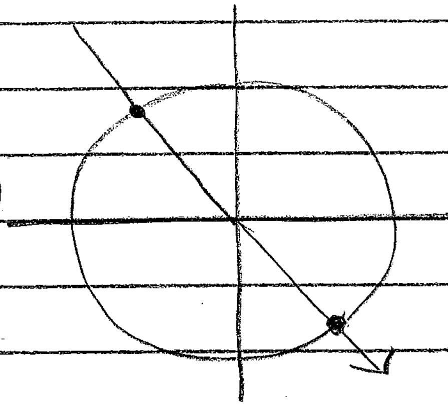
#1

§6.6

#6

(a)



$$\tan\left(\frac{3\pi}{4}\right) = -1 = \tan\left(\frac{-\pi}{4}\right).$$

Thus

$$\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right) = \tan^{-1}\left(\tan\left(\frac{-\pi}{4}\right)\right) = -\pi/4.$$

(b) $$\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}.$$

#30

$$\frac{dy}{dx} = \frac{1}{1+\left(\frac{1-x}{1+x}\right)} \; \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}} \left\{\frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}\right\}$$

$$= \left[\frac{1+x}{(1+x)+(1-x)}\right] \frac{1}{\cancel{2}} \frac{\sqrt{1+x}}{\sqrt{1-x}} \; \frac{\cancel{-2}}{(1+x)^2}$$

$$= \left(\frac{1+x}{2}\right)\frac{(1+x)^{1/2}}{(1-x)^{1/2}} \cdot \frac{-1}{(1+x)^2} = \frac{-1}{2\sqrt{(1-x)(1+x)}}$$

#62. $\displaystyle\int_0^{\sqrt{3}/4} \frac{dx}{1+16x^2} = \frac{1}{4}\int_0^{\sqrt{3}/4} \frac{4\,dx}{1+(4x)^2}$

Let $u = 4x \quad du = 4\,dx$

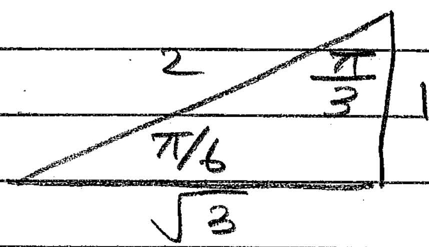
$$= \frac{1}{4}\int_0^{\sqrt{3}} \frac{1}{1+u^2}\,du$$

$$= \frac{1}{4}\tan^{-1}(u)\Big|_0^{\sqrt{3}}$$

$$= \frac{1}{4}\tan^{-1}(\sqrt{3}) - \frac{1}{4}\tan^{-1}(0)$$

$$= \frac{1}{4}\left(\frac{\pi}{3}\right)$$

$$= \frac{\pi}{12}.$$



§6.7 12. $\cosh(x)\cosh(y) + \sinh(x)\sinh(y)$

$$= \left(\frac{e^x+e^{-x}}{2}\right)\left(\frac{e^y+e^{-y}}{2}\right) + \left(\frac{e^x-e^{-x}}{2}\right)\left(\frac{e^y-e^{-y}}{2}\right)$$

$$= \frac{e^{x+y} + \cancel{e^{x-y}} + \cancel{e^{-x+y}} + e^{-x-y} + e^{x+y} - \cancel{e^{x-y}} - \cancel{e^{-x+y}} + e^{-x-y}}{4}$$

$$= \frac{2e^{x+y} + 2e^{-(x+y)}}{4} = \frac{e^{x+y} + e^{-(x+y)}}{2}$$

$$= \cosh(x+y).$$

#39.

$$G'(x) = \frac{(1+\cosh(x))(-\sinh(x)) - (1-\cosh(x))(\sinh(x))}{(1+\cosh(x))^2}$$

$$= \frac{\sinh(x)\left\{-1 - \cancel{\cosh(x)} - 1 + \cancel{\cosh(x)}\right\}}{(1+\cosh(x))^2}$$

$$= \frac{-2\sinh(x)}{(1+\cosh(x))^2}$$

#54. $$\lim_{x\to\infty} \frac{\sinh(x)}{e^x} = \lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x\, 2}$$

$$= \lim_{x\to\infty} \frac{1-e^{-2x}}{2} = \frac{1}{2}.$$

6.8. #26 $$\lim_{x\to 0} \frac{\sinh(x) - x}{x^3} \qquad \frac{0}{0} \text{ form}$$

check: $$\lim_{x\to 0} \frac{\cosh(x) - 1}{3x^2} \qquad \frac{0}{0} \text{ form}$$

check: $$\lim_{x\to 0} \frac{\sinh(x)}{6x} \qquad \frac{0}{0} \text{ form}$$

check: $$\lim_{x\to 0} \frac{\cosh(x)}{6} = \frac{1}{6}$$

$$\therefore \lim_{x\to 0} \frac{\sinh(x) - x}{x^3} = \frac{1}{6}.$$

$\infty - \infty$ form

#52 $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right) = \lim_{x\to 0}\left(\frac{\cos x}{\sin x} - \frac{1}{x}\right)$

$= \lim_{x\to 0}\left(\frac{x\cos(x) - \sin(x)}{x\cdot\sin(x)}\right)$ $\frac{0}{0}$ form

check: $\lim_{x\to 0} \frac{\cancel{\cos x} - x\sin x - \cancel{\cos x}}{\sin x + x\cos x}$ $\frac{0}{0}$

check: $\lim_{x\to 0} \frac{-\sin x - x\cos x}{\cos x + \cos x - x\sin x} = 0$

$\therefore \lim_{x\to 0}\left(\cot(x) - \frac{1}{x}\right) = 0.$

62. $\lim_{x\to\infty} e^{\frac{\ln(e^x+x)}{x}}$

consider: $\lim_{x\to\infty} \frac{\ln(e^x+x)}{x}$ $\frac{\infty}{\infty}.$

check: $\lim_{x\to\infty} \dfrac{\frac{1}{e^x+x}(e^x+1)}{1}$

$= \lim_{x\to\infty} \frac{e^x+1}{e^x+x}$ $\frac{\infty}{\infty}$

check: $\lim_{x\to\infty} \frac{e^x}{e^x+1}$ $\frac{\infty}{\infty}$

check: $\lim_{x\to\infty} \frac{e^x}{e^x} = 1 \therefore \lim_{x\to\infty}(e^x+x)^{\frac{1}{x}} = e^1.$

7.1

8. $I = \int t^2 \sin(\beta t)\,dt$

$u = t^2 \qquad dV = \sin(\beta t)\,dt$

$du = 2t \qquad V = -\dfrac{\cos(\beta t)}{\beta}$

$$I = \frac{-t^2\cos(\beta t)}{\beta} + \frac{2}{\beta}\boxed{\underbrace{\int t\cos(\beta t)\,dt}_{J}}$$

$u = t \qquad dV = \cos(\beta t)\,dt$

$du = dt \qquad V = \dfrac{\sin(\beta t)}{\beta}$

$$J = \frac{t\sin(\beta t)}{\beta} - \int \frac{\sin(\beta t)}{\beta}\,dt$$

$$= \frac{t\sin(\beta t)}{\beta} + \frac{\cos(\beta t)}{\beta^2} + C$$

$$I = -\frac{t^2\cos(\beta t)}{\beta} + \frac{2t\sin(\beta t)}{\beta^2}$$

$$+ \frac{2\cos(\beta t)}{\beta^3} + C$$

#18. $I = \int e^{-\theta}\cos(2\theta)\,d\theta$

$u = \cos(2\theta) \qquad dV = e^{-\theta}$

$du = -\sin(2\theta)\,2 \qquad V = -e^{-\theta}$

$$I = -\cos(2\theta)e^{-\theta} - 2\boxed{\underbrace{\int e^{-\theta}\sin(2\theta)\,d\theta}_{J}}$$

$$u = \sin(2\theta) \qquad dV = e^{-\theta}d\theta$$
$$du = \cos(2\theta)2 \qquad V = -e^{-\theta}$$

$$J = -\sin(2\theta)e^{-\theta} + \int 2e^{-\theta}\cos(2\theta)\,d\theta$$
$$= -\sin(2\theta)e^{-\theta} + 2I.$$

$$I = -\cos(2\theta)e^{-\theta} + 2\sin(2\theta)e^{-\theta} - 4I$$

$$I = -\tfrac{1}{5}\cos(2\theta)e^{-\theta} + \tfrac{2}{5}\sin(2\theta)e^{-\theta} + C$$

#26. $\int y^{-1/2}\ln(y)\,dy$ We will put limits on at the end.

$$u = \ln(y) \qquad dV = y^{-1/2}dy$$
$$du = \frac{1}{y} \qquad V = 2y^{1/2}$$

$$I = 2y^{1/2}\ln(y) - \int 2y^{-1/2}\,dy$$
$$= 2y^{1/2}\ln(y) - 4y^{1/2} + C$$

$$\int_4^9 y^{-1/2}\ln(y)\,dy = 2\sqrt{9}\ln(9) - 4\sqrt{9} - 2\sqrt{4}\ln(4) + 4\sqrt{4}$$
$$= 6\ln(9) - 12 - 4\ln(4) + 8$$
$$= 6\ln(9) - 4\ln(4) - 4$$

§ 7.2

#2 $\int \sin^3(\theta)\cos^4(\theta)\,d\theta$

$$= \int (1-\cos^2\theta)\cos^4(\theta)\sin\theta\,d\theta$$

$u = \cos\theta \qquad du = -\sin\theta\,d\theta$

$$= \int (1-u^2)\,u^4\,(-du)$$

$$= \int (u^6 - u^4)\,du = \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$= \frac{\cos^7\theta}{7} - \frac{\cos^5\theta}{5} + C$$

#28 $\int \tan^5(x)\sec^3(x)\,dx$

$$= \int (\tan^2 x)^2 \sec^2 x \tan x \sec x\,dx$$

$$= \int (\sec^2 x - 1)^2 \sec^2 x \tan x \sec x\,dx$$

$u = \sec x \qquad du = \sec x \tan x\,dx$

$$= \int (u^2-1)^2 u^2\,du = \int (u^4 - 2u^2 + 1)\,u^2\,du$$

$$= \int (u^6 - 2u^4 + u^2)\,du = \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} + C$$

$$= \frac{\sec^7 x}{7} - \frac{2\sec^5 x}{5} + \frac{\sec^3 x}{3} + C$$

#34 $\displaystyle\int \frac{\sin\phi}{\cos^3\phi}\,d\phi = \int \tan\phi\,\sec^2\phi\,d\phi$

$\displaystyle = \int \sec\phi\,(\tan\phi\,\sec\phi\,d\phi)$

(Let $u = \sec\phi$ $\quad du = \sec\phi\,\tan\phi\,d\phi$)

$\displaystyle = \int u\,du = \frac{u^2}{2} + C$

$\displaystyle = \frac{\sec^2\phi}{2} + C$

§ 7.3

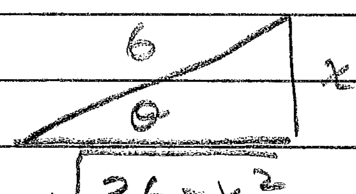
#6. $\displaystyle\int \frac{x}{\sqrt{36-x^2}}\,dx$

<u>Method #1</u>

$x = 6\sin\theta$

$dx = 6\cos\theta\,d\theta$

$\displaystyle\int \frac{6\sin\theta}{(6\cos\theta)}\,6\cos\theta\,d\theta$



$= -6\cos\theta + C$

$\displaystyle = -6\,\frac{\sqrt{36-x^2}}{6} + C = -\sqrt{36-x^2} + C$

<u>Method #2</u> Let $u = 36 - x^2$ $\quad du = -2x\,dx$

etc.

#16 $$\int_{\sqrt{2}/3}^{2/3} \frac{dx}{x^5\sqrt{9x^2-1}} = \int_{\sqrt{2}/3}^{2/3} \frac{dx}{x^5\sqrt{(3x)^2-1}}$$

Let $u = 3x$
$du = 3dx$

$$= \int_{\sqrt{2}}^{2} \frac{\frac{1}{3}du}{\left(\frac{u}{3}\right)^5\sqrt{u^2-1}}$$

$$= 81\int_{\sqrt{2}}^{2} \frac{du}{u^5\sqrt{u^2-1}}$$

$u = \sec\theta$
$du = \sec\theta\tan\theta$
$\sec\theta = 2$
$\theta = \pi/3$
$\sec\theta = \sqrt{2}$
$\theta = \pi/4$

$$= 81\int_{\pi/4}^{\pi/3} \frac{\sec\theta \cancel{\tan\theta}\, d\theta}{\sec^5\theta \cancel{\tan\theta}}$$

$$= 81\int_{\pi/4}^{\pi/3} \cos^4(\theta)\, d\theta$$

$$= 81\int_{\pi/4}^{\pi/3} \left(\frac{1+\cos 2\theta}{2}\right)\left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= 81\int_{\pi/4}^{\pi/3} \left(\frac{1}{4} + \frac{1}{2}\cos 2\theta + \frac{1}{4}\cos^2(2\theta)\right) d\theta$$

$$= 81\int_{\pi/4}^{\pi/3} \frac{1}{4} + \frac{1}{2}\cos 2\theta + \frac{1}{8} + \frac{1}{8}\cos(4\theta)\, d\theta$$

(continued)

#10

$$= 81\left(\frac{3}{8}\theta + \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin(4\theta)\right)\Bigg|_{\pi/4}^{\pi/3}$$

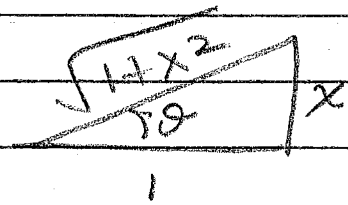
$$= 81\left(\frac{7\sqrt{3}}{64} + \frac{\pi}{8}\right) - 81\left(\frac{1}{4} + \frac{3\pi}{32}\right)$$

$$= -\frac{81}{4} + \frac{567\sqrt{3}}{64} + \frac{81\pi}{32}$$

#20 $\displaystyle\int \frac{x}{\sqrt{1+x^2}}\,dx$

Method #1
$x = \tan\theta$
$dx = \sec^2\theta\, d\theta$

$$= \int \frac{\tan\theta \sec^2\theta\, d\theta}{\sec\theta}$$



$$= \int \tan\theta \sec\theta\, d\theta = \sec\theta + C$$

$$= \sqrt{1+x^2} + C$$

Method #2 $u = 1+x^2$, $du = 2x\,dx$.

7.4 #10 $\displaystyle\frac{y}{(y+4)(2y-1)} = \frac{4}{9(4+y)} + \frac{1}{9(2y-1)}$

$$\int f(y)\,dy = \frac{4}{9}\ln|y+4| + \frac{1}{18}\ln|2y-1| + C$$

#22 $f(s) = \dfrac{1}{s^2(1-s)^2} =$

$$\frac{2}{s} + \frac{1}{s^2} - \frac{2}{s-1} + \frac{1}{(s-1)^2}$$

$$\int f(s)\,ds = 2\ln|s| - s^{-1} - 2\ln|s-1| - (s-1)^{-1} + c$$

#24 $\dfrac{x^2-x-6}{x^3+3x} = \dfrac{x^2-x-6}{x(x^2+3)}$

$$= -\frac{2}{x} + \frac{3x-1}{x^2+3}$$

$$= -\frac{2}{x} + \frac{3x}{x^2+3} - \frac{1}{x^2+3}$$

$$\int f(x)\,dx = -2\ln|x| + \frac{3}{2}\ln|x^2+3| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$