Assignment #5 Name Answer Key

Due 16 February 2015

1. Evaluate the following limits:

(a) $\lim_{x\to 0}\left(\csc(x) - \cot(x)\right) = \lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{\cos(x)}{\sin(x)}\right)$

$= \lim_{x\to 0} \frac{1-\cos(x)}{\sin(x)}$ $\frac{0}{0}$ case

check: $\lim_{x\to 0} \frac{\sin(x)}{\cos(x)} = \frac{0}{1} = 0$

Thus $\lim_{x\to 0}\left(\csc(x) - \cot(x)\right) = 0.$

(b) $\lim_{x\to +\infty}\left(1+\frac{1}{x}\right)^{x^2} = \lim_{x\to\infty} e^{x^2 \ln\left(1+\frac{1}{x}\right)}$

$\lim_{x\to\infty} x^2 \ln\left(1+x^{-1}\right) = \lim_{x\to +\infty} \frac{\ln\left(1+x^{-1}\right)}{x^{-2}}$

check: $\lim_{x\to\infty} \frac{\frac{1}{1+x^{-1}}\left(\frac{-1}{x^2}\right)}{-\frac{2}{x^3}} = \lim_{x\to +\infty} \frac{1}{1+\frac{1}{x}}\left(\frac{-1}{\cancel{x^2}}\right)\left(\frac{\cancel{x^3}^{\,x}}{-2}\right)$

$= \lim_{x\to +\infty} \frac{x}{\left(1+\frac{1}{x}\right)2} = +\infty.$

$\therefore \lim_{x\to +\infty}\left(1+\frac{1}{x}\right)^{x^2} = +\infty.$

ILATE

2. Evaluate the following integrals:

(a) $I = \int e^{-x} \cos(x) dx$

$u = \cos(x) \qquad dv = e^{-x} dx$

$du = -\sin(x) \qquad v = -e^{-x}$

$$I = -e^{-x}\cos(x) - \boxed{\int e^{-x} \sin(x)\, dx}$$

$u = \sin(x) \qquad dv = e^{-x} dx$

$du = \cos(x) \qquad v = -e^{-x}$

$$I = -e^{-x}\cos(x) - \left\{ -e^{-x}\sin(x) + I \right\}$$

$$2I = -e^{-x}\cos(x) + e^{-x}\sin(x) + C$$

$$I = -\frac{1}{2} e^{-x}\cos(x) + \frac{1}{2} e^{-x}\sin(x) + C.$$

(b) $I = \int x^2 \tan^{-1}(x) dx$

$u = \tan^{-1}(x) \qquad dv = x^2 dx$

$du = \frac{1}{1+x^2} \qquad v = \frac{x^3}{3}$

$$I = \frac{x^3 \tan^{-1}(x)}{3} - \frac{1}{3}\int \frac{x^3}{x^2+1} dx$$

$$\begin{array}{r|l} & x \\ \hline x^2+1 & x^3 \\ & x^3 + x \\ \hline & -x \end{array}$$

$$I = \frac{x^3 \tan^{-1}(x)}{3} - \frac{1}{3}\int \left(x - \frac{x}{1+x^2} \right) dx$$

$$= \frac{x^3 \tan^{-1}(x)}{3} - \frac{1}{3}\frac{x^2}{2} + \frac{1}{6}\int \frac{2x}{1+x^2} dx$$

$u = 1+x^2$

$du = 2x\, dx$

$$= \frac{x^3 \tan^{-1}(x)}{3} - \frac{x^2}{6} + \frac{1}{6}\ln(|1+x^2|) + C$$