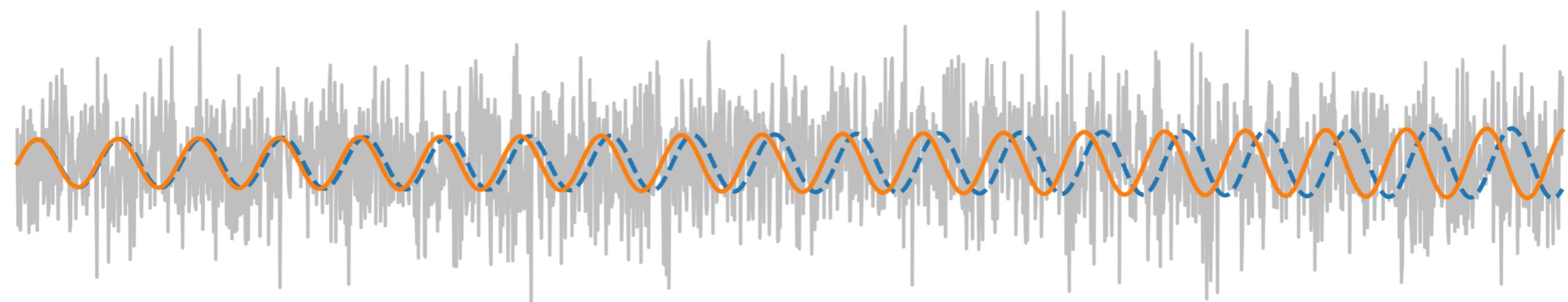
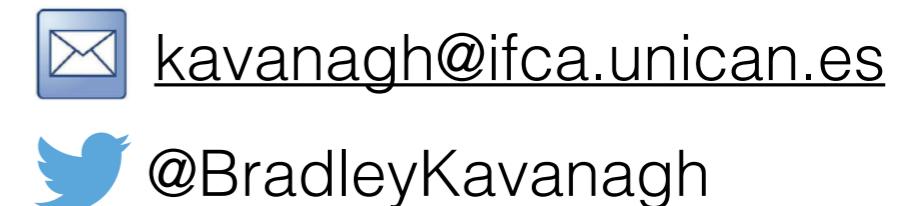


Detecting (Axion?) Dark Matter around Black Holes with Gravitational Waves



Bradley J Kavanagh
IFCA, University of Cantabria, Santander

HIDDeN Seminar, 21st April 2020

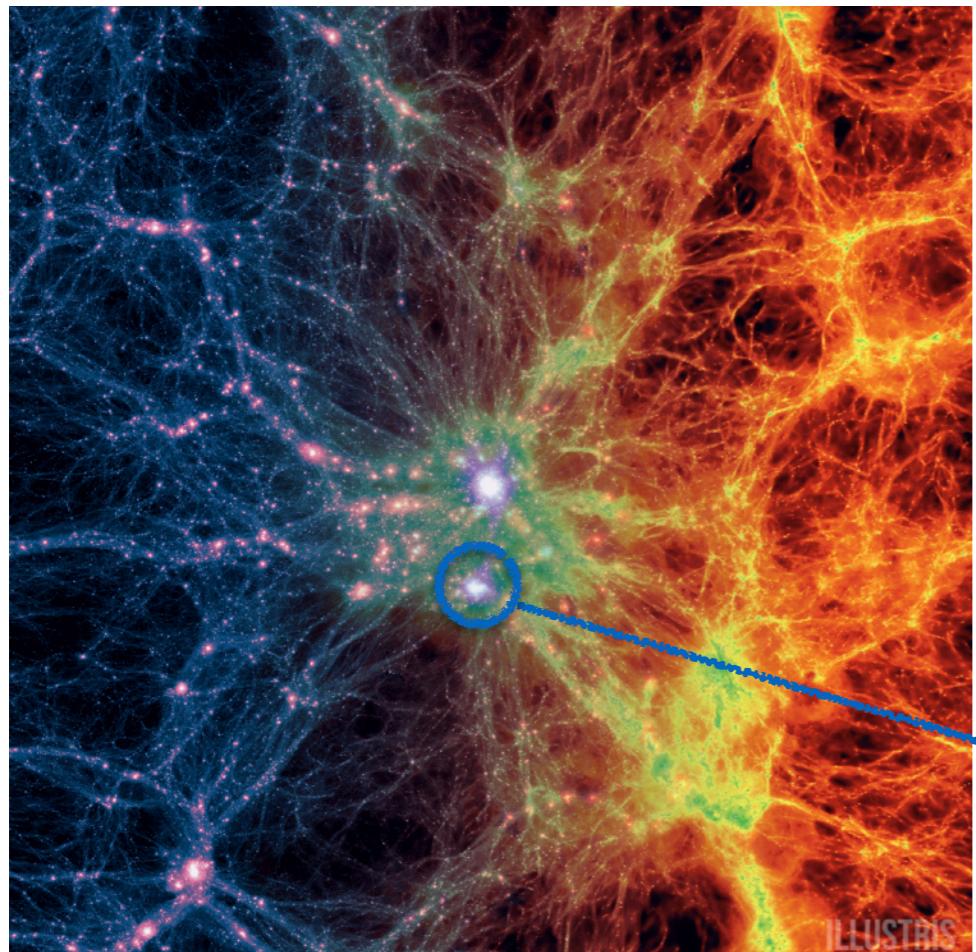


Abstract. With growing agony of not finding a dark matter (DM) particle in direct search experiments so far (for example in XENON1T), frameworks where the freeze-out of DM is driven by number changing processes within the dark sector itself and do not contribute to direct search, like Strongly Interacting Massive Particle (SIMP) are gaining more attention. In this analysis, we ideate a simple scalar DM framework stabilised by \mathbb{Z}_2 symmetry to serve

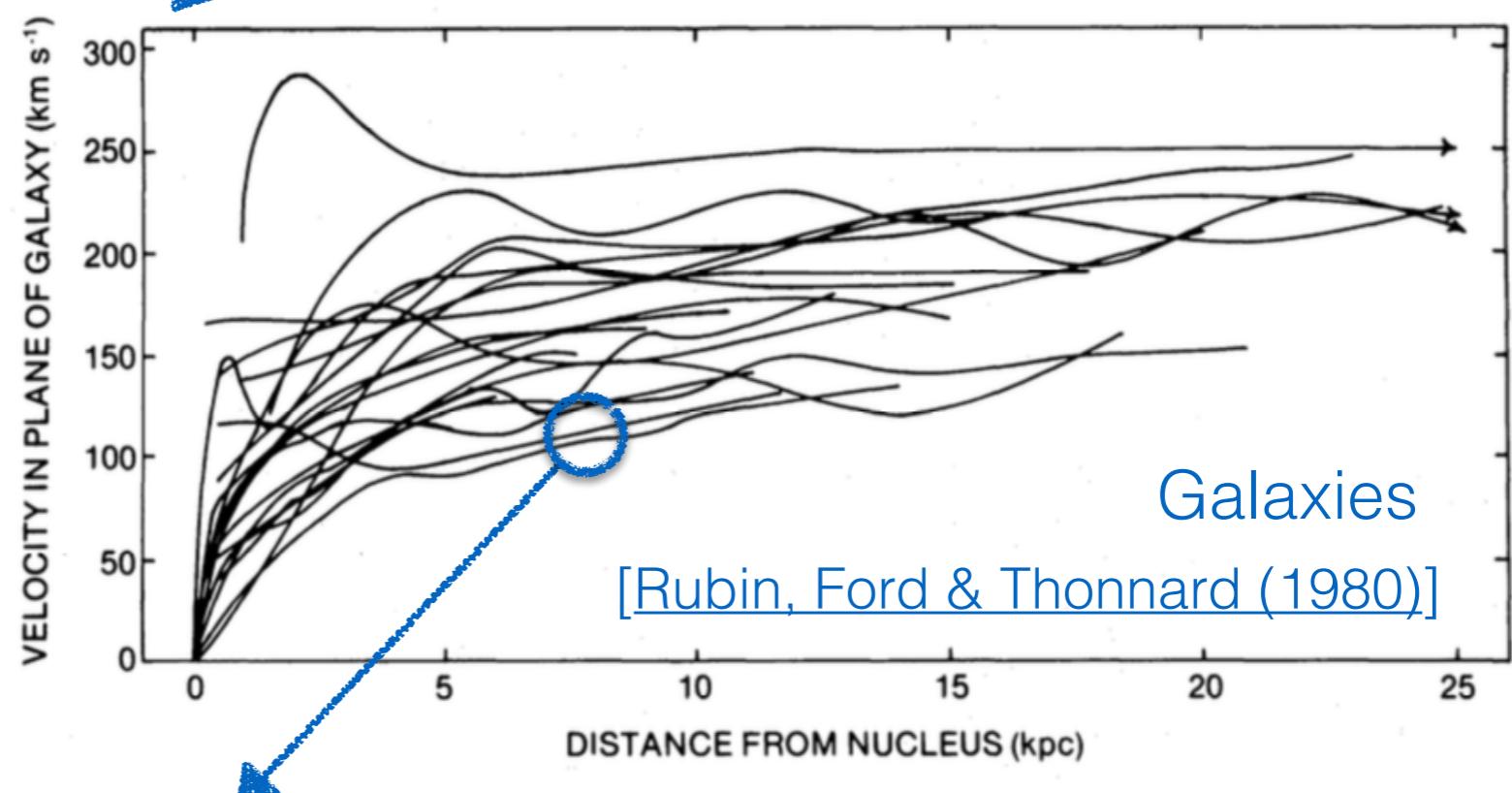
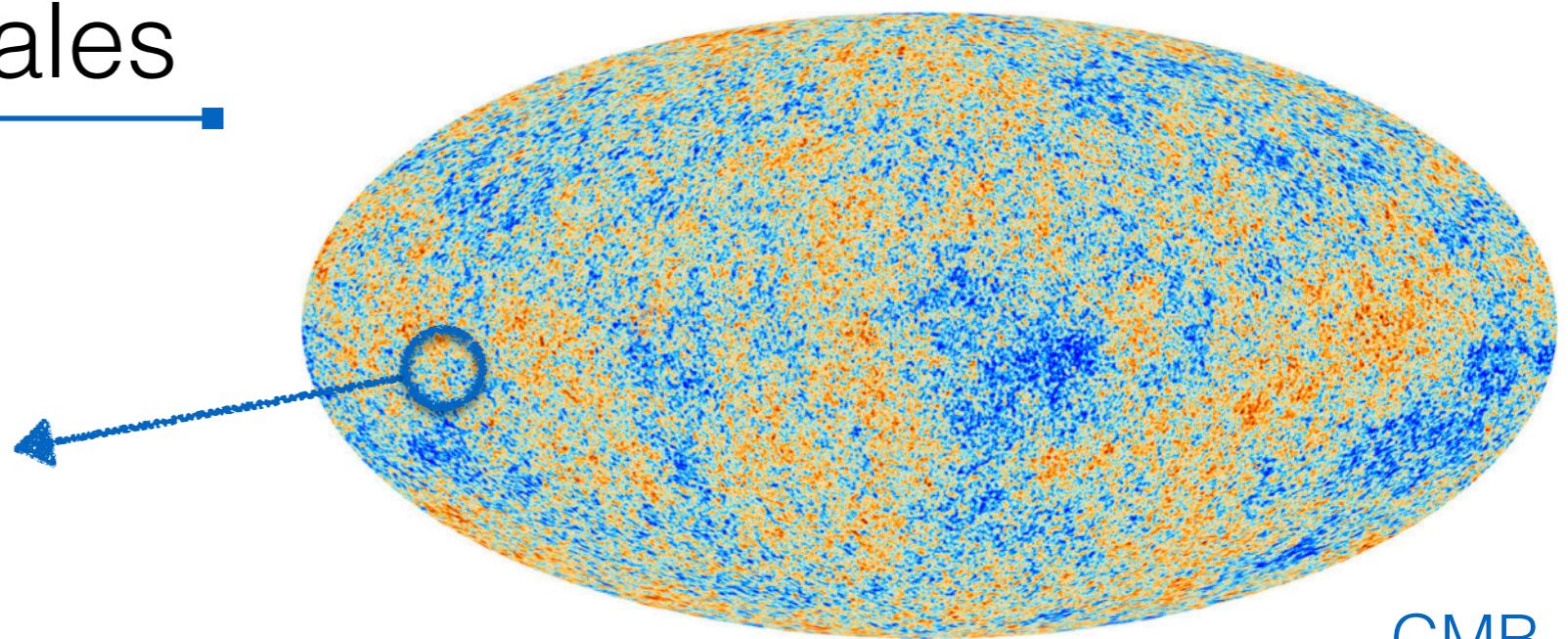
From ‘SIMPler realisation of Scalar Dark Matter’ [[1904.07562](#)]

[With thanks to @TimonEmken]

Dark Matter on all scales



Galaxy clusters
[Illustris, [1405.2921](#)]
[[astro-ph/0006397](#)]



Dark Matter at Earth

NOT TO SCALE



Global and local estimates of
DM at Solar radius give: $\rho_\chi \sim 0.2 - 0.8 \text{ GeV cm}^{-3}$

E.g. Iocco et al. [1502.03821],
Garbari et al. [1206.0015],
Read [1404.1938]

QCD Axion Dark Matter

QCD Lagrangian could have a large CP violating term:

$$\mathcal{L}_{\text{QCD}} \supset -\frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} \theta_{\text{QCD}}$$

But non-observation of neutron dipole moment implies $|\theta| \leq 10^{-10}$

Why is θ so small?

Solve dynamically, by promoting θ to a scalar field with a global U(1) symmetry

$$\theta_{\text{QCD}} \rightarrow \theta_{\text{QCD}}(x) = \frac{a(x)}{f_a} \quad \longrightarrow \quad \theta = \frac{\langle a \rangle}{f_a} \sim 0 \quad \text{at } f_a \gg 10^7 \text{ GeV}$$

[Peccei & Quinn, 1977]

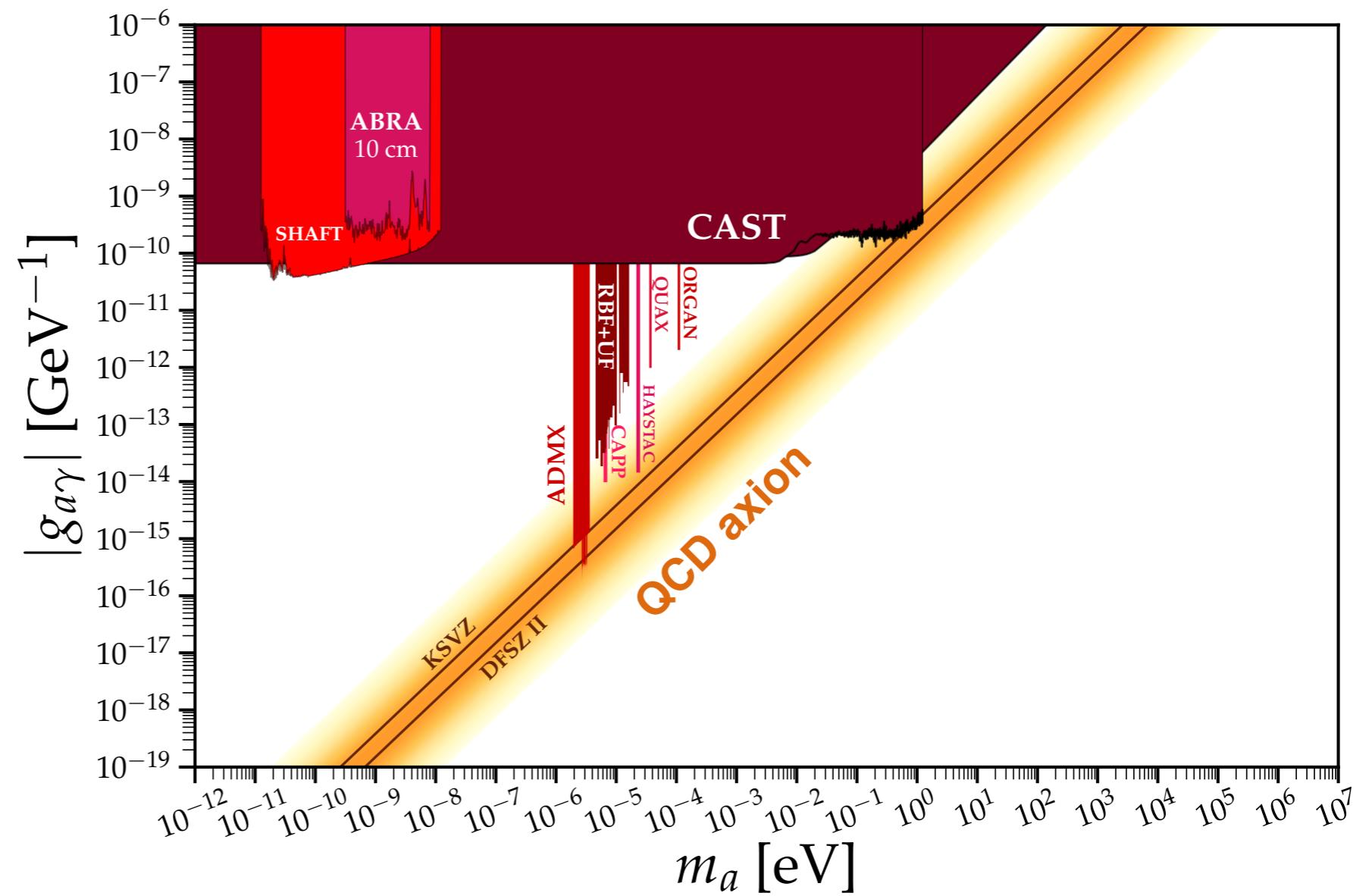
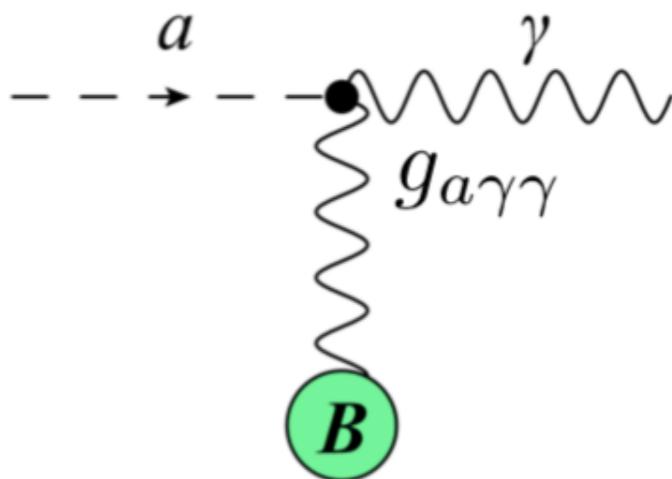
Resulting light goldstone boson - the QCD axion a - can be produced with correct DM relic abundance, e.g. through vacuum realignment or defect decay

[1510.07633]

Axion-photon Conversion

Axions can convert to photons (and vice versa) in an external magnetic field:

$$\begin{aligned}\mathcal{L} &\supset -\frac{1}{4}g_{a\gamma\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu} \\ &= -\frac{1}{4}g_{a\gamma\gamma}a\mathbf{E} \cdot \mathbf{B}\end{aligned}$$

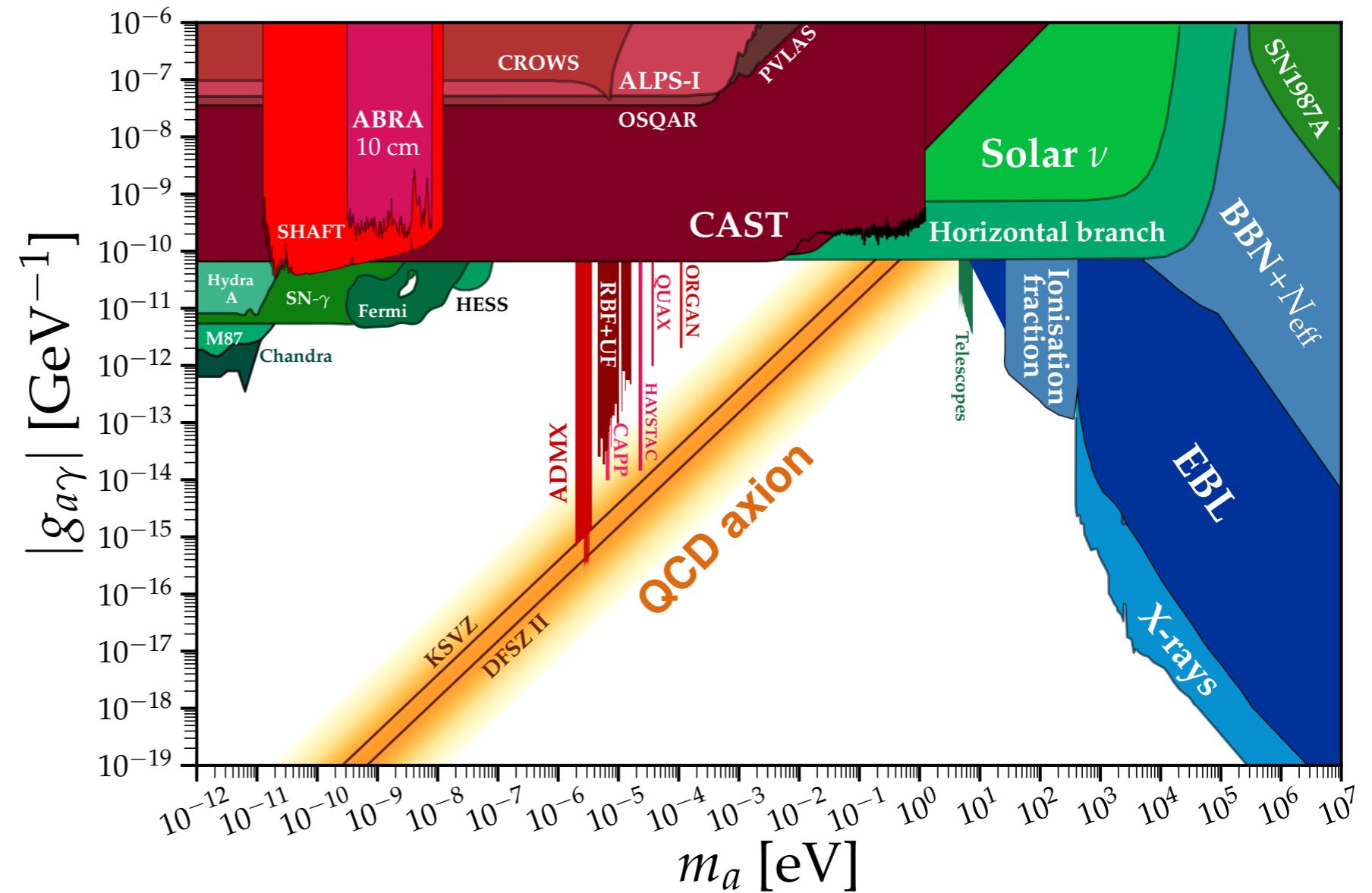
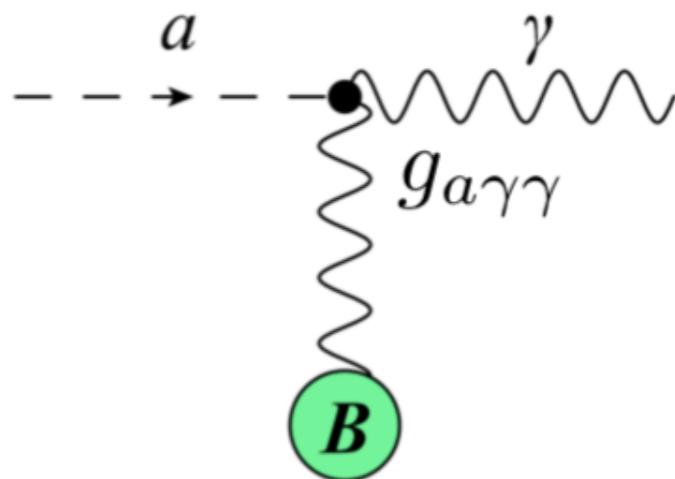


[O'Hare, <https://cajohare.github.io/AxionLimits/>]

Axion-photon Conversion

Axions can convert to photons (and vice versa) in an external magnetic field:

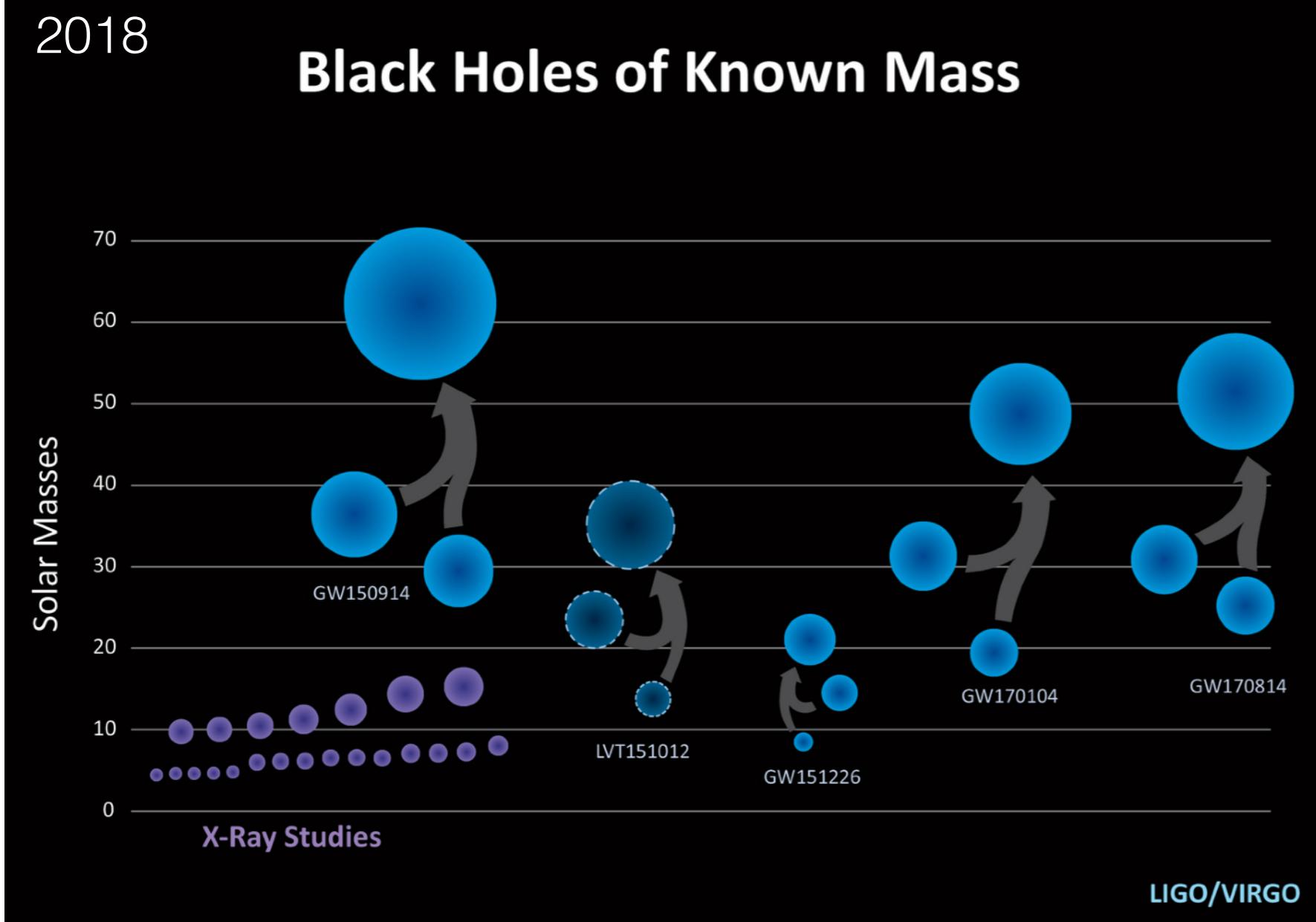
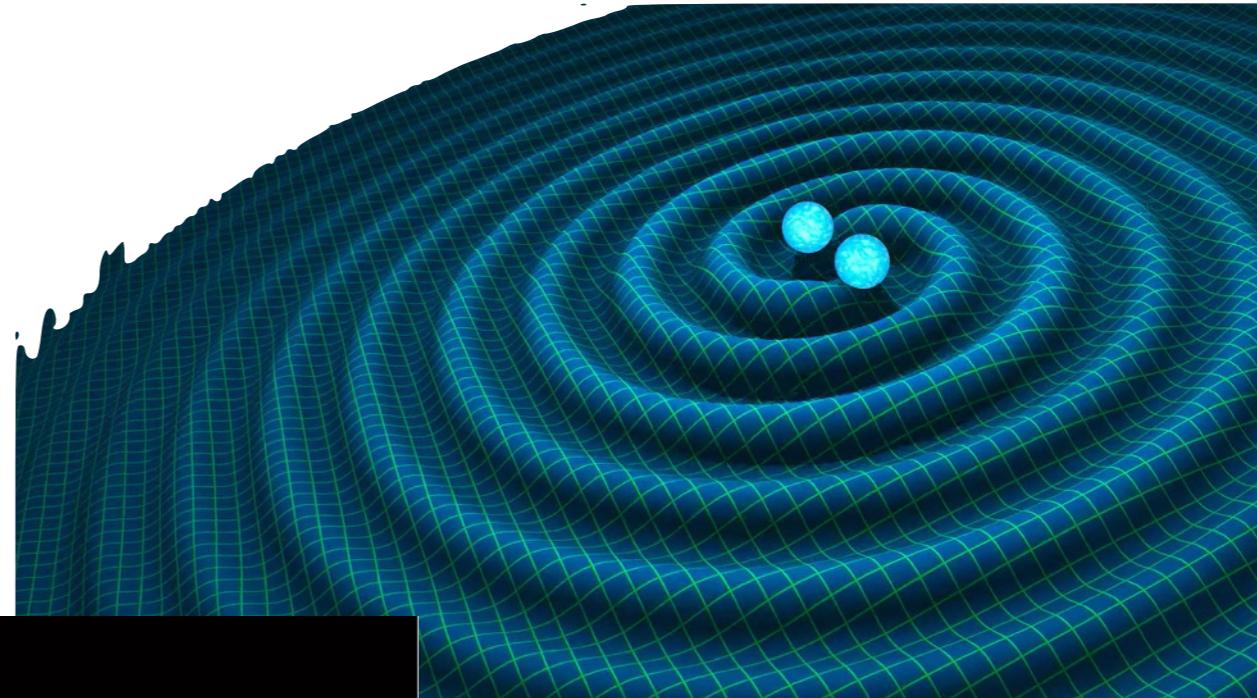
$$\begin{aligned}\mathcal{L} &\supset -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} \\ &= -\frac{1}{4} g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B}\end{aligned}$$



[O'Hare, <https://cajohare.github.io/AxionLimits/>]

Gravitational Waves (GWs)

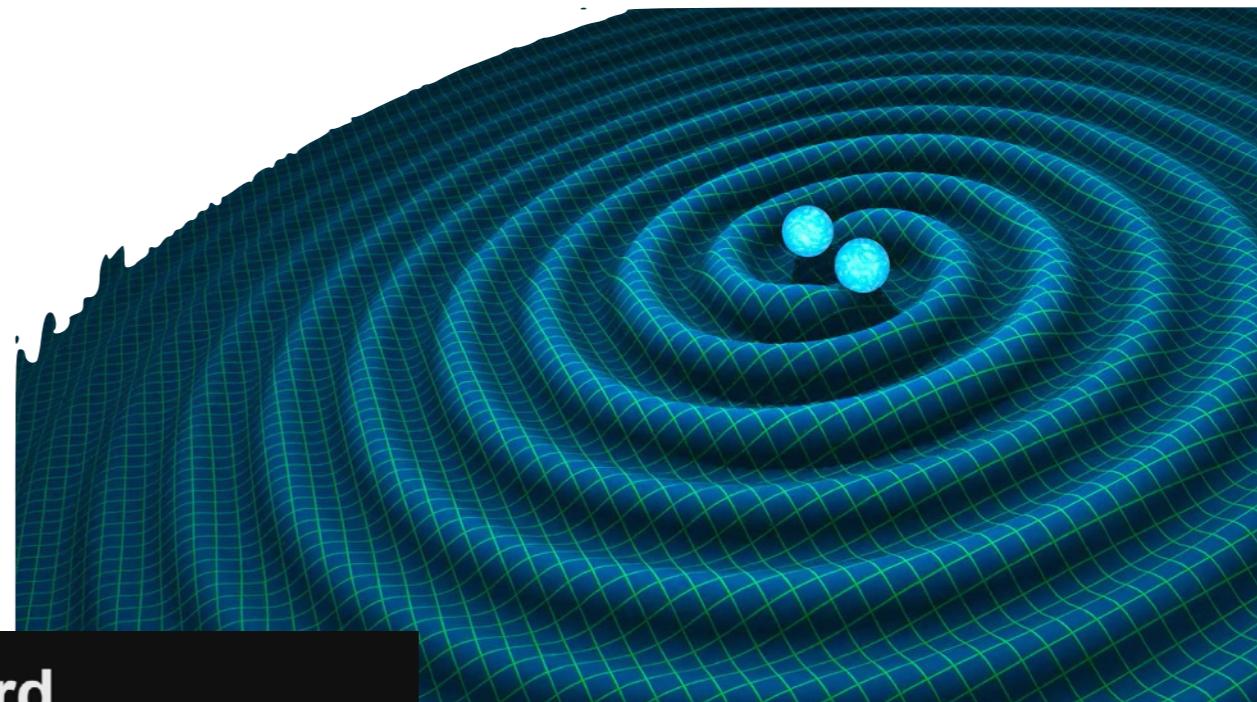
LIGO/Caltech/Sonoma State (Aurore Simonnet)



R. HURT / CALTECH-JPL /
HANDOUT/ ESA

Gravitational Waves (GWs)

LIGO/Virgo/Northwestern Univ. (Frank Elavsky)

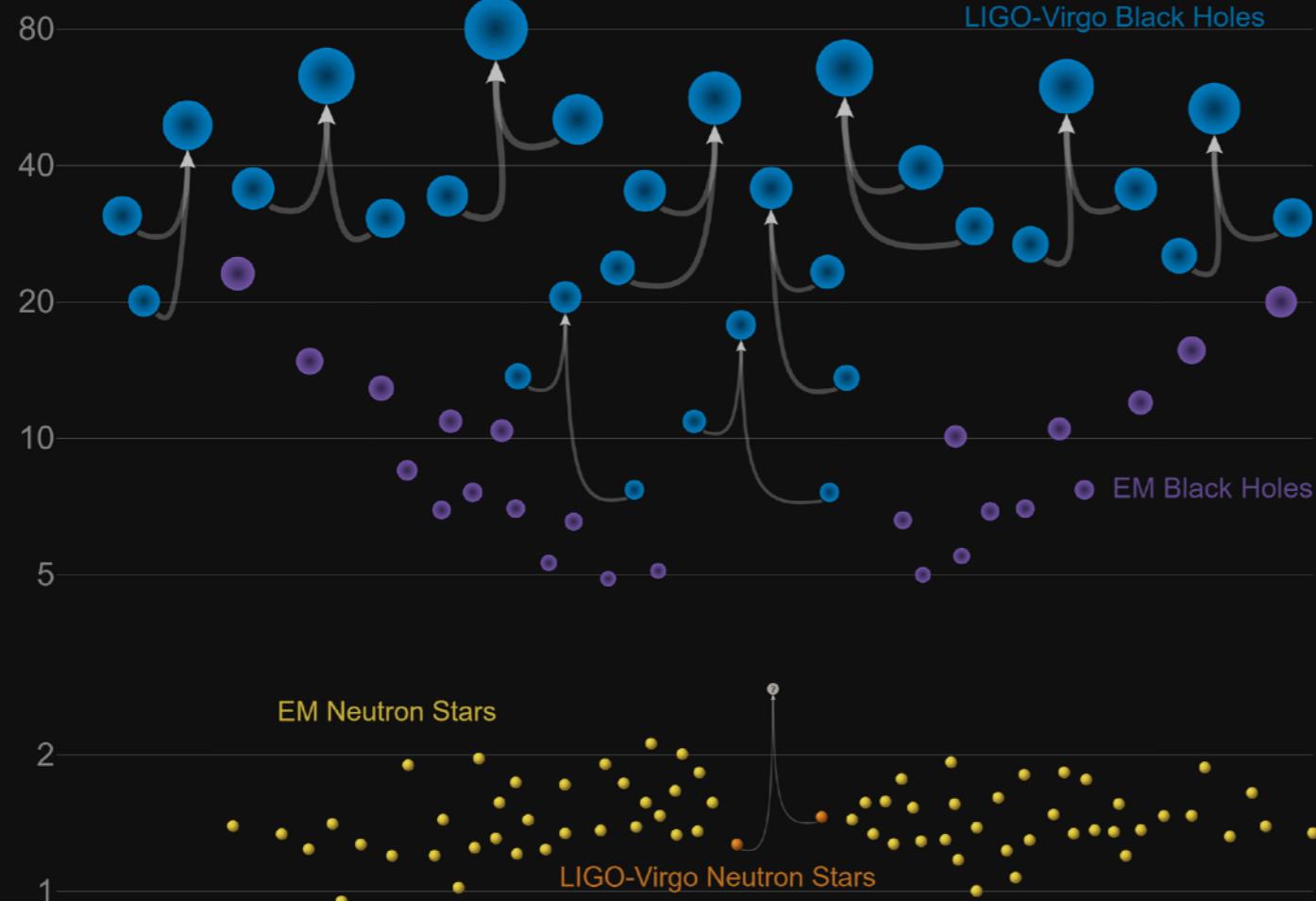


2019

Masses in the Stellar Graveyard

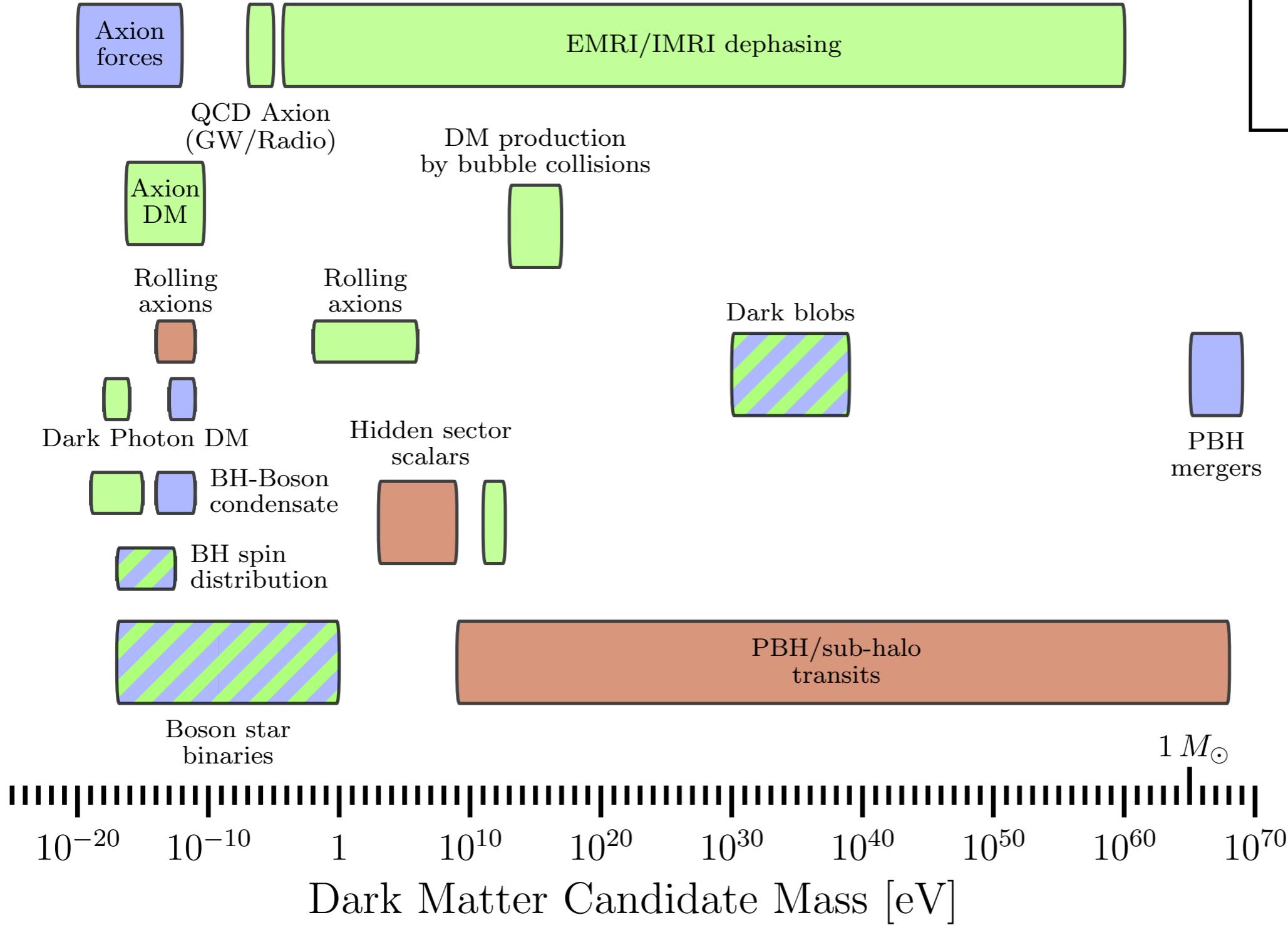
in Solar Masses

LIGO-Virgo Black Holes



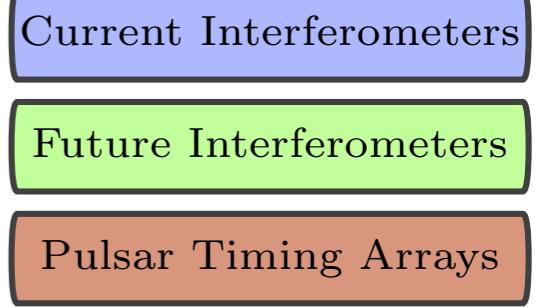
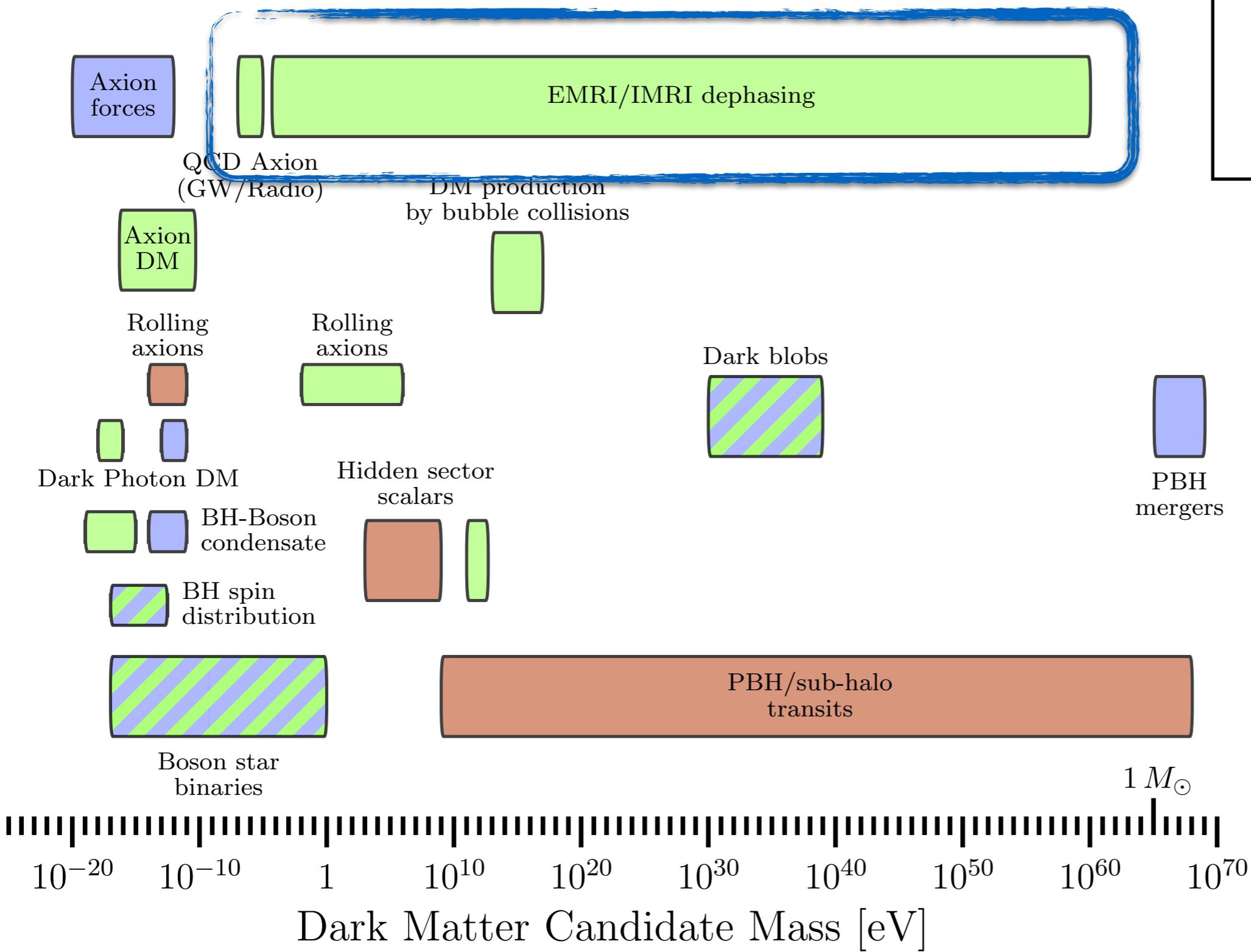
R. HURT / CALTECH-JPL /
HANDOUT / ESA

GW probes of DM



[1907.10610]

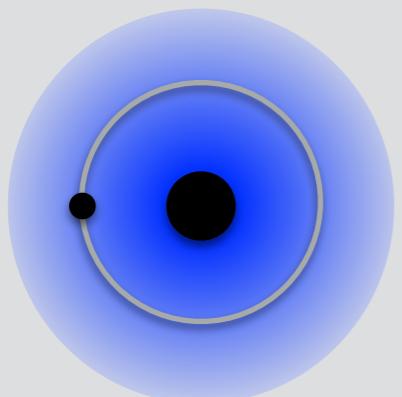
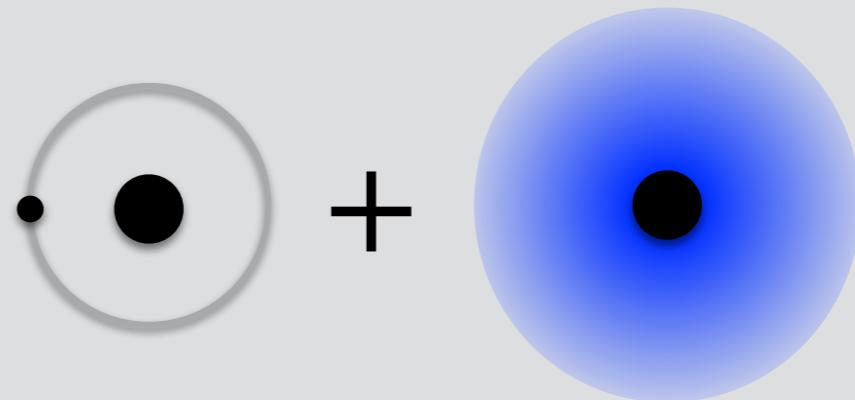
GW probes of DM



Overview

Intermediate Mass-Ratio Inspirals (IMRIs) and Dark Matter spikes

[Eda et al, [1301.5971](#), and others]

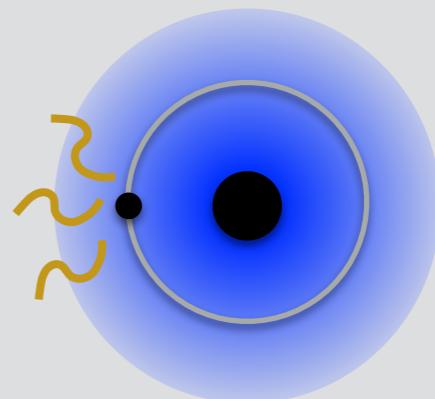


Dark Matter ‘de-phasing’ revisited

[**BJK**, Nichols, Gaggero, Bertone, [2002.12811](#)]

GW + EM signals of QCD axion Dark Matter

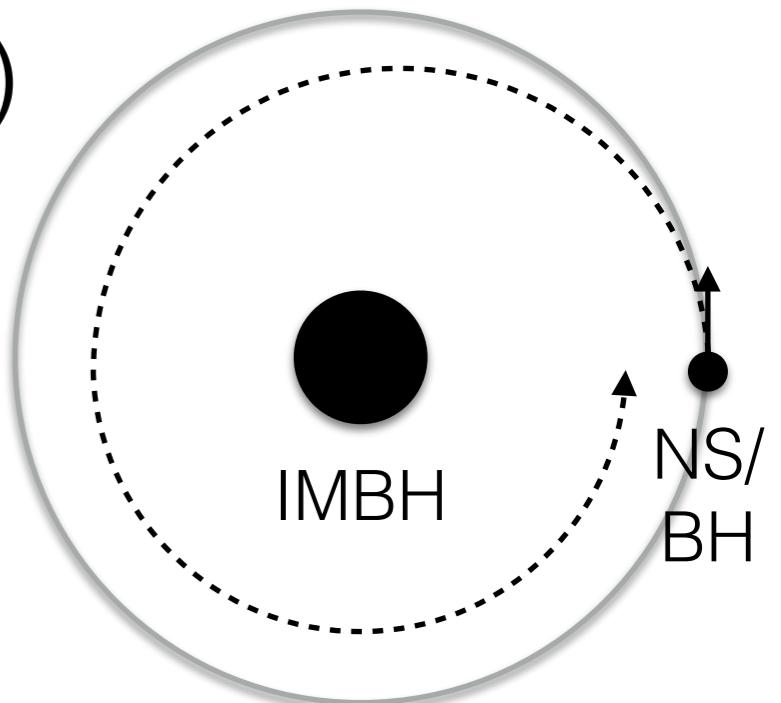
[Edwards, Chianese, **BJK**, Nissanke & Weniger,
[Phys. Rev. Lett. 124, 161101, 1905.04686](#)]



Intermediate Mass Ratio Inspiral (IMRI)

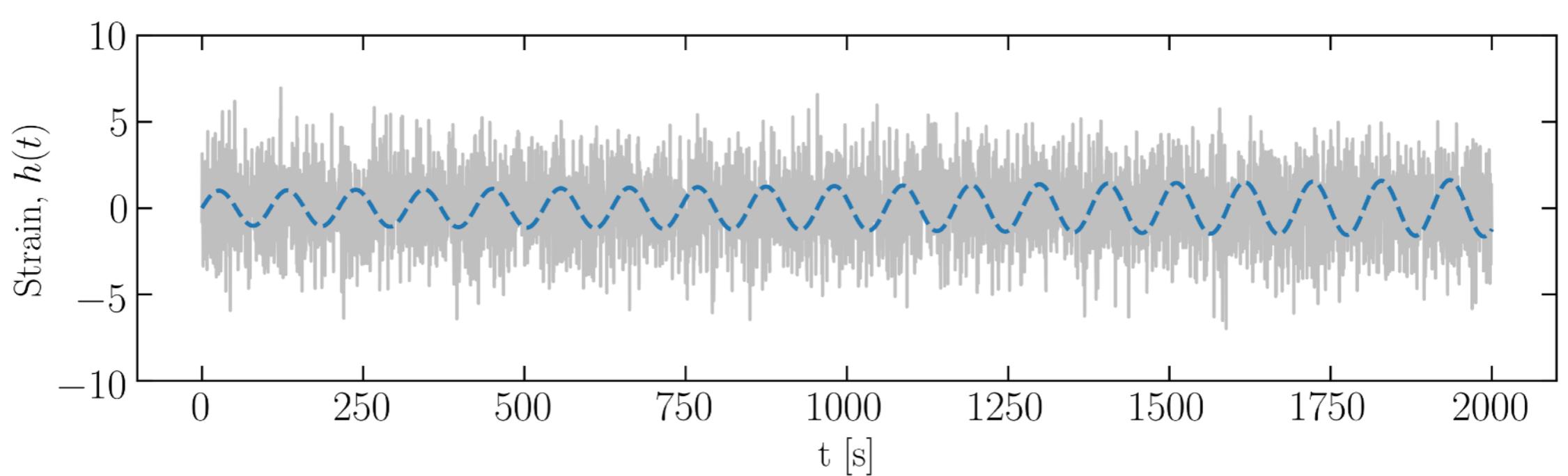
Stellar mass compact object (NS/BH) inspirals towards intermediate mass black hole (IMBH)

$$M_{\text{IMBH}} \sim 10^3 - 10^5 M_\odot$$



GW emission causes long, slow inspiral:

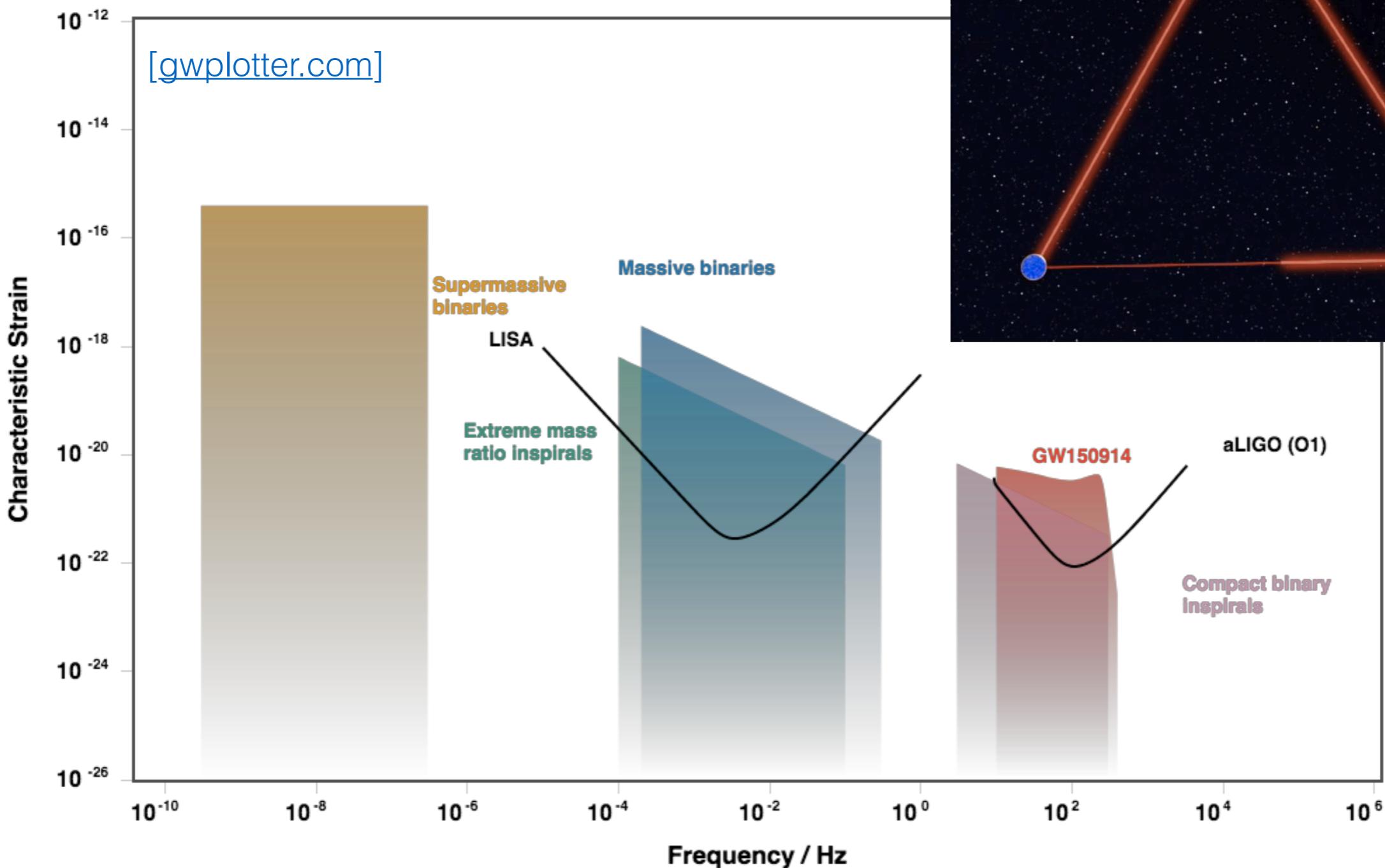
$$\dot{E}_{\text{GW}} \approx \frac{32G^4}{5c^5} \frac{M_{\text{IMBH}}^3 M_{\text{NS}}^2}{r^5} \propto (f_{\text{GW}})^{10/3}$$



LISA: GWs in Space

© AEI / MM / exozet

Laser Interferometer Space Antenna
(planned for the 2030s) [\[1702.00786\]](#)



LISA should detect $\sim 3 - 10$ IMRIs per year

[\[1711.00483\]](#)

Dark Matter ‘Mini-spikes’

Depending on the formation mechanism of the IMBH,
expect an over-density of DM:

$$\rho_{\text{DM}}(r) = \rho_{\text{sp}} \left(\frac{r_{\text{sp}}}{r} \right)^{\gamma_{\text{sp}}}$$

For BH forming in an NFW halo,
from adiabatic growth expect:

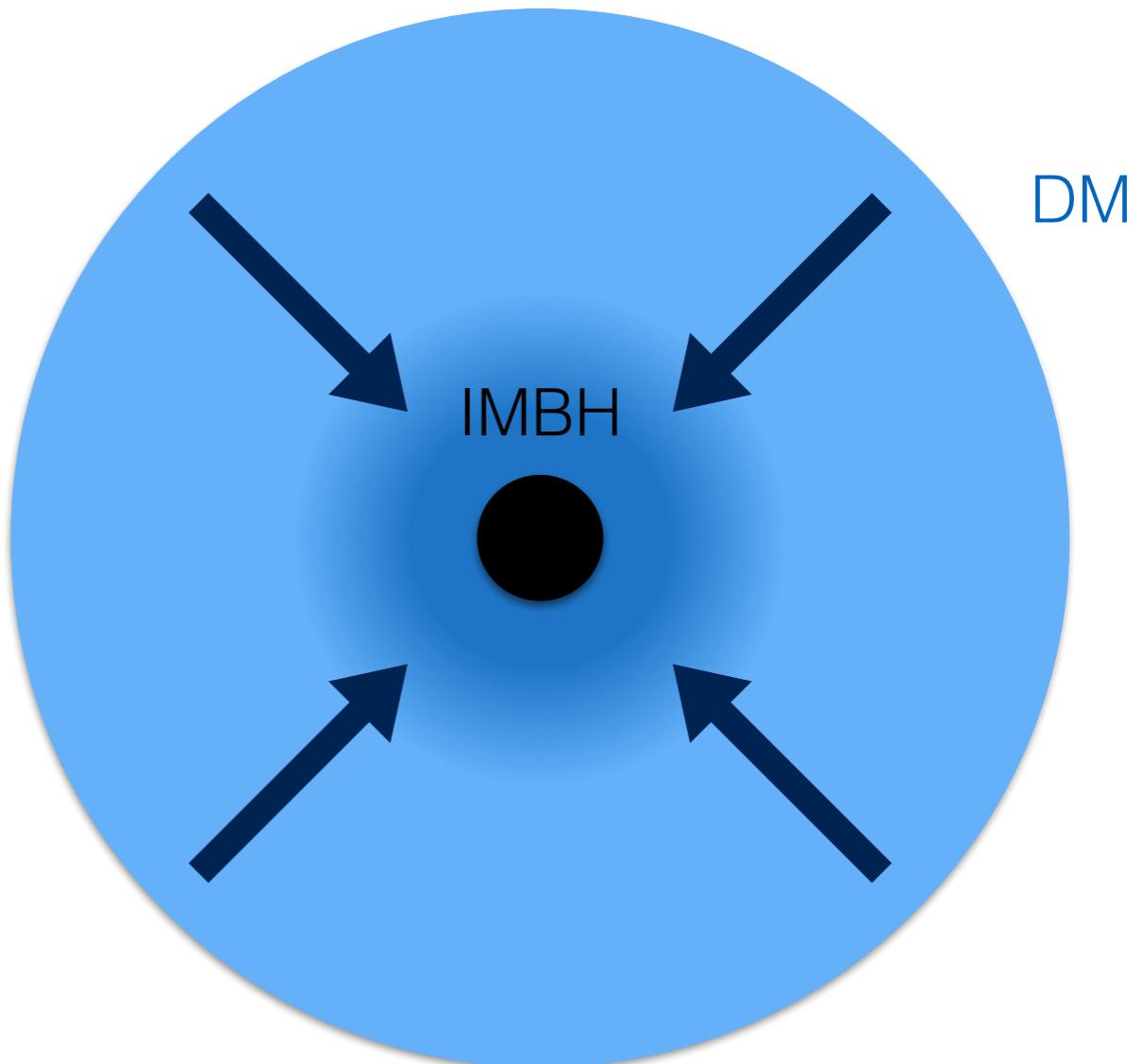
$$\gamma_{\text{sp}} = 7/3 \approx 2.333$$

For 1000 Solar mass IMBH, forming
at $z \sim 20$, get typical values:

$$\rho_{\text{sp}} = 200 M_{\odot} \text{ pc}^{-3}$$

$$r_{\text{sp}} = 0.5 \text{ pc}$$

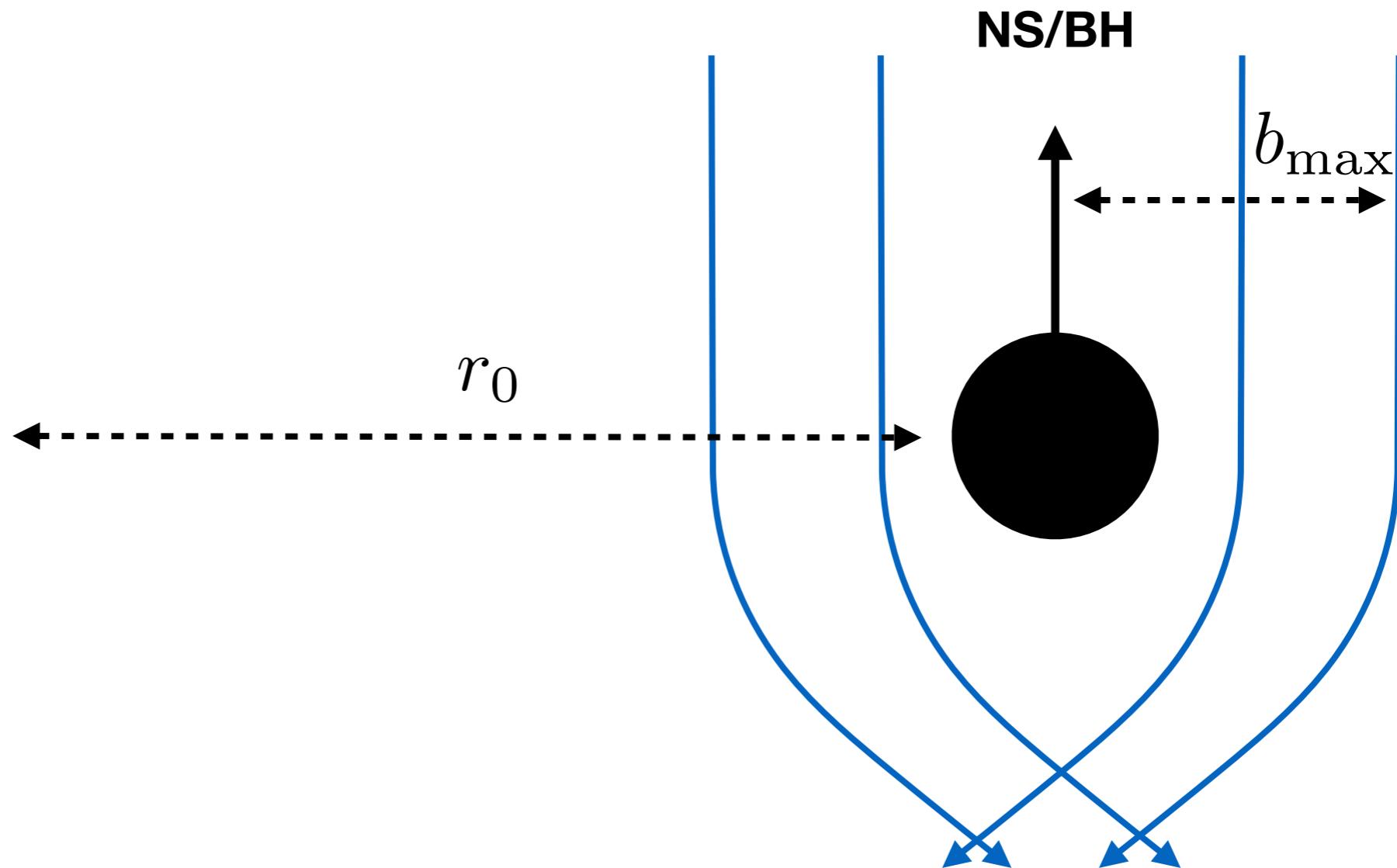
Density can reach $\rho \sim 10^{24} M_{\odot} \text{ pc}^{-3}$
($\sim 10^{24}$ times larger than local density)



[[astro-ph/9906391](#), [astro-ph/0501555](#), [astro-ph/0501625](#), [astro-ph/0509565](#), [0902.3665](#), [1305.2619](#)]

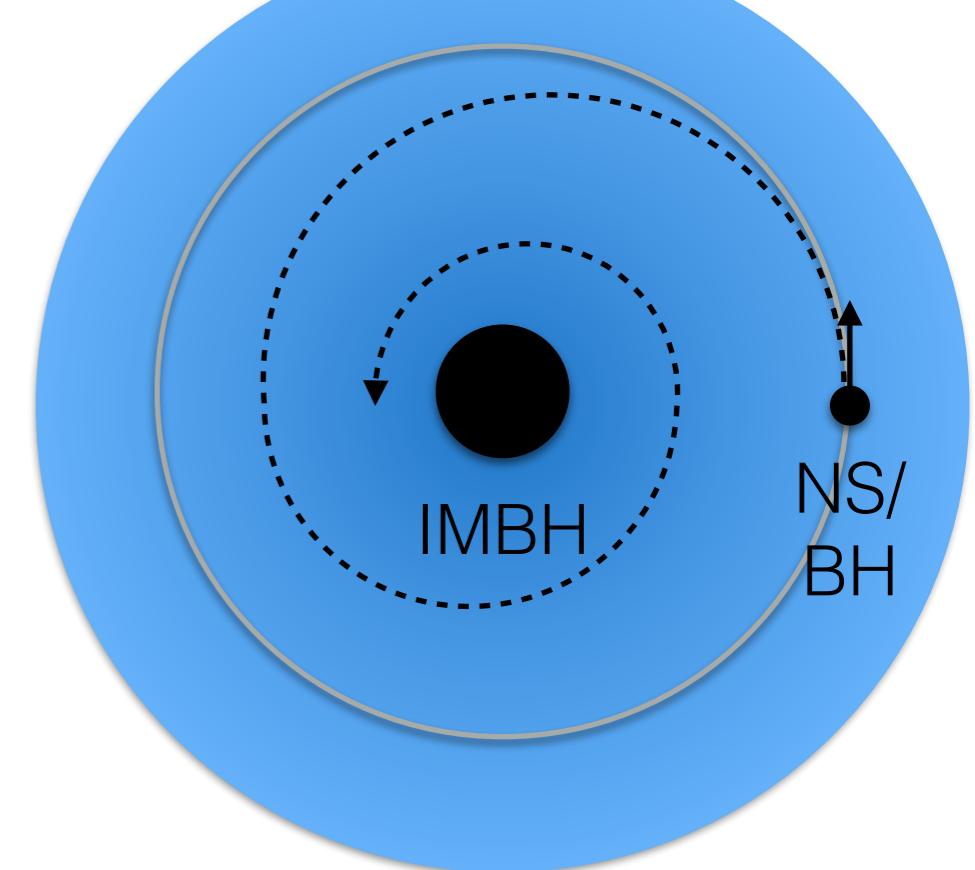
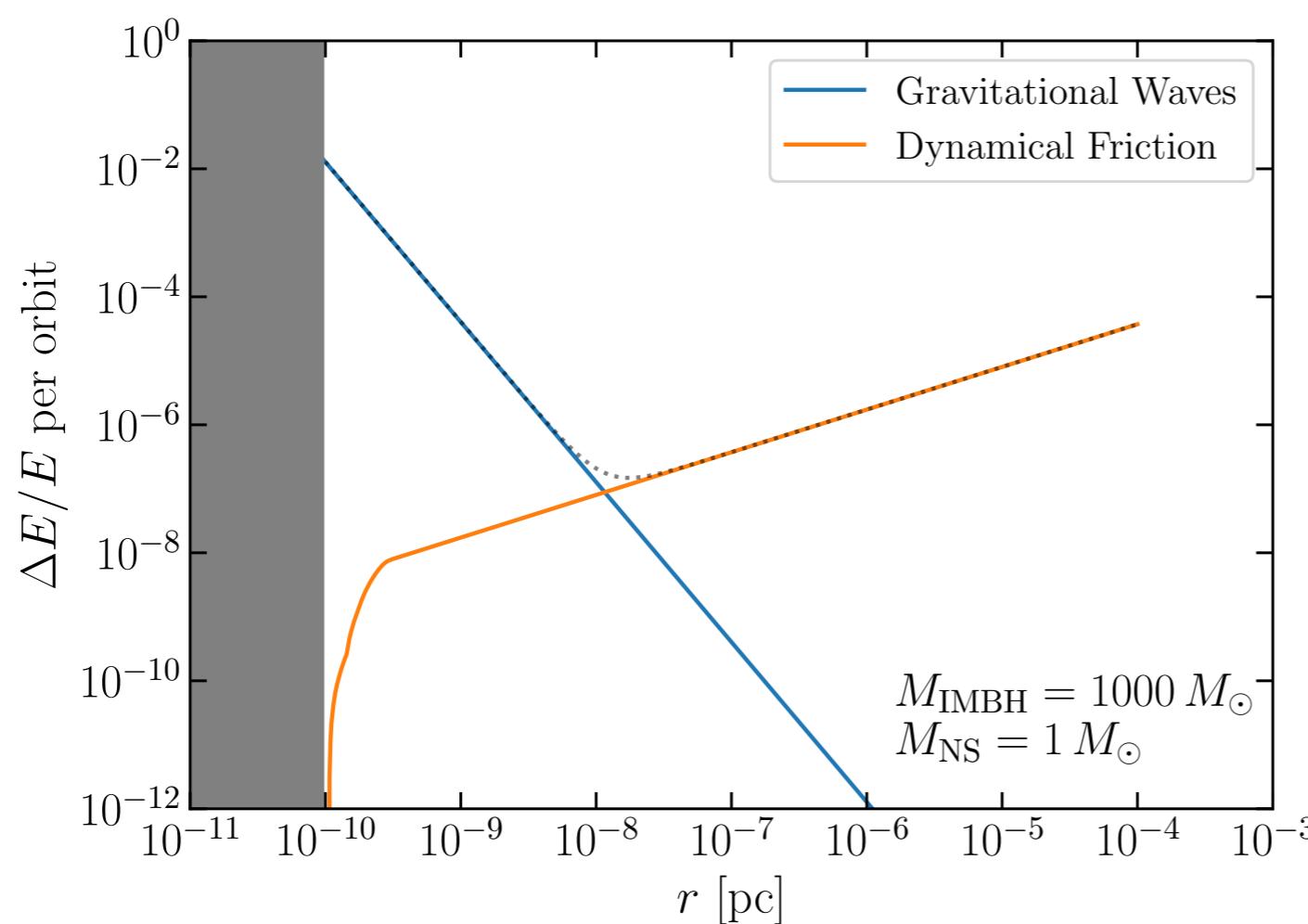
Dynamical Friction

[Chandrasekhar, 1943]

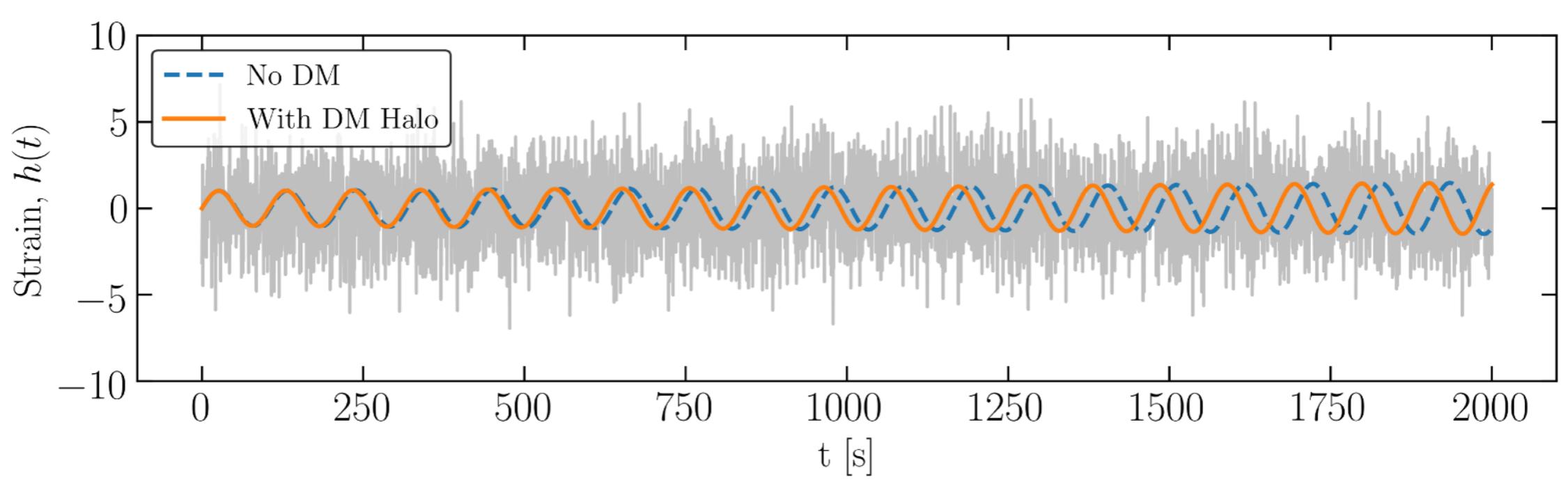


$$\dot{E}_{\text{DF}} \sim \frac{4\pi G^2 M_{\text{NS}}^2 \xi(v) \rho_{\text{DM}}(r)}{v_{\text{NS}}} \ln \Lambda \propto (f_{\text{GW}})^{\frac{2}{3}\gamma - 3}$$

IMRI + Dark Matter



$$-\dot{E}_{\text{orb}} = \dot{E}_{\text{GW}} + \dot{E}_{\text{DF}}$$



'De-phasing' signal

Benchmark:

$$M_{\text{IMBH}} = 10^3 M_{\odot}$$

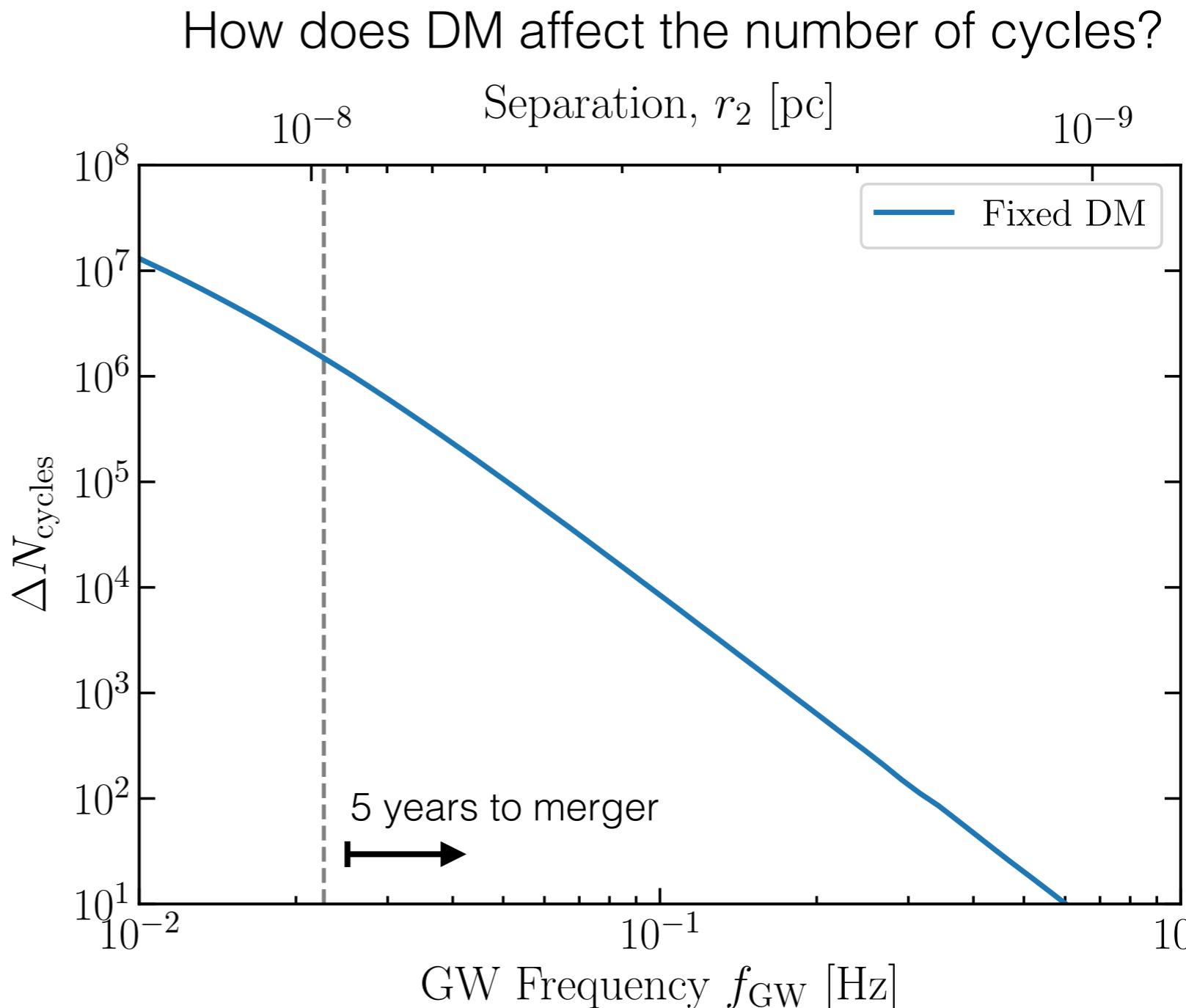
$$M_{\text{NS}} = 1 M_{\odot}$$

$$r_{\text{ini}} \sim 10^{-8} \text{ pc}$$



$$t_{\text{merge}}^{\text{vacuum}} \sim 5 \text{ yr}$$

$$N_{\text{cycles}}^{\text{vacuum}} \sim 6 \times 10^6$$



Need to know the signal to better
than ~ 1 part in 10^6 !

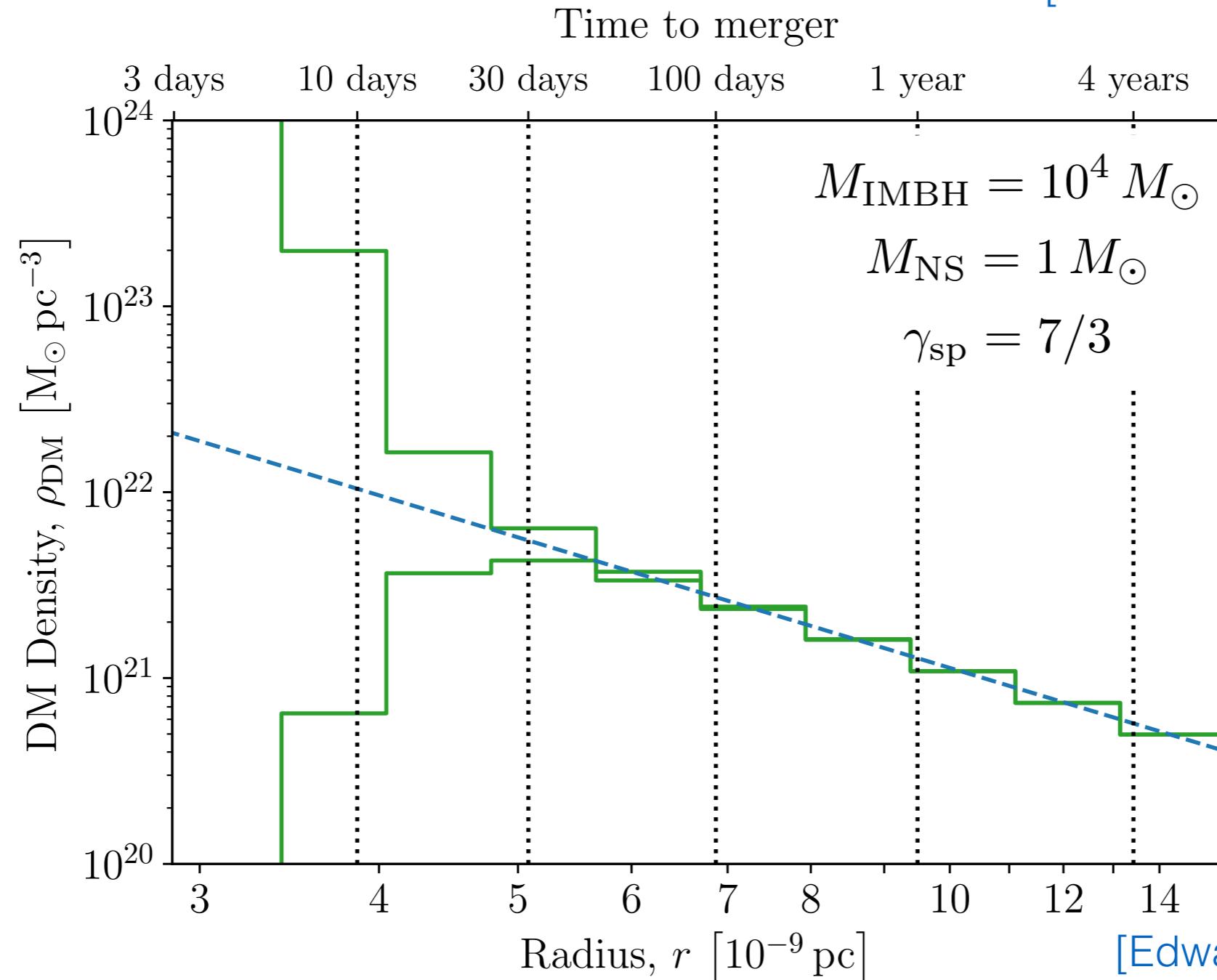
[Eda et al. [1301.5971](#), [1408.3534](#)]

[See also [1302.2646](#), [1404.7140](#), [1404.7149](#)]

Extracting DM Properties

With LISA, should be able to detect this ‘de-phasing’ and reconstruct chirp mass and DM spike properties to high precision

[Eda et al. [1301.5971](#), [1408.3534](#)]

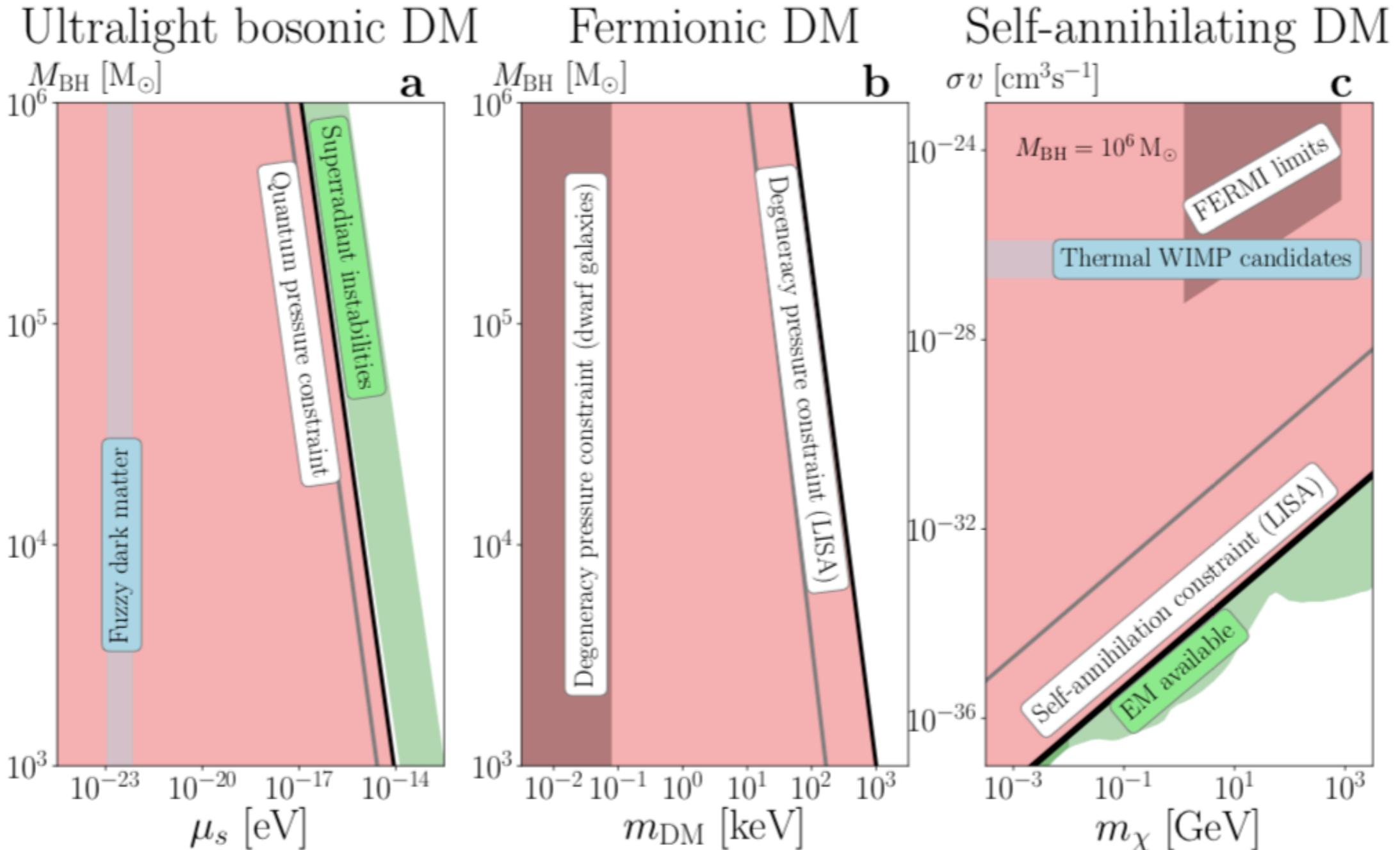


[Edwards, Chianese, **BJK**,
Nissanke & Weniger, [1905.04686](#)]

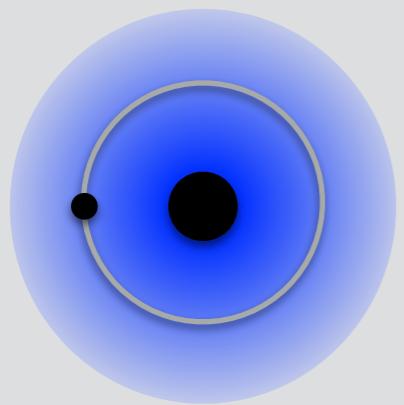
Nature of Dark Matter

Red regions would be ruled out by observation of a DM spike!

[[1906.11845](#)]

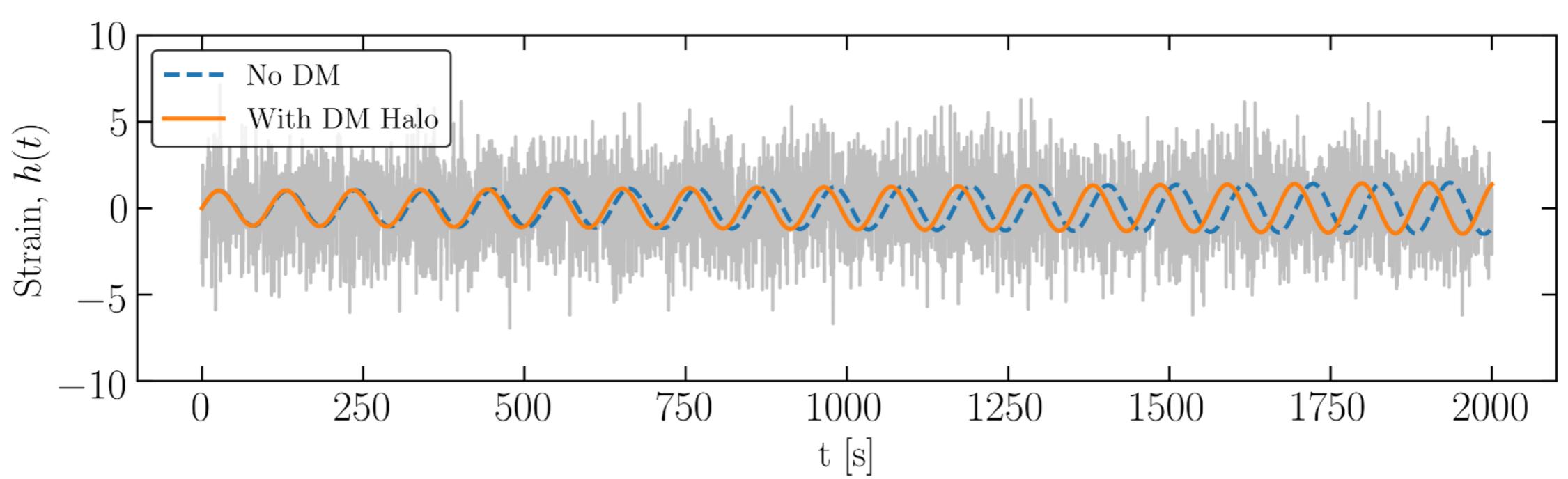


[See also Bertone, Coogan, Gaggero, **BJK** & Weniger, [1905.01238](#)]



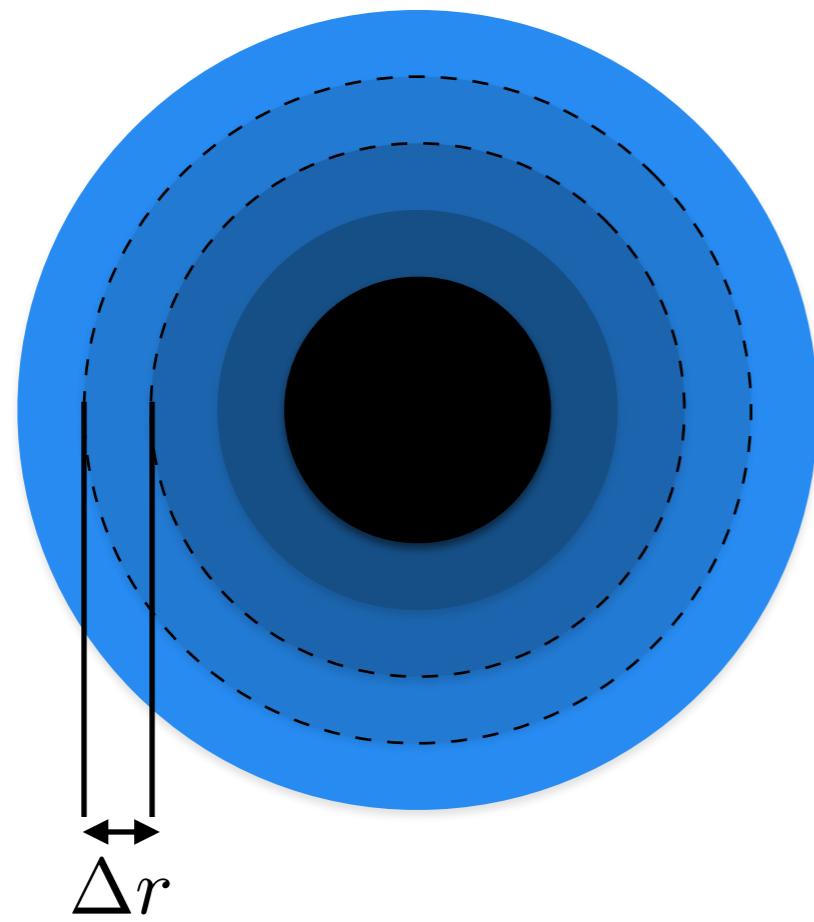
Dark Matter ‘de-phasing’ revisited

[BJK, Nichols, Gaggero, Bertone, [2002.12811](#)]

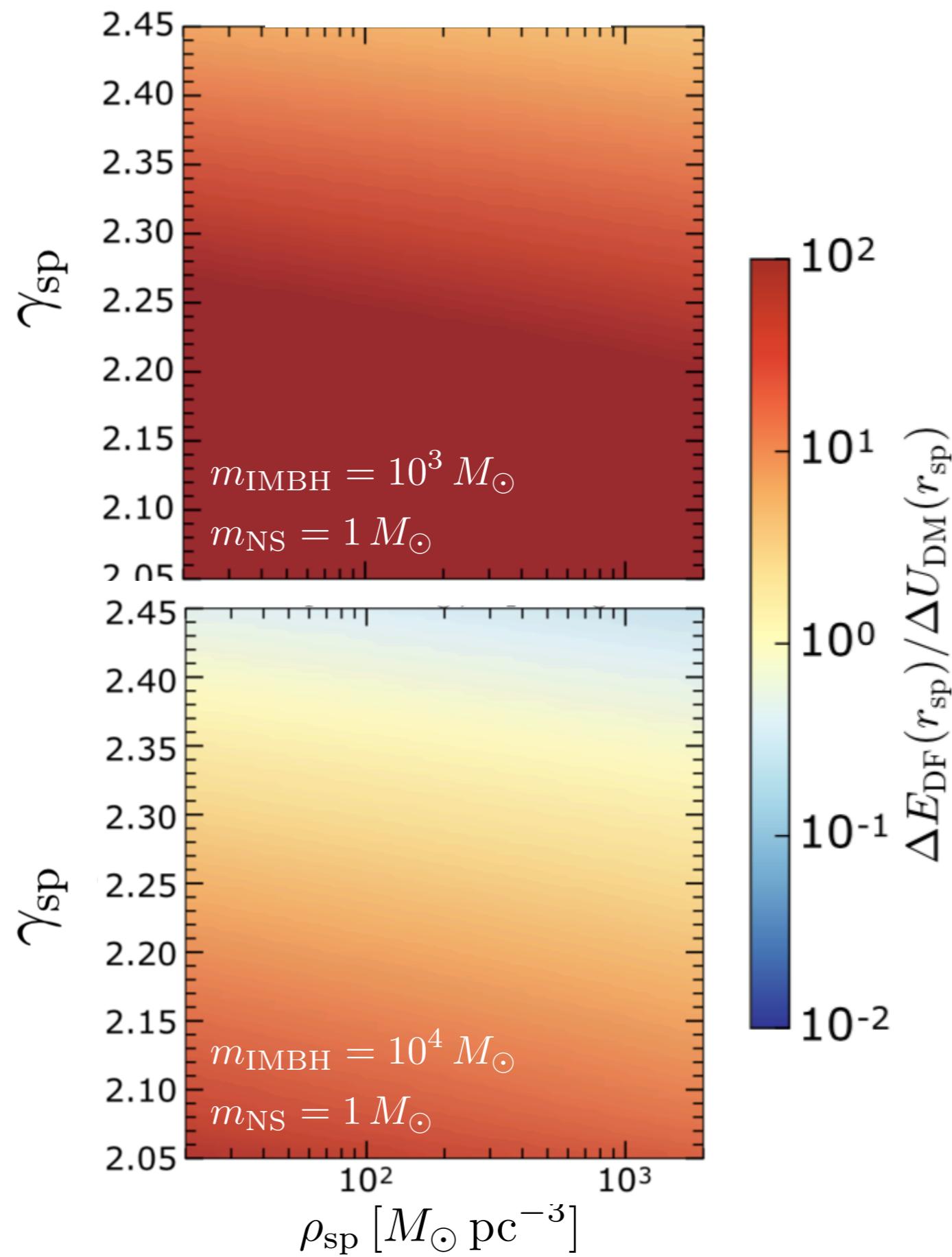


Energy Budget

Q: How much energy is *available* for dynamical friction?

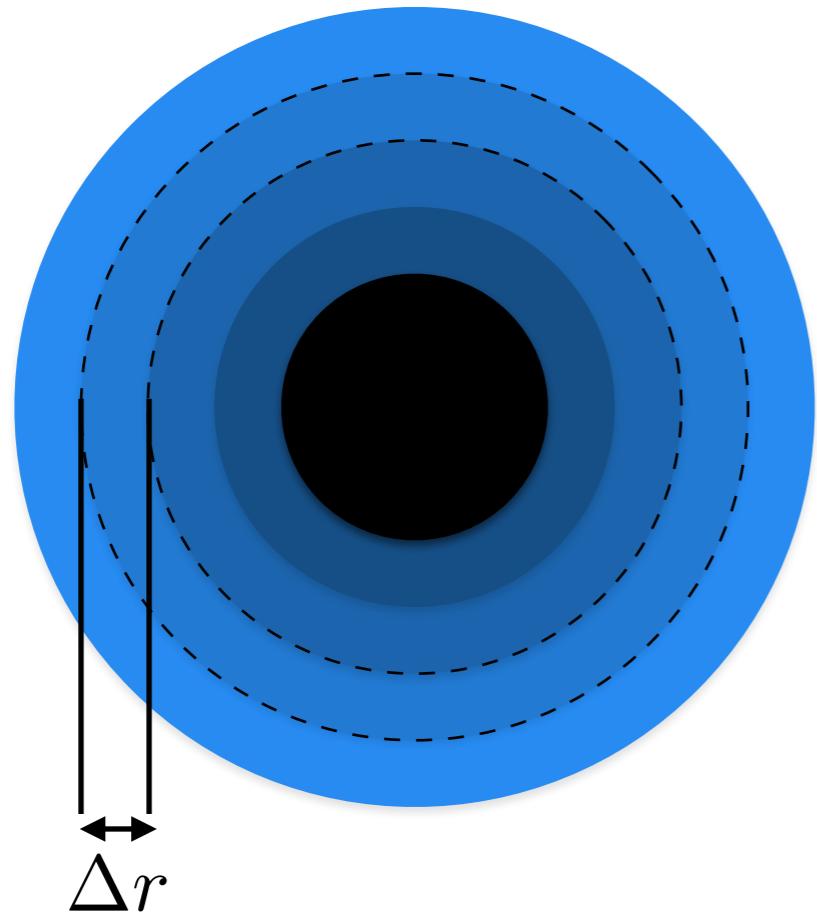


A: Binding energy of DM ΔU_{DM} over radius Δr



Energy Budget

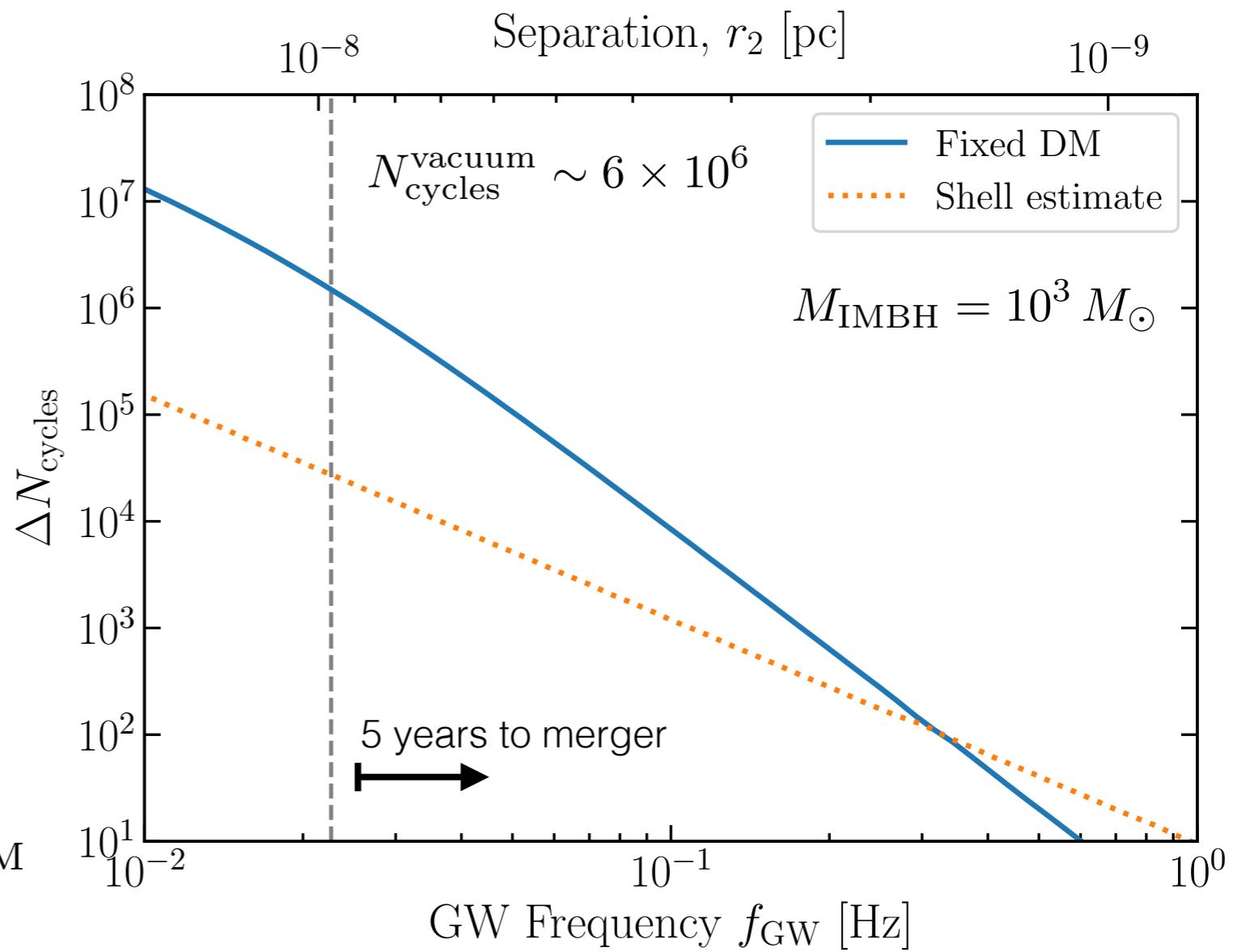
Q: How much energy is *available* for dynamical friction?



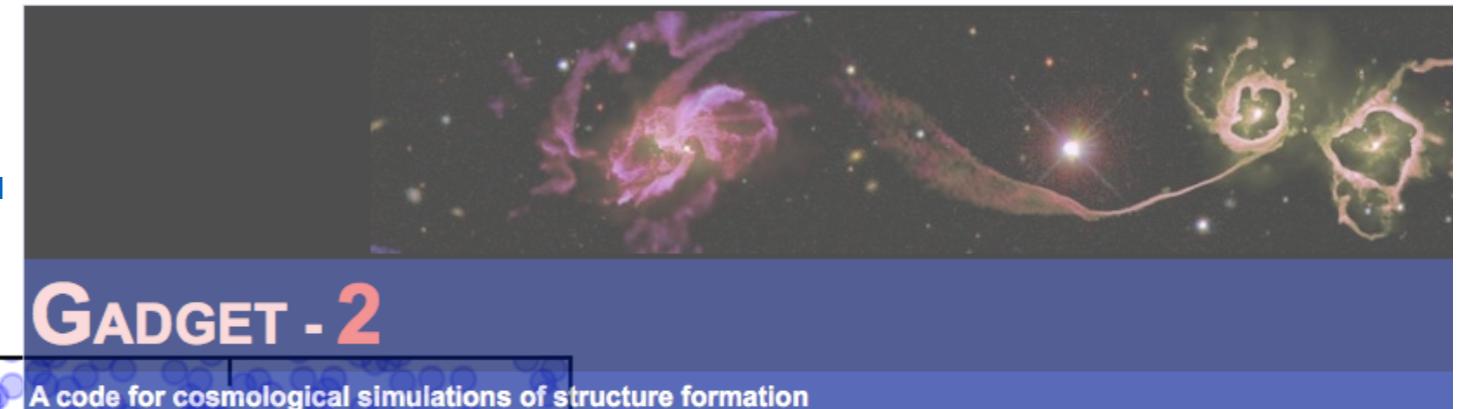
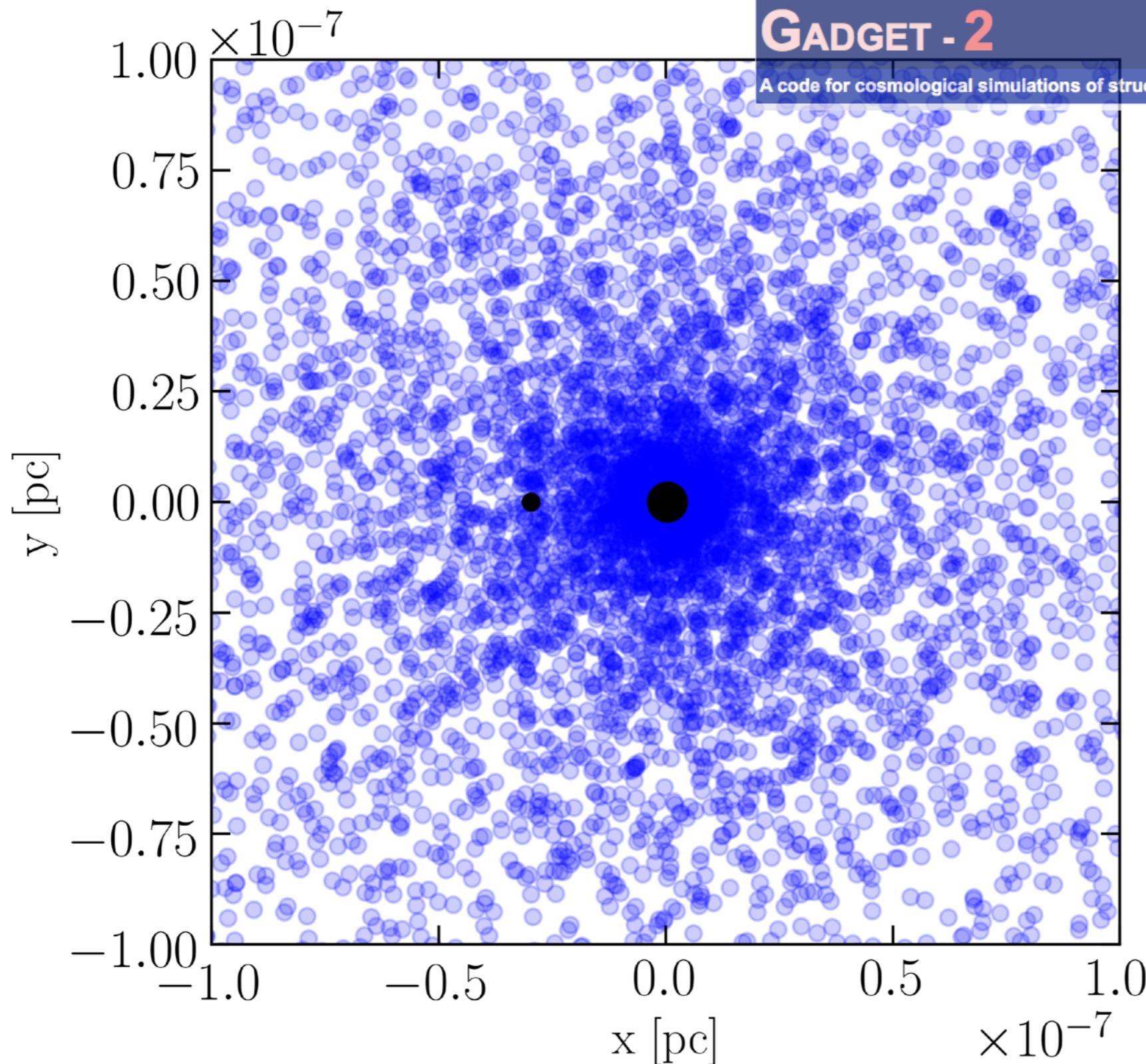
A: Binding energy of DM ΔU_{DM} over radius Δr

Evolve the system by fixing the dynamical friction force to extract *all* binding energy from a shell at a given radius:

$$\dot{E}_{\text{DF}} = \dot{r} \frac{dU_{\text{DM}}}{dr}$$



N-body Simulations



[astro-ph/0505010]

High Precision N-body Sims

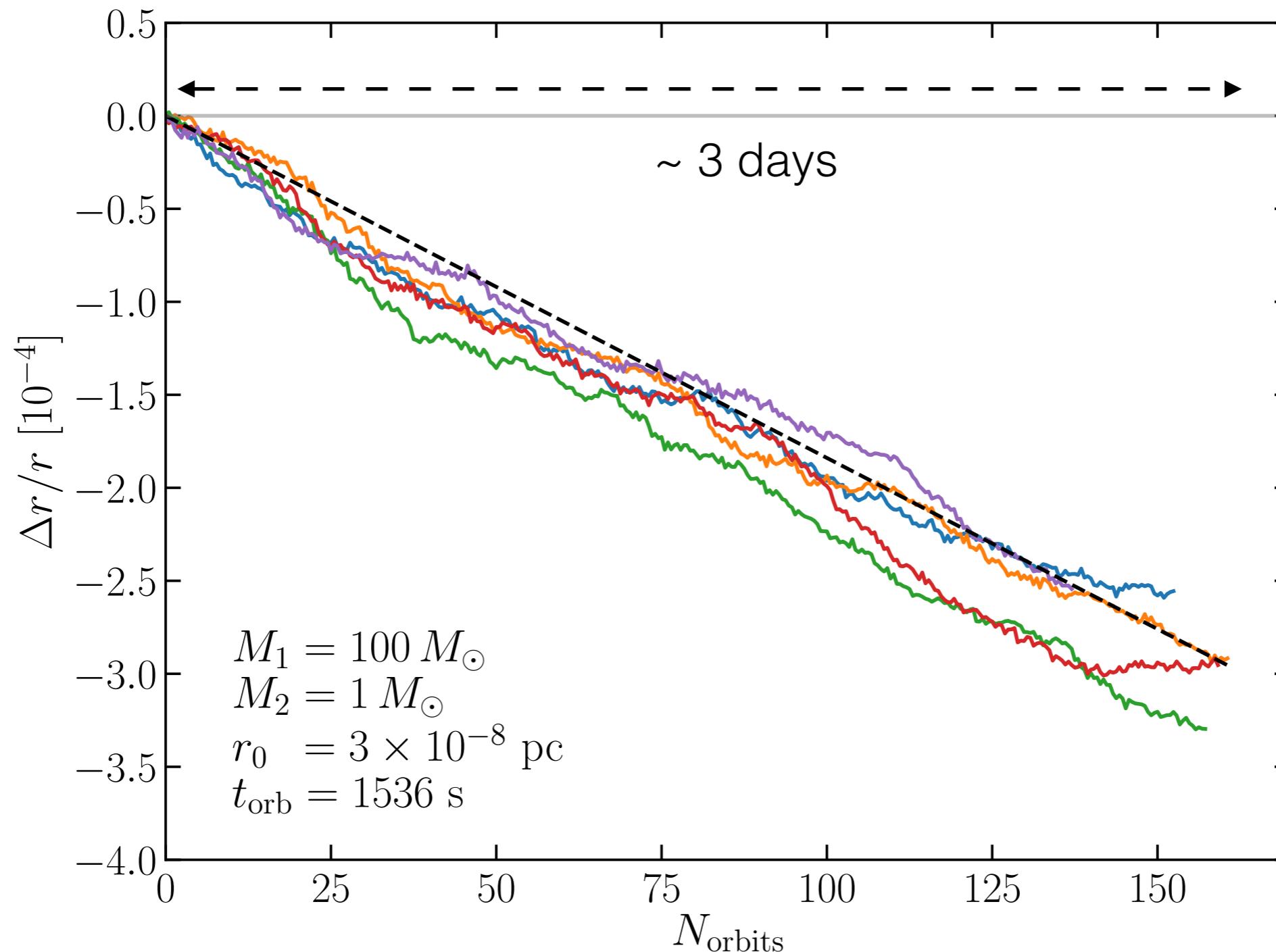
Gadget-II code:

```
58
59  /* Some physical constants in cgs units */
60
61 #define GRAVITY      6.672e-8    /*!< Gravitational constant (in cgs units) */
62 #define SOLAR_MASS   1.989e33
63 #define SOLAR_LUM    3.826e33
64 #define RAD_CONST    7.565e-15
65 #define AVOGADRO    6.0222e23
66 #define BOLTZMANN   1.3806e-16
67 #define GAS_CONST   8.31425e7
68 #define C           2.9979e10
69 #define PLANCK      6.6262e-27
70 #define CM_PER_MPC  3.085678e24
71 #define PROTONMASS  1.6726e-24
72 #define ELECTRONMASS 9.10953e-28
73 #define THOMPSON    6.65245e-25
74 #define ELECTRONCHARGE 4.8032e-10
75 #define HUBLEE     3.2407789e-18    /* in h/sec */
76
```

The Universe:

$$G_N = 6.674 \times 10^{-8} \text{ m}^3 \text{ g}^{-1} \text{ s}^{-2}$$

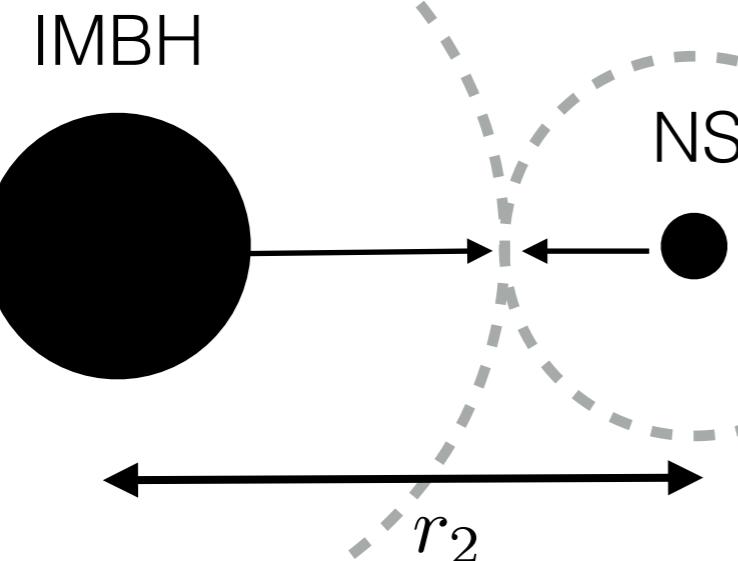
N-body Results



Allows us to check assumptions and fix normalisation of DF force ($\ln \Lambda$),
but can't simulate the whole 5 year inspiral!

N-body results

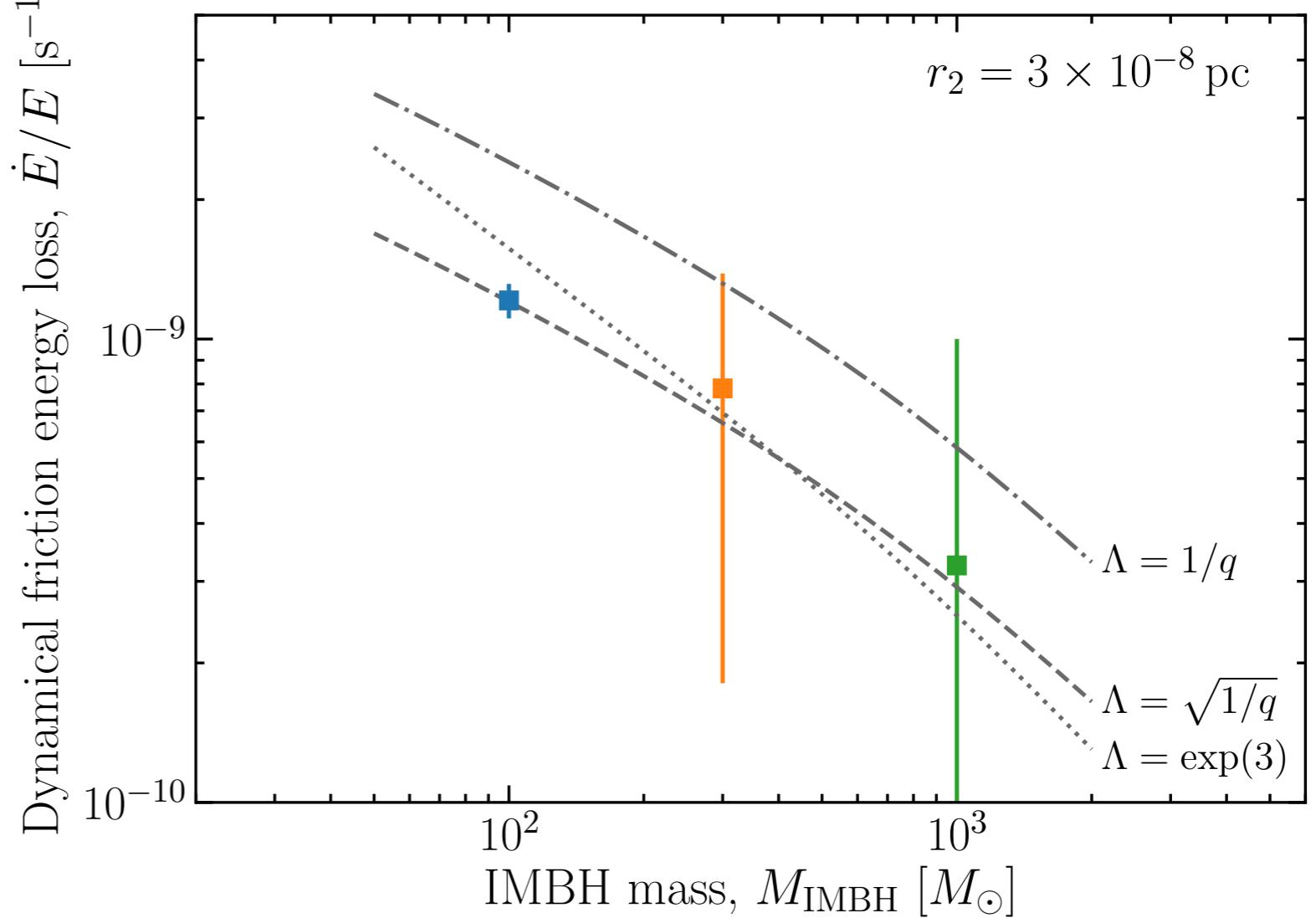
$$q \equiv m_{\text{NS}}/m_{\text{IMBH}} \ll 1$$



$$\begin{aligned}\Lambda &= b_{\max} \frac{v_0^2}{G m_{\text{NS}}} \\ &= \frac{b_{\max}}{q r_2} \\ &= 1/\sqrt{q}\end{aligned}$$

Allows us to calibrate the maximum impact parameter; tells us which particles scatter with the NS.

$$b_{\max} = \sqrt{q} r_2 \sim 3\% r_2$$



Self-consistent evolution

Follow semi-analytically the phase space distribution of DM:

$$f = \frac{dN}{d^3\mathbf{r} d^3\mathbf{v}} \equiv f(\mathcal{E})$$

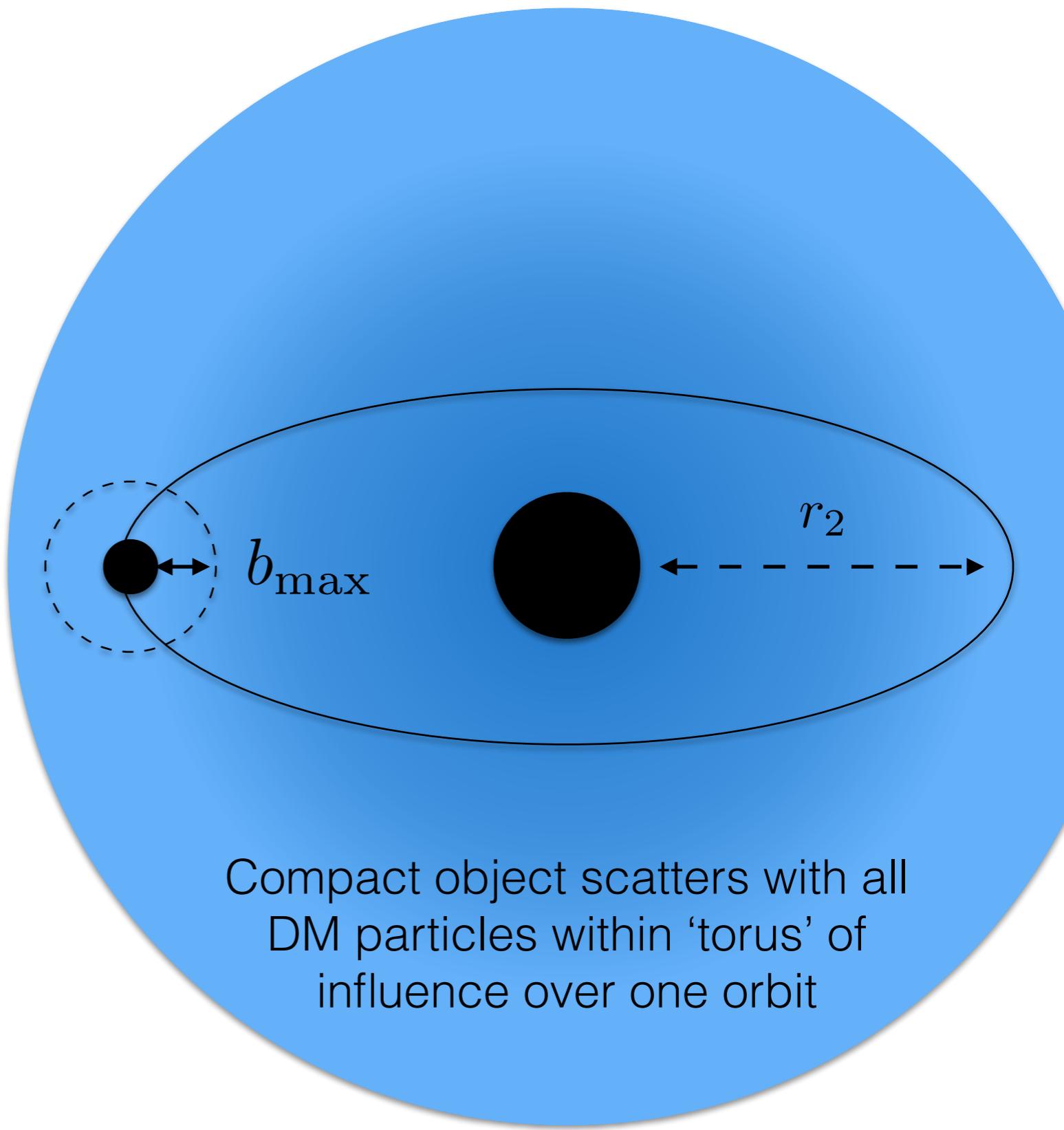
$$\mathcal{E} = \Psi(r) - \frac{1}{2}v^2$$

Each particle receives a ‘kick’ through gravitational scattering

$$\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$$

Reconstruct density from distribution function:

$$\rho(r) = \int d^3\mathbf{v} f(\mathcal{E})$$



Self-consistent evolution

Assuming everything evolves slowly compared to the orbital period:

$$\Delta f(\mathcal{E}) = -p_{\mathcal{E}} f(\mathcal{E}) + \int \left(\frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}} \right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E}-\Delta\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

$P_{\mathcal{E}}(\Delta\mathcal{E})$ - probability for a particle with energy \mathcal{E} to scatter and receive a 'kick' $\Delta\mathcal{E}$

$p_{\mathcal{E}} = \int P_{\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$ - total probability for a particle with energy \mathcal{E} to scatter

Self-consistent evolution

Assuming everything evolves slowly compared to the orbital period:

$$\Delta f(\mathcal{E}) = -p_{\mathcal{E}} f(\mathcal{E}) +$$

Particles scattering from
 $\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$

$$\int \left(\frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}} \right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E}-\Delta\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

Particles scattering from
 $\mathcal{E} - \Delta\mathcal{E} \rightarrow \mathcal{E}$

$P_{\mathcal{E}}(\Delta\mathcal{E})$ - probability for a particle with energy \mathcal{E} to scatter and receive a 'kick' $\Delta\mathcal{E}$

$$p_{\mathcal{E}} = \int P_{\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

- total probability for a particle with energy \mathcal{E} to scatter

Self-consistent evolution

Assuming everything evolves slowly compared to the orbital period:

$$T_{\text{orb}} \frac{f(\mathcal{E})}{dt} = -p_{\mathcal{E}} f(\mathcal{E}) +$$

Particles scattering from
 $\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$

$$\int \left(\frac{\mathcal{E}}{\mathcal{E} - \Delta\mathcal{E}} \right)^{5/2} f(\mathcal{E} - \Delta\mathcal{E}) P_{\mathcal{E} - \Delta\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

Particles scattering from
 $\mathcal{E} - \Delta\mathcal{E} \rightarrow \mathcal{E}$

$P_{\mathcal{E}}(\Delta\mathcal{E})$ - probability for a particle with energy \mathcal{E} to scatter and receive a 'kick' $\Delta\mathcal{E}$

$$p_{\mathcal{E}} = \int P_{\mathcal{E}}(\Delta\mathcal{E}) d\Delta\mathcal{E}$$

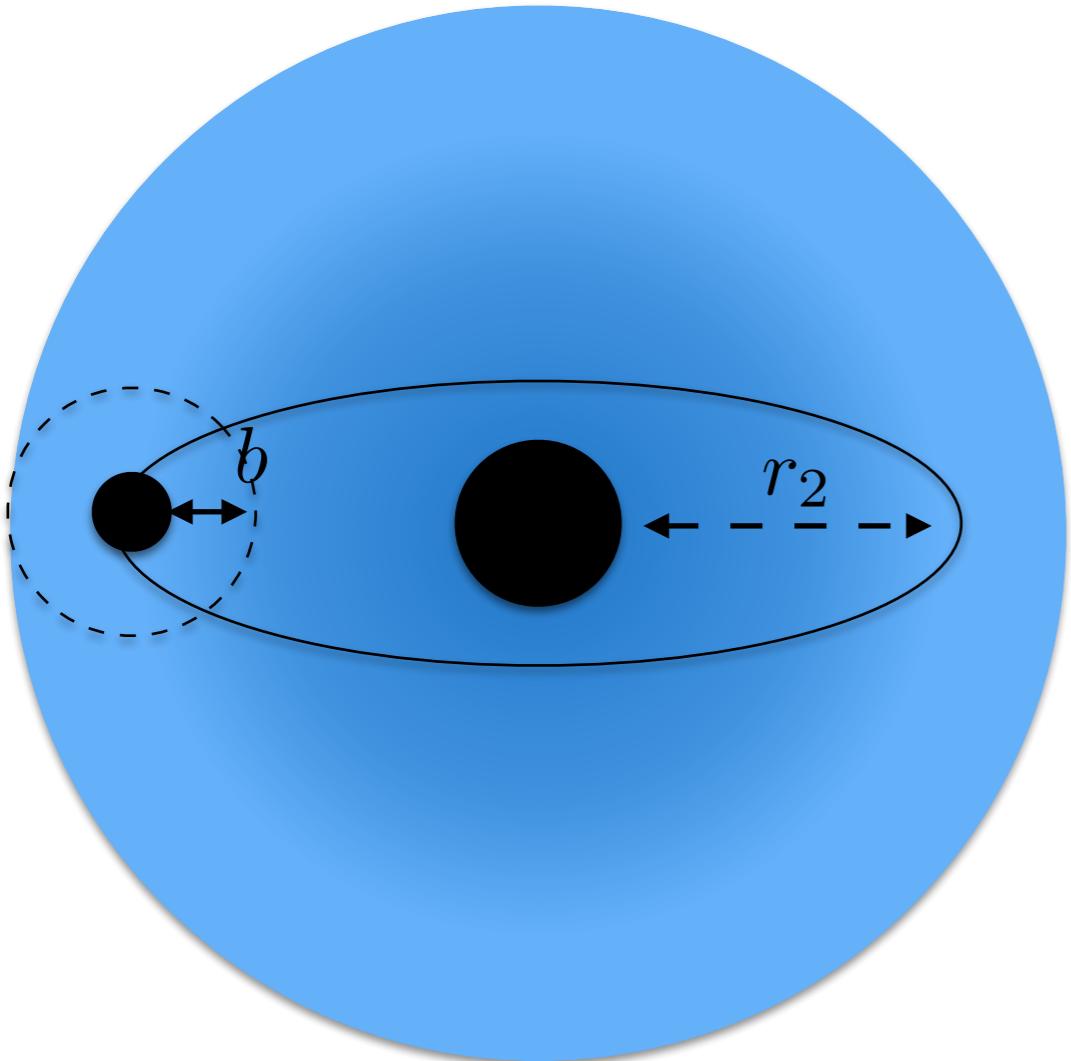
- total probability for a particle with energy \mathcal{E} to scatter

Scattering probability

Two body scattering problem relates energy exchange to impact parameter:

$$\Delta\mathcal{E}(b) = -2v_0^2 \left[1 + \frac{b^2 v_0^4}{G^2 m_2^2} \right]^{-1}$$

$$P_{\mathcal{E}}(\Delta\mathcal{E}) \propto \iint \delta(\mathcal{E}(r, v) - \mathcal{E}) \times \delta(\Delta\mathcal{E}(b) - \Delta\mathcal{E}) d^3\mathbf{r} d^3\mathbf{v}.$$



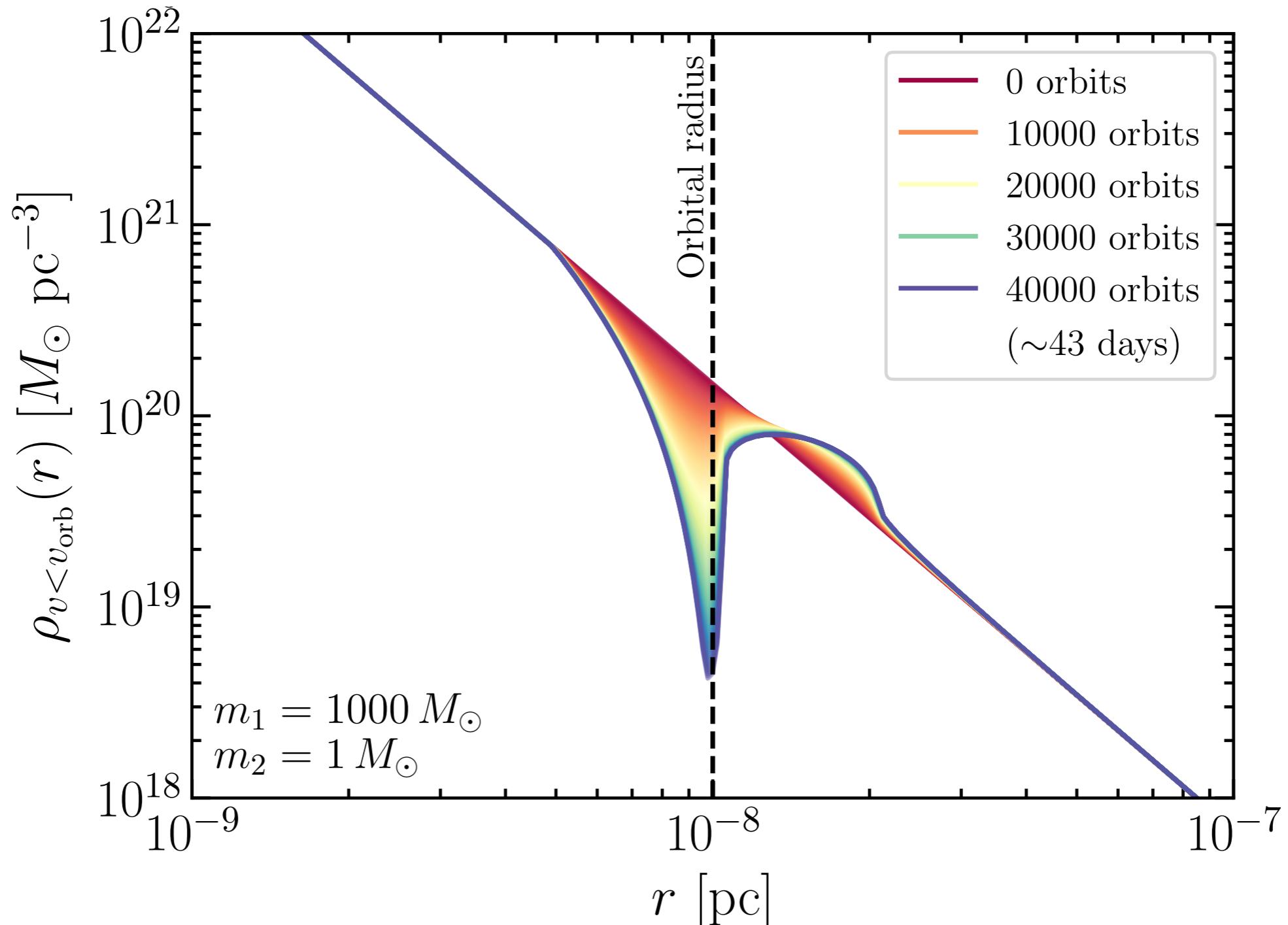
Integrate over the surface of the
'torus of influence'

Working to first order in b/r , the result
can be written in terms of elliptic integrals

Code available online:
github.com/bradkav/HaloFeedback

Evolution of density profile

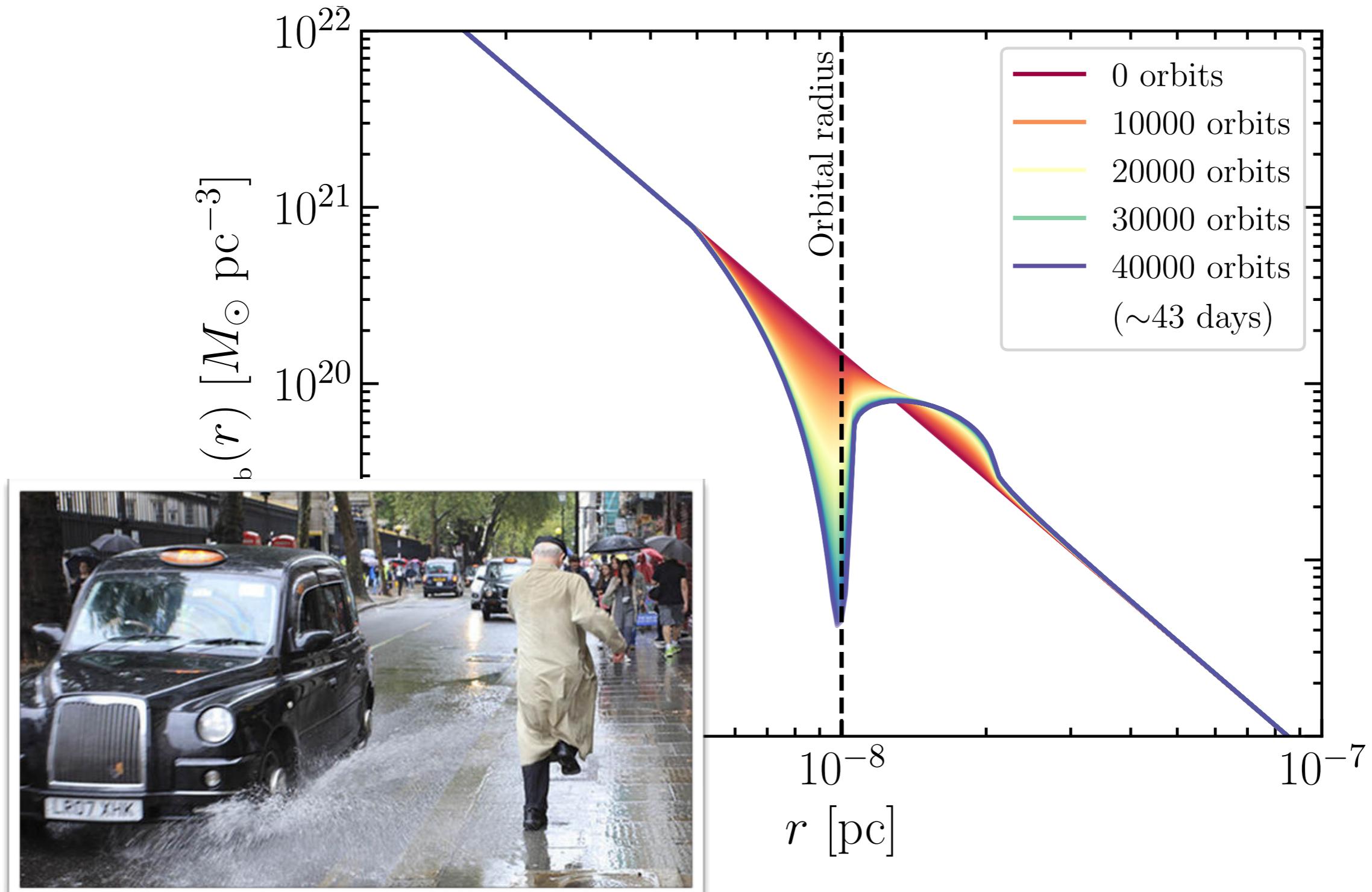
How does the DM halo ‘react’ to the orbiting compact object?



Subtlety: plotting here only ‘slow moving’ particles

Evolution of density profile

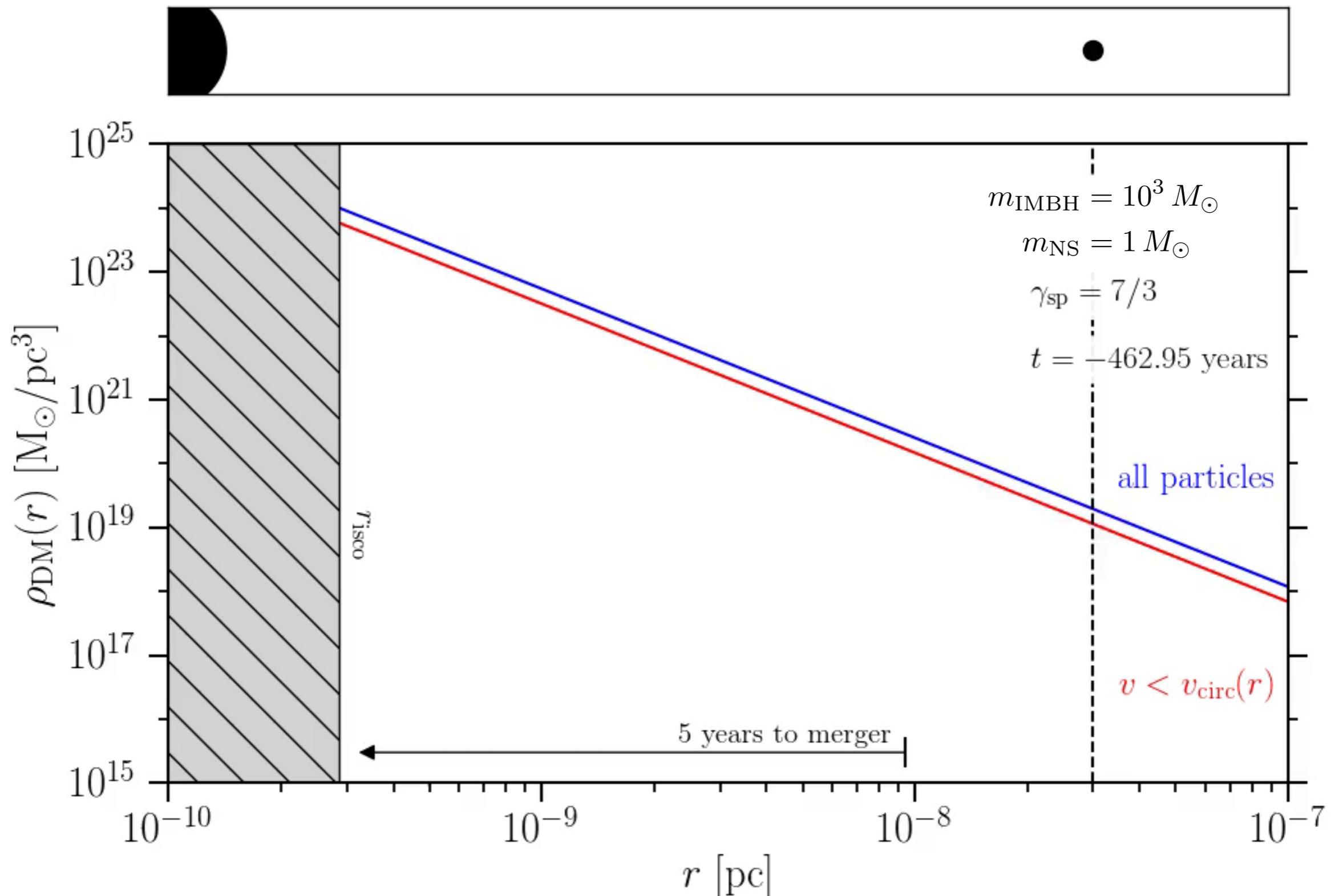
How does the DM halo ‘react’ to the orbiting compact object?



Subtlety: plotting here only ‘slow moving’ particles

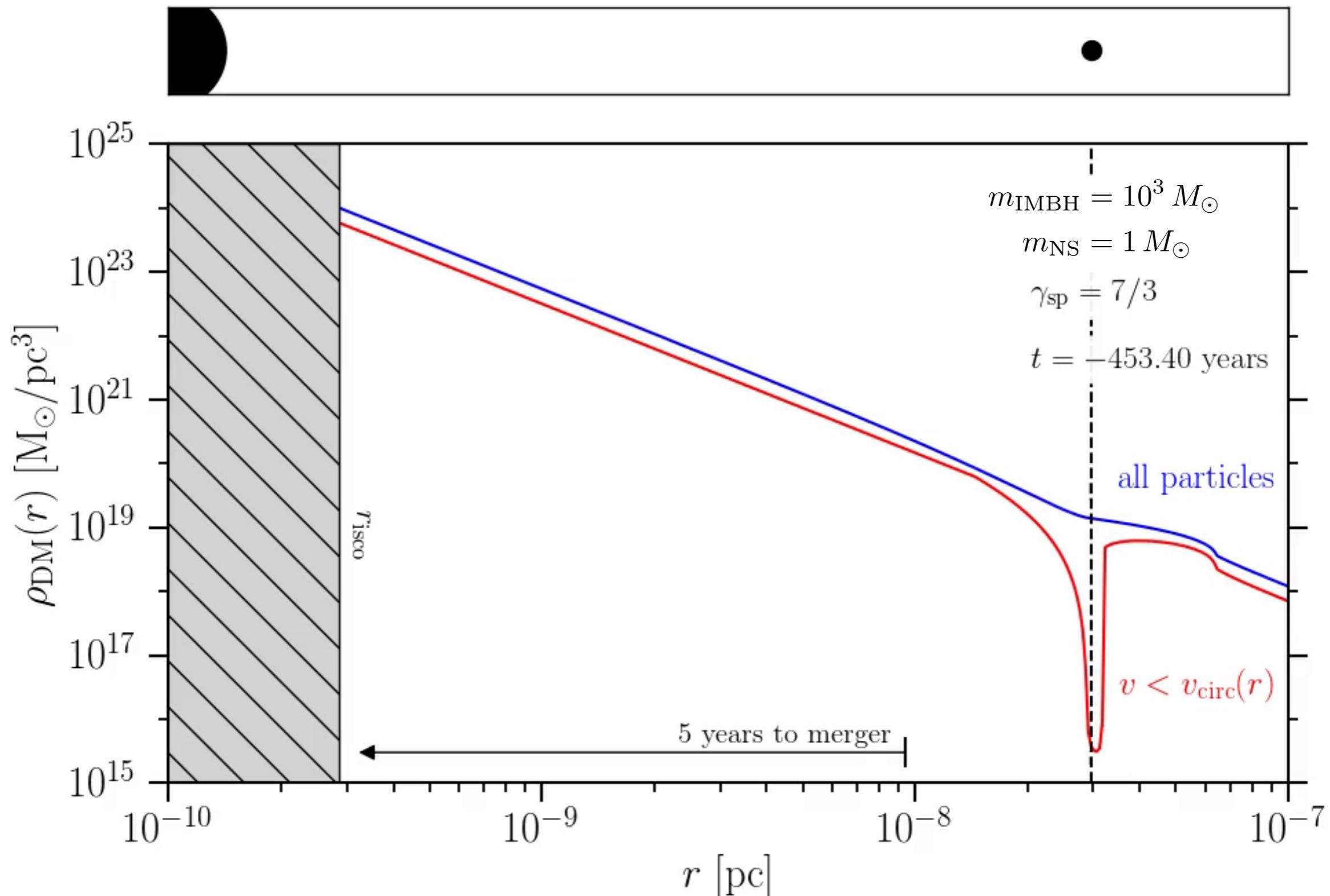
Full evolution of the system

Movies: tinyurl.com/GW4DM



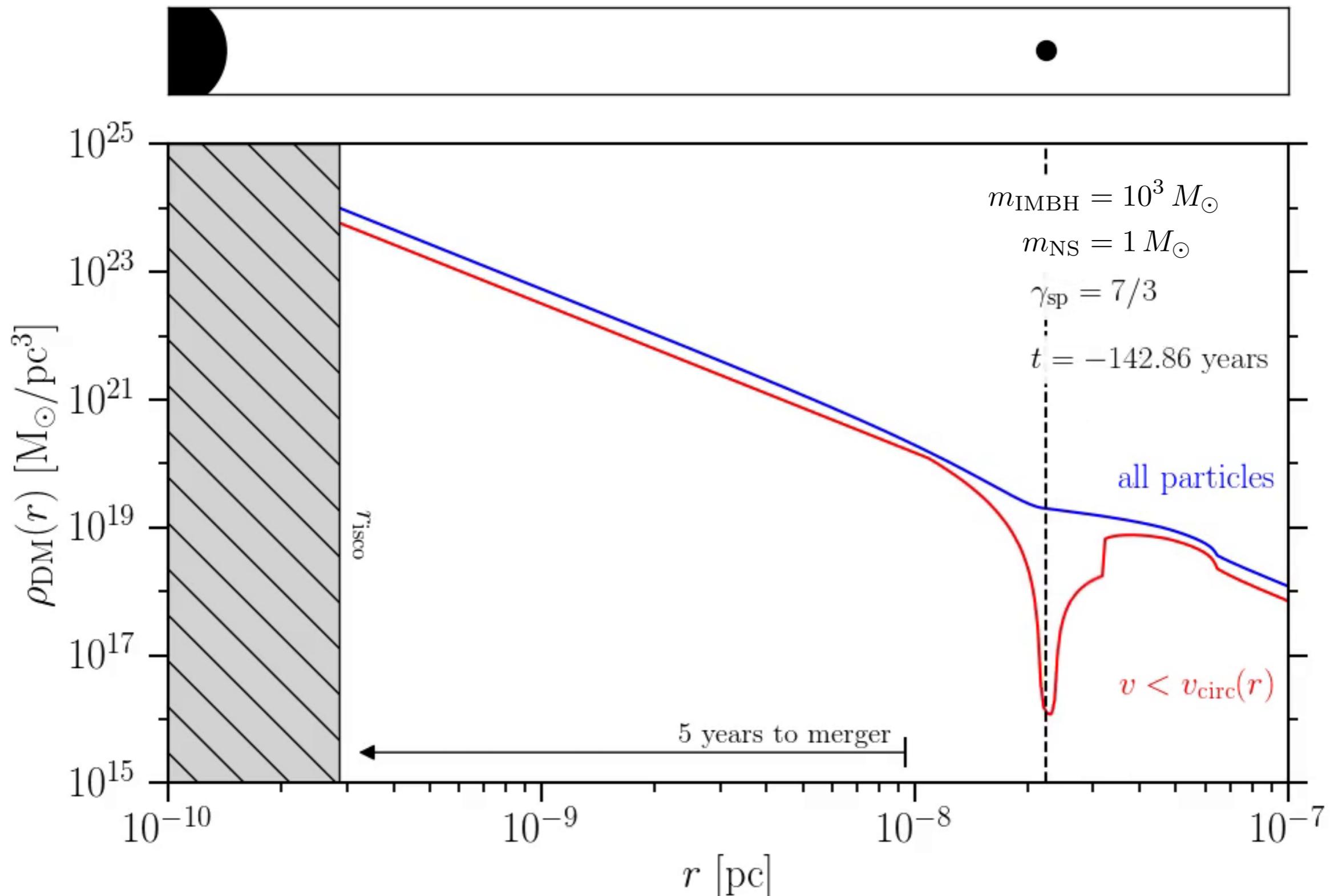
Full evolution of the system

Movies: tinyurl.com/GW4DM



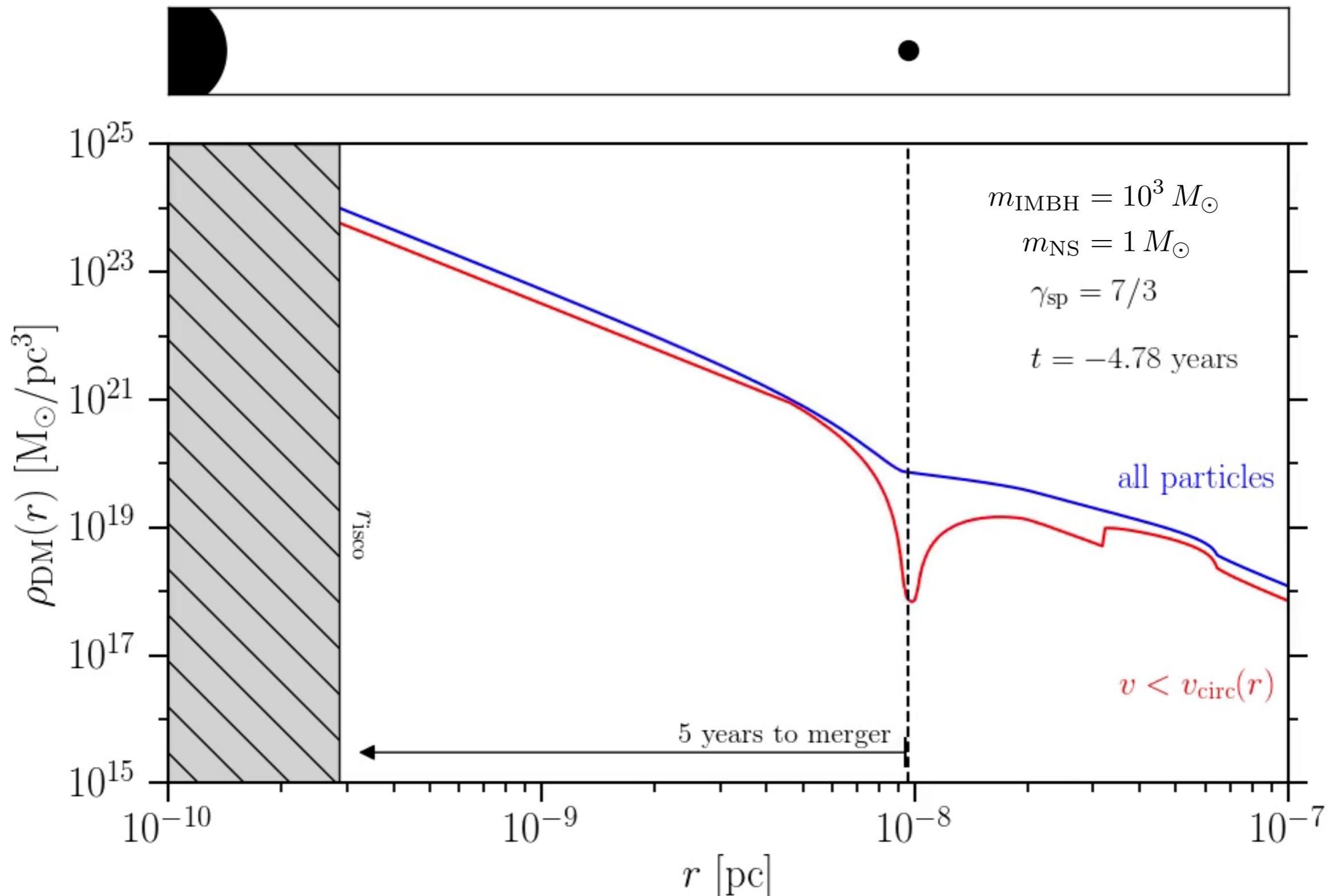
Full evolution of the system

Movies: tinyurl.com/GW4DM



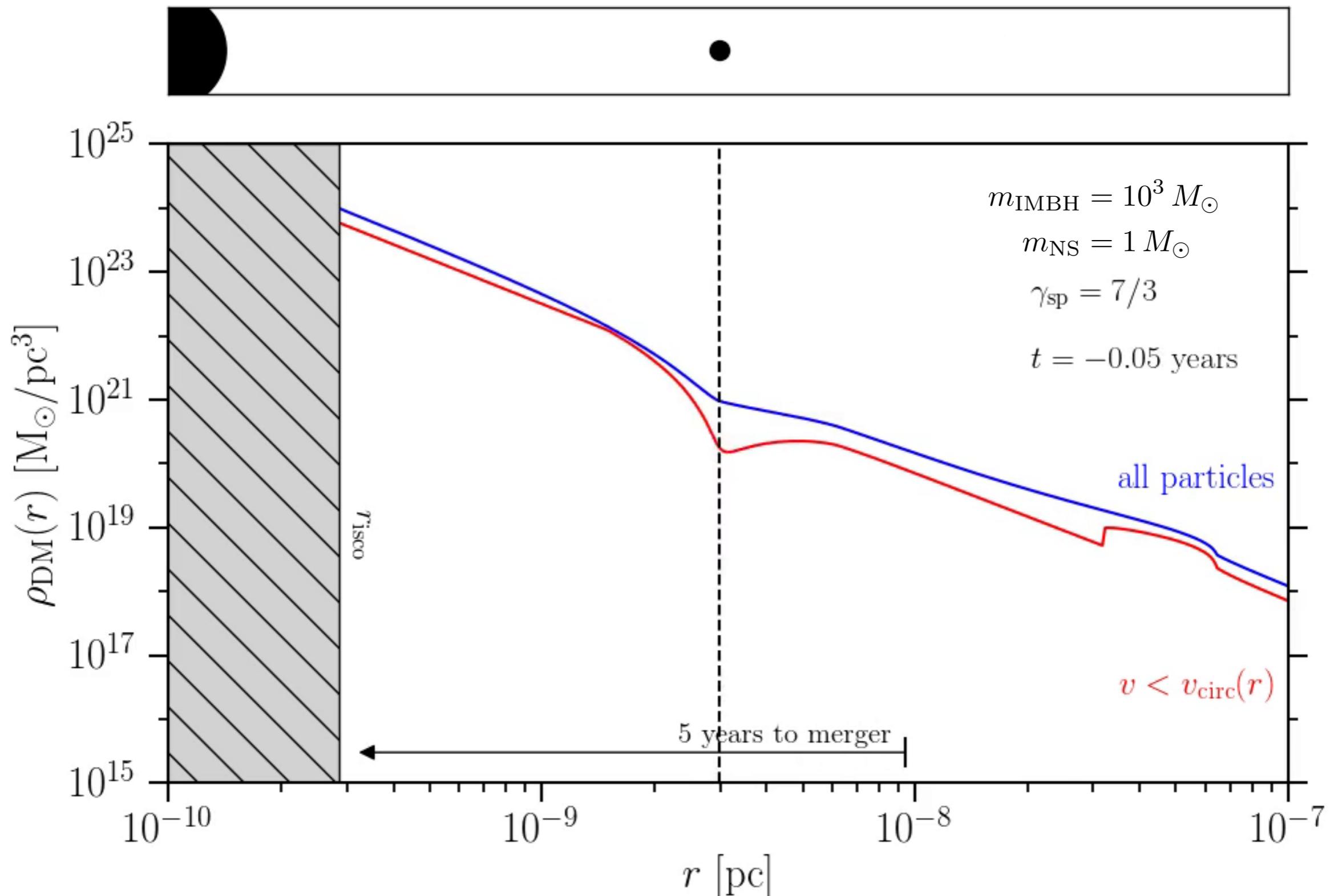
Full evolution of the system

Movies: tinyurl.com/GW4DM



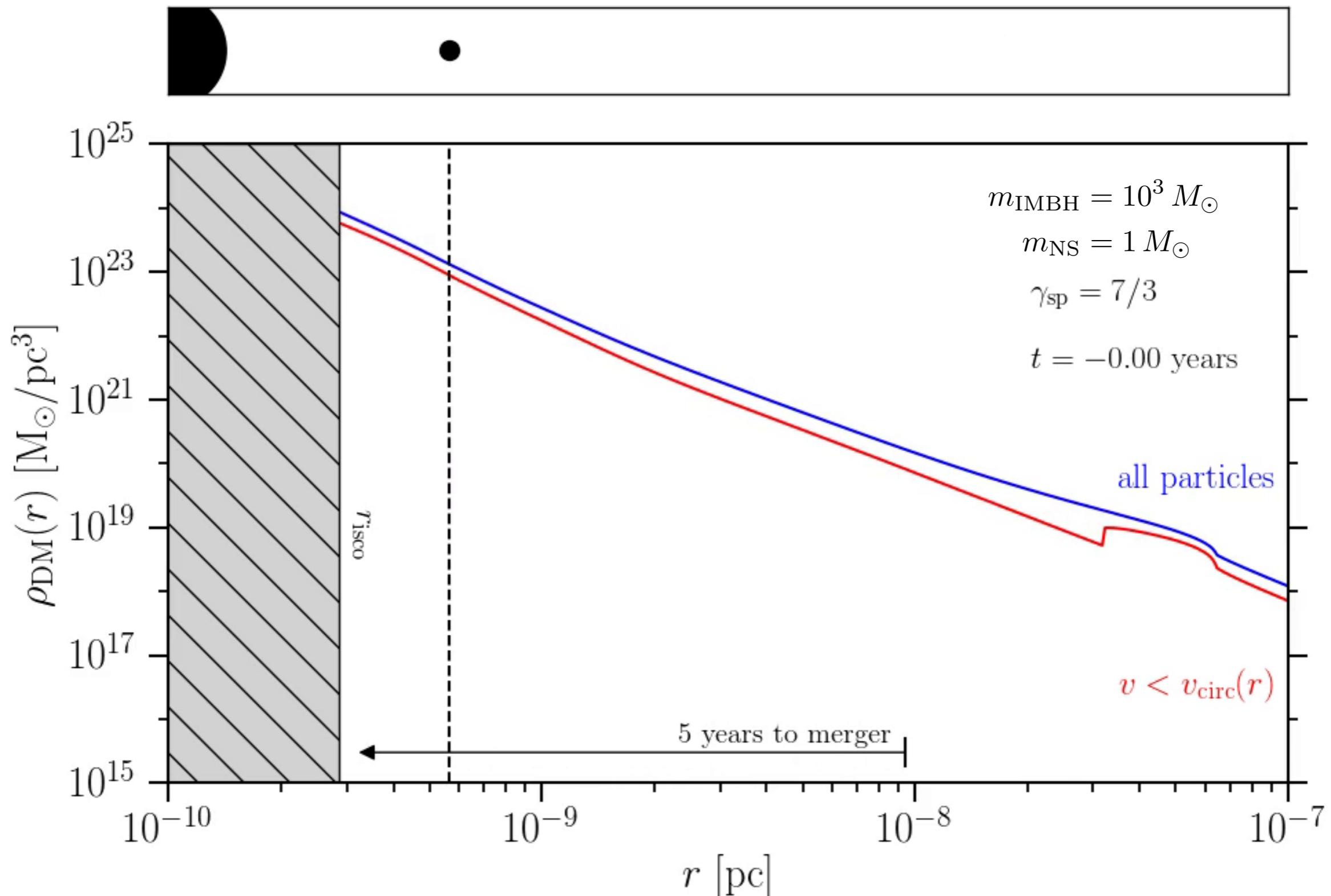
Full evolution of the system

Movies: tinyurl.com/GW4DM

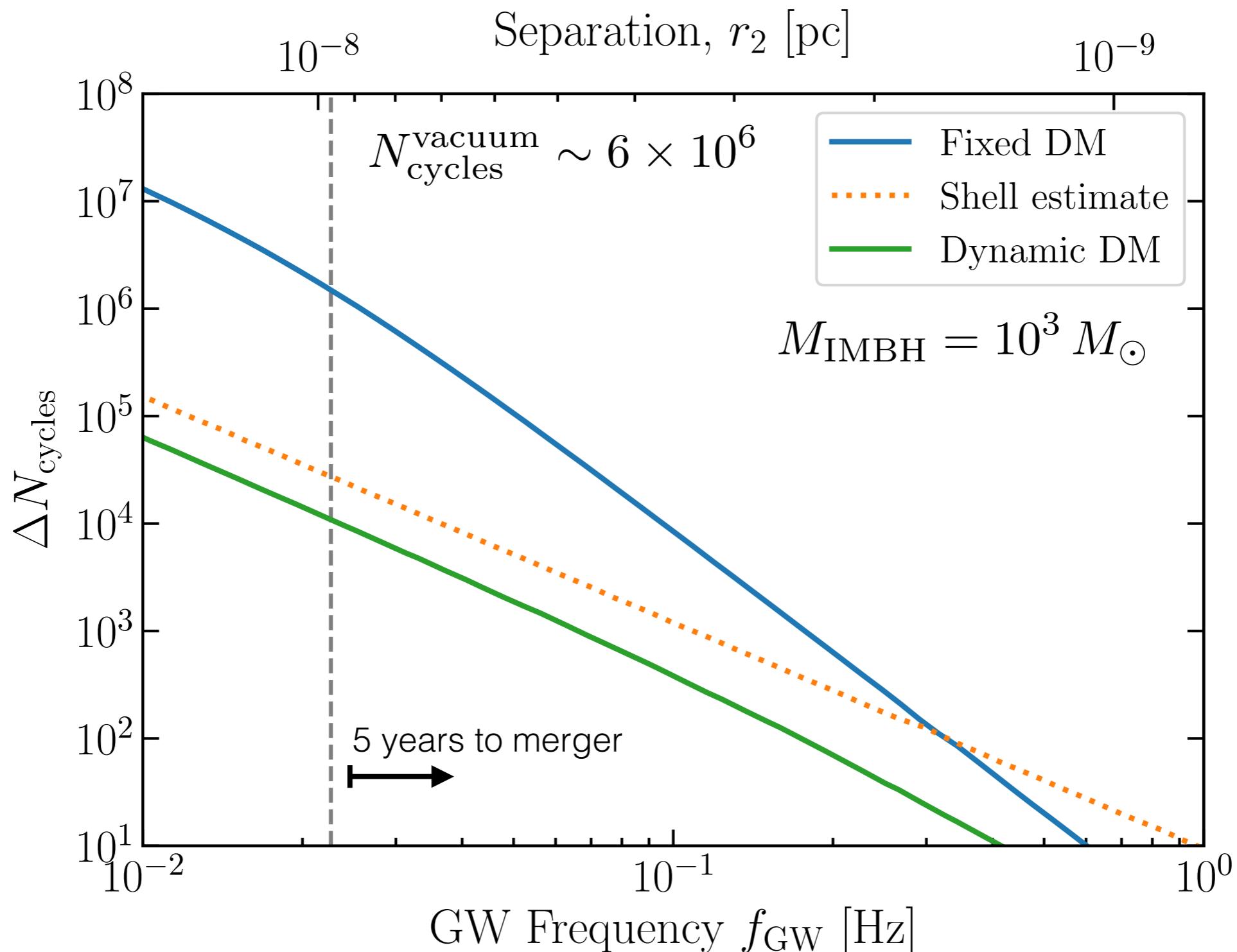


Full evolution of the system

Movies: tinyurl.com/GW4DM



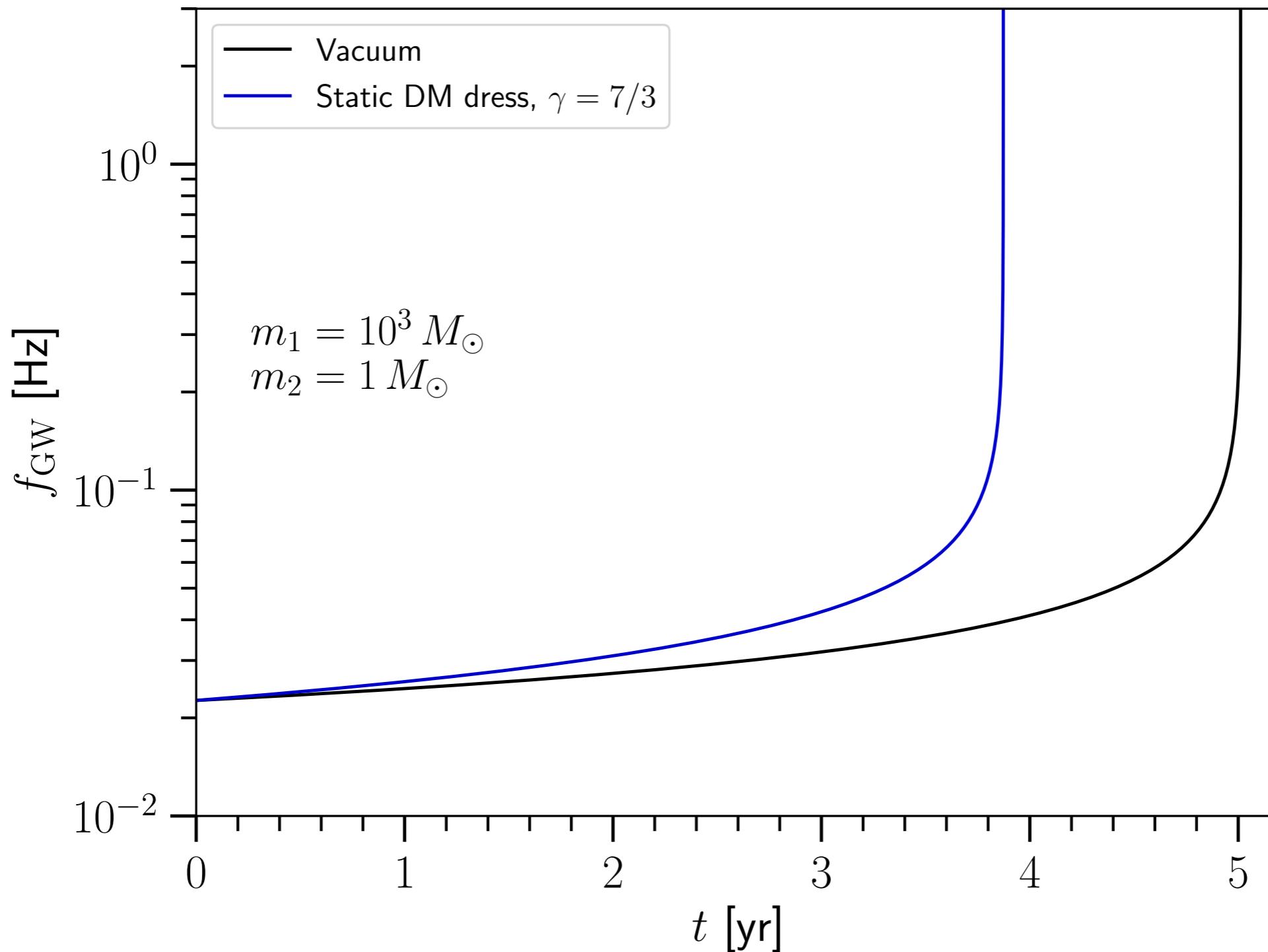
Self-consistent results



$$\Delta N_{\text{cycles}}(\text{static}) \approx 10^6 \rightarrow \Delta N_{\text{cycles}}(\text{dynamic}) \approx 10^4$$

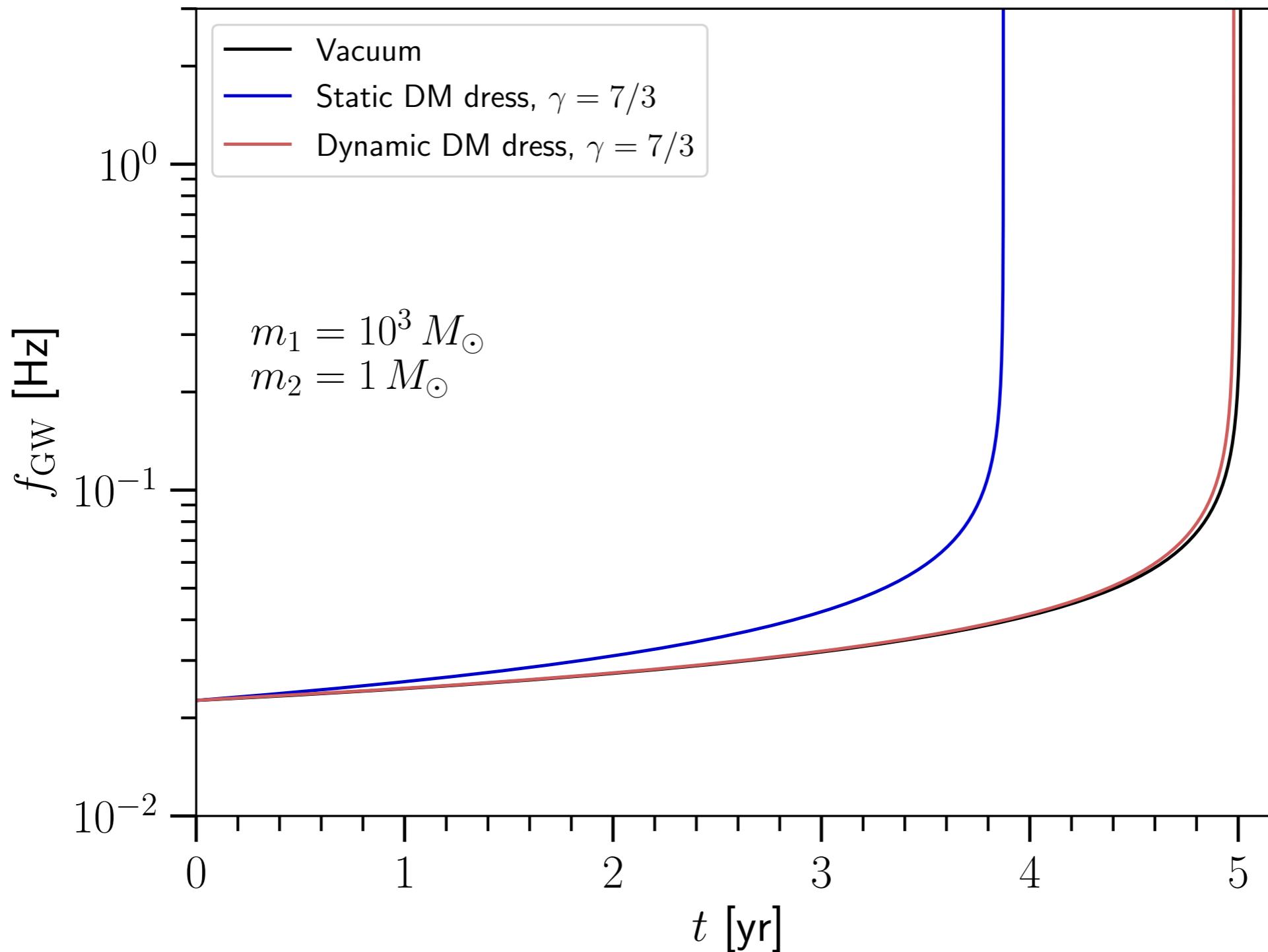
Spectrograms: $m_{\text{IMBH}} = 10^3 M_\odot$

$$\rho_{\text{DM}}(r) = \rho_{\text{sp}} \left(\frac{r_{\text{sp}}}{r} \right)^{\gamma_{\text{sp}}}$$



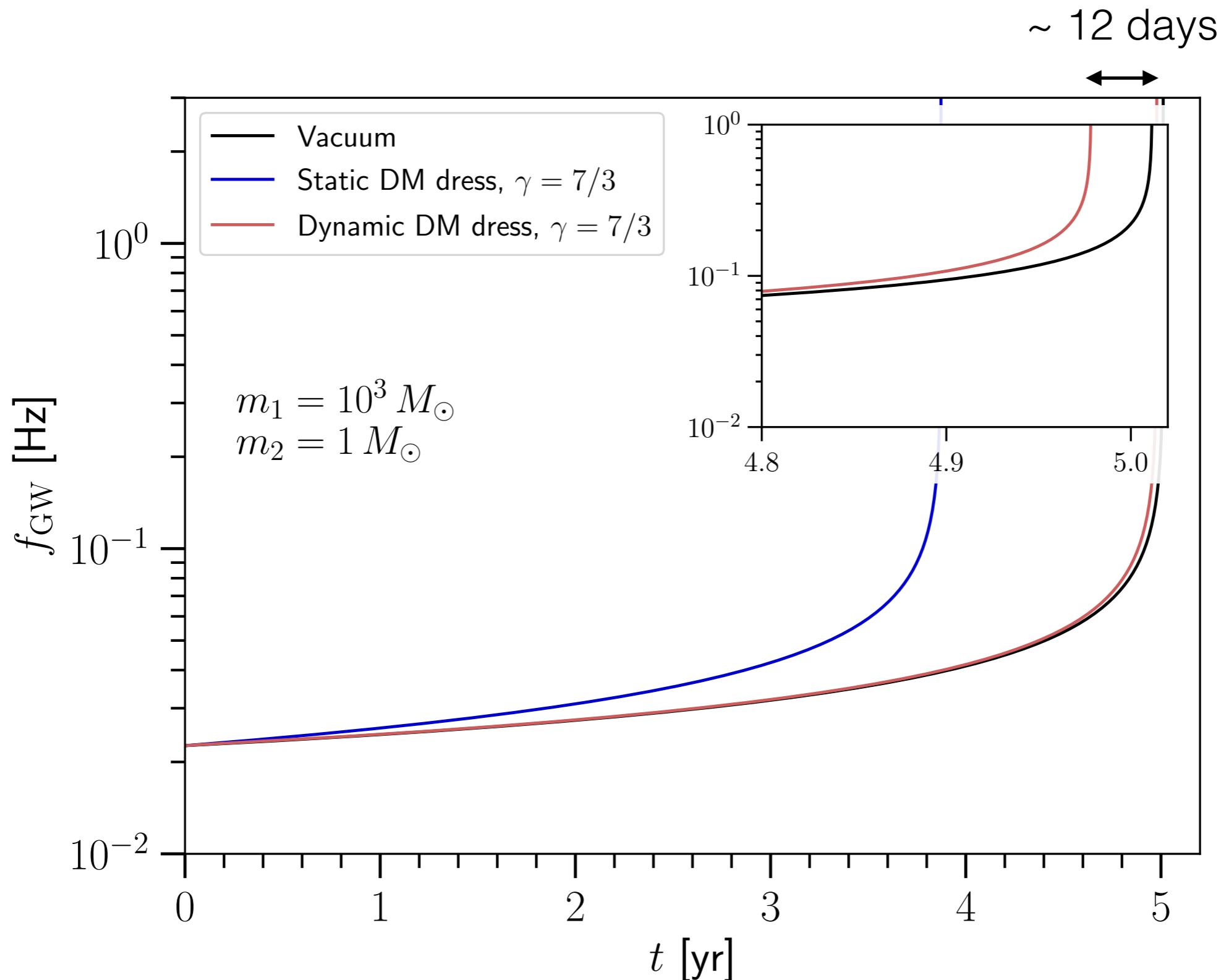
Spectrograms: $m_{\text{IMBH}} = 10^3 M_\odot$

$$\rho_{\text{DM}}(r) = \rho_{\text{sp}} \left(\frac{r_{\text{sp}}}{r} \right)^{\gamma_{\text{sp}}}$$



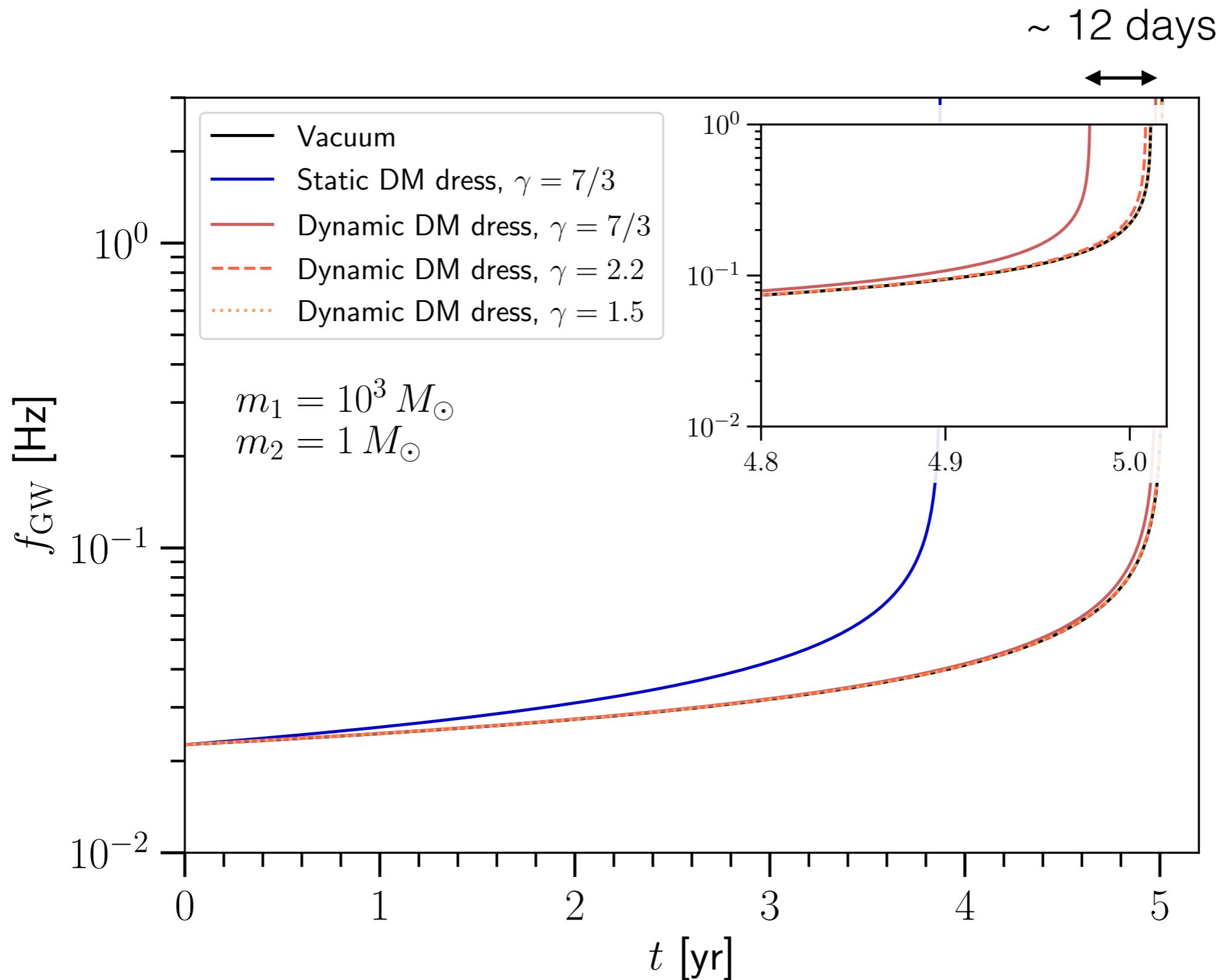
Spectrograms: $m_{\text{IMBH}} = 10^3 M_\odot$

$$\rho_{\text{DM}}(r) = \rho_{\text{sp}} \left(\frac{r_{\text{sp}}}{r} \right)^{\gamma_{\text{sp}}}$$



Spectrograms: $m_{\text{IMBH}} = 10^3 M_\odot$

$$\rho_{\text{DM}}(r) = \rho_{\text{sp}} \left(\frac{r_{\text{sp}}}{r} \right)^{\gamma_{\text{sp}}}$$

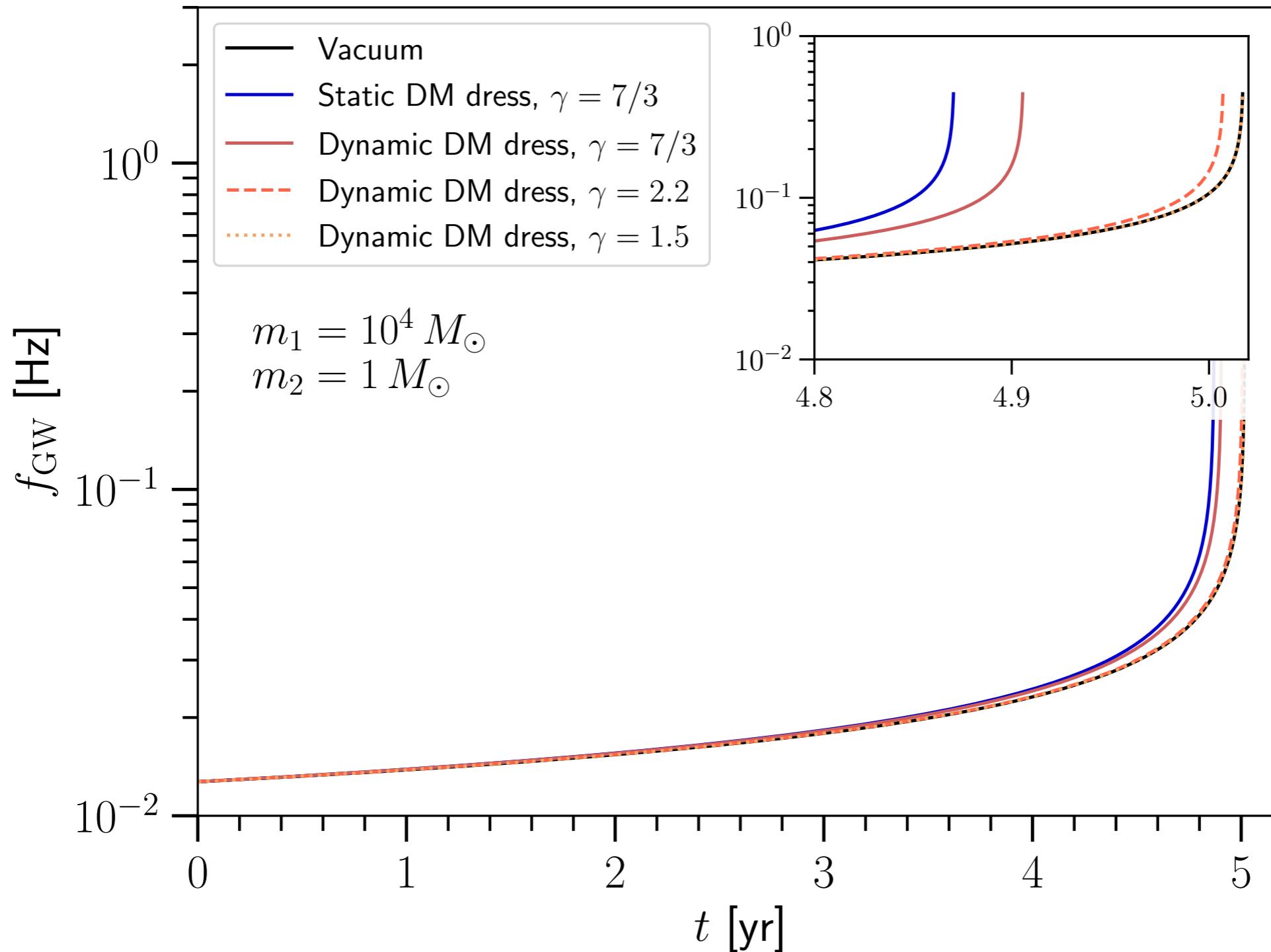


NB: $7/3 \approx 2.333$

Spectrograms: $m_{\text{IMBH}} = 10^4 M_\odot$

$$\rho_{\text{DM}}(r) = \rho_{\text{sp}} \left(\frac{r_{\text{sp}}}{r} \right)^{\gamma_{\text{sp}}}$$

As we increase the IMBH mass, the correction from having a dynamic DM halo decreases (but can still be very relevant)



Detectability

$$\Delta N_{\text{cycles}}(\text{static}) \approx 10^6 \rightarrow \Delta N_{\text{cycles}}(\text{dynamic}) \approx 10^4$$

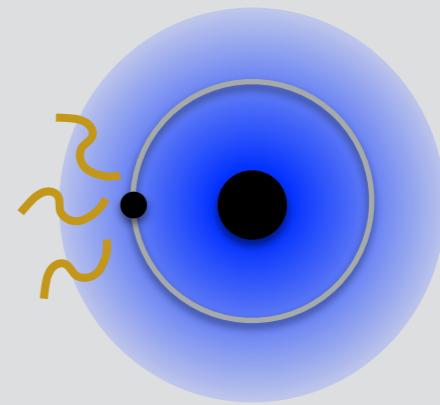
In many systems, a more realistic treatment leads to a huge reduction in the size of the ‘de-phasing’ effect.

Even for very massive BHs, small corrections can spoil our ability to find the signal in data (so they must be accounted for)

For a 1000 or 10000 solar mass IMBH, a de-phasing of ~10,000 cycles could still be detectable.

GW + EM signals of QCD axion Dark Matter

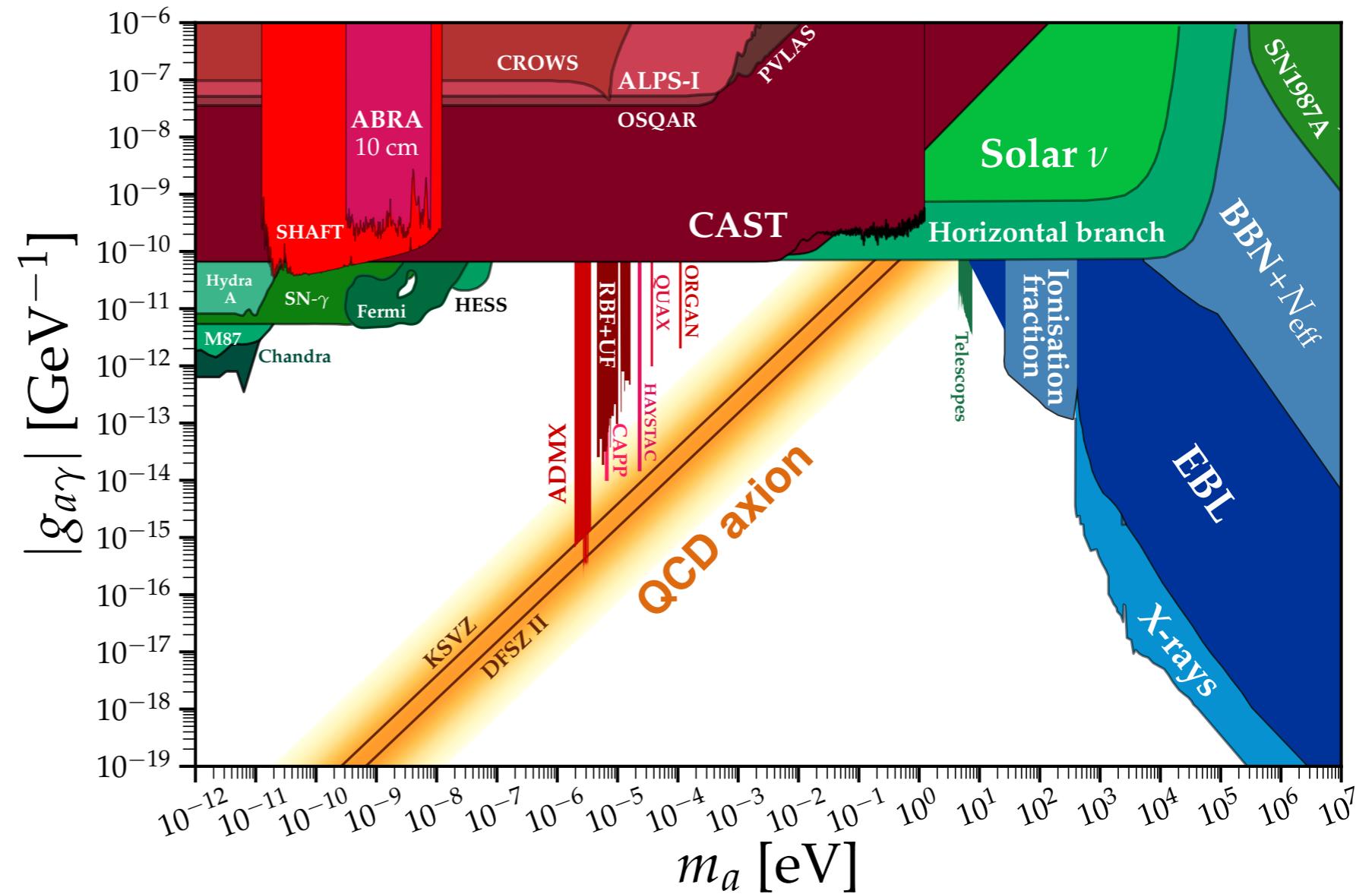
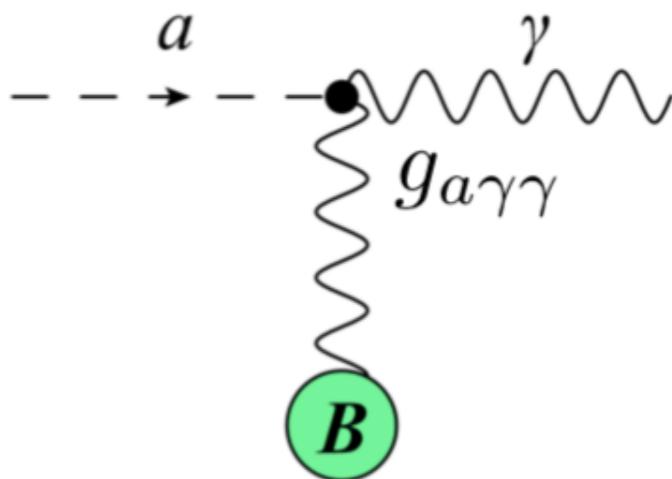
[Edwards, Chianese, **BJK**, Nissanke & Weniger,
Phys. Rev. Lett. 124, 161101, 1905.04686]



Axion-photon Conversion

Axions can convert to photons (and vice versa) in an external magnetic field:

$$\begin{aligned}\mathcal{L} &\supset -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} \\ &= -\frac{1}{4} g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B}\end{aligned}$$



[O'Hare, <https://cajohare.github.io/AxionLimits/>]

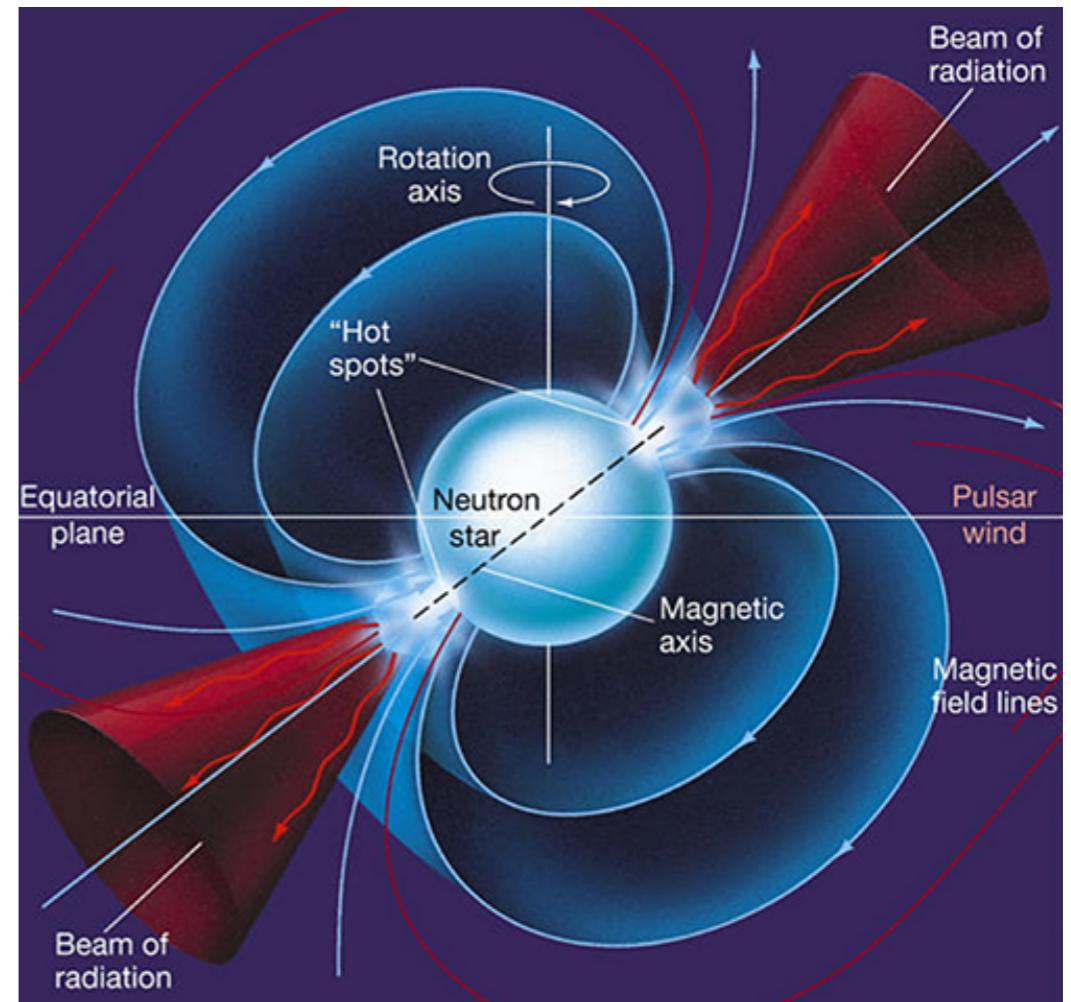
Axions and neutron stars

Old neutron stars can have extremely high magnetic fields:

$$B_0 = 10^{12} - 10^{15} \text{ G}$$

Surrounded by a dense plasma which allows 'resonant' conversion when axion mass matches plasma mass:

$$\omega_p(B_0, P) = m_a/2\pi$$



© 2005 Pearson Prentice Hall, Inc

Magnetic field and plasma frequency varies as a function of radius from NS; NS can effectively 'scan' over a range of axion masses.

[[1803.08230](#), [1804.03145](#), [1811.01020](#), [1910.11907](#)]

Axions and neutron stars

Produce a photon with axion energy $m_a \sim 10^{-6} \text{ eV} \sim 240 \text{ MHz}$



Conversion happens at a radius r_c , with probability: $p_{a\gamma} \propto \frac{g_{a\gamma\gamma}^2 B(r_c)^2}{2v_c}$

Radiated power is given by: $\frac{d\mathcal{P}}{d\Omega} \sim 2 \times p_{a\gamma} \rho_{\text{DM}}(r_c) v_c r_c^2$

Probe axions in the mass range

$$m_a \sim 10^{-7} \text{ eV} \quad \text{up to} \quad m_a \sim 10^{-5} \text{ eV}$$

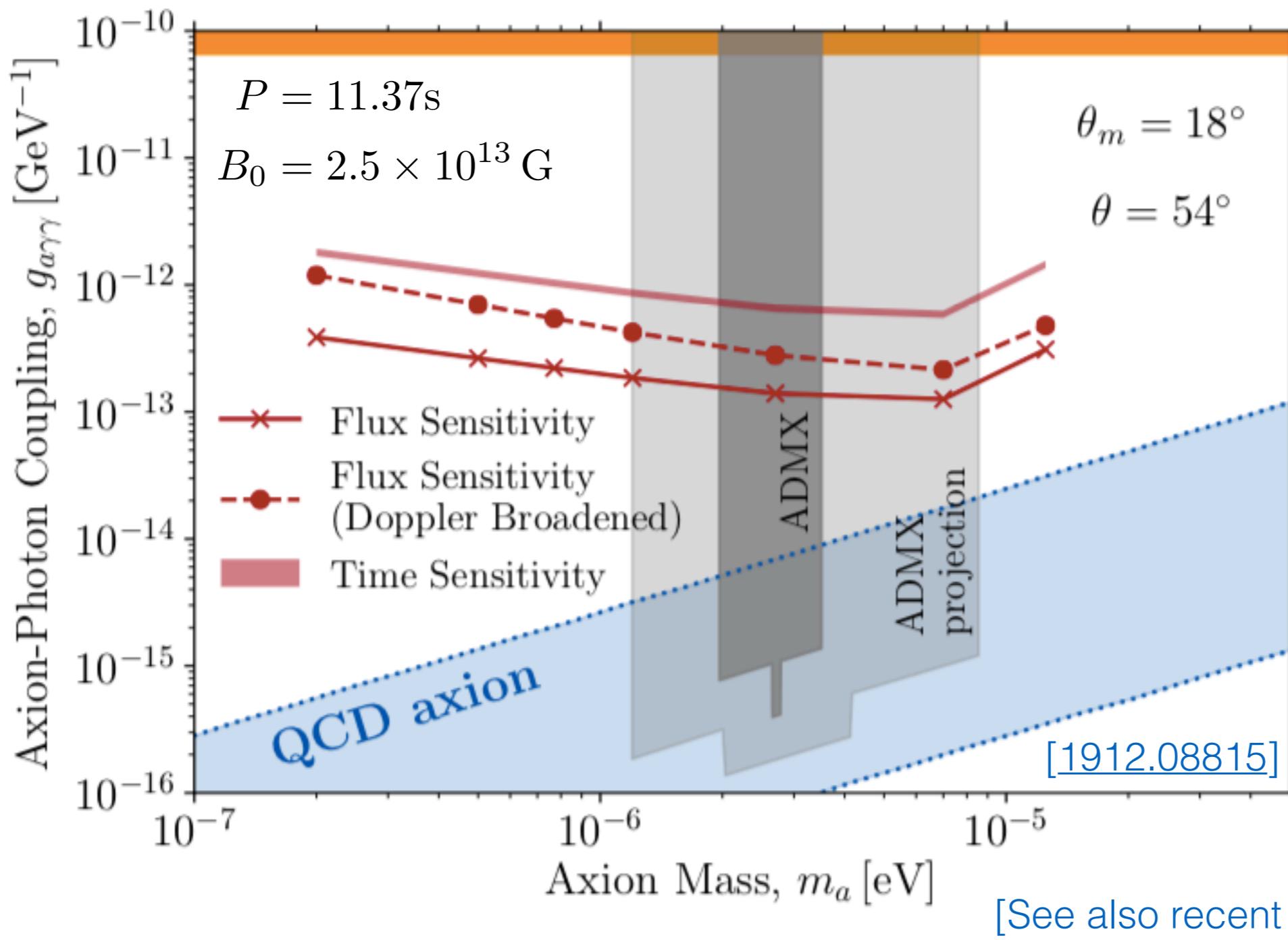
Frequency range of
radio telescopes

Require conversion
outside NS

[\[1803.08230\]](#), [\[1804.03145\]](#), [\[1811.01020\]](#), [\[1910.11907\]](#)

Axion coupling sensitivity

Consider a single isolated NS (J0806.4-412).
How strong does the coupling have to be to
give a detectable radio signal in SKA?

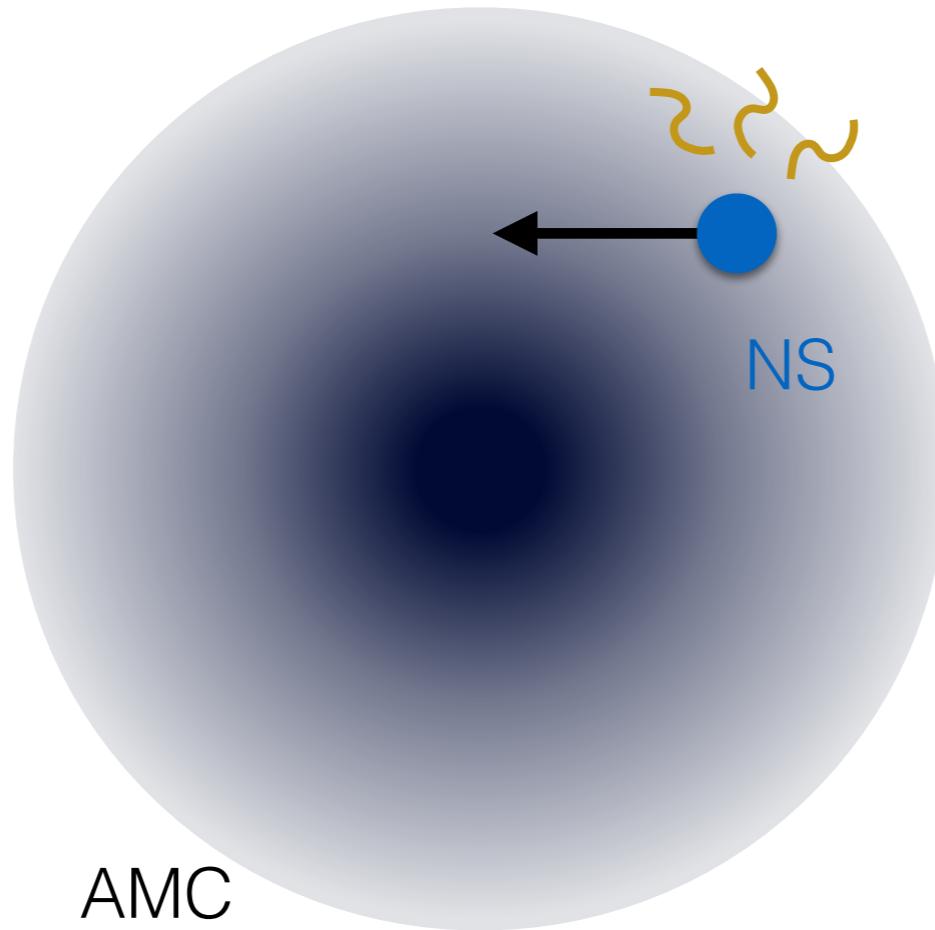


Axion Miniclusters & NSs

Increased DM density means increased signal:

Axions can form ‘miniclusters’ - diffuse substructures which survive until today.

[[Hogan & Rees \(1988\)](#), [hep-ph/9303313](#), [astro-ph/9311037](#)]

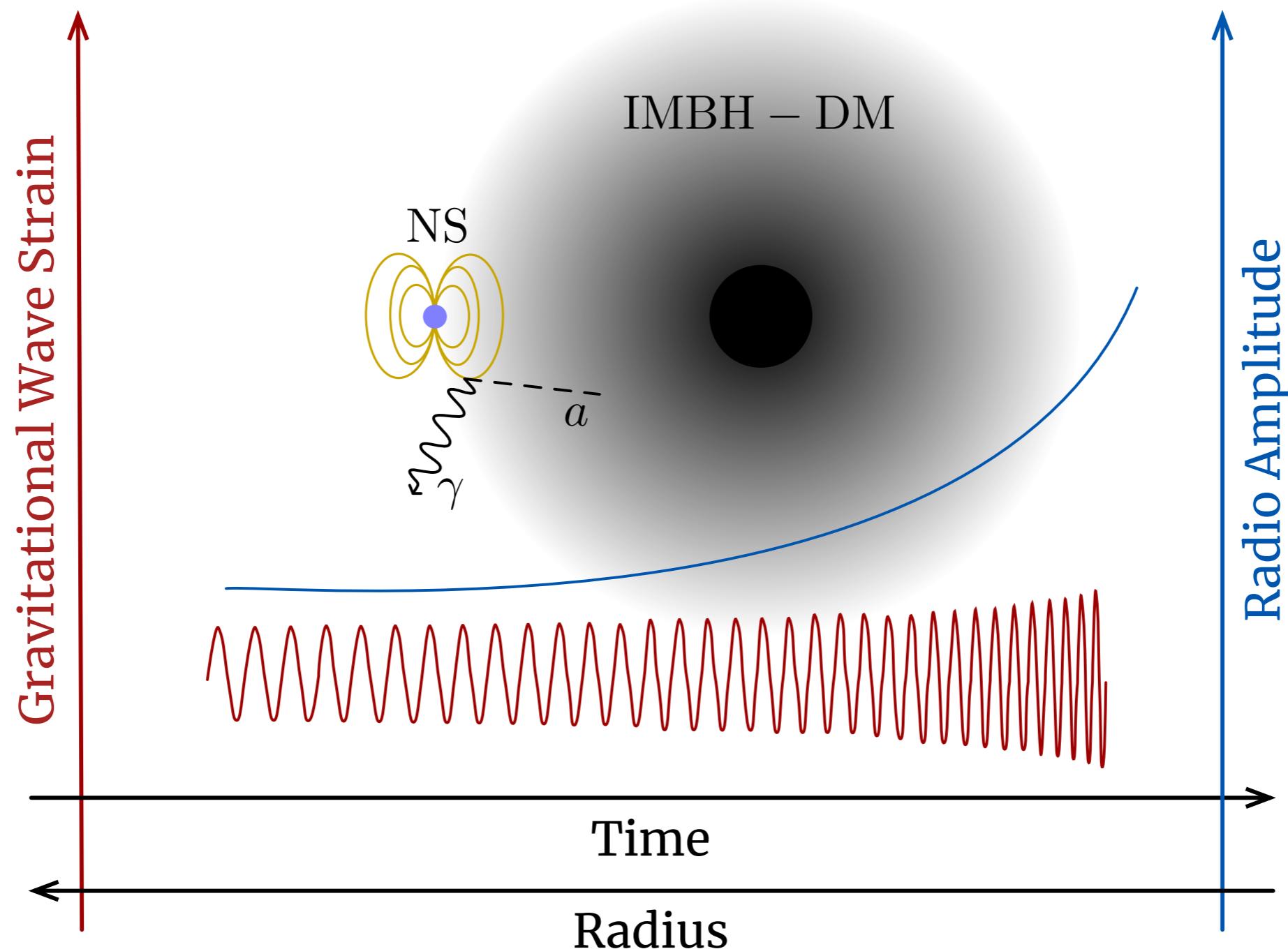


[Work in progress with Edwards, Visinelli & Weniger]

[See also [2004.06486](#)]

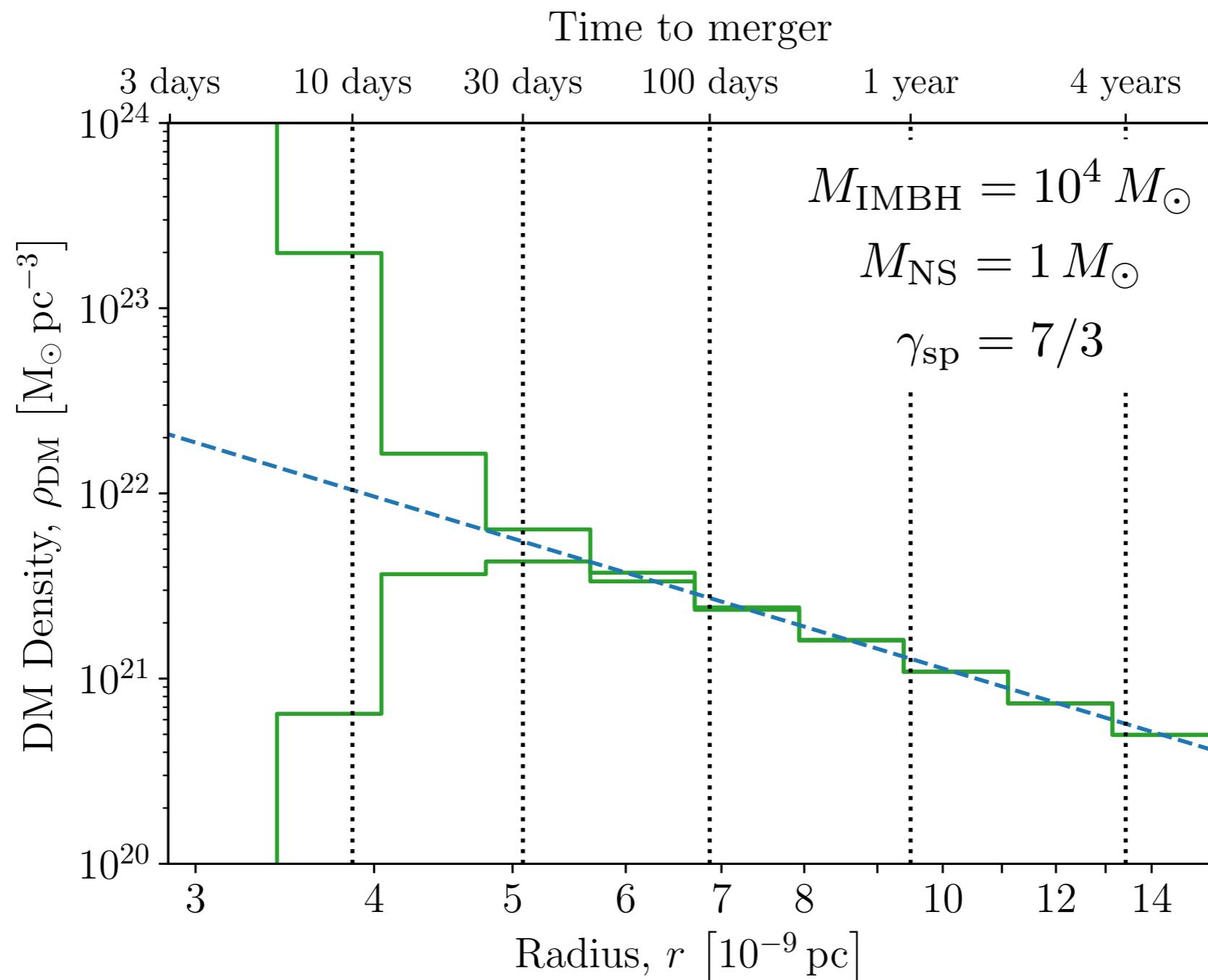
*But what about an even
more dense cloud?*

A unique signature



[Edwards, Chianese, **BJK**, Nissanke & Weniger, [1905.04686](#)]

Density reconstruction

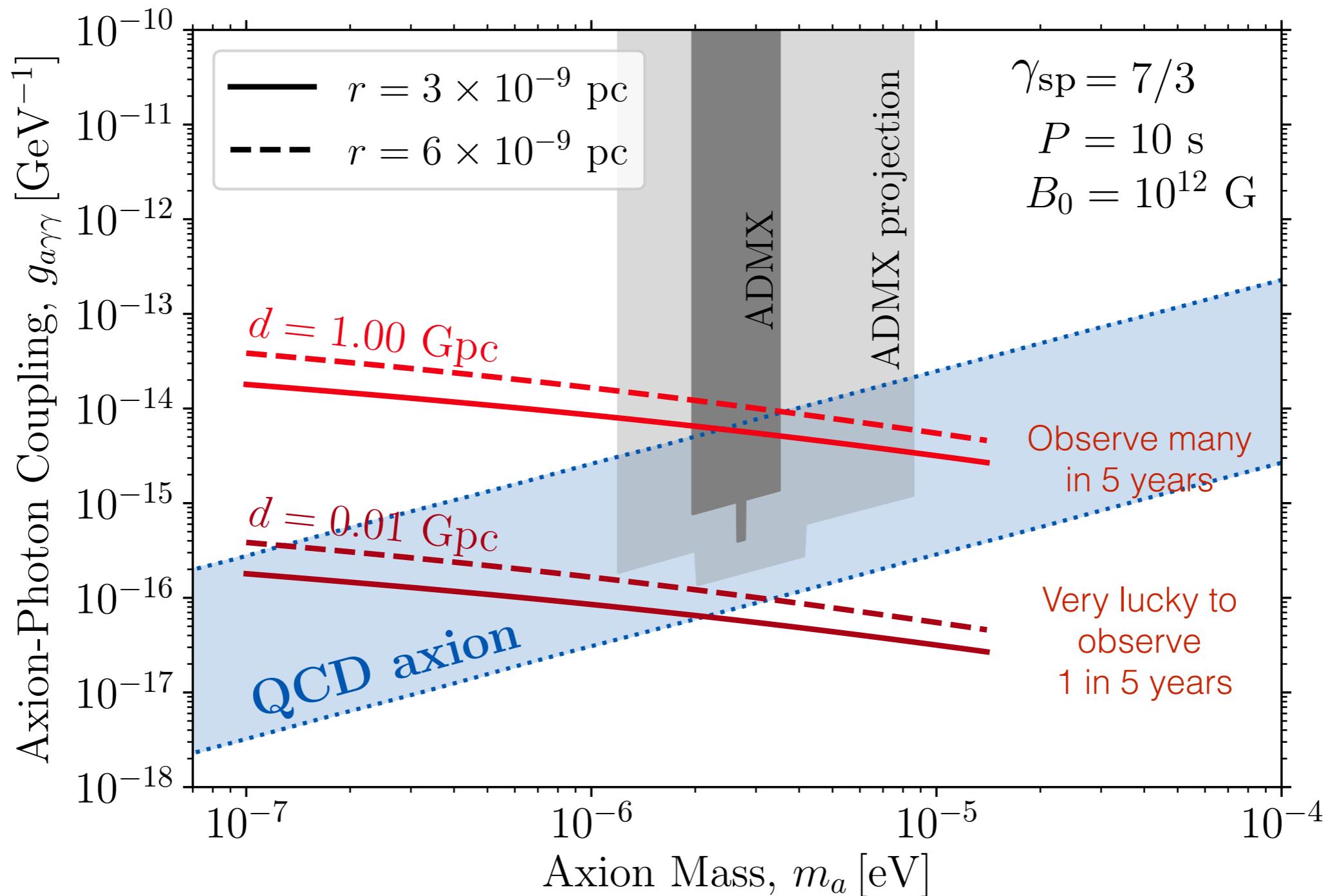


$$\frac{d\mathcal{P}}{d\Omega} \sim 2 \times p_{a\gamma} \rho_{\text{DM}}(r_c) v_c r_c^2$$

$$p_{a\gamma} \propto \frac{g_{a\gamma\gamma}^2 B(r_c)^2}{2v_c}$$

QCD Axion Reach

SKA should be able to probe QCD axion DM in the range $10^{-7} - 10^{-5}$ eV.



[Edwards, Chianese, **BJK**, Nissanke & Weniger, [1905.04686](#)]

Promising Signal

Dark Matter de-phasing is a very exciting signal. It's on long timescales (with LISA planned for 2030s), but it would allow us to:

Detect Dark Matter in
Gravitational waves

[**BJK**, Nichols, Gaggero, Bertone, [2002.12811](#)]

Probe the nature of Dark Matter

[\[1906.11845\]](#)

Predict EM signals from Dark Matter

[Edwards, Chianese, **BJK**, Nissanke
& Weniger, [1905.04686](#)]

But it can be very difficult to extract from the noise, and must be modelled very carefully...

Plans for the future

Improved modelling

- Injection and evolution of angular momentum in the DM halo
- Post-Newtonian corrections
- Better N-body approaches [[AMUSE?](#)]

Detection methods

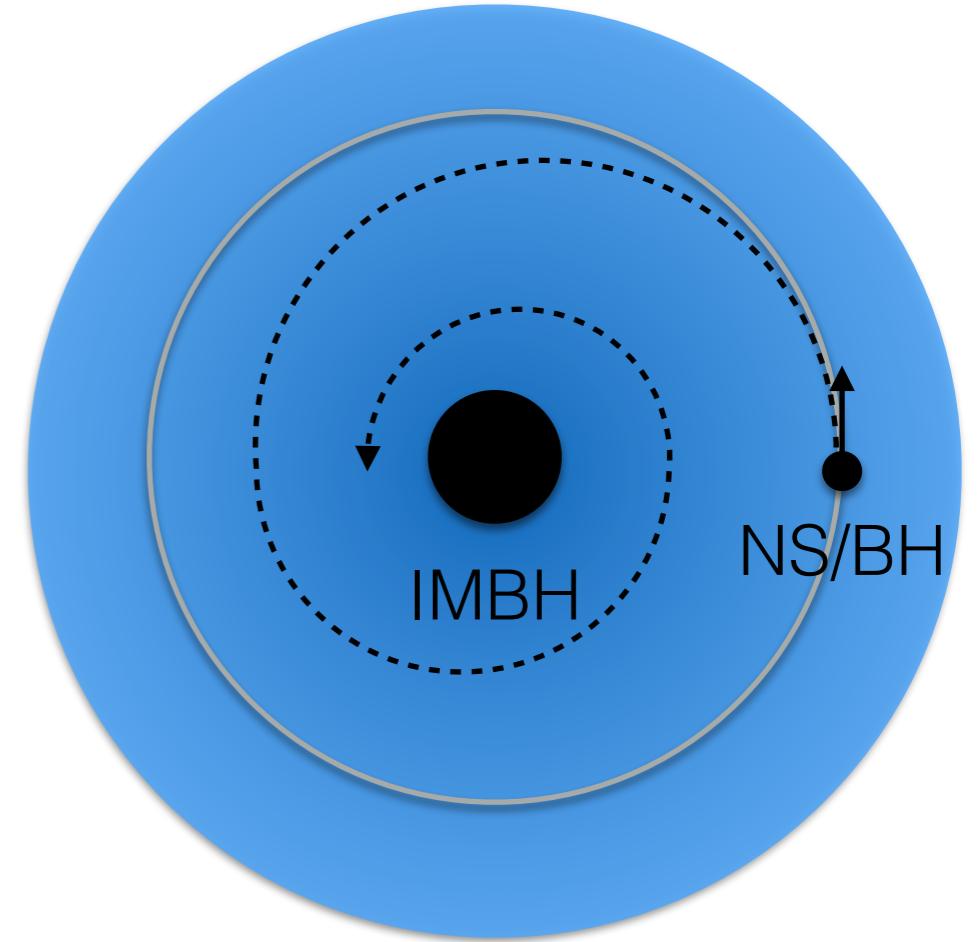
- Producing template banks for LISA searches
- Incoherent searches for continuous GWs
- ‘General’ de-phased waveform templates [[2004.06729](#)]

Detection prospects

- How many IMRI systems form? How many with NSs?
- How many systems have a (surviving) spike?
- Prospects for observing a bright enough radio signal

Conclusions

Dark Matter ‘de-phasing’ is an extremely promising GW signature, which needs to be **modelled carefully**



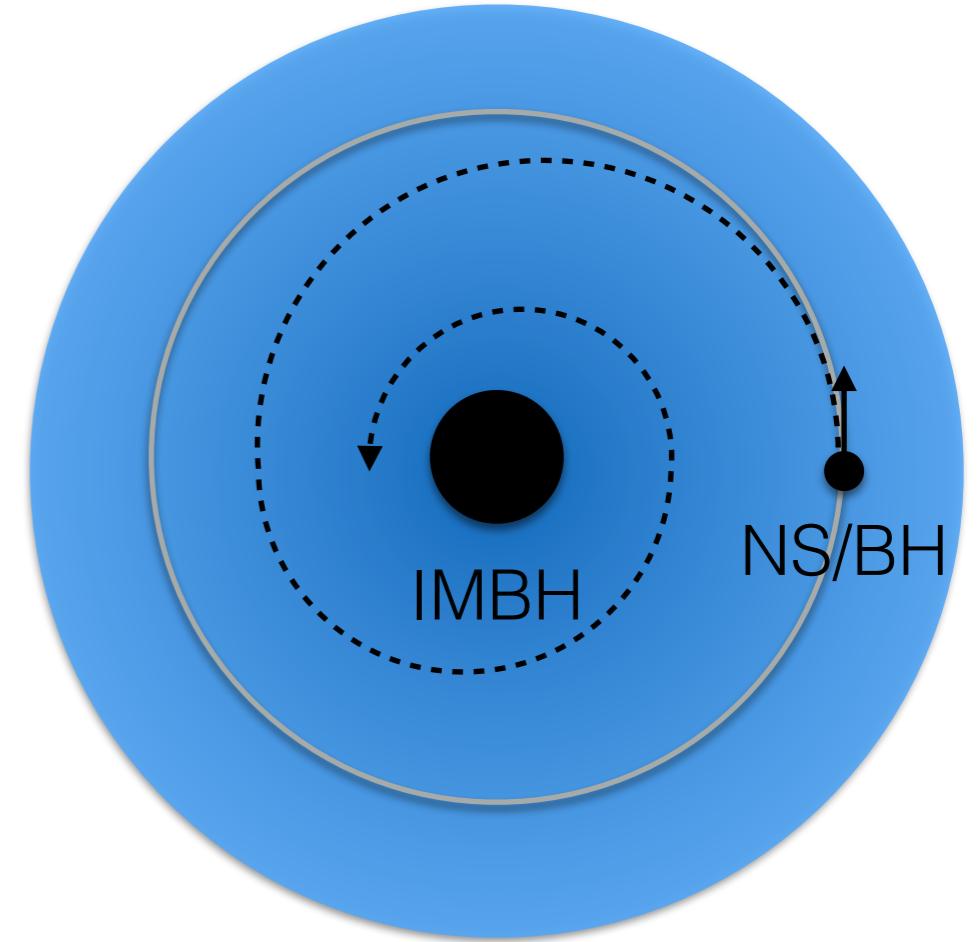
For light IMBHs, the **correction due to DM halo feedback** can be huge!

More calculations are needed to build template banks, assess detectability and parameter reconstruction

This work could pave the way towards a **multi-messenger detection** of (Axion?) Dark Matter

Conclusions

Dark Matter ‘de-phasing’ is an extremely promising GW signature, which needs to be **modelled carefully**



For light IMBHs, the **correction due to DM halo feedback** can be huge!

More calculations are needed to build template banks, assess detectability and parameter reconstruction

This work could pave the way towards a **multi-messenger detection** of (Axion?) Dark Matter

Thank you!

Backup Slides

Parameter Reconstruction

Prospects for parameter reconstruction in the *static DM* case:

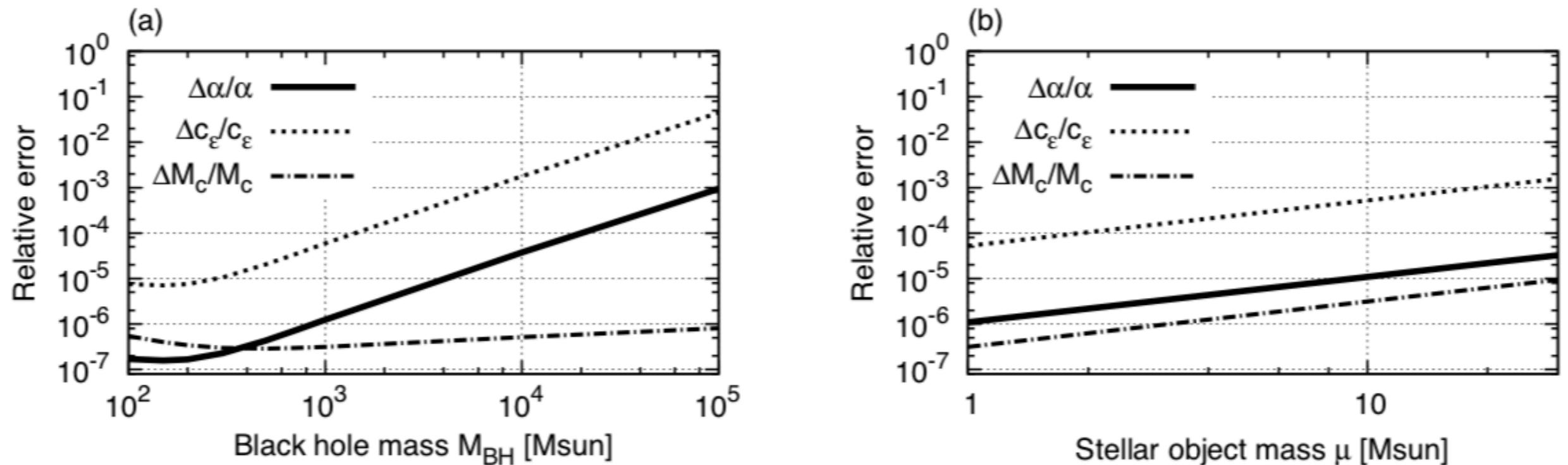


FIG. 4: The relative errors of the parameters in the phase $\tilde{\Phi}(f)$ versus (a) the central BH mass M_{BH} and (b) the stellar mass object mass μ for $S/N = 10$ and $\alpha = 7/3$. For this plot, ρ_{sp} and r_{sp} are taken from the table I. The other parameter is fixed to be $\mu = 1M_\odot$ in the left and $M_{\text{BH}} = 10^3 M_\odot$ in the right, respectively. Note that the both axes are in the logarithmic scales. The solid line, the dashed line, the dashed-dotted line correspond to $\Delta\alpha/\alpha$, $\Delta c_\varepsilon/c_\varepsilon$, $\Delta M_c/M_c$ respectively.

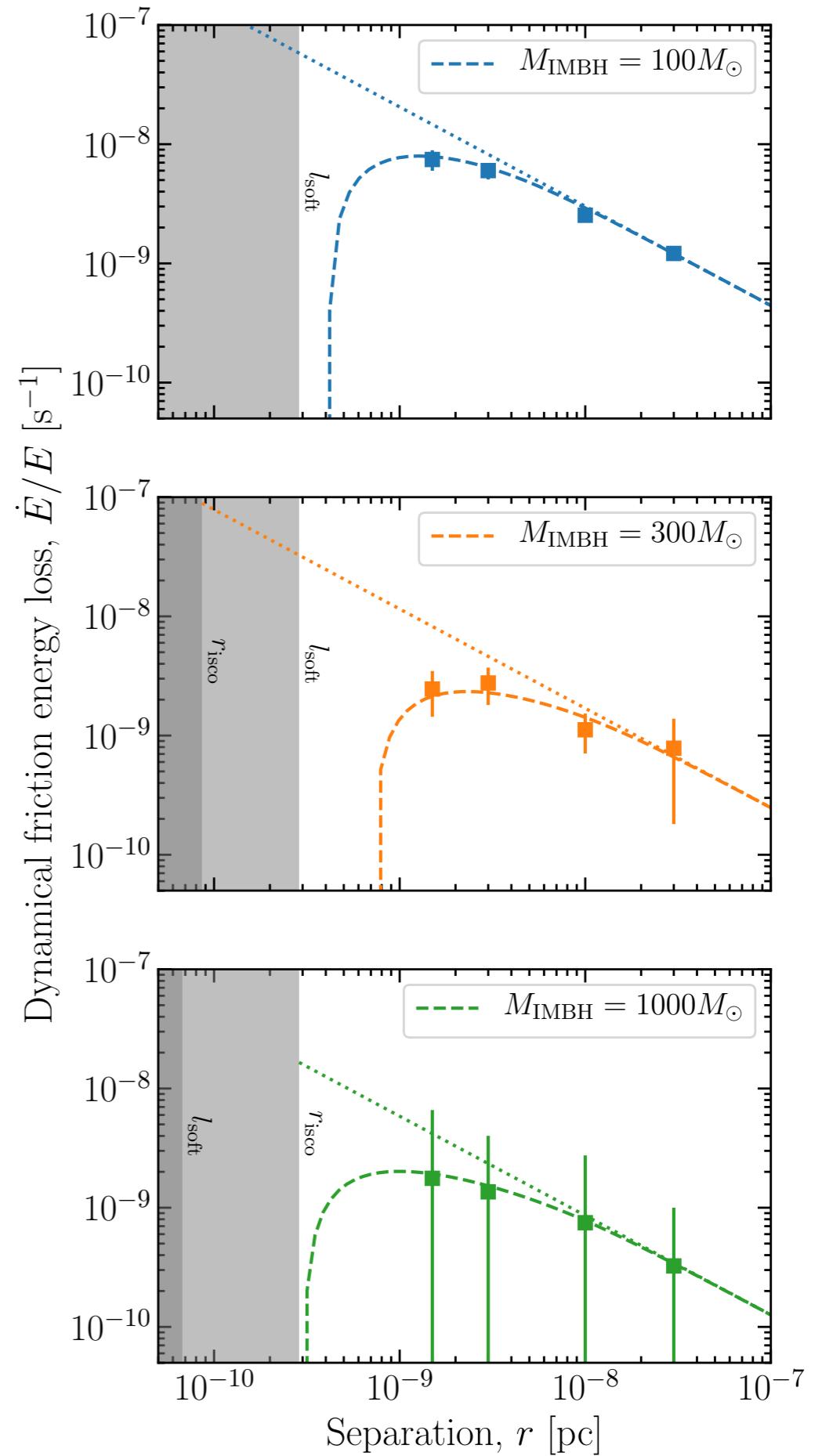
[Eda et al. 1301.5971, 1408.3534]

N-body results

Dependence of dynamical friction force on mass and separation matches expectations

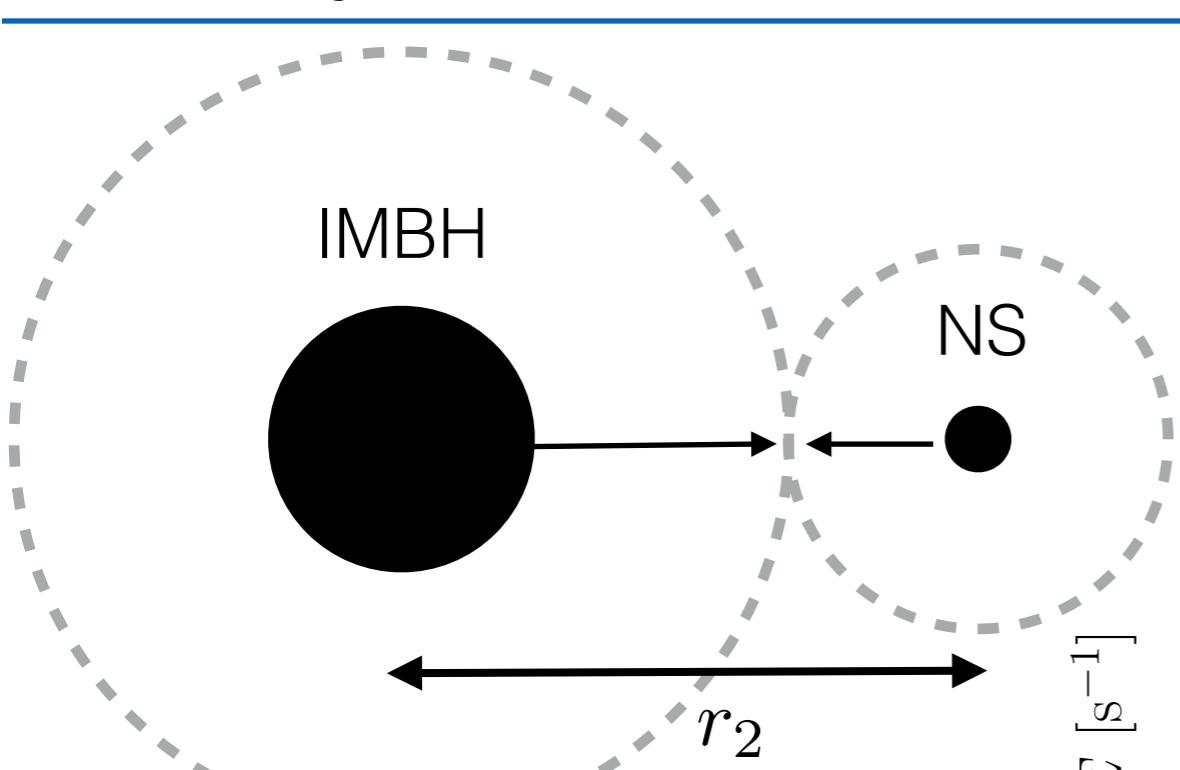
Dynamical friction traces local DM density (to better than 1%)

Drop off in DF force at small separations due to softening of simulations



N-body results

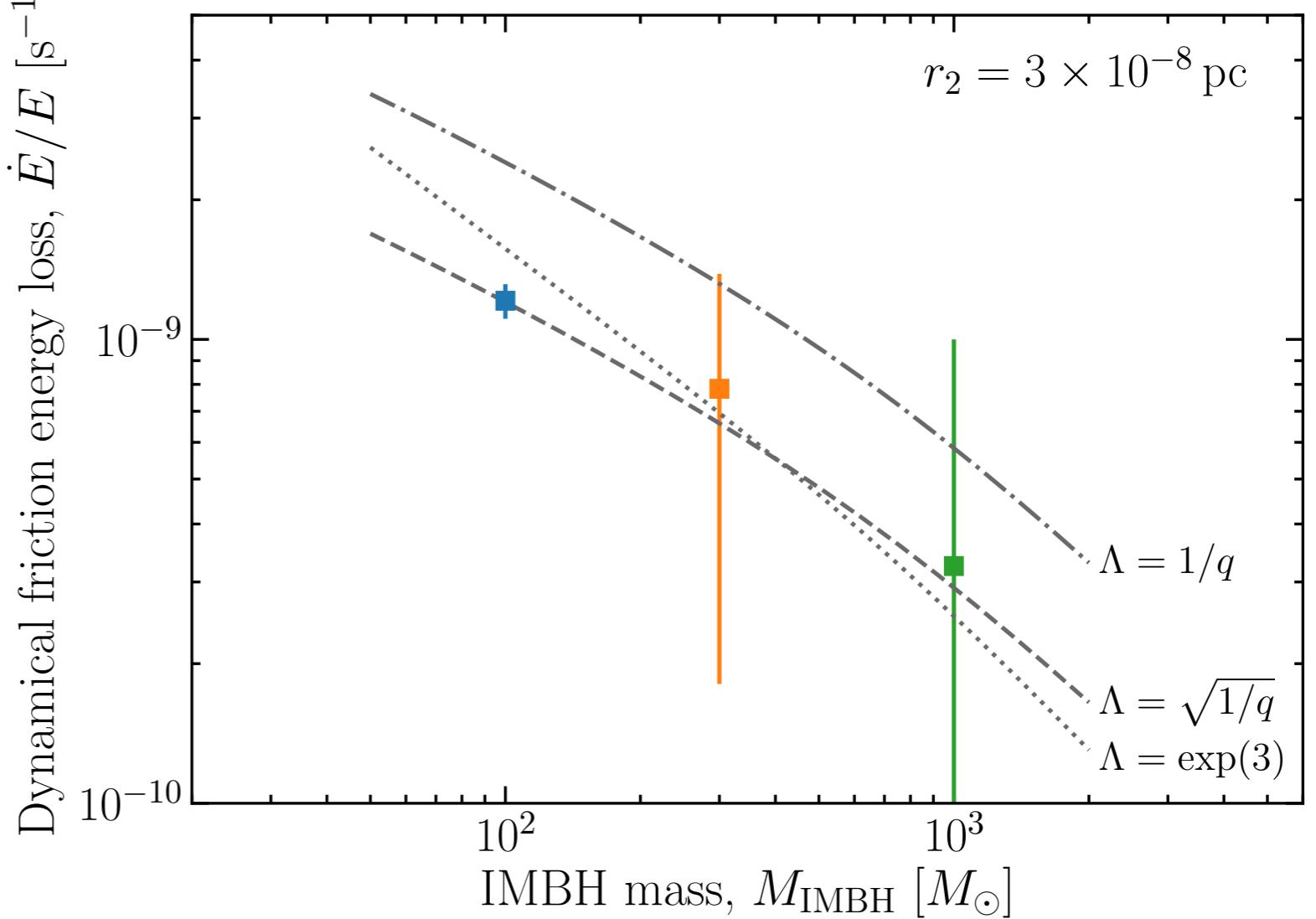
$$q \equiv m_{\text{NS}}/m_{\text{IMBH}} \ll 1$$



$$\begin{aligned}\Lambda &= b_{\max} \frac{v_0^2}{G m_{\text{NS}}} \\ &= \frac{b_{\max}}{q r_2} \\ &= 1/\sqrt{q}\end{aligned}$$

Allows us to calibrate the maximum impact parameter; tells us which particles scatter with the NS.

$$b_{\max} = \sqrt{q} r_2 \sim 3\% r_2$$



Assumptions

- Spherical symmetry and isotropy of the DM halo
- DM particles only scatter within an impact parameter

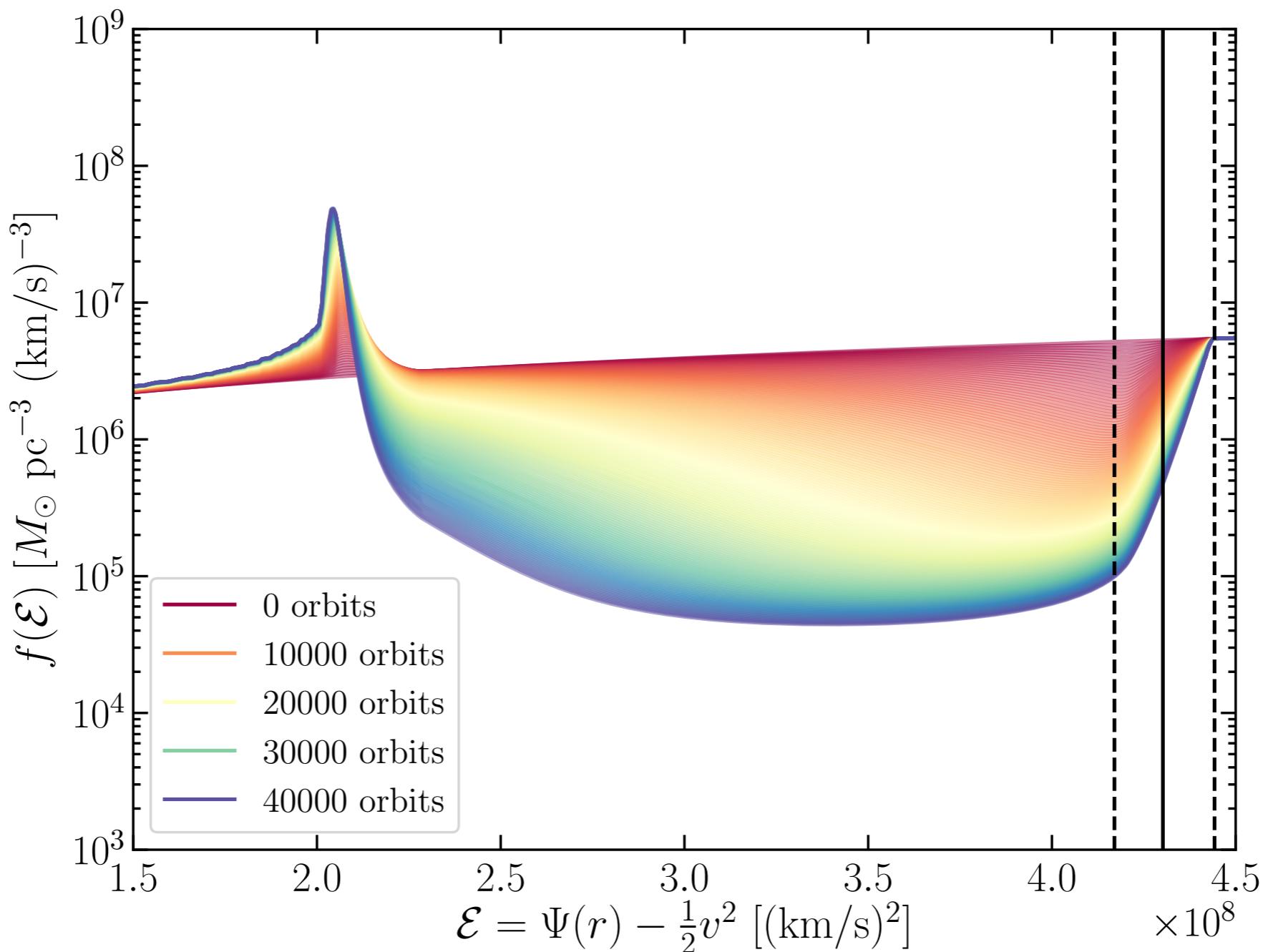
$$b < b_{\max} = \Lambda \times G_N M_{\text{NS}} / v_{\text{NS}}^2$$

- DM distribution is ‘locally’ uniform

$$b_{\max} \ll r_0$$

- Halo ‘relaxation’ is instantaneous
- Orbital properties evolve slowly compared to the orbital period

Distribution function



Self-consistently reconstruct density from distribution function:

$$\rho(r) = 4\pi \int_0^{v_{\max}(r)} v^2 f(\mathcal{E}) dv$$

Numbers of cycles

$$m_1 = 10^3 M_\odot, N_{\text{cycles}} = 5.71 \times 10^6 \text{ in vacuum}$$

	$\gamma_{\text{sp}} = 1.5$	$\gamma_{\text{sp}} = 2.2$	$\gamma_{\text{sp}} = 2.3$	$\gamma_{\text{sp}} = 2.\bar{3}$
Static	< 1	2.4×10^4	1.6×10^5	2.9×10^5
Dynamic	< 1	2.7×10^2	1.9×10^3	3.5×10^3

$$m_1 = 10^4 M_\odot, N_{\text{cycles}} = 3.20 \times 10^6 \text{ in vacuum}$$

	$\gamma_{\text{sp}} = 1.5$	$\gamma_{\text{sp}} = 2.2$	$\gamma_{\text{sp}} = 2.3$	$\gamma_{\text{sp}} = 2.\bar{3}$
Static	< 1	1.4×10^3	8.7×10^3	1.6×10^4
Dynamic	< 1	6.2×10^2	4.0×10^3	7.4×10^3

TABLE I. **Change in the number of cycles ΔN_{cycles} during the inspiral.** Change in the total number of GW cycles due to dynamical friction, starting 5 years from the merger.