

Q1: If  $X \neq Y$ . then  $P(X - Y \neq 0) > 0$ .  
 then  $\exists n \in \mathbb{N}$ . s.t.  $\{X - Y > \frac{1}{n}\}$  or  $\{Y - X > \frac{1}{n}\}$   
 is not empty.  
 Let one of non-empty set be  $A$ .  
 then  $\int_A (X(\omega) - Y(\omega)) dP \neq 0$   
 contradict.

Q3: use probability transition function  $p(t, x; T, y)$

$$P(X(t) = y | X(s) = x) = p(s, x; t, y)$$

$$P(X(r) = z | X(s) = x) = p(r, z; s, x)$$

$$E[g(X(t))h(X(r)) | X(s) = x]$$

$$= \int \int g(y) h(z) p(s, x; t, y) p(r, z; s, x) dy dz$$

$$= \int g(y) p(s, x; t, y) dy \cdot \int h(z) p(r, z; s, x) dz$$

$$= E[g(X(t)) | X(s) = x] \cdot E[h(X(r)) | X(s) = x]$$