CHAPTER 5 REVIEW ■ PAGE 378

Exercises

1. $\frac{8}{3}$ **3.** $\frac{7}{12}$ **5.** $\frac{4}{3} + 4/\pi$ **7.** $64\pi/15$ **9.** $1656\pi/5$

11. $\frac{4}{3}\pi(2ah + h^2)^{3/2}$ **13.** $\int_{-\pi/3}^{\pi/3} 2\pi\left(\pi/2 - x\right)\left(\cos^2 x - \frac{1}{4}\right) dx$

15. (a) $2\pi/15$ (b) $\pi/6$ (c) $8\pi/15$

17. (a) 0.38 (b) 0.87

19. Solid obtained by rotating the region $0 \leqslant y \leqslant \cos x$, $0 \leqslant x \leqslant \pi/2$ about the y-axis

21. Solid obtained by rotating the region $0 \leqslant x \leqslant \pi$, $0 \leqslant y \leqslant 2 - \sin x$ about the x-axis

23. 36 **25.** $\frac{125}{3}\sqrt{3}$ m^3 **27.** 3.2 J

29. (a) $8000\pi/3 \approx 8378$ ft-lb (b) 2.1 ft

31. $f(x)$

PROBLEMS PLUS ■ PAGE 380

1. (a) $f(t) = 3t^2$ (b) $f(x) = \sqrt{2x/\pi}$ **3.** $\frac{32}{27}$

5. (b) 0.2261 (c) 0.6736 m

(d) (i) $1/(105\pi) \approx 0.003$ in/s (ii) $370\pi/3$ s ≈ 6.5 min

9. $y = \frac{32}{9}x^2$

11. (a) $V = \int_0^h \pi[f(y)]^2\, dy$

(c) $f(y) = \sqrt{kA/(\pi C)}\, y^{1/4}$. Advantage: the markings on the container are equally spaced.

13. $b = 2a$

CHAPTER 6

EXERCISES 6.1 ■ PAGE 390

1. (a) See Definition 1.

(b) It must pass the Horizontal Line Test.

3. No **5.** No **7.** Yes **9.** No **11.** Yes **13.** No

15. No **17.** (a) 6 (b) 3 **19.** 4

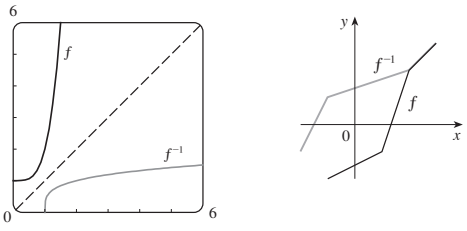
21. $F = \frac{9}{5}C + 32$; the Fahrenheit temperature as a function of the Celsius temperature; $[-273.15, \infty)$

23. $f^{-1}(x) = \frac{3}{2} - \frac{1}{2}x$ **25.** $y = \frac{1}{3}(x - 1)^2 - \frac{2}{3}$, $x \geqslant 1$

27. $y = \left(\frac{1 - x}{1 + x}\right)^2$, $-1 < x \leqslant 1$

29. $f^{-1}(x) = \sqrt[4]{x - 1}$

31.

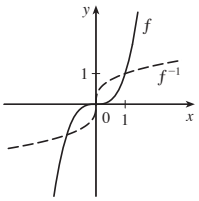


33. (a) $f^{-1}(x) = \sqrt{1 - x^2}$, $0 \leqslant x \leqslant 1$; f^{-1} and f are the same function. (b) Quarter-circle in the first quadrant

35. (b) $\frac{1}{12}$

(c) $f^{-1}(x) = \sqrt[3]{x}$, domain $= \mathbb{R} =$ range

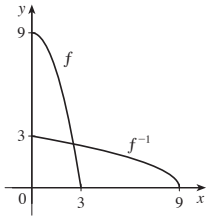
(e)



37. (b) $-\frac{1}{2}$

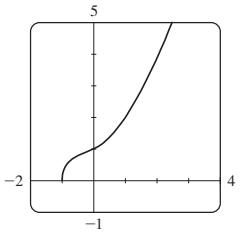
(c) $f^{-1}(x) = \sqrt{9 - x}$, domain $= [0, 9]$, range $= [0, 3]$

(e)



39. $\frac{1}{7}$ **41.** $2/\pi$ **43.** $\frac{3}{2}$ **45.** $1/\sqrt{28}$

47.



The graph passes the Horizontal Line Test.

$f^{-1}(x) = -\frac{1}{6}\sqrt[3]{4}\left(\sqrt[3]{D - 27x^2 + 20} - \sqrt[3]{D + 27x^2 - 20} + \sqrt[3]{2}\right)$, where $D = 3\sqrt{3}\sqrt{27x^4 - 40x^2 + 16}$; two of the expressions are complex.

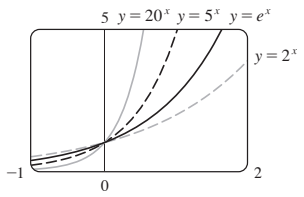
49. (a) $g^{-1}(x) = f^{-1}(x) - c$ (b) $h^{-1}(x) = (1/c)f^{-1}(x)$

EXERCISES 6.2 ■ PAGE 401

1. (a) $f(x) = a^x$, $a > 0$ (b) $\mathbb{R}$ (c) $(0, \infty)$

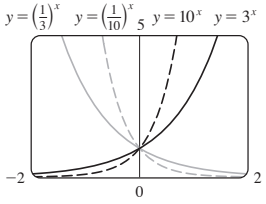
(d) See Figures 6(c), 6(b), and 6(a), respectively.

3.



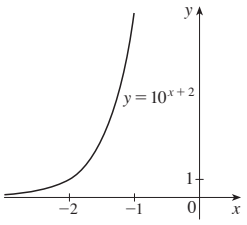
All approach 0 as $x \to -\infty$, all pass through $(0, 1)$, and all are increasing. The larger the base, the faster the rate of increase.

5.

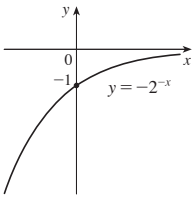


The functions with base greater than 1 are increasing and those with base less than 1 are decreasing. The latter are reflections of the former about the y-axis.

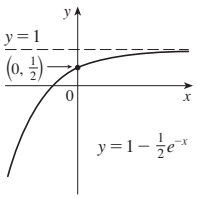
7.



9.



11.



13. (a) $y = e^x - 2$ (b) $y = e^{x-2}$ (c) $y = -e^x$
(d) $y = e^{-x}$ (e) $y = -e^{-x}$

15. (a) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ (b) $(-\infty, \infty)$

17. $f(x) = 3 \cdot 2^x$ **21.** At $x \approx 35.8$ **23.** ∞ **25.** 1

27. 0 **29.** 0 **31.** $f'(x) = 0$

33. $f'(x) = e^x(x^3 + 3x^2 + 2x + 2)$ **35.** $y' = 3ax^2e^{ax^3}$

37. $y' = e^{-kx}(-kx + 1)$ **39.** $f'(u) = (-1/u^2)e^{1/u}$

41. $F'(t) = e^{t \sin 2t}(2t \cos 2t + \sin 2t)$ **43.** $y' = \dfrac{3e^{3x}}{\sqrt{1 + 2e^{3x}}}$

45. $y' = e^{e^x}e^x$ **47.** $y' = \dfrac{(ad - bc)e^x}{(ce^x + d)^2}$

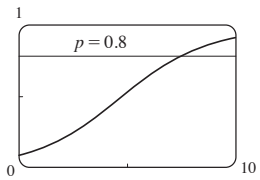
49. $y' = \dfrac{4e^{2x}}{(1 + e^{2x})^2} \sin \dfrac{1 - e^{2x}}{1 + e^{2x}}$ **51.** $y = 2x + 1$

53. $y' = \dfrac{y(y - e^{x/y})}{y^2 - xe^{x/y}}$ **57.** $-4, -2$ **59.** $f^{(n)}(x) = 2^n e^{2x}$

61. (b) -0.567143

63. (a) 1 (b) $kae^{-kt}/(1 + ae^{-kt})^2$

(c)

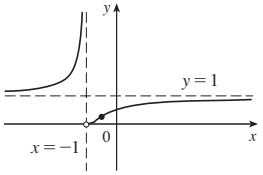


$t \approx 7.4$ h

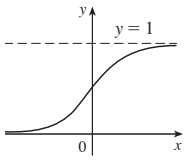
65. -1 **67.** $f(2) = 2/\sqrt{e}, f(-1) = -1/\sqrt[8]{e}$

69. (a) Inc. on $(2, \infty)$; dec. on $(-\infty, 2)$
(b) CU on $(-\infty, 3)$; CD on $(3, \infty)$ (c) $(3, -2e^{-3})$

71. A. $\{x \mid x \neq -1\}$
B. y-int. $1/e$ C. None
D. HA $y = 1$; VA $x = -1$
E. Inc. on $(-\infty, -1), (-1, \infty)$
F. None
G. CU on $(-\infty, -1), \left(-1, -\frac{1}{2}\right)$; CD on $\left(-\frac{1}{2}, \infty\right)$; IP $\left(-\frac{1}{2}, 1/e^2\right)$
H. See graph at right.

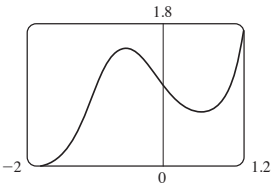


73. A. $\mathbb{R}$ B. y-int. $\frac{1}{2}$ C. None
D. HA $y = 0, y = 1$
E. Inc on $\mathbb{R}$ F. None
G. CU on $(-\infty, 0)$; CD on $(0, \infty)$; IP $\left(0, \frac{1}{2}\right)$ H. See graph at right.



75. 28.57 min, when the rate of increase of drug level in the bloodstream is greatest; 85.71 min, when rate of decrease is greatest

77. Loc. max. $f\left(-1/\sqrt{3}\right) = e^{2\sqrt{3}/9} \approx 1.5$;
loc. min. $f\left(1/\sqrt{3}\right) = e^{-2\sqrt{3}/9} \approx 0.7$;
IP $(-0.15, 1.15), (-1.09, 0.82)$



79. $\dfrac{1}{e + 1} + e - 1$ **81.** $\dfrac{1}{\pi}(1 - e^{-2\pi})$

83. $\frac{2}{3}(1 + e^x)^{3/2} + C$ **85.** $\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + C$

87. $e^{\tan x} + C$ **89.** $e - \sqrt{e}$ **91.** 4.644 **93.** $\pi(e^2 - 1)/2$

97. ≈ 4512 L **99.** $\frac{1}{2}$

EXERCISES 6.3 ■ PAGE 408

1. (a) It's defined as the inverse of the exponential function with base a, that is, $\log_a x = y \iff a^y = x$.
(b) $(0, \infty)$ (c) $\mathbb{R}$ (d) See Figure 1.

3. (a) 3 (b) -3 **5.** (a) 4.5 (b) -4

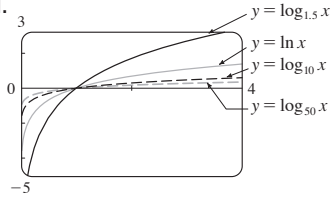
7. (a) 3 (b) -2

9. $\frac{1}{2} \ln a + \frac{1}{2} \ln b$ **11.** $2 \ln x - 3 \ln y - 4 \ln z$

13. $\ln \dfrac{x^2y^3}{z}$ **15.** $\ln 1215$ **17.** $\ln \dfrac{\sqrt{x}}{x + 1}$

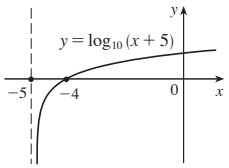
19. (a) 0.402430 (b) 1.454240 (c) 1.651496

21.

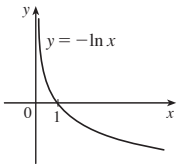


All graphs approach $-\infty$ as $x \to 0^+$, all pass through $(1, 0)$, and all are increasing. The larger the base, the slower the rate of increase.

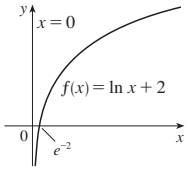
23. (a)



(b)



25. (a) $(0, \infty)$; $(-\infty, \infty)$ (b) e^{-2} (c)



27. (a) $\frac{1}{4}(7 - \ln 6)$ (b) $\frac{1}{3}(e^2 + 10)$

29. (a) $5 + \log_2 3$ or $5 + (\ln 3)/\ln 2$ (b) $\frac{1}{2}\left(1 + \sqrt{1 + 4e}\right)$

31. $-\frac{1}{2} \ln(e - 1)$ **33.** e^e **35.** $\ln 3$

37. (a) 0.5210 (b) 3.0949

39. (a) $0 < x < 1$ (b) $x > \ln 5$

41. About 1,084,588 mi **43.** 8.3

45. (a) $f^{-1}(n) = (3/\ln 2)\ln(n/100)$; the time elapsed when there are n bacteria (b) After about 26.9 hours

47. $-\infty$ **49.** 0 **51.** ∞ **53.** $(-\infty, -3) \cup (3, \infty)$

55. (a) $\left(-\infty, \frac{1}{2}\ln 3\right]$ (b) $f^{-1}(x) = \frac{1}{2}\ln(3 - x^2)$, $\left[0, \sqrt{3}\right)$

57. (a) $(\ln 3, \infty)$ (b) $f^{-1}(x) = \ln(e^x + 3)$; $\mathbb{R}$

59. $y = e^x - 3$ **61.** $f^{-1}(x) = \sqrt[3]{\ln x}$ **63.** $y = \dfrac{1}{10^x - 1}$

65. $\left(-\frac{1}{2}\ln 3, \infty\right)$

67. (b) $f^{-1}(x) = \frac{1}{2}(e^x - e^{-x})$ **69.** f is a constant function

73. $-1 \leqslant x < 1 - \sqrt{3}$ or $1 + \sqrt{3} < x \leqslant 3$

EXERCISES 6.4 ■ PAGE 418

1. The differentiation formula is simplest.

3. $f'(x) = \dfrac{\cos(\ln x)}{x}$ **5.** $f'(x) = -\dfrac{1}{x}$

7. $f'(x) = \dfrac{3x^2}{(x^3 + 1)\ln 10}$ **9.** $f'(x) = \dfrac{\sin x}{x} + \cos x \ln(5x)$

11. $G'(y) = \dfrac{10}{2y + 1} - \dfrac{y}{y^2 + 1}$ **13.** $g'(x) = \dfrac{2x^2 - 1}{x(x^2 - 1)}$

15. $f'(u) = \dfrac{1 + \ln 2}{u[1 + \ln(2u)]^2}$ **17.** $f'(x) = 5x^4 + 5^x \ln 5$

19. $y' = \sec^2(\ln(ax + b))\dfrac{a}{ax + b}$ **21.** $y' = \dfrac{-x}{1 + x}$

23. $y' = \dfrac{1}{\ln 10} + \log_{10} x$ **25.** $f'(t) = 10^{\sqrt{t}} \ln 10/\left(2\sqrt{t}\right)$

27. $y' = x + 2x\ln(2x)$; $y'' = 3 + 2\ln(2x)$

29. $y' = \dfrac{1}{\sqrt{1 + x^2}}$; $y'' = \dfrac{-x}{(1 + x^2)^{3/2}}$

31. $f'(x) = \dfrac{2x - 1 - (x - 1)\ln(x - 1)}{(x - 1)[1 - \ln(x - 1)]^2}$;
$(1, 1 + e) \cup (1 + e, \infty)$

33. $f'(x) = \dfrac{2(x - 1)}{x(x - 2)}$; $(-\infty, 0) \cup (2, \infty)$ **35.** $\frac{1}{2}$

37. $y = 3x - 9$ **39.** $\cos x + 1/x$ **41.** 7

43. $y' = (x^2 + 2)^2(x^4 + 4)^4\left(\dfrac{4x}{x^2 + 2} + \dfrac{16x^3}{x^4 + 4}\right)$

45. $y' = \sqrt{\dfrac{x - 1}{x^4 + 1}}\left(\dfrac{1}{2x - 2} - \dfrac{2x^3}{x^4 + 1}\right)$

47. $y' = x^x(1 + \ln x)$

49. $y' = x^{\sin x}\left(\dfrac{\sin x}{x} + \cos x \ln x\right)$

51. $y' = (\cos x)^x(-x\tan x + \ln\cos x)$

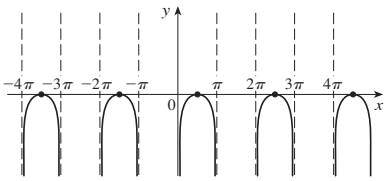
53. $y' = (\tan x)^{1/x}\left(\dfrac{\sec^2 x}{x\tan x} - \dfrac{\ln\tan x}{x^2}\right)$

55. $y' = \dfrac{2x}{x^2 + y^2 - 2y}$ **57.** $f^{(n)}(x) = \dfrac{(-1)^{n-1}(n - 1)!}{(x - 1)^n}$

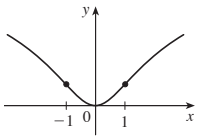
59. 2.958516, 5.290718

61. CU on $(e^{8/3}, \infty)$, CD on $(0, e^{8/3})$, IP $\left(e^{8/3}, \frac{8}{3}e^{-4/3}\right)$

63. A. All x in $(2n\pi, (2n + 1)\pi)$ (n an integer)
B. x-int $\pi/2 + 2n\pi$ C. Period 2π D. VA $x = n\pi$
E. Inc on $(2n\pi, \pi/2 + 2n\pi)$; dec on $(\pi/2 + 2n\pi, (2n + 1)\pi)$
F. Loc max $f(\pi/2 + 2n\pi) = 0$ G. CD on $(2n\pi, (2n + 1)\pi)$
H.



65. A. $\mathbb{R}$ B. y-int 0; x-int. 0
C. About y-axis D. None
E. Inc. on $(0, \infty)$;
dec. on $(-\infty, 0)$
F. Loc. min. $f(0) = 0$
G. CU on $(-1, 1)$; CD on
$(-\infty, -1)$, $(1, \infty)$;
IP $(\pm 1, \ln 2)$ H. See graph at right.



67. Inc. on (0, 2.7), (4.5, 8.2), (10.9, 14.3);
IP (3.8, 1.7), (5.7, 2.1), (10.0, 2.7), (12.0, 2.9)

69. (a) $Q = ab^t$ where $a \approx 100.01244$ and $b \approx 0.000045146$
(b) -670.63 μA

71. $3\ln 2$ **73.** $\frac{1}{3}\ln\frac{5}{2}$ **75.** $\frac{1}{2}e^2 + e - \frac{1}{2}$

77. $\frac{1}{3}(\ln x)^3 + C$ **79.** $-\ln(1 + \cos^2 x) + C$ **81.** $90/(\ln 10)$

85. $\pi\ln 2$ **87.** 45,974 J **89.** $\frac{1}{3}$

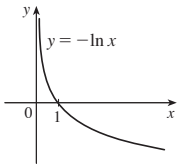
91. $0 < m < 1$, $m - 1 - \ln m$

EXERCISES 6.2* ■ PAGE 428

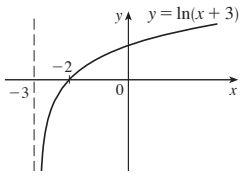
1. $\frac{1}{2}\ln a + \frac{1}{2}\ln b$ **3.** $2\ln x - 3\ln y - 4\ln z$

5. $\ln\dfrac{x^2y^3}{z}$ **7.** $\ln 1215$ **9.** $\ln\dfrac{\sqrt{x}}{x + 1}$

11.



13.



15. $-\infty$ **17.** $f'(x) = (2 + \ln x)/\left(2\sqrt{x}\right)$

19. $f'(x) = \dfrac{\cos(\ln x)}{x}$ **21.** $f'(x) = -\dfrac{1}{x}$

23. $f'(x) = \dfrac{\sin x}{x} + \cos x \ln(5x)$ **25.** $g'(x) = -\dfrac{2a}{a^2 - x^2}$

27. $G'(y) = \dfrac{10}{2y + 1} - \dfrac{y}{y^2 + 1}$ **29.** $g'(x) = \dfrac{2x^2 - 1}{x(x^2 - 1)}$

31. $f'(u) = \dfrac{1 + \ln 2}{u[1 + \ln(2u)]^2}$ **33.** $y' = \dfrac{10x + 1}{5x^2 + x - 2}$

35. $y' = \sec^2(\ln(ax + b))\dfrac{a}{ax + b}$

37. $y' = x + 2x\ln(2x)$; $y'' = 3 + 2\ln(2x)$

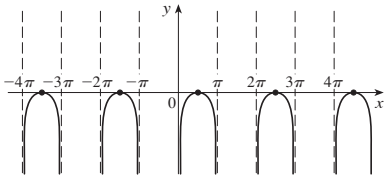
39. $f'(x) = \dfrac{2x - 1 - (x-1)\ln(x-1)}{(x-1)[1 - \ln(x-1)]^2}$; $(1, 1+e) \cup (1+e, \infty)$

41. $f'(x) = -\dfrac{1}{2x\sqrt{1 - \ln x}}$; $(0, e]$ **43.** $\frac{1}{2}$ **45.** $\cos x + 1/x$

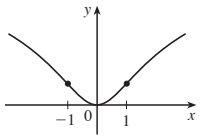
47. $y = 2x - 2$ **49.** $y' = \dfrac{2x}{x^2 + y^2 - 2y}$

51. $f^{(n)}(x) = \dfrac{(-1)^{n-1}(n-1)!}{(x-1)^n}$ **53.** 2.958516, 5.290718

55. A. All x in $(2n\pi, (2n+1)\pi)$ (n an integer)
B. x-int $\pi/2 + 2n\pi$ C. Period 2π D. VA $x = n\pi$
E. Inc on $(2n\pi, \pi/2 + 2n\pi)$; dec on $(\pi/2 + 2n\pi, (2n+1)\pi)$
F. Loc max $f(\pi/2 + 2n\pi) = 0$ G. CD on $(2n\pi, (2n+1)\pi)$
H.



57. A. $\mathbb{R}$ B. y-int 0; x-int. 0
C. About y-axis D. None
E. Inc. on $(0, \infty)$;
dec. on $(-\infty, 0)$
F. Loc. min. $f(0) = 0$
G. CU on $(-1, 1)$; CD on
$(-\infty, -1)$, $(1, \infty)$;
IP $(\pm 1, \ln 2)$ H. See graph at right.



59. Inc. on (0, 2.7), (4.5, 8.2), (10.9, 14.3);
IP (3.8, 1.7), (5.7, 2.1), (10.0, 2.7), (12.0, 2.9)

61. $y' = (x^2+2)^2(x^4+4)^4\left(\dfrac{4x}{x^2+2} + \dfrac{16x^3}{x^4+4}\right)$

63. $y' = \sqrt{\dfrac{x-1}{x^4+1}}\left(\dfrac{1}{2x-2} - \dfrac{2x^3}{x^4+1}\right)$

65. $3 \ln 2$ **67.** $\frac{1}{3}\ln\frac{5}{2}$ **69.** $\frac{1}{2}e^2 + e - \frac{1}{2}$

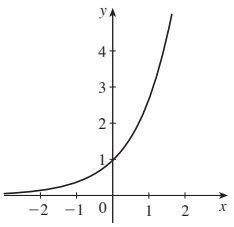
71. $\frac{1}{3}(\ln x)^3 + C$ **73.** $-\ln(1 + \cos^2 x) + C$

77. $\pi \ln 2$ **79.** 45,974 J **81.** $\frac{1}{3}$ **83.** (b) 0.405

87. $0 < m < 1$, $m - 1 - \ln m$

EXERCISES 6.3* ■ PAGE 434

1. If $f(x) = e^x$, then $f'(0) = 1$.



3. (a) $\frac{1}{25}$ (b) 10

5. (a) $\frac{1}{4}(7 - \ln 6)$ (b) $\frac{1}{3}(e^2 + 10)$

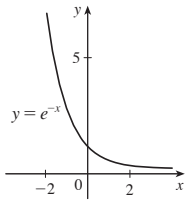
7. (a) $\frac{1}{3}(\ln k - 1)$ (b) $\frac{1}{2}\left(1 + \sqrt{1 + 4e}\right)$

9. $-\frac{1}{2}\ln(e-1)$ **11.** $\ln 3$

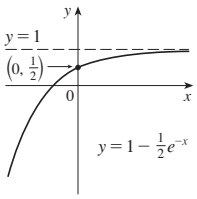
13. (a) 0.5210 (b) 3.0949

15. (a) $0 < x < 1$ (b) $x > \ln 5$

17.



19.



21. (a) $\left(-\infty, \frac{1}{2}\ln 3\right]$ (b) $f^{-1}(x) = \frac{1}{2}\ln(3 - x^2)$, $\left[0, \sqrt{3}\right)$

23. $y = e^x - 3$ **25.** $f^{-1}(x) = \sqrt[3]{\ln x}$ **27.** 1 **29.** 0

31. 0 **33.** $f'(x) = 0$ **35.** $f'(x) = e^x(x^3 + 3x^2 + 2x + 2)$

37. $y' = 3ax^2e^{ax^3}$ **39.** $y' = e^{-kx}(-kx + 1)$

41. $f'(u) = (-1/u^2)e^{1/u}$ **43.** $F'(t) = e^{t\sin 2t}(2t\cos 2t + \sin 2t)$

45. $y' = \dfrac{3e^{3x}}{\sqrt{1 + 2e^{3x}}}$ **47.** $y' = e^{e^x}e^x$ **49.** $y' = \dfrac{(ad - bc)e^x}{(ce^x + d)^2}$

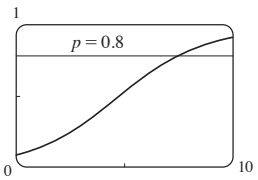
51. $y' = \dfrac{4e^{2x}}{(1 + e^{2x})^2}\sin\dfrac{1 - e^{2x}}{1 + e^{2x}}$ **53.** $y = 2x + 1$

55. $y' = \dfrac{y(y - e^{x/y})}{y^2 - xe^{x/y}}$ **59.** $-4, -2$ **61.** $f^{(n)}(x) = 2^n e^{2x}$

63. (b) -0.567143

65. (a) 1 (b) $kae^{-kt}/(1 + ae^{-kt})^2$

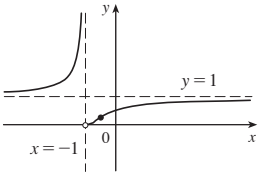
(c) $t \approx 7.4$ h



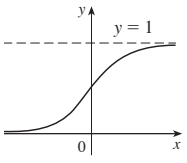
67. -1 **69.** $f(2) = 2/\sqrt{e}$, $f(-1) = -1/\sqrt[8]{e}$

71. (a) Inc. on $(2, \infty)$; dec. on $(-\infty, 2)$
(b) CU on $(-\infty, 3)$; CD on $(3, \infty)$ (c) $(3, -2e^{-3})$

73. A. $\{x \mid x \neq -1\}$
B. y-int. $1/e$ C. None
D. HA $y = 1$; VA $x = -1$
E. Inc. on $(-\infty, -1)$, $(-1, \infty)$
F. None
G. CU on $(-\infty, -1)$, $\left(-1, -\frac{1}{2}\right)$;
CD on $\left(-\frac{1}{2}, \infty\right)$; IP $\left(-\frac{1}{2}, 1/e^2\right)$
H. See graph at right.

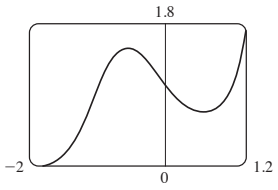


75. A. $\mathbb{R}$ B. y-int. $\frac{1}{2}$ C. None
D. HA $y = 0$, $y = 1$
E. Inc on $\mathbb{R}$ F. None
G. CU on $(-\infty, 0)$; CD on $(0, \infty)$;
IP $\left(0, \frac{1}{2}\right)$ H. See graph at right.



77. 28.57 min, when the rate of increase of drug level in the bloodstream is greatest; 85.71 min, when rate of decrease is greatest

79. Loc. max. $f(-1/\sqrt{3}) = e^{2\sqrt{3}/9} \approx 1.5$;
loc. min. $f(1/\sqrt{3}) = e^{-2\sqrt{3}/9} \approx 0.7$;
IP $(-0.15, 1.15)$, $(-1.09, 0.82)$



81. $\dfrac{1}{e+1} + e - 1$ **83.** $\dfrac{1}{\pi}(1 - e^{-2\pi})$

85. $\frac{2}{3}(1 + e^x)^{3/2} + C$ **87.** $\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + C$

89. $e^{\tan x} + C$ **91.** $e - \sqrt{e}$ **93.** 4.644 **95.** $\pi(e^2 - 1)/2$

99. ≈4512 L **101.** $\frac{1}{2}$

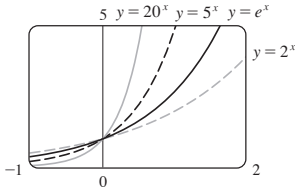
EXERCISES 6.4* ■ PAGE 444

1. (a) $a^x = e^{x \ln a}$ (b) $(-\infty, \infty)$ (c) $(0, \infty)$
(d) See Figures 1, 3, and 2.

3. $e^{-\pi \ln 4}$ **5.** $e^{x^2 \ln 10}$

7. (a) 3 (b) −3 **9.** (a) 3 (b) −2

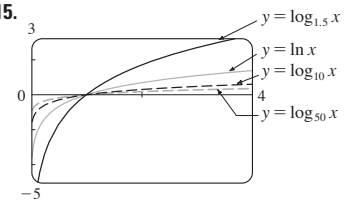
11.



All approach 0 as $x \to -\infty$, all pass through (0, 1), and all are increasing. The larger the base, the faster the rate of increase.

13. (a) 0.402430 (b) 1.454240 (c) 1.651496

15.



All graphs approach $-\infty$ as $x \to 0^+$, all pass through (1, 0), and all are increasing. The larger the base, the slower the rate of increase.

17. $f(x) = 3 \cdot 2^x$ **19.** (b) About 1,084,588 mi

21. ∞ **23.** 0 **25.** $f'(x) = 5x^4 + 5^x \ln 5$

27. $f'(t) = 10^{\sqrt{t}} \ln 10/(2\sqrt{t})$ **29.** $L'(v) = 2v \ln 4 \sec^2(4^{v^2}) \cdot 4^{v^2}$

31. $f'(x) = \dfrac{3}{(3x-1)\ln 2}$ **33.** $y' = \dfrac{1}{\ln 10} + \log_{10} x$

35. $y' = x^x(1 + \ln x)$ **37.** $y' = x^{\sin x}\left(\dfrac{\sin x}{x} + \cos x \ln x\right)$

39. $y' = (\cos x)^x(-x \tan x + \ln \cos x)$

41. $y' = (\tan x)^{1/x}\left(\dfrac{\sec^2 x}{x \tan x} - \dfrac{\ln \tan x}{x^2}\right)$

43. $y = (10 \ln 10)x + 10(1 - \ln 10)$ **45.** $90/(\ln 10)$

47. $(\ln x)^2/(2 \ln 10) + C$ $\left[\text{or } \frac{1}{2}(\ln 10)(\log_{10} x)^2 + C\right]$

49. $3^{\sin\theta}/\ln 3 + C$ **51.** $16/(5 \ln 5) - 1/(2 \ln 2)$

53. 0.600967 **55.** $y = \dfrac{1}{10^x - 1}$ **57.** 8.3

59. $10^8/\ln 10$ dB/(watt/m^2)

61. (a) $Q = ab^t$ where $a \approx 100.01244$ and $b \approx 0.000045146$
(b) −670.63 μA

EXERCISES 6.5 ■ PAGE 451

1. About 235

3. (a) $100(4.2)^t$ (b) ≈7409 (c) ≈10,632 bacteria/h
(d) $(\ln 100)/(\ln 4.2) \approx 3.2$ h

5. (a) 1508 million, 1871 million (b) 2161 million
(c) 3972 million; wars in the first half of century, increased life expectancy in second half

7. (a) $Ce^{-0.0005t}$ (b) $-2000 \ln 0.9 \approx 211$ s

9. (a) $100 \times 2^{-t/30}$ mg (b) ≈9.92 mg (c) ≈199.3 years

11. ≈2500 years **13.** (a) ≈137°F (b) ≈116 min

15. (a) 13.3°C (b) ≈67.74 min

17. (a) ≈64.5 kPa (b) ≈39.9 kPa

19. (a) (i) $3828.84 (ii) $3840.25 (iii) $3850.08
(iv) $3851.61 (v) $3852.01 (vi) $3852.08
(b) $dA/dt = 0.05A$, $A(0) = 3000$

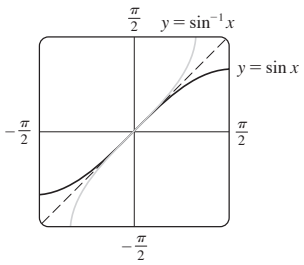
EXERCISES 6.6 ■ PAGE 459

1. (a) $\pi/6$ (b) π **3.** (a) $\pi/4$ (b) $\pi/4$

5. (a) 10 (b) $\pi/3$

7. $2/\sqrt{5}$ **9.** $\frac{2}{3}\sqrt{2}$ **13.** $x/\sqrt{1+x^2}$

15.



The second graph is the reflection of the first graph about the line $y = x$.

23. $y' = \dfrac{2\tan^{-1}x}{1+x^2}$ **25.** $y' = \dfrac{1}{\sqrt{-x^2 - x}}$ **27.** $y' = \sin^{-1}x$

29. $y' = -\dfrac{2e^{2x}}{\sqrt{1-e^{4x}}}$ **31.** $y' = -\dfrac{\sin\theta}{1+\cos^2\theta}$

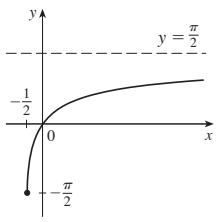
33. $h'(t) = 0$ **35.** $y' = \dfrac{\sqrt{a^2 - b^2}}{a + b\cos x}$

37. $g'(x) = \dfrac{2}{\sqrt{1-(3-2x)^2}}$; $[1, 2]$, $(1, 2)$ **39.** $\pi/6$

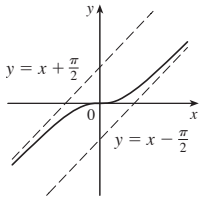
41. $1 - \dfrac{x \arcsin x}{\sqrt{1-x^2}}$ **43.** $-\pi/2$ **45.** $\pi/2$

47. At a distance $5 - 2\sqrt{5}$ from A **49.** $\frac{1}{4}$ rad/s

51. A. $\left[-\frac{1}{2}, \infty\right)$
B. y-int. 0; x-int. 0
C. None
D. HA $y = \pi/2$
E. Inc. on $\left(-\frac{1}{2}, \infty\right)$
F. None
G. CD on $\left(-\frac{1}{2}, \infty\right)$
H. See graph at right



53. A. $\mathbb{R}$
B. y-int. 0; x-int. 0
C. About $(0, 0)$
D. SA $y = x \pm \pi/2$
E. Inc. on $\mathbb{R}$ F. None
G. CU on $(0, \infty)$; CD on $(-\infty, 0)$; IP $(0, 0)$
H. See graph at right.



55. Max at $x = 0$, min at $x \approx \pm 0.87$, IP at $x \approx \pm 0.52$
57. $F(x) = \tan^{-1}x + x + C$ **59.** $4\pi/3$ **61.** $\pi^2/72$
63. $\tan^{-1}x + \frac{1}{2}\ln(1 + x^2) + C$ **65.** $\ln|\sin^{-1}x| + C$
67. $\frac{1}{3}\sin^{-1}(t^3) + C$ **69.** $2\tan^{-1}\sqrt{x} + C$ **73.** $\pi/2 - 1$

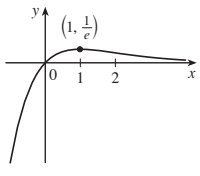
EXERCISES 6.7 ■ PAGE 467

1. (a) 0 (b) 1 **3.** (a) $\frac{3}{4}$ (b) $\frac{1}{2}(e^2 - e^{-2}) \approx 3.62686$
5. (a) 1 (b) 0
21. $\operatorname{sech} x = \frac{3}{5}$, $\sinh x = \frac{4}{3}$, $\operatorname{csch} x = \frac{3}{4}$, $\tanh x = \frac{4}{5}$, $\coth x = \frac{5}{4}$
23. (a) 1 (b) -1 (c) ∞ (d) $-\infty$ (e) 0 (f) 1 (g) ∞ (h) $-\infty$ (i) 0
31. $f'(x) = x\cosh x$ **33.** $h'(x) = \tanh x$
35. $y' = 3e^{\cosh 3x}\sinh 3x$ **37.** $f'(t) = -2e^t\operatorname{sech}^2(e^t)\tanh(e^t)$
39. $G'(x) = \dfrac{-2\sinh x}{(1 + \cosh x)^2}$ **41.** $y' = \dfrac{1}{2\sqrt{x(x-1)}}$
43. $y' = \sinh^{-1}(x/3)$ **45.** $y' = -\csc x$
51. (a) 0.3572 (b) 70.34°
53. (a) 164.50 m (b) 120 m; 164.13 m
55. (b) $y = 2\sinh 3x - 4\cosh 3x$
57. $\left(\ln(1 + \sqrt{2}), \sqrt{2}\right)$
59. $\frac{1}{3}\cosh^3 x + C$ **61.** $2\cosh\sqrt{x} + C$ **63.** $-\operatorname{csch} x + C$
65. $\ln\left(\dfrac{6 + 3\sqrt{3}}{4 + \sqrt{7}}\right)$ **67.** $\tanh^{-1}e^x + C$
69. (a) 0, 0.48 (b) 0.04

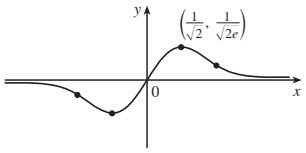
EXERCISES 6.8 ■ PAGE 477

1. (a) Indeterminate (b) 0 (c) 0
(d) ∞, $-\infty$, or does not exist (e) Indeterminate
3. (a) $-\infty$ (b) Indeterminate (c) ∞
5. $\frac{9}{4}$ **7.** 2 **9.** $-\frac{1}{3}$ **11.** $-\infty$ **13.** 2 **15.** $\frac{1}{4}$
17. 0 **19.** $-\infty$ **21.** $\frac{8}{5}$ **23.** 3 **25.** $\frac{1}{2}$ **27.** 1
29. 1 **31.** $1/\ln 3$ **33.** 0 **35.** $-1/\pi^2$ **37.** $\frac{1}{2}a(a-1)$
39. $\frac{1}{24}$ **41.** π **43.** 3 **45.** 0 **47.** $-2/\pi$ **49.** $\frac{1}{2}$
51. $\frac{1}{2}$ **53.** ∞ **55.** 1 **57.** e^{-2} **59.** $1/e$
61. 1 **63.** e^4 **65.** $1/\sqrt{e}$ **67.** e^2 **69.** $\frac{1}{4}$ **73.** 1

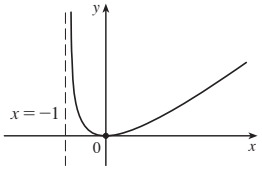
75. A. $\mathbb{R}$ B. y-int 0; x-int 0
C. None D. HA $y = 0$
E. Inc on $(-\infty, 1)$, dec on $(1, \infty)$
F. Loc max $f(1) = 1/e$
G. CU on $(2, \infty)$; CD on $(-\infty, 2)$
IP $(2, 2/e^2)$
H. See graph at right.



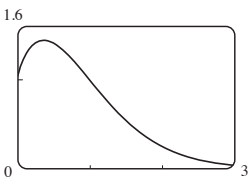
77. A. $\mathbb{R}$ B. y-int 0; x-int 0 C. About $(0, 0)$ D. HA $y = 0$
E. Inc on $(-1/\sqrt{2}, 1/\sqrt{2})$; dec on $(-\infty, -1/\sqrt{2}), (1/\sqrt{2}, \infty)$
F. Loc min $f(-1/\sqrt{2}) = -1/\sqrt{2e}$; loc max $f(1/\sqrt{2}) = 1/\sqrt{2e}$
G. CU on $(-\sqrt{3/2}, 0), (\sqrt{3/2}, \infty)$; CD on $(-\infty, -\sqrt{3/2}), (0, \sqrt{3/2})$;
IP $(\pm\sqrt{3/2}, \pm\sqrt{3/2}e^{-3/2}), (0, 0)$
H.



79. A. $(-1, \infty)$ B. y-int 0; x-int 0
C. None D. VA $x = -1$
E. Inc on $(0, \infty)$; dec on $(-1, 0)$
F. Loc min $f(0) = 0$
G. CU on $(-1, \infty)$
H. See graph at right.

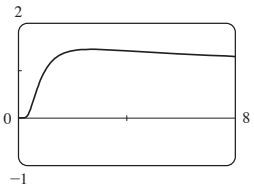


81. (a)



(b) $\lim_{x\to 0^+} x^{-x} = 1$
(c) Max value $f(1/e) = e^{1/e} \approx 1.44$ (d) 1.0

83. (a)



(b) $\lim_{x\to 0^+} x^{1/x} = 0$, $\lim_{x\to\infty} x^{1/x} = 1$
(c) Loc max $f(e) = e^{1/e}$ (d) IP at $x \approx 0.58, 4.37$
85. f has an absolute minimum for $c > 0$. As c increases, the minimum points get farther away from the origin.
91. $\pi/6$ **93.** $\frac{16}{9}a$ **95.** $\frac{1}{2}$ **97.** 56 **101.** (a) 0

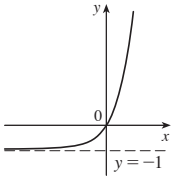
CHAPTER 6 REVIEW ■ PAGE 481

True-False Quiz

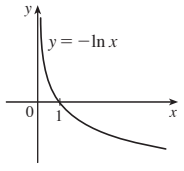
1. True **3.** False **5.** True **7.** True **9.** False
11. False **13.** False **15.** True **17.** True

Exercises

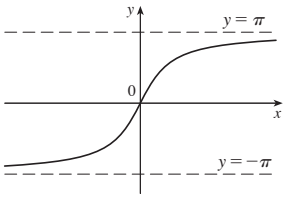
1. No **3.** (a) 7 (b) $\frac{1}{8}$
5.



7.



9.



11. (a) 9 (b) 2

13. $e^{1/3}$

15. $\ln \ln 17$ **17.** $\sqrt{1+e}$

19. $\tan 1$ **21.** $f'(t) = t + 2t \ln t$

23. $h'(\theta) = 2\sec^2(2\theta)e^{\tan 2\theta}$ **25.** $y' = 5 \sec 5x$

27. $y' = \dfrac{4x}{1+16x^2} + \tan^{-1}(4x)$ **29.** $y' = 2\tan x$

31. $y' = -\dfrac{e^{1/x}(1+2x)}{x^4}$ **33.** $y' = 3^{x \ln x}(\ln 3)(1 + \ln x)$

35. $H'(v) = \dfrac{v}{1+v^2} + \tan^{-1} v$

37. $y' = 2x^2 \cosh(x^2) + \sinh(x^2)$ **39.** $y' = \cot x - \sin x \cos x$

41. $y' = -(1/x)[1 + 1/(\ln x)^2]$

43. $y' = 3 \tanh 3x$ **45.** $y' = (\cosh x)/\sqrt{\sinh^2 x - 1}$

47. $y' = \dfrac{-3\sin\left(e^{\sqrt{\tan 3x}}\right)e^{\sqrt{\tan 3x}}\sec^2(3x)}{2\sqrt{\tan 3x}}$ **49.** $e^{g(x)}g'(x)$

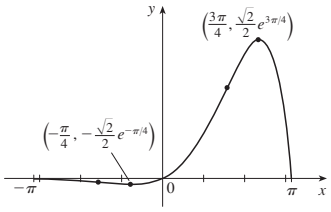
51. $g'(x)/g(x)$ **53.** $2^x(\ln 2)^n$ **57.** $y = -x + 2$ **59.** $(-3, 0)$

61. (a) $y = \frac{1}{4}x + \frac{1}{4}(\ln 4 + 1)$ (b) $y = ex$

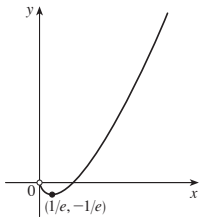
63. 0 **65.** 0 **67.** $-\infty$ **69.** -1

71. 1 **73.** 8 **75.** 0 **77.** $\frac{1}{2}$

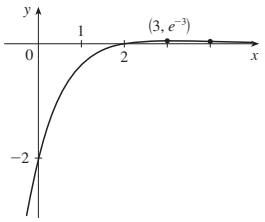
79. A. $[-\pi, \pi]$ B. y-int 0; x-int $-\pi, 0, \pi$
C. None D. None
E. Inc on $(-\pi/4, 3\pi/4)$; dec on $(-\pi, -\pi/4), (3\pi/4, \pi)$
F. Loc max $f(3\pi/4) = \frac{1}{2}\sqrt{2}e^{3\pi/4}$, loc min $f(-\pi/4) = -\frac{1}{2}\sqrt{2}e^{3\pi/4}$
G. CU on $(-\pi/2, \pi/2)$; CD on $(-\pi, -\pi/2), (\pi/2, \pi)$;
IP $(-\pi/2, -e^{-\pi/2}), (\pi/2, e^{\pi/2})$
H.



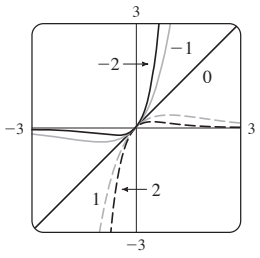
81. A. $(0, \infty)$ B. x-int 1
C. None D. None
E. Inc on $(1/e, \infty)$; dec on $(0, 1/e)$
F. Loc min $f(1/e) = -1/e$
G. CU on $(0, \infty)$
H. See graph at right.



83. A. $\mathbb{R}$
B. y-int -2; x-int 2
C. None D. HA $y = 0$
E. Inc on $(-\infty, 3)$; dec on $(3, \infty)$
F. Loc max $f(3) = e^{-3}$
G. CU on $(4, \infty)$; CD on $(-\infty, 4)$; IP $(4, 2e^{-4})$
H. See graph at right.



85.



For $c > 0$, $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = -\infty$.
For $c < 0$, $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = 0$.
As $|c|$ increases, the max and min points and the IPs get closer to the origin.

87. $v(t) = -Ae^{-ct}[c\cos(\omega t + \delta) + \omega \sin(\omega t + \delta)]$,
$a(t) = Ae^{-ct}[(c^2 - \omega^2)\cos(\omega t + \delta) + 2c\omega \sin(\omega t + \delta)]$

89. (a) $200(3.24)^t$ (b) $\approx 22{,}040$
(c) $\approx 25{,}910$ bacteria/h (d) $(\ln 50)/(\ln 3.24) \approx 3.33$ h

91. 4.32 days **93.** $\frac{1}{4}(1 - e^{-2})$ **95.** $\arctan e - \pi/4$

97. $2e^{\sqrt{x}} + C$ **99.** $\frac{1}{2}\ln|x^2 + 2x| + C$

101. $-\frac{1}{2}[\ln(\cos x)]^2 + C$ **103.** $2^{\tan\theta}/\ln 2 + C$

105. $-(1/x) - 2\ln|x| + x + C$ **109.** $e^{\sqrt{x}}/(2x)$

111. $\frac{1}{3}\ln 4$ **113.** $\pi^2/4$ **115.** $\frac{2}{3}$ **117.** $2/e$

121. $e^{2x}(1 + 2x)/(1 - e^{-x})$

PROBLEMS PLUS ■ PAGE 486

3. Abs max $f(-5) = e^{45}$, no abs min **9.** $1/\sqrt{2}$ **11.** $a = \frac{1}{2}$

15. $2\sqrt{e}$ **17.** $a \leqslant e^{1/e}$

CHAPTER 7

EXERCISES 7.1 ■ PAGE 492

1. $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$ **3.** $\frac{1}{5}x \sin 5x + \frac{1}{25}\cos 5x + C$

5. $-\frac{1}{3}te^{-3t} - \frac{1}{9}e^{-3t} + C$

7. $(x^2 + 2x)\sin x + (2x + 2)\cos x - 2\sin x + C$

9. $x \ln \sqrt[3]{x} - \frac{1}{3}x + C$ **11.** $t \arctan 4t - \frac{1}{8}\ln(1 + 16t^2) + C$

13. $\frac{1}{2}t \tan 2t - \frac{1}{4}\ln|\sec 2t| + C$

15. $x(\ln x)^2 - 2x \ln x + 2x + C$

17. $\frac{1}{13}e^{2\theta}(2\sin 3\theta - 3\cos 3\theta) + C$

19. $z^3e^z - 3z^2e^z + 6ze^z - 6e^z + C$

21. $\dfrac{e^{2x}}{4(2x+1)} + C$ **23.** $\dfrac{\pi - 2}{2\pi^2}$

25. $1 - 1/e$ **27.** $\frac{81}{4}\ln 3 - 5$ **29.** $\frac{1}{4} - \frac{3}{4}e^{-2}$

31. $\frac{1}{6}\left(\pi + 6 - 3\sqrt{3}\right)$ **33.** $\sin x(\ln \sin x - 1) + C$