§6.3*–The Natural Exponential Function

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Outline

The definition of the exponential function

exp is an exponential function

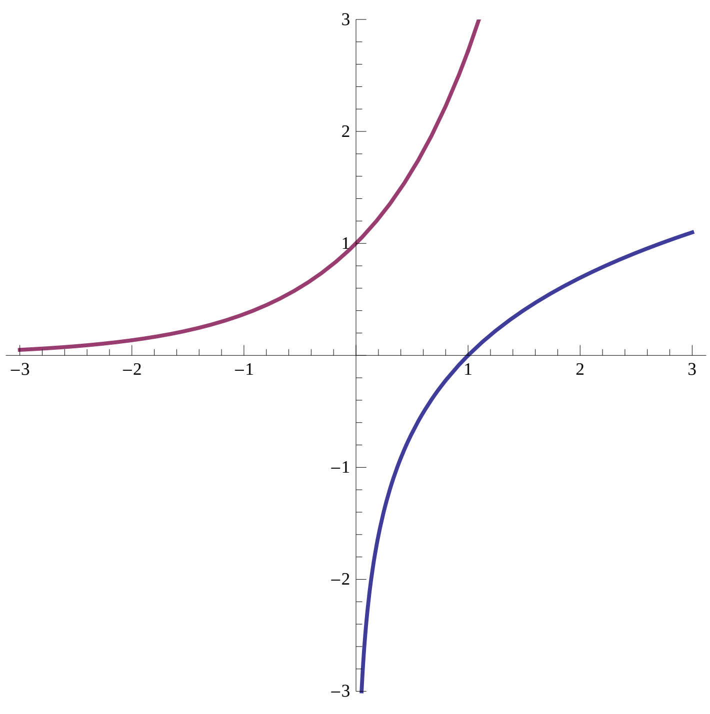
The derivative of e^x

Definition

- You will recall that ln, the natural logarithm, is an increasing function with domain $(0, \infty)$ and range $\mathbb{R}$. In particular, ln is invertible.
- Let exp denote the inverse of ln; thus, exp has domain $\mathbb{R}$ and range $(0, \infty)$.

The graph of exp

The graph of $\exp(x)$ can be obtained directly from the graph of $ln(x)$.



Some elementary properties of exp

- We have the important inverse relationships:

$$\ln\big(\exp(x)\big) = x \quad \text{and} \quad \exp\big(\ln(x)\big) = x.$$

- Since $\ln(1) = 0$, it follows that

$$\exp(0) = 1.$$

Euler's number, e

Recall the definition of e : e is the number such that $\ln(e) = 1$.
($e \approx 2.71828$)

Theorem

$\exp(x) = e^x$ *for all rational numbers* x.

Definition

- Because $\exp(r) = e^r$ for all rational numbers, we will define e^x by $\exp(x)$ for all real numbers x.
- In particular,

$$\ln e^x = x \quad \text{and} \quad e^{\ln(x)} = x.$$

Theorem (The laws of exponents)

- $e^{x+y} = e^x e^y$
- $e^{x-y} = e^x / e^y$ *(for homework)*
- $\left(e^x\right)^r = e^{rx}$ *(for homework)*

Problem

Solve the following equations:

- $e^{2x-3} = 8$
- $e^x + 2 = 8e^{-x}$

Theorem

- $\frac{d}{dx}e^x = e^x$
- $\int e^x dx = e^x + C.$

Problem

Find y' in each case:

- $y = e^{x^2}$
- $y = x^3 \exp\left(\dfrac{x+1}{x+2}\right)$
- $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Problem

Solve the following integrals:

- $\displaystyle\int \frac{e^x}{e^x + 1} dx$
- $\displaystyle\int x e^{x^2} dx$
- $\displaystyle\int_0^3 e^{x+5} dx$

Problem

Sketch the graph of $y = xe^{-x}$.