

§6.3*–The Natural Exponential Function

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Outline

The definition of the exponential function

exp is an exponential function

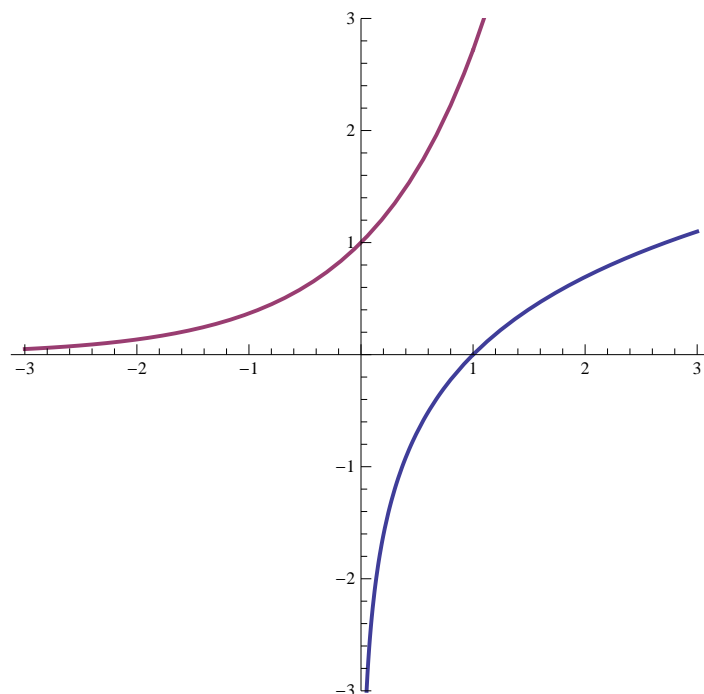
The derivative of e^x

Definition

- You will recall that \ln , the natural logarithm, is an increasing function with domain $(0, \infty)$ and range \mathbb{R} . In particular, \ln is invertible.
- Let \exp denote the inverse of \ln ; thus, \exp has domain \mathbb{R} and range $(0, \infty)$.

The graph of \exp

The graph of $\exp(x)$ can be obtained directly from the graph of $\ln(x)$.



Some elementary properties of exp

- We have the important inverse relationships:

$$\ln(\exp(x)) = x \quad \text{and} \quad \exp(\ln(x)) = x.$$

- Since $\ln(1) = 0$, it follows that

$$\exp(0) = 1.$$

Euler's number, e

Recall the definition of e : e is the number such that $\ln(e) = 1$.

($e \approx 2.71828$)

Theorem

$\exp(x) = e^x$ for all rational numbers x .

Definition

- Because $\exp(r) = e^r$ for all rational numbers, we will **define** e^x by $\exp(x)$ for all real numbers x .
- In particular,

$$\ln e^x = x \quad \text{and} \quad e^{\ln(x)} = x.$$

Theorem (The laws of exponents)

- $e^{x+y} = e^x e^y$
- $e^{x-y} = e^x / e^y$ (*for homework*)
- $(e^x)^r = e^{rx}$ (*for homework*)

Problem

Solve the following equations:

- $e^{2x-3} = 8$
- $e^x + 2 = 8e^{-x}$

Theorem

- $\frac{d}{dx}e^x = e^x$
- $\int e^x dx = e^x + C.$

Problem

Find y' in each case:

- $y = e^{x^2}$
- $y = x^3 \exp\left(\frac{x+1}{x+2}\right)$
- $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Problem

Solve the following integrals:

- $\int \frac{e^x}{e^x + 1} dx$
- $\int x e^{x^2} dx$
- $\int_0^3 e^{x+5} dx$

Problem

Sketch the graph of $y = xe^{-x}$.