BANA 420: Term Paper

Maximizing Risk-Adjusted Returns: Through Predictive and Prescriptive Analysis

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Executive Summary

This project explores the investment horizon for young investors and how they can leverage analytics-driven strategies to build an optimized long-term portfolio. Using predictive analytics based on 5 years of stock returns and variance, we can calculate expected return over a 20-year horizon and a full retirement simulation until 67. A prescriptive analytics technique uses the Sharpe ratio's maximization to measure a risk-adjusted return while adding real-world financial constraints.

Using an allocation value of an initial $10,000 investment across 40 NYSE-listed stocks, while applying a constraint of $1,000 to limit stock overexposure and promote diversification, results in a forecasted return of 24.2% and a Sharpe ratio of 2.16, indicating strong risk-adjusted returns. This outperforms both passive savings and the historical returns of the S&P 500. Using a Monte Carlo simulation, we could estimate a realistic range of market modifiers indicating both bear and bull markets, based on a realistic portfolio return rate with a 2.16 Sharpe Ratio and an annual inflation adjustment of 2.5%.

We constructed a sensitivity analysis to evaluate robustness by relaxing diversification constraints with three scenarios. These variations of stock allocation limits of 15%, 20%, and no limit have demonstrated a clear trade-off. While diversification fell oddly, the Sharpe Ratio increased up to 3.13, thus reducing the total overall expected return and increasing sector concentration, specifically in healthcare.

Our findings show that early and consistent investment is essential for young people entering the professional world, and diversification is also important to balance return and volatility. We also emphasize the importance of holding a stock portfolio rather than investing in an index fund. Yet, this model's design originates in Excel; it lays the foundation for an enhanced model using Python, allowing a stronger model as sophisticated as programs used by hedge funds and bulge-bracket banks. The further enhancement can allow dynamic rebalancing, personalized investor profiles, and stronger risk management.

This project provides a window into the finance industry's future by integrating predictive and prescriptive analytics into a wealth plan. It empowers the future for investors and financial professionals by connecting the world of finance to machine learning and data analytics, making smarter data-informed decisions.

Problem Formulation

This optimization problem uses predictive and prescriptive analytics to maximize the Sharpe ratio (asset return adjusted for risk) of the investor's portfolio over 20 years. The decision variables are the portfolio weights xi for all stocks i; {TSLA, PLTR, XOM, KO, …, AMZN}, representing the proportion of total investment allocated to each of the 40 NYSE-listed stocks. The objective function is to maximize (Portfolio Return - Risk Free Rate) / Portfolio Standard Deviation. One constraint is that the initial portfolio must not exceed $10,000, with no equity allowed to account for more than $1,000. Additionally, stocks from the same industry, such as OSCR, CPRX, OPCH, and VRTX in the healthcare sector, cannot collectively make up more than 10% of the portfolio's value. These constraints are designed to promote diversification, enhance portfolio stability, and support the goal of outperforming the S&P 500 through disciplined investment management. We aim to maximize the Sharpe ratio because it reflects strong, risk-adjusted investment growth.

Constraints

- Total Capital Invested
  - $\Sigma x_i = 1$
- Maximum Allocation per stock
  - $x_i \leq 0.10$ for all i
- Industry Diversification
  - $\Sigma x_i \leq 0.10$ for all i in a given industry
- Initial investment cap
  - $\Sigma (x_i * 10{,}000) \leq 10{,}000$
- Non - negativity (no short selling)
  - $x_i \geq 0$ for all i

Objective Function

- Maximize: Sharpe Ratio = $(E(R_p) - Rf) / \sigma_p$
  - $E(R_p)$ = expected return of the portfolio
  - Rf = risk-free rate (assumed constant over the investment horizon)
  - σ_p = standard deviation of the portfolio's return (volatility)

Decision Variable

- Let x_i = proportion of total capital allocated to stock i, where i = 1, 2, …, 40
- Each $x_i \in [0, 1]$ represents the portfolio weight of stock i

Predictive Analysis

The dataset used in this project is sourced from the NASDAQ website and includes historical stock price data for 40 publicly traded companies listed on the NYSE. Key features of the dataset include daily closing prices, which are used to calculate average returns, volatility (standard deviation), and correlations between stocks. This historical data enables predictive analytics to estimate future performance and prescriptive analytics to optimize portfolio allocation. The dataset is the foundation for constructing a well-diversified investment strategy over a 20-year horizon.

The data underwent several preprocessing steps to prepare it for analysis. Missing values in the stock price data were addressed to ensure accurate return calculations. Daily returns were computed to generate 252-day rolling averages and annualized returns. These values formed the basis for estimating volatility and Sharpe ratios. Additionally, correlation data was derived to support portfolio risk analysis. These steps ensured clean, consistent data for predictive forecasting and prescriptive portfolio optimization.

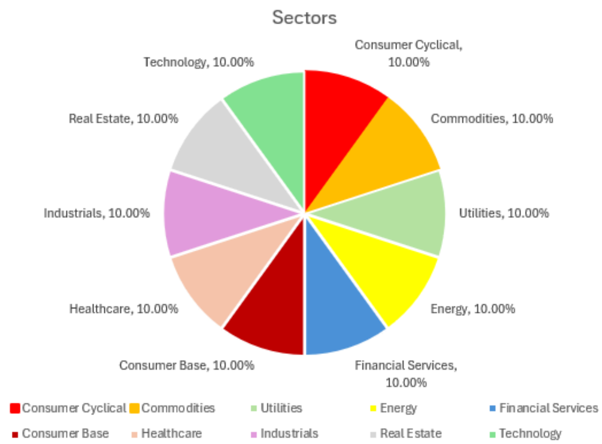
An essential enhancement in our predictive model was integrating a Monte Carlo simulation to model the market modifier (multiple applied annually to account for year-over-year valuation in investment performance). This simulation randomly generates thousands of potential market return scenarios based on a portfolio's historical volatility and returns with a Sharpe ratio of 2.16 and the S&P 500. This estimation AI provides a distribution that will show the fluctuations within the mean of the scenarios. This approach reflects real-world uncertainty more accurately than static return assumptions.

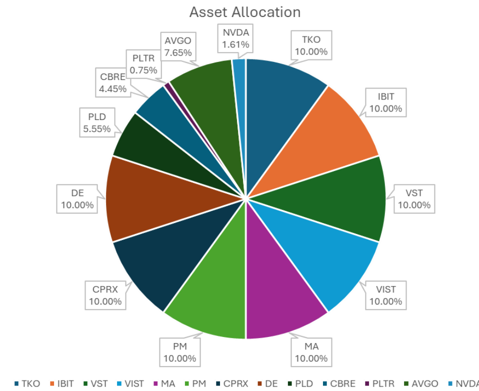
Additionally, incorporating a 2.5% annual interest rate (inflation adjustment) into our projections also compounded alongside the market's growth. This factor is applied to the future value to show the future value of the invested capital with the added adjustment of inflation. These layers of predictive modeling formed an analytical foundation for prescriptive modeling, which showed its effect on maximizing the Sharpe ratio, calculated as the expected portfolio return minus the risk-free rate divided by the portfolio’s standard deviation. A Sharpe ratio above two signals an investment-grade portfolio with strong risk-adjusted returns—our baseline target throughout the project.

Optimal Solution

The optimal solution in our model is determined by adjusting the portfolio weights to maximize the Sharpe ratio, which reflects the best possible risk-adjusted return. By optimizing these weights across selected equities, we construct a portfolio that offers the strongest return profile while maintaining diversification and adhering to real-world investment constraints. This process leads to an expected future return of approximately 24%, based on historical trends, volatility, and correlations among the assets. This projected return serves as the foundation for estimating long-term portfolio growth over a 20-year investment horizon.

Using this predicted return, we modeled the value of a retirement account assuming consistent contributions of 15% of a salary beginning at $85,000, increasing annually by 3.6%. The initial investment is $10,000 at age 21, with growth compounded over time. The simulation further extends to calculate total holdings at retirement age (67), aligning with a typical annuity or retirement planning timeline. This integrated approach allows future investors to see the power of optimized portfolio management using both predictive analytics for return estimation and prescriptive strategies for asset allocation. Ultimately, this model equips individuals with a framework for building long-term financial security through data-driven investing.

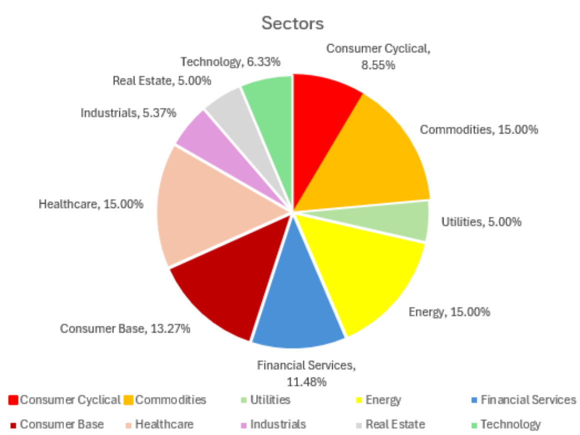


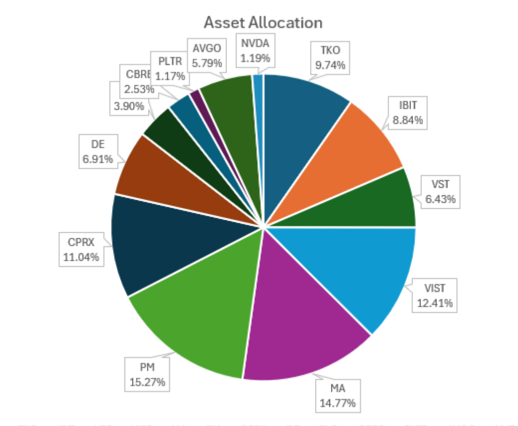


As the pie chart on the left illustrates, the constraints for each sector to have exactly 10% of the investment have been met, while the graph on the right indicates the thirteen companies that received an investment in total.

Sensitivity Analysis

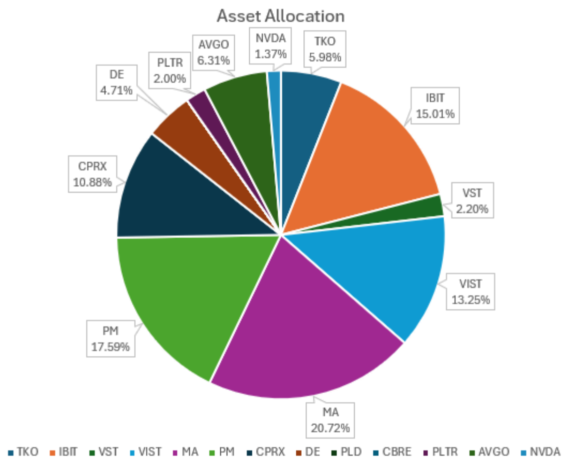
To examine the sensitivity of our model, we decided to generate three separate alternate solutions, changing the diversification constraints for each one. We structured our initial model to equally distribute the total investment among the 10 market sectors we identified, and relaxing those diversification constraints would logically be the most effective way to test our model's sensitivity. The first scenario we implemented, S1, had the same constraints as our initial problem, except now, instead of each sector being allowed exactly 10%, each sector was allowed no less than 5% of the total investment, and no more than 15%.

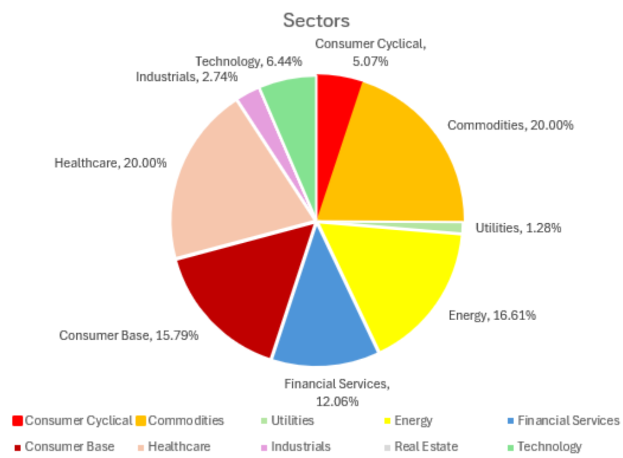




As shown above, a total of five sectors are still bound by the new constraints, indicating that the optimal solution would require further relaxation of these constraints. Additionally, S1 has a total of 20 separate companies invested in, compared to 13 in S0. While our model is designed to promote diversity across sectors, the S1 constraints allowed for more diversity in terms of the total companies invested in. Another key difference between our determined models, S0 and S1, is the forecasted future return, which is 21% in S1, 3 percentage points lower than in S0.

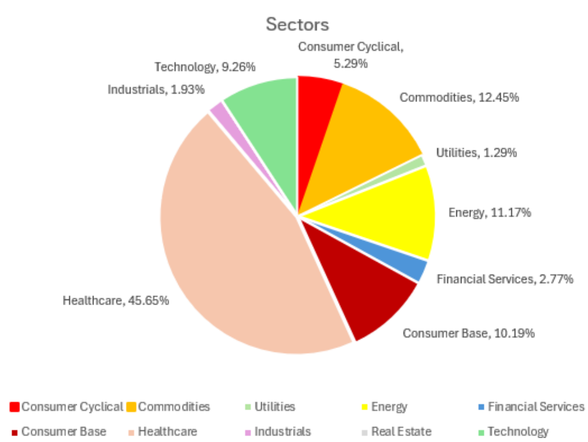
Lastly, the Sharpe ratio, our target variable, was approximately 2.16 in S0 and rose to about 2.58 in S1, representing a significant improvement in this context. The second scenario we explored, S2, had constraints similar to those in S0 and S1, with the change being that each sector could now have up to 20% invested in it, while lowering the lower constraints all the way to zero.

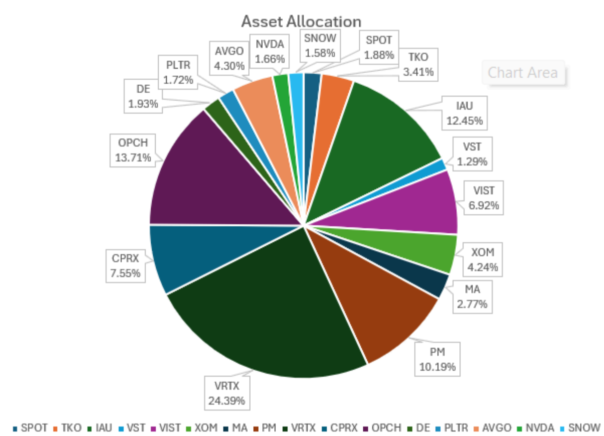




Lowering the lower constraint to zero completely removed the real estate sector from the model, while the commodities and healthcare sectors both met the new upper threshold of 20% of the total investments made. The total number of companies invested in for S2 came out to 18, with a forecasted future return of 18.64% and a Sharpe ratio of about 2.85.

The final scenario we opted to cover (S3) was one in which there were no constraints on diversification, other than that each sector couldn't be negative.





With no constraints on diversification, our model allocated 45.7% of the total investments to the healthcare sector and none to real estate. The model also allocated less than 5% each to utilities, financial services, and industrials. A total of 16 companies were invested in, with the forecasted return being about 19.7%, and the sharpe ratio coming out to approximately 2.81.

Scenario	Diversification Constraints	Sectors Invested In	Companies Invested In	Forecasted Return Rate	Sharpe Ratio
S_0	I_i=10	10	13	24.2%	2.16
S_1	5% ≤ I_i ≤ 15%	10	20	21.1%	2.58
S_2	0% ≤ I_i ≤ 20%	9	18	16.6%	2.85
S_3	0% ≤ I_i	9	16	19.7%	3.13

Table 1

Comparing each version of our model side by side, it's clear that relaxing diversification constraints has a noticeable positive effect on the Sharpe ratio, along with some negative effects on both the return rate and portfolio diversification. With just slight modifications to our model's constraints, the Sharpe ratio produced different outputs, indicating that the model is fairly sensitive to constraint changes.

Results Interpretation

The final output of our optimization model reflects a high-performing yet diversified portfolio, showing a strong balance between return and risk. The portfolio variance was calculated at 0.007996, with a corresponding standard deviation of 0.0894, indicating moderate volatility. Most notably, the Sharpe Ratio of 2.16 exceeds the industry benchmark of 2.0, confirming that our portfolio delivers excellent risk-adjusted performance.

To contextualize these results, we compared our model against two alternative strategies over a 20-year investment horizon, with all values adjusted for 2.5% annual inflation: (1) contributing with no investing, (2) investing consistently at the historical S&P 500 return of 8.84%, and (3) investing through our optimized portfolio, which assumes a 24.22% expected return. The simulation begins at age 21 with a $10,000 initial investment and 15% annual salary contributions from an $85,000 base salary, increasing 3.6% each year. By age 41, total contributions amount to $172,430.48, and the account values are:

- $444,642.74 using our optimized portfolio
- $247,522.97 if investing at the S&P 500 average return
- $198,901.49 if no investing is done

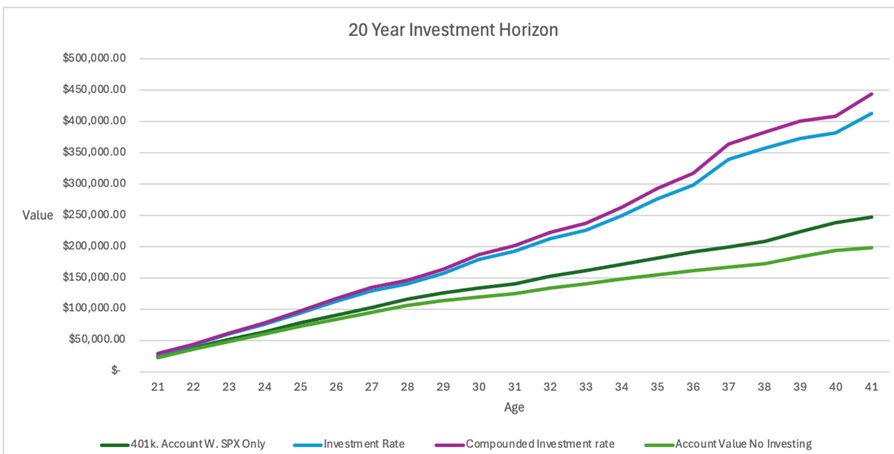
Although our model applies a fixed return rate, the actual growth curve is shaped by compound interest and the timing of contributions. Even with this moderation, our optimized strategy significantly outpaces both passive saving and traditional market level investing.

To further evaluate the flexibility of our model, we conducted a sensitivity analysis by adjusting the sector diversification constraints across three scenarios:

- S_1 (5%–15% sector cap): Sharpe ratio increased to 2.58, but expected return dropped to 21.1%
- S_2 (0%–20% sector cap): Sharpe ratio rose to 2.85, while return fell further to 16.6%
- S_3 (no constraints): Sharpe ratio peaked at 3.13, with heightened concentration, particularly in healthcare at 45.7%

These findings reveal a key trade-off: relaxing diversification constraints boosts risk-adjusted performance, but may reduce total return and increase exposure to specific sectors. From a decision-making perspective, this suggests that young investors striving to "maximize returns while minimizing risk" must weigh the benefits of Sharpe ratio optimization against the risks of overconcentration. The base model (S_0) presents a strategic balance, achieving strong returns while maintaining sector diversification, making it a robust approach for long-term investment planning.

Our analysis demonstrates the financial power of early, consistent contributions and data-driven investment strategies. Compared to passive saving, our model doubles the investor's inflation-adjusted capital and outperforms traditional benchmarks. The graphs below visually compare the three strategies, reinforcing the compounding advantages delivered by prescriptive optimization.



Trade-Off Analysis

No one measure can indicate whether a financial investment option is desirable. Finance is an unbelievably complex field, with many numbers and ratios to measure success. With our model being based on Excel, it is impossible to incorporate and optimize each desirable financial ratio; the program doesn't have the computing power to do so. To overcome this, we included constraints for diversification and provided a calculation for expected return.

Relaxing the constraints for diversification allowed us to observe how the factors we had to include manually responded to a more optimized Sharpe ratio. As shown in Table 1 above, the Sharpe ratio increases as the constraints get relaxed. With those relaxed constraints, however, the sector investment becomes less diverse, while the number of companies invested in becomes more diverse. This increase in the company's diversification will likely help the portfolio's day-to-day performance. Still, decreasing sector diversification may expose the portfolio to more macro risk, issues that impact entire industries or economies. In addition, while the Sharpe ratio increased after relaxing the constraints, the rate of return saw a noticeable decrease. This decrease could be due to the Sharpe ratio's increase, or it could be due to the lack of sector diversification, or, more likely, a combination of the two. The easiest way to increase the Sharpe ratio is to decrease its denominator, the portfolio's standard deviation, which means investing in safer stocks. Every investor wants to balance investment safety and worthwhile returns, and our model shows a clear tradeoff where increasing the Sharpe ratio and the portfolio's overall stability, leads to fewer returns.

Constraints

Several constraints were implemented in the optimization model to ensure that the simulated investment strategy aligns with real-world financial planning principles. Firstly, the full capital allocation constraint ($\sum x_i = 1$) guarantees that 100% of the investor's available capital is allocated across the selected equities. This reflects a realistic investing scenario where all funds are utilized. The model also incorporates a non-negativity constraint ($x_i \geq 0$), prohibiting short selling. This illustrates the behavior of long-term investors, particularly those preparing for retirement. A sector diversification constraint (maximum 10% per industry) was established to maintain portfolio diversification, limiting no more than 10% of the portfolio to any single sector. For example, if stocks such as OSCR, CPRX, OPCH, and VRTX are all within the healthcare sector, their cumulative allocation cannot exceed 10% of the total portfolio. This approach aids in reducing sector-specific risk and promotes balanced exposure to prevent sector volatility from damaging risk-adjusted returns. Furthermore, a maximum allocation per stock ($1,000 cap) was enforced by capping each equity at $1,000 of the initial investment. This measure prevents overconcentration in any single stock, which is particularly significant during the early stages of investing when capital is limited. The initial investment limit ($10,000) further confines the total invested capital at age 21 to $10,000, consistent with the project's assumption that the investor is in the early stages of their career and commencing with modest savings. Although not enforced through Solver, the model also considers salary-based contributions (15% of salary annually) starting from an $85,000 base salary with a 3.6% annual raise. These assumptions underpin the 20-year investment simulation and reflect typical long-term saving behavior for retirement.

- Full capital allocation ($\sum x_i = 1$)
- Non-negativity/ No short selling ($x_i \geq 0$)
- Sector diversification (Max 10% per Industry)
- Maximum allocation per stock ($1,000 Cap)
- Initial investment limit ($10,000)
- Salary-based contributions (15% of salary [$85,000 with 3.6% increase in salary per year])

Future Directions

This project set out to explore a central question: How can a young investor build a portfolio that maximizes returns while minimizing risk? By combining predictive analytics to estimate future performance and prescriptive analytics to allocate capital under real-world constraints, we developed a data-driven model for long-term investment planning.

One limitation of our current model is that it optimizes the portfolio only once at the start of the 20-year horizon. In reality, investors periodically rebalance their portfolios to manage risk and respond to changing conditions. In the case of constant rebalancing, more real-time data should be added to increase the win rate of this portfolio further and increase equity for investors who can use methods similar to ours. Future versions could incorporate rebalancing intervals to more accurately reflect this adaptive behavior. Simulating this process would align the model with real investment practices and enhance its applicability over long horizons.

While we used historical stock data to estimate annualized returns and volatility, our model assumes that past trends predict future performance. More advanced techniques like time-series forecasting or basic machine learning could improve accuracy and responsiveness to market shifts. Excel's GRG Nonlinear Solver also allowed us to optimize a nonlinear objective (Sharpe ratio), but lacked flexibility for complex constraints or complete sensitivity analysis. A more scalable version of this model could be built using Python, enabling features like transaction cost modeling, asset limits, or user-defined risk preferences as well as the addition of quadratic programming, which will connect to Modern Portfolio Theory, where investors can more accurately estimate the covariance to set up constraints based upon the investment limits and return targets. Using the hyperscalability of more sophisticated techniques, like Python, will allow macroeconomic indicators to provide a more accurate version, which will give a greater risk-adjusted return, and lead to enhancements to big data interpretation.

We also recognize that our current scenario, starting with $10,000 and contributing 15% of an $85,000 salary with 3.6% growth, represents just one investor profile. Similar to the absence of ESG (environmental, social, and governance) valuation, which creates a generalization of an investor who does not care about the holistic view of a company. A future enhancement would allow users to adjust inputs based on incomes, goals, ESG, or timelines, making the tool more customizable and relevant. Beyond investing, the prescriptive approach used here has broader applications. Defining objectives, setting constraints, and optimizing decisions are valuable in budgeting, operations, and personal finance.

This project illustrates how a young investor can build a strong, risk-aware portfolio using analytics. With further development, the model has the potential to become a versatile and personalized financial planning tool for long-term wealth creation.

Appendix:

OpenAI. *ChatGPT*. Mar. 2025, https://chat.openai.com/

```
“import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# --- Parameters ---
years = 20
simulations = 1000

# Historical performance (assumptions from your paper)
spx_return = 0.0884
spx_vol = 0.15

opt_return = 0.2422
opt_vol = 0.0894

# Store results
modifiers_spx = np.zeros((simulations, years))
modifiers_opt = np.zeros((simulations, years))
```

```
# --- Monte Carlo Simulation ---
np.random.seed(42)  # for reproducibility

for i in range(simulations):
    modifiers_spx[i] = np.random.normal(loc=spx_return, scale=spx_vol, size=years)
    modifiers_opt[i] = np.random.normal(loc=opt_return, scale=opt_vol, size=years)

# --- Calculate average modifier per year ---
avg_mod_spx = np.mean(modifiers_spx, axis=0)
avg_mod_opt = np.mean(modifiers_opt, axis=0)

# --- Plot modifiers ---
plt.figure(figsize=(10, 6))
plt.plot(range(1, years+1), avg_mod_spx, label="S&P 500", linestyle='--')
plt.plot(range(1, years+1), avg_mod_opt, label="Optimized Portfolio", linestyle='-')
plt.title("Average Market Modifier per Year (Monte Carlo Mean)")
plt.xlabel("Year")
plt.ylabel("Market Modifier")
plt.grid(True, linestyle='--', linewidth=0.5)
plt.legend()
plt.tight_layout()
plt.show()

# --- Export modifiers to CSV (for Excel use) ---
```

df_modifiers = pd.DataFrame({

'Year': np.arange(1, years+1),

'Market Modifier - S&P': avg_mod_spx,

'Market Modifier - Optimized': avg_mod_opt})

Save to CSV

df_modifiers.to_csv("market_modifiers_montecarlo.csv", index=False)”

Microsoft Excel File:

[Bana 420-20 Year Term-Retirement Plan Project.xlsx]