Assignment #10 Name Answer key.

Due 13 April 2015

1. In each case determine whether or not the series converges absolutely, converges conditionally, or diverges.

(a) $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$

since $\frac{n}{n+1} = \frac{1}{1+\frac{1}{n}} \longrightarrow 1$,

the series <u>diverges</u> by the n^{th}-term test.

(b) $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{1/3}}$

Let $T = \sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$.

The series T diverges (PST, $p = \frac{1}{3}$), but S converges by the A.S.T.

Thus S <u>converges conditionally</u>.

(c) $S = \sum_{n=1}^{\infty} (-1)^{n} \frac{n^n}{3^n n!}$

$$\frac{|a_{n+1}|}{|a_n|} = \frac{(n+1)^{n+1}}{3^{n+1}(n+1)!} \frac{3^n n!}{n^n}$$

$$= \frac{1}{3} \frac{1}{n+1} \frac{(n+1)^{n+1}}{n^n} = \frac{1}{3}\left(1+\frac{1}{n}\right)^n \longrightarrow \frac{e}{3} < 1.$$

Thus S <u>converges absolutely</u> by the ratio test.

(d) $S = \sum_{n=1}^{\infty} (-1)^n \frac{n!}{5^n}$

$$\frac{|a_{n+1}|}{|a_n|} = \frac{(n+1)!}{5^{n+1}} \frac{5^n}{n!} = \frac{n+1}{5}$$

$\longrightarrow +\infty$; thus, S diverges by the ratio test.

(e) $S = \sum_{n=1}^{\infty} \left(\frac{n+1}{n^2+4}\right)^n$

$$\sqrt[n]{|a_n|} = \left(\left(\frac{n+1}{n^2+4}\right)^n\right)^{\frac{1}{n}}$$

$$= \frac{n+1}{n^2+4} = \frac{\frac{1}{n} + \frac{1}{n^2}}{1 + \frac{4}{n^2}}$$

$$\longrightarrow 0 < 1$$

Thus S converges absolutely by the root test.

(f) $S = \frac{(1)(1)}{(1\cdot 2)} + \frac{(1\cdot 2)(1\cdot 2)}{(1\cdot 2\cdot 3\cdot 4)} + \frac{(1\cdot 2\cdot 3)(1\cdot 2\cdot 3)}{(1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6)} + \frac{(1\cdot 2\cdot 3\cdot 4)(1\cdot 2\cdot 3\cdot 4)}{(1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8)} + \cdots$
(Hint: First find a simple expression for the nth term of the this series.)

$$a_n = \frac{n!\,n!}{(2n)!} \qquad \frac{|a_{n+1}|}{|a_n|} = \frac{(n+1)!\,(n+1)!}{(2n+2)!} \frac{(2n)!}{n!\,n!}$$

$$= \frac{(n+1)(n+1)}{(2n+1)(2n+2)} = \frac{\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n}\right)}{\left(2+\frac{1}{n}\right)\left(2+\frac{2}{n}\right)} \longrightarrow \frac{1}{4}$$

Thus S converges absolutely by the ratio test