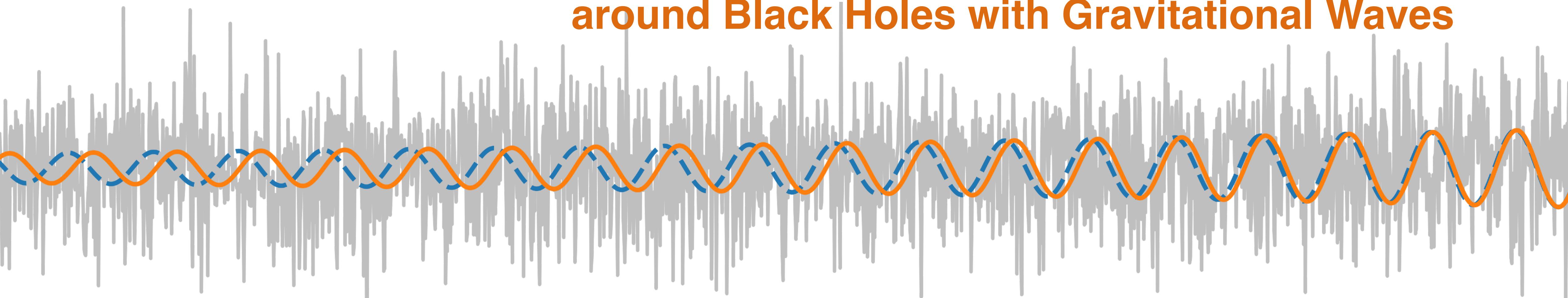


Detecting, Discovering and Measuring Dark Matter around Black Holes with Gravitational Waves

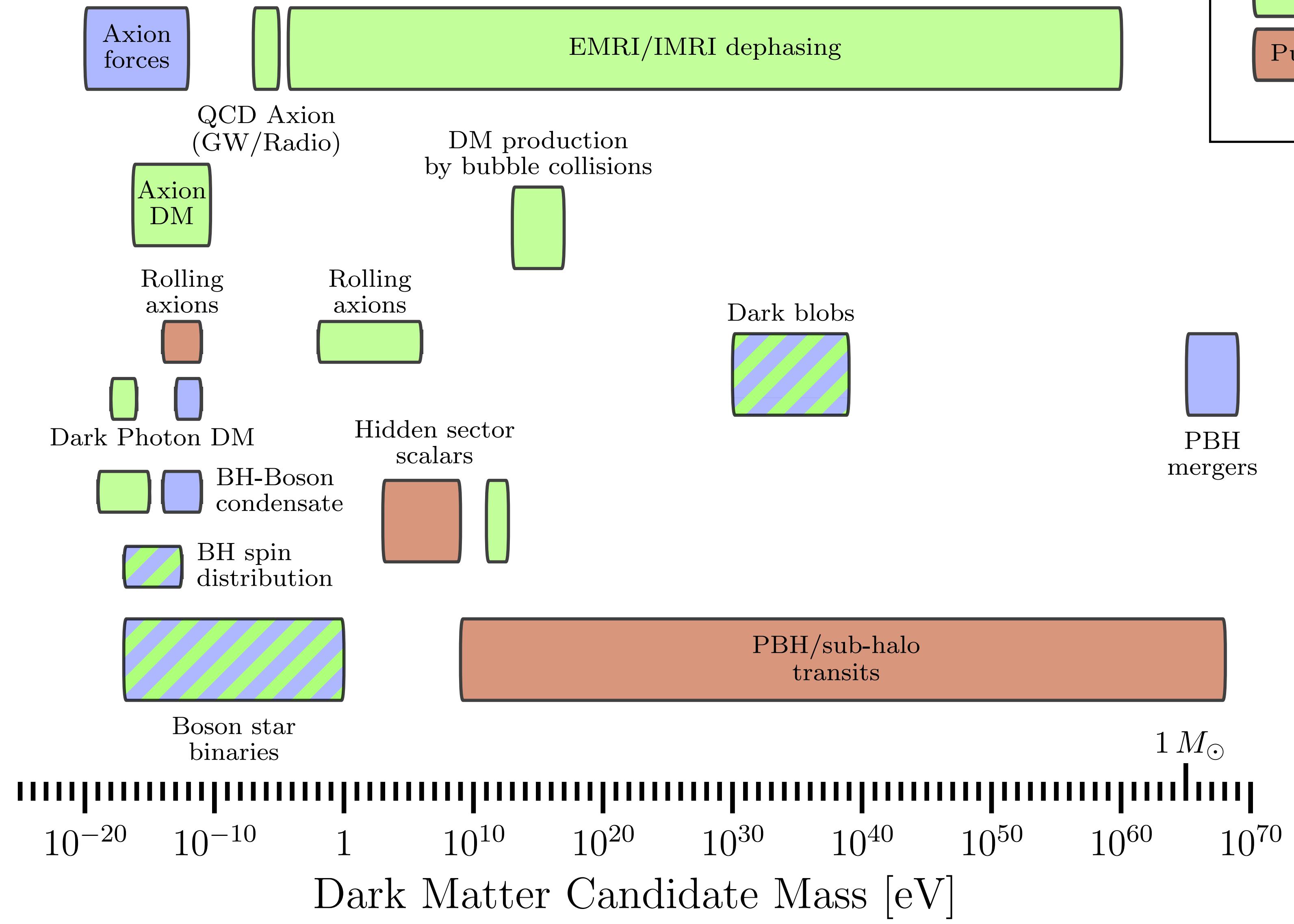


 kavanagh@ifca.unican.es

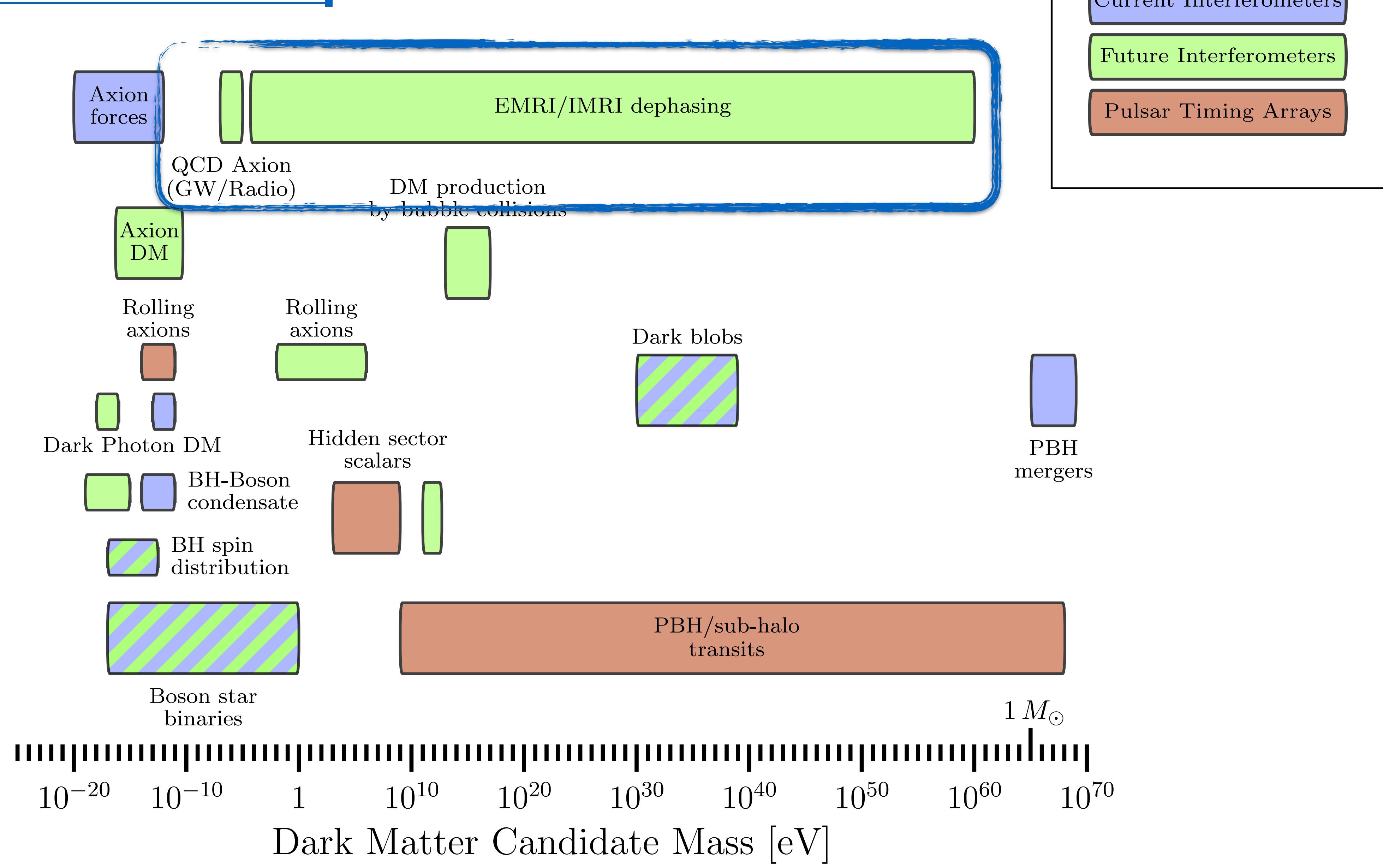
 @BradleyKavanagh

Bradley J Kavanagh
Instituto de Física de Cantabria
(CSIC-Universidad de Cantabria)
5th November 2021

GW probes of DM



GW probes of DM



Intermediate Mass Ratio Inspiral (IMRI)

Stellar mass compact object (NS/BH) inspirals towards intermediate mass black hole (IMBH)

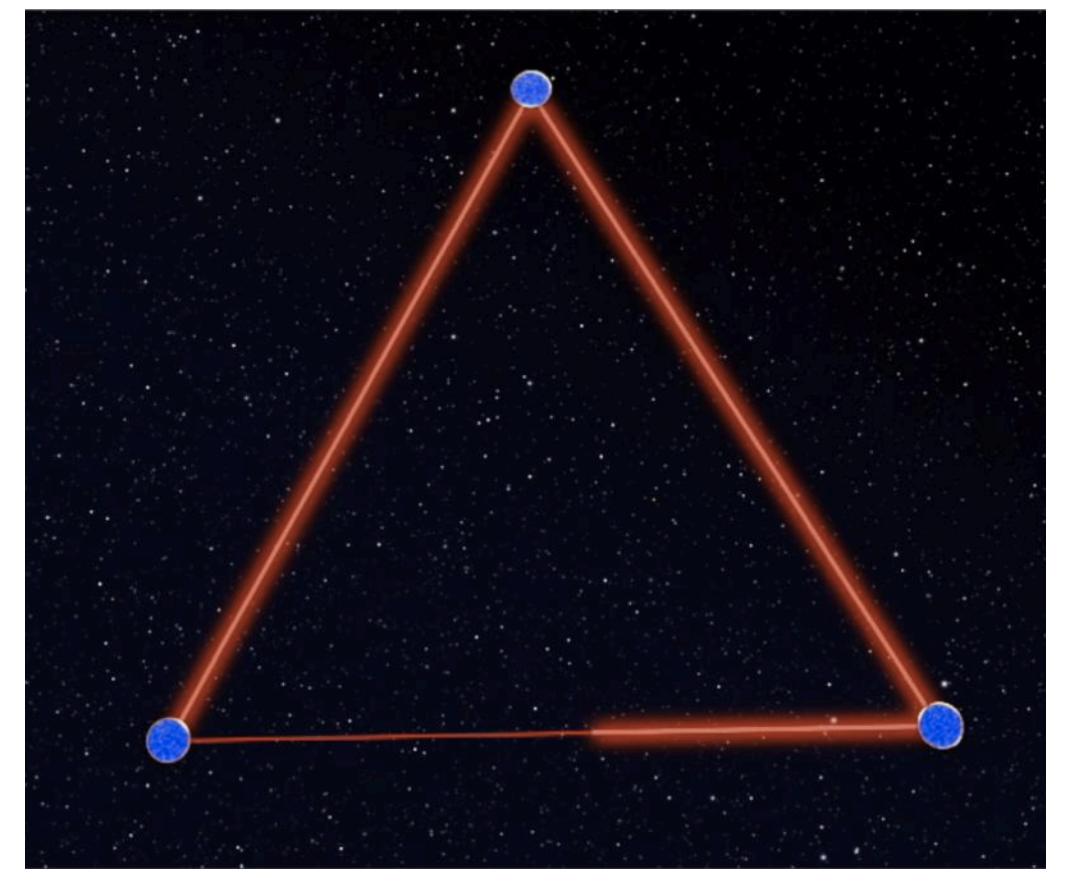
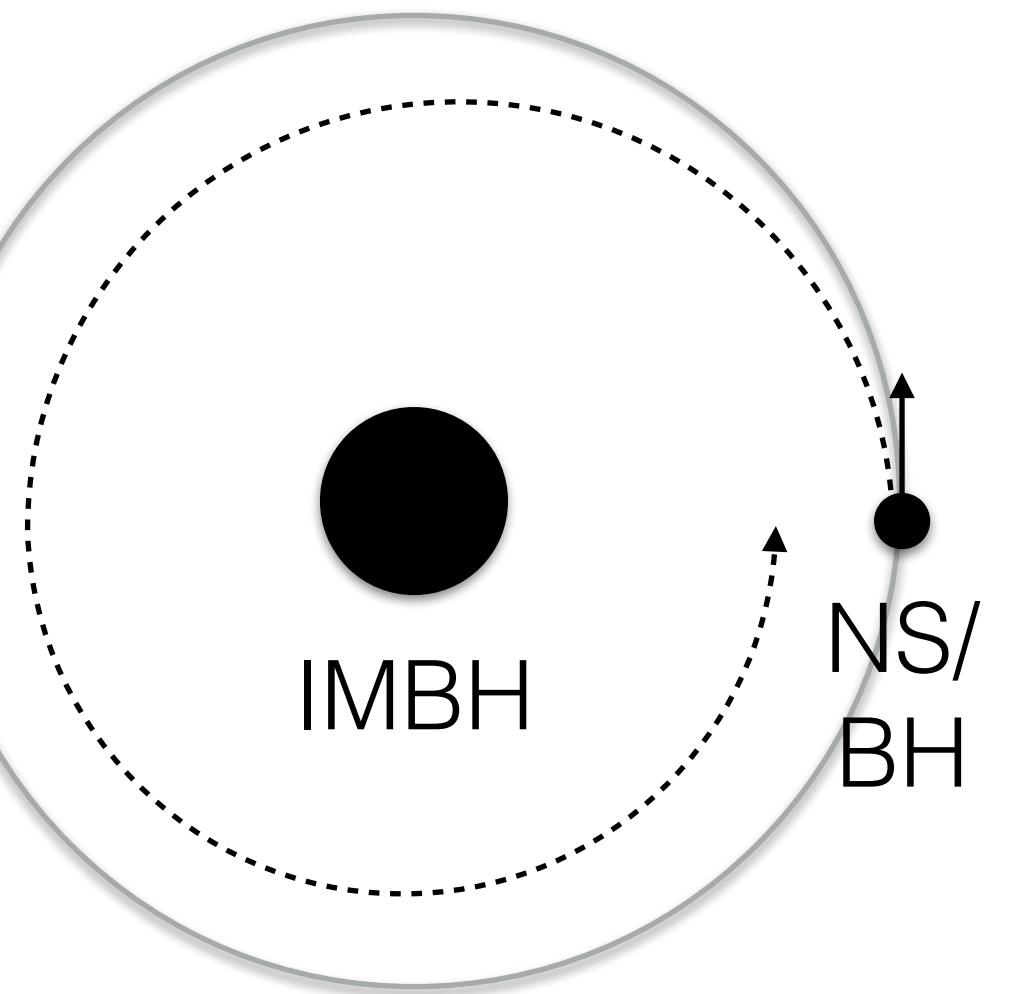
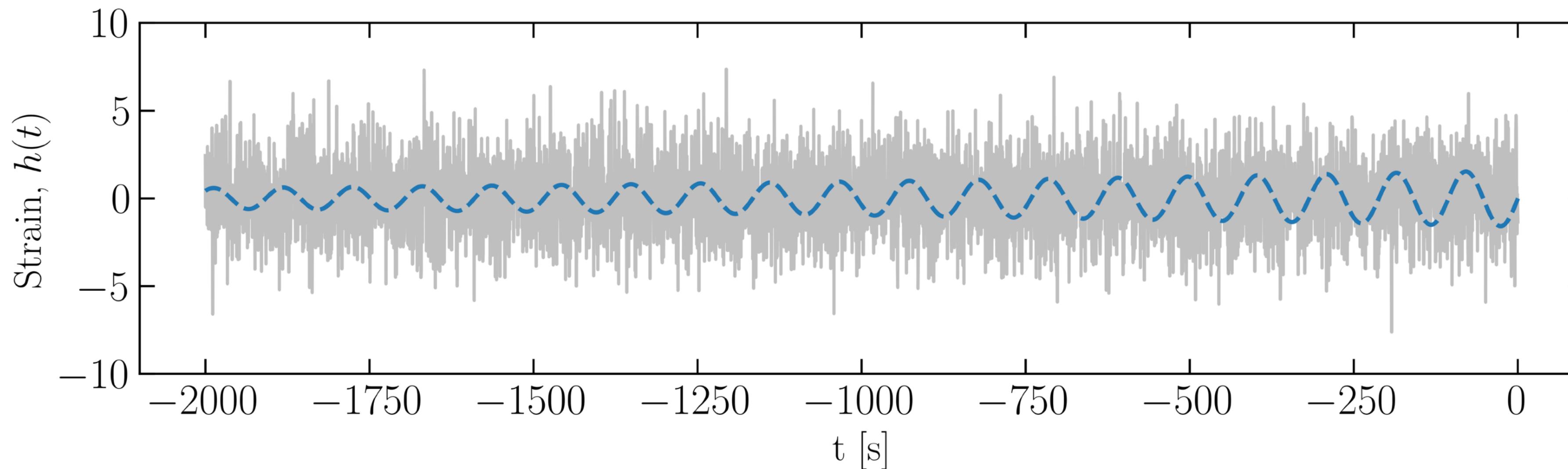
$$M_{\text{IMBH}} \sim 10^3 - 10^5 M_{\odot}$$

GW emission causes long, slow inspiral:

$$\dot{E}_{\text{GW}} \approx \frac{32G^4}{5c^5} \frac{M_{\text{IMBH}}^3 M_{\text{NS}}^2}{r^5} \propto (f_{\text{GW}})^{10/3}$$

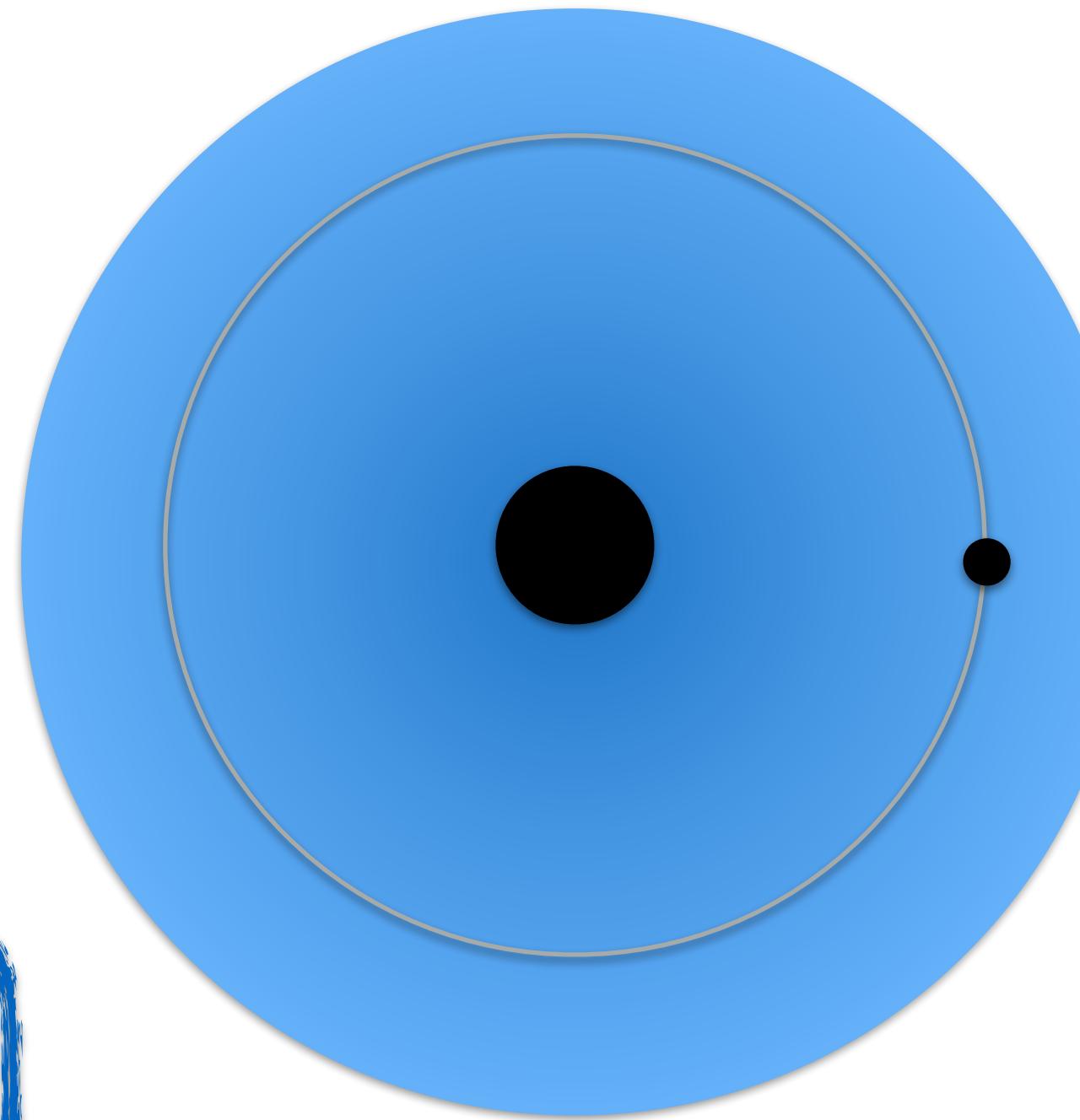
Until the innermost stable circular orbit: $f_{\text{ISCO}} = 0.44 \left(\frac{10^4 M_{\odot}}{M_1} \right) \text{ Hz}$ 

Detectable at LISA frequencies:
 $f_{\text{GW}} \sim 10^{-2} - 1 \text{ Hz}$



Dark Matter Spikes

Consider now a cold **DM ‘spike’** or **‘dress’** around the central BH (not to be confused with ultralight boson clouds).



Study the following benchmarks:

$$m_1 = 10^3 M_\odot$$

$$m_2 = 1 M_\odot$$

$$\rho_{\text{DM}} = \rho_6 \left(\frac{10^{-6} \text{ pc}}{r} \right)^{\gamma_{\text{sp}}}$$

Astrophysical scenario

$$\gamma_{\text{sp}} = 7/3 \approx 2.3333\dots$$

$$\rho_6 \approx 5.45 \times 10^{15} M_\odot \text{ pc}^{-3}$$

...depending on a number of environmental factors...

[[astro-ph/9906391](#), [astro-ph/0509565](#),
[1305.2619](#), ...]

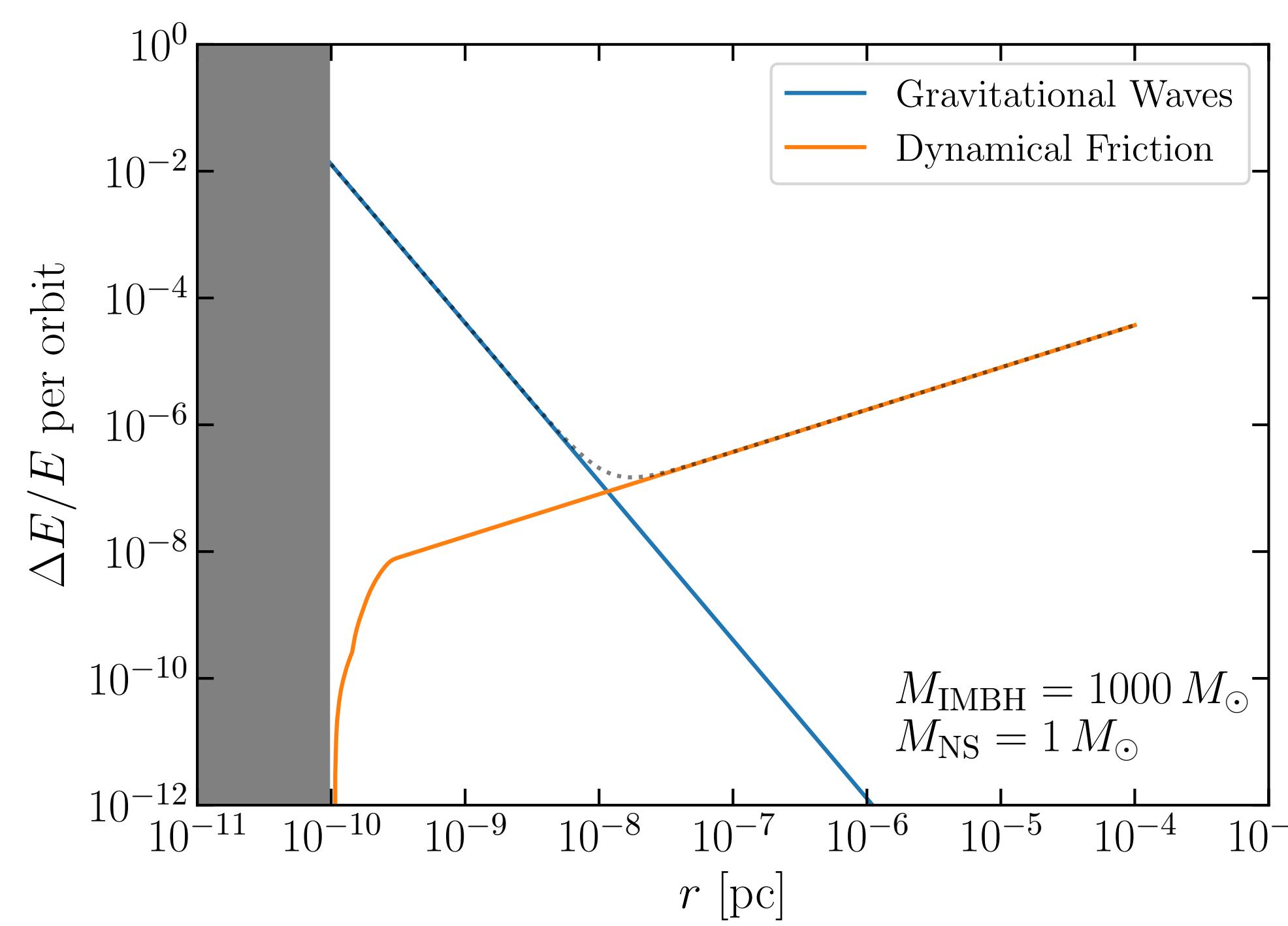
PBH scenario

$$\gamma_{\text{sp}} = 9/4 \approx 2.25$$

$$\rho_6 \approx 5.35 \times 10^{15} M_\odot \text{ pc}^{-3}$$

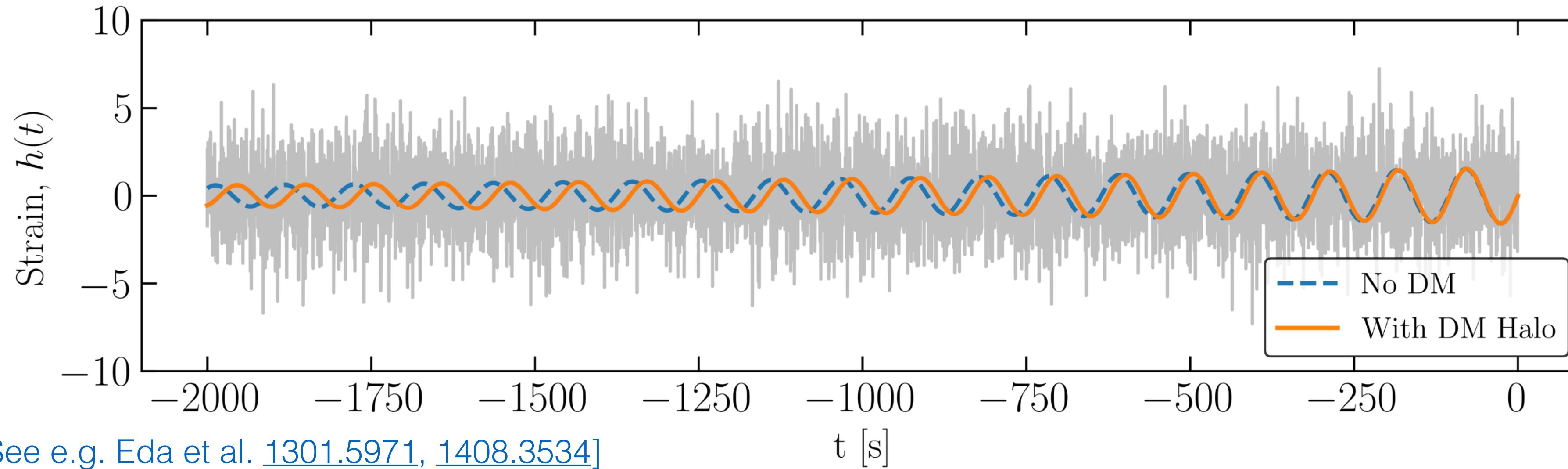
[[Bertschinger \(1985\)](#), [astro-ph/0608642](#),
[1901.08528](#), ...]

IMRI + Dark Matter



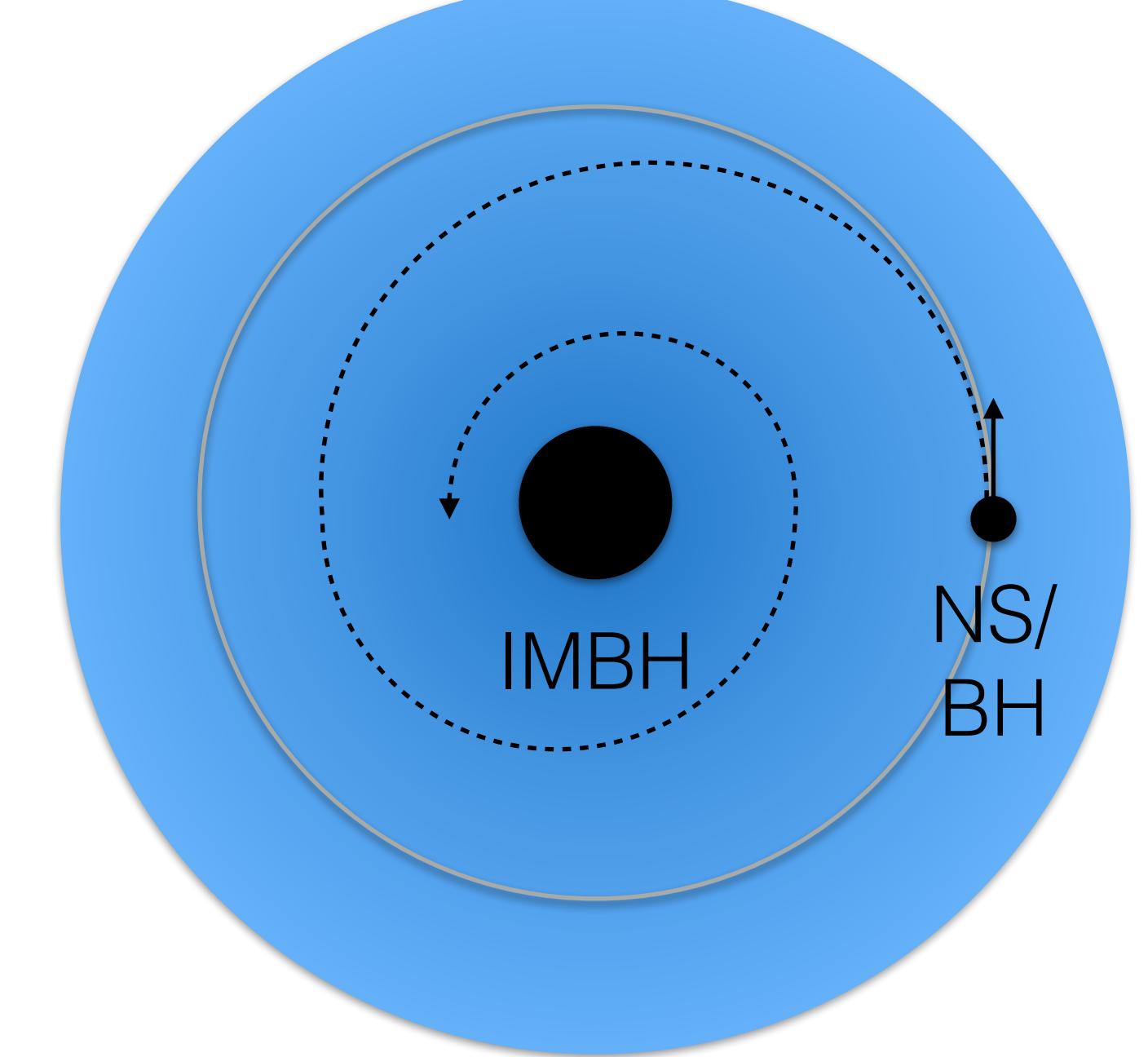
$$-\dot{E}_{\text{orb}} = \dot{E}_{\text{GW}} + \dot{E}_{\text{DF}}$$

$$\dot{E}_{\text{DF}} \sim \frac{4\pi G^2 M_{\text{NS}}^2 \xi(v) \rho_{\text{DM}}(r)}{v_{\text{NS}}} \ln \Lambda \propto (f_{\text{GW}})^{\frac{2}{3}\gamma - 3}$$



[\[Chandrasekhar, 1943\]](#)

[See e.g. Eda et al. [1301.5971](#), [1408.3534](#)]

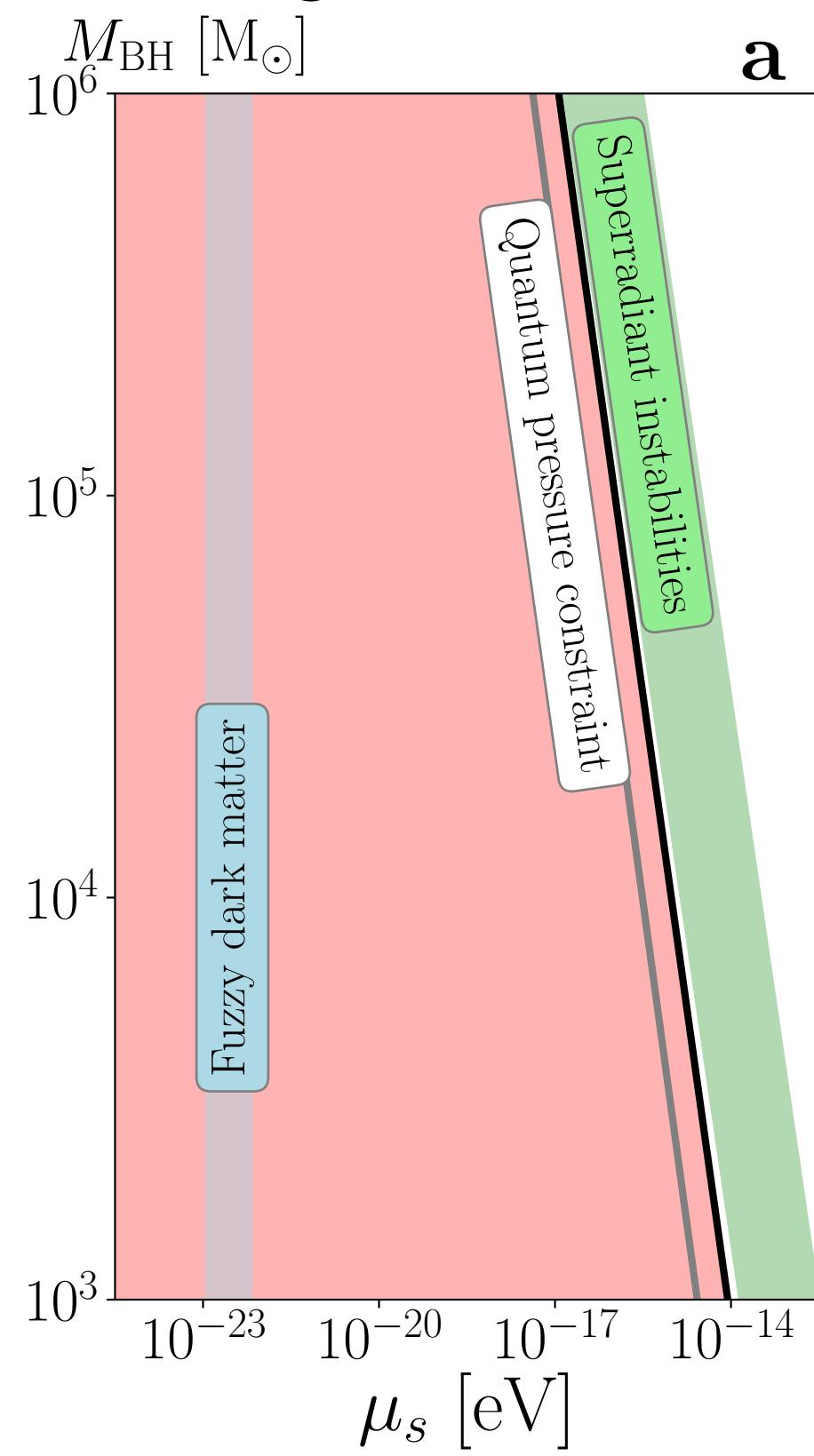


Nature of Dark Matter

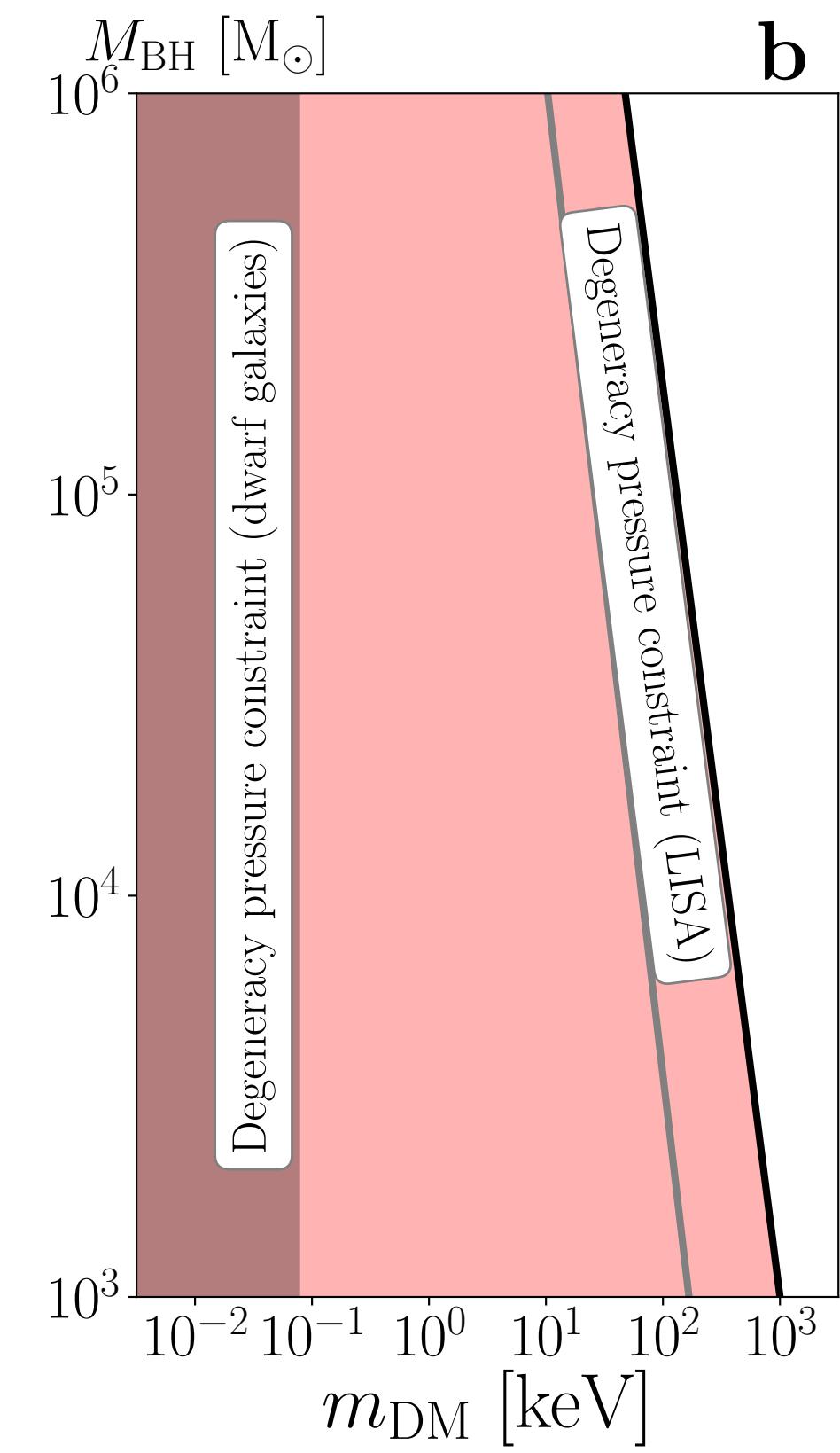
Red regions would be ruled out by observation of a DM spike!

[1906.11845]

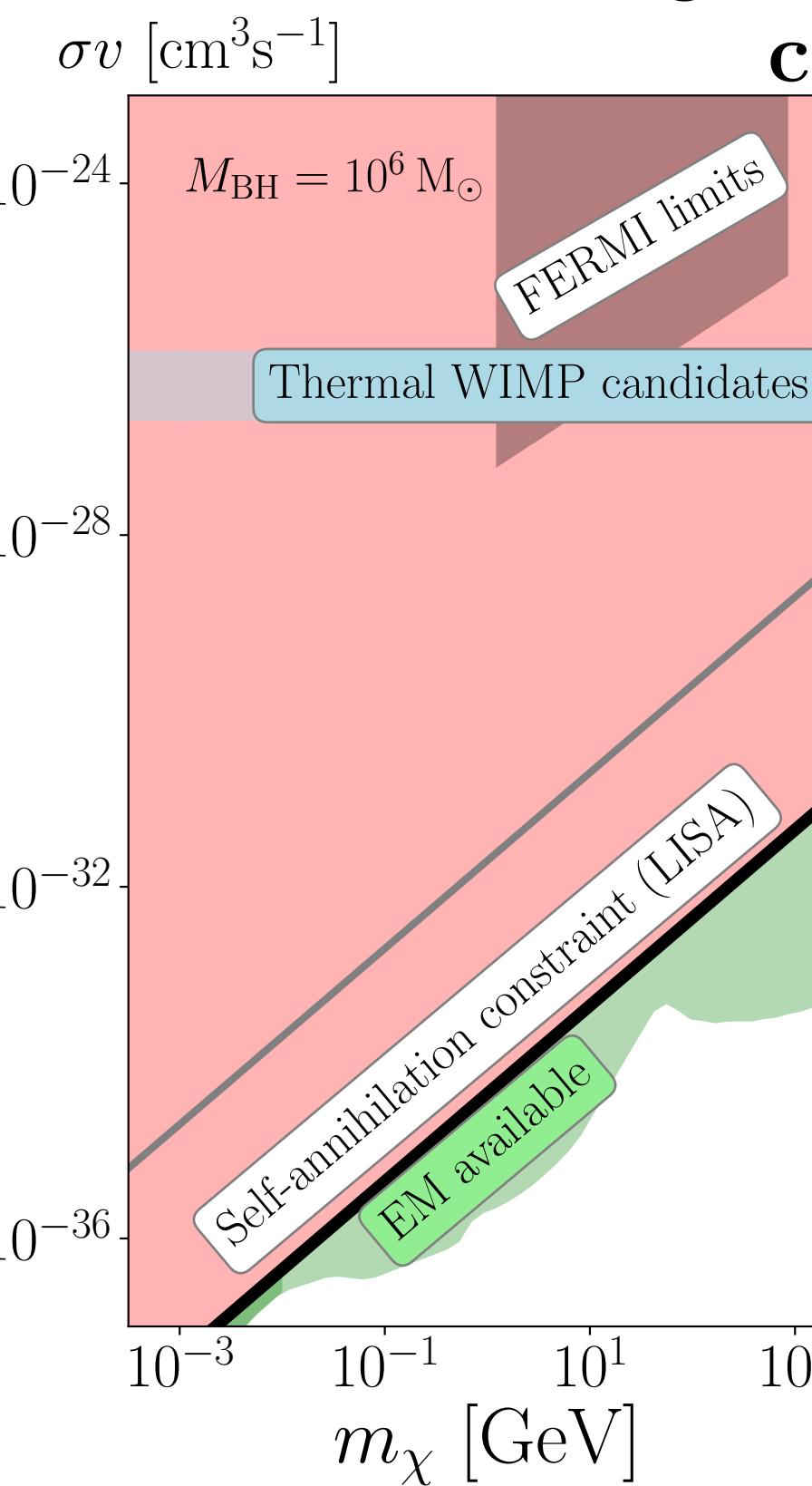
Ultralight bosonic DM



Fermionic DM

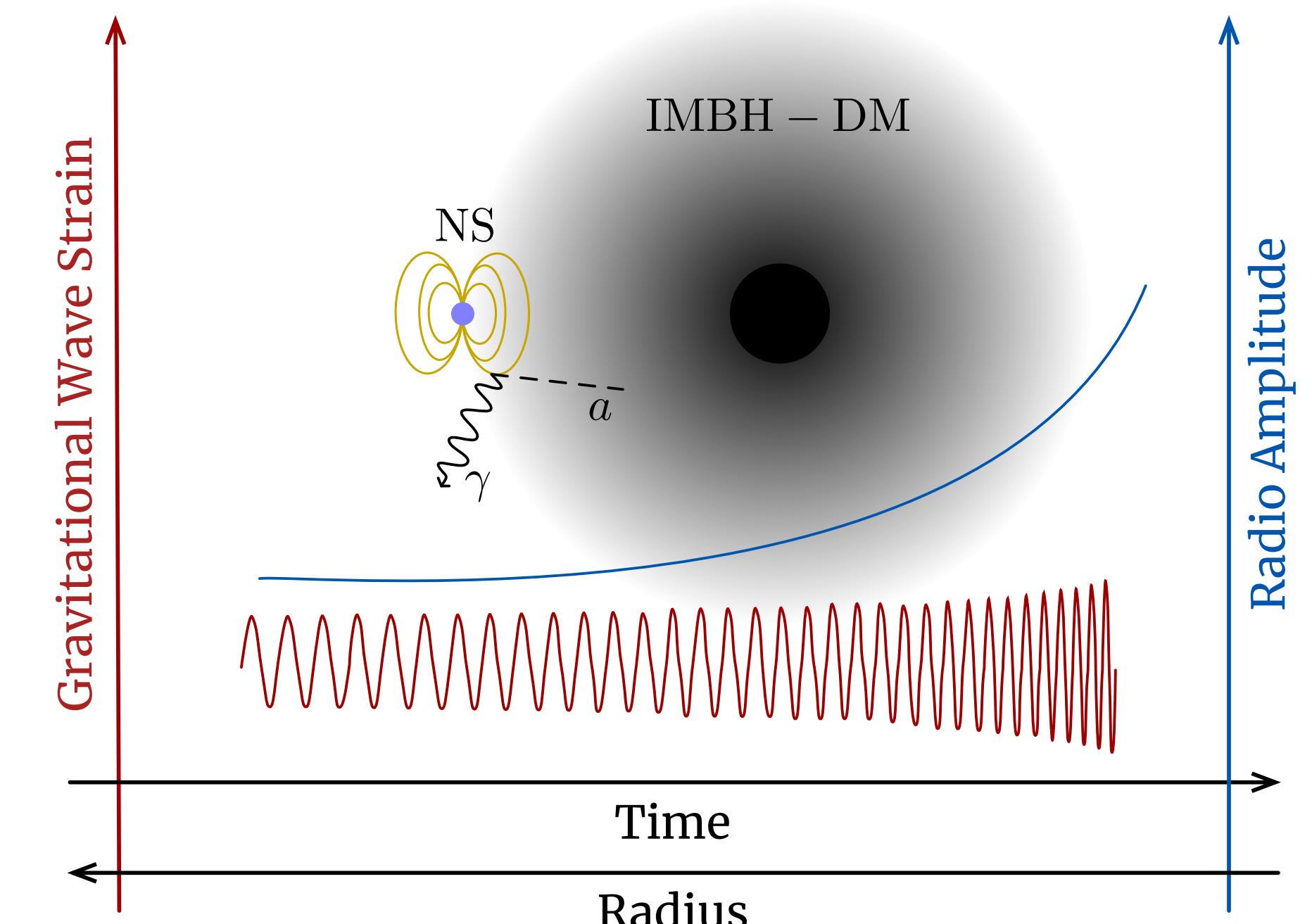


Self-annihilating DM



[See also Bertone, Coogan, Gaggero, **BJK** & Weniger, 1905.01238]

DM dephasing offers a unique multi-messenger signature of e.g. QCD axion DM



[Edwards, Chianese, **BJK**, Nissanke & Weniger, 1905.04686]

BUT - need to model the signal very carefully...

Halo Feedback

Follow semi-analytically the phase space distribution of DM:

$$f = \frac{dN}{d^3\mathbf{r} d^3\mathbf{v}} \equiv f(\mathcal{E})$$

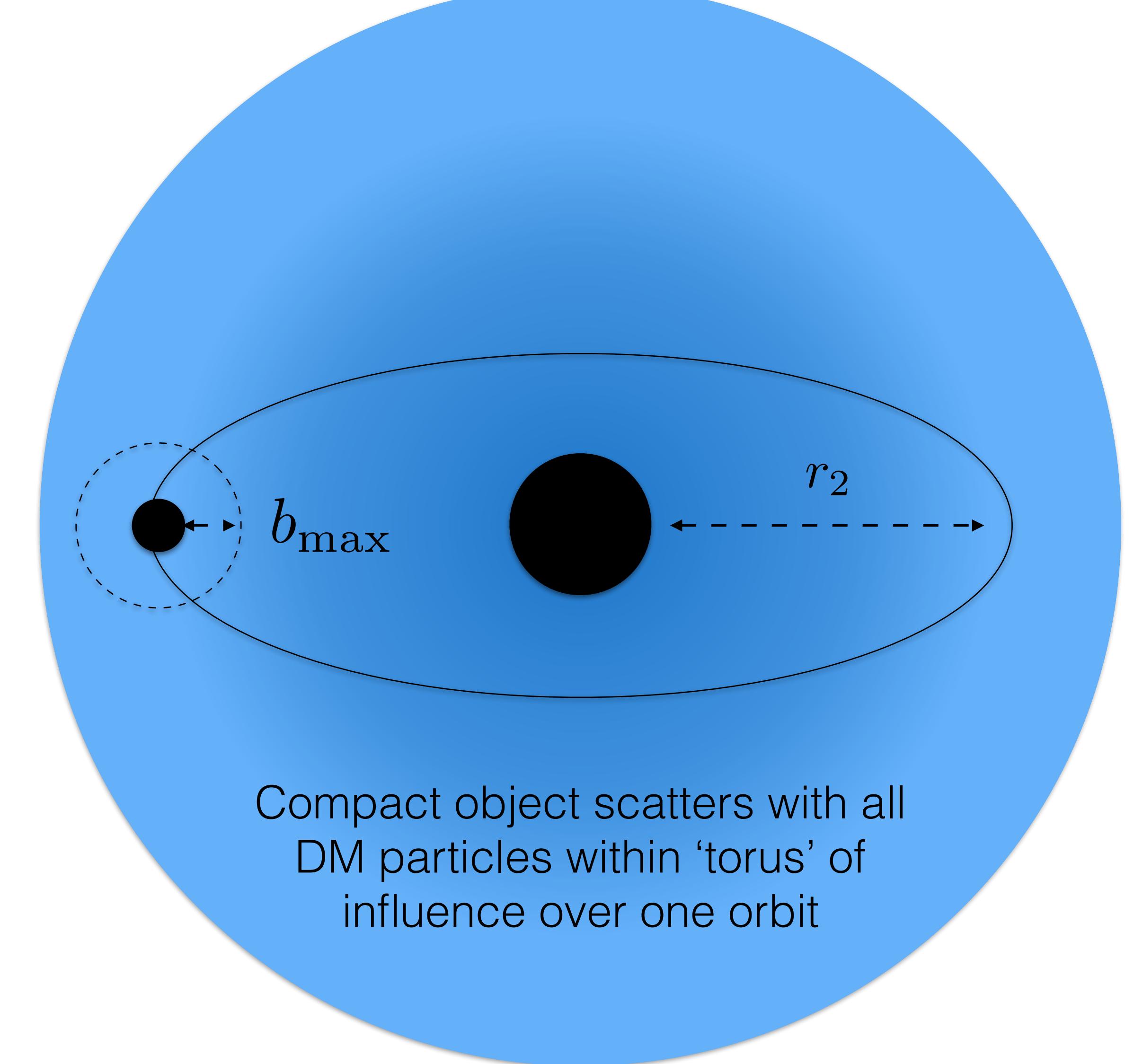
$$\mathcal{E} = \Psi(r) - \frac{1}{2}v^2$$

Each particle receives a ‘kick’ through gravitational scattering

$$\mathcal{E} \rightarrow \mathcal{E} + \Delta\mathcal{E}$$

Reconstruct density from distribution function:

$$\rho(r) = \int d^3\mathbf{v} f(\mathcal{E})$$



[**BJK**, Nichols, Gaggero, Bertone, [2002.12811](#)]

[Code available online:
github.com/bradkav/HaloFeedback]

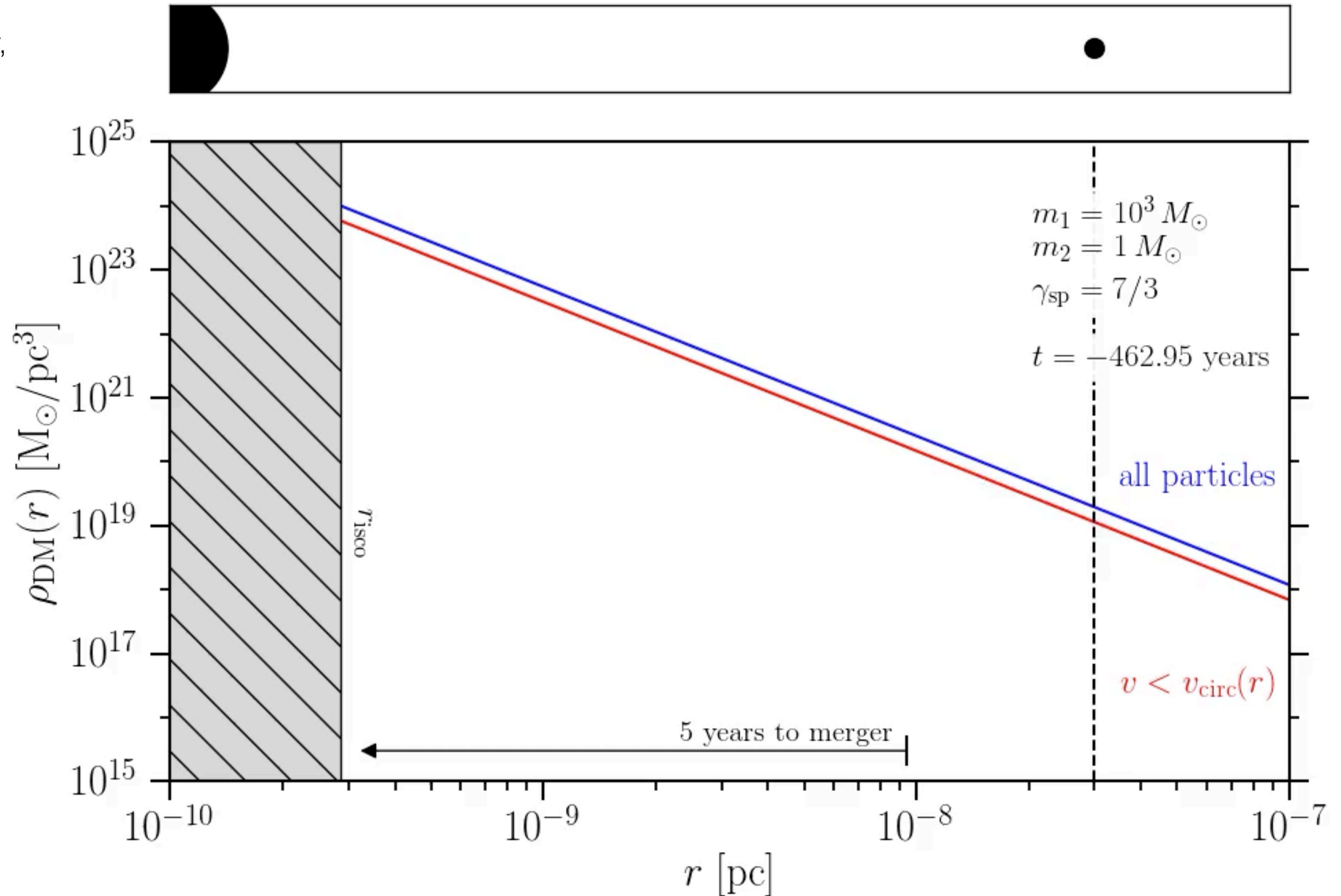
Full evolution of the system

Newtonian motion of the binary,
Taking into account:

- GW emission
- Dynamical Friction
- DM Halo Feedback

Density of the DM spike is
depleted (and replenished...)

This is one of the reasons we
want to look at IMRIs/EMRIs...



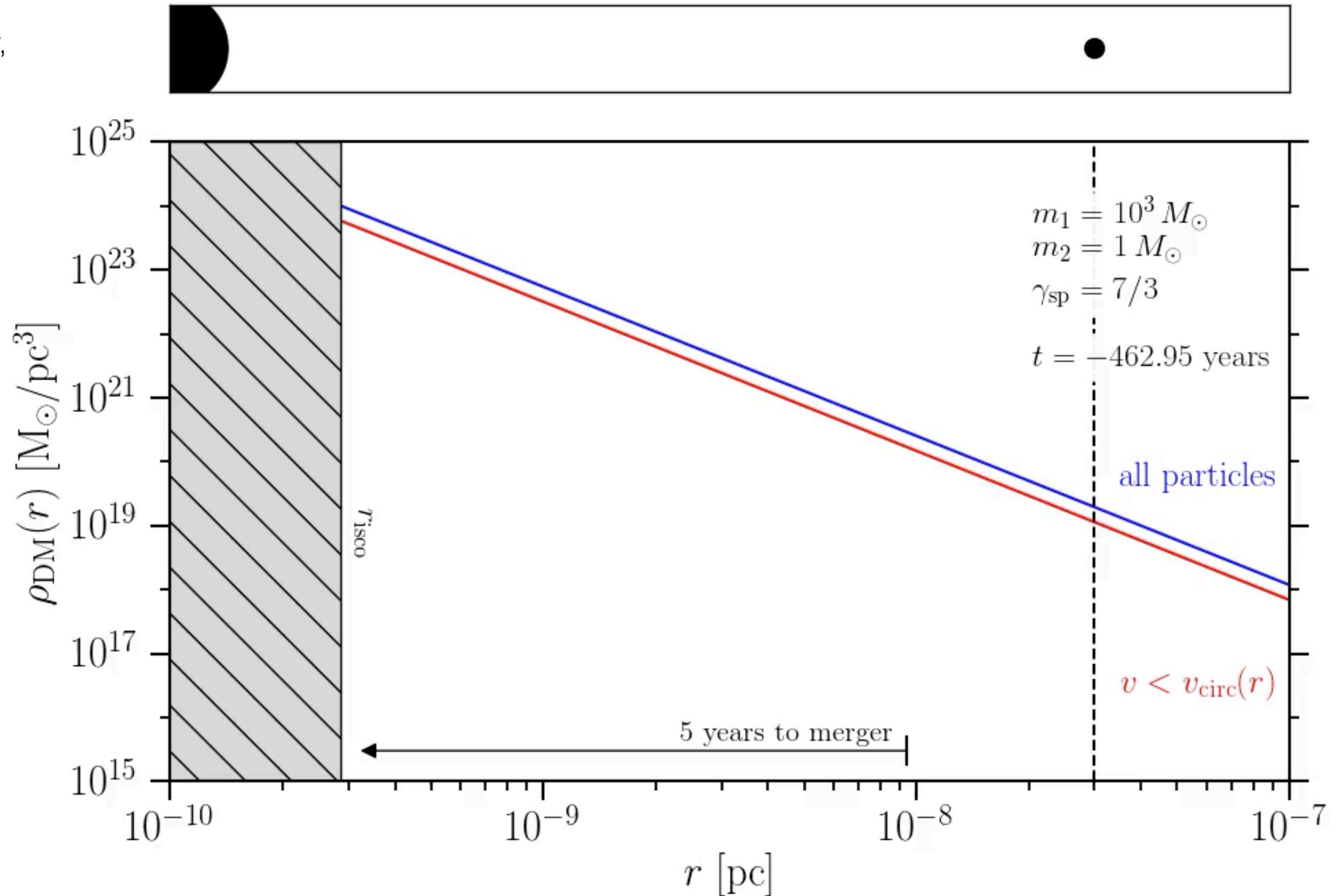
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Self-consistent dephasing

[BJK, Nichols, Gaggero, Bertone, [2002.12811](#)]

Consider our astro benchmark system,
starting at some initial separation:

$$r_{\text{ini}} \sim 10^{-8} \text{ pc}$$

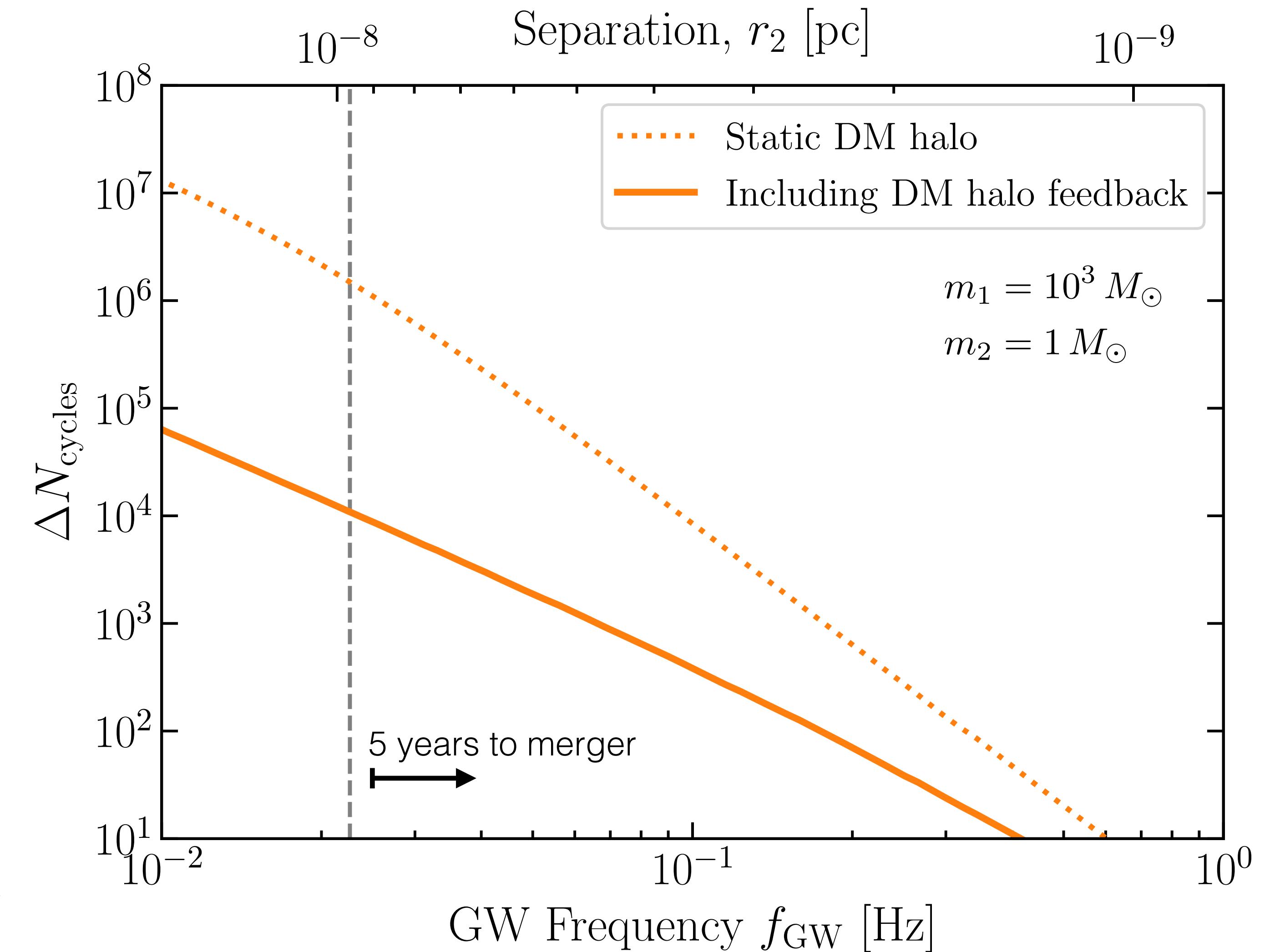
$$t_{\text{merge}}^{\text{vacuum}} \sim 5 \text{ yr}$$

$$N_{\text{cycles}}^{\text{vacuum}} \sim 6 \times 10^6$$

DM dephasing

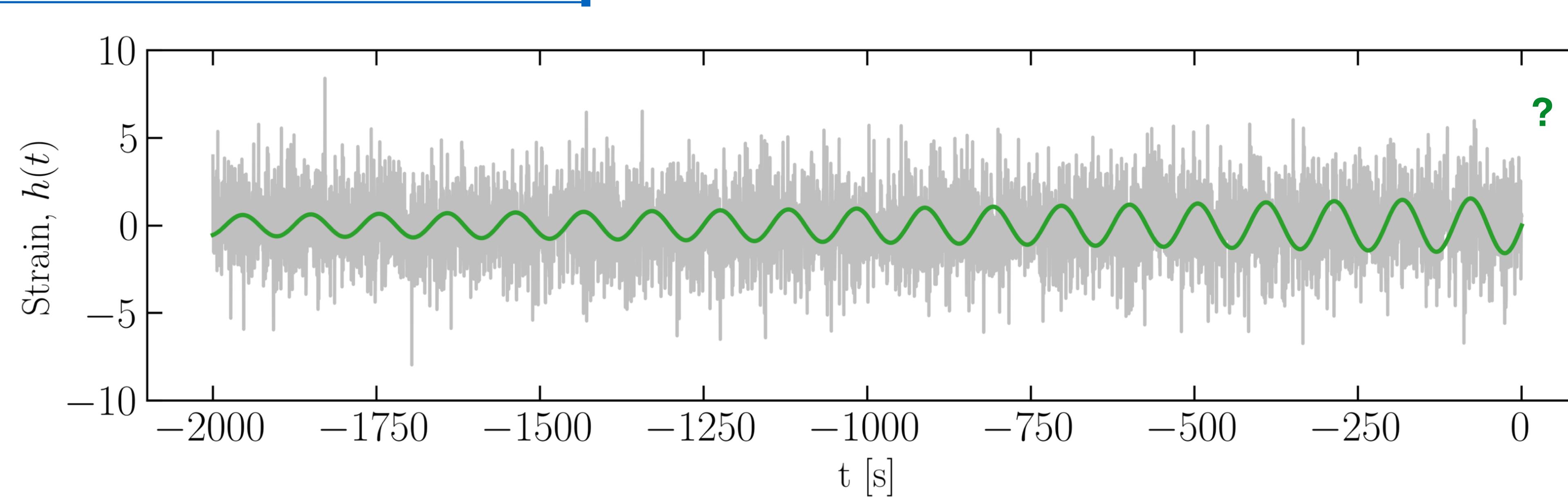
$$\Delta N_{\text{cycles}} \sim \mathcal{O}(10^4) \text{ cycles} \sim \% \text{ level}$$

Change in the number of GW cycles to merger,
starting at some initial frequency/separation:



A more realistic scenario

[Coogan, Bertone, Gaggero, **BJK** & Nichols, [2108.04154](#)]



Want to address questions of:

- **Detectability** - is the event loud enough to detect?
- **Discoverability** - can we tell it apart from a *GR-in-vacuum* waveform?
- **Measurability** - can we pin down the properties of the system (*especially the DM*)?

Detectability

A signal may be **detectable** with LISA using matched filtering with a signal-to-noise ratio (SNR) $\gtrsim 15\dots$

[\[1905.11998\]](#)

Match between waveforms a and b
defined as:

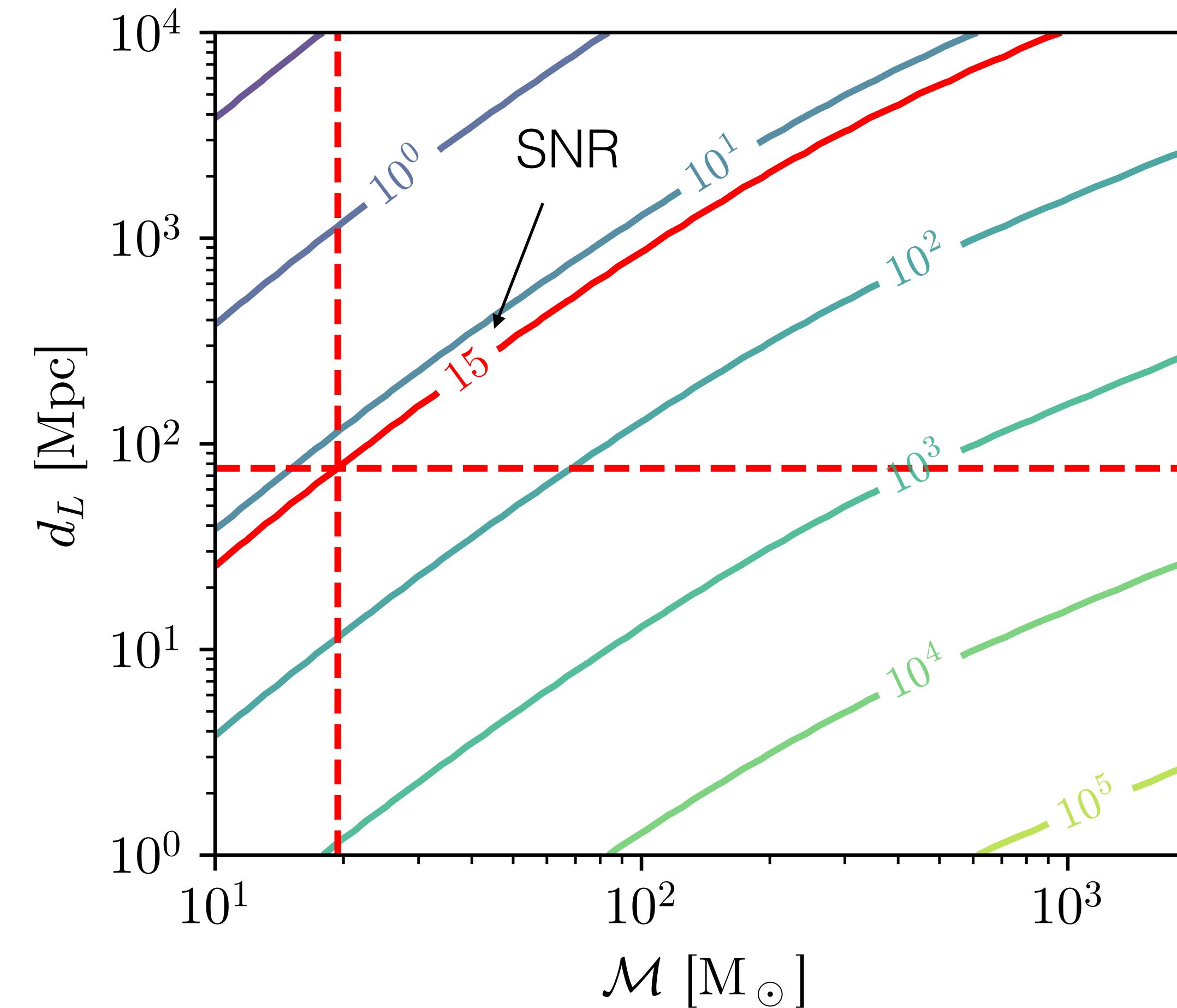
$$\langle a | b \rangle = 4 \operatorname{Re} \int_0^\infty df \frac{\tilde{a}(f)^* \tilde{b}(f)}{S_n(f)}$$

LISA noise curve

Optimal SNR for waveform s is then:

$$\text{SNR}(s) = \sqrt{\langle s | s \rangle}$$

NB: Presence of the dark dress
does not substantially affect SNR



$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

Discoverability

$$q = m_2/m_1$$

We'll call a DM spike **discoverable** if it can be distinguished from a GR-in-vacuum system.

Compare Bayesian evidence for
Vacuum and **D**ressed systems:

$$\theta_V = \{\mathcal{M}\}$$

$$\theta_D = \{\gamma_{\text{sp}}, \rho_6, \mathcal{M}, \log_{10} q\}$$

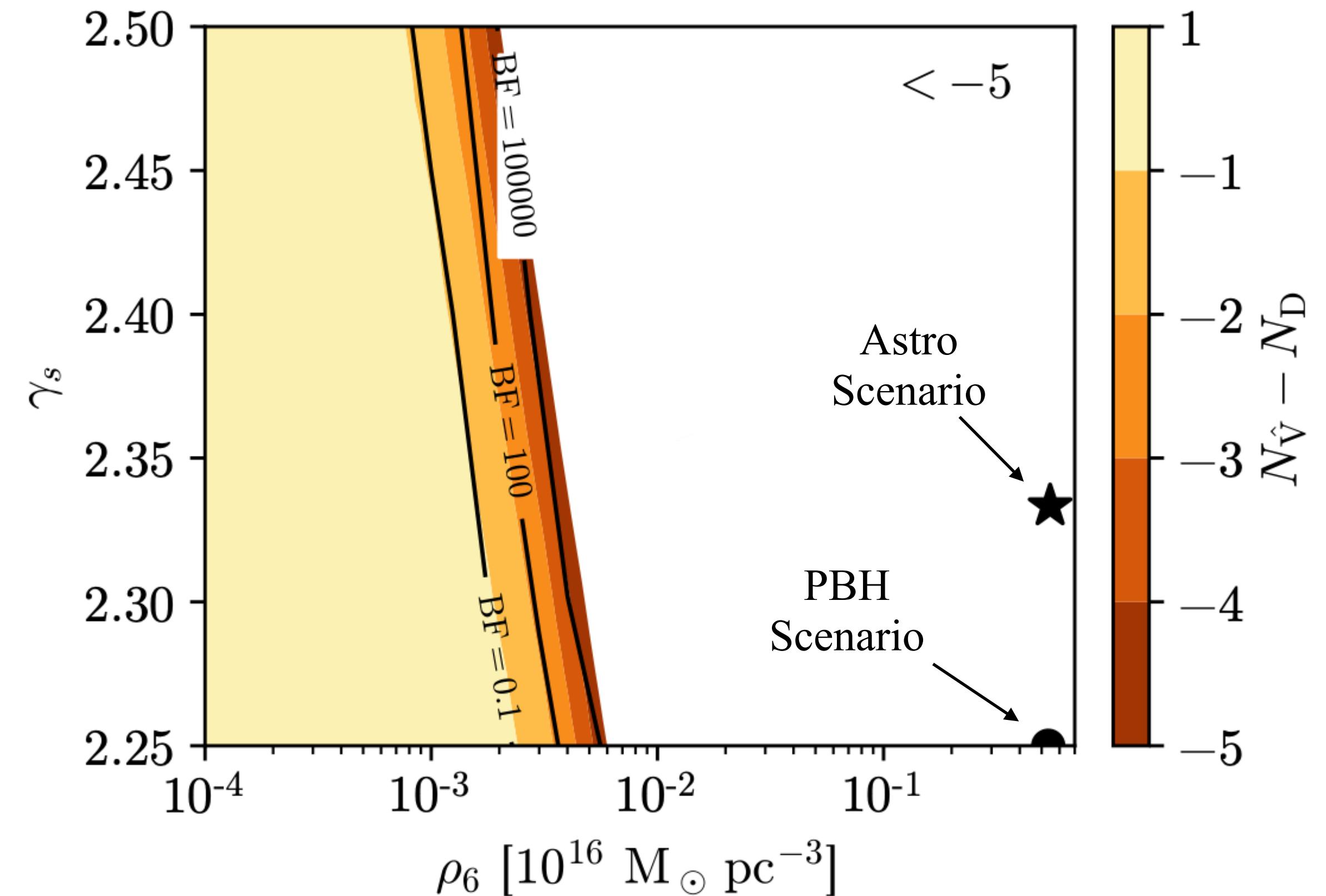
$$\theta_{\text{ext}} \equiv \{D_L, \phi_c, \tilde{t}_c\}$$

Use an approximate waveform
parametrisation in terms of θ_D

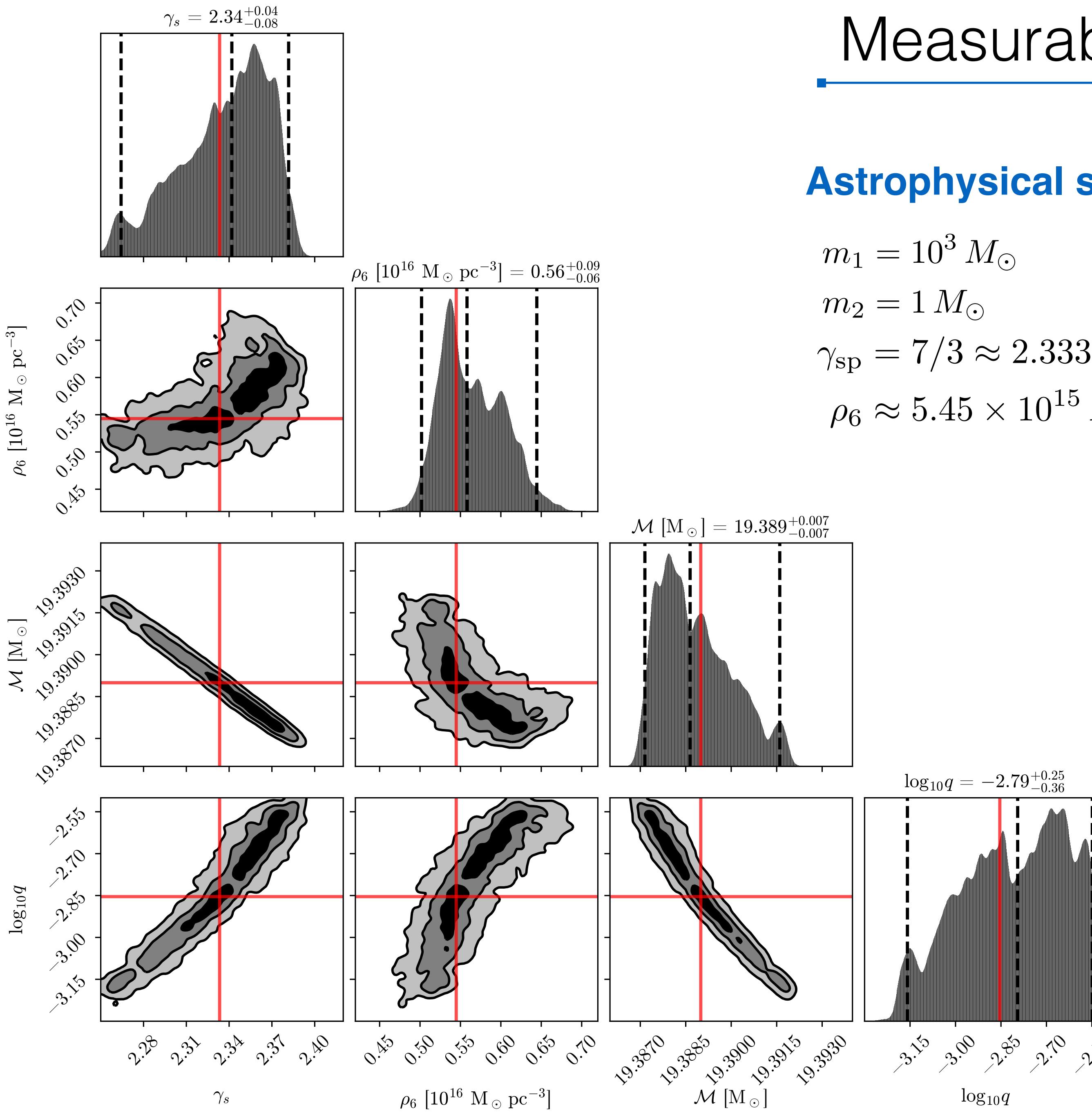
[Code available online:
<https://github.com/adam-coogan/pydd>]

$$\text{BF}(d) \equiv \frac{p(d|\text{D})}{p(d|\text{V})} \quad p(d) = \int d\theta \mathcal{L}(\theta) p(\theta)$$

Likelihood
Prior



Measurability



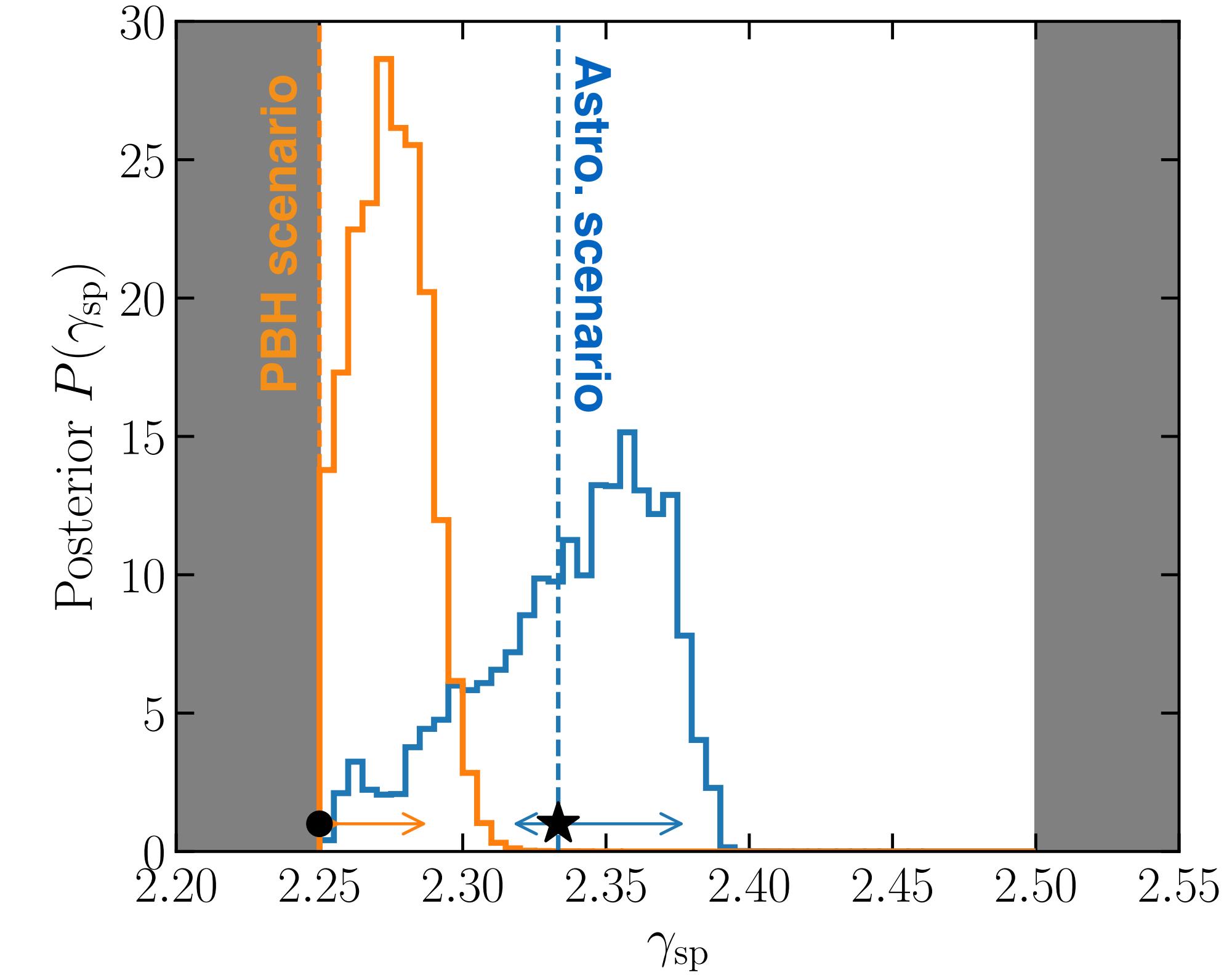
Astrophysical scenario

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We may be able to distinguish different *shapes* of spike
 → Different DM models and formation mechanisms!

Plans for the future

Improved modelling

- Injection and evolution of angular momentum in the spike
- Orbital eccentricity
- Post-Newtonian corrections
- Better N-body approaches [[AMUSE?](#)]

Detection methods

- Producing template banks for LISA searches
- Surrogate models for waveform generation
- Incoherent searches for continuous GWs
- ‘General’ de-phased waveform templates [[2004.06729](#)]

Detection prospects

- How many IMRI systems form? How many with BH/NSs?
- How many systems have a (surviving) spike?
- Comparison with dephasing due to baryons, or due to ultralight bosons (gravitational atoms)
- *What about ground-based detectors? Low mass PBHs?*



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Pippa Cole
(GRAPPA, Amsterdam)



Adam Coogan
(Mila, Montreal)



Pratibha Jangra
(IFCA, Santander)



Daniele Gaggero
(IFT, Madrid)



Jose Maria Diego
(IFCA, Santander)



David Nichols
(U. Virginia)



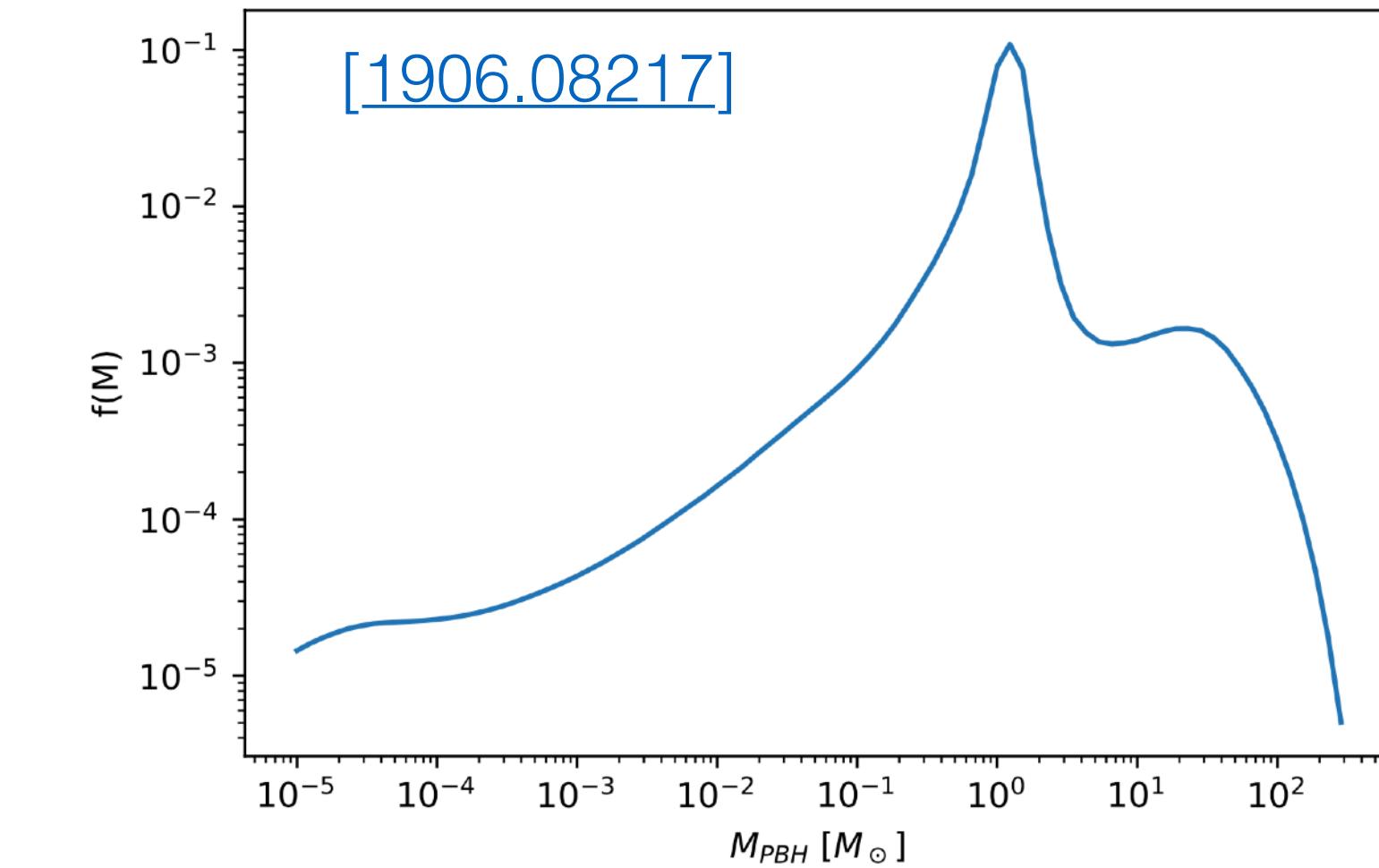
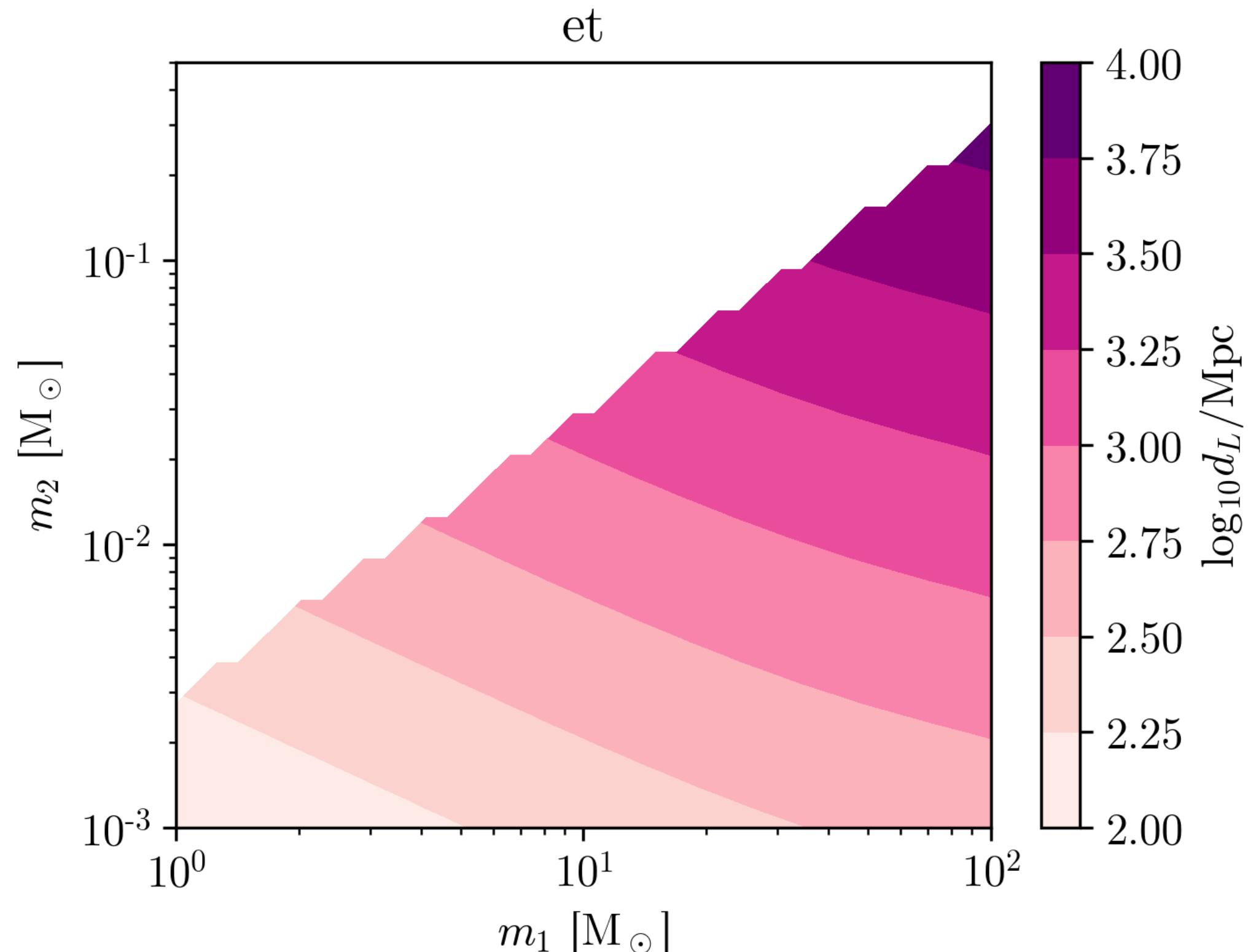
Francesca Scarcella
(IFT, Madrid)

...and others...

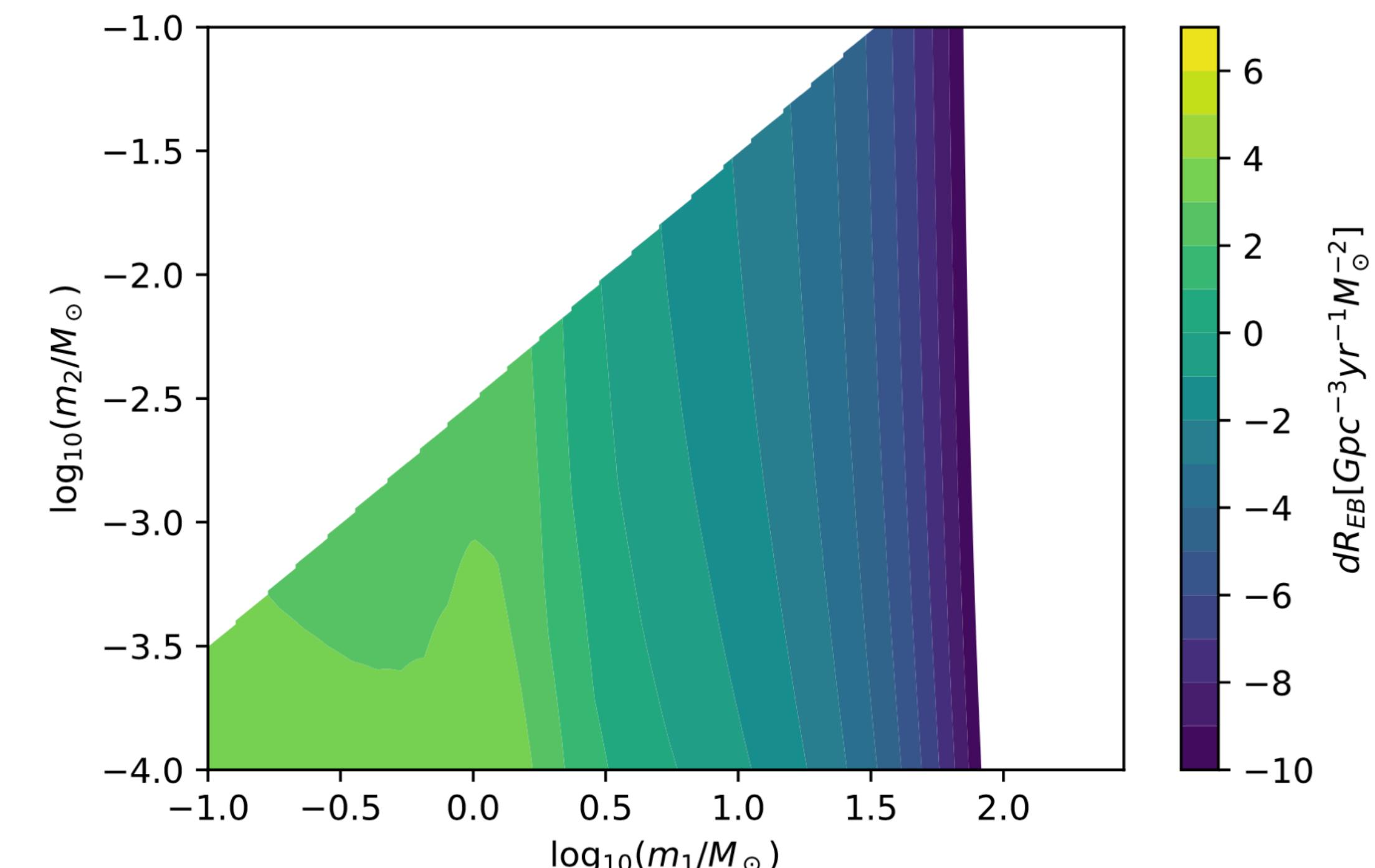
Low mass PBH binaries

Low mass PBH binaries could be detected with ground based detectors such as LIGO or Einstein Telescope

PRELIMINARY



(b) $k_p = 5 \times 10^5 \text{ Mpc}^{-1}$, $f_{PBH} = 0.085$.



Conclusions

Dark Matter ‘de-phasing’ is an extremely promising GW signature, which needs to be **modelled carefully**

[**BJK**, Nichols, Gaggero, Bertone, [2002.12811](#)]

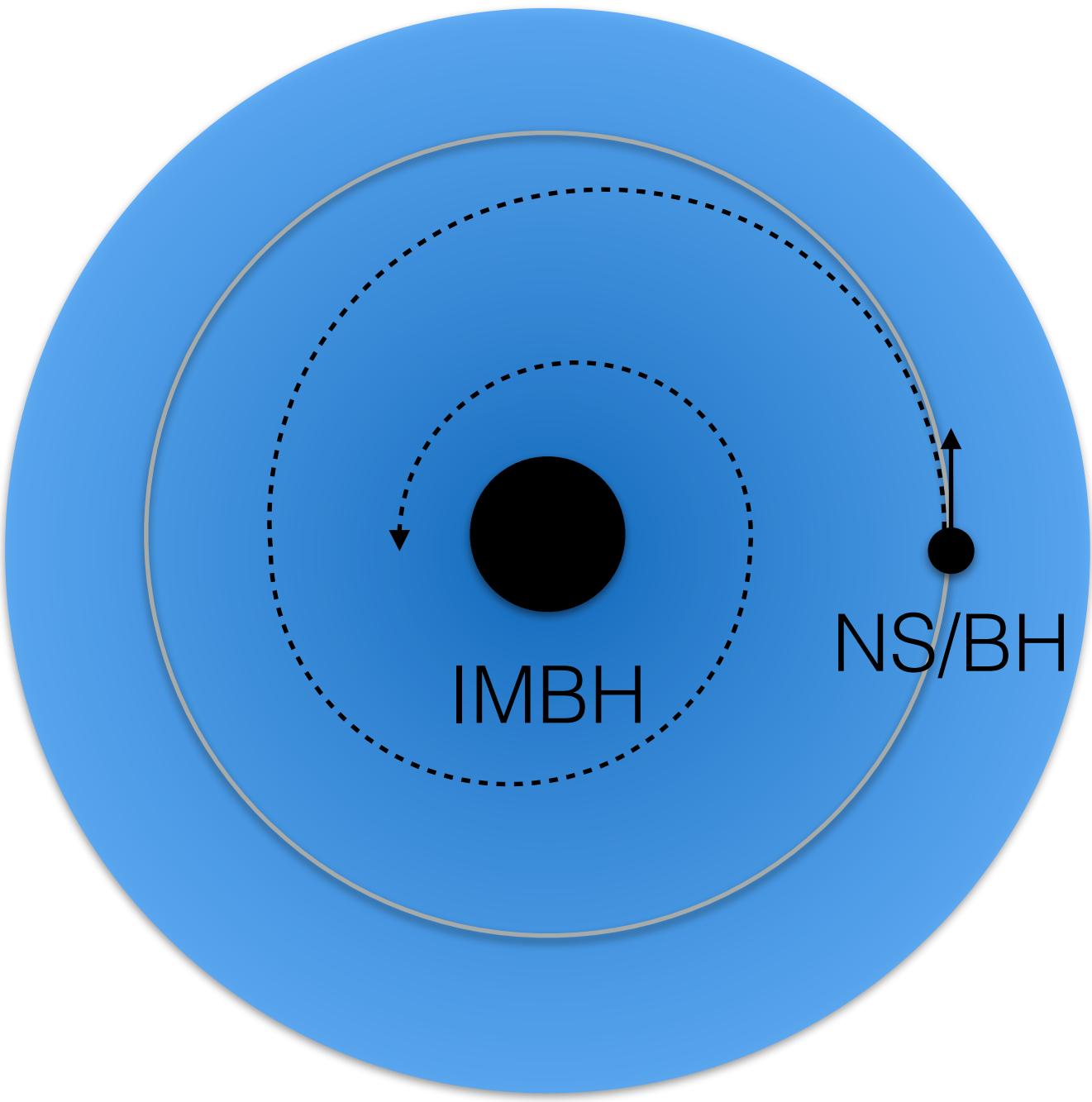
With LISA, such systems should be **detectable, discoverable** against vacuum-only systems, and the properties **measurable**.

[Coogan, Bertone, Gaggero, **BJK** & Nichols, [2108.04154](#)]

These signals could probe the **nature of Dark Matter** and pave the way towards a **multi-messenger detection** of Dark Matter

[Edwards, Chianese, **BJK**, Nissanke & Weniger, [1905.04686](#)]

**There are lots of open questions remaining,
but they’re well worth answering!**



Conclusions

Dark Matter ‘de-phasing’ is an extremely promising GW signature, which needs to be **modelled carefully**

[**BJK**, Nichols, Gaggero, Bertone, [2002.12811](#)]

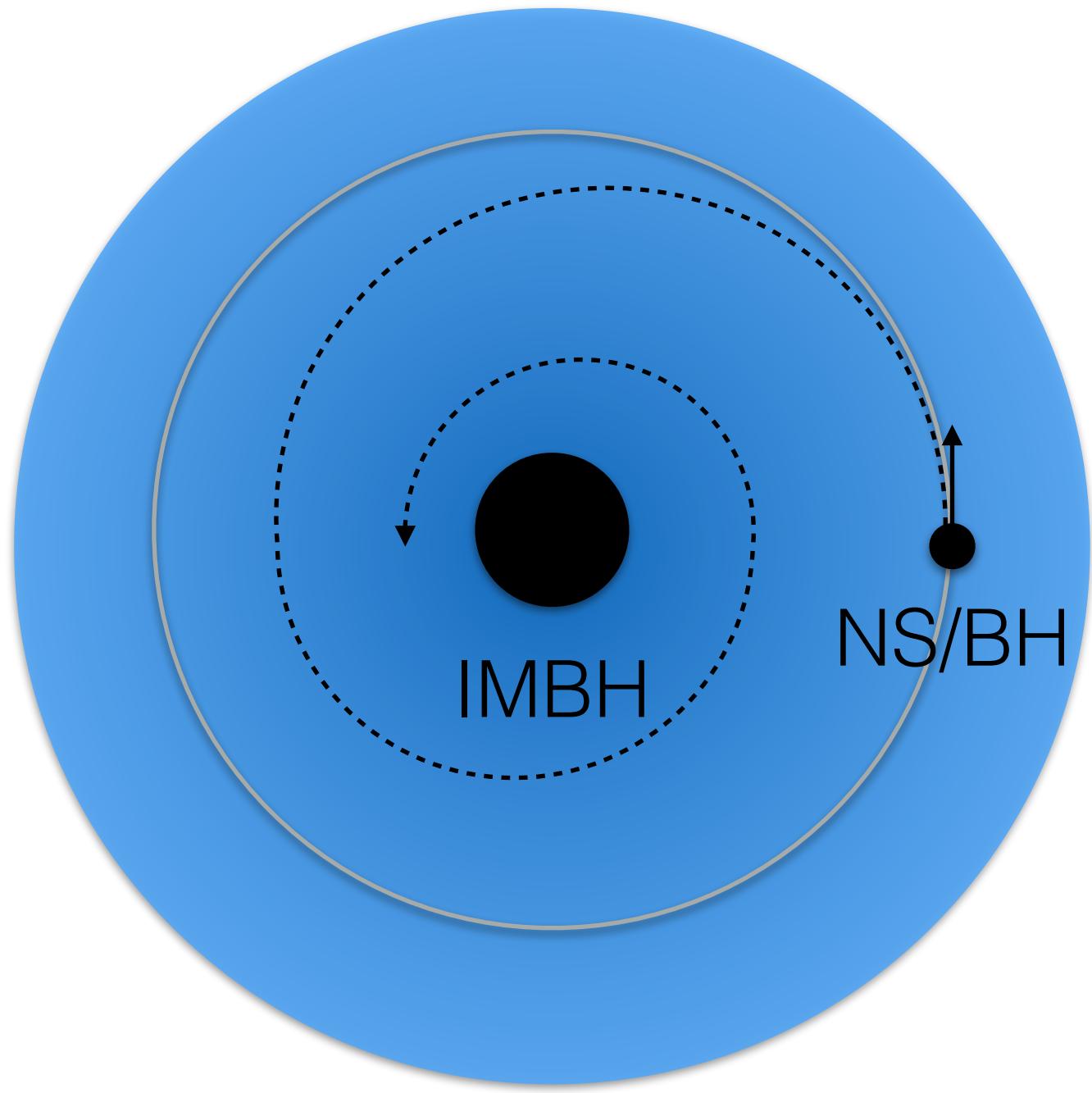
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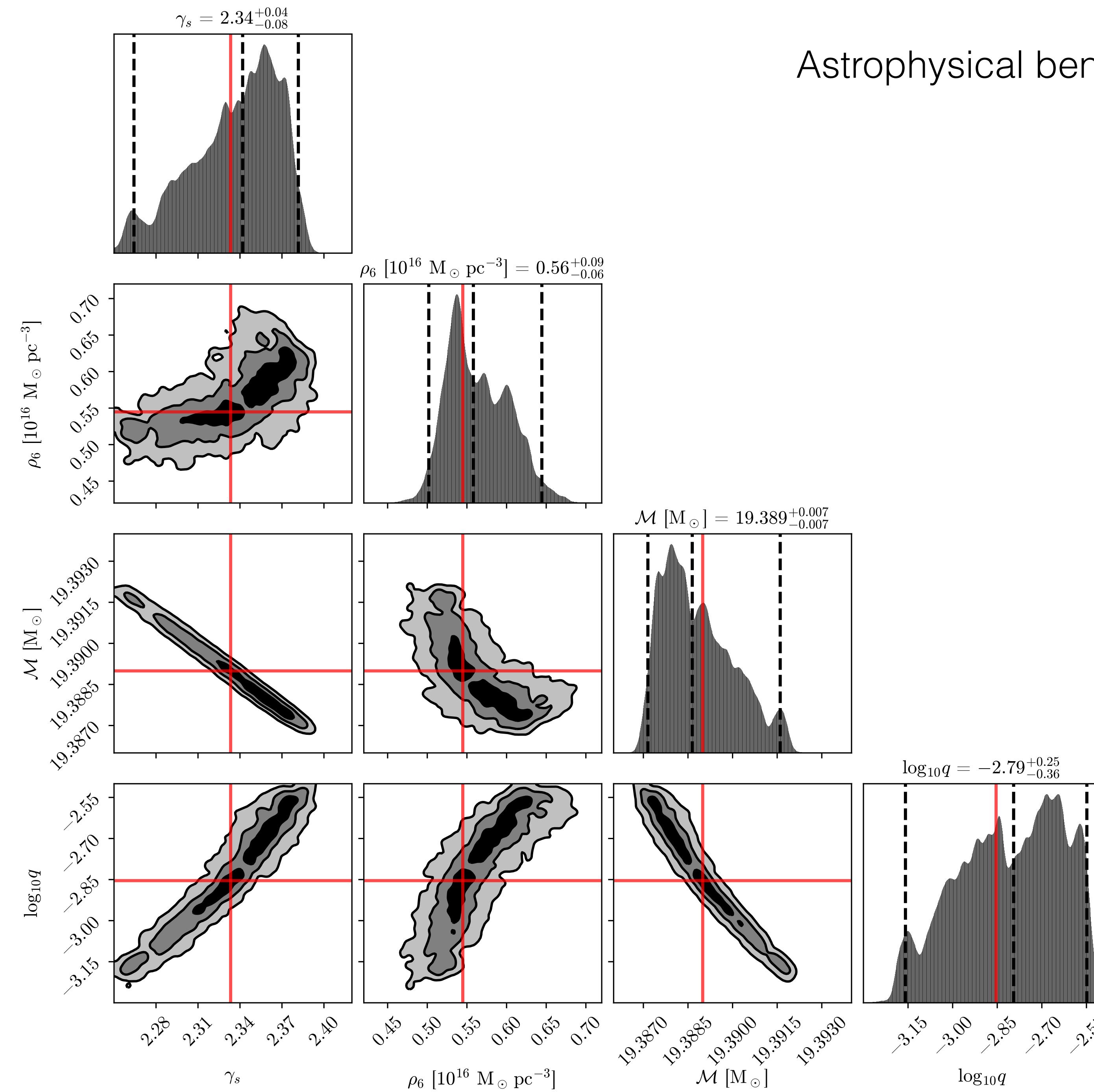


Thank you!

Backup Slides

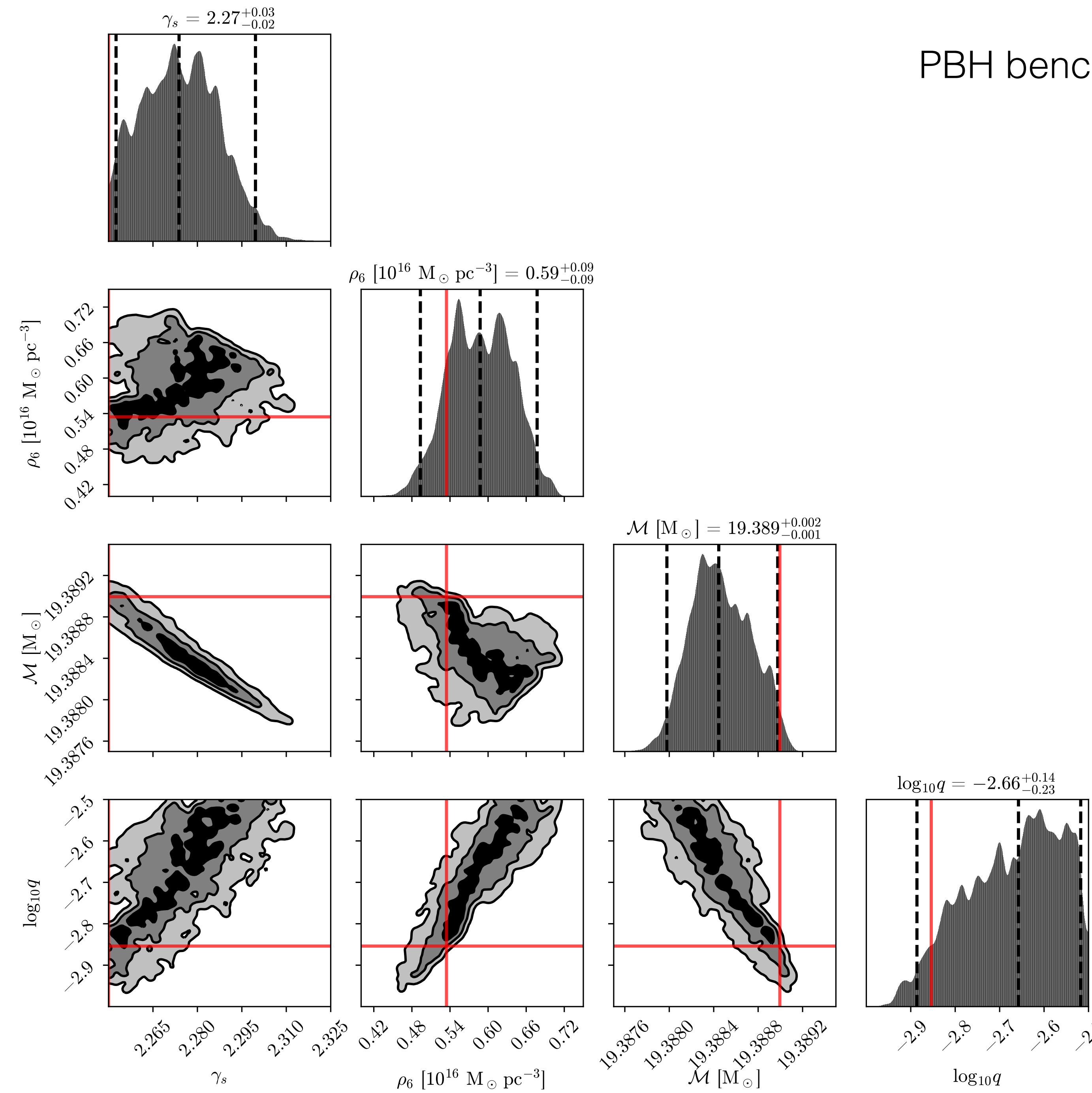
Measurability

Astrophysical benchmark

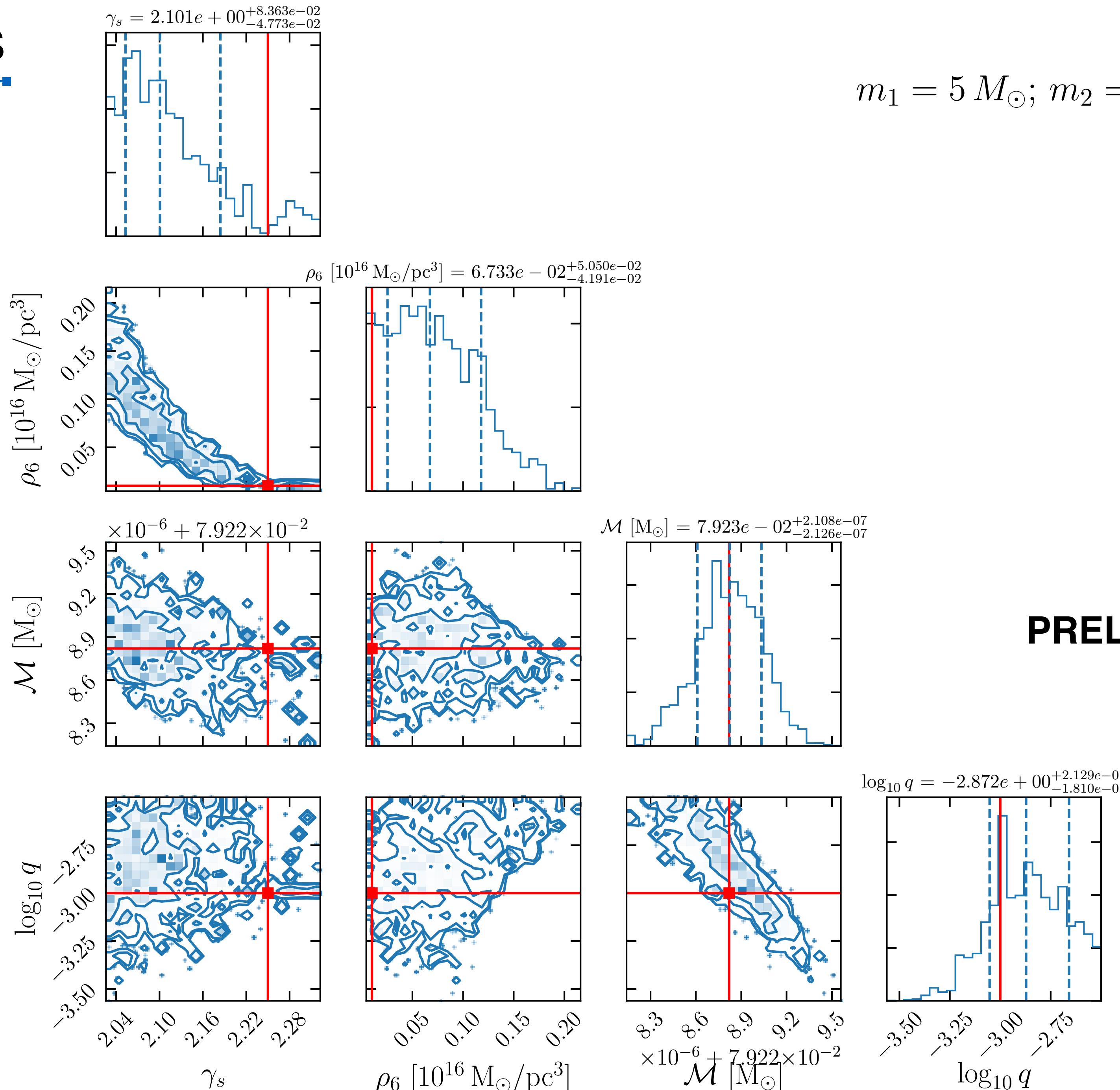


Measurability

PBH benchmark



Low mass PBHs



$m_1 = 5 M_\odot$; $m_2 = 0.005 M_\odot$; aLIGO

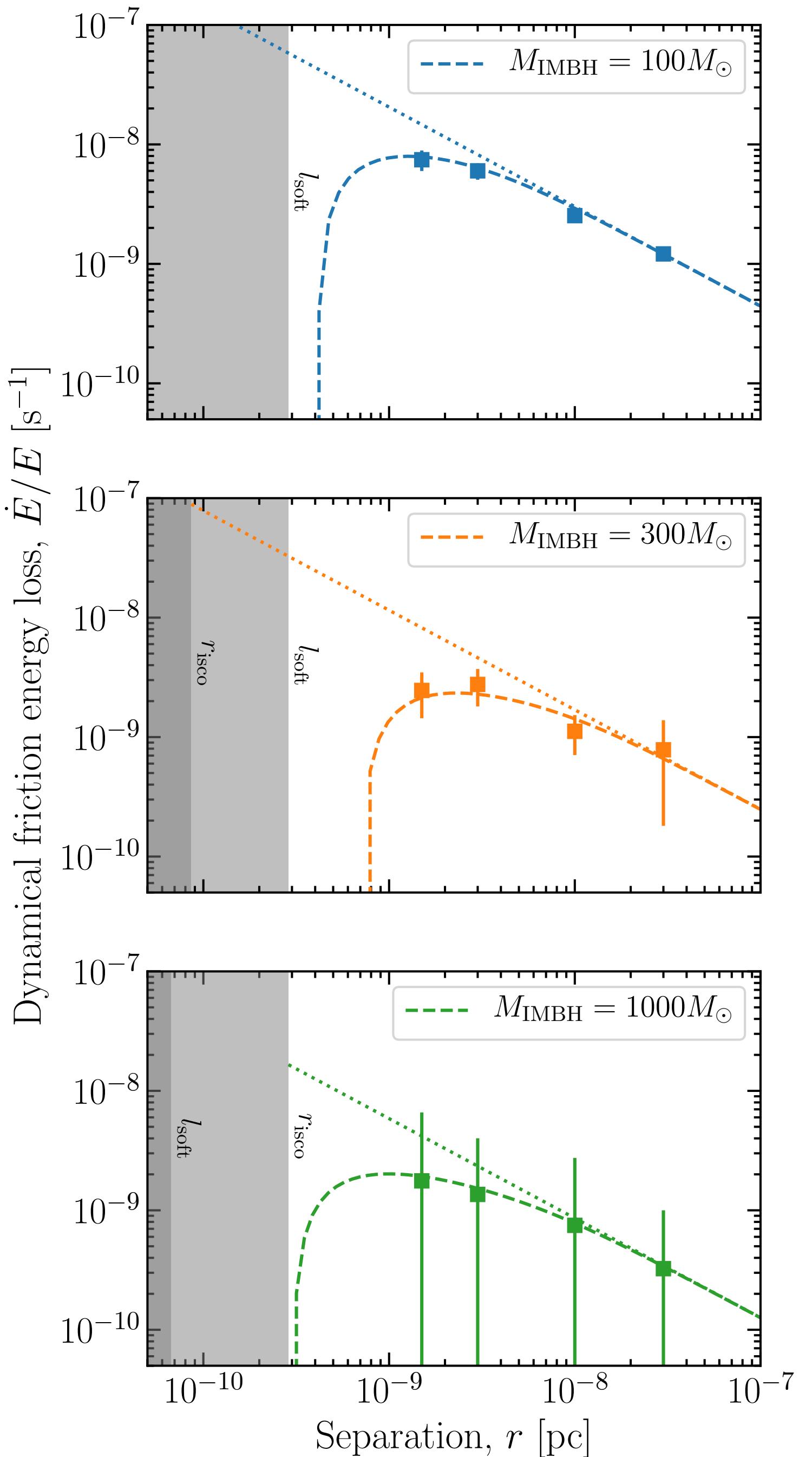
PRELIMINARY

N-body results

Dependence of dynamical friction force on mass and separation matches expectations

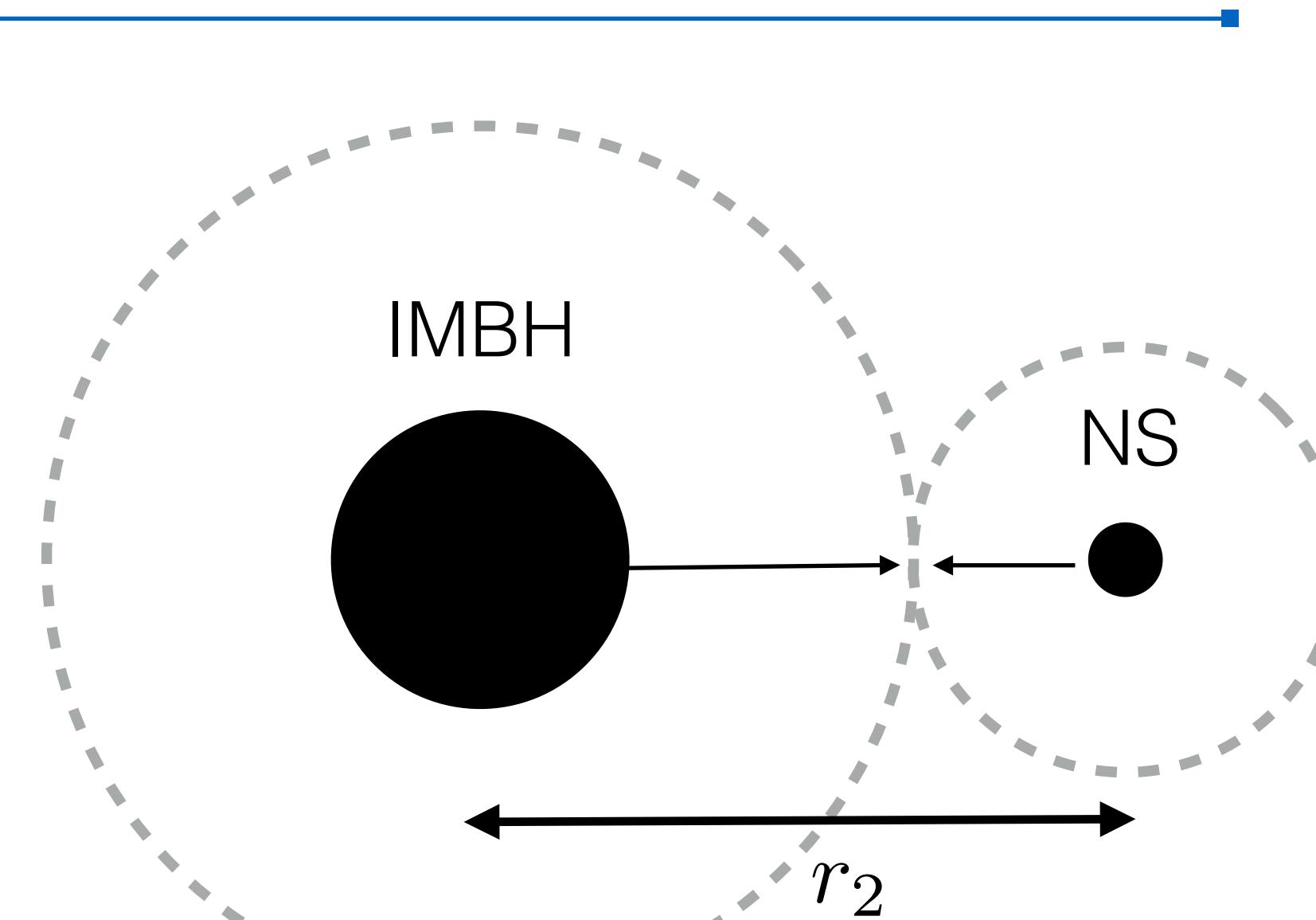
Dynamical friction traces local DM density (to better than 1%)

Drop off in DF force at small separations due to softening of simulations



N-body results

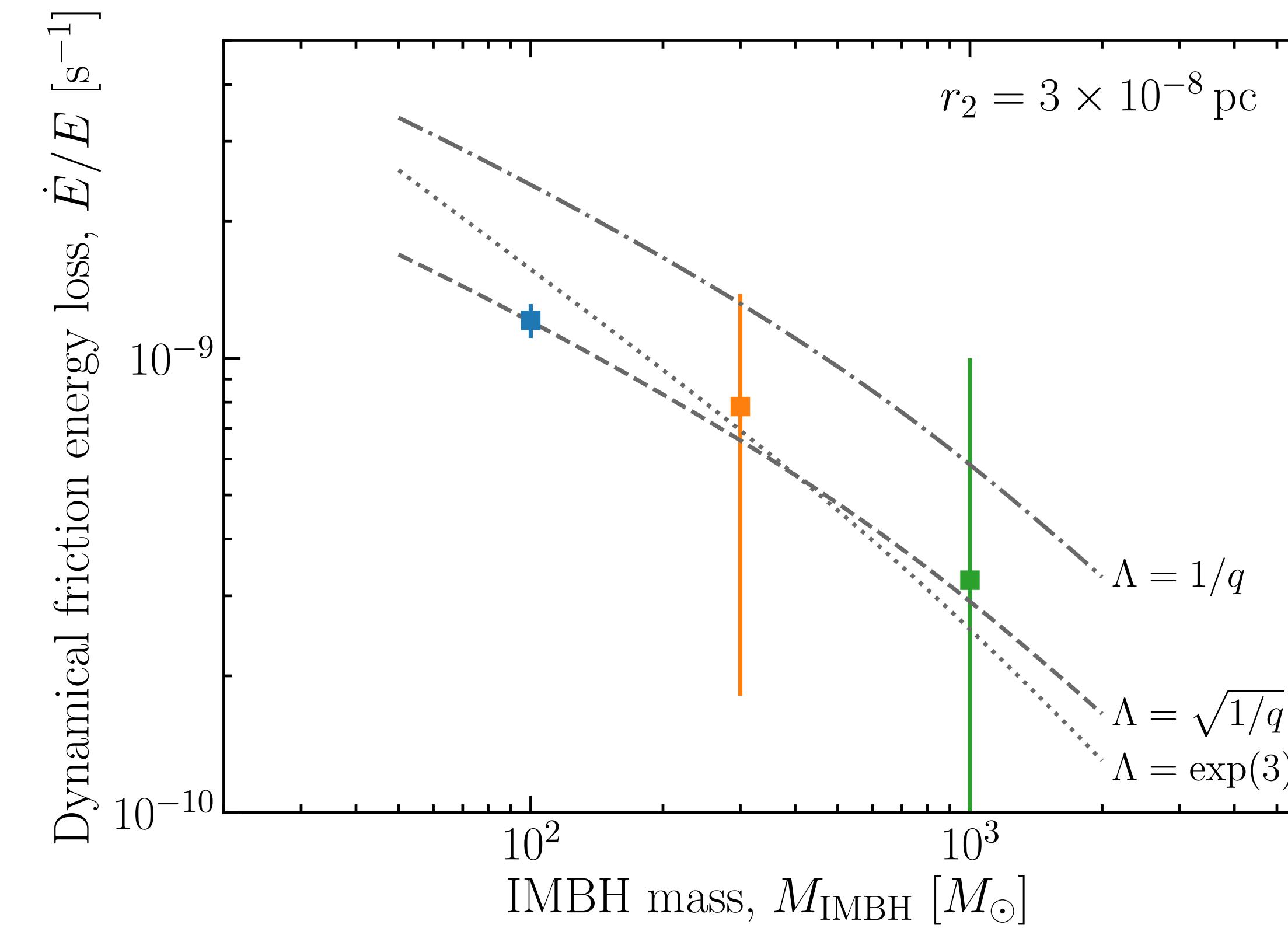
$$q \equiv m_{\text{NS}}/m_{\text{IMBH}} \ll 1$$



$$\begin{aligned}\Lambda &= b_{\max} \frac{v_0^2}{G m_{\text{NS}}} \\ &= \frac{b_{\max}}{q r_2} \\ &= 1/\sqrt{q}\end{aligned}$$

Allows us to calibrate the maximum impact parameter; tells us which particles scatter with the NS.

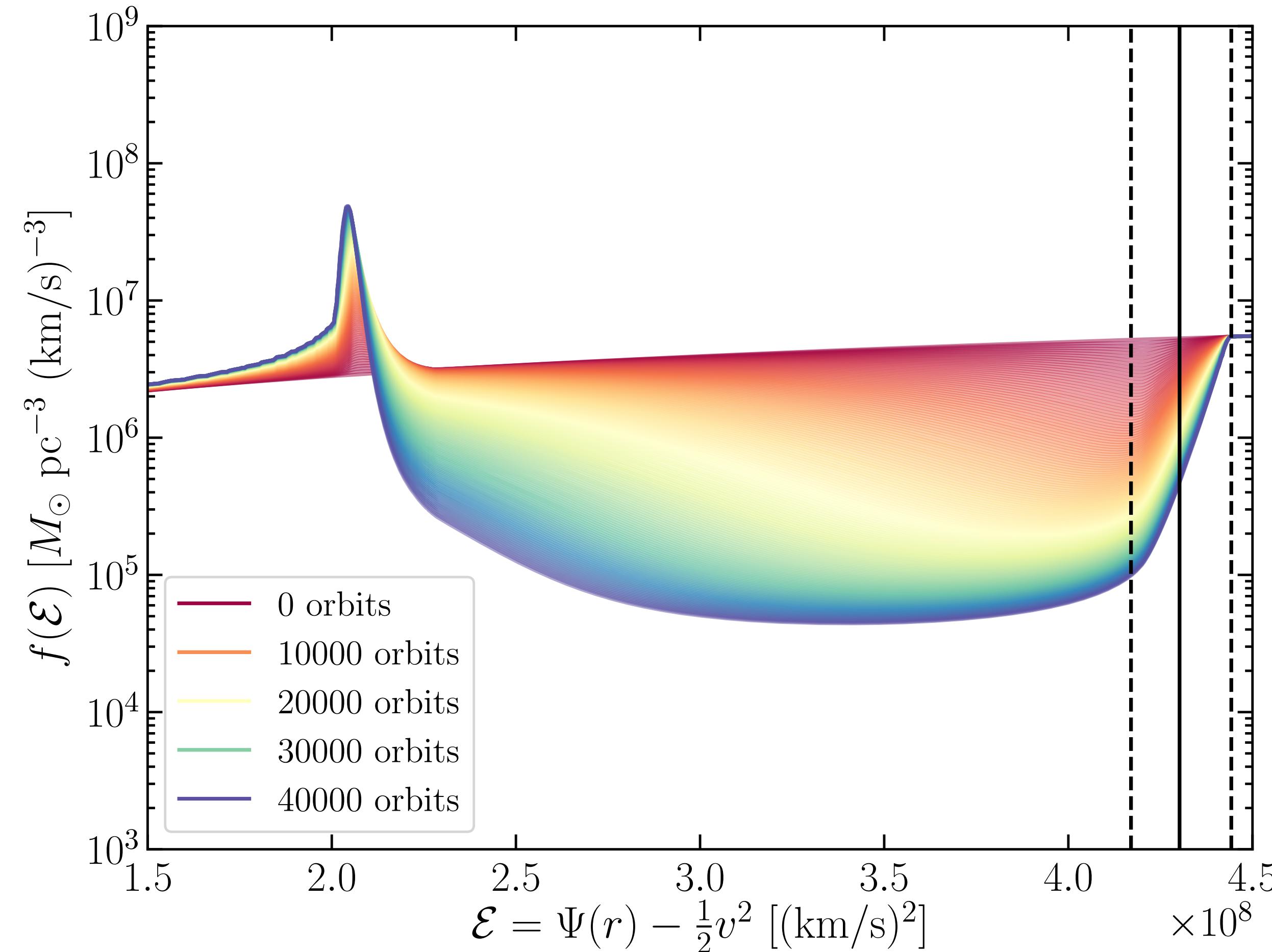
$$b_{\max} = \sqrt{q} r_2 \sim 3\% r_2$$



Assumptions

- Spherical symmetry and isotropy of the DM halo
- DM particles only scatter within an impact parameter
$$b < b_{\max} = \Lambda \times G_N M_{\text{NS}} / v_{\text{NS}}^2$$
- DM distribution is ‘locally’ uniform
$$b_{\max} \ll r_0$$
- Halo ‘relaxation’ is instantaneous
- Orbital properties evolve slowly compared to the orbital period

Distribution function



Self-consistently reconstruct density from distribution function:

$$\rho(r) = 4\pi \int_0^{v_{\max}(r)} v^2 f(\mathcal{E}) dv$$

Numbers of cycles

$$m_1 = 10^3 M_\odot, N_{\text{cycles}} = 5.71 \times 10^6 \text{ in vacuum}$$

	$\gamma_{\text{sp}} = 1.5$	$\gamma_{\text{sp}} = 2.2$	$\gamma_{\text{sp}} = 2.3$	$\gamma_{\text{sp}} = 2.\bar{3}$
Static	< 1	2.4×10^4	1.6×10^5	2.9×10^5
Dynamic	< 1	2.7×10^2	1.9×10^3	3.5×10^3

$$m_1 = 10^4 M_\odot, N_{\text{cycles}} = 3.20 \times 10^6 \text{ in vacuum}$$

	$\gamma_{\text{sp}} = 1.5$	$\gamma_{\text{sp}} = 2.2$	$\gamma_{\text{sp}} = 2.3$	$\gamma_{\text{sp}} = 2.\bar{3}$
Static	< 1	1.4×10^3	8.7×10^3	1.6×10^4
Dynamic	< 1	6.2×10^2	4.0×10^3	7.4×10^3

TABLE I. **Change in the number of cycles ΔN_{cycles} during the inspiral.** Change in the total number of GW cycles due to dynamical friction, starting 5 years from the merger.

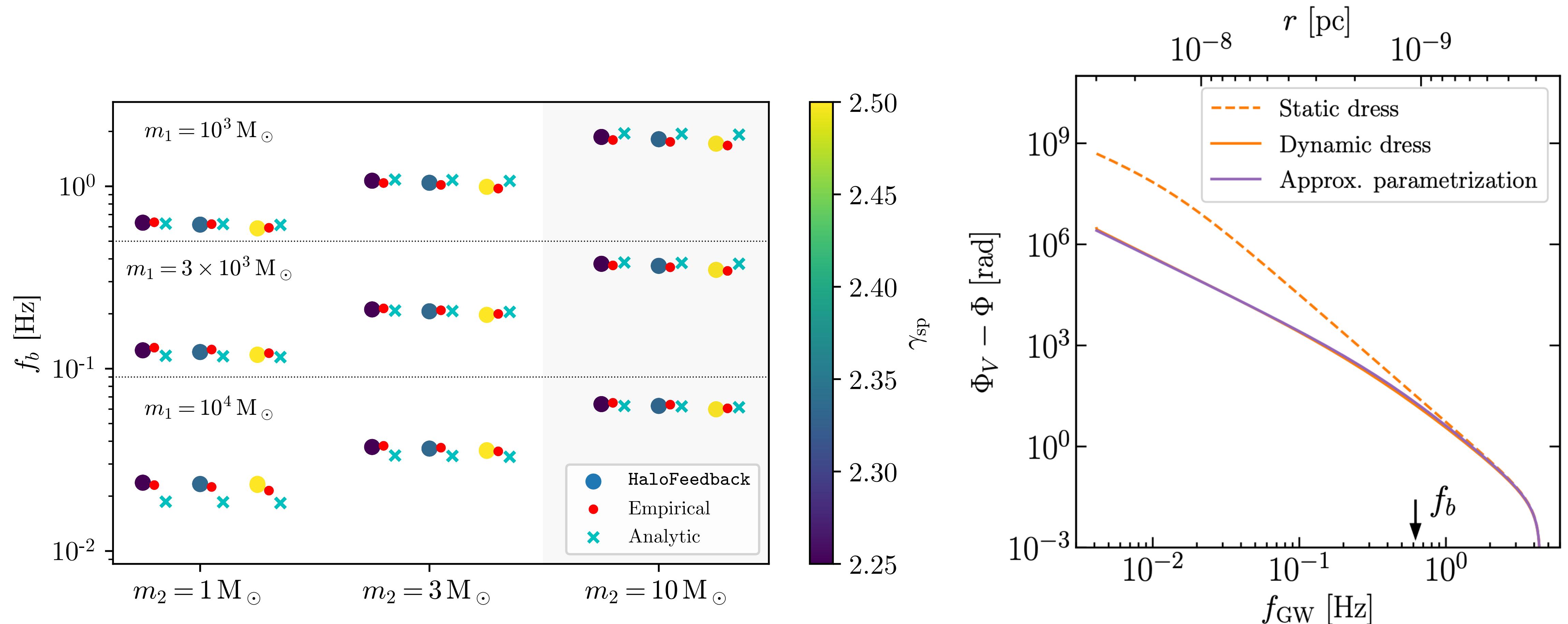
Phase parametrisation

$$\vartheta = \frac{5}{2\gamma_e}, \quad \lambda = \frac{11 - 2(\gamma_{\text{sp}} + \gamma_e)}{3}, \quad \eta = \frac{5 + 2\gamma_e}{2(8 - \gamma_{\text{sp}})} \left(\frac{f_{\text{eq}}}{f_b} \right)^{\frac{11 - 2\gamma_{\text{sp}}}{3}}$$

$$\hat{\Phi}(f) \equiv \Phi^V(f) \\ \times \left\{ 1 - \eta y^{-\lambda} \left[1 - {}_2 F_1 \left(1, \vartheta, 1 + \vartheta, -y^{-\frac{5}{3\vartheta}} \right) \right] \right\}$$

$$f_b = \beta \left(\frac{m_1}{1000 M_\odot} \right)^{-\alpha_1} \left(\frac{m_2}{M_\odot} \right)^{\alpha_2} \left[1 + \zeta \log \frac{\gamma_{\text{sp}}}{\gamma_r} \right], \quad (35)$$

where $\alpha_1 = 1.4412$, $\alpha_2 = 0.4511$, $\beta = 0.8163 \text{ Hz}$, $\zeta = -0.4971$ and $\gamma_r = 1.4396$.



Axions and neutron stars

Produce a photon with axion energy $m_a \sim 10^{-6} \text{ eV} \sim 240 \text{ MHz}$



Conversion happens at a radius r_c , with probability: $p_{a\gamma} \propto \frac{g_{a\gamma\gamma}^2 B(r_c)^2}{2v_c}$

Radiated power is given by: $\frac{d\mathcal{P}}{d\Omega} \sim 2 \times p_{a\gamma} \rho_{\text{DM}}(r_c) v_c r_c^2$

Probe axions in the mass range

$$m_a \sim 10^{-7} \text{ eV} \quad \text{up to} \quad m_a \sim 10^{-5} \text{ eV}$$

Frequency range of
radio telescopes

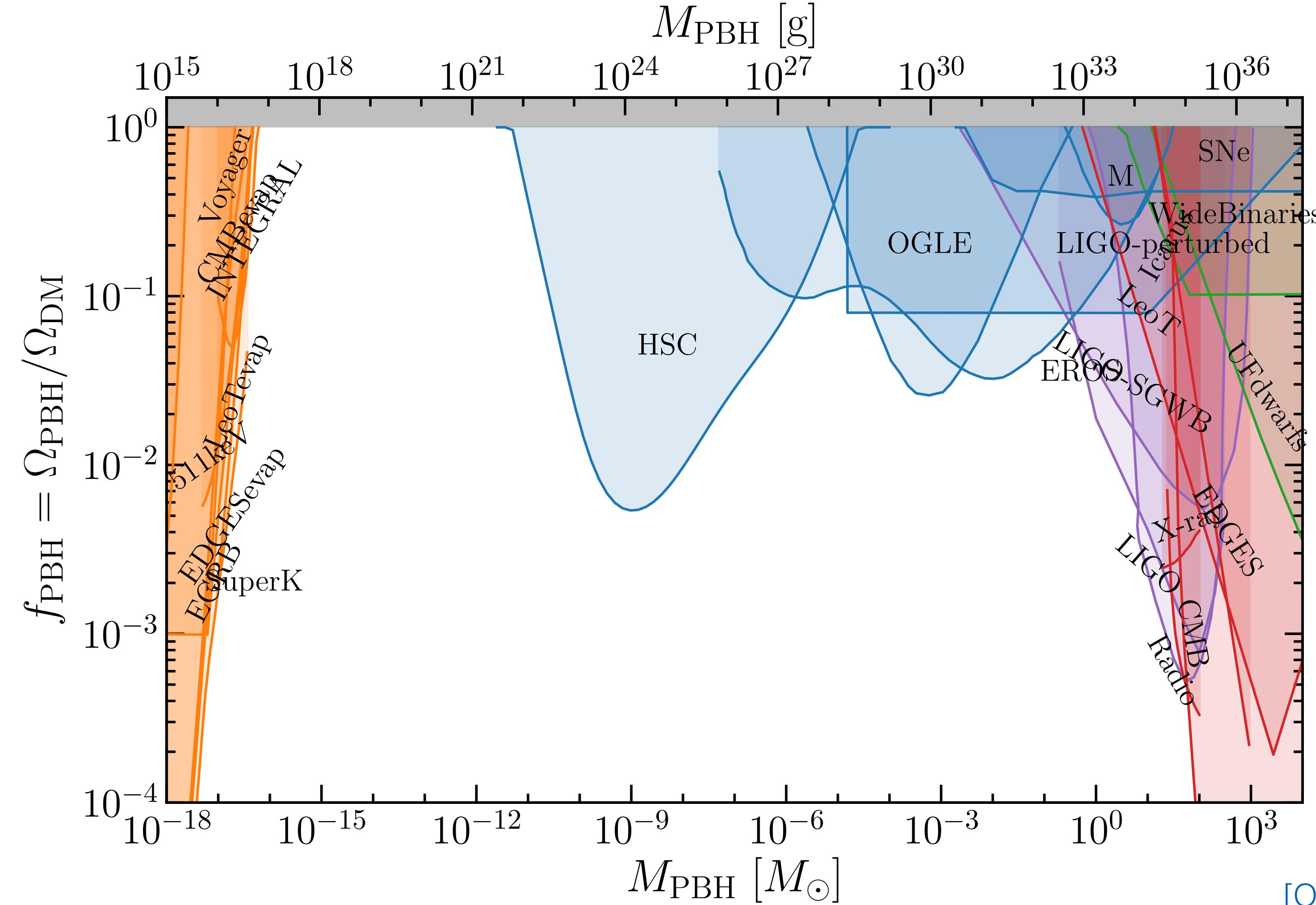
Require conversion
outside NS

[[1803.08230](#), [1804.03145](#), [1811.01020](#), [1910.11907](#)]

PBH Constraints

[Green & **BJK**, 2007.10722]

[Code online: github.com/bradkav/PBHbounds]

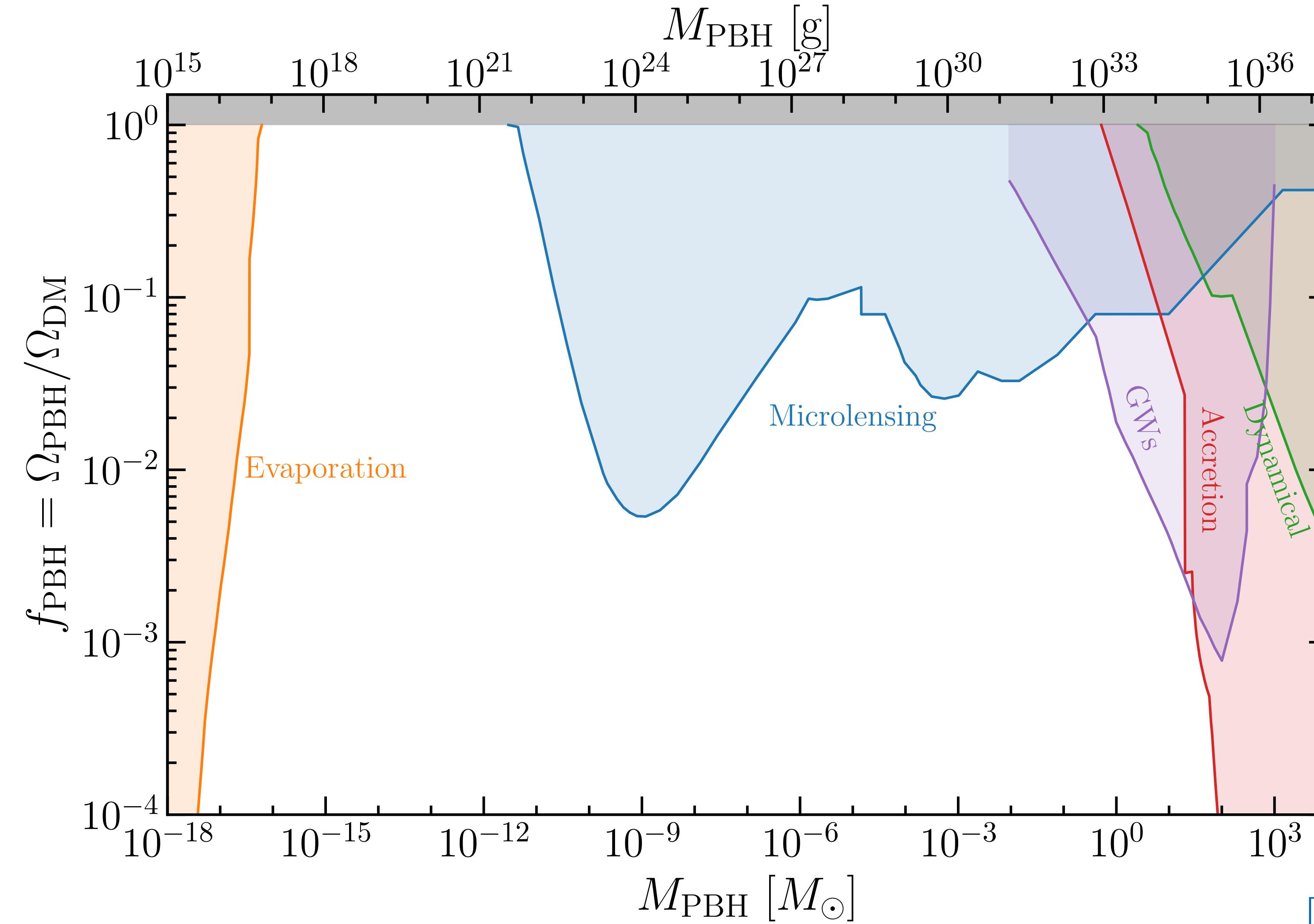


[Other reviews: [1801.05235](https://arxiv.org/abs/1801.05235),
[2002.12778](https://arxiv.org/abs/2002.12778), [2006.02838](https://arxiv.org/abs/2006.02838)]

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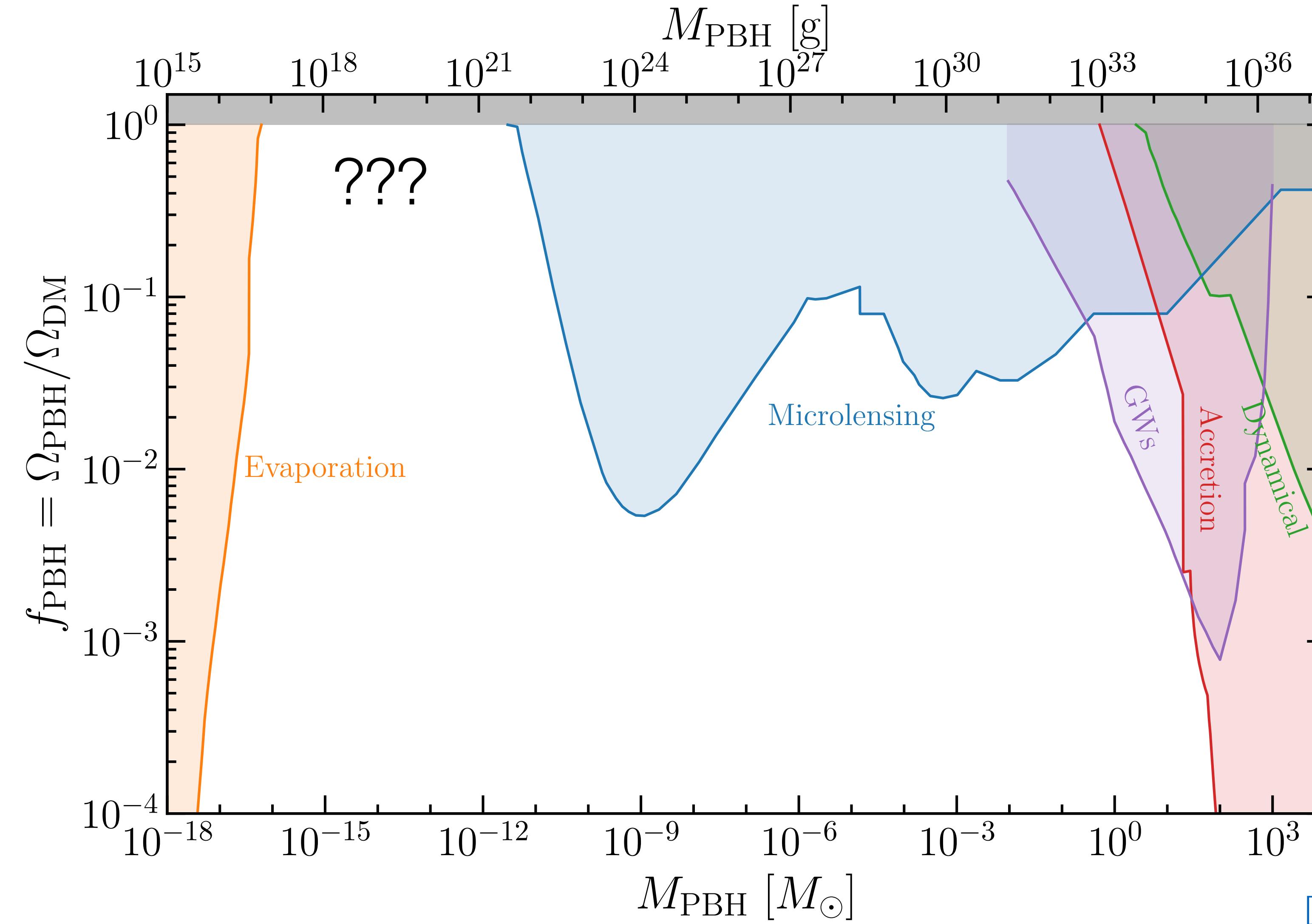


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