

Probing the properties of dark matter beyond the discovery era

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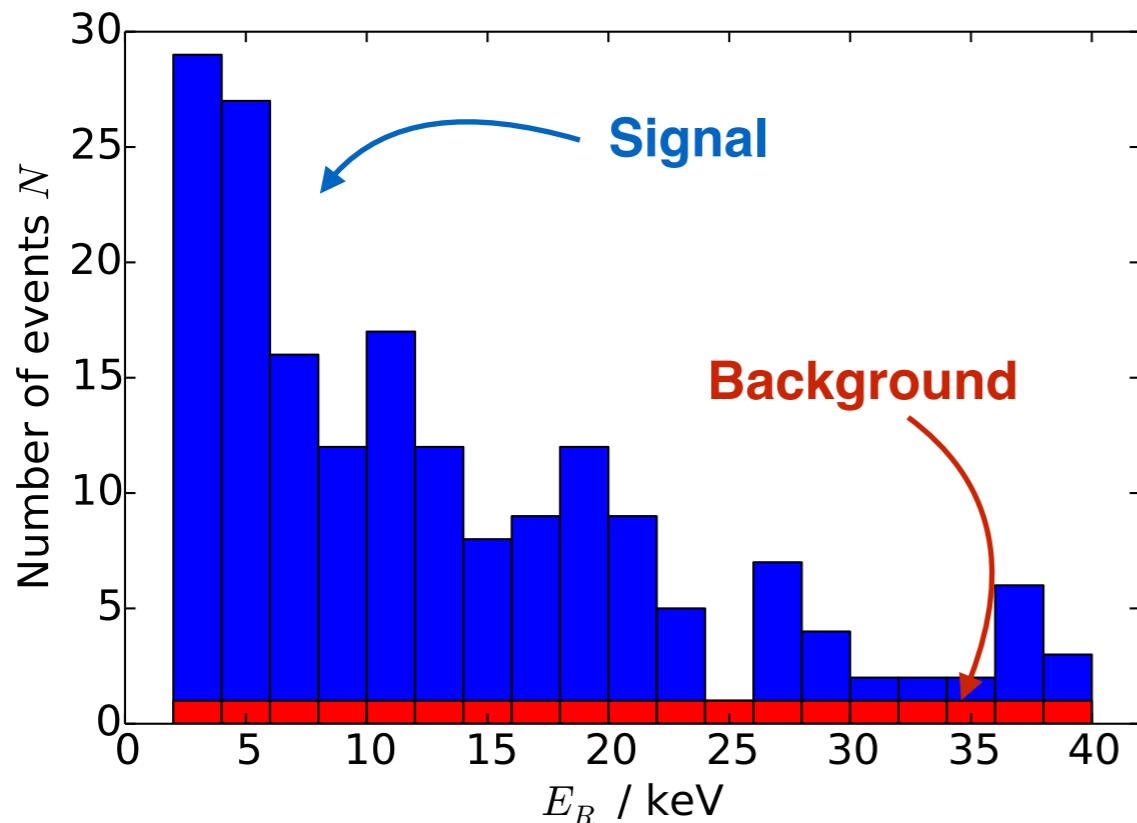
Based on work with A M Green and M Fornasa:

[arXiv:1303.6868](https://arxiv.org/abs/1303.6868), [arXiv:1312.1852](https://arxiv.org/abs/1312.1852), [arXiv:1410.8051](https://arxiv.org/abs/1410.8051)



The problem

Consider data from a future direct detection experiment:



In order to extract the properties of dark matter (DM), we need to know the *expected* event rate.

This requires us to make some assumptions.

**If we get those assumptions wrong,
we get the DM properties wrong too.**

Outline

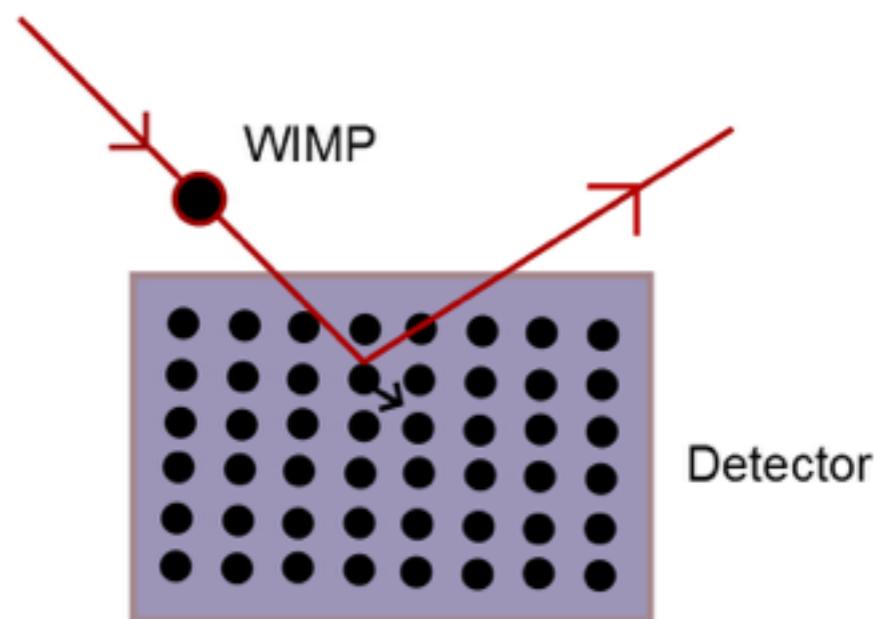
1. Direct detection of dark matter
2. Astrophysical uncertainties
3. How to deal with these uncertainties
4. Combining direct detection with neutrino telescopes

1. Direct detection of dark matter



Direct detection of dark matter

- If DM is made up of Weakly Interacting Massive Particles (WIMPs), they should interact with ordinary matter
- Aim to measure keV-scale nuclear recoils cause by DM interactions in dedicated detectors
- Expected rate is very small. Require:
 - Large target mass
 - Low backgrounds
 - Low energy thresholds



Event Rate

- Flux of DM particles with speed v is $v \left(\frac{\rho_\chi}{m_\chi} \right) f_1(v) dv$
- Need to integrate over all DM speeds, above minimum required to excite a recoil of energy E_R
- Event rate per unit mass is then

$$\frac{dR}{dE_R} = \frac{\rho_\chi}{m_\chi m_N} \int_{v_{\min}}^{\infty} v f_1(v) \frac{d\sigma}{dE_R} dv$$

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Astrophysics

Particle and nuclear physics

The diagram illustrates the components of the event rate formula. The formula is shown as:

$$\frac{dR}{dE_R} = \frac{\rho_\chi}{m_\chi m_N} \int_{v_{\min}}^{\infty} v f_1(v) \frac{d\sigma}{dE_R} dv$$

Two arrows point from labels to specific parts of the formula:

- A green arrow points from the label "Astrophysics" to the term $\frac{\rho_\chi}{m_\chi m_N}$.
- A red arrow points from the label "Particle and nuclear physics" to the term $\frac{d\sigma}{dE_R}$.

Cross section

Typically assume contact interactions.

In the non-relativistic limit, obtain two main contributions.

Write in terms of DM-proton cross section σ^p :

Spin-independent (SI)

$$(\bar{\chi}\chi)(\bar{n}n) \rightarrow \frac{d\sigma_{SI}^N}{dE_R} \propto \frac{\sigma_{SI}^p}{v^2} A^2 F_{SI}^2(E_R)$$

Nuclear physics

Spin-dependent (SD)

$$(\bar{\chi}\gamma_5\gamma_\mu\chi)(\bar{n}\gamma_5\gamma^\mu n) \rightarrow \frac{d\sigma_{SD}^N}{dE_R} \propto \frac{\sigma_{SD}^p}{v^2} \frac{J+1}{J} F_{SD}^2(E_R)$$

But more general interactions have been considered
e.g. Del Nobile et al. [arXiv:1307.5955]

The final event rate

Combining the various components for the event rate, we obtain:

$$\frac{dR}{dE_R} = \frac{\rho_\chi \sigma_i^p}{m_\chi \mu_{\chi p}^2} \mathcal{C}_i F_i^2(E_R) \eta(v_{\min}) \quad i = SI, SD$$

- Enhancement factor: \mathcal{C}_i
- Form factor: $F_i^2(E_R)$
- Mean inverse speed:

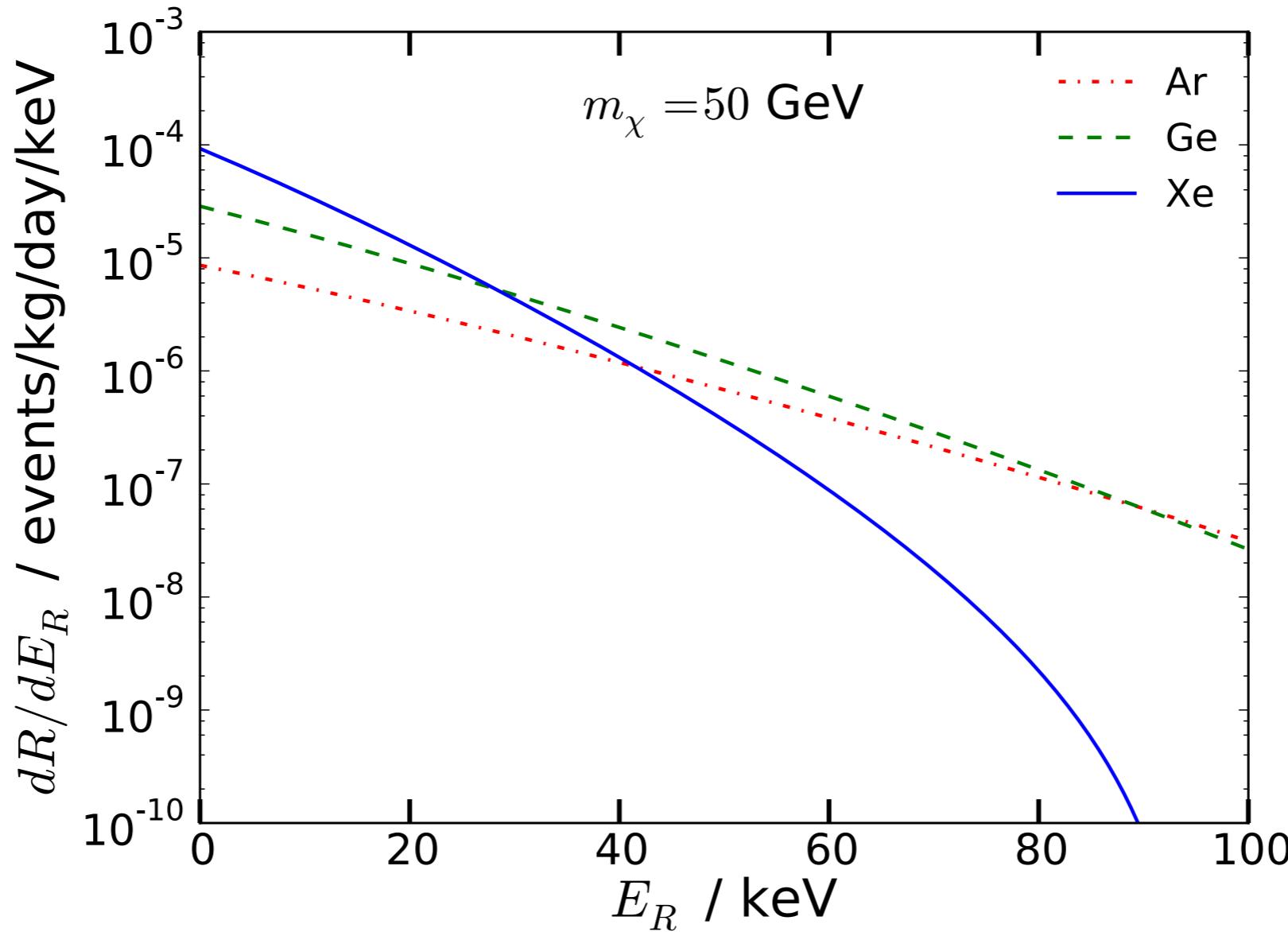
$$\eta(v_{\min}) = \int_{v_{\min}}^{\infty} \frac{f_1(v)}{v} dv$$

$$v_{\min} = \sqrt{\frac{m_N E_R}{2 \mu_{\chi N}^2}}$$

$$\mu_{AB} = \frac{m_A m_B}{m_A + m_B}$$

Typical spectra

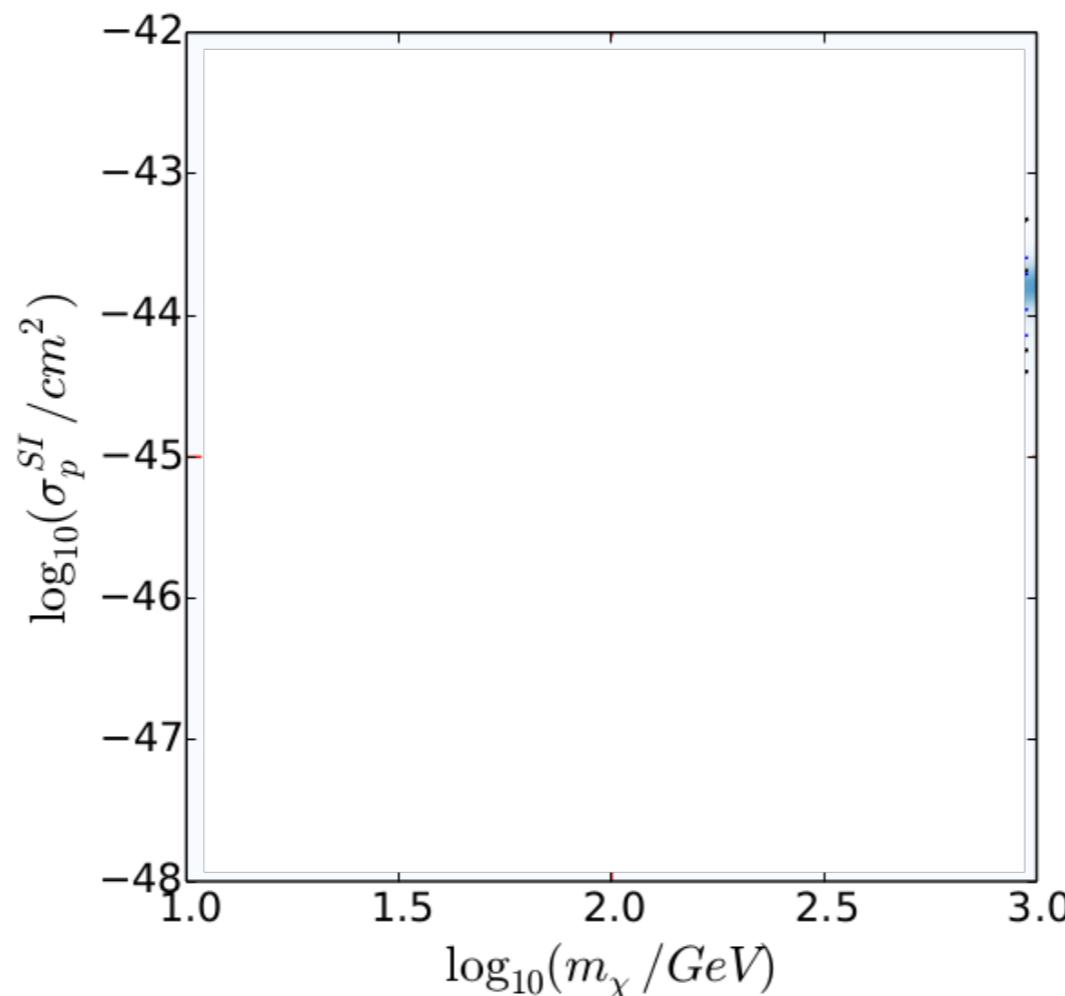
Spin-independent (SI) event rates:



...assuming Standard Halo Model [see later...]

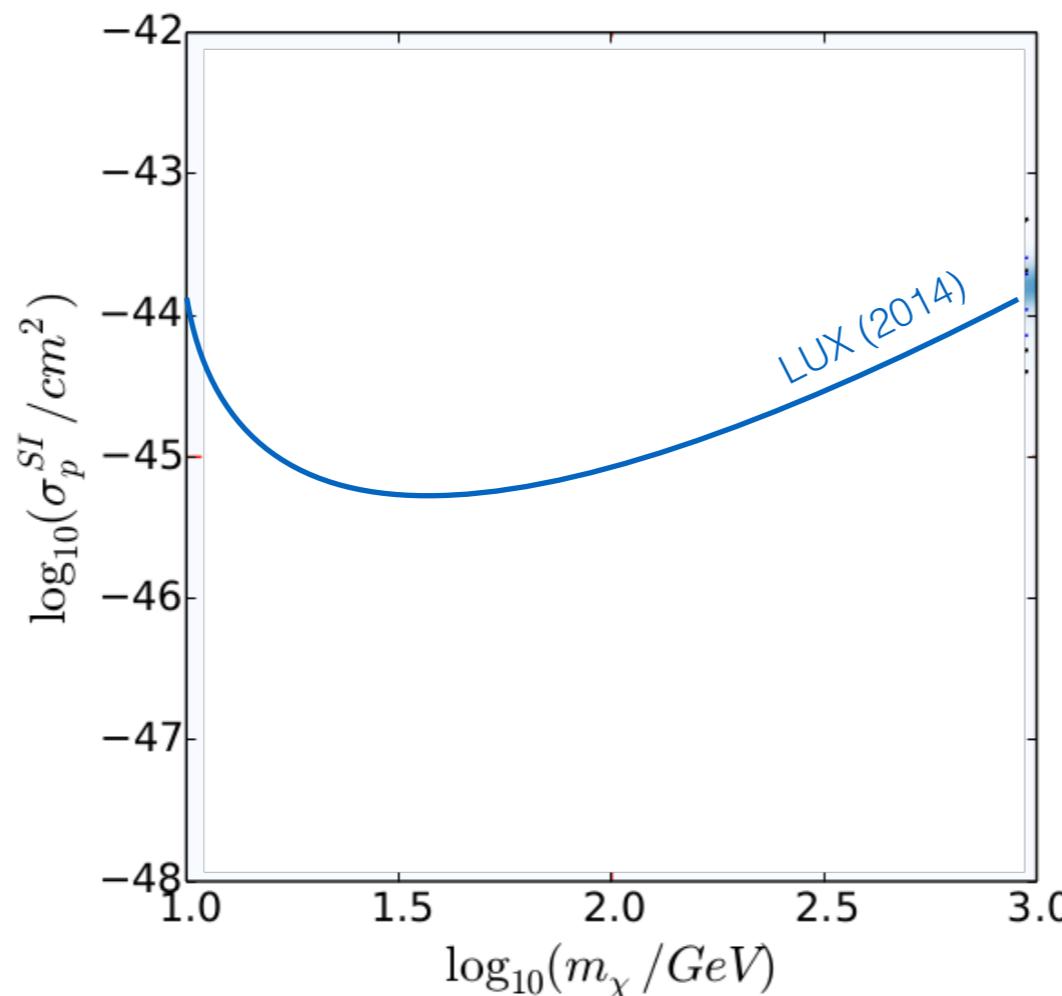
Direct detection constraints

- Using direct detection experiments, we can place constraints on the parameter space $(m_\chi, \sigma_{SI}^p, \sigma_{SD}^p)$
- Typically SI interaction dominates due to A^2 enhancement



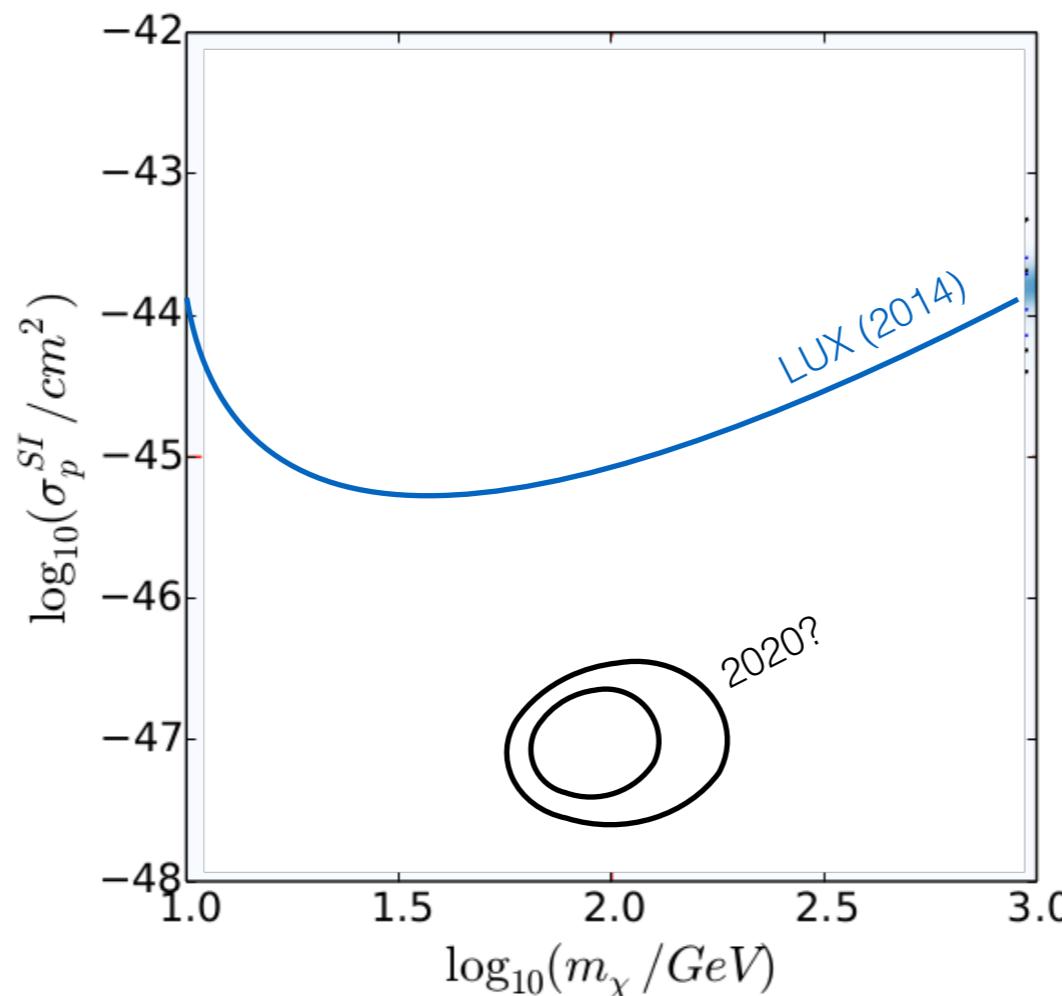
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2. Astrophysical uncertainties



Astrophysical uncertainties

The spectra, limits and contours of the previous slides are based a several assumptions:

1. A fixed value of the **local DM density** - which controls the overall normalisation of the rate
2. A fixed shape for the **DM speed distribution** - which influences the shape of the recoil spectrum

1. Local DM density

Can be measured using two methods:

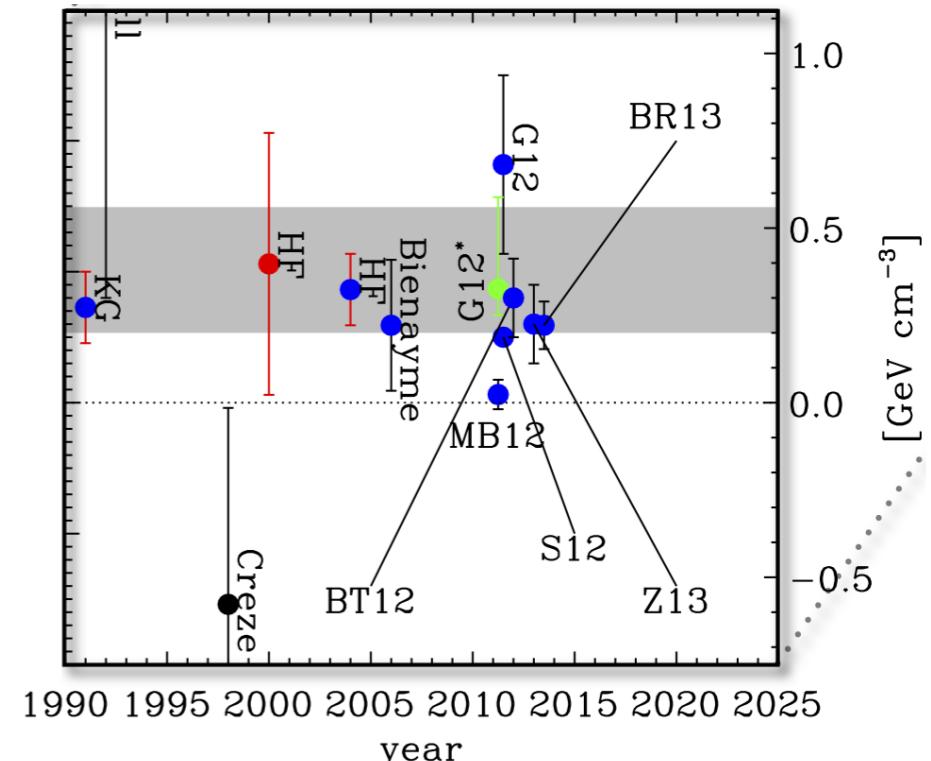
- **Global** - build a global mass model of the MW (including bulge, disk, halo...) and fit to available data
- **Local** - use kinematics of local *tracers* to reconstruct potential and therefore mass in Solar neighbourhood

[Read \(2014\) \[arXiv:1404.1938\]](#)

Different measurements using both techniques give results in broad agreement

$$\rho_\chi \sim 0.2 - 0.6 \text{ GeV cm}^{-3}$$

A factor of 2-3 uncertainty in the overall normalisation of the rate



2. DM speed distribution

- Obtained from *velocity* distribution $f(\mathbf{v})$:

$$f(v) = \oint f(\mathbf{v}) d\Omega_v \quad f_1(v) = v^2 f(v)$$

- $f_1(v)$ describes the fraction of dark matter particles with speed in range $v \rightarrow v + dv$
- Depends on the formation and merger history of the Milky Way
- Form of $f_1(v)$ can be estimated by making simplifying assumptions, or it can be extracted from N-body simulations
- However, ultimately, the form of $f_1(v)$ is *a priori* unknown

Standard Halo Model (SHM)

Speed distribution obtained for a spherical, isotropic and isothermal halo, with density profile $\rho(r) \propto r^{-2}$:

Leads to Maxwell-Boltzmann distribution:

$$f_1(v) \propto v^2 \oint \exp\left(-\frac{(\mathbf{v} - \mathbf{v}_e)^2}{2\sigma_v^2}\right) d\Omega_v \Theta(|\mathbf{v} - \mathbf{v}_e| - v_{\text{esc}})$$

Even within the SHM, there are still some parameter uncertainties:

$$v_e \sim 220 - 250 \text{ km s}^{-1}$$

E.g. [Feast et al. \(1997\) \[astro-ph/9706293\]](#),
[Bovy et al. \(2012\) \[arXiv:1209.0759\]](#)

$$\sigma_v \sim 155 - 175 \text{ km s}^{-1}$$

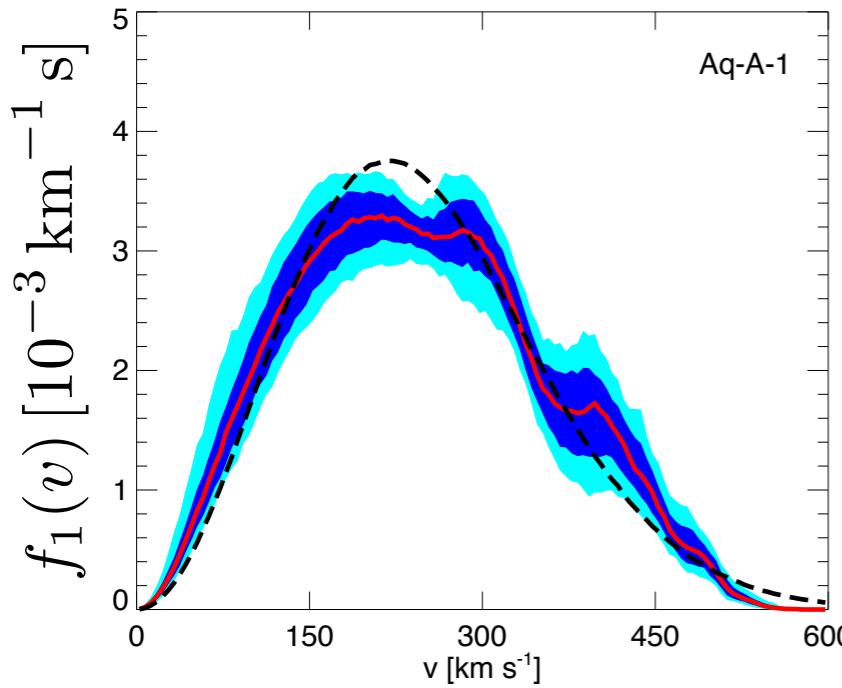
$$v_{\text{esc}} = 533^{+54}_{-41} \text{ km s}^{-1}$$

[Piffl et al. \(RAVE, 2013\) \[arXiv:1309.4293\]](#)

N-body simulations

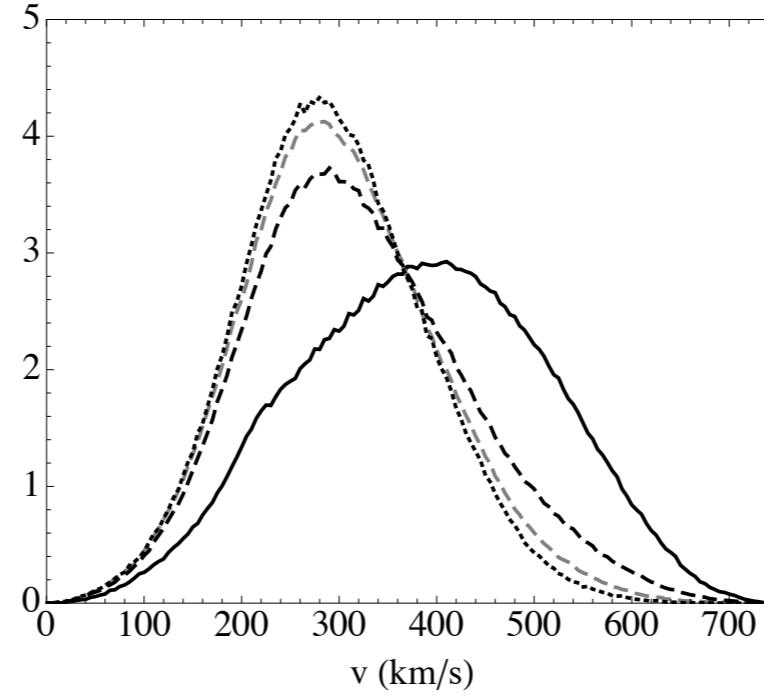
High resolution N-body simulations can be used to extract the DM speed distribution

Non-maxwellian structure



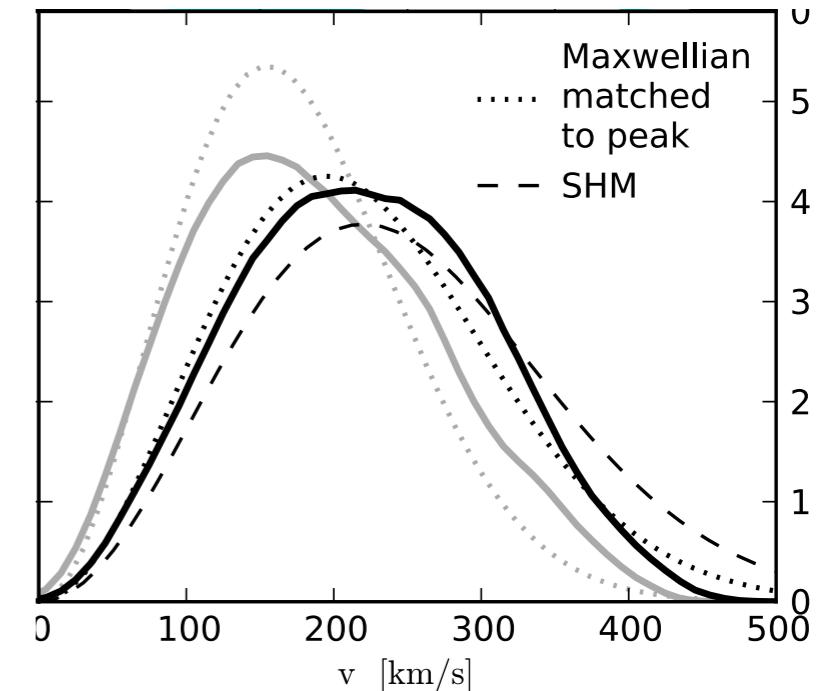
Vogelsberger et al. (2009)
[arXiv:0812.0362]

Debris flows



Kuhlen et al. (2012)
[arXiv:1202.0007]

Dark disk



Pillepich et al. (2014)
[arXiv:1308.1703]

However, N-body simulations cannot probe down to the sub-milliparsec scales probes by direct detection...

Local substructure

May want to worry about ultra-local substructure - subhalos and streams which are not completely phase-mixed.

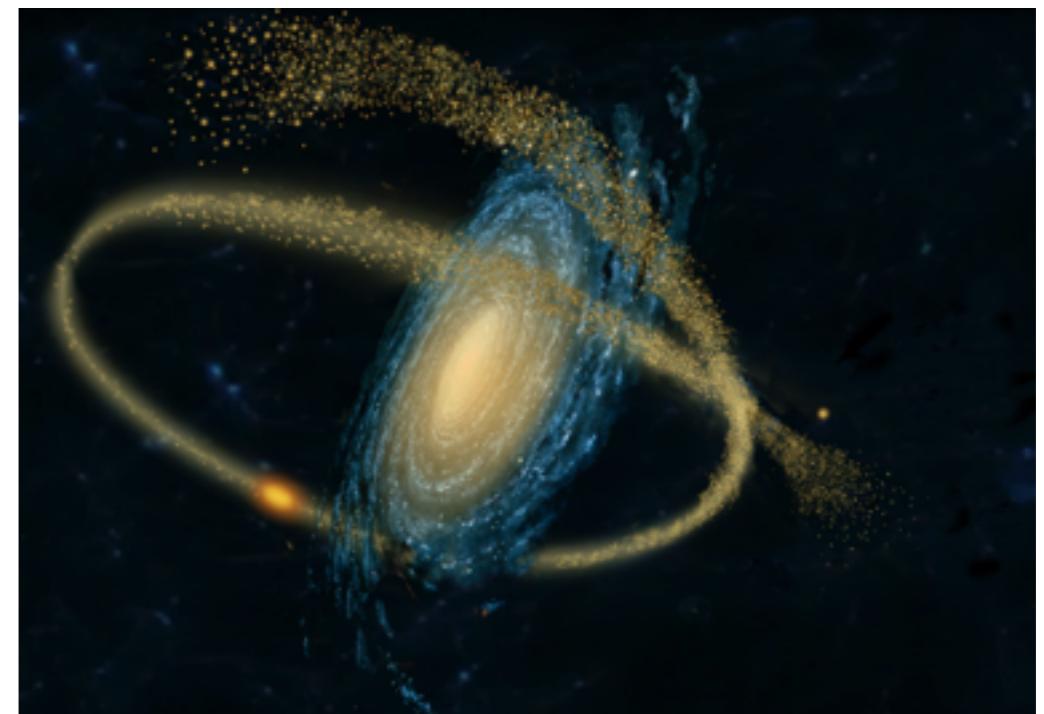
Analysis of N-body simulations indicate that it is unlikely for a single stream to dominate the local density - lots of different 'streams' contribute to make a smooth halo.

[Helmi et al. \(2002\) \[astro-ph/0201289\]](#)

[Vogelsberger et al. \(2007\) \[arXiv:0711.1105\]](#)

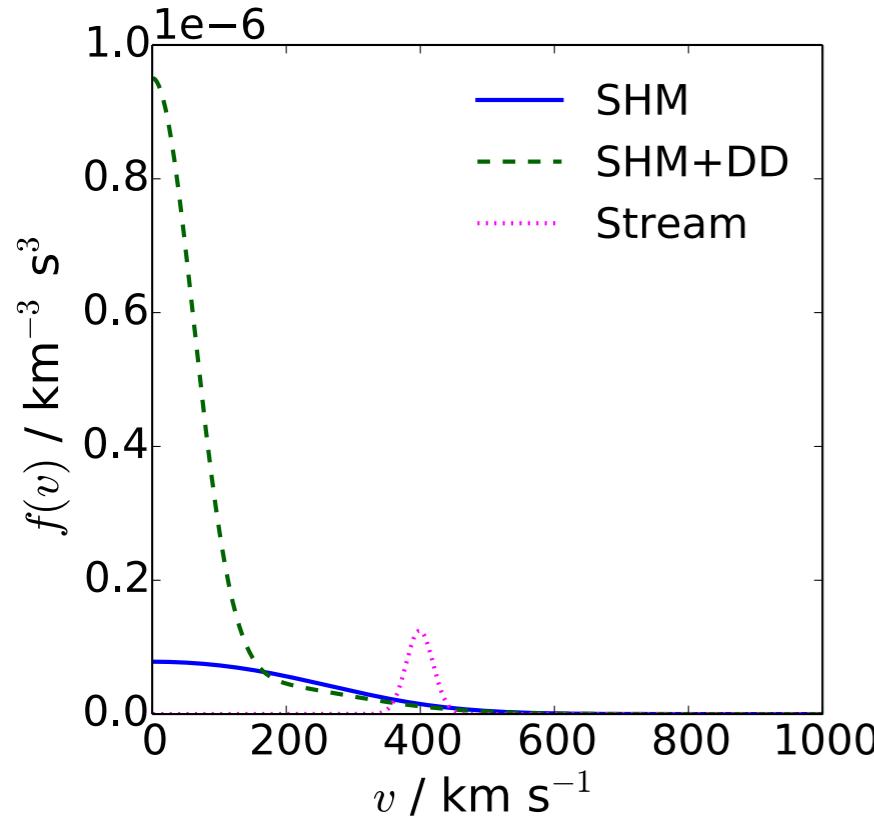
However, this does not exclude the possibility of a stream - e.g. due to the ongoing tidal disruption of the Sagittarius dwarf galaxy.

[Freese et al. \(2004\) \[astro-ph/0309279\]](#)

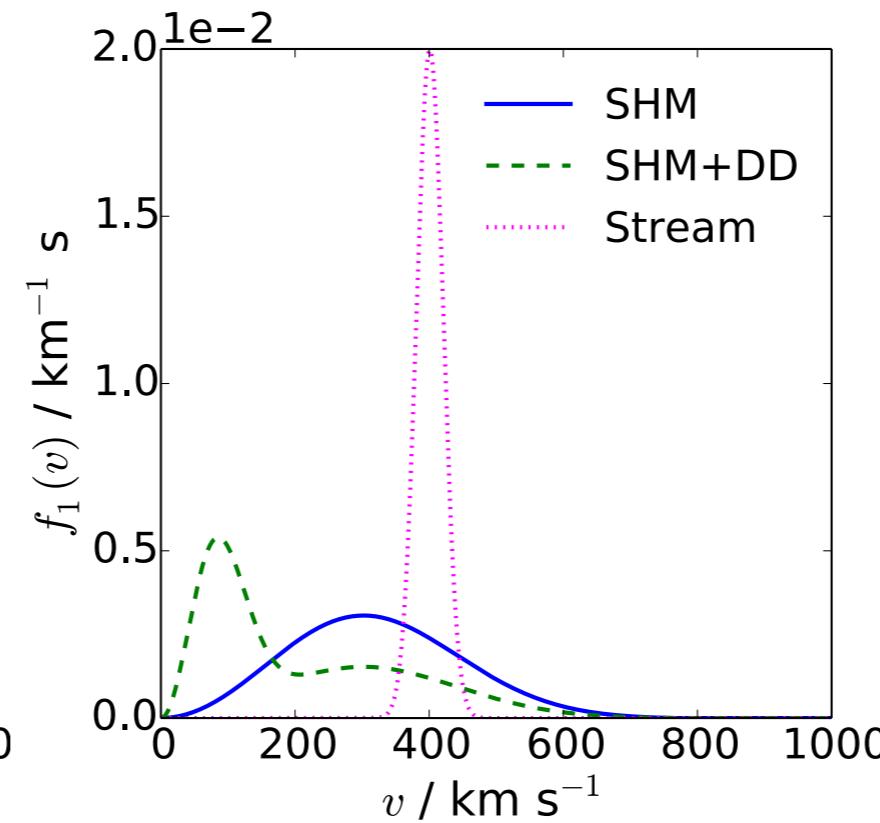


Examples

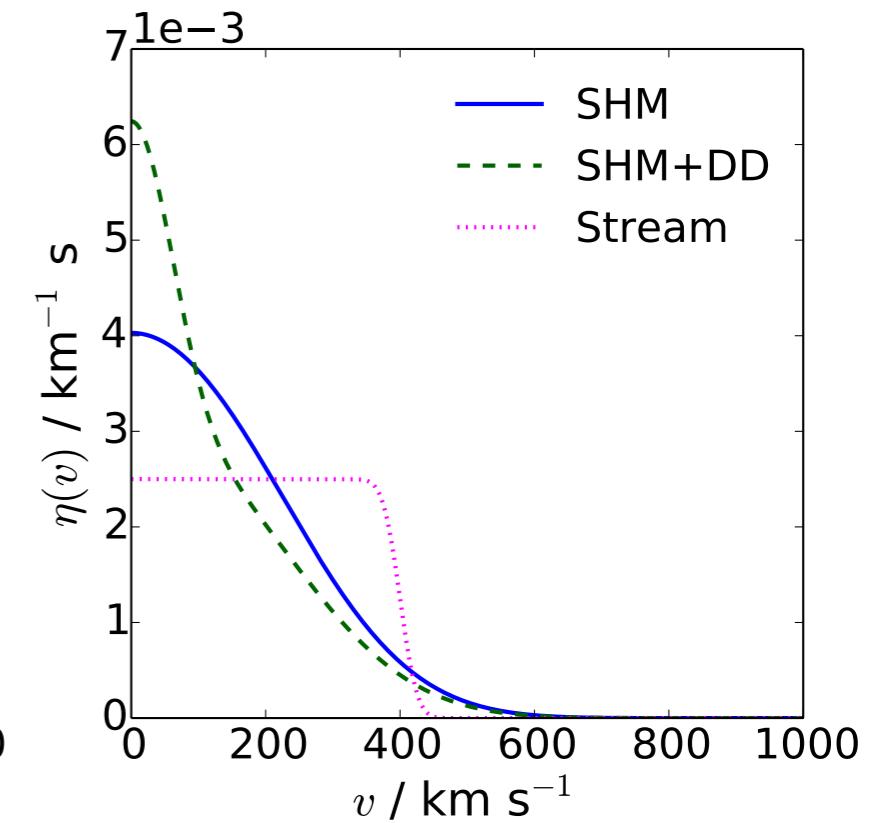
$$f(v) = \oint f(\mathbf{v}) d\Omega_v$$



$$f_1(v) = v^2 f(v)$$



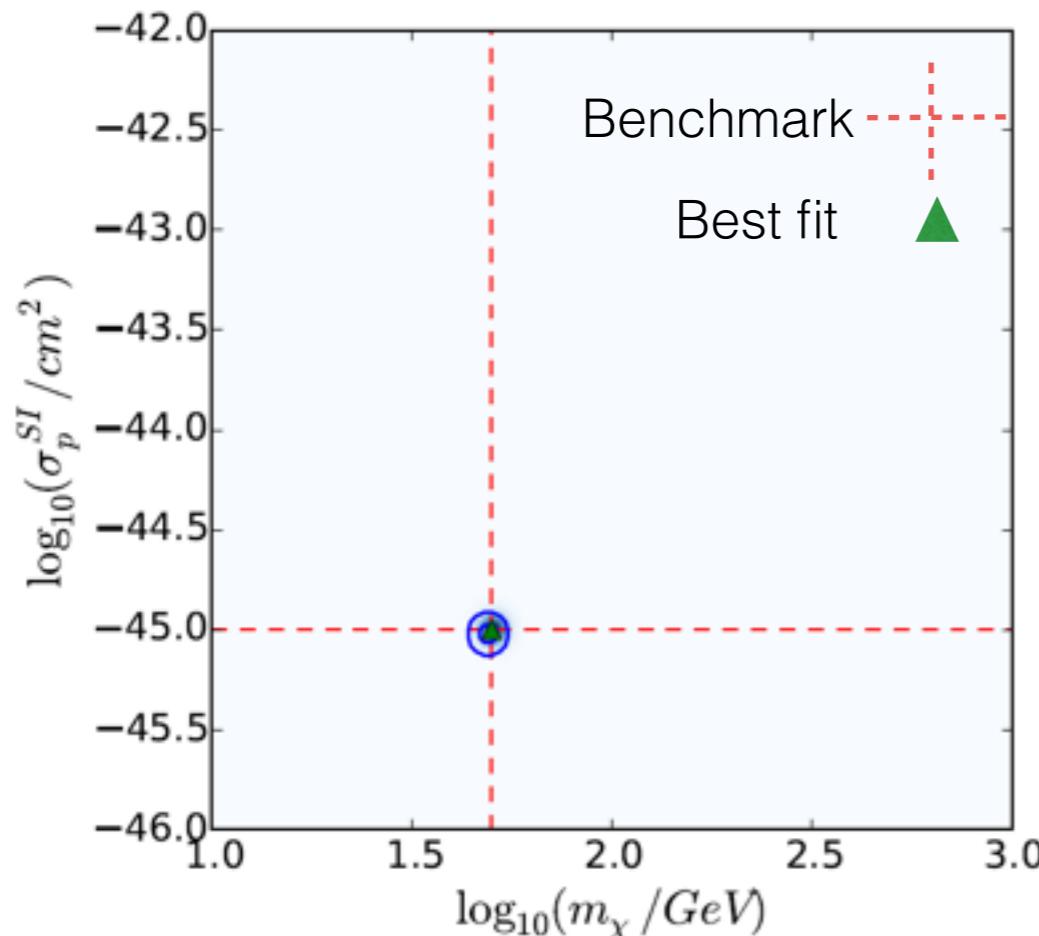
$$\eta(v) = \int_v^\infty \frac{f_1(v')}{v'} dv'$$



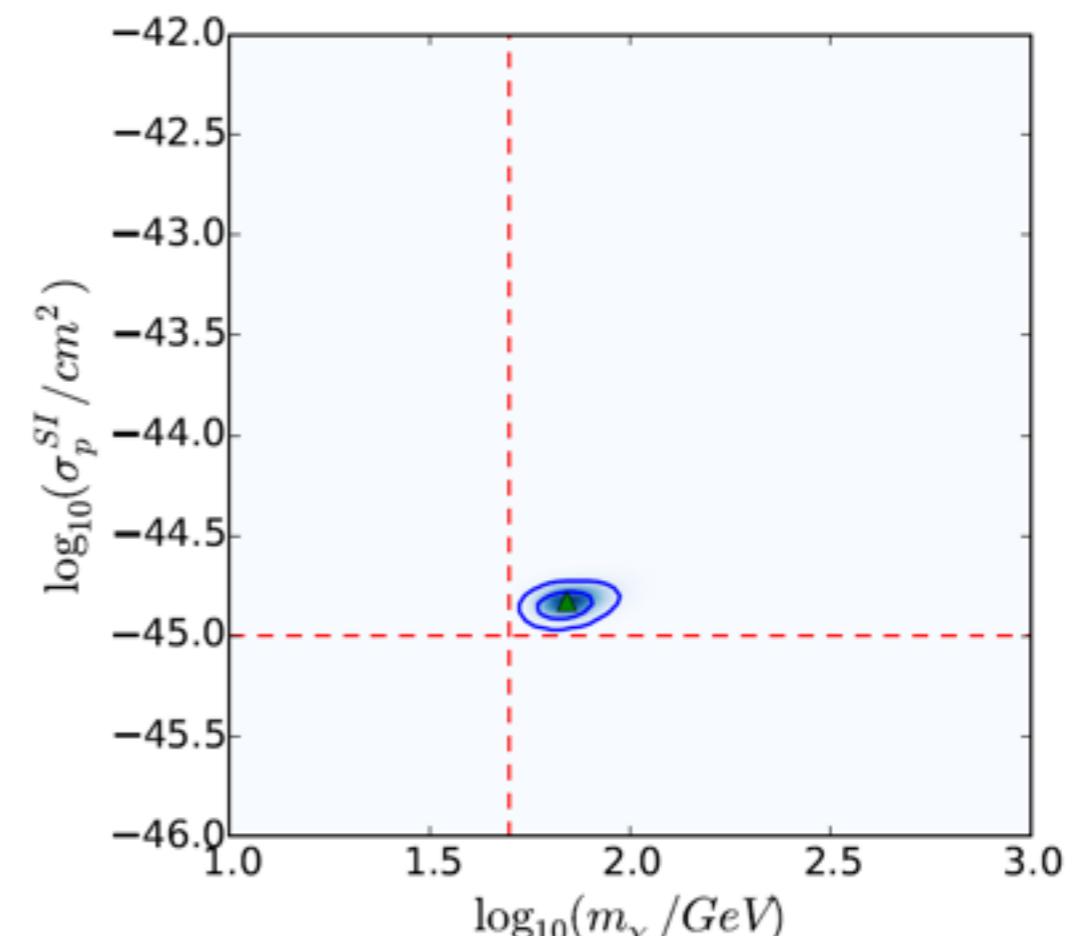
What could possibly go wrong?

Generate mock data for 3 future experiments - Xe, Ar, Ge - for a given (m_χ, σ_{SI}^p) assuming a **stream** distribution function. Then construct confidence contours for these parameters, assuming:

(correct) **stream** distribution



(incorrect) **SHM** distribution

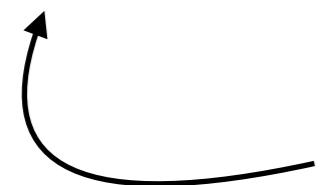


3. How to deal with these uncertainties



Fixing it: previous approaches

- Incorporate uncertainties in SHM parameters
[Strigari & Trotta \[arXiv:0906.5361\]](#)
- Attempt to measure $\eta(v_{\min})$ directly from the data (assuming a particular value for m_χ)
[Fox, Liu & Weiner \[arXiv:1011.915\]](#)
[Frandsen et al. \[arXiv:1111.0292\]](#)
- Write $\eta(v_{\min})$ as a large number of steps and optimise the step heights
[Feldstein & Kahlhoefer \[arXiv:1403.4606\]](#)
- Write down a general parametrisation for $f(v)$ and fit the parameters to the data
[Peter \[arXiv:1103.5145\]](#)



Our approach - but need to be careful
which parametrisation to use

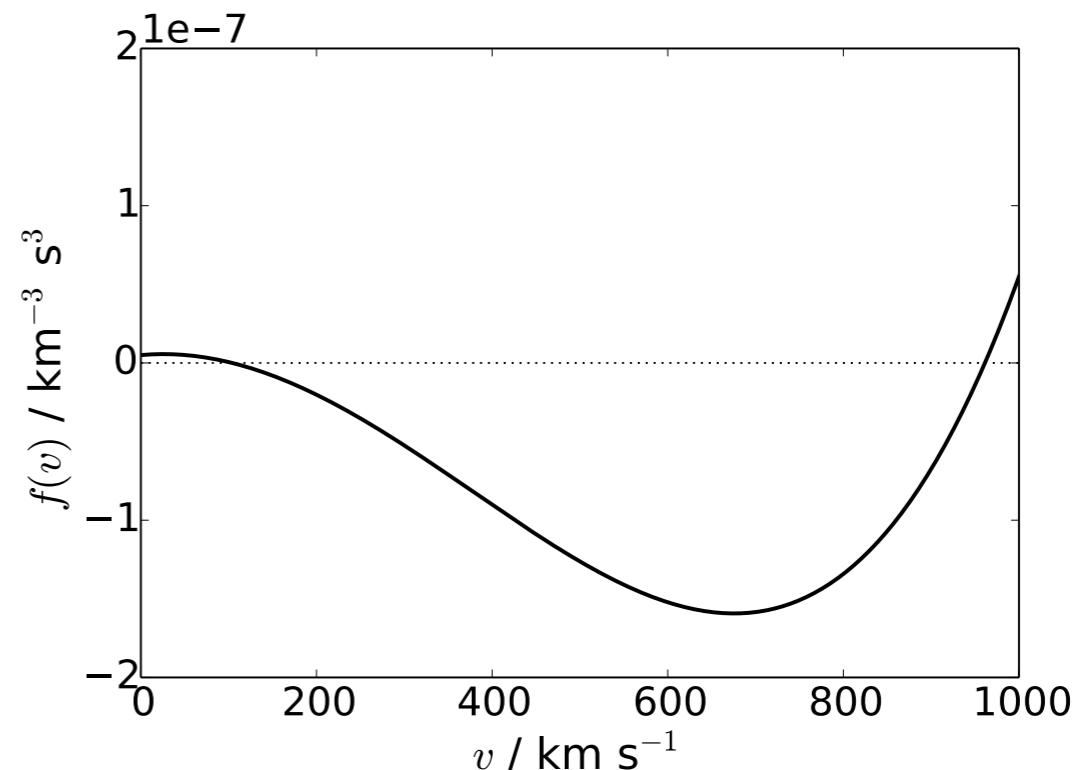
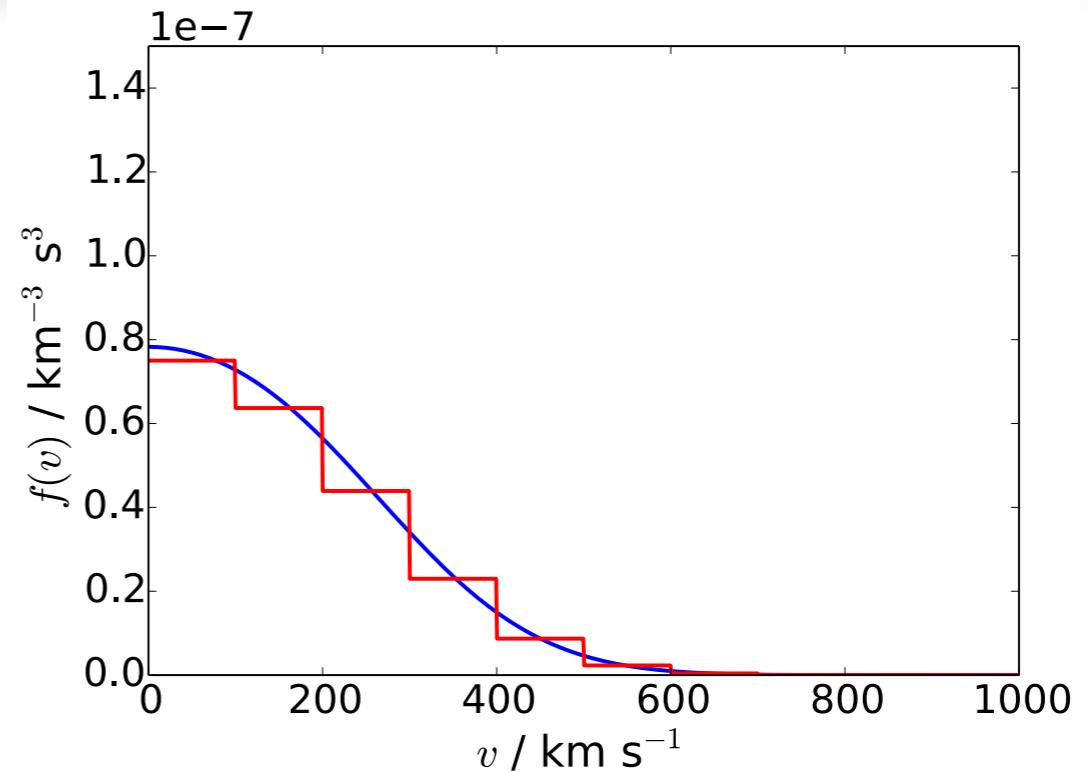
Possible parametrisations

Previous work of Peter considered a *binned* parametrisation for $f(v)$. However, the fixed width of the bins leads to a bias in the reconstructed WIMP mass.

Could also consider a polynomial parametrisation:

$$f(v) = \sum_{k=0}^{N-1} a_k v^k = a_0 + a_1 v + a_2 v^2 + \dots$$

However - this does not give us a *physical* distribution.



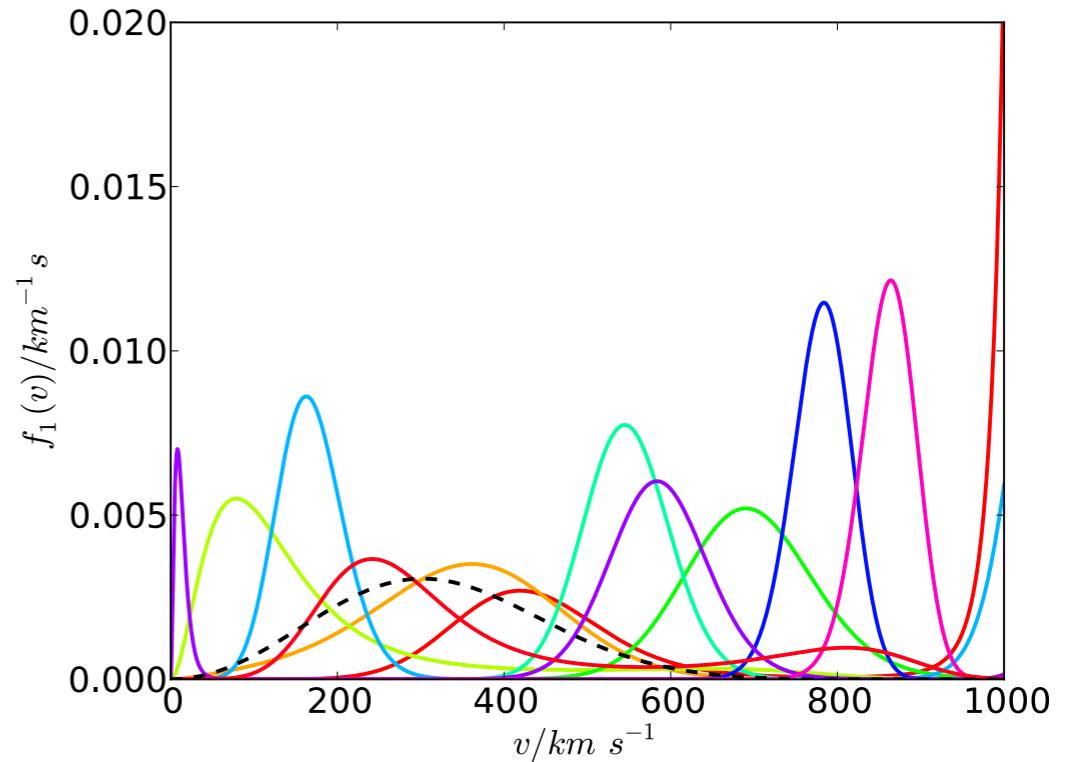
Our parametrisation

Need a parametrisation that is *general* and which is *everywhere positive*:

$$f(v) = \exp \left(\sum_{k=0}^{N-1} a_k P_k(v) \right)$$

for some polynomial basis $P_k(v)$.

Now we attempt to fit the particle physics parameters (m_χ, σ^p) , as well as the astrophysics parameters $\{a_k\}$.

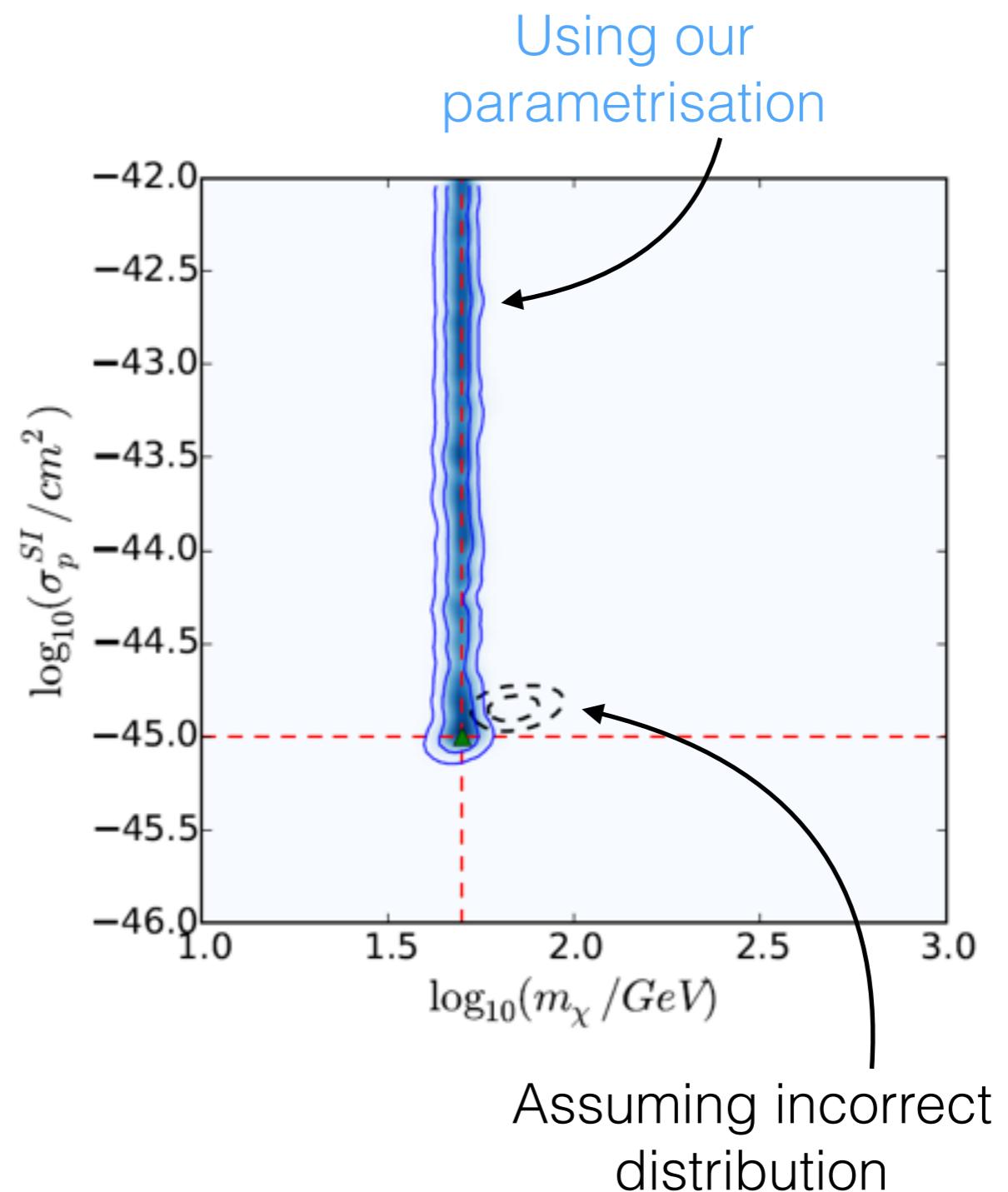


$$f_1(v) = v^2 f(v)$$

Our previous example

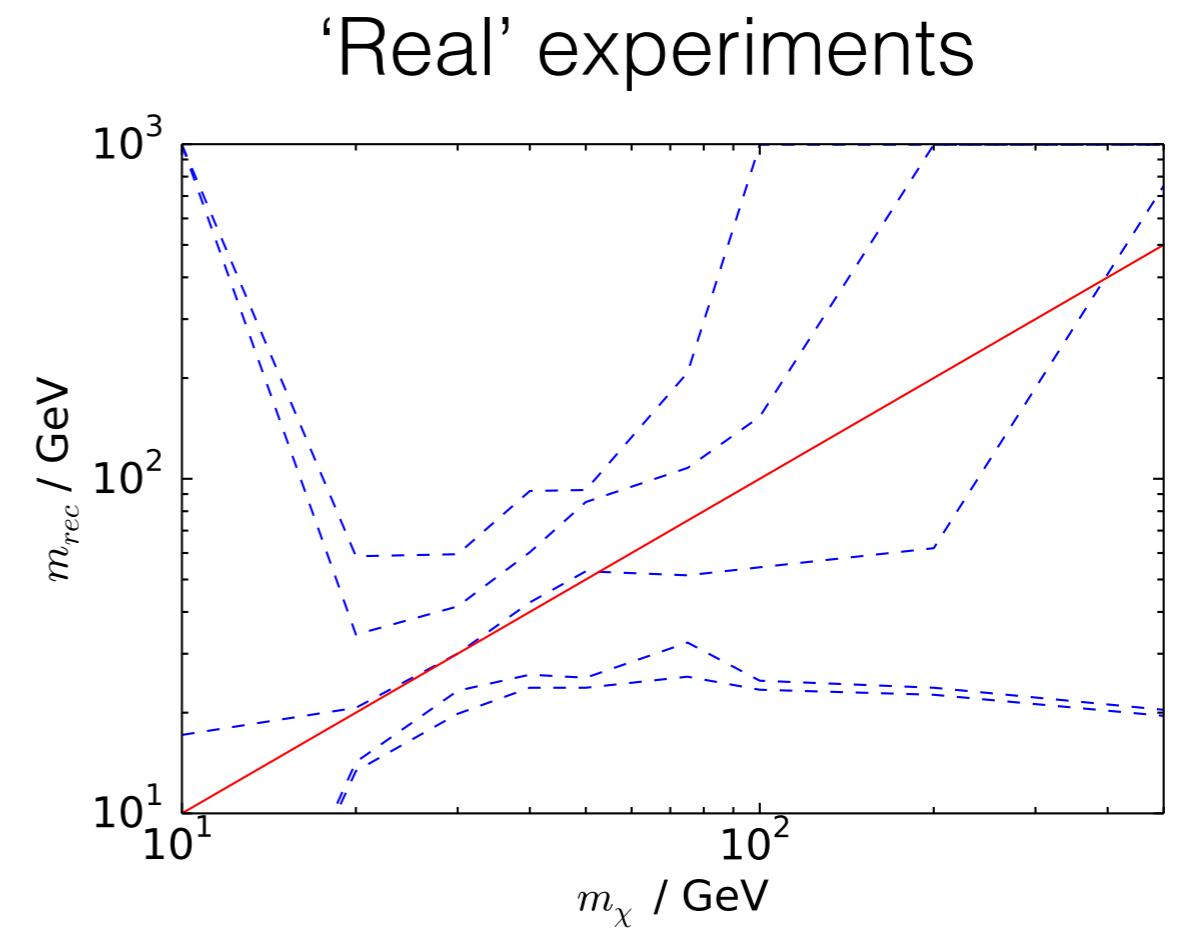
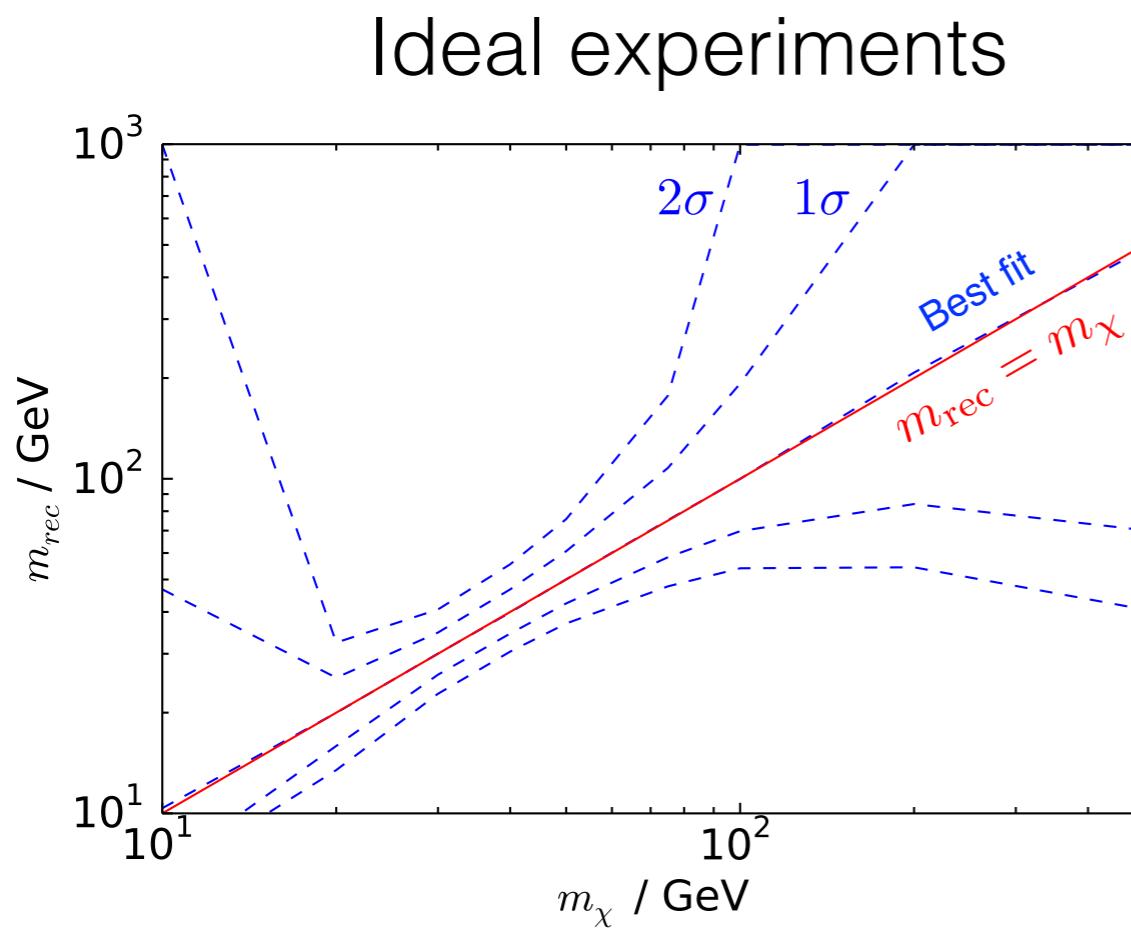
DM mass is accurately and precisely reconstructed - *without any assumptions.*

But, there is now a strong degeneracy in the reconstructed cross section [which we'll get back to shortly...]



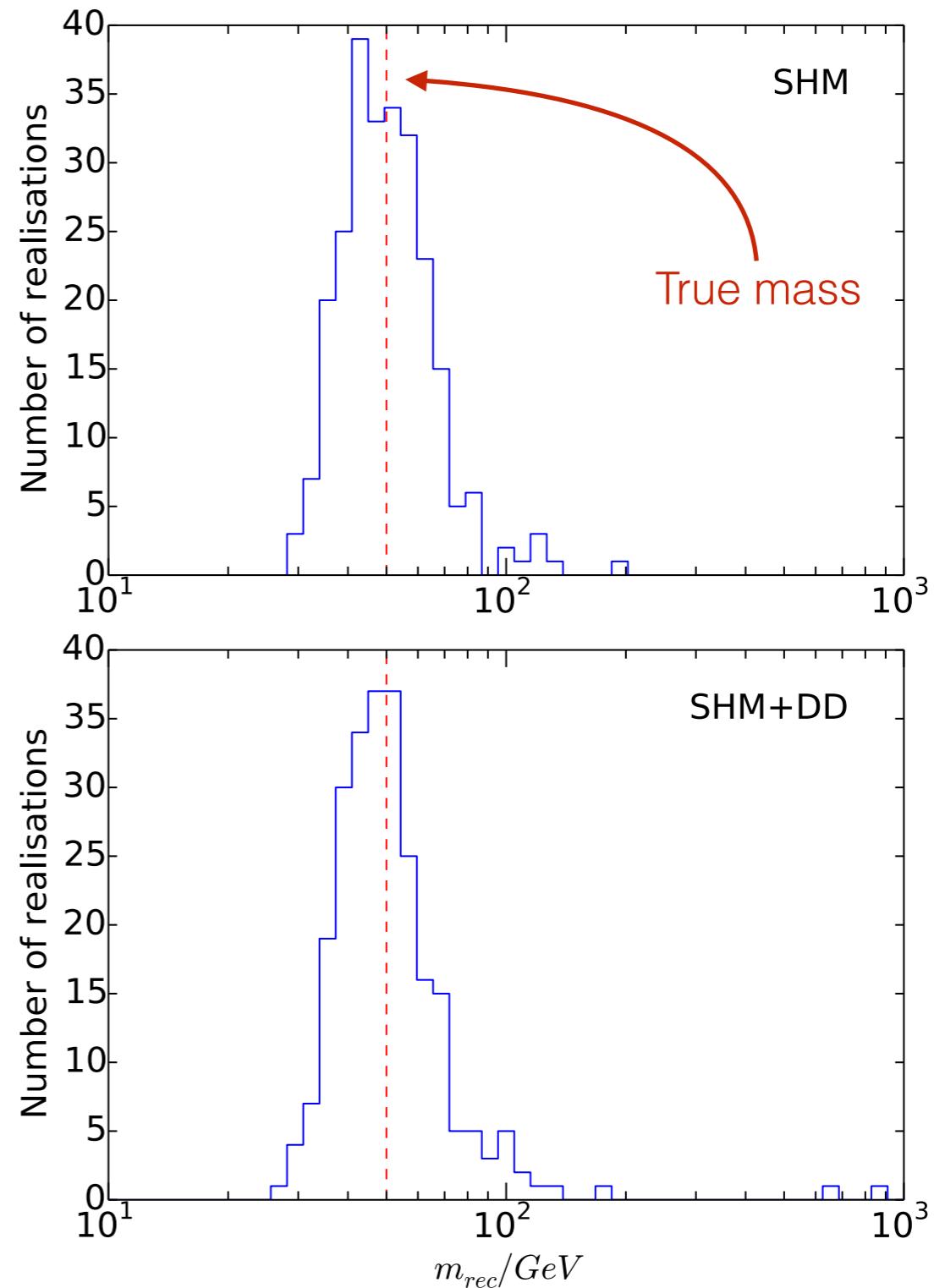
Reconstructing the WIMP mass

This method allows an unbiased reconstruction of the WIMP mass over a wide range of parameter space - including when realistic detector properties (background, energy resolution) are taken into account.

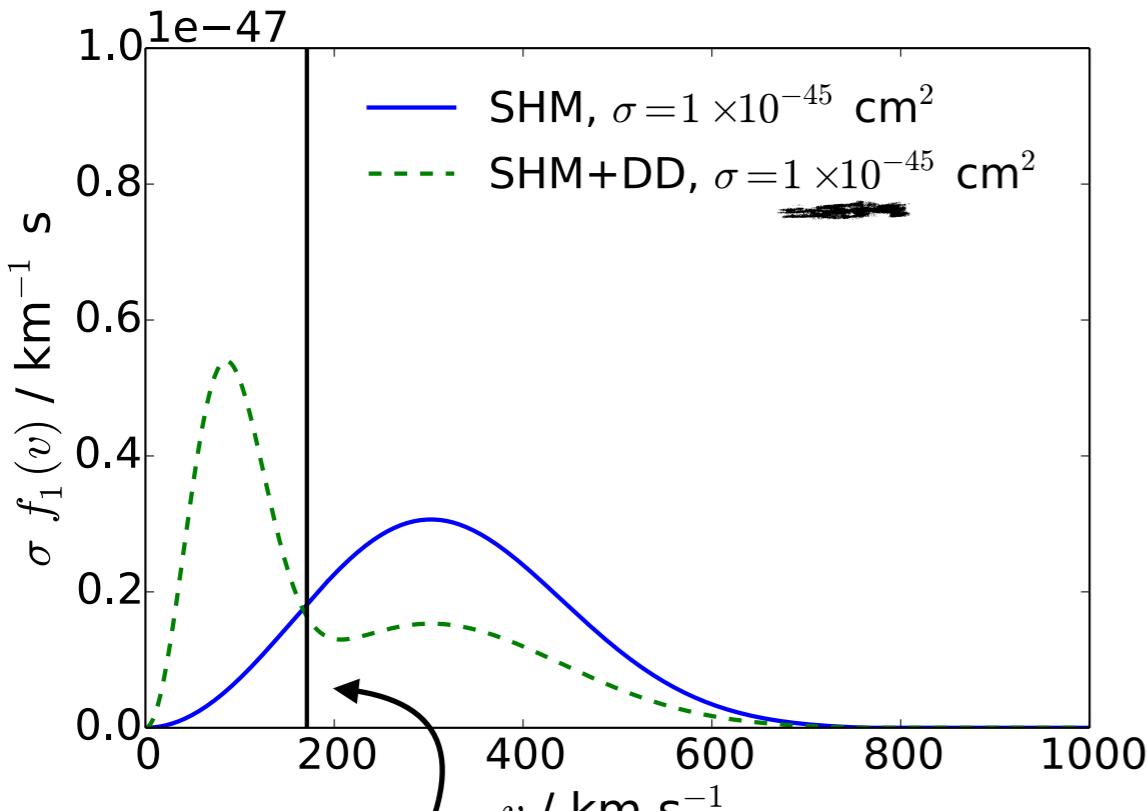


Different speed distributions

- Generate 250 mock data sets
- Reconstruct mass and obtain confidence intervals for each data set
- True mass reconstructed well (independent of speed distribution)
- Can also check that 68% intervals *are really 68% intervals*



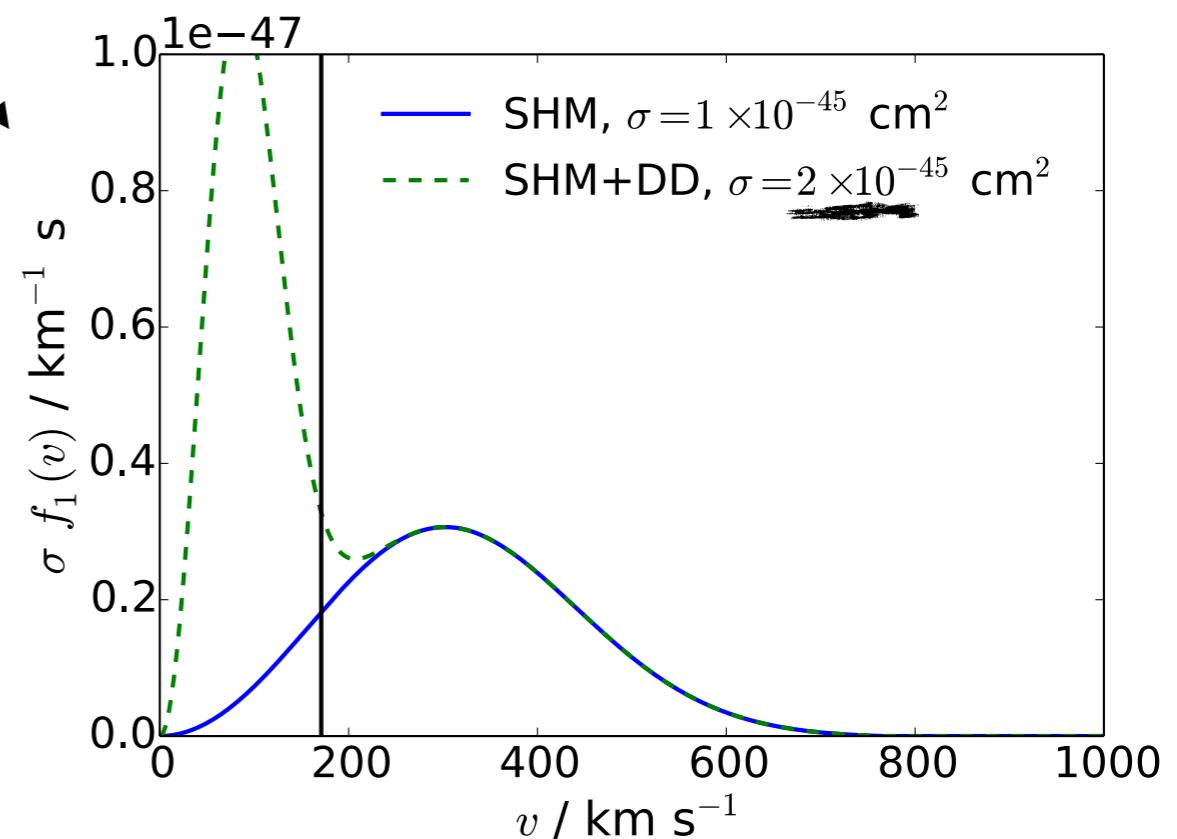
Cross section degeneracy



Minimum WIMP speed probed
by a typical Xe experiment

This is a problem for *any*
astrophysics-independent method!

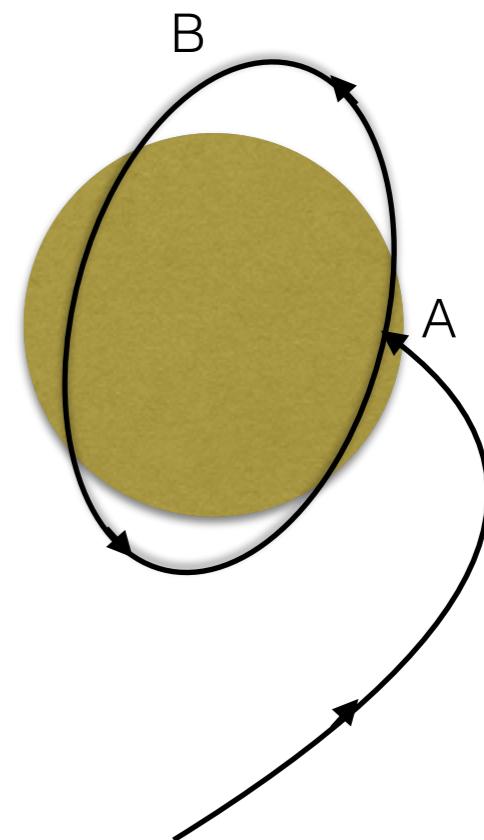
$$\frac{dR}{dE_R} \propto \sigma \int_{v_{\min}}^{\infty} \frac{f_1(v)}{v} dv$$



4. Combining direct detection with neutrino telescopes

Solar Capture

- Need to find a way of probing the low-speed WIMP population
- WIMPs scatter with nuclei in the Sun (A), losing energy and entering a bound orbit (B)
- WIMPs thermalise and eventually annihilate
- We can measure the neutrinos produced using neutrino telescope experiments (e.g. IceCube) and therefore probe the capture rate
- Crucially, it is the low energy - *low speed* - WIMPs which are preferentially captured!



Incorporating IceCube

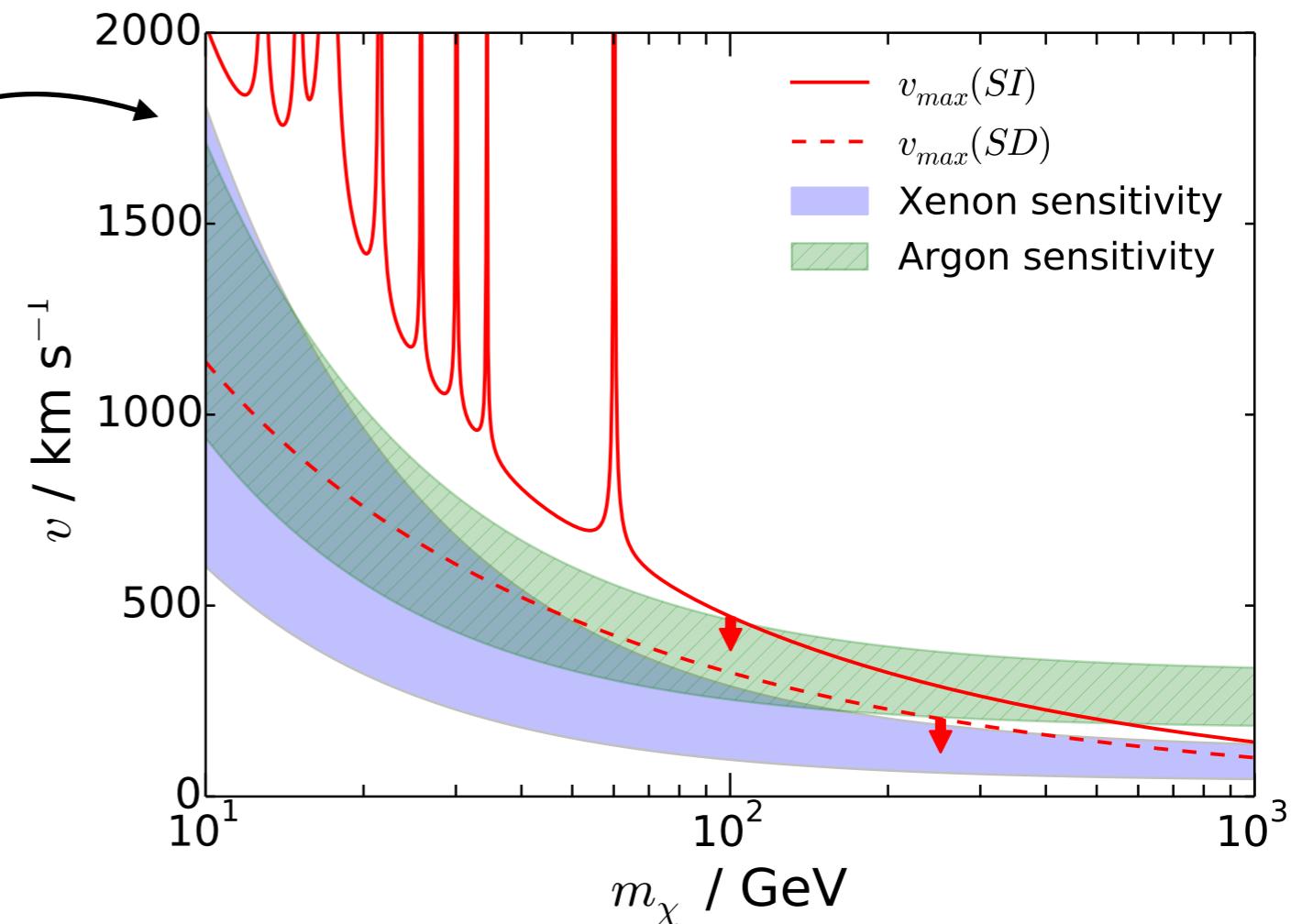
Solar capture rate per unit volume:

$$\frac{dC_i}{dV} = \int_0^{v_{\max}} dv \frac{f_1(v)}{v} w \Omega_{v_{\text{esc}}, i}^-(w)$$

Good overlap between speeds probed by different experiments

Generate mock data from IceCube and include it in the reconstruction

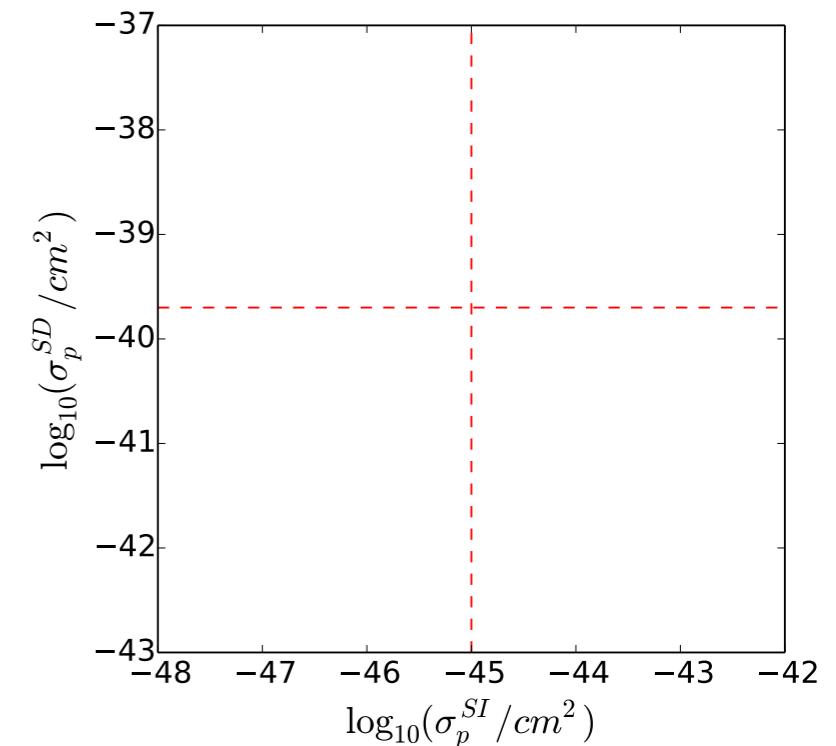
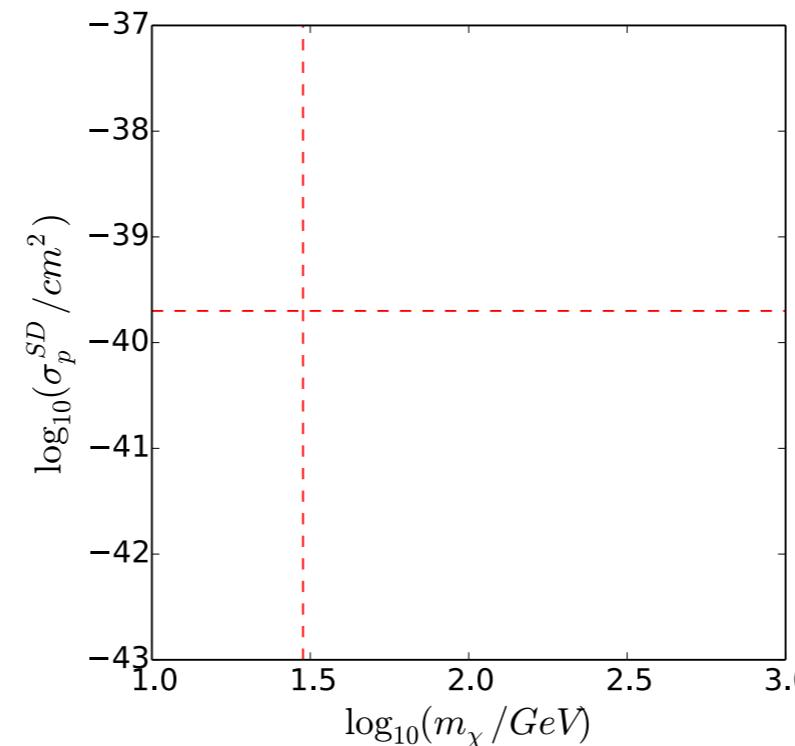
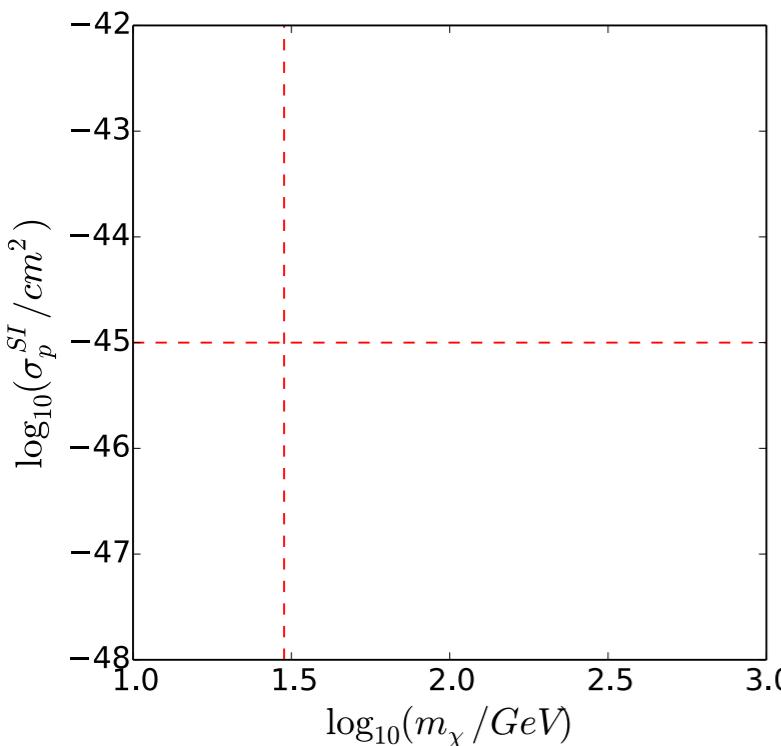
But Sun is mainly spin-1/2 Hydrogen - need to include SD interactions



Direct detection only

Consider a single benchmark:

$m_\chi = 30 \text{ GeV}$; $\sigma_p^{SI} = 10^{-45} \text{ cm}^2$; $\sigma_p^{SD} = 2 \times 10^{-40} \text{ cm}^2$
annihilation to $\nu_\mu \bar{\nu}_\mu$, SHM+DD distribution

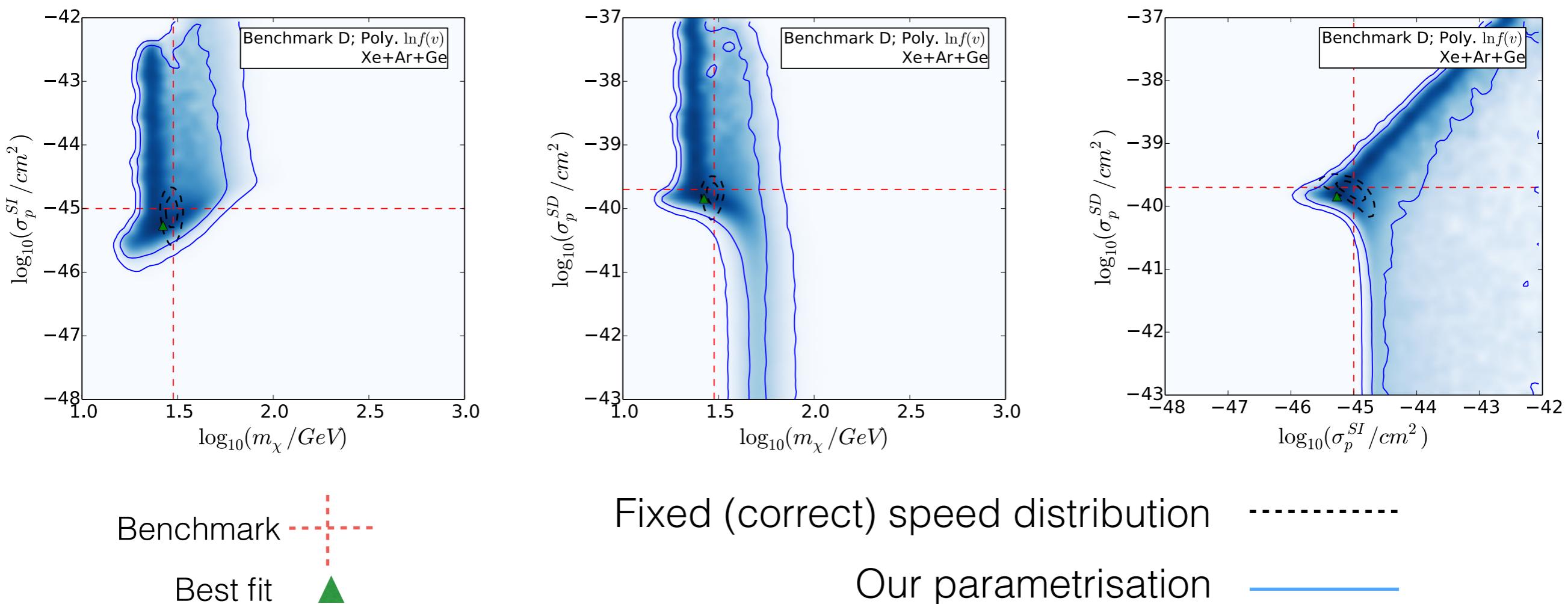


Benchmark

Direct detection only

Consider a single benchmark:

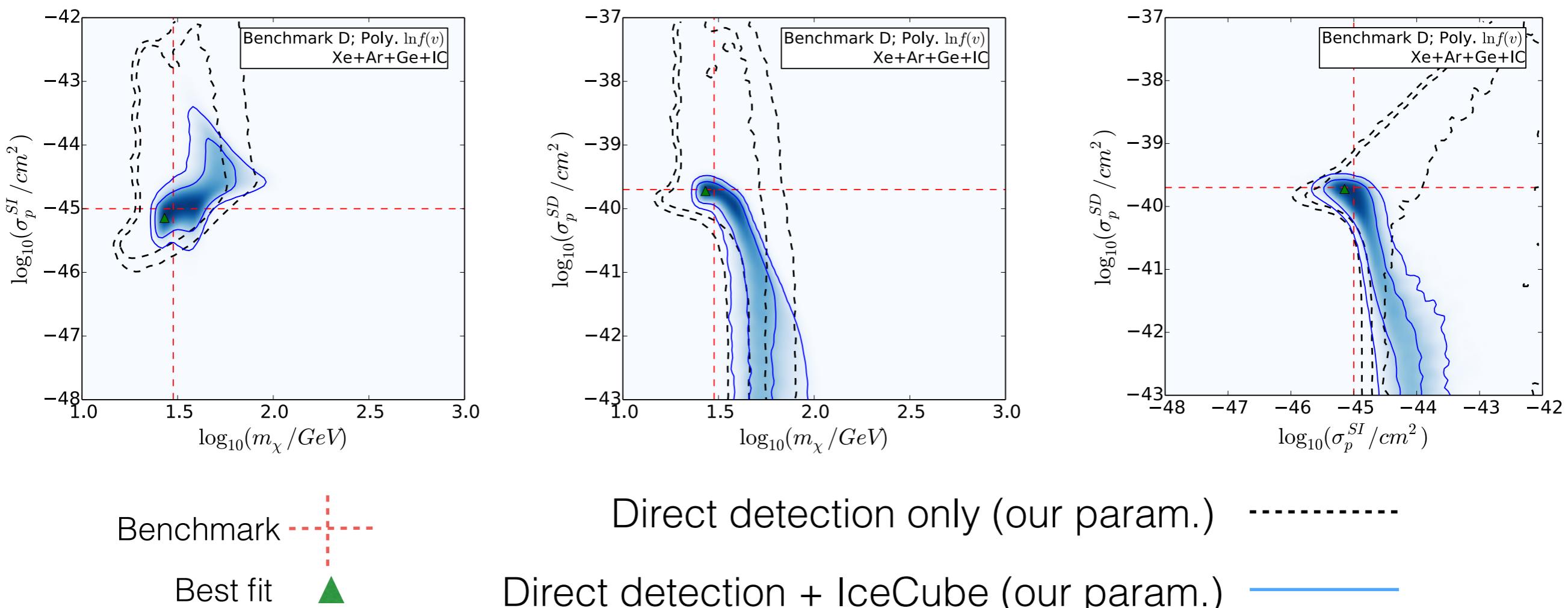
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Direct detection and IceCube

Consider a single benchmark:

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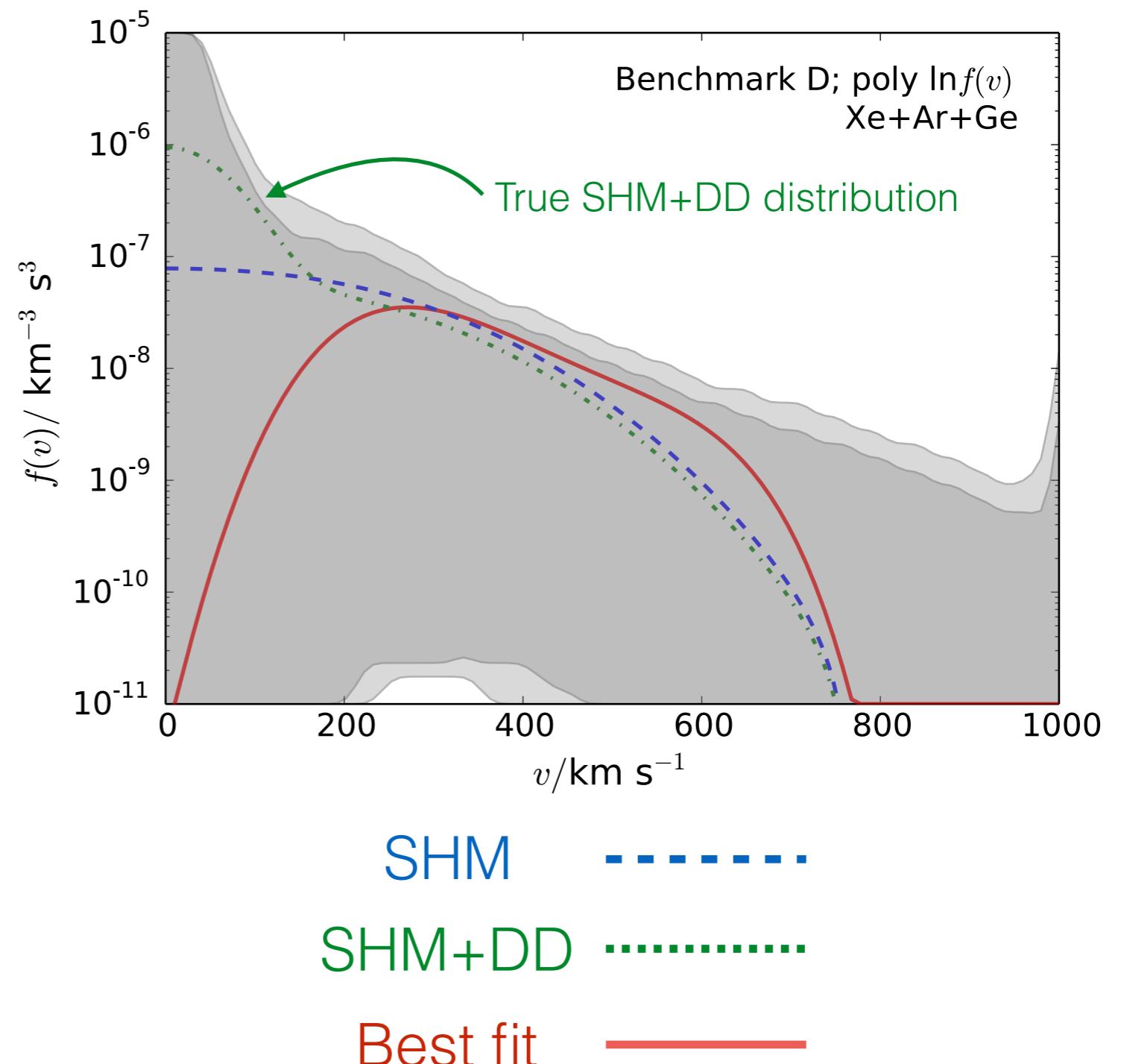


Reconstructing $f(v)$ - DD only

Use constraints on $\{a_k\}$ to construct confidence intervals on $f(v)$

Note: strong correlations between intervals at different values of v

With direct detection only, constraints are very weak



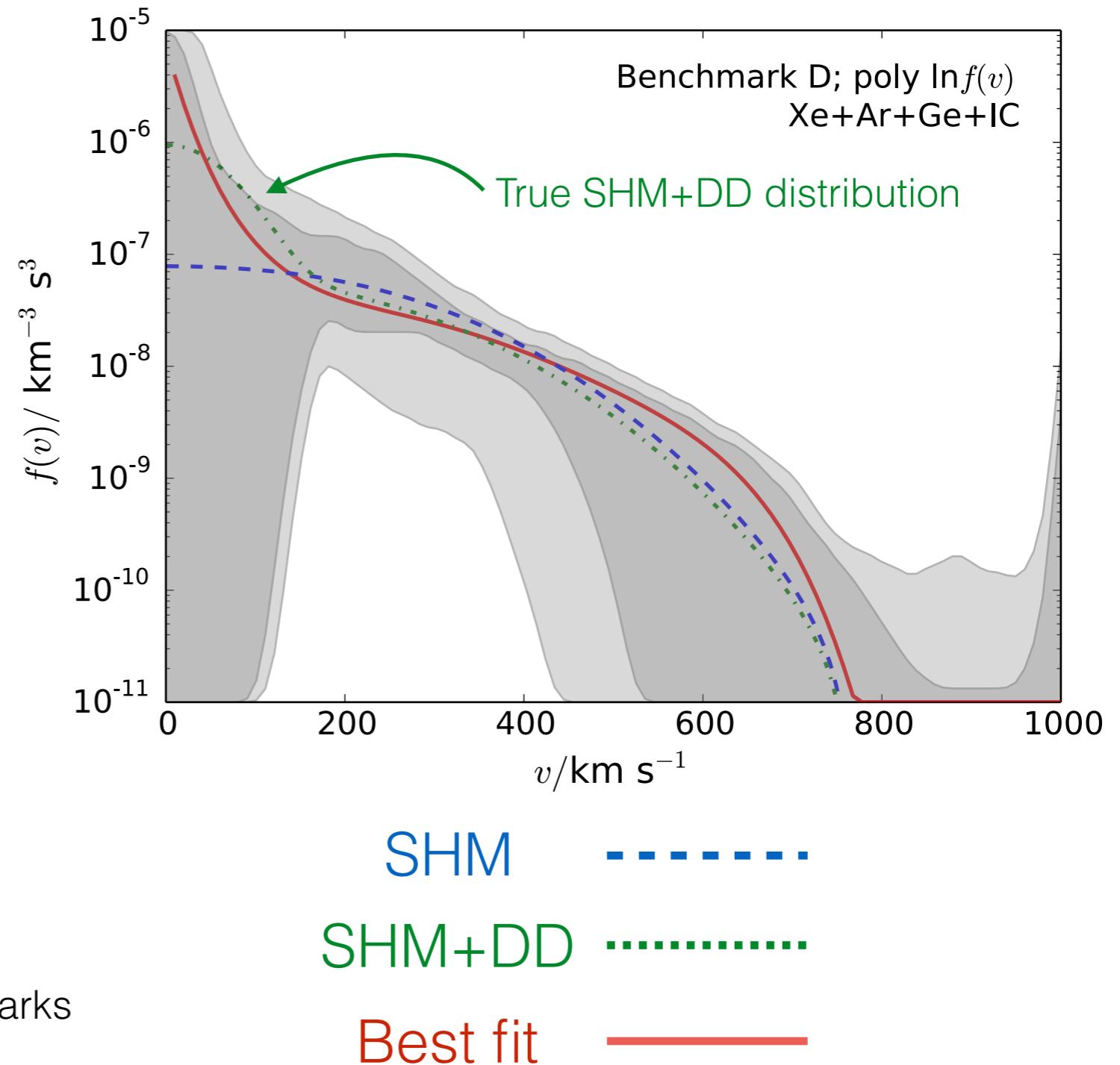
Reconstructing $f(v)$ - DD + IceCube

Addition of IceCube significantly improves constraints on $f(v)$ (factor of ~ 4 at 300 km/s)

Best fit now traces true distribution closely over all speeds

Performing full likelihood analysis, we can exclude SHM at 3σ level.

[Using 3-5 years exposure, for benchmarks just below current sensitivity]



What next?

This method shows that astrophysical uncertainties in canonical direct detection scenarios can be entirely controlled!

So what next?

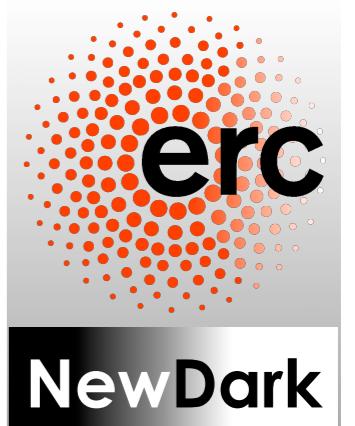
- **Directional detection** - can we extend this parametrisation to cover the full 3-D dimensional *velocity* distribution?
- **Non-standard interactions** - does this technique also work successfully for more complex (velocity dependent?) interactions?
- **Other probes** - can we constrain this parametrisation with other probes sensitive to the speed distribution (e.g. mass modelling of the Milky Way)?

Conclusions

- In the post-discovery era, we want to extract WIMP physics from direct detection experiments
- Astrophysical uncertainties were previously a serious problem in the analysis of future data
- We have presented **a new, general parametrisation** for the speed distribution, which allows us to reconstruct the WIMP mass - and the speed distribution itself
- As with all astrophysics-independent methods, we cannot pin down the cross section without information about **low-speed WIMPs**
- Neutrino telescopes should provide us with that information - allowing us to extract the **WIMP mass and cross section** in the years after the discovery of Dark Matter

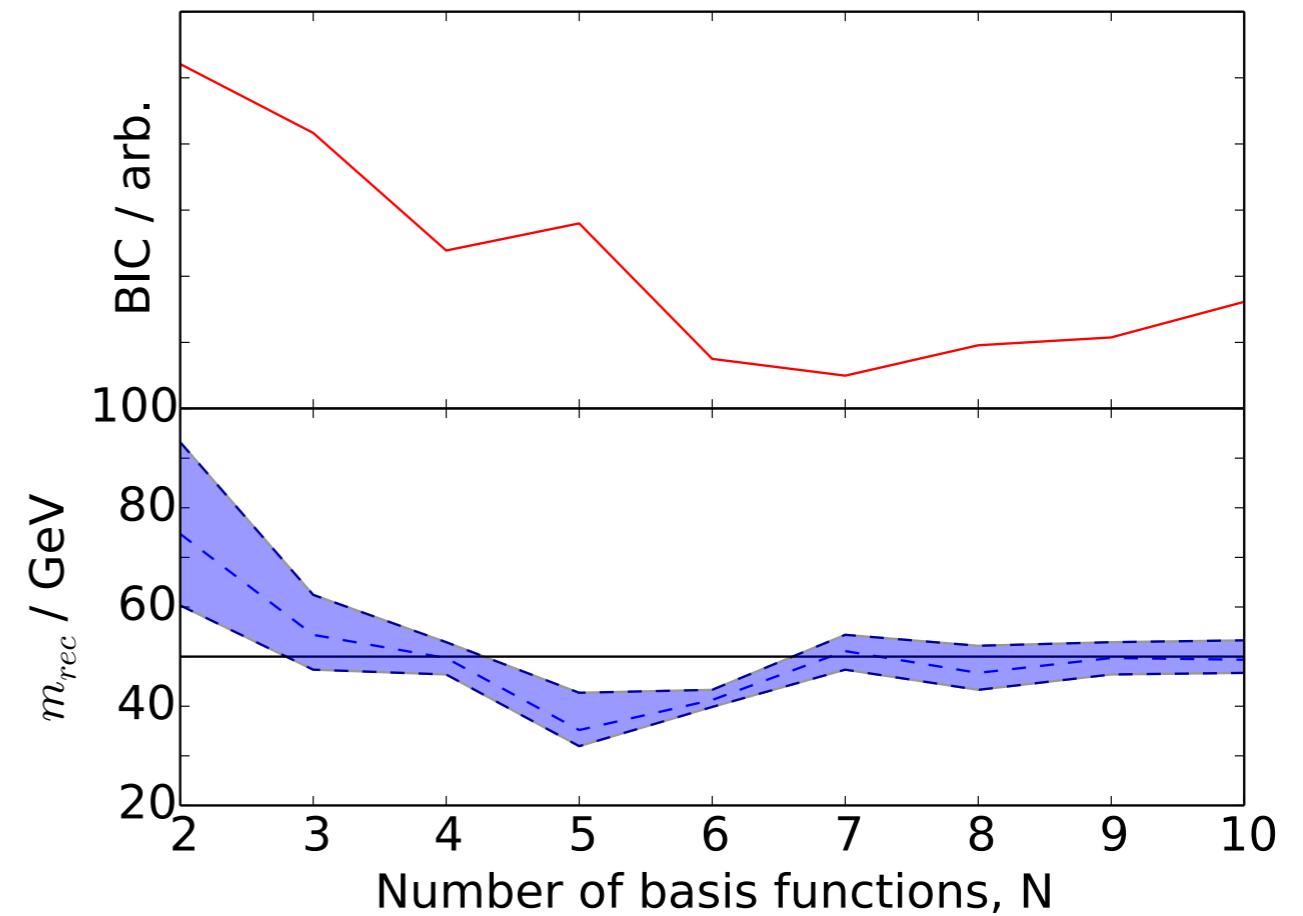
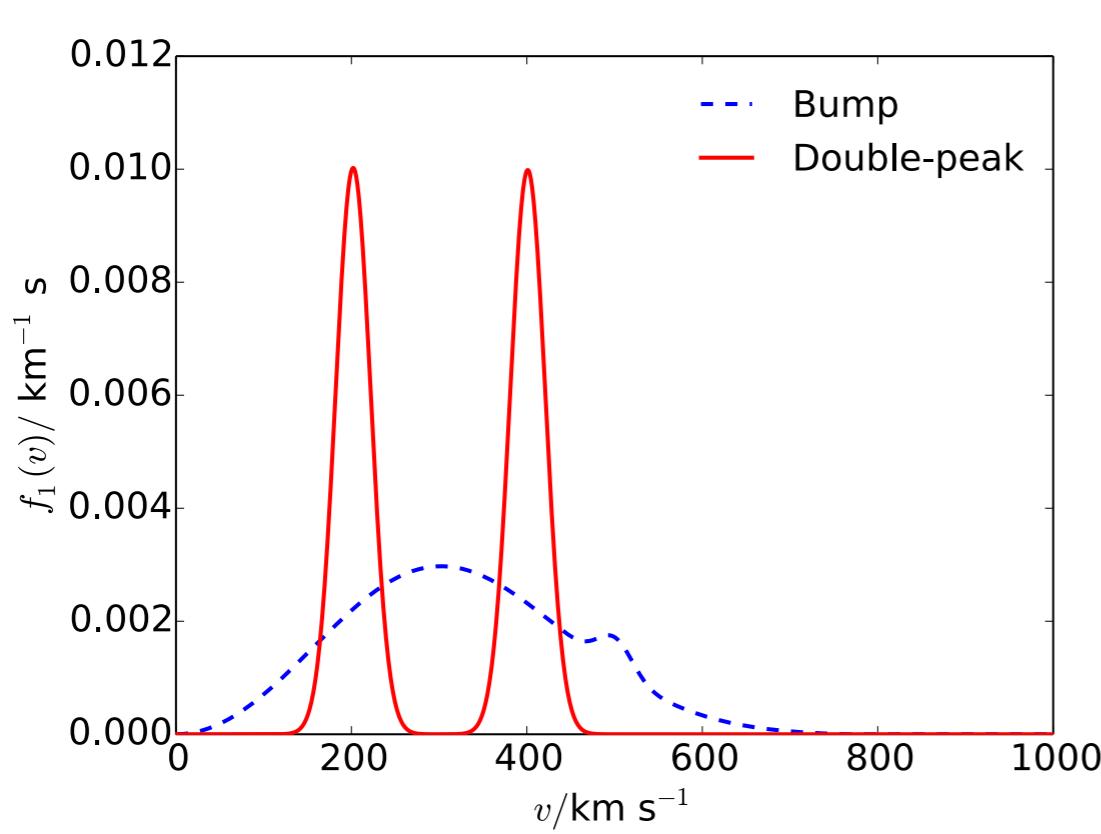
Thank you

Questions?

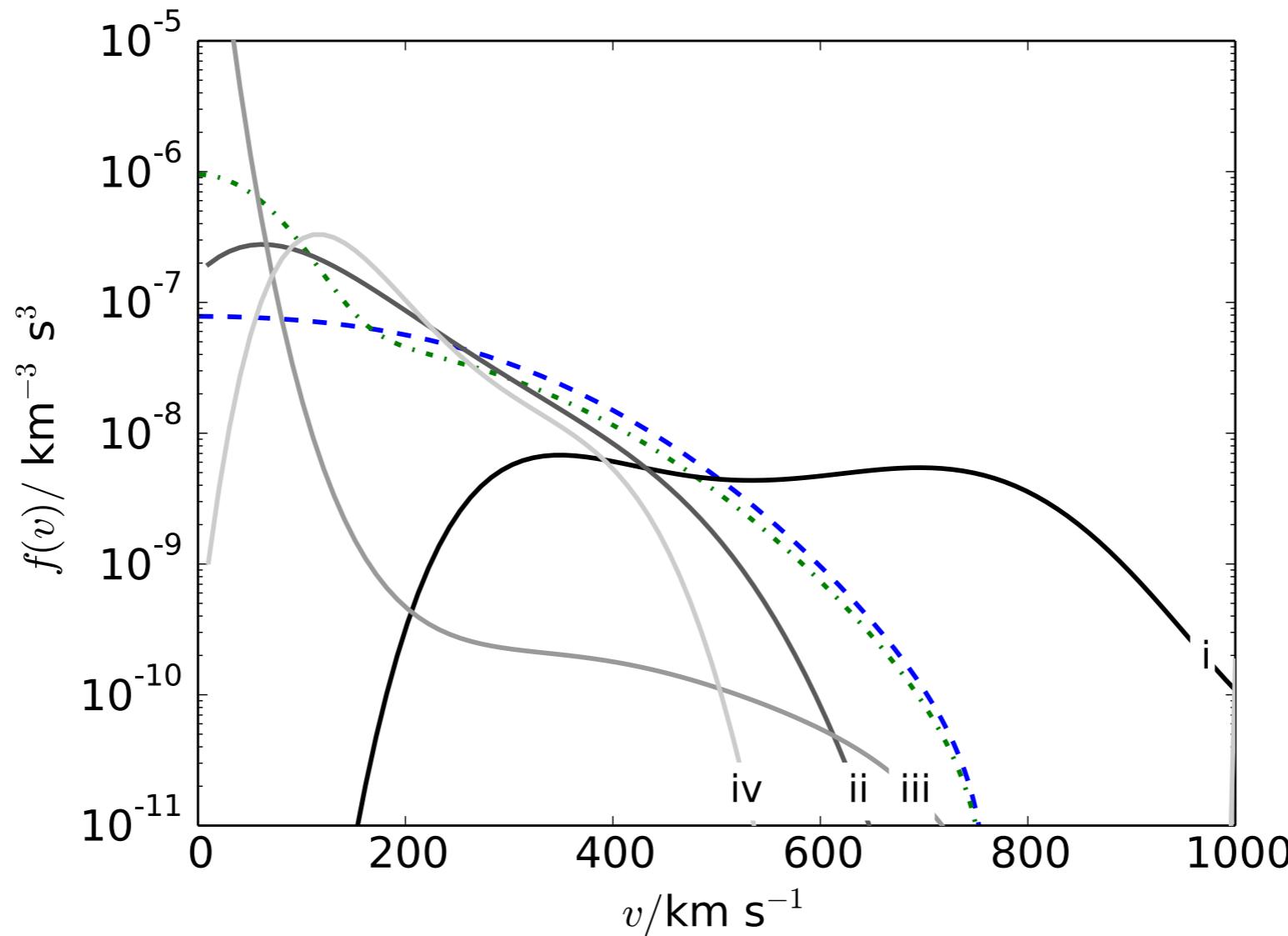


Backup Slides

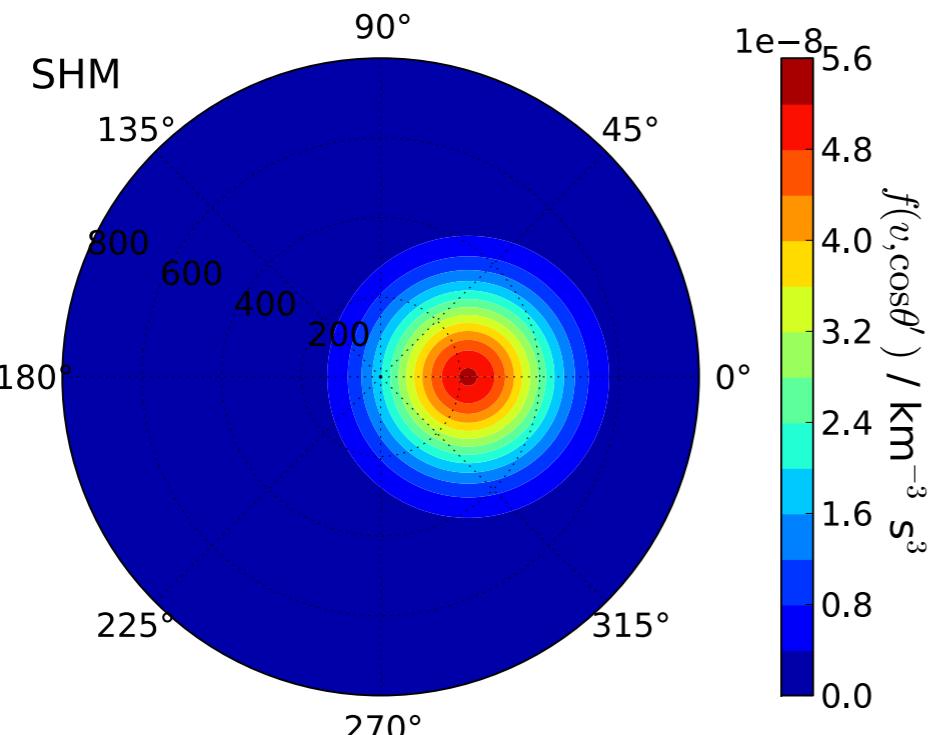
How many terms?



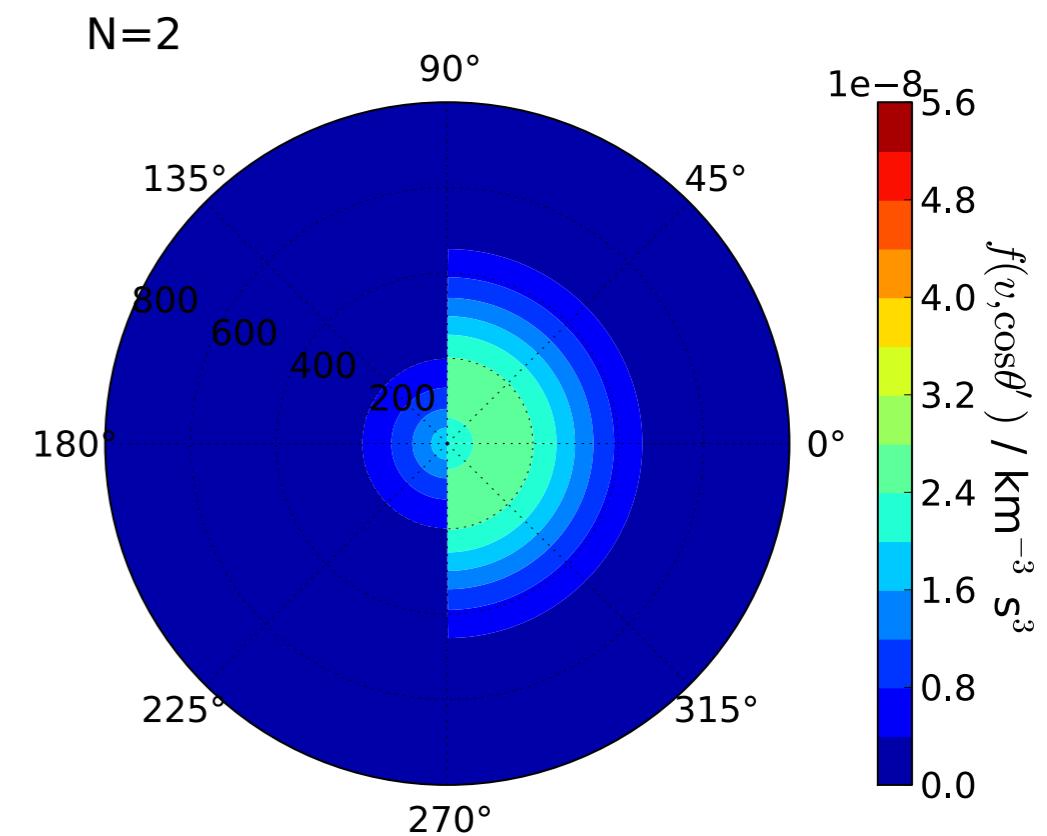
‘Shapes’ of the speed distribution



Directional detection



Discretise into forward and backwards distributions



Results

Distribution of recoils using the exact velocity distribution and the approximate (discretized) distribution (for SHM)

