Introduction

Transduction in the dynamic loudspeaker

The dynamic loudspeaker system is often refered to as a 'transducer,' and this is true as it transforms electrical power into acoustic power. However, there are infact two stages of transduction that occur within the system. These are electrical to mechanical power, and mechanical to acoustic power.

Each stage is governed by physical laws that describe the key parameters and relationships involved. To achieve an accurate equivalent circuit of the system one must have the knowledge of these laws and how each parameter relates to its curcuit equivaletnt.

Equations of transduction

When a current is presented to the speaker voice coil the electrons in motion interact with the stationary field in the magnetic gap. A force is produced which is perpendicular to both the current flow and the stationary magnetic field. The magnetic filed is in the radial direction while the current is always tangent to the voice coil circumference. The resulting force is always axial with respect to the voice coil, pushing either inward or outward. The relationship is described in the following equation

$$\vec{F} = \vec{B}\ell \times \vec{i}$$

Where $\vec{B}$ and ℓ are the magnetic gap field and length of wire in the magnetic gap respectively. The multiplicaion here is a vector cross product, which has the magnitude

$$|\vec{F}| = |B\ell i sin(\theta)| = B\ell i$$

where θ is the angle between $\vec{B}$ and $\vec{i}$. As stated these two vectors are perpendicular so the $|sin(\theta)|$ term must be unity. The direction of $\vec{F}$ is, therefore, either inwards or outwards and determined by application of the right hand rule.

Transduction between the mechanical and acoustical domains is more straightforward. An acoustic pressure is generated due to the mechanical force derived above acting over the area of the speaker cone or

$$\vec{P} = \vec{F} * S_D$$

where S_D is the effective cone surface area. This is essentially the area calculated by considering the cone to be a circle with a diameter stretching from one surround peak across to the other.

With these laws of transduction in hand we will be able to translate between electrical, mechanical, and acoustic domains in the eventual analogous circuit

Method of construction

We now consider the method by which an analogous circuit may be constructed. A logical first question would be whether or not there is a unique circuit representation for the mechanical analogy. As it turns out there are two distinct possibilities.

Dual circuits

Consider the development of the L-C-R cicuit response as given previously. When constructed as a series circuit it was clear that current obeyed the same underlying differential equation as required by displacement in the mechanical system and a formal parallel was inferred.

In contrast, let us consider the parallel L-C-R circuit

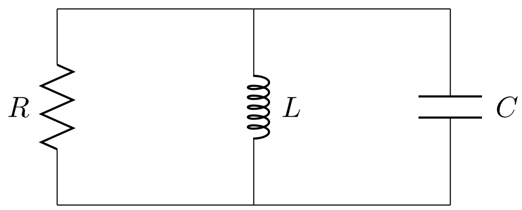


Figure 1: Sourceless parallel L-C-R circuit

We determine the voltage response of this circuit by considering the current through each leg.

$$C\frac{de}{dt} + \frac{1}{R}e + \frac{1}{L}\int e\,dt = 0$$

taking the time derivative

$$C\frac{d^2e}{dt^2} + \frac{1}{R}\frac{de}{dt} + \frac{1}{L}e = 0$$

and again we have found the equation for harmonic oscillation, albeit with some swapping and inverting of constant values. With enough care in mapping mass, damping, and spring constants we may just as correctly use the parallel circuit to represent mechanical displacement in terms of voltage. In general the mapping now becomes

$$\Lambda_2 \Rightarrow \text{Mass} \Rightarrow \text{Capacitance}$$
$$\Lambda_1 \Rightarrow \text{Damping Constant} \Rightarrow 1/\text{Resistance}$$
$$\Lambda_0 \Rightarrow \text{Spring Constant} \Rightarrow 1/\text{Inductance}$$

Circuits with this relationship are said to be duals of one another. As we have seen this transformation takes the current behavior of one circuit and transforms it to an identical voltage behavior in another. As a last point we also note that, should a driving voltage or current source have been included, these would also swapped one for the other. A voltage source becomes a current source of the same magnitude and vice versa.

Back to our analogy construction we see we have a choice to make. Should the equivalent circuit be constructed as a parallel or series L-C-R type? The decision is arbitrary and often a matter of convenience though care is needed to keep track of the underlying parameter mappings.

Choice of variables

In our analagous circuit we are most concerned with how electrical power at the input is translated to acoustical power at the output. In fact, the electrical circuit gives us power directly by considering the product

$$P_E = ei$$

Ideally our mechanical circuit should map equivalent parameters whose product also indicates mechanical power. This will become important when the domains are linked via the equations of transduction above. At present we have been concerned only with mechanical displacement. If displacement is mapped either to current (or voltage) in an equivalent circuit we may ask what mechanical equivalent is represented by voltage (or current). Further, would the product of displacement and this variable yield mechanical power?

This line of reasoning suggests we try another approach. Let us consider the mass-spring system equation in terms of velcity instead of displacement

$$m\frac{dv}{dt} + bv + k\int v\,dt = 0$$

Already we can see a closer similarity to the L-C-R circuit equivalent. Taking the time derivative reveals the same equation of harmonic oscillation

$$m\frac{d^2v}{dt^2} + b\frac{dv}{dt} + kv = 0$$

As a result we see that mechanical velocity may be used as the analog to electrical current or voltage in our chosen analogy. Taking the dual of this equation transforms the dependant variable into its power product partner

$$\frac{1}{k}\frac{d^2\Delta}{dt^2} + \frac{1}{b}\frac{d\Delta}{dt} + \frac{1}{m}\Delta = 0$$

For the sake of clarity we integrate both sides

$$\frac{1}{k}\frac{d\Delta}{dt} + \frac{1}{b}\Delta + \frac{1}{m}\int \Delta\, dt = 0$$

Now each term here must be a velocity for the dual relationship to hold. Using this information we may now determine the variable mapped to Δ. Take, for instance, the integral term

$$\frac{1}{m}\int \Delta\, dt = v$$

so that $\int \Delta\, dt$ must have units of mass times velocity or momentum. Straight away this imples that

$$\Delta \quad \Rightarrow \quad Force$$

and as required mechanical power is indeed given by

$$P_M = Fv$$

We will proceed in using velocity and force to represent current and voltage in our equivalent circuit.

Impedance and mobility analogies

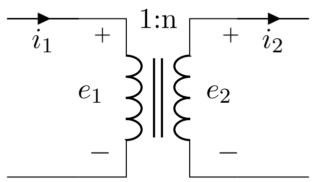
The two equivalent circuit analogies above have specific names. These are the impedance type analogy and the mobility type analogy. To distinguish between the two we look at the fundamental differential equation for the system.

If we were to choose to map velocity to current and force to voltage this would correspond to the series L-C-R configuration. This stems from the fact that a common velcity must be shared by both the spring and mass just as a common current is shared among the series cicuit elements. The system equation for this scenario exhibits the damping term bv so that b is interpereted as a mechanical impedance. Hence this is the impedance analogy

Conversely if velocity is mapped to voltage the restulting circuit is the parallel L-C-R configuration. Again the velcity is common to all mechanical elements in the same way that the voltage is common across all three circuit elements. The system equation here, as shown, has a damping term of $\frac{1}{b}F$ so that b is interpreted as an inverse impedance, also called mobility. This is the mobility analogy.

Transformation between domains

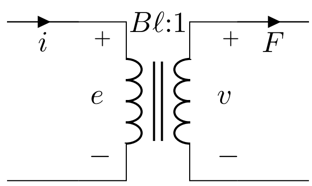
The dynamic loudspeaker system contains three physical domains namely electrical, mechanical, and acoustical. Transduction occurs when moving from one domain to another and is governed by the equations already given. If each domain is to have an equivalent circuit then we must determine how to join these three circuits into one model. At the transition point power from one domain is converted into an equal power in the next. In traditional circuit theory the coupling of power from one circuit to another may be accomplished by the transformer element.



For a windings ratio of n the voltage and current ratios are as follows

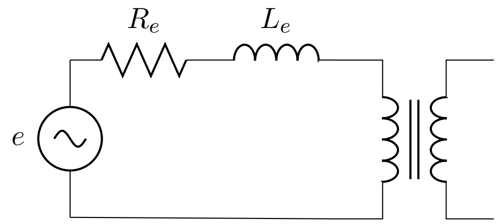
$$v_2 = nv_1$$
$$i_2 = \frac{i_1}{n}$$

and by the electical power equation P_E is conserved in the translation. For our analgous circuit the windings ratio is replaced by the current or voltage conversion factor. For, example mapping mechanical force to current and mechanical velocity to voltage and using the equation of transduction between electrical and mechanical domains gives



The driver electrical circuit

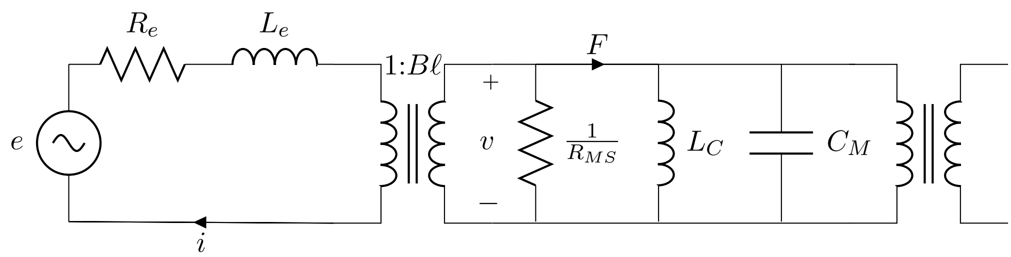
It is finally time to put pen to paper to form our circuit model. We begin with the driver electrical domain. Physically this consists of the voice coil only. Electrically this breaks down into two elements. As a length of copper wire the voice coil has resistance in ohms determined by its length and diameter. This is commonly known as R_e. Because this wire is wound in loops with a core of air and steel (pole piece) it also behaves as an inductor. Putting these together with an input voltage source, an amplifier for instance, we obtain



Here the transformer has been added in anticipation of adding the mechanical domain

Adding the mechanical domain

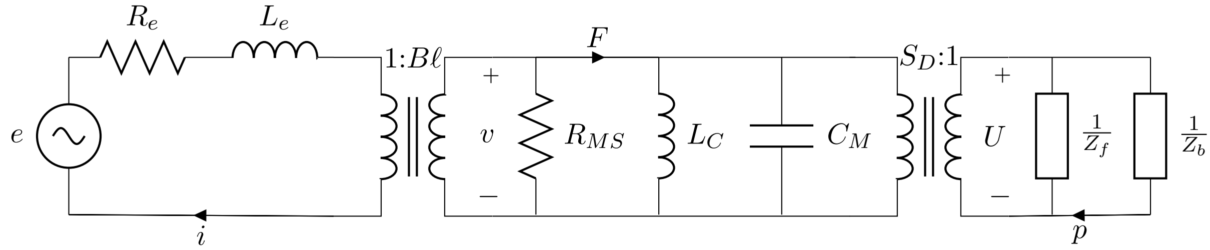
Since our equation of transduction between electrical and mechanical domains relates current to mechanical force we choose the mobility analogy. The quantity $B\ell$ in the electrical domain must map to unity in the mechancial domain giving



As discussed mass corresponds to capacitance in the model and inductance corresponds to compliance.

Adding the acoustic domain

In the acoustic domain we are dealing with pressure and volume velocity as the power product pair. It is most intuitive to map pressure to voltage and volume velocity to current. This is the impedance analogy and takes the form



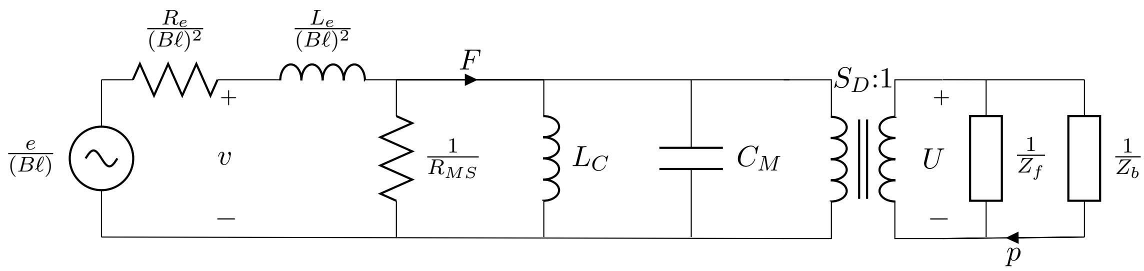
Here Z_f and Z_b correspond to the acoustic radiation impedances of the front and back of the driver respectively. These are, generally, complex but will be left in these simplified terms for the time being.

Completing the analogy

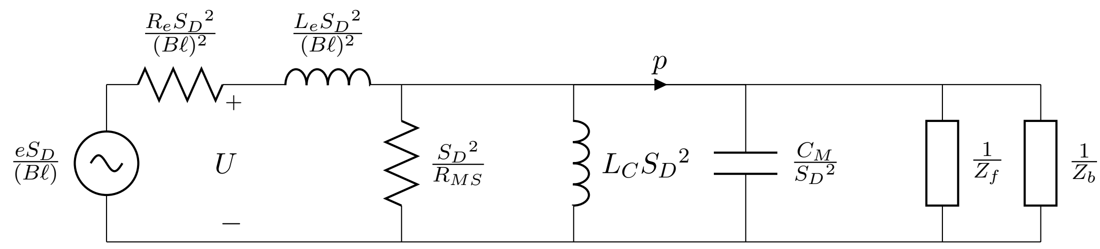
We have essentially arrived at our goal of a circuit based system model. At this point we may transform this cicuit into a form more suitable for analysis. Our transformer notation shows the underlying relationships between domains. For the sake of analysis, however, it would be better if they were not required. Each circuit element represents an impedance relating current to voltage. The transformer equations developed above tell us how to transform impedances thereby obviating the use of the transformer element. The impedance relationship becomes

$$Z = e/i \quad \Rightarrow \quad Z_T = (n * e)/\left(\frac{i}{n}\right) = Zn^2$$

Moving back to our circuit we wish to convert the entire model into the acoustic domain. This may be accomplished in two steps. First we combine the mechanical and electrical domains by dividing the electrical domain impedances by the traformer conversion factor $(B\ell)^2$ giving



The electro-mechanical domain may then be combined with the acoustical domain using the transformation factor ${S_D}^2$ giving

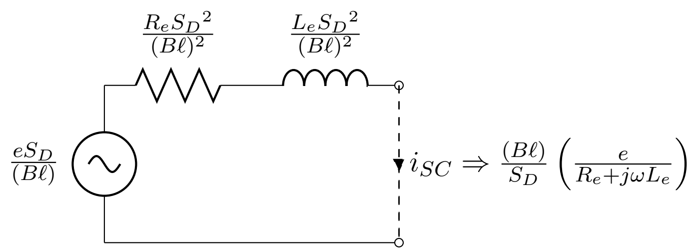


Note that the mechanical mass-capacitance takes the factor $\frac{1}{S_D{}^2}$ because the impedance of the capacitor is proportional to the inverse of capacitance, namely $\frac{1}{j\omega C}$

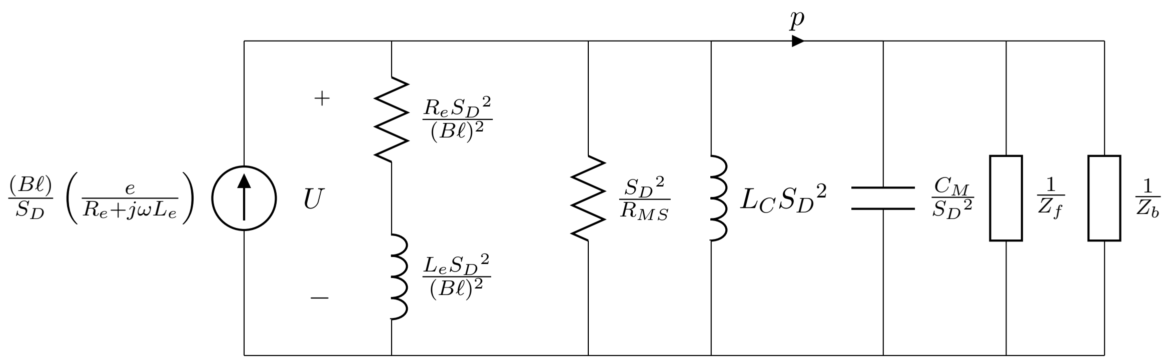
One more transformation is now considered. In present form we have mapped volume velocity to voltage and pressure to current. It will be preferrable in the analysis to come to explore the opposite mapping. We need to map volume velocity to current and pressure to voltage. This will give the more intuitive impedance analogy of the entire circuit.

This may be accomplished in two steps. First the driving voltage, voice coil resistance, and inductance are taken into the mobility analogy. After this is complete the entire circuit will be of the mobility form. Then we may take the dual of the entire circuit transforming from mobility to impedance analogies and obtaining the desired voltage and current mapping.

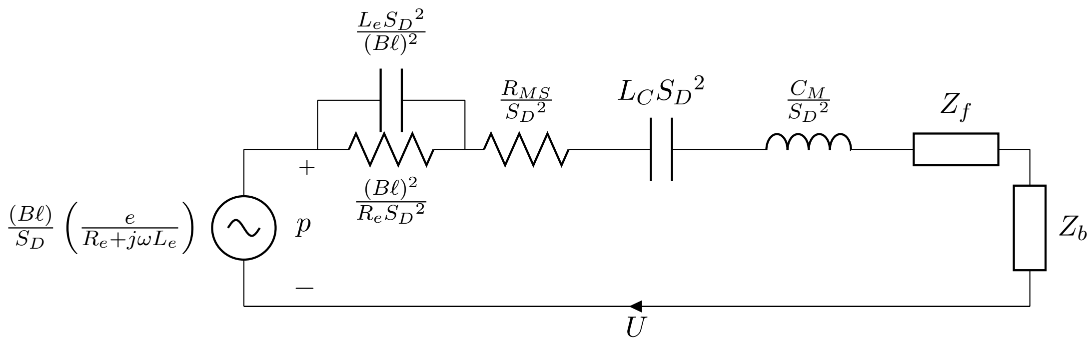
By Norton's Theorm of equivalence the series voice coil resistance and inductance may be represented as an equivalent parallel circuit. The voltage source is replaced by a parallel current source whose value is the same as the short circuit current of the orignal series configuration. Calculating the short circuit current in series gives



Replacing this section with its parallel equivalent gives



We are now ready for the final step. Taking the dual of our circuit gives the desired relationships, voltage to pressure and current to volume velocity.



For a given front and back acoustic impedance we may now relate the input electrical voltage e to the generated acoustic volume velocity and pressure. This is the form we shall use for further analysis of the loudspeaker system behavior.