$$f(x \pm n\delta) = f^{(0)}(x)\Big[1\Big] + f^{(1)}(x)\Big[\pm n\delta\Big] + f^{(2)}(x)\Big[\frac{n^2\delta^2}{2}\Big] + f^{(3)}(x)\Big[\pm \frac{n^3\delta^3}{6}\Big] + \mathcal{O}\big(\delta^4\big)$$

Five Point Symmetric Stencil:

$$\begin{aligned} f(x) &= f^{(0)}(x) \\ f(x+\delta) - f(x-\delta) &= f^{(1)}(x)\Big[2\delta\Big] + f^{(3)}(x)\Big[\frac{\delta^3}{3}\Big] + \mathcal{O}(\delta^5) \\ f(x+2\delta) - f(x-2\delta) &= f^{(1)}(x)\Big[4\delta\Big] + f^{(3)}(x)\Big[\frac{8\delta^3}{3}\Big] + \mathcal{O}(\delta^5) \end{aligned}$$

$$8\left(f(x+\delta) - f(x-\delta)\right) - \left(f(x+2\delta) - f(x-2\delta)\right) = f^{(1)}(x)\Big[12\delta\Big] + \mathcal{O}(\delta^5) \tag{1}$$

$$f^{(1)}(x) \approx \frac{8\left(f(x+\delta) - f(x-\delta)\right) - \left(f(x+2\delta) - f(x-2\delta)\right)}{12\delta} \tag{2}$$

Assymmetric Stencils:

$$\begin{aligned} f^{(1)}(x) &\approx \frac{-10f(x) + 18f(x+\delta) - 3f(x-\delta) - 6f(x+2\delta) + f(x+3\delta)}{12\delta} \\ &\approx \frac{1}{12\delta}\Bigg[-10f(x) + \Big(18f(x) + 18f^{(1)}(x)\delta + 9f^{(2)}(x)\delta^2 + 3f^{(3)}(x)\delta^3 + \frac{3}{4}f^{(4)}(x)\delta^4\Big) \\ &\qquad + \Big(-3f(x) + 3f^{(1)}(x)\delta - \frac{3}{2}f^{(2)}(x)\delta^2 + \frac{1}{2}f^{(3)}(x)\delta^3 - \frac{1}{8}f^{(4)}(x)\delta^4\Big) \\ &\qquad + \Big(-6f(x) - 12f^{(1)}(x)\delta - 12f^{(2)}(x)\delta^2 - 8f^{(3)}(x)\delta^3 - 4f^{(4)}(x)\delta^4\Big) \\ &\qquad + \Big(f(x) + 3f^{(1)}(x)\delta + \frac{9}{2}f^{(2)}(x)\delta^2 + \frac{9}{2}f^{(3)}(x)\delta^3 + \frac{27}{8}f^{(4)}(x)\delta^4\Big)\Bigg] \\ &= \frac{1}{12\delta}\Big[12f^{(1)}(x)\delta\Big] = f^{(1)}(x) \end{aligned} \tag{3}$$

$$\begin{aligned} f^{(1)}(x) &\approx \frac{-25f(x) + 48f(x+\delta) - 36f(x+2\delta) + 16f(x+3\delta) - 3f(x+4\delta)}{12\delta} \\ &\approx \frac{1}{12\delta}\Bigg[-25f(x) + \Big(48f(x) + 48f^{(1)}(x)\delta + 24f^{(2)}(x)\delta^2 + 8f^{(3)}(x)\delta^3 + 2f^{(4)}(x)\delta^4\Big) \\ &\qquad + \Big(-36f(x) - 72f^{(1)}(x)\delta - 72f^{(2)}(x)\delta^2 - 48f^{(3)}(x)\delta^3 - 24f^{(4)}(x)\delta^4\Big) \\ &\qquad + \Big(16f(x) + 48f^{(1)}(x)\delta + 72f^{(2)}(x)\delta^2 + 72f^{(3)}(x)\delta^3 + 54f^{(4)}(x)\delta^4\Big) \\ &\qquad + \Big(-3f(x) - 12f^{(1)}(x)\delta - 24f^{(2)}(x)\delta^2 - 32f^{(3)}(x)\delta^3 - 32f^{(4)}(x)\delta^4\Big)\Bigg] \\ &= \frac{1}{12\delta}\Big[12f^{(1)}(x)\delta\Big] = f^{(1)}(x) \end{aligned} \tag{4}$$