Holt D208 Predictive Modeling Task 1

July 15, 2021

1 Part I: Research Question

1.1 A1: Question

What customer qualities or factors from the data set have a significant effect on a customer's length of tenure?

1.2 A2: Objective and Goals

The objective of an analysis of the data is to determine what features, if any, have a significant relationship with the length of tenure of a customer.

2 Part II: Method Justification

2.1 B1: Summary of Assumptions

2.1.1 1) Independent and Dependent Variables Have a Linear Relationship

Multiple linear regression analysis assumes that independent variables have a linear relationship with the target variable. If variables have a non-linear relationship with the target variable the regression results will under-estimate the true relationship. The preferable method to detect non-linearity is to examine residual plots (plots of the standardized residuals as a function of standardized predicted values). (Osborne, et al., 2002)

2.1.2 2) Assumption of Homoscedasticity

The multiple linear regression model assumes that the errors between the predicted value and the observed value are constant along the time series. Heteroscedasticity creates problems when using ordinary least-squares(OLS) regression because OLS attempts to minimize errors and gives equal weights across the errors. Observations with larger error variances would be given an unequal weight. Creating and chosing a model without checking for homoscedasticity can lead to negative consequences and could invalidate the model. Homoscedasticity can be checked by plotting the independent variable's residuals against the target variable.

2.1.3 3) No Multicollinearity

Multicollinearity in the model occurs when two or more independent variables share the same correlation with the target variable. In cases where a perfect correlation between two or more variables is present, multicollinearity can mean that no unique least-squares solution to a regression analysis can be computed. When less severe multicollinearity is present, as is more likely, the coefficient

can be unreliable and the standard errors and confidence intervals for the coefficient estimates will be inflated leading to incorrect conclusions that a variable is statistically insignificant. (Williams, et al., 2013) Additionally, having two or more variables in the model with the same correlation factors can unnecessarily overcomplicate the model leading to inaccurate predictions. Using the variance inflation factor(VIF) results of each variable can help determine if multicollinearity exists within the model.

2.1.4 4) No Autocorrelation

Autocorrelation measures the relationship between a variable's current value and its past values. When variables have measurements that have a relationship or are dependent with previously measured values in the same variable it is deemed to be autocorrelated. This most often occurs in time series measured data where values that were measured closer together are more similar than values measured at a later time. Autocorrelation violates the linear model assumption that variable measurements are independent.

2.1.5 5) Normally distributed residuals (errors).

Finally, the multiple linear regression model assumes that independent variable residuals (errors) are normally distributed. With large sample sizes, normally distributed residuals are not required to estimate unbiased and efficient coefficients. However, with small samples sizes, it is the case that normally distributed residuals are required as significance tests and confidence intervals can be untrustworthy. (Williams, et al., 2013) A histogram of the variable's residuals or P-P plot can be created to observe the normality of the errors.

2.2 B2: Benefits of Using Python

By using Python, data can be easily cleaned, explored, and prepared for use in predictive model building. The models themselves can be created using Python. Plots, charts, and graphs can be created to visualize the data and better understand relationships within datasets. This creates opportunites to provide detailed visual information for presentations. Python contains many packages built by data scientist that help with the previously mentioned tasks. Some packages that will be used are Numpy, Pandas, Matplotlib, Seaborn, Pyplot, Statsmodels, Sklearn and Yellowbrick.

2.3 B3: Why Multiple Linear Regression?

Using the question in part I, it was determined that multiple linear regression was an appropriate model to build. The predictor variable (Tenure) contains continuous values making it a prime target to use multiple linear regression. The dataset also contains many other continuous and categorical variables that can be used to build the model in relation to the predictor variable. While an initial model could be built using every variable in the dataset, by choosing specific variables using experience, sense, and previous familiarization with the dataset, unneeded variables can be weeded out to start with a simpler model. At this point in the workflow, however, we have yet to determine whether each assumption listed above has been determined to be true. There now is a clean dataset, a question to answer, and a decision of a model type to build. Now we move into the next portion of the analysis, data preparation.

3 Part III: Data Preparation

3.1 C1: Data Preparation Goals and Manipulation

The overall goal of data preparation is to ensure that the data we use for our linear regression model is complete, accurate, and efficiently used. If the data used to create and input into the model are garbage, garbage will be returned from the model. Some data manipulation tasks that need to be completed for data preparation to conduct multiple linear regression are: - Identify and handle missing data - Identify and handle outliers or strange values - Ensure all variables are numerical and transform those that are not into numerical values - Ensure the target variable is numerical - Scale numeric variables if needed - Check if any linear model or multiple linear model assumptions are in violation

3.2 C2: Summary Statistics Needed

The below statistical factors will be needed to help answer the research question and were derived from the course textbook:

- Partial correlation correlations coefficients between all predictor variables and target variables:
 Along with visual scatterplots, the partial correlation matrix helps determine if there are
 colinear relationships between a predictor variable and the target variable and between each
 predictor variable.
- Coefficients of all predictor variables the coefficients of each independent variable will need to be determined to build the linear regression model.
- R-squared statistic: The R-squared statistic is the proportion of variation in the dependent variable accounted for by the independent variable(s).
- Adjusted R-squared: The same statistic as the R-squared statistic but adjusted for the number
 of independent variables in the model. This statistic will always be lower than the R-squared.
- F-statistic This statistic indicates whether a linear regression model provides a better fit to the data than a model that contains no independent variables.
- F-statistic probability the probability that the F-statistic was obtained by chance. (p-value)
- Variance inflation factor(VIF) provides an index that measures how much the variance (the square of the estimate's standard deviation) of an estimated regression coefficient is increased because of collinearity

3.3 C3: Steps to Prepare the Data for Analysis & C4: Univariate and Bivariate Visualizations

- 1) Generate a list of potential variables; independent and dependent
- 2) Collect data on the variables (Import the Data), and transform as needed
- 3) Check the relationship between each independent variable and the dependent variable using sand correlations
- 4) Check the relationships among the independent variables using scatterplots and correlations
- 5) Use the non-redundant independent variables in the analysis to find the best fitting model

The above list was generated from Brandon Foltz's website. (2014)

3.3.1 1) Generate a list of potential variables; independent and dependent

List of variables to be explored:

Independent Variables - Population (Continuous)

- Area (Categorical)
- Children (Continuous)
- Age (Continuous)
- Income (Continuous)
- Marital (Categorical)
- Gender (Categorical)
- Churn (Categorical)
- Outage_sec_perweek (Continuous)
- Email (Continuous)
- Contacts (Continuous)
- Yearly_equip_failure (Continuous)
- Techie (Categorical)
- Contract (Categorical)
- Port_modem (Categorical)
- Tablet (Categorical)
- InternetService (Categorical)
- Phone (Categorical)
- Multiple (Categorical)
- OnlineSecurity (Categorical)
- OnlineBackup (Categorical)
- DeviceProtection (Categorical)
- TechSupport (Categorical)
- StreamingTV (Categorical)
- StreamingMovies (Categorical)
- PaperlessBilling (Categorical)
- PaymentMethod (Categorical)
- Tenure (Categorical)
- MonthlyCharge (Continuous)
- Bandwidth_GB_Year (Continuous)

- Item1 (Categorical)
- Item2 (Categorical)
- Item3 (Categorical)
- Item4 (Categorical)
- Item5 (Categorical)
- Item6 (Categorical)
- Item7 (Categorical)
- Item8 (Categorical)

Target Variable

• Tenure (Continuous)

3.3.2 2) Import the data and transform as required

```
[1]: #Load packages
   import numpy as np
   import pandas as pd
   import matplotlib.pyplot as plt
   import seaborn as sns
   import pylab

import statsmodels.api as sm
   from statsmodels.stats import diagnostic as diag
   from statsmodels.stats.outliers_influence import variance_inflation_factor

from sklearn.linear_model import LinearRegression, Ridge, RidgeCV
   from sklearn.model_selection import train_test_split, cross_val_predict
   from sklearn.metrics import mean_squared_error, r2_score, mean_absolute_error

from yellowbrick.regressor import AlphaSelection, PredictionError, ResidualsPlot
   %matplotlib inline
```

1_____

Population	False
Area	False
Children	False
Age	False
Income	False
Marital	False
Gender	False
Churn	False
Outage_sec_perweek	False
Email	False
Contacts	False
Yearly_equip_failure	False
Techie	False
Contract	False
Port_modem	False
Tablet	False
InternetService	False
Phone	False
Multiple	False
OnlineSecurity	False
OnlineBackup	False
DeviceProtection	False
TechSupport	False
StreamingTV	False
StreamingMovies	False
PaperlessBilling	False
PaymentMethod	False
Tenure	False
MonthlyCharge	False
Bandwidth_GB_Year	False
Item1	False
Item2	False
Item3	False
Item4	False
Item5	False
Item6	False
Item7	False

Item8 False

dtype: bool

·-----

```
[2]:
       Population
                      Area Children Age
                                             Income
                                                       Marital Gender Churn \
               38
                      Urban
                                       68 28561.99
                                                                  Male
                                                                         No
    0
                                   0
                                                       Widowed
    1
            10446
                      Urban
                                    1
                                       27 21704.77
                                                       Married Female
                                                                        Yes
    2
             3735
                      Urban
                                   4
                                      50
                                           9609.57
                                                       Widowed Female
                                                                        No
    3
            13863 Suburban
                                   1
                                       48 18925.23
                                                       Married
                                                                  Male
                                                                         No
            11352 Suburban
                                   0
                                       83 40074.19 Separated
                                                                  Male
                                                                        Yes
       Outage_sec_perweek Email ... MonthlyCharge Bandwidth_GB_Year Item1
    0
                 7.978323
                                       172.455519
                                                          904.536110
                                                                         5
                              10 ...
                                                          800.982766
                11.699080
    1
                              12 ...
                                       242.632554
                                                                         3
    2
                10.752800
                              9 ...
                                       159.947583
                                                         2054.706961
    3
                14.913540
                              15 ...
                                       119.956840
                                                         2164.579412
                 8.147417
                              16 ...
                                       149.948316
                                                          271.493436
      Item2 Item3 Item4 Item5 Item6 Item7 Item8
    0
          5
                5
                      3
                            4
                                 4
                                       3
    1
                3
                      3
                            4
                                 3
                                       4
    2
          4
                2
                      4
                                 3
                                       3
                                             3
                           4
    3
          4
                4
                      2
                                 4
                                       3
                                             3
                            5
```

[5 rows x 38 columns]

```
[3]: #Transform categorical variables to numeric using dummy variables and dropping

→ one column to meet n-1

churn_LRM_transdata = pd.get_dummies(churn_LRM_data, drop_first=True)

#Check that dummy variables properly transformed

churn_LRM_transdata.head(5)
```

[3]:	Population	Children	Age	Inco	me	Outage_sec	_pe	rweek	Email	Contac	ts	\
0	38	0	68	28561.	99		7.9	78323	10		0	
1	10446	1	27	21704.	77	1	1.6	99080	12		0	
2	3735	4	50	9609.	57	1	0.7	52800	9		0	
3	13863	1	48	18925.	23	1	4.9	13540	15		2	
4	11352	0	83	40074.	19		8.1	47417	16		2	
	Yearly_equi	p_failure	T	enure	Mon	thlyCharge	•••	Onlin	eSecuri	ty_Yes	\	
0		1	6.7	95513		172.455519	•••			1		
1		1	1.1	56681		242.632554	•••			1		
2		1	15.7	54144		159.947583	•••			0		

```
3
                          17.087227
                                          119.956840 ...
                                                                             1
4
                            1.670972
                                          149.948316 ...
                                                                             0
   OnlineBackup_Yes
                      DeviceProtection_Yes
                                              TechSupport_Yes StreamingTV_Yes
0
1
                   0
                                           0
                                                             0
                                                                                1
2
                   0
                                           0
                                                             0
                                                                                0
3
                   0
                                           0
                                                             0
                                                                                1
4
                   0
                                           0
                                                             1
                                                                                1
   StreamingMovies_Yes PaperlessBilling_Yes
0
                      1
                                              1
1
2
                      1
                                              1
3
                      0
                                              1
4
                      0
                                              0
   PaymentMethod_Credit Card (automatic) PaymentMethod_Electronic Check \
0
                                          0
                                                                             0
1
2
                                          1
                                                                             0
                                          0
3
                                                                             0
4
                                          0
                                                                             0
   PaymentMethod_Mailed Check
0
                              0
                              0
1
2
                              0
3
                              1
                              1
```

[5 rows x 47 columns]

Summary of data to obtain univariate statistics of each variable:

```
[4]: #Display univariate statistics of each numberical variable churn_statistics = churn_LRM_transdata.describe() churn_statistics.round(2)
```

[4]:		Population	Children	Age	Income	Outage_sec_perweek	\
	count	10000.00	10000.00	10000.00	10000.00	10000.00	
	mean	9756.56	2.09	53.08	39806.93	10.00	
	std	14432.70	2.15	20.70	28199.92	2.98	
	min	0.00	0.00	18.00	348.67	0.10	
	25%	738.00	0.00	35.00	19224.72	8.02	
	50%	2910.50	1.00	53.00	33170.60	10.02	
	75%	13168.00	3.00	71.00	53246.17	11.97	
	max	111850.00	10.00	89.00	258900.70	21.21	

```
Email
                  Contacts
                             Yearly_equip_failure
                                                       Tenure
                                                                MonthlyCharge
       10000.00
                  10000.00
                                          10000.00
                                                     10000.00
                                                                     10000.00
count
                                              0.40
           12.02
                       0.99
                                                        34.53
                                                                        172.62
mean
std
            3.03
                       0.99
                                              0.64
                                                        26.44
                                                                         42.94 ...
            1.00
                       0.00
                                              0.00
                                                         1.00
                                                                         79.98
min
                                                                        139.98 ...
25%
           10.00
                       0.00
                                              0.00
                                                         7.92
50%
           12.00
                       1.00
                                              0.00
                                                        35.43
                                                                        167.48
75%
                       2.00
           14.00
                                              1.00
                                                        61.48
                                                                        200.73 ...
           23.00
                      7.00
                                              6.00
                                                        72.00
                                                                        290.16 ...
max
       OnlineSecurity_Yes
                             OnlineBackup_Yes
                                                DeviceProtection_Yes
                  10000.00
                                      10000.00
                                                              10000.00
count
                       0.36
                                          0.45
                                                                  0.44
mean
std
                       0.48
                                          0.50
                                                                  0.50
                       0.00
                                          0.00
                                                                  0.00
min
25%
                       0.00
                                          0.00
                                                                  0.00
50%
                       0.00
                                          0.00
                                                                  0.00
75%
                       1.00
                                          1.00
                                                                  1.00
                       1.00
max
                                          1.00
                                                                  1.00
       TechSupport_Yes
                         StreamingTV_Yes
                                           StreamingMovies_Yes
count
               10000.00
                                 10000.00
                                                        10000.00
                   0.38
                                      0.49
                                                            0.49
mean
std
                   0.48
                                      0.50
                                                             0.50
                   0.00
min
                                      0.00
                                                            0.00
25%
                   0.00
                                                             0.00
                                      0.00
50%
                   0.00
                                      0.00
                                                             0.00
75%
                   1.00
                                      1.00
                                                             1.00
                   1.00
                                      1.00
                                                             1.00
max
       PaperlessBilling_Yes
                               PaymentMethod_Credit Card (automatic)
                    10000.00
                                                               10000.00
count
                         0.59
                                                                   0.21
mean
                         0.49
                                                                   0.41
std
min
                         0.00
                                                                   0.00
25%
                         0.00
                                                                   0.00
50%
                         1.00
                                                                   0.00
75%
                         1.00
                                                                   0.00
max
                         1.00
                                                                   1.00
       PaymentMethod_Electronic Check PaymentMethod_Mailed Check
count
                               10000.00
                                                              10000.00
mean
                                    0.34
                                                                  0.23
std
                                    0.47
                                                                  0.42
                                    0.00
                                                                  0.00
min
                                    0.00
25%
                                                                  0.00
```

```
50% 0.00 0.00
75% 1.00 0.00
max 1.00 1.00

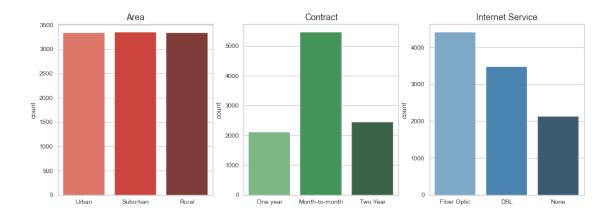
[8 rows x 47 columns]

[5]: print("\n")
```

Bar charts of each categorical variable:

```
[6]: fig, axs = plt.subplots(1,3, figsize=(15,5))
     for i in range(3):
         ax = axs[i]
         if i == 0:
             title = ax.set_title("Area",
                                    loc='center',
                                    size = 14,
                                    y=1.0),
         if i == 1:
             title = ax.set_title("Contract",
                                    loc='center',
                                    size = 14,
                                    y=1.0)
         if i == 2:
             title = ax.set_title("Internet Service",
                                    size = 14,
                                    loc='center',
                                    y=1.0)
     sns.set_style('whitegrid')
     \#Create\ barcharts\ of\ each\ catergorical\ variable\ using\ the\ non-transformed_{\sqcup}
      \hookrightarrow dataset
     c1 = sns.countplot(x = 'Area',
                         data = churn_LRM_data,
                         palette="Reds_d",
                         ax = axs[0])
     c2 = sns.countplot(x = 'Contract',
                         data = churn_LRM_data,
                         palette="Greens_d",
                         ax = axs[1])
     c3 = sns.countplot(x = 'InternetService',
                         data = churn_LRM_data,
                         palette="Blues_d",
                         ax = axs[2])
```

```
#Remove xlabels
c1.set(xlabel=None);
c2.set(xlabel=None);
c3.set(xlabel=None);
print("\n")
```



- 3.3.3 3) Check relationships between each independent variable and the dependent variable using scatterplots and correlations
- 3.3.4 &
- 3.3.5 4) Check the relationships among the independent variables using scatterplots and correlations

```
[27]: #I marked out the correlation matrix as the matrix took up too much space in the PDF as was not a requirement.

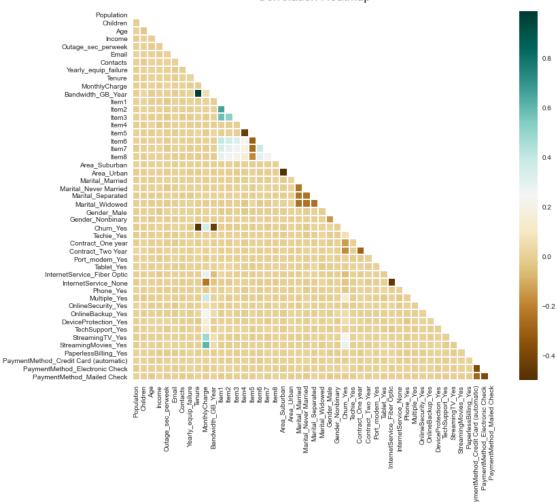
#corr = churn_LRM_transdata.corr()
#display(corr)
```

```
[8]: mask = np.triu(np.ones_like(churn_LRM_transdata.corr()))
```

```
[9]: plt.figure(figsize=(12, 10))
sns.heatmap(corr, xticklabels=corr.columns, yticklabels=corr.columns, mask =

→mask, linewidth=.01,cmap='BrBG')
plt.title('Correlation Heatmap', fontdict={'fontsize':18}, pad=16);
```

Correlation Heatmap

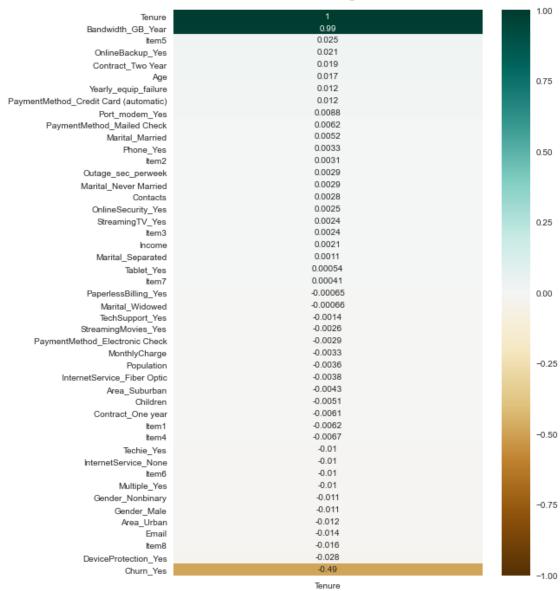


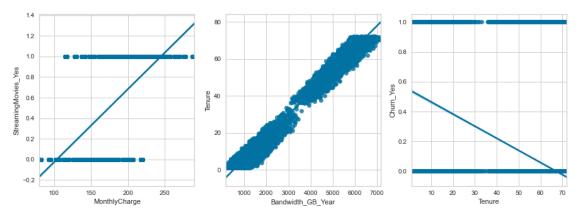
```
[10]: plt.figure(figsize=(8, 12))
sns.heatmap(churn_LRM_transdata.corr()[['Tenure']].sort_values(by='Tenure',

→ascending=False), vmin=-1, vmax=1,

annot=True, cmap='BrBG')
plt.title('Features Correlating with Tenure', fontdict={'fontsize':18}, pad=16);
```

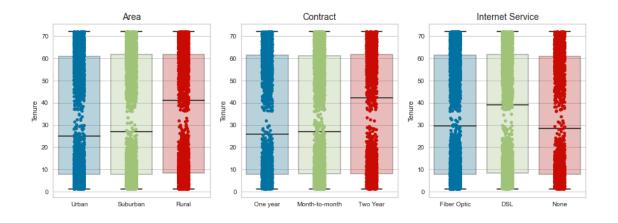
Features Correlating with Tenure





```
[12]: fig, axs = plt.subplots(1,3, figsize=(15,5))
      for i in range(3):
          ax = axs[i]
          if i == 0:
              title = ax.set_title("Area",
                                   loc='center',
                                   size = 14,
                                   y=1.0),
          if i == 1:
              title = ax.set_title("Contract",
                                   loc='center',
                                   size = 14,
                                   y=1.0)
          if i == 2:
              title = ax.set_title("Internet Service",
                                   size = 14,
                                   loc='center',
                                   y=1.0)
      #Create stripplots overlapped with boxplots of each catergorical variable
      g1 = sns.boxplot(x='Area',
                  y='Tenure',
                  data = churn_LRM_data,
                  boxprops=dict(alpha=.3),
```

```
ax = axs[0])
sns.stripplot(x='Area',
              y='Tenure',
              data = churn_LRM_data,
              ax = axs[0]
g2 = sns.boxplot(x='Contract',
            y='Tenure',
            data = churn_LRM_data,
            boxprops=dict(alpha=.3),
            ax = axs[1]
sns.stripplot(x='Contract',
              y='Tenure',
              data = churn_LRM_data,
              ax = axs[1]
g3 = sns.boxplot(x='InternetService',
            y='Tenure',
            data = churn_LRM_data,
            boxprops=dict(alpha=.3),
            ax = axs[2])
sns.stripplot(x='InternetService',
              y='Tenure',
              data = churn_LRM_data,
              ax = axs[2])
#Remove xlabels
g1.set(xlabel=None);
g2.set(xlabel=None);
g3.set(xlabel=None);
print("\n")
```



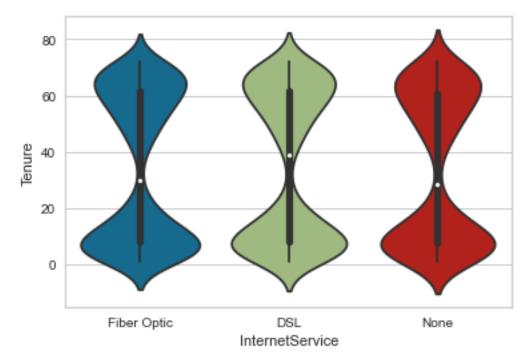
```
[13]: #Created a violin plot to better visualize the distibution of the

→InternetService variable

sns.violinplot(x='InternetService',

y='Tenure',

data = churn_LRM_data);
```



3.3.6 5) Use the non-redudant independent variables in the analysis to find the best fitting model

Data Before Removing Target and Redundant Variables

const	238.984679	
Population	1.003163	
Children	1.005223	
Age	1.004917	
Income	1.004313	
Outage_sec_perweek	1.005126	
Email	1.004522	
Contacts	1.004216	
Yearly_equip_failure	1.004513	
MonthlyCharge	24.383308	
Bandwidth_GB_Year	1.486010	
Item1	2.218863	
Item2	1.936285	
Item3	1.608640	
Item4	1.280036	
Item5	1.376610	
Item6	1.484337	
Item7	1.315291	
Item8	1.192026	
Area_Suburban	1.337302	
Area_Urban	1.339328	
Marital_Married	1.553599	
Marital_Never Married	1.562971	
Marital_Separated	1.572499	
Marital_Widowed	1.579742	
Gender_Male	1.029279	

Gender_Nonbinary	1.025488
Churn_Yes	1.966203
Techie_Yes	1.010727
Contract_One year	1.191609
Contract_Two Year	1.204735
Port_modem_Yes	1.002303
Tablet_Yes	1.006094
<pre>InternetService_Fiber Optic</pre>	2.680909
InternetService_None	1.652055
Phone_Yes	1.006150
Multiple_Yes	4.435773
OnlineSecurity_Yes	1.027237
OnlineBackup_Yes	2.644752
DeviceProtection_Yes	1.505744
TechSupport_Yes	1.488305
StreamingTV_Yes	6.779135
StreamingMovies_Yes	9.881864
PaperlessBilling_Yes	1.005103
PaymentMethod_Credit Card (automatic)	1.537269
PaymentMethod_Electronic Check	1.675824
PaymentMethod_Mailed Check	1.570644
dtype: float64	

Data After Removing Target and Redundant Variables

const	159.189361
Population	1.003147
Children	1.005126
Age	1.004854
Income	1.004180
Outage_sec_perweek	1.005030
Email	1.004505
Contacts	1.004130
Yearly_equip_failure	1.004434
MonthlyCharge	2.085597
Bandwidth_GB_Year	1.485036
Item1	2.218140
Item2	1.935335
Item3	1.608234
Item4	1.279988
Item5	1.376347
Item6	1.484187
Item7	1.315162
Item8	1.191926

```
Area_Suburban
                                             1.337271
Area_Urban
                                             1.339255
Marital_Married
                                             1.553586
Marital_Never Married
                                             1.562880
Marital Separated
                                             1.572453
Marital Widowed
                                             1.579559
Gender Male
                                             1.028943
Gender_Nonbinary
                                             1.025177
Churn Yes
                                             1.965478
Techie_Yes
                                             1.010350
Contract_One year
                                             1.190871
Contract_Two Year
                                             1.204389
Port_modem_Yes
                                             1.002122
Tablet_Yes
                                            1.005818
InternetService_Fiber Optic
                                             1.512357
InternetService_None
                                             1.313499
Phone_Yes
                                             1.005838
Multiple_Yes
                                             1.239551
OnlineSecurity_Yes
                                             1.007713
OnlineBackup Yes
                                             1.115532
DeviceProtection Yes
                                             1.045398
TechSupport Yes
                                             1.037459
PaperlessBilling_Yes
                                            1.004773
PaymentMethod_Credit Card (automatic)
                                            1.536866
PaymentMethod_Electronic Check
                                            1.675565
PaymentMethod_Mailed Check
                                            1.570557
dtype: float64
```

3.4 C5: Provide a copy of the data set:

```
[15]: churn_LRM_transdata.to_csv('C:/Users/holtb/Data/D208/Task1/churn_LRM_transdata. 

⇔csv')
```

4 Part IV: Model Comparison Analysis

4.1 D1: Initial Linear Regression Model

The model is first built with all variables determined in part C3:

```
[16]: # define our input variable (X) & output variable
    churn_LRM_data_initial = churn_LRM_transdata
    X = churn_LRM_data_initial.drop('Tenure', axis = 1)
    Y = churn_LRM_data_initial[['Tenure']]

# Split X and y into X_
```

```
→random_state=1)
[17]: #define the input
     X2 = sm.add_constant(X)
     #create an OLS model
     initial_model = sm.OLS(Y, X2)
     #fit the data
     initial_est = initial_model.fit()
     #Summarize the output
     initial_est.summary()
[17]: <class 'statsmodels.iolib.summary.Summary'>
                             OLS Regression Results
     Dep. Variable:
                                Tenure R-squared:
                                                                    1.000
    Model:
                                  OLS Adj. R-squared:
                                                                    1.000
    Method:
                         Least Squares F-statistic:
                                                                1.316e+07
    Date:
                       Sun, 11 Jul 2021 Prob (F-statistic):
                                                                     0.00
     Time:
                              14:33:57 Log-Likelihood:
                                                                   8140.4
     No. Observations:
                                10000 AIC:
                                                               -1.619e+04
    Df Residuals:
                                 9953
                                      BIC:
                                                                -1.585e+04
    Df Model:
                                   46
     Covariance Type:
                            nonrobust
     _____
     _____
                                           coef
                                                  std err
     P>|t| [0.025 0.975]
     const
                                        -3.8431
                                                   0.017 -231.335
          -3.876 -3.811
     0.000
    Population
                                      -7.836e-08 7.46e-08 -1.051
     0.293
            -2.25e-07 6.78e-08
     Children
                                        -0.3755
                                                    0.001
                                                          -748.384
     0.000
              -0.377 -0.375
                                         0.0400
                                                5.2e-05
                                                           768.285
     Age
     0.000
               0.040
                         0.040
     Income
                                       1.564e-08 3.82e-08
                                                             0.410
     0.682
           -5.92e-08
                       9.05e-08
     Outage_sec_perweek
                                         0.0003
                                                   0.000
                                                             0.809
     0.419
             -0.000
                         0.001
     Email
                                      -6.71e-05
                                                   0.000
                                                            -0.189
```

X_train, X_test, Y_train, Y_test = train_test_split(X, Y, test_size=0.20, u)

0.850 -0.001	0.001			
Contacts	0.002	-0.0006	0.001	-0.545
0.586 -0.003	0.002			
Yearly_equip_failure		0.0002	0.002	0.113
0.910 -0.003	0.004			
MonthlyCharge		-0.0352	0.000	-284.931
0.000 -0.035	-0.035			
Bandwidth_GB_Year		0.0122	5.99e-07	2.04e+04
0.000 0.012	0.012			
Item1		0.0023	0.002	1.503
0.133 -0.001	0.005			
Item2		-0.0007	0.001	-0.484
0.628 -0.004	0.002			
Item3		0.0010	0.001	0.758
0.448 -0.002	0.004			
Item4		0.0004	0.001	0.377
0.706 -0.002	0.003			
Item5		-0.0006	0.001	-0.483
0.629 -0.003	0.002			
Item6		-0.0013	0.001	-1.007
0.314 -0.004	0.001			
Item7		-0.0011	0.001	-0.898
0.369 -0.003	0.001			
Item8		7.959e-05	0.001	0.070
0.944 -0.002	0.002			
Area_Suburban		-0.0068	0.003	-2.593
0.010 -0.012	-0.002			
Area_Urban		-0.0033	0.003	-1.251
0.211 -0.008	0.002			
Marital_Married		-0.0006	0.003	-0.166
0.868 -0.007	0.006			
Marital_Never Married		-0.0011	0.003	-0.329
0.742 -0.008	0.006			
Marital_Separated		0.0026	0.003	0.785
0.432 -0.004	0.009			
Marital_Widowed		3.45e-05	0.003	0.010
0.992 -0.007	0.007			
Gender_Male		-0.7924	0.002	-362.915
0.000 -0.797	-0.788			
Gender_Nonbinary		0.2618	0.007	36.143
0.000 0.248	0.276			
Churn_Yes		0.0020	0.003	0.571
0.568 -0.005	0.009			
Techie_Yes		-1.104e-05	0.003	-0.004
0.997 -0.006	0.006	2.2020	3.000	3.001
Contract_One year	0.000	0.0011	0.003	0.389
0.697 -0.005	0.007	0.0011	0.000	0.000
0.000	0.001			

Contract_Two Year	0.0023	0.003	0.855	
0.393 -0.003 0.008				
Port_modem_Yes	0.0027	0.002	1.248	
0.212 -0.002 0.007				
Tablet_Yes	0.0007	0.002	0.277	
0.781 -0.004 0.005				
<pre>InternetService_Fiber Optic</pre>	5.7542	0.004	1623.665	
0.000 5.747 5.761				
InternetService_None	4.6006	0.003	1363.478	
0.000 4.594 4.607				
Phone_Yes	0.0017	0.004	0.457	
0.648 -0.006 0.009				
Multiple_Yes	0.2684	0.005	59.120	
0.000 0.260 0.277				
OnlineSecurity_Yes	-0.8313	0.002	-365.801	
0.000 -0.836 -0.827				
OnlineBackup_Yes	-0.3552	0.004	-101.115	
0.000 -0.362 -0.348				
DeviceProtection_Yes	-0.5971	0.003	-224.678	
0.000 -0.602 -0.592				
TechSupport_Yes	0.3850	0.003	142.188	
0.000 0.380 0.390				
StreamingTV_Yes	-1.2984	0.006	-232.008	
0.000 -1.309 -1.287				
StreamingMovies_Yes	-0.7227	0.007	-106.938	
0.000 -0.736 -0.709				
PaperlessBilling_Yes	-0.0037	0.002	-1.675	
0.094 -0.008 0.001				
<pre>PaymentMethod_Credit Card (automatic)</pre>	0.0021	0.003	0.628	
0.530 -0.004 0.008				
PaymentMethod_Electronic Check	0.0029	0.003	0.975	
0.329 -0.003 0.009				
PaymentMethod_Mailed Check	0.0073	0.003	2.283	
0.022 0.001 0.014				
Omnibus: 34822.579	====== Durbin-Wats	======= on:		002
Prob(Omnibus): 0.000	Jarque-Bera		1633.	
	Prob(JB):	, .		0.00
Kurtosis: 1.021	Cond. No.		8.316	

Notes

^[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

^[2] The condition number is large, 8.31e+05. This might indicate that there are strong multicollinearity or other numerical problems.

4.2 D2: Linear Regression Model with Colinear Independent Variables Removed

As described in the notes section of the model, the condition number calculated indicates there are strong multicollinearity issues within the model. This can be confirmed by looking at the variance inflation factors (VIF) that were calculated in section C. VIF values above 5 to 10 are usually indicators of multicollinearity. Variables identified as having colinearity were dropped in each category until the VIF calculation returned to an acceptable value. In this case, the 'StreamingMovies_Yes' and 'StreamingTV_Yes' variables were dropped which reduced "MonthlyCharge" VIF to less than 2. These same variables will be dropped to create the next model.

```
[19]: #define the input
    next2_X2 = sm.add_constant(next2_X)

#create an OLS model
    next2_model = sm.OLS(next2_Y, next2_X2)

#fit the data
    next2_est = next2_model.fit()

#Summarize the output
    next2_est.summary()
```

[19]: <class 'statsmodels.iolib.summary.Summary'>

OLS Regression Results

Dep. Variable:	Tenure	R-squared:	1.000
Model:	OLS	Adj. R-squared:	1.000
Method:	Least Squares	F-statistic:	1.320e+06
Date:	Sun, 11 Jul 2021	Prob (F-statistic):	0.00
Time:	14:35:00	Log-Likelihood:	-3581.3
No. Observations:	10000	AIC:	7253.
Df Residuals:	9955	BIC:	7577.
Df Model:	44		
Covariance Type:	nonrobust		

coef std err

P> t	[0.025	0.975]				
const			-2.3651	0.044	-54.027	
	-2.451	-2.279				
Populat			-1.522e-08	2.41e-07	-0.063	
	-4.87e-07	4.57e-07	0.0744	0.000	000 004	
Children 0.000	n -0.377	-0.371	-0.3741	0.002	-230.896	
Age	-0.377	-0.371	0.0401	0.000	238.707	
0.000	0.040	0.040	0.0401	0.000	200.101	
Income	0.020	0.010	9.543e-08	1.23e-07	0.774	
0.439	-1.46e-07	3.37e-07				
Outage_	sec_perweek		0.0002	0.001	0.134	
0.894	-0.002	0.002				
Email			-0.0005	0.001	-0.438	
0.661	-0.003	0.002	0.0000	0.004	0 005	
Contact: 0.948	s -0.007	0.007	0.0002	0.004	0.065	
	-0.007 equip_failur		-0.0038	0.005	-0.696	
0.486		0.007	0.0000	0.005	0.050	
Monthly			-0.0542	0.000	-464.833	
0.000	-0.054	-0.054				
Bandwid [.]	th_GB_Year		0.0122	1.93e-06	6305.369	
0.000	0.012	0.012				
Item1			-0.0062	0.005	-1.246	
0.213	-0.016	0.004	0.000		0 544	
Item2 0.477	0.006	0.010	0.0033	0.005	0.711	
Item3	-0.006	0.012	0.0066	0.004	1.540	
0.124	-0.002	0.015	0.0000	0.001	1.010	
Item4			4.772e-05	0.004	0.012	
0.990	-0.007	0.008				
Item5			0.0046	0.004	1.159	
0.246	-0.003	0.012				
Item6			-0.0011	0.004	-0.262	
0.793	-0.009	0.007	0 0005	0.004	0.654	
Item7 0.513	-0.005	0.010	0.0025	0.004	0.654	
Item8	0.005	0.010	-0.0031	0.004	-0.834	
0.404	-0.010	0.004	0.0001	0.001	0.001	
Area_Su			-0.0031	0.009	-0.362	
0.717	-0.020	0.014				
Area_Ur	ban		0.0021	0.009	0.247	
0.805	-0.015	0.019				
	_Married	0.000	0.0018	0.011	0.160	
0.873	-0.020	0.023				

Marital_Never Married -0.0089 0.011 -0.817 0.414 -0.030 0.013 -0.0024 0.011 -0.223 Marital_Separated 0.019 -0.0105 0.011 -0.964 0.823 -0.032 0.011 -0.8021 0.007 -113.809 0.000 -0.816 -0.788 -0.8021 0.007 -113.809 0.000 -0.816 -0.788 -0.8021 0.003 9.638 0.000 -0.816 -0.788 -0.8021 0.023 9.638 0.000 0.180 0.271 0.0181 0.011 -1.642 0.101 -0.040 0.004 0.0055 0.009 0.593 0.553 -0.013 0.024 0.0055 0.009 0.593 0.553 -0.013 0.024 0.007 0.009 -1.688 0.091 -0.034 0.003 0.009 -0.875 0.382 -0.025 0.010 0.007 0.069 0.945 -0.013 0.014 0.007 0.0063 0.008 -0.829
Marital_Separated
0.823 -0.024 0.019 Marital_Widowed -0.0105 0.011 -0.964 0.335 -0.032 0.011 -0.8021 0.007 -113.809 0.000 -0.816 -0.788 666er -0.2254 0.023 9.638 0.000 0.180 0.271 0.011 -1.642 Churn_Yes -0.0181 0.011 -1.642 0.101 -0.040 0.004 0.0055 0.009 0.593 0.553 -0.013 0.024 0.0055 0.009 0.593 0.553 -0.013 0.024 0.0055 0.009 -1.688 0.091 -0.034 0.003 0.009 -1.688 0.091 -0.034 0.003 0.009 -0.875 0.382 -0.025 0.010 0.007 0.069 0.945 -0.013 0.014 0.007 0.009 Tablet_Yes -0.001 0.009 713.355 0.000 6.114 6.147 0.007 0.009 InternetService_None 4.3502 0.010 4
Marital_Widowed
0.335 -0.032 0.011 Gender_Male -0.8021 0.007 -113.809 0.000 -0.816 -0.788 -0.023 9.638 0.000 0.180 0.271 0.011 -1.642 Churn_Yes -0.0181 0.011 -1.642 0.101 -0.040 0.004 0.0055 0.009 0.593 0.553 -0.013 0.024 0.0055 0.009 0.593 0.553 -0.013 0.024 0.007 0.009 -1.688 0.091 -0.034 0.003 0.009 -0.875 0.382 -0.025 0.010 0.007 0.069 0.945 -0.013 0.014 0.005 0.007 0.069 0.407 -0.013 0.014 0.0063 0.008 -0.829 0.407 -0.021 0.009 0.0063 0.009 713.355 0.000 6.114 6.147 0.007 0.012 0.007 1.000 4.331 4.369 0.012 0.007 0.095 0.095 -0.023
Gender_Male
0.000 -0.816 -0.788 Gender_Nonbinary 0.2254 0.023 9.638 0.000 0.180 0.271 0.011 -1.642 Churn_Yes -0.0181 0.011 -1.642 0.101 -0.040 0.004 0.005 0.009 0.593 0.553 -0.013 0.024 0.005 0.009 -1.688 0.091 -0.034 0.003 0.009 -1.688 0.091 -0.034 0.003 0.009 -0.875 0.382 -0.025 0.010 0.007 0.069 0.945 -0.013 0.014 0.007 0.069 0.407 -0.021 0.009 0.008 -0.829 0.407 -0.021 0.009 0.009 713.355 0.000 6.114 6.147 0.007 0.007 0.007 1nternetService_None 4.3502 0.010 447.834 0.000 4.331 4.369 9 7.915e-05 0.012 0.007 0.995 -0.023 0.024 0.8915 0.008
0.000 0.180 0.271 Churn_Yes -0.0181 0.011 -1.642 0.101 -0.040 0.004 -0.0055 0.009 0.593 Centing Yes 0.0055 0.009 0.593 0.553 -0.013 0.024 Contract_One year -0.0157 0.009 -1.688 0.091 -0.034 0.003 0.007 0.009 -0.875 0.382 -0.025 0.010 0.007 0.069 O.945 -0.013 0.014 0.007 0.069 Tablet_Yes -0.0063 0.008 -0.829 0.407 -0.021 0.009 0.009 713.355 0.000 6.114 6.147 0.009 0.009 713.355 0.000 4.331 4.369 0.012 0.007 0.007 Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 0.8915 0.008 115.042 0.000 0.876 0.997 0.07776 0.007 <t< td=""></t<>
Churn_Yes
0.101 -0.040 0.004 Techie_Yes 0.0055 0.009 0.593 0.553 -0.013 0.024 Contract_One year -0.0157 0.009 -1.688 0.091 -0.034 0.003 Contract_Two Year -0.0078 0.009 -0.875 0.382 -0.025 0.010 Port_modem_Yes 0.0005 0.007 0.069 0.945 -0.013 0.014 Tablet_Yes -0.0063 0.008 -0.829 0.407 -0.021 0.009 InternetService_Fiber Optic 6.1306 0.009 713.355 0.000 6.114 6.147 InternetService_None 4.3502 0.010 447.834 0.000 4.331 4.369 Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
Techie_Yes 0.0055 0.009 0.593 0.553 -0.013 0.024 Contract_One year -0.0157 0.009 -1.688 0.091 -0.034 0.003 Contract_Two Year -0.0078 0.009 -0.875 0.382 -0.025 0.010 Port_modem_Yes 0.005 0.007 0.069 0.945 -0.013 0.014 Tablet_Yes -0.021 0.009 InternetService_Fiber Optic 6.1306 0.009 713.355 0.000 6.114 6.147 InternetService_None 4.3502 0.010 447.834 0.000 4.331 4.369 Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
0.553 -0.013 0.024 Contract_One year -0.0157 0.009 -1.688 0.091 -0.034 0.003 0.009 -1.688 Contract_Two Year -0.0078 0.009 -0.875 0.382 -0.025 0.010 0.0005 0.007 0.069 Port_modem_Yes 0.0005 0.007 0.069 0.945 -0.013 0.014 -0.0063 0.008 -0.829 0.407 -0.021 0.009 -0.009 -0.829 0.407 -0.021 0.009 -0.009 713.355 0.000 6.114 6.147 -0.000 447.834 0.000 4.331 4.369 -0.012 0.007 0.995 -0.023 0.024 -0.000 0.876 0.907 0nlineSecurity_Yes -0.7776 0.007 -107.001
Contract_One year
0.091 -0.034 0.003 Contract_Two Year -0.0078 0.009 -0.875 0.382 -0.025 0.010 Port_modem_Yes 0.0005 0.007 0.069 0.945 -0.013 0.014 Tablet_Yes -0.0063 0.008 -0.829 0.407 -0.021 0.009 InternetService_Fiber Optic 6.1306 0.009 713.355 0.000 6.114 6.147 InternetService_None 4.3502 0.010 447.834 0.000 4.331 4.369 Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
Contract_Two Year
0.382 -0.025 0.010 Port_modem_Yes 0.0005 0.007 0.069 0.945 -0.013 0.014 Tablet_Yes -0.0063 0.008 -0.829 0.407 -0.021 0.009 InternetService_Fiber Optic 6.1306 0.009 713.355 0.000 6.114 6.147 InternetService_None 4.3502 0.010 447.834 0.000 4.331 4.369 Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
Port_modem_Yes 0.0005 0.007 0.069 0.945 -0.013 0.014 Tablet_Yes -0.0063 0.008 -0.829 0.407 -0.021 0.009 InternetService_Fiber Optic 6.1306 0.009 713.355 0.000 6.114 6.147 InternetService_None 4.3502 0.010 447.834 0.000 4.331 4.369 Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
0.945 -0.013 0.014 Tablet_Yes -0.0063 0.008 -0.829 0.407 -0.021 0.009 InternetService_Fiber Optic 6.1306 0.009 713.355 0.000 6.114 6.147 InternetService_None 4.3502 0.010 447.834 0.000 4.331 4.369 Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
Tablet_Yes -0.0063 0.008 -0.829 0.407 -0.021 0.009 InternetService_Fiber Optic 6.1306 0.009 713.355 0.000 6.114 6.147 InternetService_None 4.3502 0.010 447.834 0.000 4.331 4.369 Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
0.407 -0.021 0.009 InternetService_Fiber Optic 6.1306 0.009 713.355 0.000 6.114 6.147 InternetService_None 4.3502 0.010 447.834 0.000 4.331 4.369 Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
InternetService_Fiber Optic 6.1306 0.009 713.355 0.000 6.114 6.147
0.000 6.114 6.147 InternetService_None 4.3502 0.010 447.834 0.000 4.331 4.369 Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
InternetService_None 4.3502 0.010 447.834 0.000 4.331 4.369 Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
0.000 4.331 4.369 Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
Phone_Yes 7.915e-05 0.012 0.007 0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
0.995 -0.023 0.024 Multiple_Yes 0.8915 0.008 115.042 0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
0.000 0.876 0.907 OnlineSecurity_Yes -0.7776 0.007 -107.001
OnlineSecurity_Yes -0.7776 0.007 -107.001
v –
0.000 -0.792 -0.763
0.000
OnlineBackup_Yes 0.0799 0.007 10.852
0.000 0.065 0.094
DeviceProtection_Yes -0.3542 0.007 -49.549
0.000 -0.368 -0.340
TechSupport_Yes 0.6221 0.007 85.224
0.000 0.608 0.636
PaperlessBilling_Yes 0.0083 0.007 1.175
0.240 -0.006 0.022 PaymentMethod_Credit Card (automatic) 0.0051 0.011 0.483
PaymentMethod_Credit Card (automatic) 0.0051 0.011 0.483 0.629 -0.016 0.026
PaymentMethod_Electronic Check 0.0109 0.009 1.146
0.252 -0.008 0.029
PaymentMethod_Mailed Check 0.0131 0.010 1.263
0.207 -0.007 0.033

Omnibus:	1428.031	Durbin-Watson:	2.002
<pre>Prob(Omnibus):</pre>	0.000	Jarque-Bera (JB):	1425.968
Skew:	-0.859	Prob(JB):	2.26e-310
Kurtosis:	2.315	Cond. No.	6.31e+05

Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 6.31e+05. This might indicate that there are strong multicollinearity or other numerical problems.

4.3 D2 Cont: Linear Regression Model Variables with P-Values greater than .05 Removed

A p-value determines the significance between a null hypothesis that a coefficient is zero and the alternate hypothesis that the coefficient is not equal to zero. In the model, the normal significance threshold of p < .05 was used. The variable with the largest p-value was removed one at a time and the model was reran using stepwise until all variable's p-values were less than .05.

```
[20]: churn LRM rmvhighpvalues = churn LRM nomulti.

¬drop(['Population', 'Contacts', 'Item4', 'Phone_Yes',
                                                'Port_modem_Yes', __
      →'Marital_Separated','Outage_sec_perweek',
     →'Marital Married', 'Item6', 'Area Suburban', 'Yearly equip failure',
     'Contract Two,,

¬Year', 'Marital_Never Married', 'Marital_Widowed',
                                                'PaymentMethod_Electronic_
     → Check', 'PaymentMethod_Mailed Check',
     → 'PaperlessBilling_Yes', 'Item1', 'Churn_Yes', 'Contract_One year',
                                                'Contract_One⊔
     next3_X = churn_LRM_rmvhighpvalues.drop('Tenure', axis = 1)
     next3_Y = churn_LRM_rmvhighpvalues[['Tenure']]
     # Split X and y into X_{-}
     next3_X_train, next3_X_test, next3_Y_train, next3_Y_test =_
      →train_test_split(next3_X, next3_Y, test_size=0.20, random_state=1)
```

```
[21]: #define the input
      next3_X2 = sm.add_constant(next3_X)
      #create an OLS model
      next3_model = sm.OLS(next3_Y, next3_X2)
      #fit the data
      next3_est = next3_model.fit()
      \#Summarize\ the\ output
      next3_est.summary()
```

[21]:

<pre><class 'statsmodels.iolib.summary.summary'=""></class></pre>										
OLS Regression Results										
Dep. Variable: Model: Method: Date: Time: No. Observations: Df Residuals: Df Model: Covariance Type:		Leas Sun, 11	Tenure OLS st Squares Jul 2021 14:35:56 10000 9986 13 nonrobust	Adj. R-squared: F-statistic: Prob (F-statistic): Log-Likelihood:		1.000 1.000 4.472e+06 0.00 -3590.7 7209. 7310.				
[0.025	====			std err		P> t				
const -2.376	-2.300		-2.3378		-121.206	0.000				
Children -0.377 Age	-0.371		-0.3742 0.0401	0.002	-231.428 239.136	0.000				
0.040 MonthlyChar	0.040 ge -0.054		-0.0543	0.000	-540.082	0.000				
Bandwidth_GB_Year 0.012 0.012			0.0122	1.6e-06	7616.847	0.000				
Gender_Male	-0.789		-0.8025		-114.275	0.000				
Gender_Nonb 0.178 InternetSer 6.119	0.270	optic	0.2240 6.1348	0.023	9.592 751.051	0.000				
	3.131									

InternetService_None		4.3514	0.010	450.093	0.000
4.332	4.370				
Multiple_Yes		0.8924	0.008	115.759	0.000
0.877	0.908				
OnlineSecurity_Yes		-0.7775	0.007	-107.279	0.000
-0.792	-0.763				
OnlineBackup_Yes		0.0815	0.007	11.127	0.000
0.067	0.096				
DeviceProtection_Yes		-0.3536	0.007	-49.622	0.000
-0.368	-0.340				
TechSupport_Yes		0.6231	0.007	85.781	0.000
0.609	0.637				
	=========				
Omnibus:		1442.743	Durbin-Watson:		2.001
Prob(Omnibus):		0.000	Jarque-Bera (JB):		1436.185
Skew:		-0.862	Prob(JB):		0.00
Kurtosis:		2.311	Cond. No.		2.74e+04
=======		=========	=======		

Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 2.74e+04. This might indicate that there are strong multicollinearity or other numerical problems.

4.4 D2 Cont: Linear Regression Model Variables without Linear Relationships Removed

By plotting the scatterplots of the remaining variables against the target variable and by referencing the partial correlation matrix in section C, it can be determined that there are variables within the model that are violating the linear relationship assumption. Additionally, removing the variables has little affect on the R-squared value meaning that they can be removed without compromising the accuracy of the model. Some variables remained in the model to meet task requirement of having two continuous and two categorical variables in the model.

```
→train_test_split(final_X, final_Y, test_size=0.20, random_state=1)
[24]: #define the input
    final_X2 = sm.add_constant(final_X)
    #create an OLS model
    final_model = sm.OLS(final_Y, final_X2)
    #fit the data
    final est = final model.fit()
    #Summarize the output
    final_est.summary()
[24]: <class 'statsmodels.iolib.summary.Summary'>
                          OLS Regression Results
                                                             0.985
    Dep. Variable:
                            Tenure R-squared:
    Model:
                              OLS Adj. R-squared:
                                                             0.985
    Method:
                      Least Squares F-statistic:
                                                         1.621e+05
    Date:
                   Sun, 11 Jul 2021 Prob (F-statistic):
                                                              0.00
    Time:
                           14:37:50 Log-Likelihood:
                                                           -26002.
    No. Observations:
                             10000 AIC:
                                                          5.201e+04
    Df Residuals:
                              9995 BIC:
                                                          5.205e+04
    Df Model:
    Covariance Type:
                         nonrobust
    ______
                         coef std err t
                                                 P>|t|
                                                           Γ0.025
    0.975]
    const
                       -4.4968 0.078 -57.444 0.000
                                                          -4.650
    -4.343
                -0.3743 0.015 -24.649 0.000 -0.404
    Children
    -0.345
    Bandwidth_GB_Year 0.0120 1.49e-05 804.600 0.000
                                                          0.012
    0.012
    OnlineBackup_Yes -1.0870
                                 0.066 -16.579 0.000
                                                           -1.216
    -0.959
    DeviceProtection_Yes -1.0626 0.066 -16.177 0.000
                                                          -1.191
    -0.934
    ______
    Omnibus:
                            470.196 Durbin-Watson:
                                   Jarque-Bera (JB): 329.077
```

final_X_train, final_X_test, final_Y_train, final_Y_test =

0.000

Prob(Omnibus):

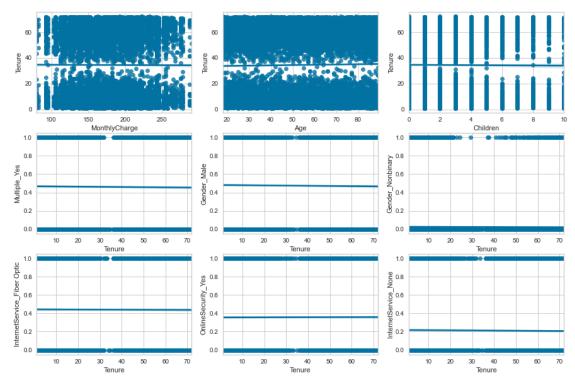
 Skew:
 -0.337
 Prob(JB):
 3.48e-72

 Kurtosis:
 2.421
 Cond. No.
 1.11e+04

Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 1.11e+04. This might indicate that there are strong multicollinearity or other numerical problems.

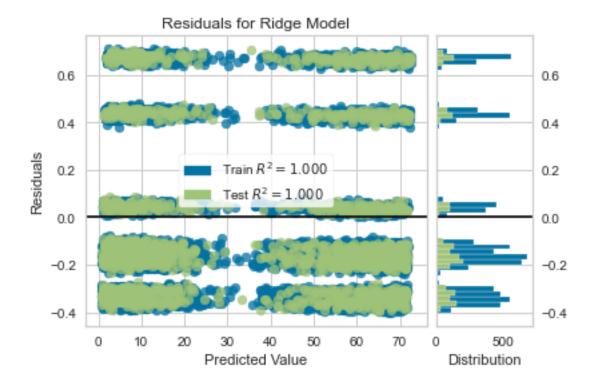
```
[25]: fig, axs = plt.subplots(3,3,figsize=(15,10))
      pl1 = sns.regplot(x='MonthlyCharge',
                       y='Tenure',
                       data=churn_LRM_transdata,
                       ax = axs[0,0])
      pl2 = sns.regplot(x='Age',
                       y = 'Tenure',
                       data=churn_LRM_transdata,
                       ax = axs[0,1]);
      pl3 = sns.regplot(x='Children',
                       y='Tenure',
                       data=churn_LRM_transdata,
                       ax = axs[0,2]
      pl4 = sns.regplot(x='Tenure',
                       y='Multiple_Yes',
                       data=churn_LRM_transdata,
                       logistic=True,
                       ax = axs[1,0]
      pl5 = sns.regplot(x='Tenure',
                       y='Gender_Male',
                       logistic=True,
                       data=churn_LRM_transdata,
                       ax = axs[1,1])
      pl6 = sns.regplot(x='Tenure',
                       y='Gender_Nonbinary',
                       logistic=True,
                       data=churn_LRM_transdata,
                       ax = axs[1,2])
      pl7 = sns.regplot(x='Tenure',
```



```
[22]: #initial model residual plot
model = Ridge()
visualizer = ResidualsPlot(model)

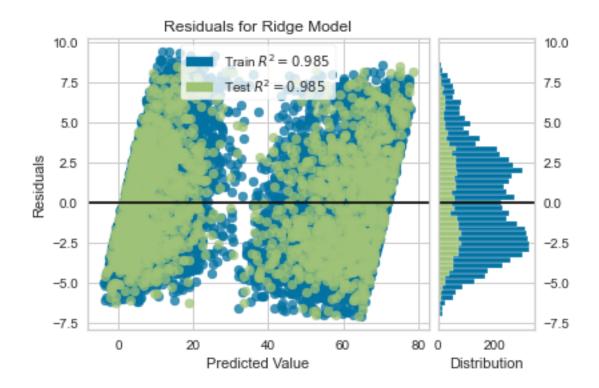
visualizer.fit(next3_X_train, next3_Y_train)
visualizer.score(next3_X_test, next3_Y_test)
```

g = visualizer.poof()



```
[26]: model = Ridge()
visualizer = ResidualsPlot(model)

visualizer.fit(final_X_train, final_Y_train)
visualizer.score(final_X_test, final_Y_test)
g = visualizer.poof()
```



4.5 Part E: Analyze the Dataset Using the Reduced Multiple Regression Model

The logic of the variable selection technique was to ensure each variable in the model met each of the multiple linear regression assumptions. If an assumption was violated it was removed. Each violation is described in section D2 as variables were removed.

With the exception of the amount of variables included, both the initial model and reduced multiple regression models are very similar. This is because the variable "Bandwidth_GB_Year", which is in both models accounts for the majority of the R-squared value. The independent variables that were removed had very little effect on the R-squared value but were removed to avoid assumption violations and legitimize the reduced model. Comparison of the initial and final models are made with the following metrics:

R-Squared/R-Squared adjusted: Both models have an extremely high R-Squared statistic that defines $\sim 99\%$ of the target variable's variance. The final model has a slightly lower R-squared value. These high R-squared indicate that both models are well fitted.

One concerning statistic in both models is the condition number. While significantly reduced in the final model these numbers are extremely large suggesting multicollinearity among the variables. However, the VIF numbers calculated early suggest that multicollinearity is insignificant. This large number may be caused by the near-perfect colinearity between the "Bandwidth_GB_Year" independent variable and the target variable.

As shown above using the Yellowbrick package, the inital models' errors were not normally distributed and did not meet the normally distributed errors assumption. After removing non colinear variables, the distribution normalized. However, the assumption of homoscedasticity among the

residuals is not met. As visualized above between the predicted values of 20 and 40 the error distribution shrinks into a gap of errors between approximately 30 and 35. This violation of homoscedasticity suggests that the test results and confidence intervals are unreliable.

5 Part V: Data Summary and Implications

5.0.1 F1 Results of the Data Analysis

Regression equation:

```
y = -0.3743x_1 + 0.120x_2 - 1.0870x_3 - 1.0626x_4 - 4.4968
```

where

 $x_1 = \text{Children}$

 $x_2 = Bandwidth_GB_Year$

 $x_3 = \text{OnlineBackup_Yes}$

 $x_4 = \text{DeviceProtection_Yes}$

Coefficient interpretation

- Children: This coefficient suggests that for each child the customer has reduce the estimated Tenure by .3743 months.
- Bandwidth_GB_Year: Even while being the smallest coefficient in the model, this variable has the most weight on the model. This is due to (1) the variable being the only continuous variable in the model and (2) the variable having the largest values between 155.51 and 7158.98 with a mean value of 3392.34. Ultimately, this independent variable is the best predictor variable for the target variable.
- OnlineBackup_Yes: As a categorical variable, this variable only has two potential values, 1 or
 If the variable exists as a 1 in the observation it will decrease the target variable prediction (Tenure) by -1.0870 months.
- DeviceProtection_Yes: Again, as a categorical variable, this variable only has two potential values, 1 or 0. If the variable exists as a 1 in the observation it will decrease the target variable prediction (Tenure) by -1.0626 months.

Statistical and Practical Significance of the Model

Looking at the statistical significance of the model we determined that the R-squared value is .985. This means that the model can explain ~99% of the target variable's variance with a 95% confidence level. At a statistical level, the model can very accurately predict the tenure of a customer.

Practically, the model makes sense at first. Customers with a longer tenure would be much more likely to use a higher amount of data because they've been using the service longer. However, the Bandwidth_GB_Year variable isn't cumulative and continues the near-perfect positive colinear relationship with Tenure even past the 12-month mark. The model suggests there is a high correlation between the amount of data a customer uses and their tenure and further analysis suggests the causation is different than just the length of time the customer has used the service.

Limitations

A significant limitation of the model is the fact that the independent variable "Bandwidth_GB_Year" has such a large overshadowing effect over the other variables. The model is basically a simple linear model because of the strong correlation of this variable with the target variable. The other two remaining variables could be removed and there would be almost no change to the outcome of the model.

Another limitation of the model is, due to multicollinearity, some independent variables needed to be removed from the model because they were redundant. By returning to the initial model, redundant variables can be replaced to determine what other independent variables had similar effects on the target variables.

5.0.2 F2 Recommended Course of Action

The recommended course of action is to conduct further exploration and analysis on the correlation of the amount of data customers use over time and their length of tenure. Looking at the data, there is a cluster of customers who use a larger amount of data per year and stay with the customers longer. It would be beneficial to find out why these customers use more data to attempt to target potential customers with the same qualities.

6 Part VI: Demonstration

6.1 G. Video

https://wgu.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=b066ba2f-6981-4845-8eab-ad6200cc72d3

6.2 H. Code Sources

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