§6.5–Exponential Growth and Decay

Tom Lewis

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The general model Examples Doubling-time and half-life Compound interest

Outline

The law of natural growth

- Let $y(t)$ denote a population at time t. Then we can write this mathematically as
$$\begin{cases} y' = ky \\ y(0) = A \end{cases}$$
where k is the constant of proportionality.
- If $k > 0$, then this is a model for population growth; if $k < 0$, then this is a model for population decay.

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Theorem

The unique solution of the equation

$$\begin{cases} y' = ky \\ y(0) = A \end{cases}$$

is $y(t) = Ae^{kt}$.

Problem

Suppose that a certain culture of bacteria grew from 500 to 800 in 2 hours. Use an exponential growth model to predict the number of bacteria in 5 hours.

Definition (Doubling-time and half-life)

- When $k > 0$ (growth model), we will let T denote the time for a population to be doubled. T is called the *doubling-time*.
- When $k < 0$ (decay model), we will let T denote the time for a population to be halved. T is called the *half-life*.

Theorem (Doubling time & half life)

1. *If $k > 0$, then the doubling time for the population is $T = \ln(2)/k$.*
2. *If $k < 0$, then the half life for the population is $T = \ln(.5)/k$.*

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Problem

The half-life of radium-226 is 1590 years. A certain sample has a weight of 2g. How much will this sample weigh in 5000 years?

Compound interest formula

Suppose that you invest amount A_0 dollars into an account with annual rate r which compounds interest n times per year. Then after t years you will have

$$A(t) = A_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{dollars}$$

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Problem

$500 is invested into an account which pays 8% compounded yearly. How much will you have in your account at the end of the twelfth year?

Problem

Suppose that you invest \$150 into an account with rate $r = .06$ with monthly compoundings ($n = 12$). How much will you have in your account in 7 years?

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Problem

If we let $n \to \infty$, then the account is said to pay interest continuously. Show that $A(t) = A_0 e^{rt}$ in this case.

Problem

Suppose that you invest \$150 into an account with rate $r = .06$ with continuous compounding. How much will you have in your account in 7 years?