Assignment #11 **Name** Answer Key

Due 22 April 2015

1. Find the radius of convergence and the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{1}{2^n\sqrt{n}}(x-4)^n$.

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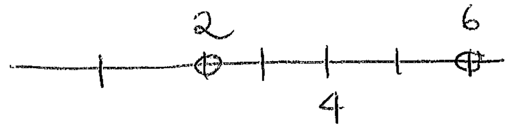
$$\frac{|x-4|^{n+1}}{2^{n+1}\sqrt{n+1}} \cdot \frac{\sqrt{n}\ 2^n}{|x-4|^n} = \sqrt{\frac{n}{n+1}}\ \frac{|x-4|}{2} \to \frac{|x-4|}{2}$$

If $|x-4| \begin{cases} <2 & \text{conv. abs.} \\ =2 & ? \\ >2 & \text{div.} \end{cases}$



$x=6$: $\sum_{n=0}^{\infty} \frac{1}{2^n\sqrt{n}} 2^n = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n}}$ diverges, p-series $p=\frac{1}{2}$.

$x=2$: $\sum_{n=0}^{\infty} \frac{1}{2^n\sqrt{n}} (-1)^n 2^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges by A.S.T.

$R=2$ Int. of conv. $=[2,6)$

2. Find the sum of the series $\sum_{n=2}^{\infty} n(n-1)(3/4)^n$.

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$$(1-x)^{-1} = \sum_{n=0}^{\infty} x^n$$

$$+1(1-x)^{-2}(+1) = \sum_{n=1}^{\infty} n x^{n-1}$$

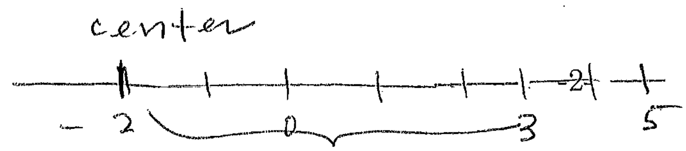
$$+2(1-x)^{-3}(+1) = \sum_{n=2}^{\infty} n(n-1) x^{n-2}.$$

$$\frac{2}{(1-x)^3} = \sum_{n=2}^{\infty} n(n-1) x^{n-2}$$

$$\frac{2x^2}{(1-x)^3} = \sum_{n=2}^{\infty} n(n-1) x^n.$$

plug in $x = 3/4$:

$$\frac{2(3/4)^2}{(1-3/4)^3} = \sum_{n=2}^{\infty} n(n-1)\left(\frac{3}{4}\right)^n = 72$$



3. Suppose that the series $\sum_{n=0}^{\infty} c_n(x+2)^n$ converges at $x = 3$. Which of the following statements is *necessarily* true:

3 (a) The radius of convergence of the power series is at most 5. Not necessarily

(b) The series converges at $x = -6$. Yes.

(c) The series $\sum_{n=0}^{\infty} c_n 2^n$ converges. Yes.

4. Develop a power series centered at 0 for each of the following functions. Indicate the interval on which the series represents the function.

(a) $\frac{1}{1+3x} = \frac{1}{1-(-3x)} = \sum_{n=0}^{\infty} (-3x)^n = \sum_{n=0}^{\infty} (-3)^n x^n.$

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converges for $|-3x| < 1$ or $\underline{|x| < \frac{1}{3}.}$

(b) $\frac{1}{9-x^2} = \frac{1}{9} \frac{1}{1-\left(\frac{x}{3}\right)^2} = \frac{1}{9} \cdot \sum_{n=0}^{\infty} \left(\left(\frac{x}{3}\right)^2\right)^n$

$= \frac{1}{9} \cdot \sum_{n=0}^{\infty} \frac{x^{2n}}{9^n} = \sum_{n=0}^{\infty} \frac{x^{2n}}{9^{n+1}}$

converges for $\left|\left(\frac{x}{3}\right)^2\right| < 1$ or $\underline{|x| < 3.}$

(c) $\frac{1}{(1+x)^2}$ $\quad \frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-1)^n x^n.$

$-1(1+x)^{-2} = \sum_{n=1}^{\infty} (-1)^n n x^{n-1}$

$\frac{1}{(1+x)^2} = \sum_{n=1}^{\infty} (-1)^{n+1} n \cdot x^{n-1}$ or

$= \sum_{n=0}^{\infty} (-1)^n (n+1) x^n.$