$$\frac{1}{\sqrt{2\pi(T-t)}} \int_{-\infty}^{\infty} e^{-y^2} \cdot e^{\frac{(y-x)^2}{2(T-t)}} dy$$

$$= \frac{1}{\sqrt{2\pi(T-t)}} \int_{-\infty}^{\infty} e^{-\frac{(2(T-t)+1)y^2 - 2xy + x^2}{2(T-t)}}$$

$$= \frac{1}{\sqrt{2\pi(T-t)}} \int_{-\infty}^{\infty} e^{-\frac{(2(T-t)+1)\left(y - \frac{x}{2(T-t)+1}\right)^2 + x^2}{2(T-t)} - \frac{x^2}{2(T-t)+1}} dy$$

$$= \frac{1}{\sqrt{2\pi(T-t)}} e^{-\frac{x^2}{2(T-t)+1}} \int_{-\infty}^{\infty} e^{-\frac{2(T-t)+1}{2(T-t)}\left(y - \frac{x}{2(T-t)+1}\right)^2} dy$$

$$= \frac{1}{\sqrt{2\pi(T-t)}} e^{-\frac{x^2}{2(T-t)+1}} \int_{-\infty}^{\infty} e^{-\frac{2(T-t)+1}{2(T-t)} z^2} dz$$

$$= \frac{1}{\sqrt{2\pi(T-t)}} e^{-x^2/(2(T-t)+1)} \cdot \sqrt{\frac{2(T-t)\pi}{2(T-t)+1}}$$

$$= \frac{1}{\sqrt{2(T-t)+1}} e^{-\frac{x^2}{2(T-t)+1}}$$