

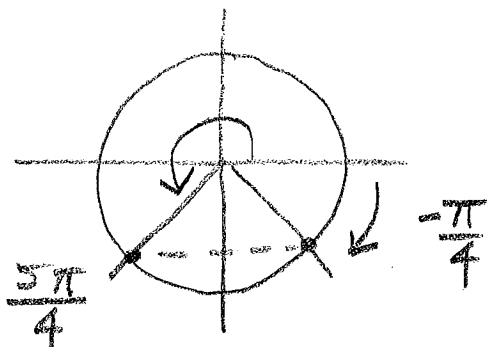
Test #2

Name Answer Key

Show all of your work

1. In each case, find the exact value:

3 (a)  $\sin^{-1}(\sin(5\pi/4)) = -\frac{\pi}{4}$



3 (b)  $\cos(\sin^{-1}(.6)) = \sqrt{1 - (.6)^2}$   
 $= \sqrt{.64}$   
 $= .8$

2. Show that  $2 \sinh(x) \cosh(x) = \sinh(2x)$ .

5 
$$2 \left( \frac{e^x - e^{-x}}{2} \right) \left( \frac{e^x + e^{-x}}{2} \right) = \frac{e^{2x} + 1 - 1 - e^{-2x}}{2}$$
  

$$= \frac{e^{2x} - e^{-2x}}{2}$$
  

$$= \sinh(2x).$$

5 3. Express the **form** of the partial fraction decomposition of  $f(x) = \frac{x^2}{(x+3)(x^2+2x-3)(x^2+2x+2)}$ .  
 You do not have to find the coefficients.

$$(x+3)(x-1) \uparrow$$

Irreducible  
 $2^2 - 4(1)(2) < 0$

$$\frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x-1} + \frac{Dx+E}{x^2+2x+2}$$

4. Evaluate the following limits:

3 (a)  $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x \sin(x)} = \frac{1}{2} \leftarrow$   
 check:  $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x + x \cos x} = \frac{0}{0}$

check:  $\lim_{x \rightarrow 0} \frac{e^x}{\cos x + \cos x - x \sin x} = \frac{1}{2}$

3 (b)  $\lim_{x \rightarrow \infty} x \ln \left(1 + \frac{2}{x}\right) = \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{2}{x}\right)}{\frac{1}{x}} = \frac{0}{0}$   
 check:  $\lim_{x \rightarrow \infty} \frac{\frac{1}{\left(1 + \frac{2}{x}\right)} \left(-\frac{2}{x^2}\right)}{\frac{1}{x^2}} = 2$

5. (a) Let  $f(x) = \frac{1}{2}\sqrt{1-4x^2} + x \sin^{-1}(2x)$ . Show that  $f'(x) = \sin^{-1}(2x)$ .

3  $f'(x) = \frac{1}{4}(1-4x^2)^{-\frac{1}{2}}(-8x) + \sin^{-1}(2x) + x \frac{1}{\sqrt{1-4x^2}}(2)$   
 $= \frac{-2x}{\sqrt{1-4x^2}} + \sin^{-1}(2x) + \frac{2x}{\sqrt{1-4x^2}}$   
 $= \sin^{-1}(2x) \checkmark$

(b) Let  $f(x) = \tan^{-1}\left(\frac{x-1}{x+1}\right)$ . Show that  $f'(x) = \frac{1}{1+x^2}$ 

3  $f'(x) = \frac{1}{1 + \left(\frac{x-1}{x+1}\right)^2} \cdot \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$   
 $= \frac{2}{(x+1)^2 + (x-1)^2} = \frac{2}{x^2 + 2x + 1 + x^2 - 2x + 1}$   
 $= \frac{1}{x^2 + 1}$

6. In each case, evaluate the indicated integral.

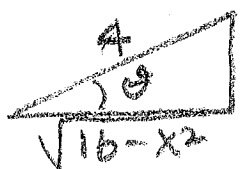
4 (a)  $I = \int_0^{\pi/2} \sin^5(x) \cos^3(x) dx = \int_0^{\pi/2} \sin^4 x (1 - \sin^2 x) \cos(x) dx$

$u = \sin x$   
 $du = \cos x dx$

$= \int_0^1 u^4 (1 - u^2) du$

$= \int_0^1 (u^4 - u^6) du = \left. \frac{u^5}{5} - \frac{u^7}{7} \right|_0^1$

$= \frac{1}{5} - \frac{1}{7} = \frac{2}{35}$

4 (b)  $I = \int \frac{x^2}{\sqrt{16-x^2}} dx$   $x = 4 \sin \theta$  

$dx = 4 \cos \theta d\theta$

$= \int \frac{16 \sin^2 \theta \cdot 4 \cos \theta d\theta}{4 \cos \theta} = 16 \int \frac{1 - \cos(2\theta)}{2} d\theta$

$= 8\theta - 4 \sin(2\theta) + C = 8\theta + 8 \sin \theta \cos \theta + C$

$= 8 \sin^{-1}\left(\frac{x}{4}\right) + 8 \cdot \frac{x}{4} \cdot \frac{\sqrt{16-x^2}}{4} + C$

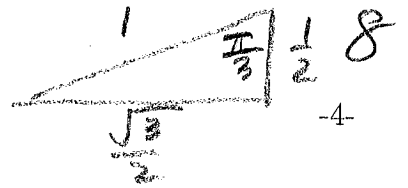
$= 8 \sin^{-1}\left(\frac{x}{4}\right) + \frac{x \sqrt{16-x^2}}{2} + C$

4 (c)  $\int \sqrt{x} \ln(x) dx$

$u = \ln(x)$   $dv = x^{1/2}$   
 $du = \frac{1}{x} dx$   $v = \frac{2}{3} x^{3/2}$

$= \frac{2}{3} x^{3/2} \ln(x) - \int \frac{2}{3} x^{1/2} dx$

$= \frac{2}{3} x^{3/2} \ln(x) - \frac{4}{9} x^{3/2} + C$



4 (d)  $I = \int_1^2 \frac{\sqrt{x^2-1}}{x} dx$

$$x = \sec \theta.$$

$$dx = \sec \theta \tan \theta d\theta$$

$$= \int_0^{\pi/2} \frac{\tan \theta \tan \theta \sec \theta}{\sec \theta} d\theta. \quad \sqrt{x^2-1} = \tan \theta.$$

$$\sec \theta = 1$$

$$\theta = 0$$

$$= \int_0^{\pi/2} \tan^2 \theta d\theta. \quad 2.5 \text{ pts}$$

$$\sec \theta = 2$$

$$\theta = \frac{\pi}{3}$$

$$= \int_0^{\pi/2} (\sec^2 \theta - 1) d\theta = \tan \theta - \theta \Big|_0^{\pi/3} = \sqrt{3} - \frac{\pi}{3} - 0 = \sqrt{3} - \frac{\pi}{3}.$$

4 (e)  $I = \int \frac{5x^2-9}{x^3-9x} dx$

$$\frac{5x^2-9}{x(x-3)(x+3)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+3}$$

$$5x^2-9 = A(x-3)(x+3) + Bx(x+3) + Cx(x-3)$$

$$x=0: -9 = -9A \quad (A=1)$$

$$x=3: 36 = B(18) \quad (B=2)$$

$$x=-3: 36 = C(18) \quad (C=2)$$

$$\int \left( \frac{1}{x} + \frac{2}{x-3} + \frac{2}{x+3} \right) dx$$

$$= \ln|x| + 2\ln|x-3| + 2\ln|x+3| + C.$$