Assignment #9 **Name** Answer Key

Due 25 March 2015

1. Consider the infinite series $\sum_{k=1}^{\infty} ke^{-k}$.

3 (a) Evaluate $\int xe^{-x}dx$.

$u = x \qquad dv = e^{-x}\,dx$

$du = dx \qquad v = -e^{-x}$

$$\int xe^{-x}dx = -xe^{-x} + \int e^{-x}dx$$
$$= -xe^{-x} - e^{-x} + c$$
$$= -e^{-x}(x+1) + c$$

(b) Show that the two conditions of the integral test are satisfied by the series and show that the series converges. Let $f(x) = xe^{-x}$.

2

① f is non-negative because $x \geq 1$ and $e^{-x} > 0$.

② $f'(x) = (1-x)e^{-x}$

$+ \quad | \quad -$

$0 \quad 1$ dec.

f is decreasing on $[1,\infty)$.

$$\int_1^{\infty} xe^{-x}dx = \lim_{b\to\infty}\int_1^b xe^{-x}dx = \lim_{b\to\infty}\left(\frac{-(b+1)}{e^b} + \frac{2}{e}\right)$$

$$= 0 + 2 = 2, \text{ by l'hôpital.}$$

Since $\int_1^{\infty} xe^{-x}dx$ converges, $\sum_{k=1}^{\infty} ke^{-k}$ converges.

(c) Let L denote the sum of the series and let $s_n = \sum_{k=1}^{n} ke^{-k}$ be the nth partial sum of the series. Compute s_4, expressing your answer as a decimal.

3

$$s_4 = 1e^{-1} + 2e^{-2} + 3e^{-3} + 4e^{-4}$$

$$\approx .861174$$

Let $R = L - s_4$, the remainder.

(d) Find upper and lower bounds for the error in approximating L by s_4.

2

$$\int_5^\infty xe^{-x}dx \leq R \leq \int_4^\infty xe^{-x}dx$$

$$\text{But } \int_4^\infty xe^{-x}dx = \lim_{b\to\infty}\left(-\frac{(b+1)}{e^b} + \frac{5}{e^4}\right) = \frac{5}{e^4}$$

$$\int_5^\infty xe^{-x}dx = \lim_{b\to\infty}\left(\frac{-(b+1)}{e^b} + \frac{6}{e^5}\right) = \frac{6}{e^5}$$

$$\frac{6}{e^5} \leq R \leq \frac{5}{e^4}$$

$$.0404 \leq R \leq .0915$$