Assignment #6 **Name** Answer Key

Due 23 February 2015

1. Evaluate the following integrals:

2 (a) $\int_0^{\pi/4} \tan^5(x)\sec^3(x)dx$ $= \int_0^{\pi/4} (\tan^2 x)^2 \sec^2 x \ \sec(x)\tan(x)\,dx$

$$= \int_0^{\pi/4} (\sec^2(x) - 1)^2 \sec^2 x \ \sec(x)\tan(x)\,dx$$

Let $u = \sec(x)$

$du = \sec(x)\tan(x)\,dx$

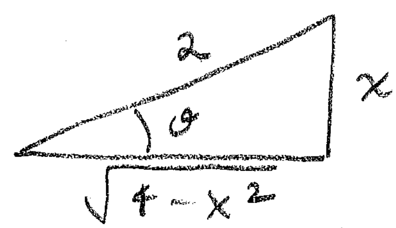
$$= \int_1^{\sqrt{2}} (u^2 - 1)^2 u^2\,du = \int_1^{\sqrt{2}} (u^6 - 2u^4 + u^2)\,du$$

$$= \left.\frac{u^7}{7} - 2\frac{u^5}{5} + \frac{u^3}{3}\right|_1^{\sqrt{2}} = \frac{22\sqrt{2} - 8}{105}$$

3 (b) $\int \frac{1}{x^2\sqrt{4-x^2}}dx$ Let $x = 2\sin(\theta)$ $dx = 2\cos(\theta)\,d\theta$

$$= \int \frac{\cancel{2\cos(\theta)}\,d\theta}{4\sin^2(\theta)\,\cancel{2\cos(\theta)}} = \frac{1}{4}\int \csc^2(\theta)\,d\theta$$

$$= -\frac{1}{4}\cot(\theta) + C$$



$$= -\frac{\sqrt{4-x^2}}{4x} + C$$

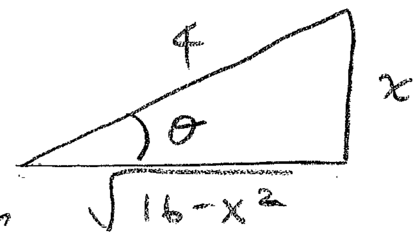
$$\cot(\theta) = \frac{\sqrt{4-x^2}}{x}$$

2 (c) $\int \sqrt{16-x^2}dx$ $\qquad x = 4\sin(\theta) \qquad dx = 4\cos(\theta)\,d\theta$

$$= \int 4\cos(\theta)\, 4\cos(\theta)\, d\theta = 16 \int \cos^2(\theta)\, d\theta.$$

$$= 16 \int \frac{1+\cos(2\theta)}{2} d\theta = 16\left(\frac{1}{2}\theta + \frac{\sin(2\theta)}{4}\right) + C$$

$$= 8\theta + 8\cos(\theta)\sin(\theta) + C$$



$$= 8\sin^{-1}\left(\frac{x}{4}\right) + 8\left(\frac{x}{4}\right)\left(\frac{\sqrt{16-x^2}}{4}\right) + C$$

$$= 8\sin^{-1}\left(\frac{x}{4}\right) + \frac{1}{2}x\sqrt{16-x^2} + C$$

3 (d) $\int_0^3 \frac{1}{(x^2+9)^{5/2}}dx$ $\qquad x = 3\tan(\theta) \qquad dx = 3\sec^2(\theta)\,d\theta.$

$$\int_0^{\pi/4} \frac{1}{3^5 \sec^5(\theta)} 3\sec^2(\theta)\, d\theta = \frac{1}{81}\int_0^{\pi/4} \cos^3(\theta)\, d\theta$$

$$= \frac{1}{81}\int_0^{\pi/4} (1-\sin^2(\theta))\cos(\theta)\, d\theta. \qquad u = \sin(\theta) \quad du = \cos(\theta).$$

$$= \frac{1}{81}\int_0^{\sqrt{2}/2} (1-u^2)\, du = \frac{1}{81}\left(u - \frac{u^3}{3}\right)\Big|_0^{\sqrt{2}/2}$$

$$= \frac{5\sqrt{2}}{972}$$