§6.2*-The Natural Logarithm Function

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Spring Semester 2015

The log base 10 The natural logarithm Further properties of the natural logarithm The graph of the log function The number e, Eu

Outline

The log base 10

The natural logarithm

Further properties of the natural logarithm

The graph of the log function

The number e, Euler's number

Further derivative problems

Integration

Logarithmic differentiation

Properties of the base 10 logarithm function

Here are some familiar properties of the base 10 logarithm function.

- $\log_{10}(1) = 0$, $\log_{10}(10) = 1$, $\log_{10}(100) = 2$.
- $\log_{10}(ab) = \log_{10}(a) + \log_{10}(b)$, a, b > 0.
- $\log_{10}(1/a) = -\log_{10}(a)$, a > 0.
- $\log_{10}(a/b) = \log_{10}(a) \log_{10}(b)$.
- $\log_{10}(a^r) = r \log_{10}(a)$, a > 0, r rational.

The log base 10 The natural logarithm Further properties of the natural logarithm The graph of the log function The number e, Eu

Definition (The natural logarithm function)

For x > 0, let

$$\ln(x) = \int_{1}^{x} \frac{1}{t} dt.$$

This function is called the natural logarithm.

Theorem (Elementary properties of In)

- ln(1) = 0
- ln(x) < 0 if 0 < x < 1
- ln(x) > 0 if x > 1
- $D_x \ln(x) = 1/x$. In particular, $\ln(x)$ is an increasing function.

The log base 10 The natural logarithm Further properties of the natural logarithm The graph of the log function The number e, Eu

Problem

Show that $.5 \leqslant ln(2) \leqslant 1$.

Theorem

- ln(ab) = ln(a) + ln(b), a, b > 0.
- ln(a/b) = ln(a) ln(b).
- $ln(a^p) = p ln(a)$, for p > 0, p rational. (Homework)

The log base 10 The natural logarithm Further properties of the natural logarithm The graph of the log function The number e, Eu

Theorem

- In is increasing and concave down.
- As $x \to +\infty$, $\ln(x) \to +\infty$.
- As $x \to 0^+$, $\ln(x) \to -\infty$.

Problem

Sketch the graph of ln(x).

Definition

• The range of ln(x) is $(-\infty, \infty)$. Since ln(x) is increasing, there exists a unique number e such that

$$ln(e) = 1.$$

• The number e is called *Euler's number*. Note that

$$e \approx 2.71828$$

• Since ln(e) = 1, e is called the *base* of the natural logarithm function.

The log base 10 The natural logarithm Further properties of the natural logarithm The graph of the log function The number e, Eu

Theorem (The chain rule)

If f is a positive, differentiable function, then

$$\frac{d}{dx}\ln f(x) = \frac{1}{f(x)}f'(x).$$

Problem

Find $\frac{dy}{dx}$ in each case:

- $y = \ln(x^2)$
- $y = \ln(|x|)$

The log base 10 The natural logarithm Further properties of the natural logarithm The graph of the log function The number e, Eu

Theorem

$$\int \frac{1}{x} dx = \ln(|x|) + C.$$

Problem

Evaluate the following integrals:

- $\int \frac{x^2}{x^3+1} dx$
- $\int_{-3}^{-2} (x+1)^{-1} dx$
- $\int \tan(x) dx$

The log base 10 The natural logarithm Further properties of the natural logarithm The graph of the log function The number e, Eu

Problem

Use the logarithm function and its properties to evaluate the derivative of

$$f(x) = \frac{x^2(x-4)^3}{(x^2+1)^4}.$$