

$$f(x \pm n\delta) = f^{(0)}(x) \left[ 1 \right] + f^{(1)}(x) \left[ \pm n\delta \right] + f^{(2)}(x) \left[ \frac{n^2\delta^2}{2} \right] + f^{(3)}(x) \left[ \pm \frac{n^3\delta^3}{6} \right] + \mathcal{O}(\delta^4)$$

Five Point Symmetric Stencil:

$$\begin{aligned} f(x) &= f^{(0)}(x) \\ f(x + \delta) - f(x - \delta) &= f^{(1)}(x) \left[ 2\delta \right] + f^{(3)}(x) \left[ \frac{\delta^3}{3} \right] + \mathcal{O}(\delta^5) \\ f(x + 2\delta) - f(x - 2\delta) &= f^{(1)}(x) \left[ 4\delta \right] + f^{(3)}(x) \left[ \frac{8\delta^3}{3} \right] + \mathcal{O}(\delta^5) \end{aligned}$$


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$$8(f(x + \delta) - f(x - \delta)) - (f(x + 2\delta) - f(x - 2\delta)) = f^{(1)}(x) \left[ 12\delta \right] + \mathcal{O}(\delta^5) \quad (1)$$

$$f^{(1)}(x) \approx \frac{8(f(x + \delta) - f(x - \delta)) - (f(x + 2\delta) - f(x - 2\delta))}{12\delta} \quad (2)$$