48 + 2 16

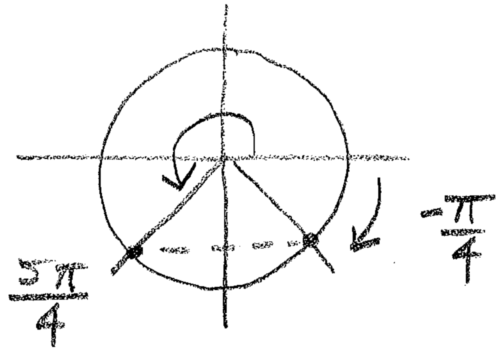
Test #2 Name Answer Key

Show all of your work

1. In each case, find the exact value:

   (a) $\sin^{-1}(\sin(5\pi/4))$ $= -\dfrac{\pi}{4}$

3



   (b) $\cos(\sin^{-1}(.6))$ $= \sqrt{1-(.6)^2}$

3

$= \sqrt{.64}$

$= .8$

2. Show that $2\sinh(x)\cosh(x) = \sinh(2x)$.

5

$$\cancel{2}\left(\frac{e^x - e^{-x}}{\cancel{2}}\right)\left(\frac{e^x + e^{-x}}{2}\right) = \frac{e^{2x} + 1 - 1 - e^{-2x}}{2}$$

$$= \frac{e^{2x} - e^{-2x}}{2}$$

$$= \sinh(2x).$$

5 3. Express the **form** of the partial fraction decomposition of $f(x) = \dfrac{x^2}{(x+3)(x^2+2x-3)(x^2+2x+2)}$. You do not have to find the coefficients.

$(x+3)(x-1)$ ↑

Irreducible

$2^2 - 4(1)(2) < 0$

$$\frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x-1} + \frac{Dx+E}{x^2+2x+2}$$

4. Evaluate the following limits:

(a) $\lim_{x\to 0} \frac{e^x - 1 - x}{x\sin(x)}$ $= 1/2$

3

check: $\lim_{x\to 0} \frac{e^x - 1}{\sin x + x\cos x}$ $\frac{0}{0}$

check: $\lim_{x\to 0} \frac{e^x}{\cos x + \cos x - x\sin x} = \frac{1}{2}$

(b) $\lim_{x\to\infty} x\ln\left(1+\frac{2}{x}\right)$ $= \lim_{x\to\infty} \frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}}$ $\frac{0}{0}$ $= 2$

3

check: $\lim_{x\to 0} \frac{\frac{1}{\left(1+\frac{2}{x}\right)}\left(\frac{-2}{\cancel{x^2}}\right)}{\cancel{-\frac{1}{x^2}}} = 2$

5. (a) Let $f(x) = \frac{1}{2}\sqrt{1-4x^2} + x\sin^{-1}(2x)$. Show that $f'(x) = \sin^{-1}(2x)$.

3 $f'(x) = \frac{1}{4}(1-4x^2)^{-\frac{1}{2}}(-8x) + \sin^{-1}(2x) + x\frac{1}{\sqrt{1-4x^2}}(2)$

$= \frac{-2x}{\sqrt{1-4x^2}} + \sin^{-1}(2x) + \frac{2x}{\sqrt{1-4x^2}}$

$= \sin^{-1}(2x)$ ✓

(b) Let $f(x) = \tan^{-1}\left(\frac{x-1}{x+1}\right)$. Show that $f'(x) = \frac{1}{1+x^2}$

3 $f'(x) = \frac{1}{1+\left(\frac{x-1}{x+1}\right)^2} \frac{(x+1)(1)-(x-1)(1)}{(x+1)^2}$

$= \frac{2}{(x+1)^2+(x-1)^2} = \frac{2}{x^2+2x+1+x^2-2x+1}$

$= \frac{1}{x^2+1}$

12

6. In each case, evaluate the indicated integral.

(a) $I = \int_0^{\pi/2} \sin^5(x)\cos^3(x)dx$ $= \int_0^{\pi/2} \sin^5 x\,(1-\sin^2 x)\cos(x)\,dx$

4

$u = \sin x$

$du = \cos x\,dx$

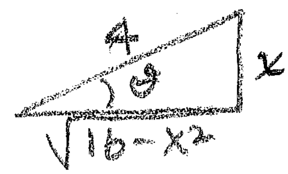
$= \int_0^1 u^5(1-u^2)\,du$

$= \int_0^1 (u^5 - u^7)\,du = \frac{u^6}{6} - \frac{u^8}{8}\Big|_0^1$

$= \frac{1}{6} - \frac{1}{8} = \frac{1}{24}$

(b) $I = \int \frac{x^2}{\sqrt{16-x^2}}dx$ $\quad x = 4\sin\theta$

$dx = 4\cos\theta\,d\theta.$



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2.5

$= \int \frac{16\sin^2\vartheta\ \cancel{4\cos\theta}}{\cancel{4\cos\theta}}\,d\vartheta = 16\int \frac{1-\cos(2\theta)}{2}\,d\vartheta$

$= 8\theta - 4\sin(2\theta) + C = 8\theta + 8\sin\theta\cos\theta + C$

$= 8\sin^{-1}\left(\frac{x}{4}\right) + 8\cdot\frac{x}{4}\cdot\frac{\sqrt{16-x^2}}{4} + C$

$= 8\sin^{-1}(x/4) + \frac{x\sqrt{16-x^2}}{2} + C$

(c) $\int \sqrt{x}\ln(x)dx$

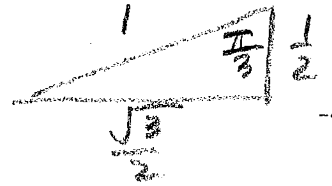
4

$u = \ln(x) \quad dv = x^{1/2}$

$du = \frac{1}{x}dx \quad v = \frac{2}{3}x^{3/2}$

$= \frac{2}{3}x^{3/2}\ln(x) - \int \frac{2}{3}x^{1/2}dx$

$= \frac{2}{3}x^{3/2}\ln(x) - \frac{4}{9}x^{3/2} + C$



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4 (d) $I = \int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx$

$x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-1} = \tan\theta$

$$= \int_0^{\pi/3} \frac{\tan\theta\tan\theta\,\cancel{\sec\theta}}{\cancel{\sec\theta}}\,d\theta$$

$\sec\theta = 1$
$\theta = 0$

$\sec\theta = 2$
$\theta = \frac{\pi}{3}$

$$= \int_0^{\pi/3} \tan^2\theta\,d\theta$$ 2.5 pts

$$= \int_0^{\pi/3} (\sec^2\theta - 1)\,d\theta = \tan\theta - \theta\Big|_0^{\pi/3} = \sqrt{3} - \frac{\pi}{3} - 0$$

$$= \sqrt{3} - \frac{\pi}{3}$$

4 (e) $I = \int \frac{5x^2-9}{x^3-9x}\,dx$

$$\frac{5x^2-9}{x(x-3)(x+3)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+3}$$

$$5x^2 - 9 = A(x-3)(x+3) + Bx(x+3) + Cx(x-3)$$

$x=0$: $-9 = -9A$ $A = 1$

$x=3$: $36 = B(18)$ $B = 2$

$x=-3$: $36 = C(18)$ $C = 2$

$$\int\left(\frac{1}{x} + \frac{2}{x-3} + \frac{2}{x+3}\right)dx$$

$$= \ln|x| + 2\ln|x-3| + 2\ln|x+3| + C.$$