§11.4–The Comparison Tests

Tom Lewis

Spring Semester
2015

Outline

The setting for the comparison tests

- In this section we will only consider series with positive terms.
- Recall that a series of positive terms is convergent if and only if it is bounded above. This simple idea is the basis for the comparison tests of this section.

The main idea

The idea behind the comparison test is this: given a series $\sum a_n$ construct a reference series $\sum b_n$ in such a way that the convergence or divergence of $\sum a_n$ can be inferred from that of $\sum b_n$.

Theorem (Simple Comparison Test (SCT))

Suppose that $0 \leqslant a_n \leqslant b_n$ *for* $n \geqslant 1$.

- *If* $\sum b_n$ *converges, then* $\sum a_n$ *converges.*
- *If* $\sum a_n$ *diverges, then* $\sum b_n$ *diverges.*

Problem

In each case, determine the convergence or divergence of the following series:

- $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n+2)}{n}$
- $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+9}$
- $\displaystyle\sum_{n=1}^{\infty} \frac{2-\sin(n)}{n}$

Theorem (Limit Comparison Test (LCT))

Let $\sum a_n$ and $\sum b_n$ be series consisting of positive numbers with $a_n/b_n \to c$. If $0 < c < \infty$, then either both series converge or both series diverge.

Problem

In each case, determine the convergence or divergence of the following series:

- $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$
- $\displaystyle\sum_{n=1}^{\infty} \frac{2n+4}{\sqrt{n^4+2}}$
- $\displaystyle\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n^2+9}$