

HW6 Singular Value Decomposition

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Abstract

This homework requires us to use Singular Value Decomposition to decompose an image. Assuming we want to reduce the size of the image to compress the image and at the same time keep certain quality of the one.

- 1 A plot includes curve describing the relation of k and approximation error

SVD

- For any $m \times n$ matrix A , we can decompose it into three components U , Σ and V .

$$A = U \Sigma V^T$$
$$A_i = \sigma_i \vec{u}_i \vec{v}_i^T$$

A
 $m \times n$

=

\vec{u}_i

U
 $m \times m$

x

σ_i

Σ
 $m \times n$
diagonal
matrix
 $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_k$

x

\vec{v}_i

V^T
 $n \times n$

Figure 1: SVD on image

U and V^T are orthogonal matrix, and Σ is diagonal matrix used as an index to enlarge/retract the vector. To reconstruct the image, we use matrix multiplication on U , V^T and Σ . However, if we want to compress the image, we can only use part of the matrix. The Σ matrix from top left to right bottom represents the significance priority, we can take only one singular to construct matrix with U shape (2880, 1), Σ shape (1, 1) and V^T shape (1, 1620), so on so forth.

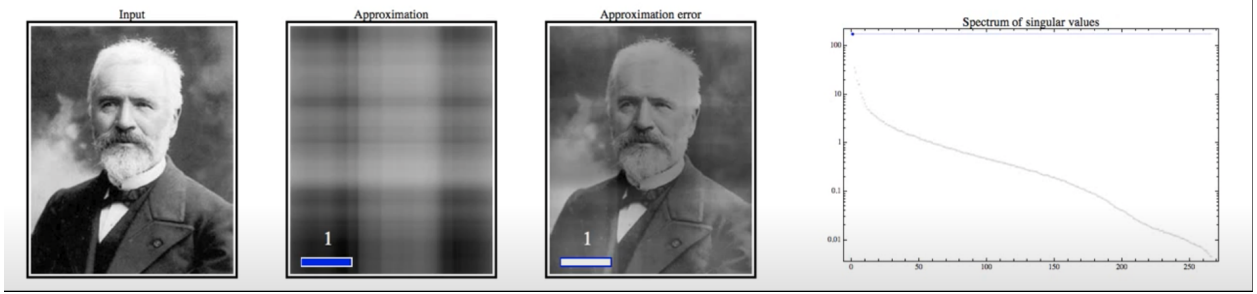


Figure 2: Regression Error with $k = 1$

This is the approximation and approximation error with only use one singular value.

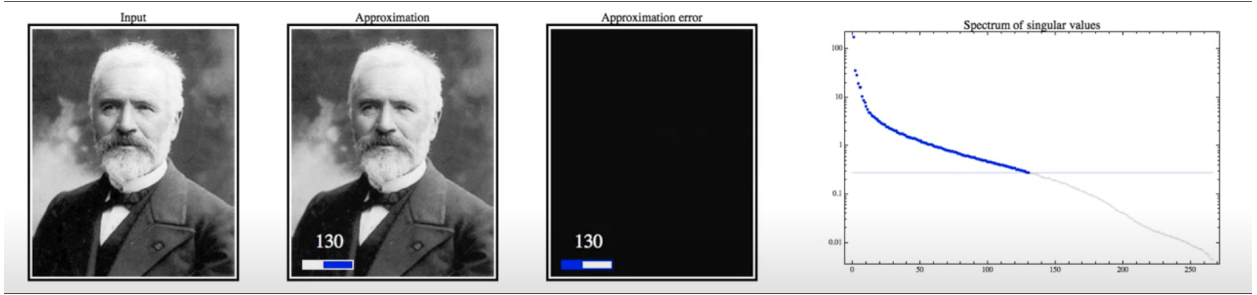


Figure 3: Regression Error with $k = 130$

This is the approximation and approximation with use 130 singular value.

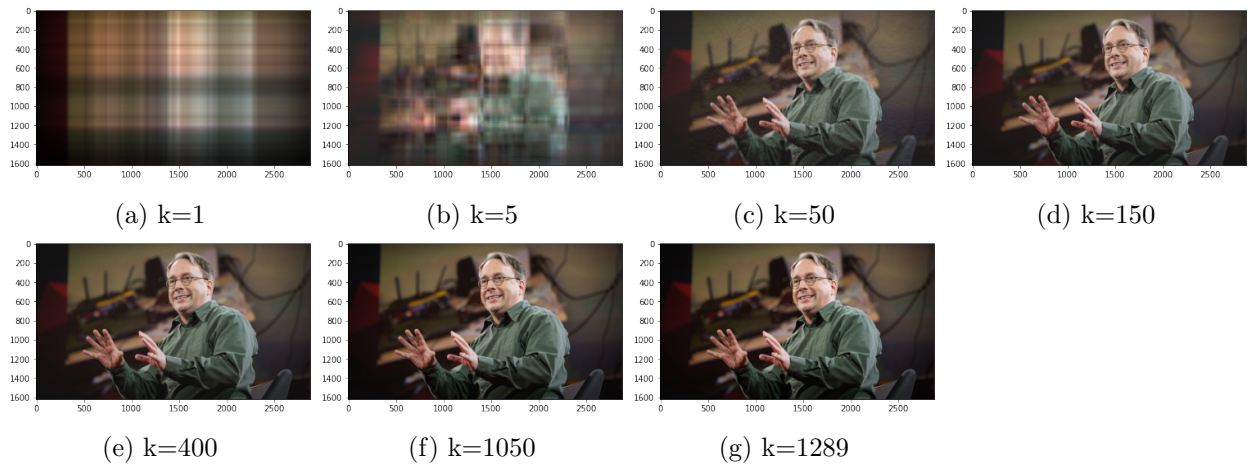


Figure 4: Approximation Image with different k

We can graph the approximation error depends on k value, and normally with larger k the approximation error is smaller.

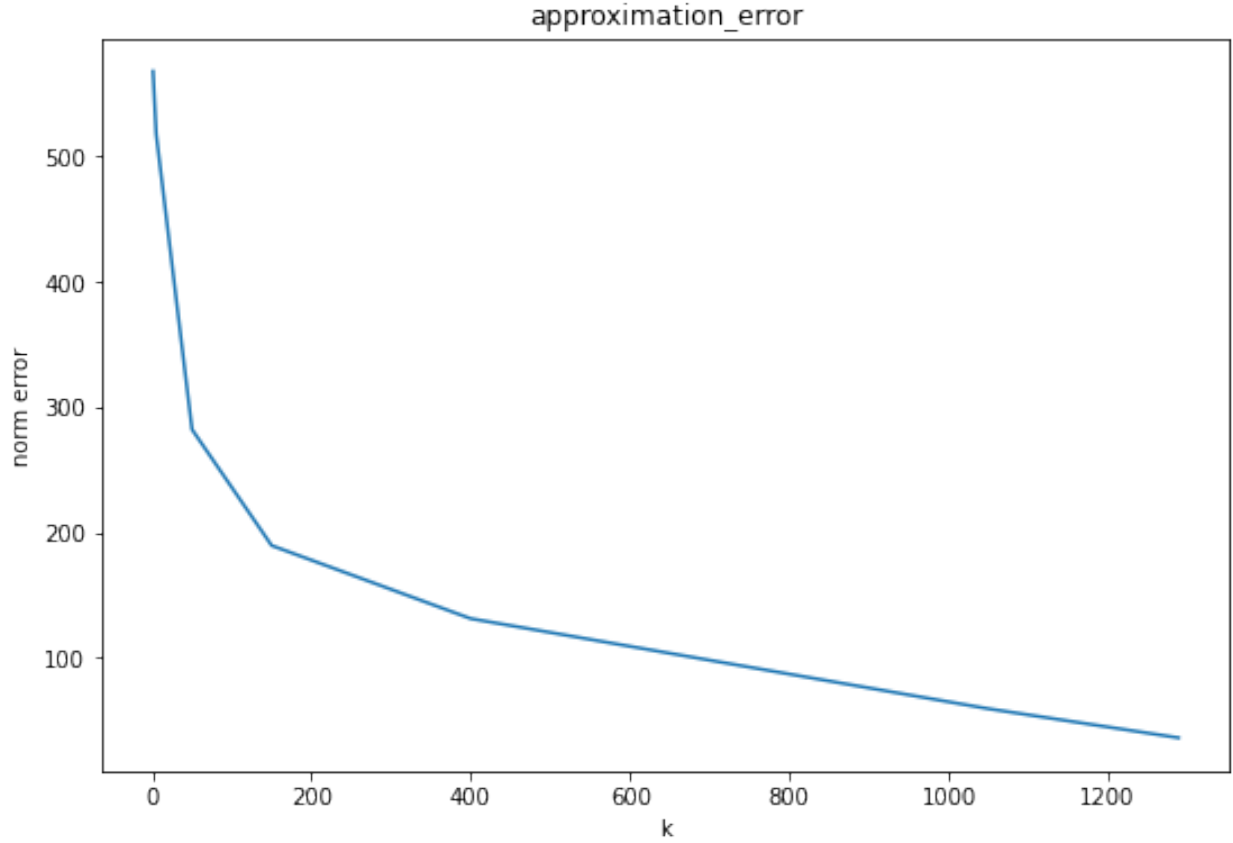


Figure 5: Approximation Error on k

2 Analyze the rank of R channel of the provided image and explain how you analyze.

The non-zero number of Σ equals the rank of the image, which is 1680.

3 Plots in page 6 but on G channel($A_{i,G}, \forall 1 \leq i \leq 5$)

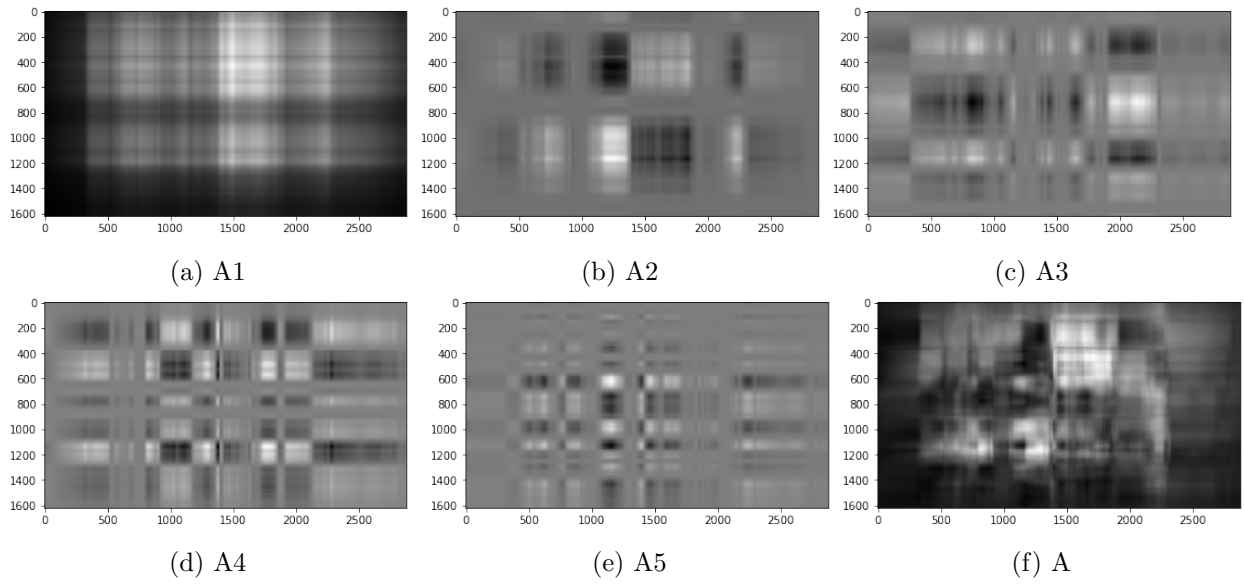


Figure 6: Approximation Image with different singular value