§7.1–Integration by Parts

Tom Lewis

Spring Semester
2015

Outline

The formula (long form)

Here is the integration by parts formula in its long form:

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

The formula (short form)

Let

$$u = f(x) \qquad dv = g'(x)dx$$
$$du = f'(x)dx \qquad v = g(x)$$

$$\int udv = uv - \int vdu,$$

which is a short form of the *integration by parts formula*.

Problem

Evaluate the following integrals:

- $\int x \sin(x) dx$
- $\int x^2 \cos(x)$
- $\int x^3 \ln(x) dx$
- $\int x \tan^{-1}(x) dx$
- $\int_0^1 x e^x dx$
- $\int e^x \sin(2x) dx$

Problem

Show that

$$\int \sin^n(x) dx = -\frac{1}{n} \cos(x) \sin^{n-1}(x) + \frac{n-1}{n} \int \sin^{n-2}(x) dx.$$

This is called a reduction formula: the power of the sine in the integral is reduced from n to $n-2$.

Problem

Use the reduction formula to evaluate $\int_0^\pi \sin^5(x)dx$ *and* $\int_0^\pi \sin^6(x)dx$.