Assignment #3 **Name** Answer Key

Due 30 January 2015

1. Solve the equations or evaluate some integrals?

2. The half-life of Beryllium-11 (^{11}Be) is 13.81 seconds.

   (a) How much of a 150g sample of Beryllium-11 sample remains after 10 seconds?

$$k = \frac{\ln(.5)}{13.81} \cong -.0519$$

$$A(10) = 150\, e^{(-.0519)(10)} = 90.8\text{ g.}$$

   (b) How long will it take for a sample of Beryllium-11 to be reduced to one-third of its original mass?

$$50\text{g} = 150\, e^{(-.0519)(t)}$$

$$\frac{1}{3} = e^{(-.0519)(t)}$$

$$\ln\left(\frac{1}{3}\right) = (-.0519)t$$

$$t = \frac{\ln(1/3)}{-.0519}$$

$$t \approx 21.2\text{ secs.}$$

3. You invest \$500.00 into an account paying an annual interest rate of 6%. Calculate how much money is in the account after 5 years under each of the following compounding schemes:

   (a) the interest is compounded monthly;

$$A(5) = 500\left(1 + \frac{.06}{12}\right)^{(12)(5)} \cong 674.43.$$

   (b) the interest is compounded daily;

$$A(5) = 500\left(1 + \frac{.06}{365}\right)^{\overbrace{(365)(5)}^{1825}} \cong 674.91$$

   (c) the interest is compounded continuously.

$$A(5) = 500\, e^{(.06)(5)} \cong 674.93.$$

4. Use properties of logs and exponential functions to evaluate the following:

(a) $\log_2(\sqrt{32}) + \log_2(16^{2/3})$ $= \log_2(2^{5/2}) + \log_2(2^{8/3})$

$$= \frac{5}{2} + \frac{8}{3} = \frac{31}{6}$$

(b) $100^{\log_{10}(3)}$ $= (10^2)^{\log_{10}(3)} = 10^{2\log_{10}(3)}$

$$= 10^{\log_{10}(9)}$$

$$= 9.$$

5. Evaluate $\displaystyle\int_0^1 \frac{4^x}{4^x+1}dx.$ $\displaystyle = \int_2^5 \frac{1}{\ln(4)} \frac{du}{u} = \frac{1}{\ln(4)} \ln(|u|)\Big|_1^5$

$u = 4^x + 1$

$du = 4^x \ln(4)\, dx$

$$= \frac{\ln(5)}{\ln(4)} - \frac{\ln(1)}{\ln(4)}$$

$$= \frac{\ln(5)}{\ln(4)} \text{ or}$$

$$= \log_4(5).$$

6. Rewrite the function $f(x) = x^{\ln(x)}$ and evaluate $f'(x)$.

$$f(x) = e^{\ln(x^{\ln(x)})} = e^{\ln(x)\ln(x)} = e^{(\ln(x))^2}$$

$$\text{thus}\quad f'(x) = e^{(\ln x)^2} \cdot 2(\ln(x))^1 \cdot \frac{1}{x}.$$

or

$$= \frac{2\ln(x)}{x} \cdot x^{\ln(x)}$$