§11.5–Alternating Series

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Outline

Problem

Study the partial sums of the series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots$$

Definition

Let $\{a_n\}$ be a sequence of positive numbers. Infinite series of the form

$$\sum_{n=1}^{\infty}(-1)^{n+1}a_n = a_1 - a_2 + a_3 - a_4 + \cdots$$

$$\sum_{n=1}^{\infty}(-1)^{n}a_n = -a_1 + a_2 - a_3 + a_4 - \cdots$$

are called *alternating series*.

Disclaimer

We will concentrate on alternating series of the form $a_1 - a_2 + a_3 - \cdots$. All of our results apply to the series $-a_1 + a_2 - a_3 + \cdots$ as well.

Theorem (Alternating Series Test (AST))

Let $\{a_n\}$ be a decreasing sequence of positive terms. If $a_n \to 0$, then the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

converges.

Problem

Show that $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n-1}$ *converges by the alternating series test.*

Problem

Show that the conditions of the alternating series test do not apply to the series $\sum_{n=1}^{\infty}(-1)^n n^4 e^{-n}$*. Show that we can nonetheless snatch victory from the jaws of defeat.*

Theorem

Let $\{a_n\}$ be a decreasing sequence of positive terms with $a_n \to 0$ and let

$$S = \sum_{n=1}^{\infty} (-1)^{n+1} a_n.$$

Then

$$\left| S - \sum_{n=1}^{N} (-1)^{n+1} a_n \right| \leqslant a_{N+1}$$

In other words...

The error in using a partial sum to approximate the full sum is no greater than the size of the next term of the series.

Problem

Let $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n!}$. Compute $s_6 = \sum_{n=1}^{6} (-1)^{n+1} \frac{1}{n!}$ and estimate the error in the approximation $S \approx s_6$.