§11.6–The Absolute Convergence, and the Ratio and Root Tests

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Outline

Theorem (Absolute Convergence Test (ACT))

If $\sum |a_n|$ converges, then $\sum a_n$ converges.

Problem

Examine the convergence of the following series:

1. $$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$
2. $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+2}{\sqrt{n^5+8}}$$

Definition

1. If the series $\sum |a_n|$ converges, then we say that the series $\sum a_n$ converges absolutely.
2. if the series $\sum a_n$ converges but $\sum |a_n|$ diverges, then we say that the series $\sum a_n$ converges conditionally.

Problem

Does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ *converge absolutely, converge conditionally, or diverge?*

Theorem (The Ratio Test (RT))

Let $\{a_n\}$ be a sequence of nonzero real numbers and suppose that

$$\frac{|a_{n+1}|}{|a_n|} \to L$$

1. *If $L < 1$, then $\sum a_n$ converges absolutely.*
2. *If $L > 1$, then $\sum a_n$ diverges.*
3. *If $L = 1$, then the test is inconclusive; the series may or may not converge.*

Problem

Determine the convergence or divergence of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n!}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

Problem

Examine the convergence of each of the following series using the ratio test.

1. $\sum_{n=1}^{\infty} \frac{1}{n^2}$
2. $\sum_{n=1}^{\infty} \frac{1}{n}$

Theorem (The Root Test)

Suppose $|a_n|^{1/n} \to L$.

1. *If $L < 1$, then $\sum a_n$ converges absolutely.*
2. *If $L > 1$, then $\sum a_n$ diverges.*
3. *If $L = 1$, then the test is inconclusive; the series may or may not converge.*

Problem

Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3n}{5n+6}\right)^{n}$ *converges.*