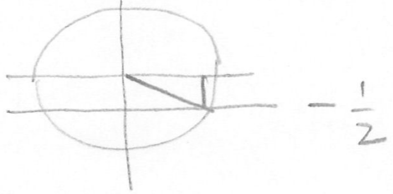
**Assignment #4** Name Answer Key

**Due 9 February 2010**

1. Find the exact values in each case:

   (a) $\sin^{-1}(-1/2) = -\frac{\pi}{6}.$




   (b) $\sin^{-1}(\sin(20\pi/3)) = \pi/3.$

$$\frac{20\pi}{3} = \frac{18\pi}{3} + \frac{2\pi}{3} = 6\pi + \frac{2\pi}{3}$$

   (c) $\tan(\tan^{-1}(16)) = 16.$

   (d) $\cos\left(\underbrace{\sin^{-1}(.3)}_{\alpha} + \underbrace{\cos^{-1}(.2)}_{\beta}\right) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$\left.\begin{aligned} \cos\alpha &= \sqrt{1-(.3)^2} = \sqrt{.91} \\ \cos\beta &= .2 \\ \sin\alpha &= .3 \\ \sin\beta &= \sqrt{1-(.2)^2} = \sqrt{.96} \end{aligned}\right\} = .2\sqrt{.91} - .3\sqrt{.96}$$

2. Evaluate $\lim_{x\to+\infty} e^{-2x}\sinh(x)$.

$$= \lim_{x\to+\infty} e^{-2x}\left(\frac{e^x - e^{-x}}{2}\right) = \lim_{x\to+\infty}\left(\frac{e^{-x} - e^{-3x}}{2}\right)$$

$$= 0.$$

3. Evaluate the following integrals: $u = e^x \quad du = e^x\,dx$

(a) $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$

1

$$= \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C$$
$$= \sin^{-1}(e^x) + C.$$

(b) $\displaystyle\int_0^3 \frac{1}{9+x^2} = \int_0^3 \frac{1}{9}\,\frac{1}{1+\left(\frac{x}{3}\right)^2}\,dx$ $\quad u = \frac{x}{3} \quad du = \frac{dx}{3}$

1

$$= \frac{1}{3}\int_0^1 \frac{1}{1+u^2}\,du = \frac{1}{3}\tan^{-1}(u)\Big|_0^1 = \frac{1}{3}\left(\frac{\pi}{4}\right) = \frac{\pi}{12}.$$

4. Let $y = \tan^{-1}(x) + \ln\left(\sqrt{\dfrac{x-1}{x+1}}\right)$. Show that $y' = \dfrac{2x^2}{x^4-1}$.

2

$$y = \tan^{-1}(x) + \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x+1)$$
$$y' = \frac{1}{1+x^2} + \frac{1}{2}\,\frac{1}{x-1} - \frac{1}{2}\,\frac{1}{x+1} = \frac{1}{1+x^2} + \frac{1}{x^2-1}$$
$$= \frac{(x^2-1)+(x^2+1)}{(x^2+1)(x^2-1)} = \frac{2x^2}{x^4-1}.$$

5. Show that $\dfrac{1+\tanh(x)}{1-\tanh(x)} = e^{2x}$.

1

$$\frac{1+\frac{\sinh(x)}{\cosh(x)}}{1-\frac{\sinh(x)}{\cosh(x)}} = \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)} = \frac{\frac{e^x+e^{-x}}{\cancel{2}} + \frac{e^x-e^{-x}}{\cancel{2}}}{\frac{e^x+e^{-x}}{\cancel{2}} - \left(\frac{e^x-e^{-x}}{\cancel{2}}\right)}$$
$$= \frac{\cancel{2}e^x}{\cancel{2}e^{-x}} = e^{2x}.$$