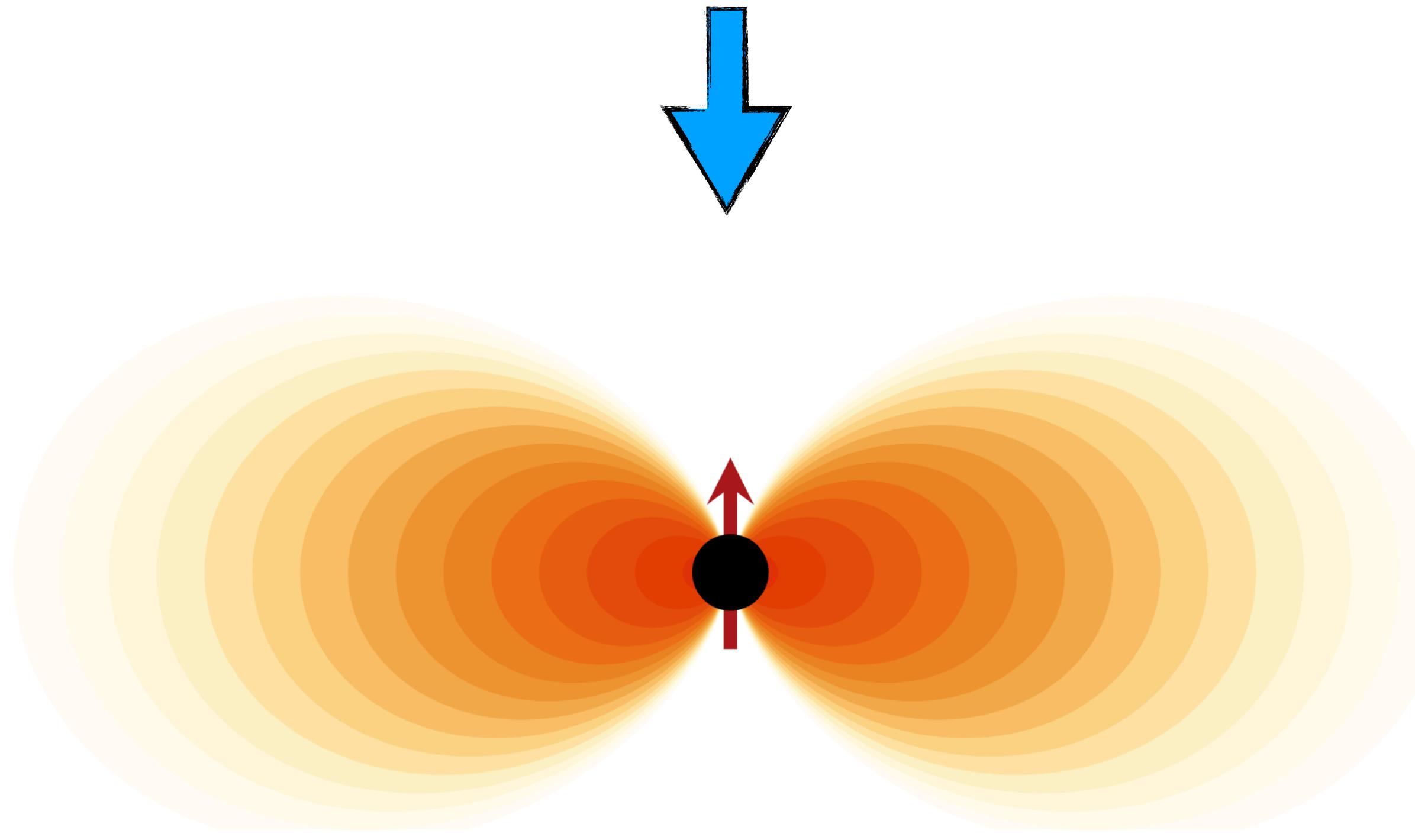
Energy loss mechanisms in an accretion disk are very different!



Signals **very hard to confuse** in 1 year of LISA data (huge Bayes factors!)

[See also Becker & Sagunski, [2211.05145](#)]

# Gravitational Atoms



Compton wavelength of a light scalar field:

$$\lambda_c \simeq 2 \text{ km} \left( \frac{10^{-10} \text{ eV}}{\mu} \right)$$

Super-radiant growth of a scalar cloud extracts spin from the BH when:

$$r_g \sim GM_{\text{BH}}/c^2 < \lambda_c$$

[Zel'dovich (1972), Starobinsky (1973)]

$$M_{\text{BH}} \in [1, 10^{10}] M_{\odot}$$

$$\rightarrow m_{\phi} \in [10^{-20}, 10^{-10}] \text{ eV}$$

[Chia, 2012.09167]

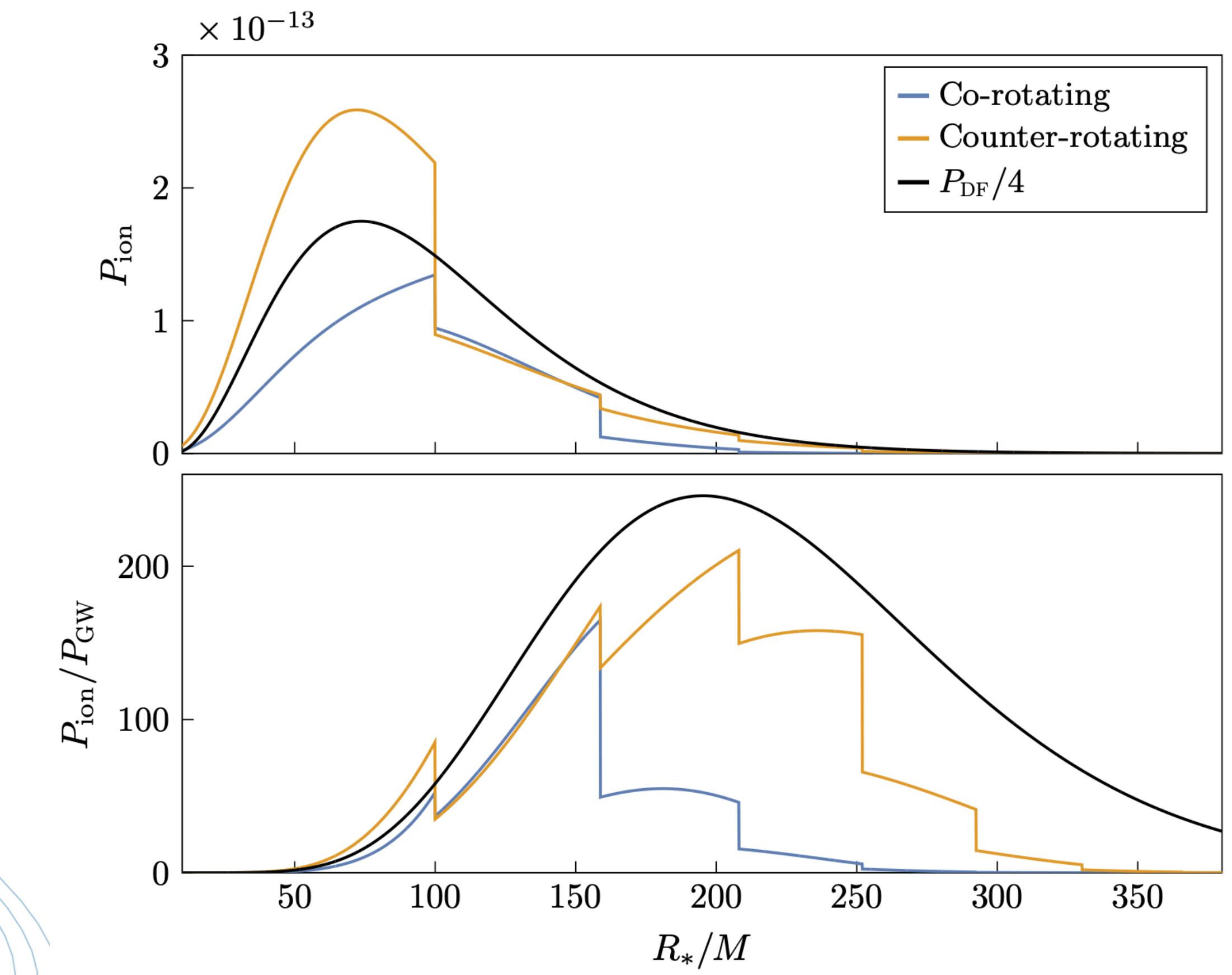
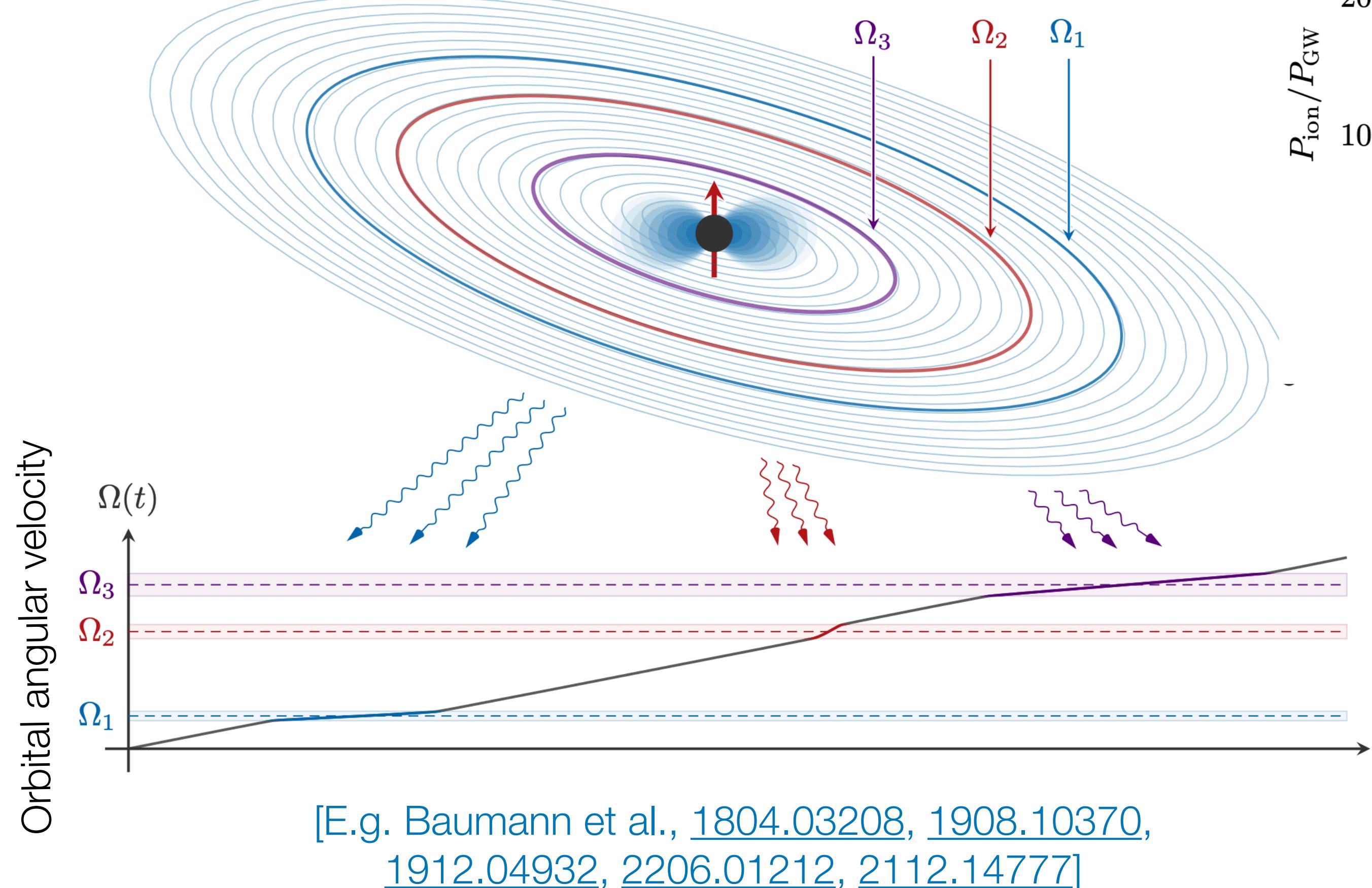
Bound states of the scalar field look like those of an electron in a hydrogen atom:

**'gravitational atom'**

# Gravitational Atoms

Orbiting BH can induce resonant transitions in the boson cloud.

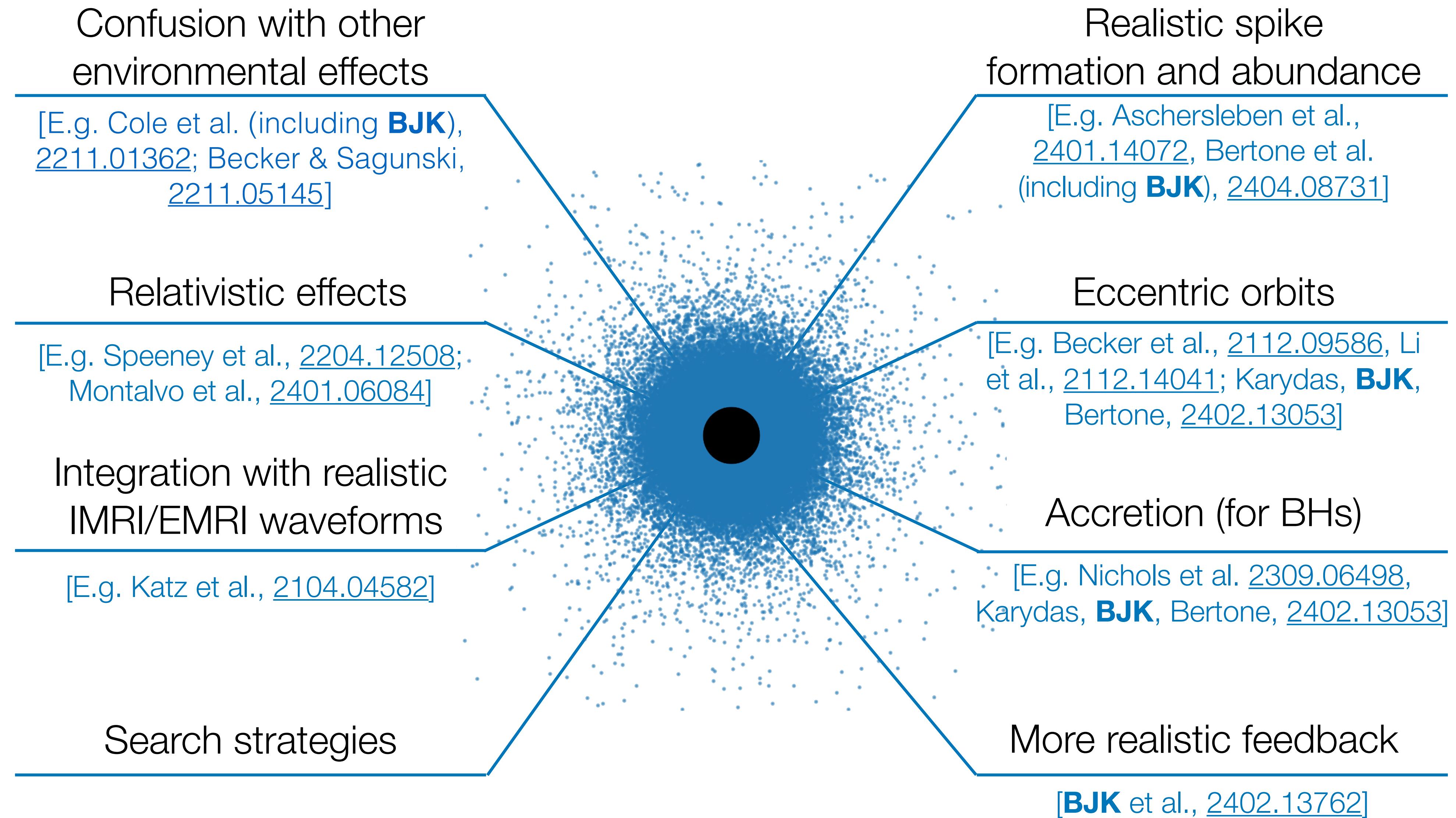
Presence of a gravitational atom can lead to ‘floating’ or ‘sinking’ orbits.



[Tomaselli, Spieksma, Bertone,  
[2305.15460](#), [2403.03147](#)]

Also easily distinguished from cold DM spikes and accretion disks.

[Cole et al. (including **BJK**),  
[2211.01362](#), Nature Astronomy]



## Confusion with other environmental effects

[E.g. Cole et al. (including **BJK**), [2211.01362](#); Becker & Sagunski, [2211.05145](#)]

## Relativistic effects

[E.g. Speeney et al., [2204.12508](#); Montalvo et al., [2401.06084](#)]

## Integration with realistic IMRI/EMRI waveforms

[E.g. Katz et al., [2104.04582](#)]

## Search strategies

## Realistic spike formation and abundance

[E.g. Aschersleben et al., [2401.14072](#), Bertone et al. (including **BJK**), [2404.08731](#)]

## Eccentric orbits

[E.g. Becker et al., [2112.09586](#), Li et al., [2112.14041](#); Karydas, **BJK**, Bertone, [2402.13053](#)]

## Accretion (for BHs)

[E.g. Nichols et al. [2309.06498](#), Karydas, **BJK**, Bertone, [2402.13053](#)]

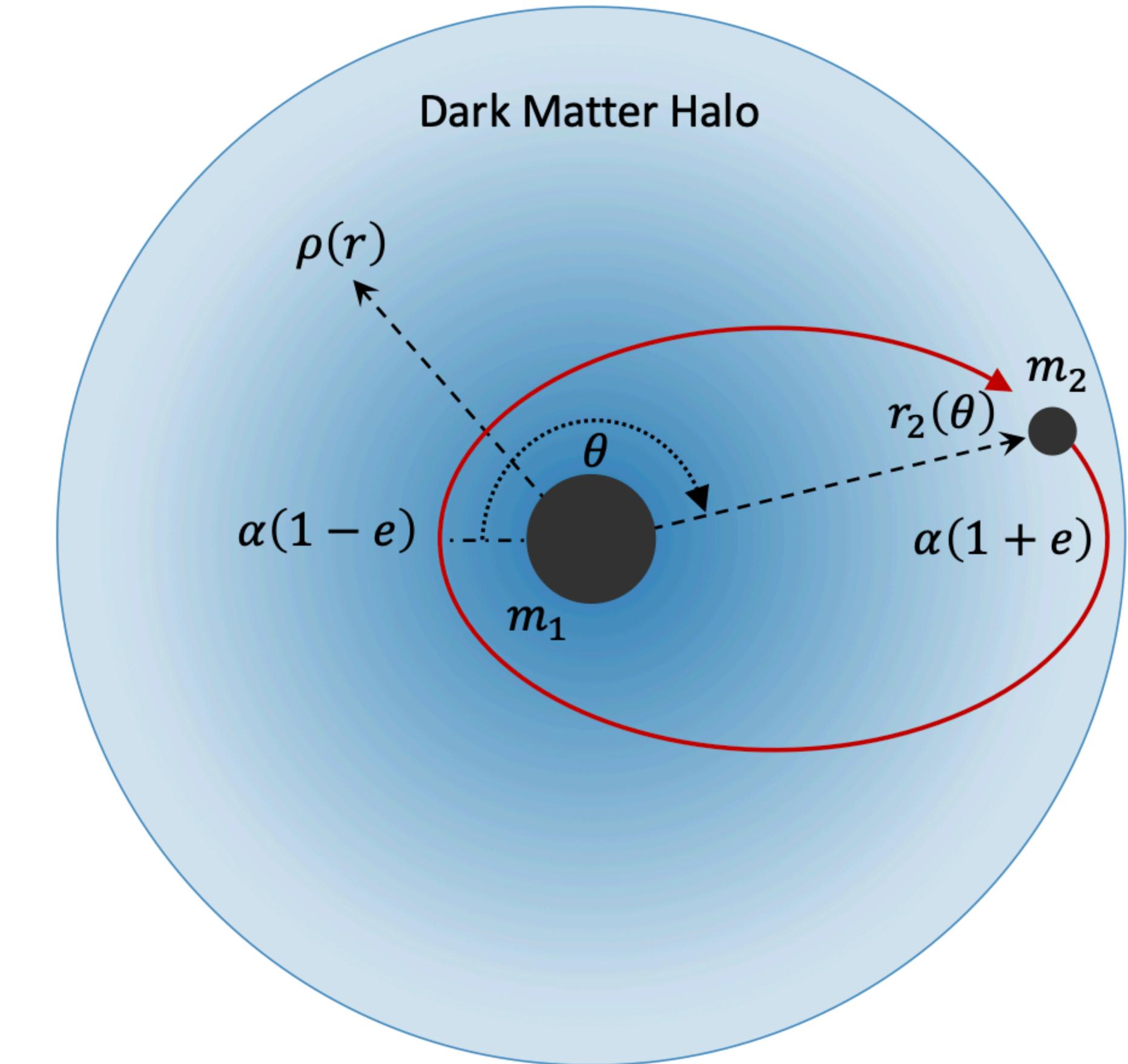
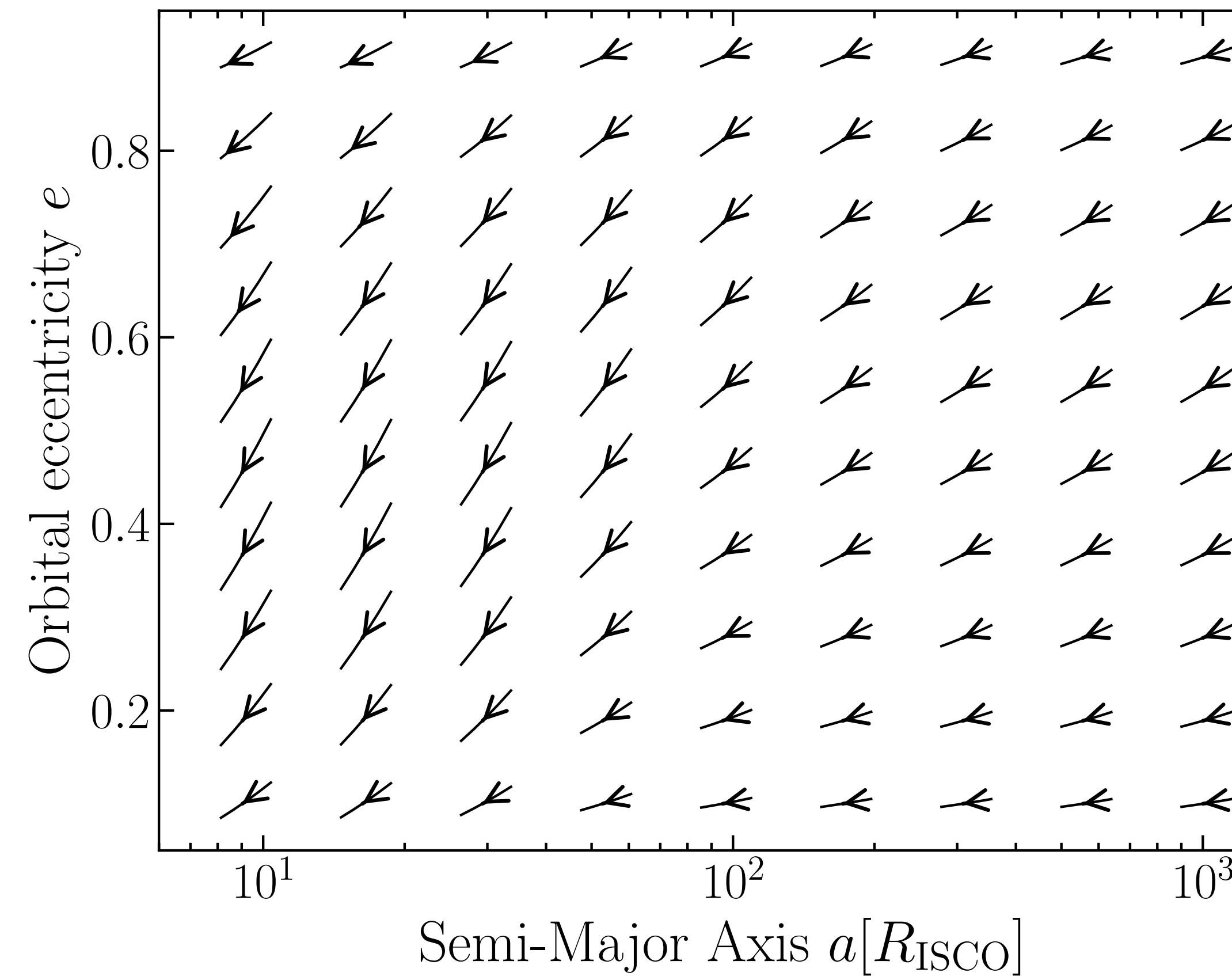
## More realistic feedback

[**BJK** et al., [2402.13762](#)]

# Eccentricity and Accretion

Recently generalised feedback formalism to account for eccentric orbits and accretion onto the orbiting compact object. [Karydas, BJK, Bertone, [2402.13053](#)]

Details are messy (need to co-evolve  $a$ ,  $e$  and DM distribution), but ultimately find that the orbits tend to circularise during the inspiral:

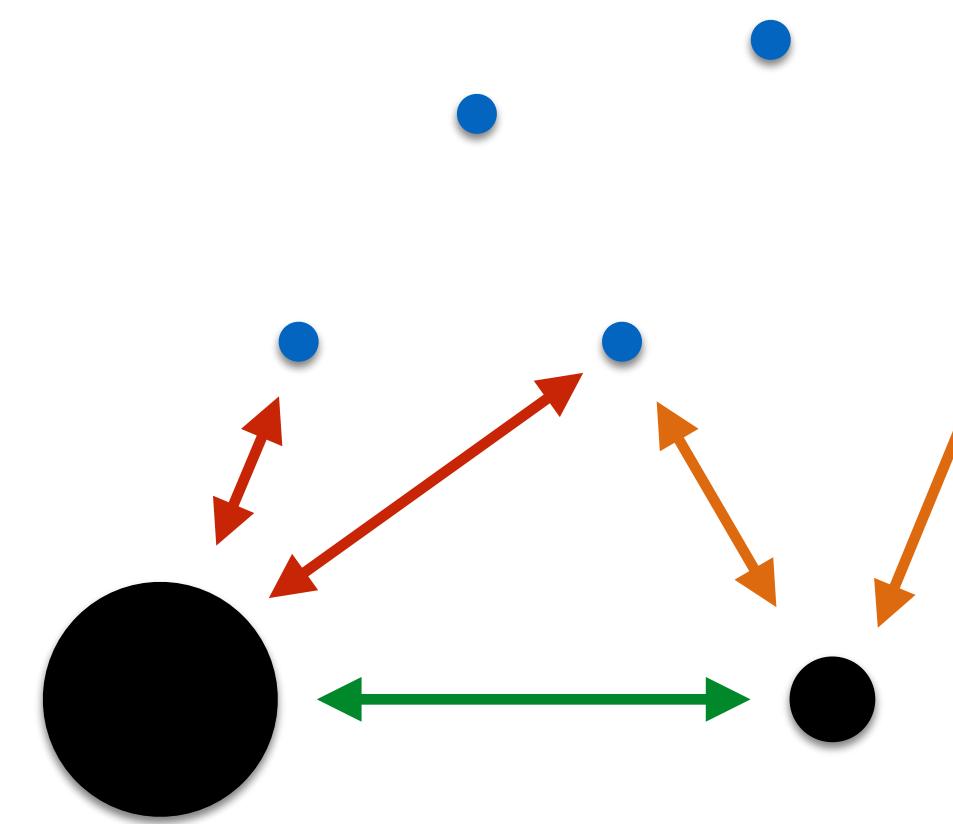


[More details on accretion in backup slides]

# N-Body Simulations

Eventually need to expand and verify our description of dynamical friction and feedback in the DM spike

**NbodyIMRI**: Newtonian N-body solver tailored to DM spikes.

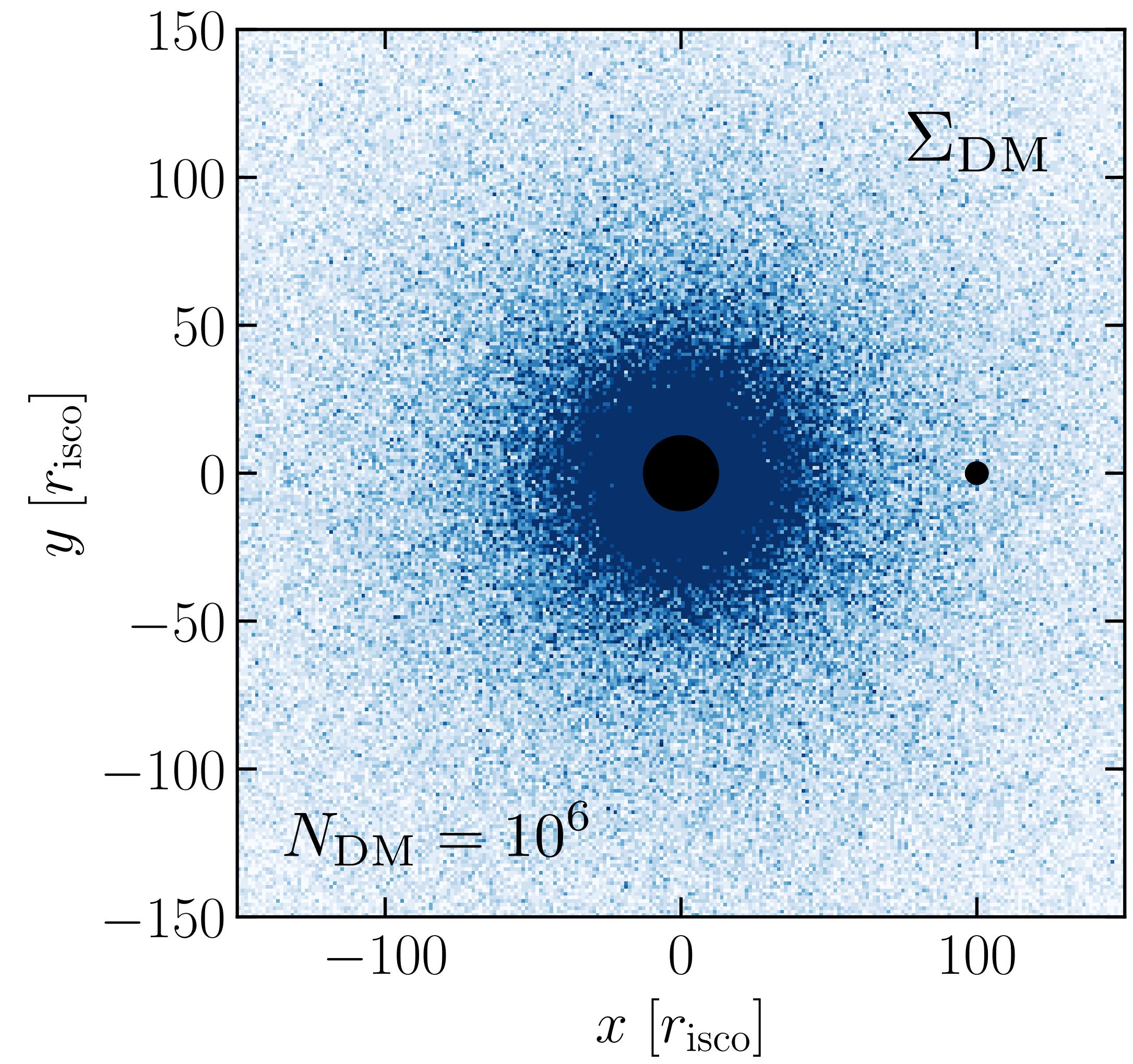


Neglect GW emission and *ignore DM-DM interparticle forces*.

Focus on understanding co-evolution of spike and binary.

[BJK et al., 2402.13762]

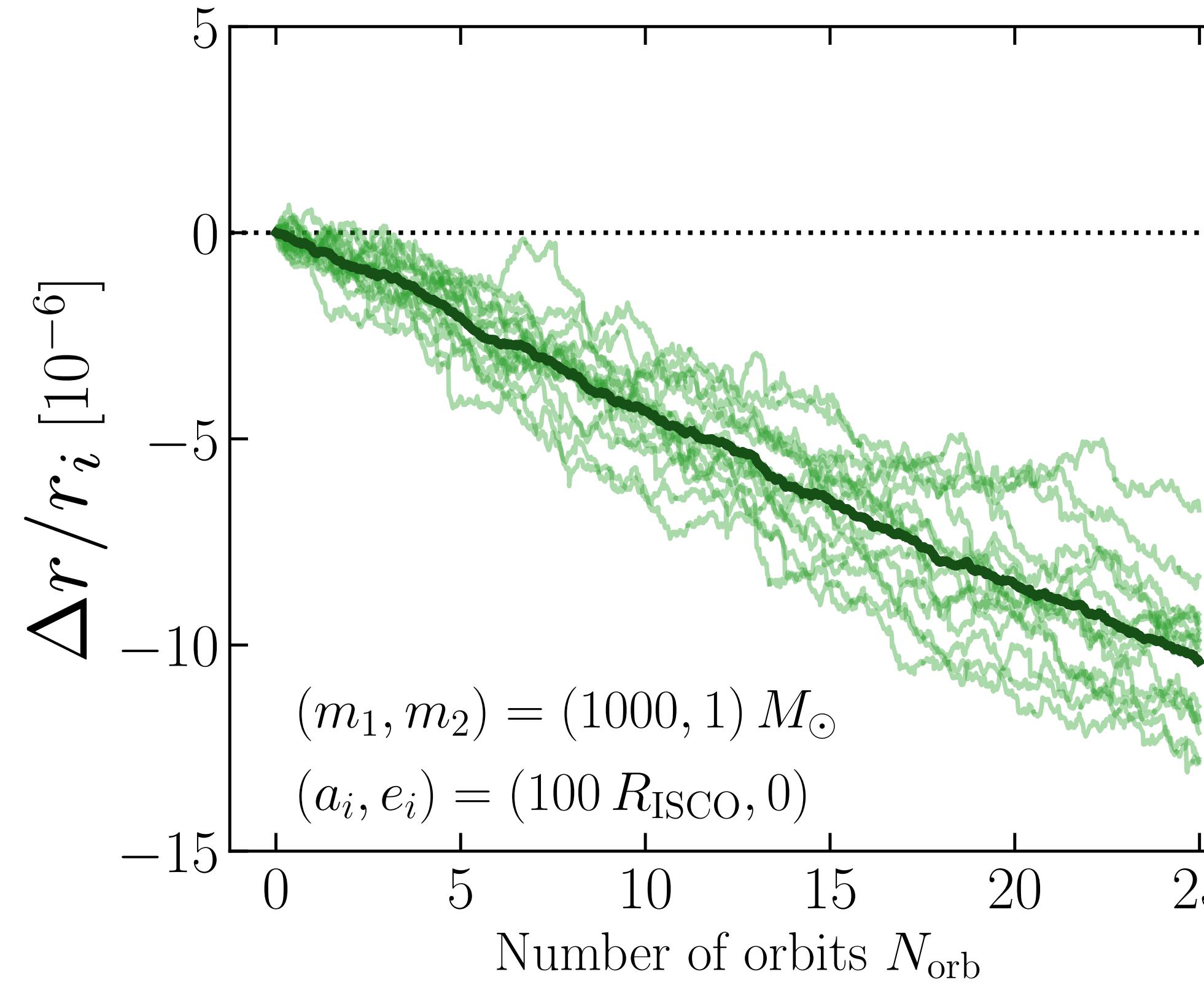
[Code here: [github.com/bradkav/NbodyIMRI](https://github.com/bradkav/NbodyIMRI)]



[See also Mukherjee et al., 2312.02275]

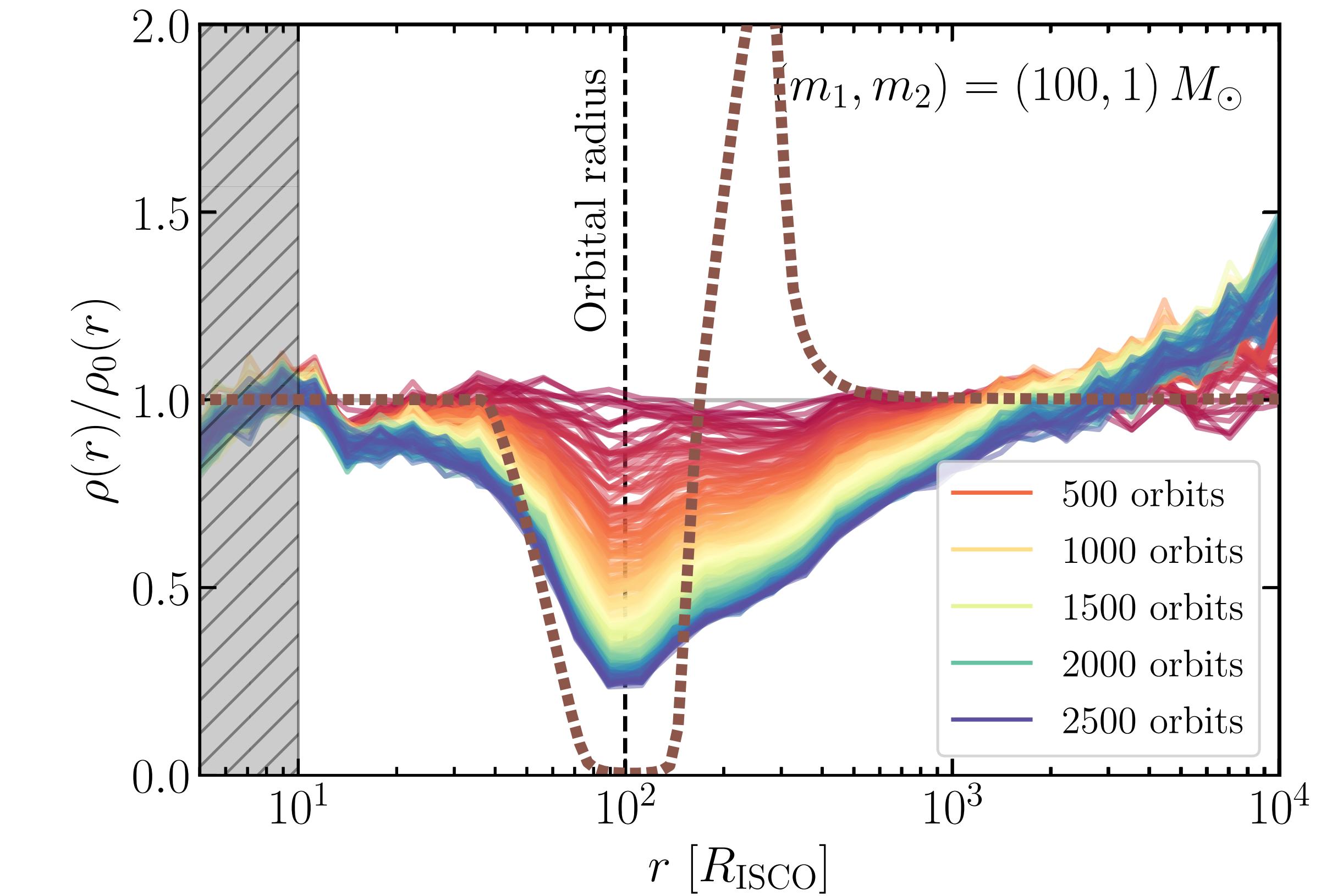
# N-body Verification

Can use **NbodyMRI** to verify the strength of the dynamical friction force...



$$\dot{E}_{\text{DF}} \sim \frac{4\pi G^2 m_2^2 \rho_{\text{DM}}(r) \xi(v)}{v} \ln \Lambda$$

...and to study the depletion of the DM spike ‘in full’.



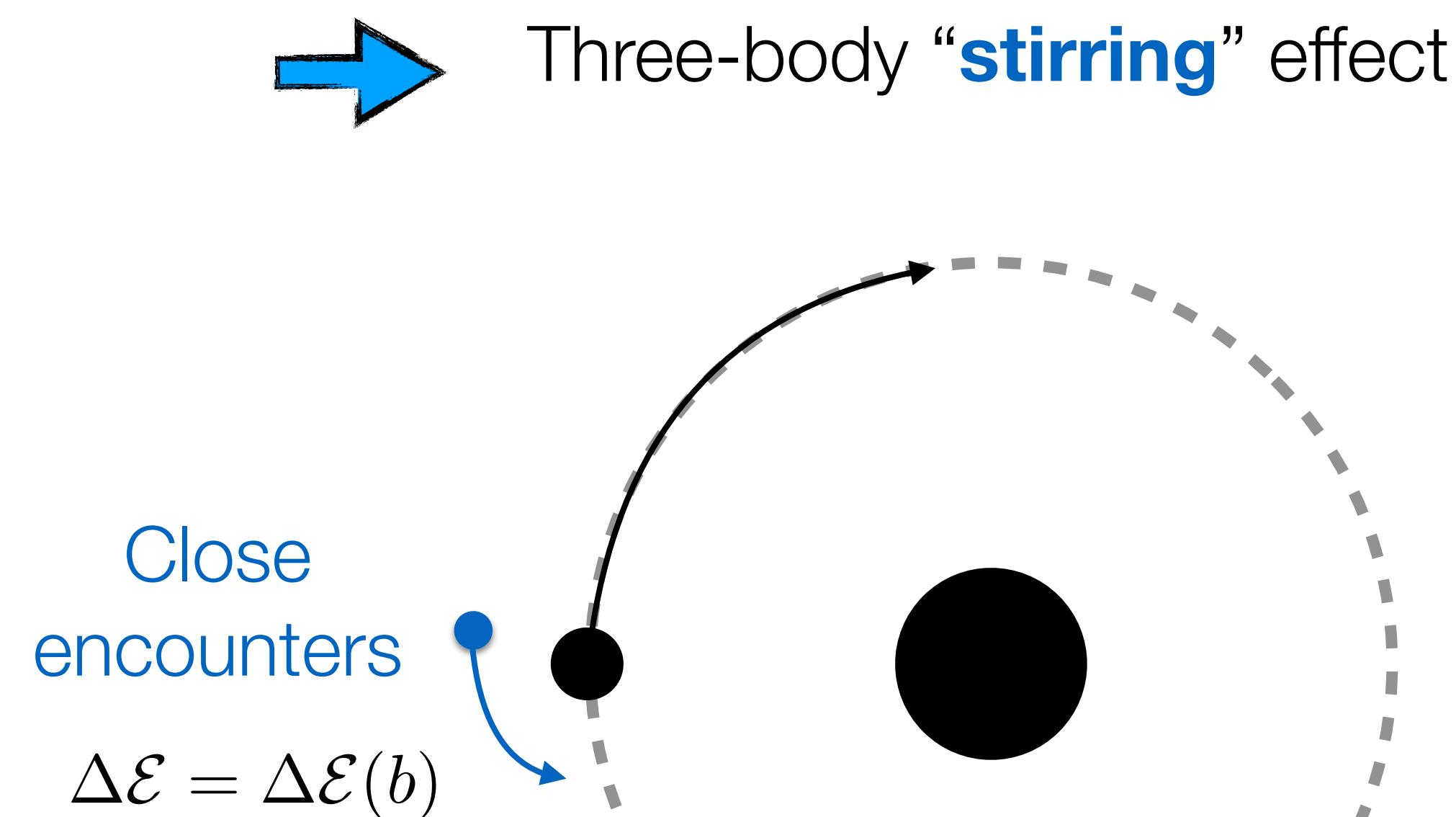
Find that the depletion is slower than using the previous **HaloFeedback formalism!**

# Three-body 'stirring' effects

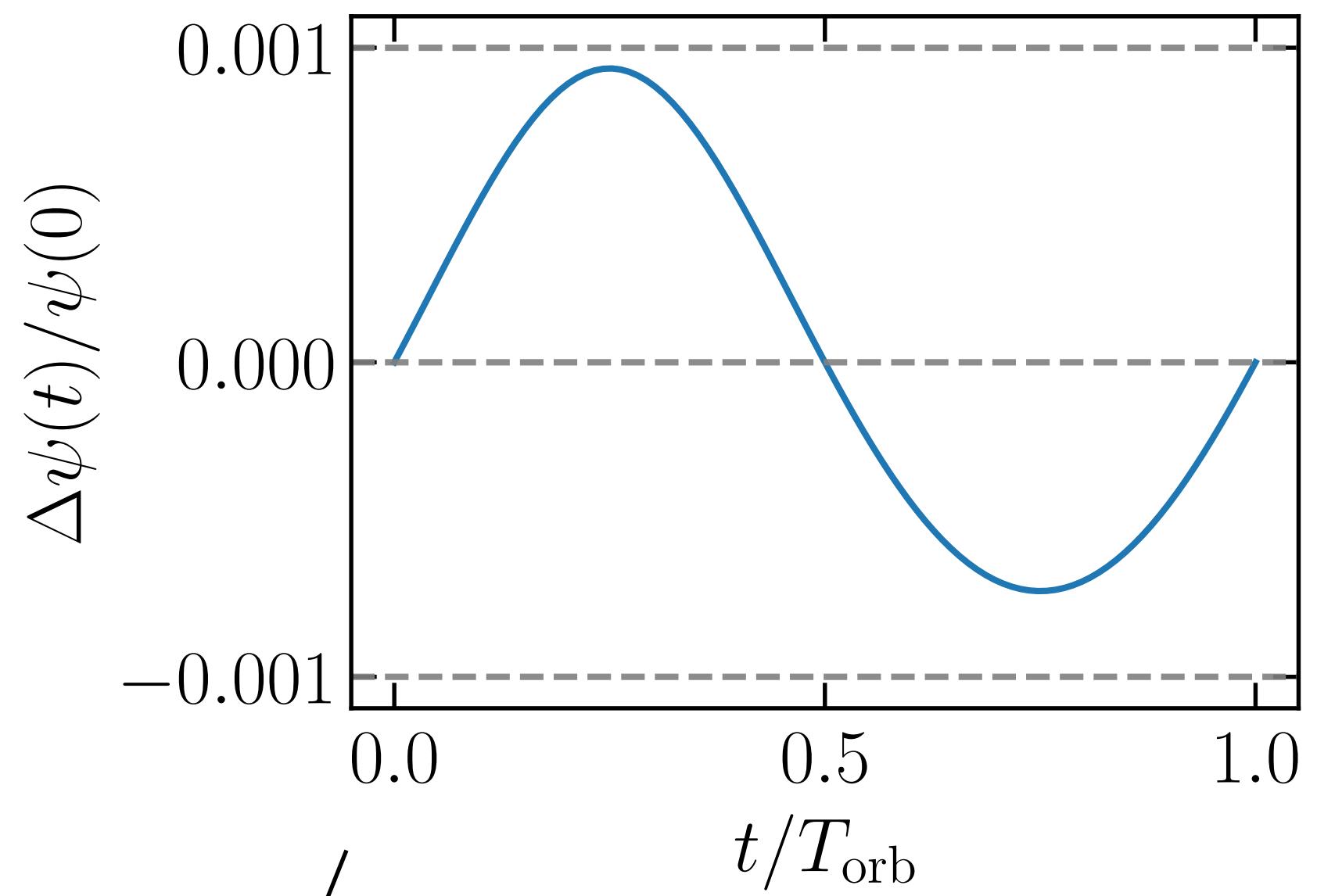
Depletion of spike is not as rapid due to a new feedback mechanism, coming from the time dependent potential of the binary.

[BJK et al., 2402.13762]

[Similar results found by Mukherjee et al., 2312.02275]



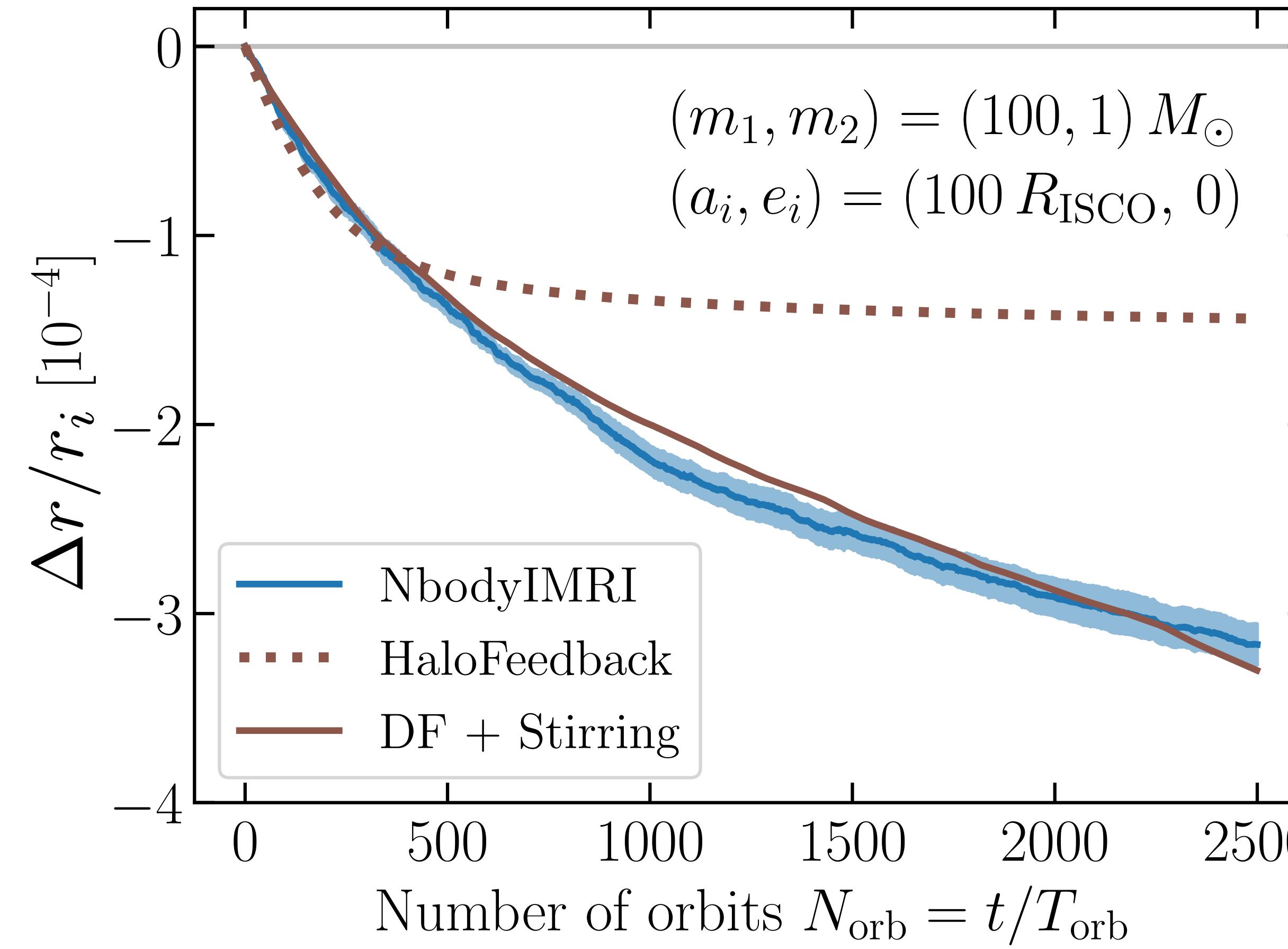
Orbits of distant particles are ‘redistributed’, slowing down the depletion.



‘Distant’  
particles

$$\Delta\mathcal{E} = \int_0^{\Delta t} \frac{\partial\psi}{\partial t} dt$$

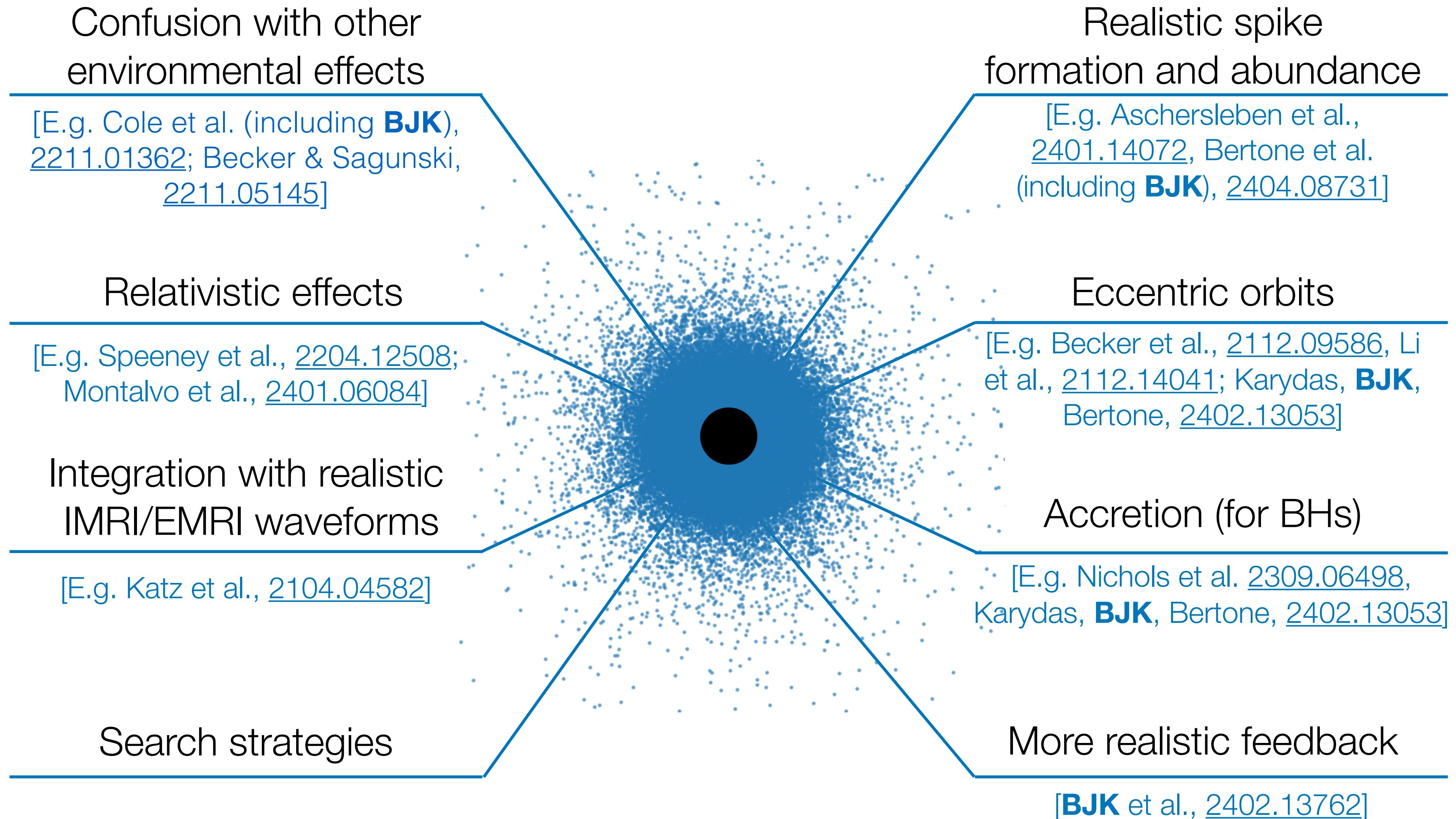
# Is dephasing larger than we thought?



Inspiral of the binary is faster than predicted by previous ‘HaloFeedback’ estimates.

Dark Matter dephasing may be bigger than we thought!

Working on understanding simulations over much longer timescales ( $N_{\text{orb}} \gtrsim 10^5$ ) and extending the feedback formalism to incorporate this ‘stirring’ effect.



# Towards better DM spikes

Gianfranco Bertone  
(GRAPPA, Amsterdam)



Pippa Cole  
(UNIMIB, Milan)



Adam Coogan  
(Mila, Montreal)



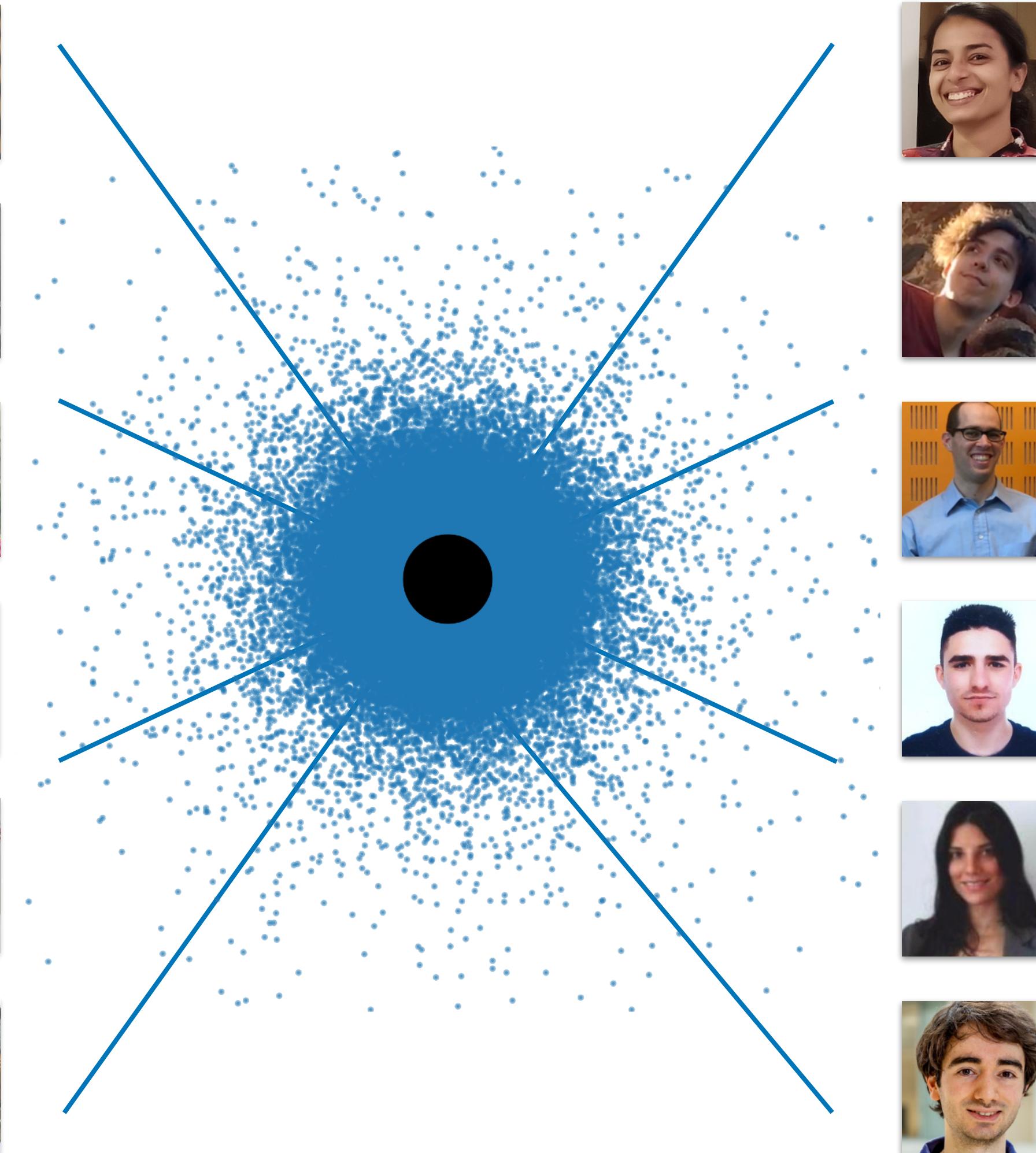
Pratika Dayal  
(Groningen University)



Jose Maria Diego  
(IFCA, Santander)



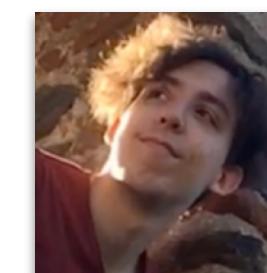
Daniele Gaggero  
(INFN, Pisa)



Pratibha Jangra  
(IFCA, Santander)



Theophanes Karydas  
(GRAPPA, Amsterdam)



David Nichols  
(U. Virginia)



Abram Perez Herrero  
(IFCA, Santander)



Francesca Scarcella  
(Montpellier)



Gimmy Tomaselli  
(GRAPPA, Amsterdam)



[Special thanks also to Sonic Adventure 2 for graphic design inspiration]

...and others.

# Towards better DM spikes

**Thank you!**

Gianfranco Bertone  
(GRAPPA, Amsterdam)



Pippa Cole  
(UNIMIB, Milan)



Adam Coogan  
(Mila, Montreal)



Pratika Dayal  
(Groningen University)



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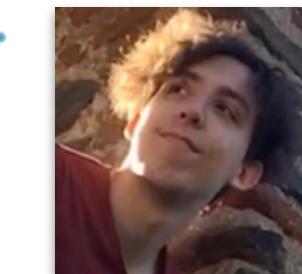
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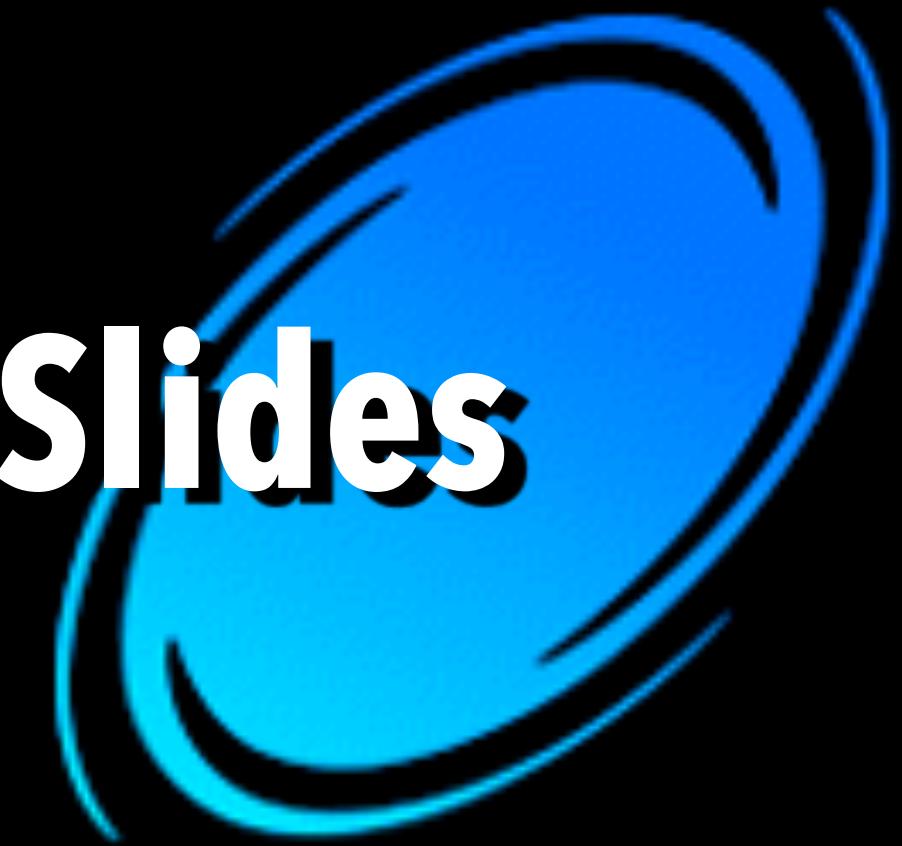
Gimmy Tomaselli  
(GRAPPA, Amsterdam)



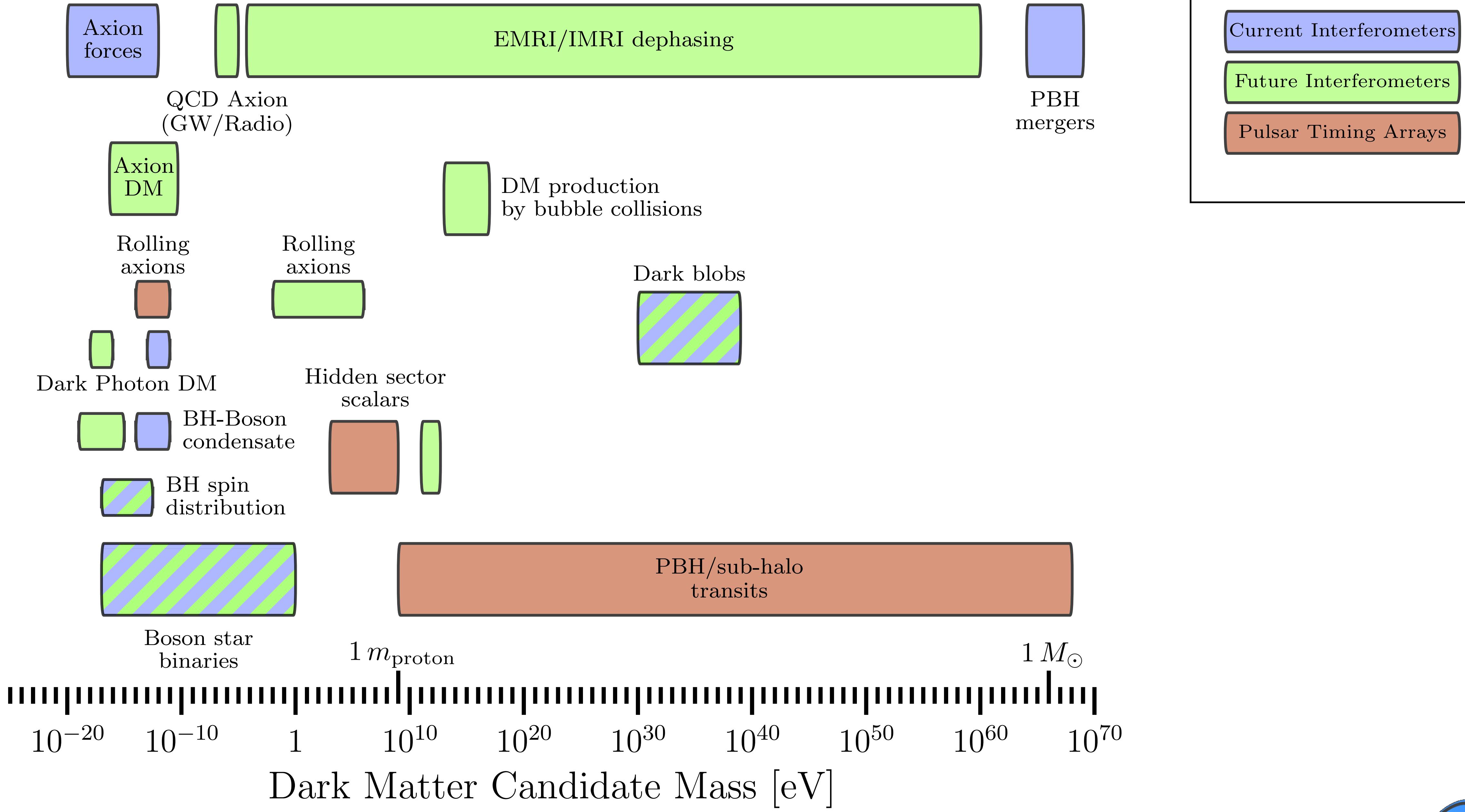
[Special thanks also to Sonic Adventure 2 for graphic design inspiration]

...and others.

# Backup Slides



# GW Probes



Follow semi-analytically the phase space distribution of DM:

$$f = \frac{dN}{d^3\mathbf{r} d^3\mathbf{v}} \equiv f(\mathcal{E})$$

$$\mathcal{E} = \Psi(r) - \frac{1}{2}v^2$$

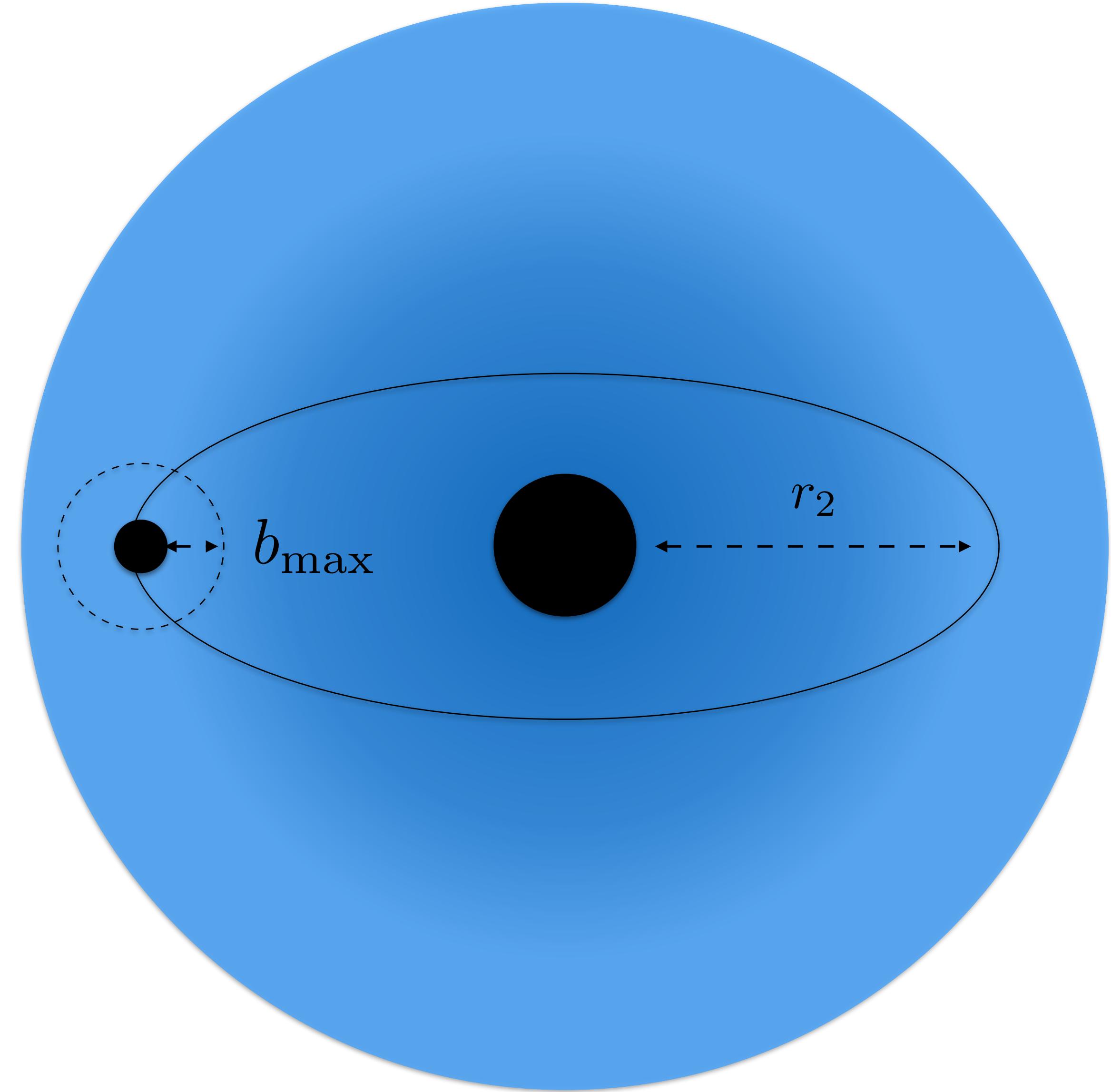
Each particle receives a ‘kick’ through gravitational scattering

$$\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$$

Reconstruct density from distribution function:

$$\rho(r) = \int d^3\mathbf{v} f(\mathcal{E})$$

[BJK, Nichols, Gaggero, Bertone, 2002.12811]



Compact object scatters with all DM particles within ‘torus’ of influence over one orbit

[Code available online: [github.com/bradkav/HaloFeedback](https://github.com/bradkav/HaloFeedback)]

Assuming everything evolves slowly compared to the orbital period:

$$T_{\text{orb}} \frac{df(\mathcal{E})}{dt} = -p_{\mathcal{E}} f(\mathcal{E}) + \int \left( \frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}} \right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E}-\Delta\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

$P_{\mathcal{E}}(\Delta\mathcal{E})$  - probability for a particle with energy  $\mathcal{E}$  to scatter and receive a ‘kick’  $\Delta\mathcal{E}$

$$p_{\mathcal{E}} = \int P_{\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$
 - total probability for a particle with energy  $\mathcal{E}$  to scatter

Assuming everything evolves slowly compared to the orbital period:

$$T_{\text{orb}} \frac{df(\mathcal{E})}{dt} = -p_{\mathcal{E}} f(\mathcal{E}) + \int \left( \frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}} \right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E}-\Delta\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

Particles scattering from  
 $\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$

Particles scattering from  
 $\mathcal{E} - \Delta\mathcal{E} \rightarrow \mathcal{E}$

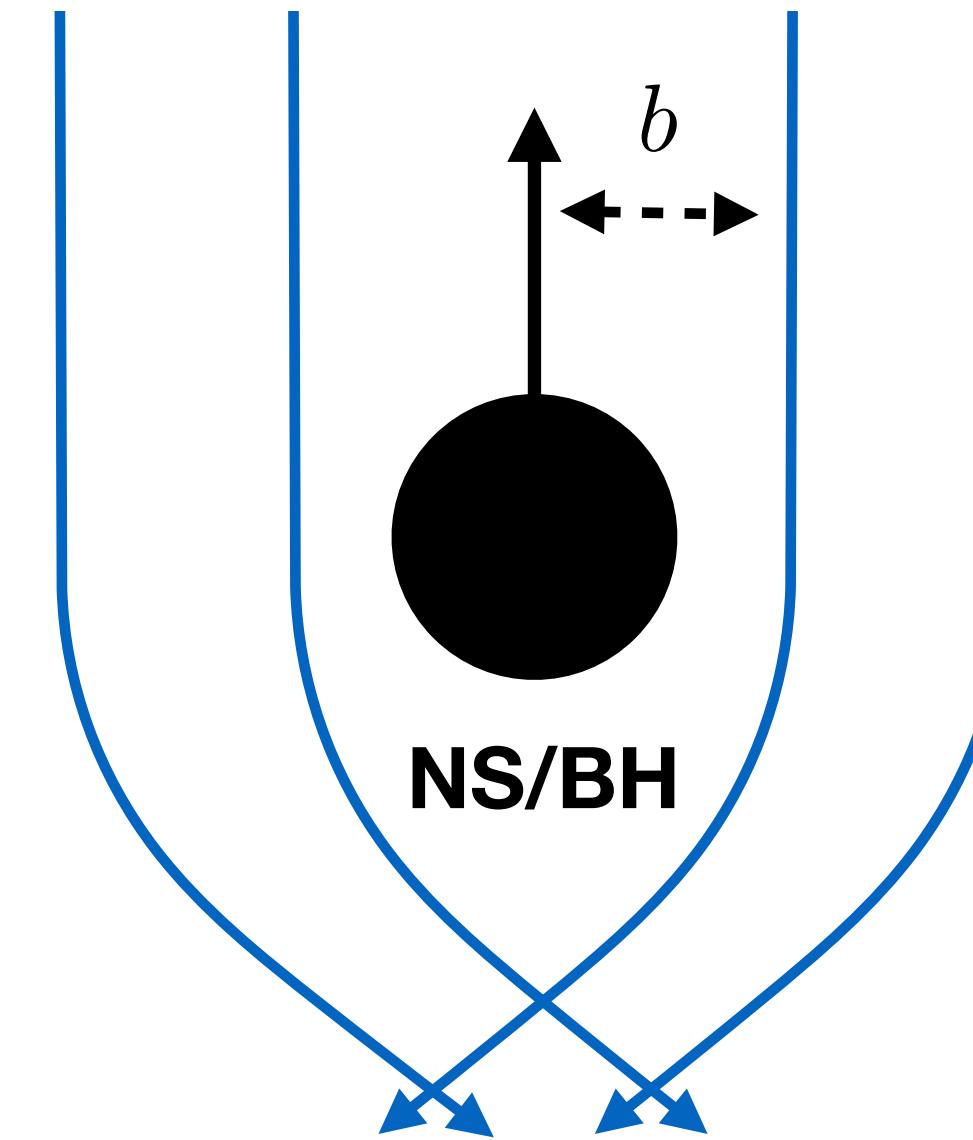
$P_{\mathcal{E}}(\Delta\mathcal{E})$  - probability for a particle with energy  $\mathcal{E}$  to scatter and receive a ‘kick’  $\Delta\mathcal{E}$

$$p_{\mathcal{E}} = \int P_{\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

- total probability for a particle with energy  $\mathcal{E}$  to scatter

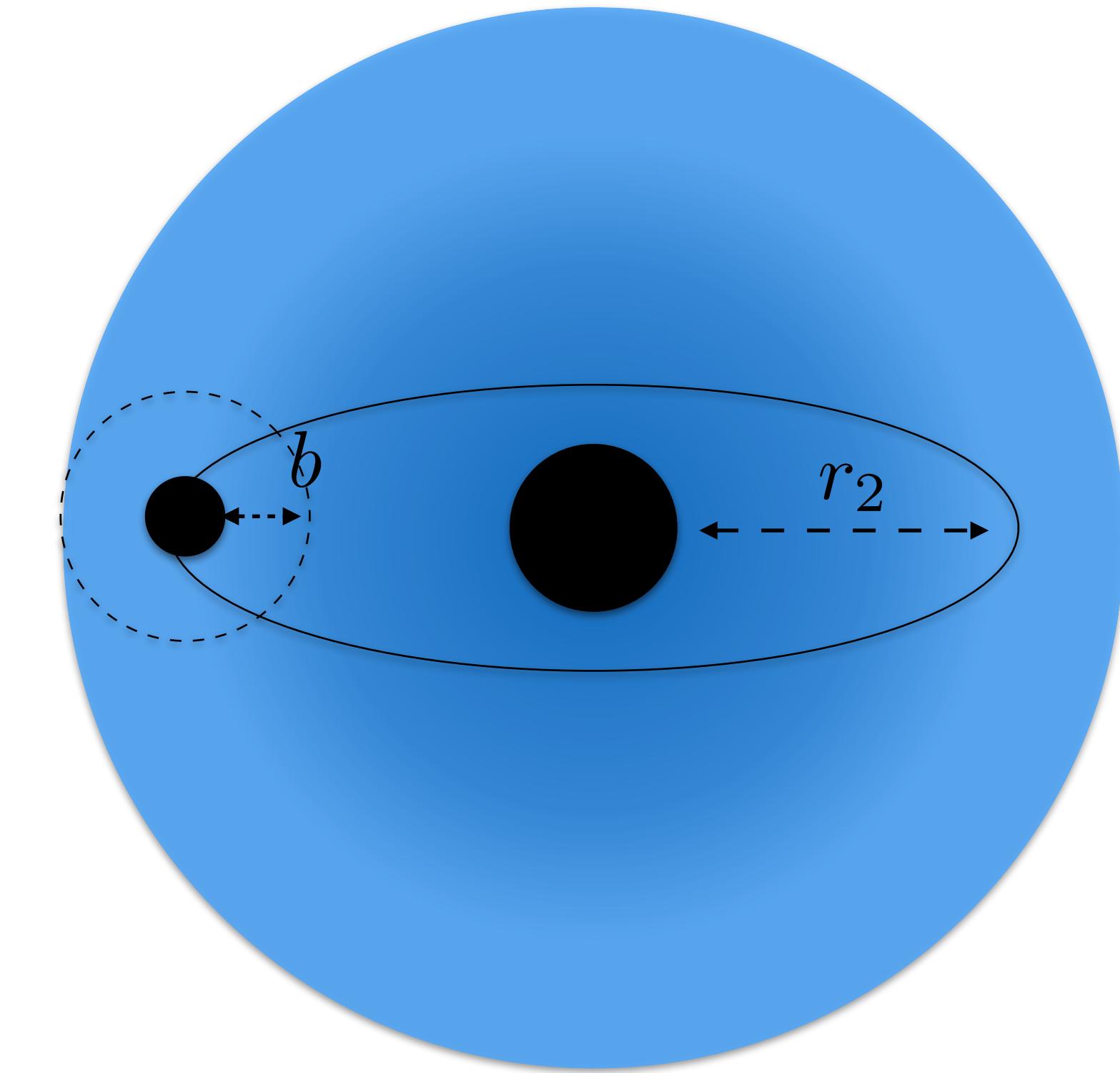
Two body scattering problem relates energy exchange to impact parameter:

$$\Delta\mathcal{E}(b) = -2v_0^2 \left[ 1 + \frac{b^2 v_0^4}{G^2 m_2^2} \right]^{-1}$$



Scattering probability becomes a *geometric* problem:

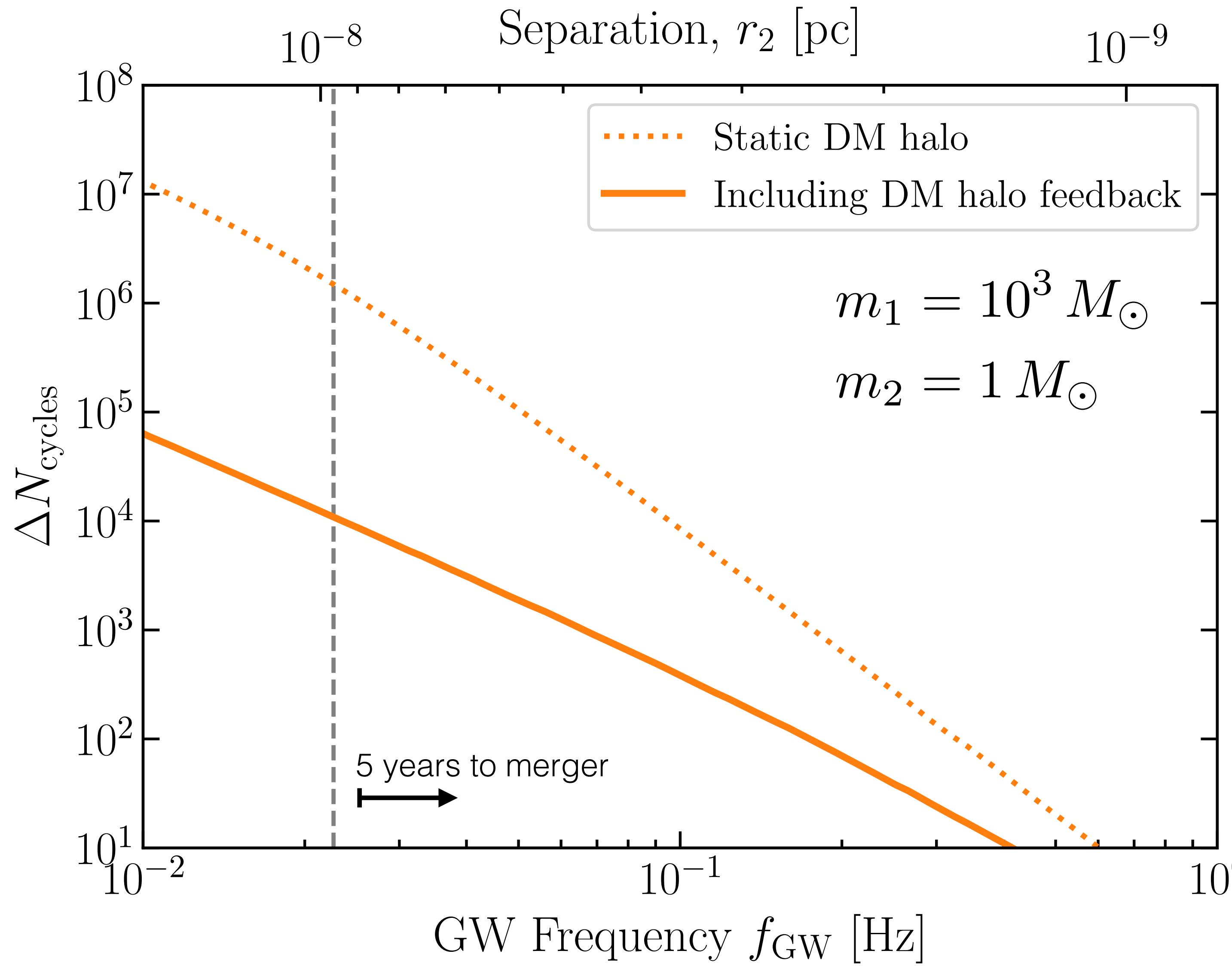
$$P_{\mathcal{E}}(\Delta\mathcal{E}) \propto P(b|\mathcal{E})$$



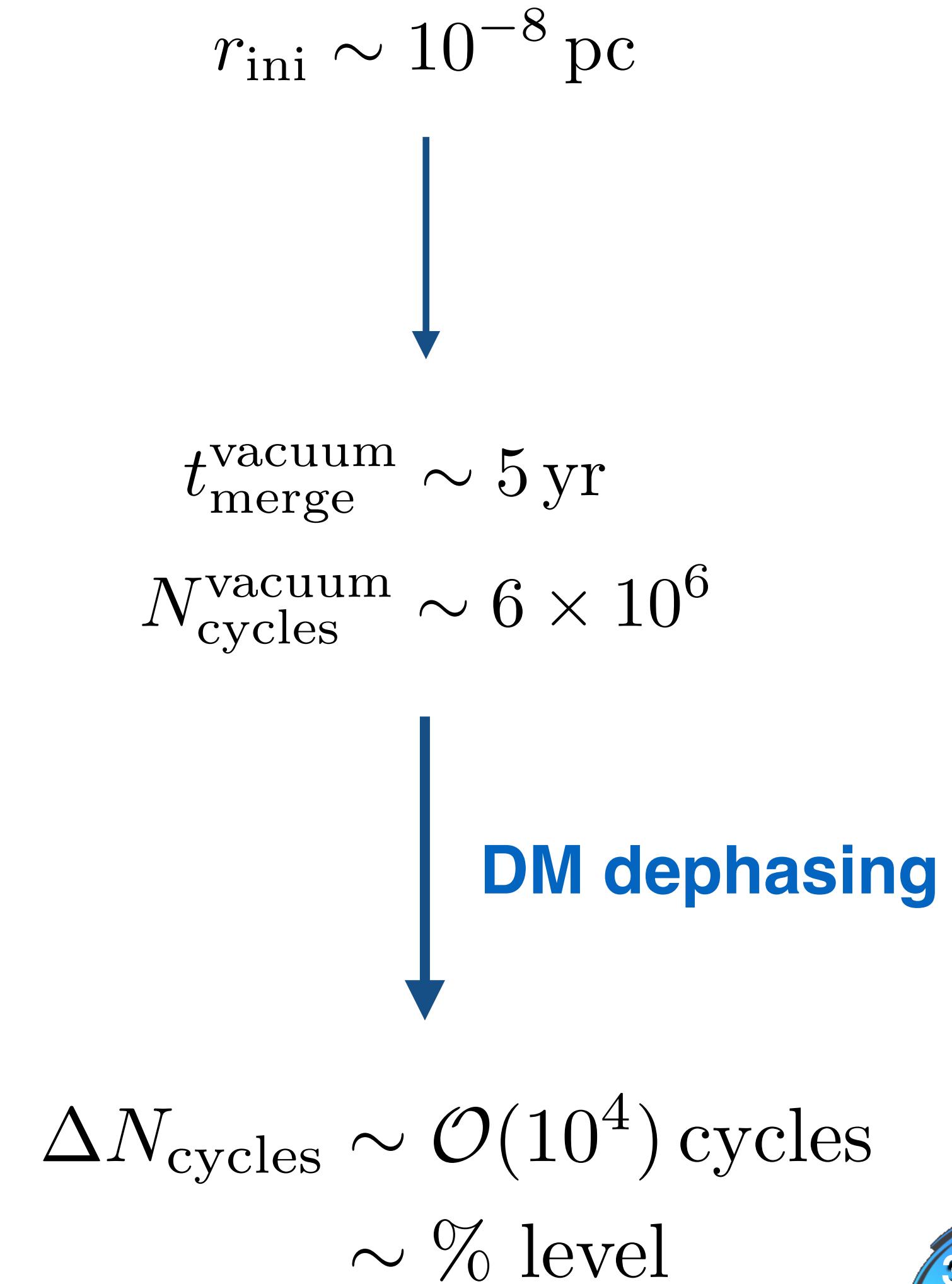
Code available online: [github.com/bradkav/HaloFeedback](https://github.com/bradkav/HaloFeedback)  
(See also <https://github.com/DMGW-Goethe/imripy>)

# Size of the dephasing

Change in the number of GW cycles to merger,  
starting at some initial frequency/separation:



Consider our astro benchmark  
system, starting at some initial  
separation:



# What about baryons?

Risk of confusion between New Physics  
and accretion disks?

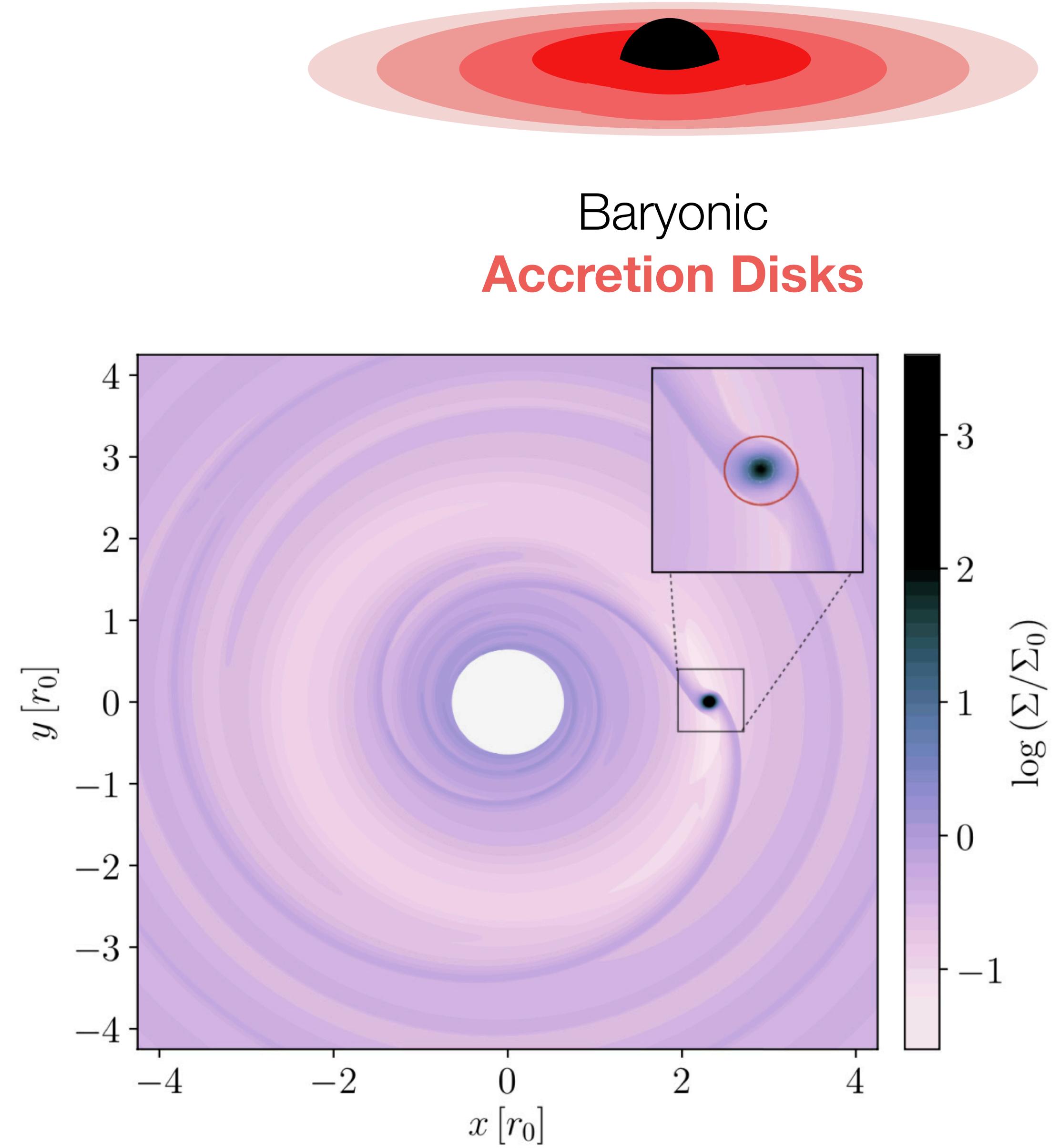
Many possible geometries and parameters,  
but focus on dense, thin disks.

Dominant cause of dephasing is *not*  
dynamical friction but *gas torques*.

Perturbation of the disk leads to a build up of  
gas inward or outward of the inspiraling BH.

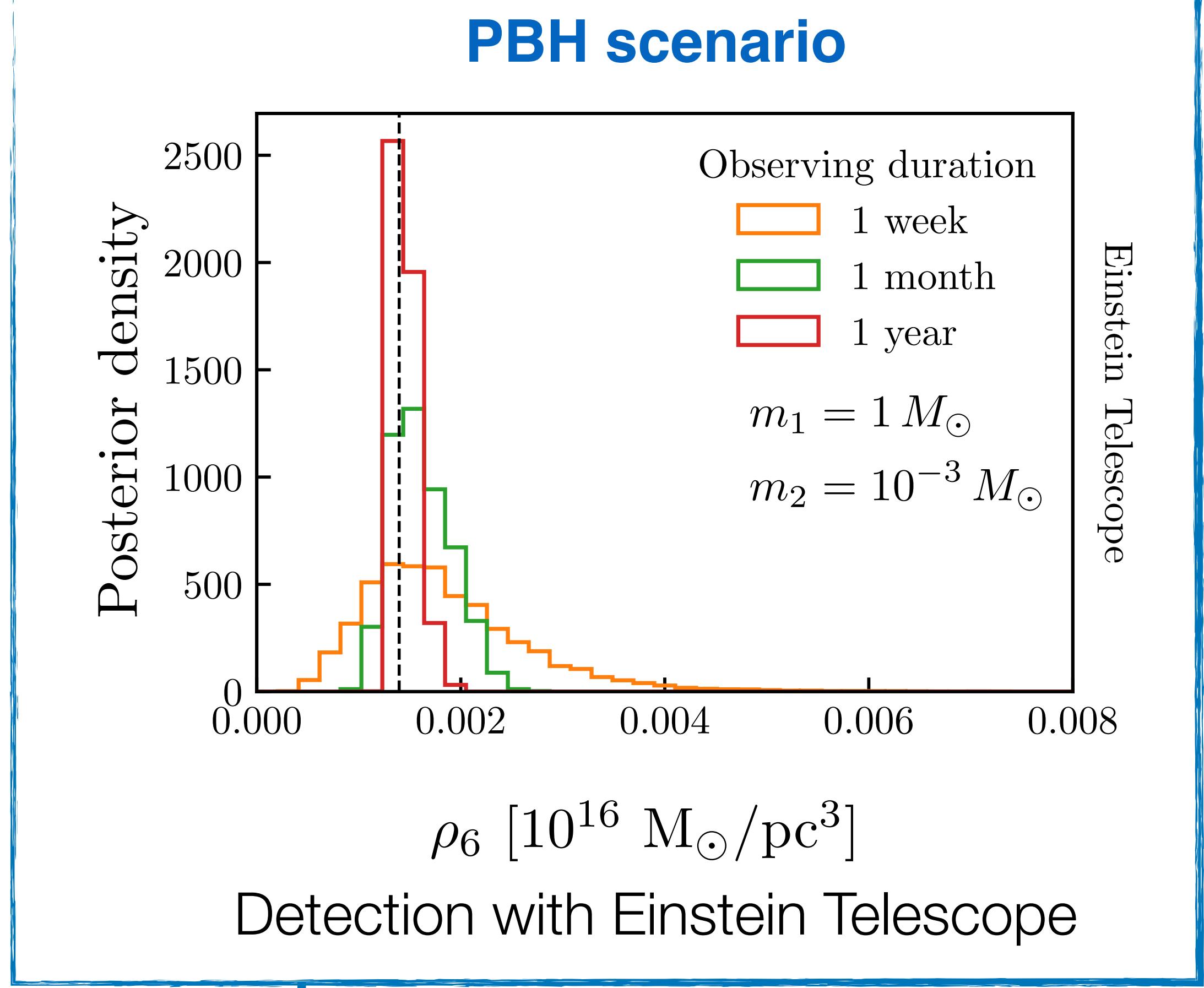
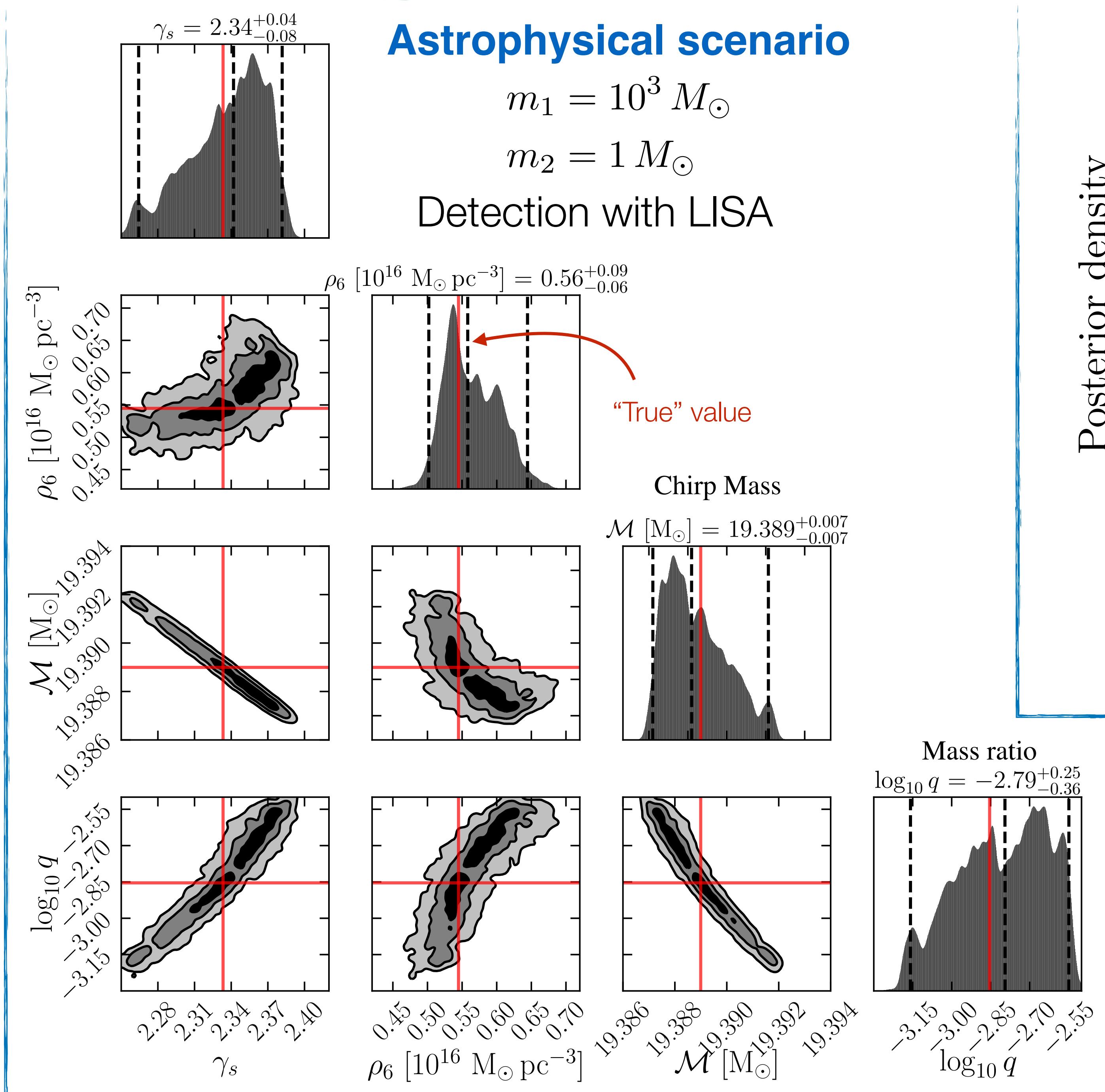


Dependence of energy losses on BH  
separation is *different* from DM spikes!



[Derdzinski et al., 2005.11333]

# Measurability



[Cole, Coogan, **BJK**, Bertone, [2207.07576](#)]

[Coogan, Bertone, Gaggero, **BJK** & Nichols, [2108.04154](#)]

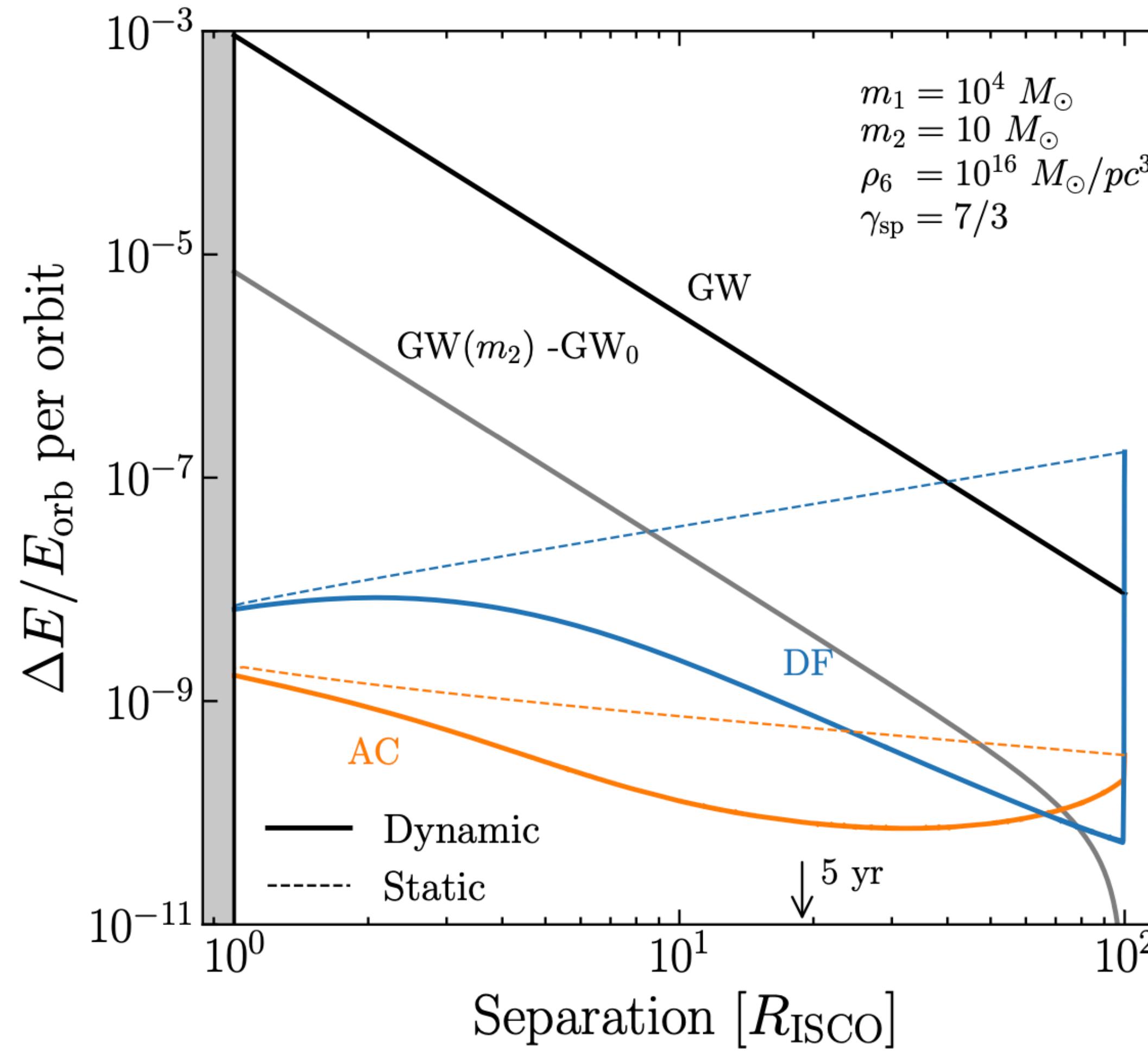
[Code: [github.com/adam-coogan/pydd](https://github.com/adam-coogan/pydd)]

Accretion of DM by the secondary BH leads to an additional accretion force:

$$F_{\text{acc}} = \int d^3v \pi b_{\text{acc}}^2(v_{\text{rel}}) v_{\text{rel}} \rho_\chi v_{\text{rel}}$$

$$\mathbf{v}_{\text{rel}} = \mathbf{v}_{\text{orb}} - \mathbf{v}$$

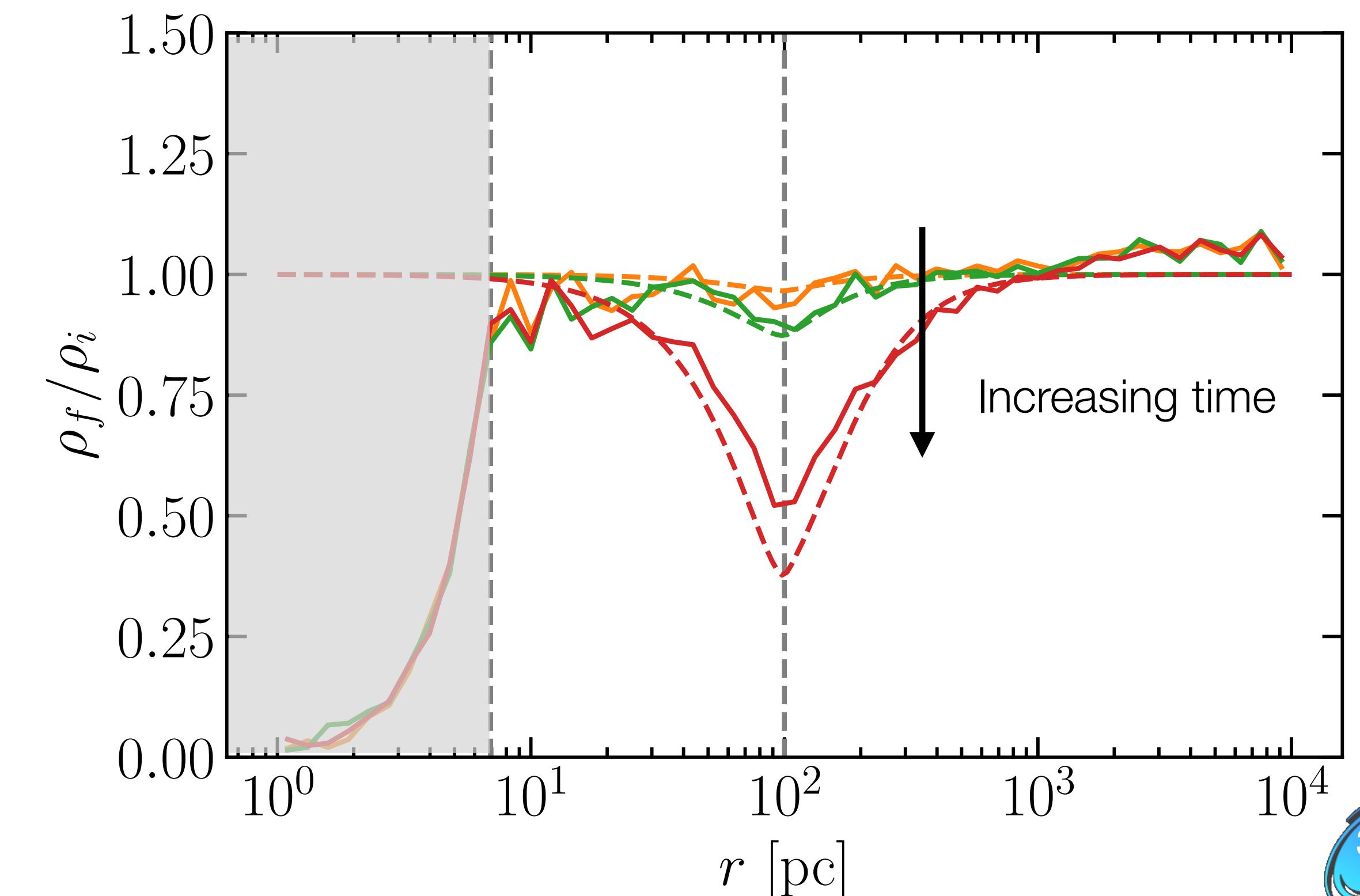
## DM Accretion



[Karydas, **BJK**, Bertone, [2402.13053](#)]

[See also Nichols et al. [2309.06498](#)]

Use simulations to validate accretion force  
and DM spike depletion:



Change in energy due to time-dependent potential:

$$\Delta\mathcal{E} = \int_0^{\Delta t} \frac{\partial\psi}{\partial t} dt$$

where the integral is taken over the path of the particle during a time  $\Delta t$

For particles at large distances from the binary ( $r \gg a$ ), we can estimate the change in energy over a single orbit of the binary:

$$\Delta\mathcal{E} = \left( \frac{Gm_2}{r} \frac{a^2}{r^2} \right) \left( \frac{vT_{\text{orb}}}{r} \right) \mathcal{A}(\hat{\mathbf{r}}, \hat{\mathbf{v}})$$

Time-dependent component  
of the binary of the potential  
(Gravitational quadrupole  $\sim 1/r^3$ )

Fractional change in DM  
position of DM particle with  
speed v

Factor depending on  
orientation of the particle  
position and velocity with  
respect to the binary

The factor  $\mathcal{A}(\hat{\mathbf{r}}, \hat{\mathbf{v}})$  can take positive or negative values -> these ‘three-body’ effects redistribute orbits and tend to smooth out the density profile. Though they don’t extract any net energy from the binary (at least for circular binaries).