TeRA: Vector-based Random Tensor Network for High-Rank Adaptation of Large Language Models

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Abstract

Parameter-Efficient Fine-Tuning (PEFT) methods, such as Low-Rank Adaptation (LoRA), have significantly reduced the number of trainable parameters needed in fine-tuning large language models (LLMs). Subsequent developments of LoRA-style adapters have diverged into two main directions: (1) enhancing model expressivity with high-rank adapters, and (2) pushing for further parameter reduction, as exemplified by vector-based methods. However, these approaches present a trade-off, as achieving the expressivity of highrank weight updates typically comes at the cost of sacrificing the extreme parameter efficiency offered by vector-based techniques. To address this issue, we propose a vector-based random Tensor network for high-Rank Adaptation (TeRA), a novel PEFT method that achieves high-rank weight updates while retaining the parameter efficiency of vector-based PEFT adapters. This is achieved by parameterizing the tensorized weight update matrix as a Tucker-like tensor network (TN), in which large randomly initialized factors are frozen and shared across layers, while only small layer-specific scaling vectors, formed by entries in diagonal factor matrices, are trained. This design effectively decouples the rank of the weight update matrix from the number of trainable parameters. Comprehensive experiments demonstrate that TeRA matches or even outperforms high-rank adapters, while requiring a trainable parameter count similar to vector-based methods. Theoretical analysis and ablation studies further validate the effectiveness of our approach.

Introduction

Foundation models, such as the LlaMA (Touvron et al. 2023; Grattafiori et al. 2024) and GPT (Brown et al. 2020; Achiam et al. 2023) series, have revolutionized the field of natural language processing (NLP) by demonstrating strong generalization abilities across a diverse range of tasks. Although these models have been pre-trained on a large-scale corpus of textual data, supervised fine-tuning (SFT) is often employed to improve their performance on specific downstream tasks. However, the large number of trainable parameters in full-parameter SFT can become computationally prohibitive in resource-constrained environments.

Parameter-Efficient Fine-Tuning (PEFT) methods, particularly Low-Rank Adaptation (LoRA) (Hu et al. 2022), have become the standard for efficiently adapting large language

models (LLMs) to downstream tasks. These methods work by freezing the original pre-trained weights and training only the adapter weight updates, which are then added back to the original weights. The core assumption of LoRA is that these weight updates can be effectively approximated by a low-rank decomposition, $\Delta \mathbf{W} = \mathbf{A}\mathbf{B}$, where $\mathbf{A} \in \mathbb{R}^{J_1 \times r}$ and $\mathbf{B} \in \mathbb{R}^{r \times J_2}$, with r being the hyperparameter controlling the rank upper bound. This substantially reduces the number of trainable parameters compared to full fine-tuning.

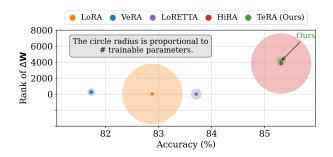


Figure 1: TeRA exhibits superior performance, high-rank and parameter efficiency trade-off. On commonsense reasoning benchmarks using Llama-3-8B, TeRA obtains high-rank query weight updates and on-par performance as high-rank methods such as HiRA, while maintaining a number of trainable parameters similar to the more parameter-efficient vector-based methods like VeRA. See Table 2 for details.

However, the low rank assumption of $\Delta \mathbf{W}$ can limit its expressivity when adapting to more complex downstream tasks (Huang et al. 2025). To address this, Huang et al. (2025) proposed Hadamard High-Rank Adaptation (HiRA), which enhances expressivity by introducing an element-wise (Hadamard) product between the learned weight update matrix, \mathbf{AB} , and the frozen pre-trained weight matrix, \mathbf{W}_0 . Specifically, HiRA defines $\Delta \mathbf{W} = (\mathbf{AB}) \odot \mathbf{W}_0$, where \odot denotes the Hadamard product (element-wise multiplication), and \mathbf{W}_0 is the frozen pre-trained weight matrix. Similar to LoRA, HiRA still requires training all parameters in \mathbf{A} and \mathbf{B} .

Methods like VeRA (Kopiczko, Blankevoort, and Asano 2024) focus on reducing the number of trainable parameters

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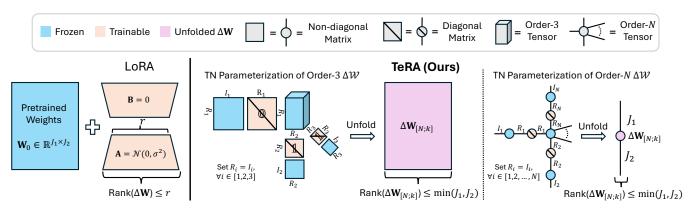


Figure 2: A comparison between LoRA (Hu et al. 2022) and our proposed TeRA method. LoRA represents the weight update matrix using two smaller matrices, while TeRA employs a Tucker-like (Tucker et al. 1964) tensor network (TN) to parameterize the tensorized ΔW . This design allows TeRA to achieve high-rank updates with much fewer trainable parameters than LoRA.

in LoRA by freezing $\bf A$ and $\bf B$, while training only two scaling vectors, $\bf b$ and $\bf d$, but they remain constrained by the lowrank assumption. Specifically, VeRA weight update is parameterized as $\Delta {\bf W} = {\bf \Lambda}_b {\bf B} {\bf \Lambda}_d {\bf A}$, where ${\bf b} \in \mathbb{R}^{J_1}$ and ${\bf d} \in \mathbb{R}^r$ are the two trainable diagonal entries in ${\bf \Lambda}_b \in \mathbb{R}^{J_1 \times J_1}$ and ${\bf \Lambda}_d \in \mathbb{R}^{r \times r}$, respectively. Thus, VeRA requires only a fraction of the trainable parameters in LoRA. In parallel, multi-linear tensor-based PEFT methods, such as Low-Rank Economic Tensor-Train Adaptation (LoRETTA) (Yang et al. 2024) which assumes a higher-order low-rank weight update matrix (Oseledets 2011), have demonstrated similar performance while requiring much fewer trainable parameters than in LoRA.

It is usually possible to obtain high-rank weight updates with low-rank adapters. However, this leads to an explosion in the number of trainable parameters. At the same time, the low-rank assumption in the weight update matrices has been shown to restrict their performance in more complex tasks, such as arithmetic and commonsense reasoning (Huang et al. 2025). Naturally, we propose the following question:

• Is it possible to achieve the high performance that is typically associated with high- (or full-) rank weight updates, while still keeping number of trainable parameters low, similar to vector-based PEFT adapters like VeRA?

To resolve this trade-off, we propose a vector-based random $\underline{\mathbf{Te}}$ nsor network for high- $\underline{\mathbf{R}}$ ank $\underline{\mathbf{A}}$ daptation (TeRA), a novel PEFT method that achieves high-rank weight updates using very few trainable parameters. The core idea is to tensorize the weight update matrix, $\Delta \mathbf{W}$, into a higher-order tensor, which is then parameterized using a Tucker-like (Tucker et al. 1964) tensor network (Cichocki et al. 2015) as shown in Figure 2. Within this tensor network, we freeze large randomly initialized factors that are shared across all layers and train only the small layer-specific scaling vectors formed by entries in diagonal factor matrices. This design effectively decouples the rank of the update matrix from the number of trainable parameters, allowing for high-rank adaptation with a parameter count similar to vector-based methods.

Extensive experiments demonstrate that TeRA establishes

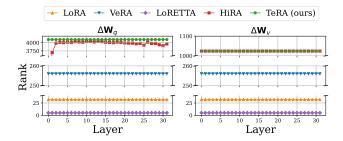


Figure 3: Rank analysis of $\Delta \mathbf{W}_q$ (max allowed rank of 4096) and $\Delta \mathbf{W}_v$ (max allowed rank of 1024) across Llama-3-8B layers. TeRA consistently maintains a high (near-full) rank. In contrast, methods like LoRA and VeRA have lower-rank weight updates, limiting their expressivity.

a superior trade-off between model performance, high rank, and parameter efficiency. As shown in Figure 1, TeRA matches the accuracy of HiRA while using orders of magnitude fewer parameters. Compared to parameter-matched methods like VeRA and LoRETTA (Yang et al. 2024), TeRA yields a significant accuracy improvement. This performance is due to the high-rank weight updates in TeRA across all model layers (See Figure 3), a property that low-rank methods inherently lack. Figure 3 presents the rank of the query and value weight updates, $\Delta \mathbf{W}_q$ and $\Delta \mathbf{W}_v$, across all transformer layers for different PEFT methods. TeRA consistently achieves the highest (almost full) rank across layers, allowing for potentially more expressive weight updates. In summary, our contributions are as follows:

- We propose TeRA, a new PEFT method that uses a multilinear Tucker-like tensor network to parameterize the tensorized high-rank weight updates. TeRA adapters can be merged with the original weights at inference time, incurring zero computational and latency overhead.
- We provide a theoretical analysis demonstrating that TeRA can achieve high-rank weight updates with provably fewer parameters than existing high-rank methods. Our analysis also formalizes the trade-off between the

- performance and the trainable parameter count of TeRA.
- We conduct extensive experiments to compare TeRA with baseline methods. TeRA is shown to exhibit superior performance while requiring a similar number of trainable parameters as vector-based PEFT adapters.

Related Work

Prompt-based Methods. One category of PEFT methods comprises prompt-based methods, such as Prompt Tuning (Lester, Al-Rfou, and Constant 2021) and P-Tuning (Liu et al. 2022), which introduce additional trainable virtual tokens into the input of LLMs and optimize only these tokens. Prompt-based methods are often sensitive to initialization schemes, increase computational costs during inference, and reduce the effective context length of the model.

Low-rank Adaptation (LoRA). Introduced by Hu et al. (2022), LoRA employs two matrices, A and B, to parameterize the weight update matrix, ΔW , as a low-rank decomposition, thereby significantly reducing the number of trainable parameters. Building on LoRA, VeRA (Kopiczko, Blankevoort, and Asano 2024) proposes to randomly initialize and freeze the A and B matrices, which are shared across layers, and instead train only two scaling vectors, b and d, thus significantly reducing the trainable parameter count needed for low-rank weight updates. Recently, tensorbased methods, which operate on tensorized neural network weights (Gu et al. 2025), have also been shown to be effective in fine-tuning LLMs (Yang et al. 2024; Bershatsky et al. 2024). More specifically, they focus on reducing the number of trainable parameters compared to LoRA while assuming higher-order low-rank structures (Oseledets 2011; Cichocki et al. 2015), which may fail to capture high-rank updates for complex tasks such as reasoning (Huang et al. 2025).

High-rank Adaptation. To overcome the limited expressivity of low-rank adaptation, high-rank variants of LoRA have been proposed. MoRA (Jiang et al. 2024) employs a square matrix to achieve high-rank updates, while HiRA (Huang et al. 2025) uses the Hadamard product to learn a high-rank weight update matrix. Both methods maintain the same level of trainable parameter count as LoRA. Different from these methods, TeRA effectively achieves high-rank weight updates while using a similar number of trainable parameters as vector-based PEFT adapters such as VeRA.

Preliminaries

The mathematical notations used in this paper are listed in Table 1. This is consistent with the notation used in Cichocki et al. (2015).

LoRA-style PEFT Adapters

LoRA-style PEFT adapter (Hu et al. 2022) obtains a weight update matrix, $\Delta \mathbf{W} \in \mathbb{R}^{J_1 \times J_2}$, which is added to the pretrained weight matrix, $\mathbf{W} \in \mathbb{R}^{J_1 \times J_2}$, for fine-tuning. The weight update matrix is usually explicitly designed to have a low-rank structure, e.g., $\Delta \mathbf{W} = \mathbf{A} \times \mathbf{B}$ in LoRA, where $\mathbf{A} \in \mathbb{R}^{J_1 \times r}$, and $\mathbf{B} \in \mathbb{R}^{r \times J_2}$ ($r \ll J_1, J_2$) are the two trainable matrices. Additionally, since the adapter weight matrix

Symbol	Meaning					
$a, \mathbf{a}, \mathbf{A}, \mathcal{A}$	Scalar, vector, matrix, tensor					
$(\cdot)^{\top}$	Matrix transpose					
$(\cdot)^{\dagger}$	Matrix pseudoinverse					
$\ \cdot\ _F$	Frobenius norm					
$\mathcal{A}(i_1,\ldots,i_N)$	The (i_1, \ldots, i_N) -th element of \mathcal{A}					
$\mathcal{A}\times_n\mathbf{B}$	Mode-n product					
$\operatorname{diag}(\mathbf{a})$	A diagonal matrix whose diagonal is a					

Table 1: Mathematical notations

shares the same shape as the pre-trained weight matrix, it can be merged back into the pre-trained weight during inference, eliminating any additional inference overhead. Variants of LoRA impose different algebraic structures within $\Delta \mathbf{W} \in \mathbb{R}^{J_1 \times J_2}$ to achieve further trainable parameter reduction or high-rank adaptations (Yang et al. 2024; Bershatsky et al. 2024; Huang et al. 2025; Kopiczko, Blankevoort, and Asano 2024).

Tensors and Multi-linear Algebra

A tensor is a multi-dimensional array and a higher-order generalization of vectors and matrices. E.g., A vector, $\mathbf{a} \in \mathbb{R}^{I_1}$ is an order-1 tensor. A matrix, $\mathbf{A} \in \mathbb{R}^{I_1 \times I_2}$, is an order-2 tensor. An order-N tensor is denoted by $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$.

Tensorization and Matricization. Tensorization (Folding) is the process to fold (reshape) a lower-dimensional tensor into a higher-dimensional one. For a matrix $\mathbf{A} \in \mathbb{R}^{J_1 \times J_2}$, we can reshape it into an order-N tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times \cdots \times I_N}$, provided that $\prod_{i=1}^k I_i = J_1$ and $\prod_{i=k+1}^N I_i = J_2$ for some split point k. Its inverse operation is termed matricization (unfolding). Unfolding operation converts an order-N tensor, $\mathcal{A} \in \mathbb{R}^{I_1 \times \cdots \times I_N}$, into a matrix, $\mathbf{A}_{[N;k]} \in \mathbb{R}^{\prod_{i=1}^k I_i \times \prod_{i=k+1}^N I_i}$, whose element-wise definition is given by

$$\mathbf{A}_{[N;k]}(\overline{i_1\cdots i_k},\overline{i_{k+1}\cdots i_N}) = \mathcal{A}(i_1,i_2,\ldots,i_N), \quad (1)$$

where $1 \leq k \leq N-1$. The corresponding tensorization operation is denoted by $\operatorname{Fold}_{[N:k]}(\mathbf{A}_{[N:k]}) = \mathcal{A}$.

Mode-n **product.** Mode-n product between a tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ and a matrix $\mathbf{B} \in \mathbb{R}^{J_n \times I_n}$ yields a tensor $\mathcal{C} \in \mathbb{R}^{I_1 \times \cdots \times I_{n-1} \times J_n \times I_{n+1} \times \cdots \times I_N}$. This operation is denoted as

$$C = A \times_n \mathbf{B}. \tag{2}$$

The element-wise definition of C is

$$C(i_1,\cdots,i_{n-1},j_n,i_{n+1},\cdots,i_N)$$

$$= \sum_{i_n=1}^{I_N} \mathcal{A}(i_1, \dots, i_{n-1}, i_n, i_{n+1}, \dots, i_N) \mathbf{B}(j_n, i_n).$$
(3)

Tucker Decomposition. Tucker decomposition (Tucker et al. 1964) is a generalization of Singular Value Decomposition (SVD) to higher-order tensors (De Lathauwer, De Moor, and Vandewalle 2000a) and a cornerstone of multi-linear tensor network (Cichocki et al. 2015). Given an order-N tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$, Tucker decomposition expresses it

using a smaller order-N core tensor $\mathcal{G} \in \mathbb{R}^{R_1 \times R_2 \times \cdots \times R_N}$, where $R_i \ll I_i$, and N factor matrices $\{\mathbf{B}^{(i)} \in \mathbb{R}^{R_i \times I_i}\}_{i=1}^N$. Tucker decomposition is defined as

$$\mathcal{X} = \mathcal{G} \times_1 \mathbf{B}^{(1)} \times_2 \mathbf{B}^{(2)} \cdots \times_N \mathbf{B}^{(N)}. \tag{4}$$

Identifying the optimal set of Tucker ranks, $[R_1, \ldots, R_N]$, efficiently is an active area of research, with numerous recent studies focusing on advanced methods for tensor rank search (Iacovides, Zhou, and Mandic 2024; Li et al. 2023; Iacovides et al. 2025).

Methodology

TeRA parameterizes the tensorized weight update matrix $\Delta \mathbf{W}$ using a Tucker-like tensor network, as shown in Figure 2. The method involves two steps: first, the weight update matrix is tensorized into a higher-order tensor $\Delta \mathcal{W} \in \mathbb{R}^{I_1 \times \cdots \times I_N}$. Then, this tensor is parameterized as a Tucker-like tensor network structure whose majority of factors are frozen, and only small diagonal matrices are trainable. After training, $\Delta \mathcal{W}$ is unfolded to obtain $\Delta \mathbf{W}_{[N:k]}$.

Specifically, given a weight update matrix, TeRA tensorizes it into an order-N tensor $\Delta \mathcal{W} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N} = \operatorname{Fold}_{[N;k]}(\Delta \mathbf{W}_{[N;k]} \in \mathbb{R}^{J_1 \times J_2})$, where $\prod_{i=1}^k I_i = J_1$, $\prod_{i=k+1}^N I_i = J_2$, $I_i \geq 2 \ \forall \ i = 1, \ldots, N$, and $1 \leq k < N$. For example, the attention weight matrices in Llama-2-7B have size 4096×4096 , which can be tensorized into $64 \times 64 \times 64 \times 64$, $16 \times 16 \times \cdots \times 16$, etc.

TeRA Formulation. We parameterize the weight update tensor $\Delta \mathcal{W}$ as the mode-n product of a frozen core tensor $\mathcal{G} \in \mathbb{R}^{R_1 \times R_2 \times \cdots \times R_N}$, a set of N frozen non-diagonal factor matrices $\{\mathbf{A}^{(i)} \in \mathbb{R}^{R_i \times I_i}\}_{i=1}^N$, and a set of N trainable vectors $\{\mathbf{d}^{(i)} \in \mathbb{R}^{R_i}\}_{i=1}^N$, which are diagonal entries of $\{\operatorname{diag}(\mathbf{d}^{(i)}) \in \mathbb{R}^{R_i \times R_i}\}_{i=1}^N$. The definition of TeRA is

$$\Delta \mathcal{W}(i_1, \dots, i_N) = \sum_{r_1=1}^{R_1} \sum_{r_2=1}^{R_2} \dots \sum_{r_N=1}^{R_N} \mathcal{G}(r_1, r_2, \dots, r_N)$$

$$\mathbf{d}^{(1)}(r_1) \mathbf{d}^{(2)}(r_2) \dots \mathbf{d}^{(N)}(r_N)$$
(5)

$$\mathbf{d}^{(1)}(r_1)\mathbf{d}^{(2)}(r_2)\cdots\mathbf{d}^{(N)}(r_N) \\ \mathbf{A}^{(1)}(r_1,i_1)\mathbf{A}^{(2)}(r_2,i_2)\cdots\mathbf{A}^{(N)}(r_N,i_N),$$

or equivalently

$$\Delta \mathcal{W} = \mathcal{G} \times_1 \operatorname{diag}(\mathbf{d}^{(1)}) \times_2 \operatorname{diag}(\mathbf{d}^{(2)}) \times_3$$
$$\cdots \times_N \operatorname{diag}(\mathbf{d}^{(N)}) \times_1 \mathbf{A}^{(1)}$$
$$\times_2 \mathbf{A}^{(2)} \times_3 \cdots \times_N \mathbf{A}^{(N)}.$$
 (6)

During fine-tuning, the core tensor $\mathcal G$ and the factor matrices $\{\mathbf A^{(i)}\}_{i=1}^N$ are randomly initialized and kept frozen. These matrices are shared between all the adapted layers of the LLM. The only trainable components are the diagonal entries of the matrices $\{\operatorname{diag}(\mathbf d^{(i)})\}_{i=1}^N$. All $\operatorname{diag}(\mathbf d^{(i)})$ are initialized as identity matrices, except for one $\operatorname{diag}(\mathbf d^{(i)})$, which is initialized as a zero matrix to ensure $\Delta \mathbf W_{[N;k]}$ is zero at initialization. This reduces the number of trainable parameters to just $\sum_{i=1}^N R_i$ per TeRA adapter, where $[R_1,\ldots,R_N]$ is the rank vector and a hyperparameter.

TeRA introduces zero computational overhead during inference. After fine-tuning, the TeRA weight update $\Delta \mathbf{W}_{[N;k]}$ is unfolded from $\Delta \mathcal{W}$ and can be merged with the pre-trained weights, \mathbf{W}_0 , to obtain the following

$$\mathbf{W}_{\text{final}} = \mathbf{W}_0 + \Delta \mathbf{W}_{[N;k]}. \tag{7}$$

Theorem 1. Let $\Delta \mathbf{W} \in \mathbb{R}^{J_1 \times J_2}$ be the weight update matrix, and $\Delta \mathcal{W} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N} = \operatorname{Fold}_{[N;k]}(\Delta \mathbf{W}_{[N;k]})$ be its folded weight update tensor, parameterized by TeRA as in Eq. (6). The following inequality holds

$$\operatorname{rank}\left(\Delta\mathbf{W}_{[N;k]}\right) \le \min\left(\prod_{i=1}^{k} R_i, \prod_{i=k+1}^{N} R_i\right). \quad (8)$$

This allows for a full-rank update matrix under any tensorization (folding) schemes if $R_i = I_i \ \forall i = 1, ..., N$, i.e., rank $(\Delta \mathbf{W}_{[N:k]}) \leq \min(J_1, J_2)$.

Note that in LoRA or VeRA, it is also possible to obtain high-rank weight update matrices, but this undesirably leads to an explosion in trainable parameter count. In contrast, TeRA not only can enable high-rank adaptation, but also requires a very small number of trainable parameters. For example, to allow for a full-rank weight update matrix of size $J_1 \times J_2$ ($J_1 \ge J_2$), we need $J_1 \cdot J_2 + J_2 \cdot J_2$ trainable parameters in LoRA and at least $J_1 + J_2$ trainable parameters in both VeRA and HiRA. However, TeRA only requires $\sum_{i=1}^N I_i$ trainable parameters, such that $\prod_{i=1}^k I_i = J_1$, $\prod_{i=k+1}^N I_i = J_2$, $I_i \ge 2 \ \forall i=1,\ldots,N$, and $1 \le k < N$.

Theorem 2. TeRA is more parameter-efficient than VeRA and HiRA when a full-rank weight update matrix is allowed, i.e., when $R_i = I_i, \forall i = 1, ..., N$, the following holds

$$\sum_{i=1}^{N} R_i \le J_1 + J_2,\tag{9}$$

where $\prod_{i=1}^k I_i = J_1$, $\prod_{i=k+1}^N I_i = J_2$, $I_i \geq 2 \ \forall i = 1, \ldots, N$, and $1 \leq k < N$.

Theorem 2 establishes that TeRA is provably more parameter-efficient than previous methods when a full-rank weight update matrix is allowed. Specifically, TeRA requires at most J_1+J_2 trainable parameters to parameterize a full-rank update matrix. The number of trainable parameters used in TeRA can also be further reduced by tensorizing the weight matrix to higher-dimensions. For example, a 4096×4096 matrix can be tensorized to $64\times64\times64\times64$. A full-rank update with HiRA or VeRA would require at least 4096+4096=8192 parameters, while TeRA requires only 64+64+64+64=256 parameters. By increasing the tensor order N to 24 (e.g., tensor size of $2\times2\times\ldots\times2$), the number of trainable parameters in TeRA can be reduced to as few as $2\times24=48$.

Theorem 3. Consider the optimal weight update $\mathbf{W}^* \in \mathbb{R}^{J_1 \times J_2}$ and the TeRA weight update, $\mathbf{W}_{TeRA} \in \mathbb{R}^{J_1 \times J_2}$, whose tensorized format is defined as in Eq. (6). Denote $\bigotimes_{i=1}^k \mathbf{A}^{(i)}$ as $\mathbf{L}^\top \in \mathbb{R}^{\prod_{i=1}^k R_i \times J_i}$, $\bigotimes_{i=k+1}^N \mathbf{A}^{(i)}$ as $\mathbf{M} \in \mathbb{R}^{K}$

Model	Method	Params (%)	BoolQ	PIQA	SIQA	ARC-c	ARC-e	OBQA	HellaS	WinoG	Average
	Prompt Tuning	0.0012	55.93	12.35	30.50	6.06	8.63	9.40	6.91	40.57	21.29
	P-Tuning	0.7428	58.75	36.02	0.20	0.17	1.98	0.80	0.01	0.00	12.24
Llama-2-7B	LoRA	0.2484	67.65	79.22	78.20	69.20	83.88	78.60	81.05	80.98	77.35
	HiRA (r=1)	0.0078	65.35	77.97	72.42	62.03	81.48	64.80	79.90	70.01	71.74
	HiRA (r=32)	0.2484	69.39	83.24	78.86	71.33	86.57	81.40	87.23	81.69	79.97
	LoRETTA	0.0052	67.52	79.33	76.31	65.70	84.34	76.00	86.32	80.98	77.06
	VeRA	0.0041	64.59	78.84	76.56	65.44	83.63	73.20	82.71	77.90	75.36
	TeRA (Ours)	0.0039	65.78	81.23	78.92	<u>69.45</u>	84.05	81.00	85.94	82.64	78.63
	Prompt Tuning	0.0010	56.85	45.05	36.13	31.57	32.74	29.20	14.01	50.12	36.96
	P-Tuning	0.6240	59.97	11.64	8.19	7.42	8.63	9.60	1.77	37.65	18.11
Llama-3-8B	LoRA	0.1695	71.99	85.91	79.58	76.19	88.55	82.60	92.54	85.63	82.88
	HiRA (r=1)	0.0053	68.04	85.75	75.54	76.96	90.32	77.00	89.04	77.43	80.01
	HiRA (r = 32)	0.1695	73.09	88.85	81.06	80.38	92.68	86.20	94.37	85.87	85.31
	LoRETTA	0.0045	65.17	89.01	79.53	79.86	91.54	84.40	94.40	84.69	83.58
	VeRA	0.0022	67.95	85.64	76.51	77.22	91.29	81.00	91.96	82.24	81.73
	TeRA (Ours)	0.0033	70.70	88.08	81.58	80.89	92.00	88.00	94.92	86.27	85.31

Table 2: Accuracy comparison of different PEFT methods on Commonsense170k dataset. The percentage of trainable parameters is calculated as #trainable params, where trainable parameters refers to those requiring gradient updates, and total parameters include both the frozen and trainable parameters across all layers in the LLM. The best and second best values among LoRA-style adapters are in bold and underlined, respectively.

 $\mathbb{R}^{J_2 \times \prod_{i=K+1}^N R_i}$, and $\mathbf{Z} = \mathbf{L}^{\dagger} \mathbf{W}^{\star} \mathbf{M}^{\dagger} \oslash \mathbf{G}_{[N;k]}$, where \oslash is the element-wise division. Then, we have

$$\min_{\{\operatorname{diag}(\mathbf{d}^{(i)})\}_{i=1}^{N}} \|\mathbf{W}^{\star} - \mathbf{W}_{TeRA}\|_{F}^{2}$$

$$\leq \|\mathbf{W}^{\star} - \mathbf{L}\mathbf{L}^{\dagger}\mathbf{W}^{\star}\mathbf{M}^{\dagger}\mathbf{M}\|_{F}^{2}$$

$$+ g_{max}(\|\mathbf{Z}\|_{F}^{2} - \|\operatorname{Fold}_{[N;k]}(\mathbf{Z})\|_{2}^{2})\|\mathbf{L}\|_{F}^{2}\|\mathbf{M}\|_{F}^{2},$$
(10)

where g_{max} is the largest entry in \mathcal{G} , and $\|\cdot\|_2$ denotes the tensor spectral norm.

Theorem 3 provides a theoretical upper bound on the approximation error between the optimal weight update and the TeRA weight update. This bound consists of two terms. The first term, $\|\mathbf{W}^* - \mathbf{L} \mathbf{L}^\dagger \mathbf{W}^* \mathbf{M}^\dagger \mathbf{M}\|_F^2$, quantifies the portion of the optimal update \mathbf{W}^* , which lies outside the subspace characterized by the frozen factor matrices $\{\mathbf{A}^{(i)}\}$. This error can only be minimized by expanding the subspace through increasing the ranks, $\{R_i\}_{i=1}^N$. Consequently, the bound provides direct theoretical motivation to maximize the ranks to their corresponding tensor dimension sizes, *i.e.*, $R_i = I_i \ \forall i = 1, 2, \dots, N$. As a benefit, this also reduces the number of hyperparameters in TeRA by eliminating the need to choose the tensor network ranks $[R_1, R_2, \dots, R_N]$.

The second error term establishes the trade-off between parameter efficiency and approximation accuracy (expressivity) in TeRA. It is bounded by a term dependent on the tensor spectral norm of \mathbf{Z} (De Lathauwer, De Moor, and Vandewalle 2000b). The tensor spectral norm has been shown to decrease as the order of the tensor, N, increases (Wang et al. 2017). Therefore, assuming $R_i = I_i \ \forall i = 1, \ldots, N$, this indicates that using a higher-order tensorization (a larger N in $\Delta \mathcal{W} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$) reduces the number of trainable parameters $(\sum_i^N R_i)$, but also increases the upper bound of the approximation error. Con-

versely, lower-order tensorization may result in lower approximation errors, but require more trainable parameters. The proofs for the three theorems are provided in the technical appendix.

Experiments

We conducted extensive experiments to demonstrate the effectiveness of TeRA across a diverse set of reasoning and generation tasks. We also performed a series of ablation studies to validate our model design and analyze the impact of hyperparameter choices.

Implementation Details. All experiments were conducted using one NVIDIA A100 (80GB) GPU. The AdamW optimizer (Loshchilov and Hutter 2019) was employed with 100 warm-up steps. We used two LLMs as the base models for fine-tuning: Llama-2-7B and Llama-3-8B. The percentage of trainable parameters is calculated as $\frac{\# \operatorname{Trainable params}}{\# \operatorname{Total params}}$. In practice, we find that only tensorizing one dimension of the weight update matrix in TeRA yields a good trade-off between performance and the number of trainable parameters. To reduce the hyperparameter space, we always set $R_i = I_i \ \forall i = 1, \dots, N$ and tensorize each dimension into equal-sized modes. E.g., a dimension size of 4096 can be tensorized into $64 \times 64, 16 \times 16 \times 16,$ etc. We report the average performance of TeRA over 5 independent runs.

Baseline Methods. We benchmark TeRA against two main categories of PEFTs: prompt-based methods (Prompt-Tuning, P-Tuning) and LoRA-style adapters with no inference overhead (LoRA, HiRA, LoRETTA, VeRA). To ensure fairness in terms of number of trainable parameters, we apply all adapter methods, including LoRA, HiRA, VeRA, LoRETTA and TeRA, to the query and value weights in the attention modules. We also report HiRA with two rank con-

Model	Method	Params (%)	BLEU	BERT F1	BERT-R	BERT-P	Meteor	R-L	Average
	Prompt Tuning	0.0012	0.04	72.44	77.38	68.23	0.80	0.80	36.62
	P-Tuning	0.7428	0.60	83.29	83.33	83.28	15.11	12.36	46.33
Llama-2-7B	LoRA	0.2484	2.59	85.25	85.31	85.21	14.14	13.28	47.63
Liailia-2-7D	HiRA (r = 1)	0.0078	2.39	84.84	84.93	84.78	13.35	12.60	47.15
	HiRA (r = 32)	0.2484	2.67	85.26	85.37	85.17	13.94	13.19	47.60
	LoRETTA	0.0052	2.22	84.99	84.97	85.03	12.37	11.85	46.91
	VeRA	0.0041	2.27	85.01	85.04	85.02	12.64	12.55	47.09
	TeRA (Ours)	0.0039	<u>2.62</u>	85.28	85.40	<u>85.19</u>	14.26	13.39	47.69
	Prompt Tuning	0.0010	1.45	82.99	82.99	83.05	14.72	13.13	46.39
	P-Tuning	0.6240	1.50	81.52	81.07	82.01	15.49	13.55	45.86
Llama-3-8B	LoRA	0.1695	2.59	85.25	85.31	85.21	14.14	13.28	47.63
Liailia-3-8D	HiRA (r = 1)	0.0053	3.36	84.81	84.40	85.26	15.12	14.19	47.86
	HiRA (r = 32)	0.1695	3.22	84.58	84.35	84.87	14.90	13.72	47.61
	LoRETTA	0.0045	2.97	84.77	84.26	85.31	13.78	13.15	47.37
	VeRA	0.0022	3.12	84.61	84.27	85.01	14.56	13.72	47.55
	TeRA (Ours)	$\overline{0.0021}$	3.38	<u>85.03</u>	<u>84.53</u>	85.57	15.32	14.59	48.07

Table 3: Evaluation results of different PEFT methods on the ConvAI2 dataset. Metrics include BLEU, BERTScore (F1/R/P), Meteor, and ROUGE-L. The best and second best values among LoRA-style adapters are in bold and underlined, respectively.

figurations: r=1 and r=32, to show how it performs under different numbers of trainable parameters.

Commonsense Reasoning

We evaluated TeRA on eight challenging commonsense reasoning tasks using the Commonsense170k benchmark (Hu et al. 2023), which has 170, 420 query-answer pairs, covering questions from physical commonsense, social reasoning, multi-step reasoning, etc. The eight sub-tasks include: BoolQ (Clark et al. 2019), PIQA (Bisk et al. 2020), SIQA (Sap et al. 2019), HellaSwag (Zellers et al. 2019), Wino-Grande (Sakaguchi et al. 2020), ARC-e and ARC-c (Clark et al. 2018), and OBQA (Mihaylov et al. 2018). We fine-tune each model for 3 epochs on the official training split and select the checkpoint with the highest accuracy on a 120-example validation set. We report the test accuracy for the eight sub-tasks in Table 2.

Results. As shown in Table 2, TeRA consistently outperforms baseline methods which require similar number of trainable parameters, such as HiRA (r = 1), LoRETTA, and VeRA, in terms of average accuracy. Specifically, TeRA achieves an average accuracy of 78.63% on Llama-2-7B (followed by 77.06% with LoRETTA) and 85.31% on Llama-3-8B (followed by 83.58% with LoRETTA). TeRA matches the performance of the best performing high-rank adapter, HiRA (r=32), while having $64 \times$ fewer trainable parameters in the Llama-2-7B model and $51 \times$ less trainable parameters in the Llama-3-8B model. Remarkably, TeRA requires fewer parameters than even the lowest-rank HiRA configuration (r = 1), yet consistently delivers superior performance. These results demonstrate that TeRA achieves the performance benefits of high-rank adapters, while offering significantly improved parameter efficiency.

High-Rank Weight Updates of TeRA. To visualize the high-rank nature of TeRA, Figure 3 shows the ranks of

the update matrices across different layers obtained in the commonsense reasoning task for Llama-3-8B. TeRA consistently obtains higher-rank updates than HiRA (r=32), demonstrating its ability to capture high-rank optimal weight updates. Additionally, the weight updates of TeRA often reach full-rank, validating Theorem 1 in practice. In comparison, the low-rank weight updates of methods such as LoRA, LoRETTA, and VeRA may limit their expressivity. For a visual comparison, see Figure 3.

Personalized Dialogue Generation

We evaluated the ability of fine-tuned models to engage in natural, persona-consistent conversations using the ConvAI2 dataset (Dinan et al. 2019) with 17,878 training and 1,000 testing multi-turn conversations. Following the experimental setup of (Huang et al. 2025), where only the speaker's persona is revealed (self-persona setting), we report the quality of the generated responses using standard metrics, including BLEU, BERTScore (Zhang et al. 2020), METEOR (Banerjee and Lavie 2005), and ROUGE (Lin 2004) in Table 3.

Results. As shown in Table 3, TeRA consistently achieves the highest average score among all baseline methods in conversational tasks and uses the least number of trainable parameters among LoRA-style adapters. More specifically, TeRA achieves an average score of 47.69% with Llama-2-7B and 48.07% with Llama-3-8B. This shows the superior performance and parameter efficiency of TeRA in conversation fine-tuning tasks.

Arithmetic Reasoning

We evaluated arithmetic reasoning capabilities of the finetuned model through the Math10k dataset from (Hu et al. 2023), consisting of 10,000 mathematical reasoning examples (Cobbe et al. 2021; Koncel-Kedziorski et al. 2016; Ling et al. 2017). Following the experimental setup in (Hu et al. 2023), we select the best model checkpoint, based on the lowest validation loss, and evaluate it on two sub-tasks: AQuA (Ling et al. 2017) and SVAMP (Patel, Bhattamishra, and Goyal 2021). Test accuracies are reported in Table 4.

Model	Method	Params (%)	AQuA	SVAMP
Llama -2-7B	HiRA (r = 1)	0.0078	18.90	27.40
	HiRA (r = 32)	0.2484	18.90	46.50
	LoRETTA	0.0052	12.60	47.50
	VeRA	0.0041	18.50	26.60
	TeRA (Ours)	0.0039	24.41	49.70
Llama -3-8B	HiRA (r = 1)	0.0053	27.95	57.70
	HiRA (r = 32)	0.1695	29.92	72.80
	LoRETTA	0.0045	18.50	56.50
	VeRA	0.0022	25.59	55.70
	TeRA (Ours)	$\overline{0.0021}$	30.71	73.10

Table 4: Performance comparison on arithmetic reasoning datasets. Test accuracy is reported. The best and second best values are in bold and underlined, respectively.

Results. TeRA consistently outperforms all baseline methods in AQuA and SVAMP, while requiring the least number of trainable parameters. As shown in Table 4, with the Llama–2-7B backbone, TeRA achieves accuracies of 24.41% on AQuA and 49.7% on SVAMP, exceeding the strongest baseline by 5.51% and 1.7% absolute points, respectively. When applied to Llama–3-8B, TeRA attains 30.71% accuracy on AQuA and 73.1% on SVAMP, outperforming HiRA (r=32) by 0.79% and 0.3% absolute points while using $80\times$ fewer parameters.

Ablation study

Impact of Tensorization on Performances. Different tensorizations of the weight matrices lead to different trade-offs between the number of trainable parameters and the model performance, as formalized in Theorem 3. As shown in Figure 4 (left), when both dimensions of the original weight matrix are tensorized into higher dimensions, the performance decreases as the number of trainable parameters decreases rapidly. We usually achieve an optimal trade-off between performance and parameter efficiency by tensorizing only one dimension of the original weight matrix, while keeping the other dimension unchanged. Under this approach, the performance of TeRA remains robust across different tensorization sizes as shown in Figure 4 (right).

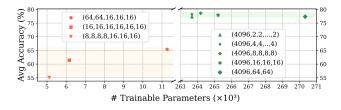


Figure 4: Average accuracy across eight commonsense reasoning tasks against number of trainable parameters under different tensorization strategies in Llama-2-7B.

Impact of Tensorization on Ranks. The high-rank property of TeRA is insensitive to the specific choice of tensorization schemes, as formalized in Theorem 1. We evaluated the ranks of the query weight updates, $\Delta \mathbf{W}_q$, and the value weight updates, $\Delta \mathbf{W}_v$, across different layers under various tensorizations. Figure 5 shows that TeRA obtains high-rank (near full-rank) updates across different tensorization choices.

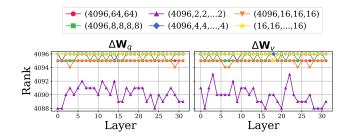


Figure 5: Rank of $\Delta \mathbf{W}_q$ and $\Delta \mathbf{W}_v$ (Max possible rank = 4096) across different layers in Llama-2-7B under different tensorization schemes in the commonsense reasoning task.

Initialization of Frozen Factor Matrices. We explore different initialization choices for the frozen factor matrices. Specifically, we compare TeRA with a variant, $TeRA_{iden}$, where its frozen factor matrices are all identity matrices. Note that $TeRA_{iden}$ has the same number of trainable parameters as TeRA. As shown in Figure 6, TeRA (solid lines) consistently outperforms $TeRA_{iden}$ (dashed lines) in terms of average accuracy with identical hyperparameters, highlighting the effectiveness of the random tensor network initialization scheme in TeRA.

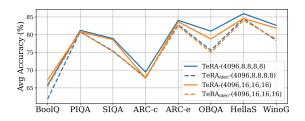


Figure 6: Comparison between TeRA and TeRA_{iden} on the commonsense reasoning dataset with Llama-2-7B.

Conclusion

We have introduced TeRA, a high-rank PEFT adapter which utilizes a tensor network to parameterize the tensorized weight updates. In this way, TeRA offers a more effective alternative to existing vector-based adapters, achieving much better performances and high-rank updates but with a similar amount of trainable parameters. Extensive experiments demonstrate the effectiveness of the proposed TeRA method. Our future work aims to apply TeRA to applications beyond large language models.

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