Mathematics 151 **Practice Test #1**

6.1.18 We need to solve $f(x) = x^5 + x^3 + x = 3$. We can find the solution by inspection: $x = 1$. Thus $f^{-1}(3) = 1$. By the cancellation equations, $f(f^{-1}(2)) = 2$.

6.1.20

1. The function passes the horizontal line test.
2. $D = [-3, 3]$ and $R = [-1, 3]$.
3. $f^{-1}(2) = 0$ (since $f(0) = 2$).
4. $f^{-1}(0) \approx -2.6$.

6.1.26 Let $f(x) = x^2 - x$ for $x \geq 1/2$. We need to solve $y^2 - y = x$ for y. Equivalently, we must solve $y^2 - y - x = 0$ for y. From the quadratic formula, we have two solutions

$$y = \frac{1}{2}\left(1 - \sqrt{4x+1}\right) \quad \text{and} \quad y = \frac{1}{2}\left(1 + \sqrt{4x+1}\right),$$

only one of which is $f^{-1}(x)$. *Since the domain of f is $[1/2, \infty)$, the range of f^{-1} is $[1/2, \infty)$.* This shows that

$$f^{-1}(x) = \frac{1}{2}\left(1 + \sqrt{4x+1}\right).$$

6.1.38

1. Since $f'(x) = -(1/(-1+x)^2)$, we see that $f'(x) < 0$ on the interval $x > 1$. Thus f is decreasing hence one-to-one.
2. The equation $1/(x-1) = 2$ has solution $x = 3/2$. Thus
$$(f^{-1})'(2) = \frac{1}{f'(3/2)} = -1/4.$$
3. Solving $f(y) = x$ for y yields $f^{-1}(x) = \dfrac{x+1}{x}$.
4. From our explicit formula for $f^{-1}(x)$, we obtain $(f^{-1})'(x) = -1/x^2$; thus, $(f^{-1})'(2) = -1/(2)^2 = -1/4$.
5. Sketch omitted.

6.2*.8 We have
$$\ln(3) + \frac{1}{3}\ln(8) = \ln(3 \cdot 8^{1/3}) = \ln(6).$$

6.2*.24 We have
$$h'(x) = \left(\frac{1}{x + \sqrt{x^2 - 1}}\right)\left(1 + \frac{1}{2}(x^2 - 1)^{-1/2}(2x)\right).$$

6.2*.28 We can use properties of logarithms to simplify H.
$$H(z) = \frac{1}{2}\ln(a^2 - z^2) - \frac{1}{2}\ln(a^2 + z^2).$$

Thus
$$H'(z) = \frac{1}{2}\frac{1}{a^2 - z^2}(-2z) - \frac{1}{2}\frac{1}{a^2 + z^2}(2z).$$

6.2*.62 By properties of logarithms,
$$\ln y = 4\ln(x + 1) + 3\ln(x - 5) - 8\ln(x - 3).$$

Thus
$$\frac{1}{y}y' = 4\frac{1}{x + 1} + 3\frac{1}{x - 5} - 8\frac{1}{x - 3}.$$

Finally,
$$y' = \frac{(x + 1)^4(x - 5)^3}{(x - 3)^8}\left(4\frac{1}{x + 1} + 3\frac{1}{x - 5} - 8\frac{1}{x - 3}\right).$$

6.2*.68 First, expand the integrand, obtaining
$$\int_4^9 \left(x + 2 + \frac{1}{x}\right) dx = \frac{x^2}{2} + 2x + \ln(x)\bigg|_4^9 = \frac{85}{2} + \ln\left(\frac{9}{4}\right)$$

6.2*.72 Let $u = 2 + \sin(x)$ and $du = \cos(x)$. The integral is of the form
$$\int \frac{1}{u}du = \ln(|u|) + C = \ln(|2 + \sin(x)|) + C.$$

Since $2 + \sin(x)$ is always positive, it would be appropriate to drop the absolute value signs in the answer.

6.3*.6

1. Exponentiating both sides yields $x^2 - 1 = e^3$ or $x^2 = e^3 - 1$. This equation has two solutions, $x = \pm\sqrt{e^3 - 1}$.
2. This equation is "quadratic type." Letting $u = e^x$, we obtain $u^2 - 3u + 2 = (u-2)(u-1) = 0$. This equation has solutions $u = 2$ and $u = 1$ or, equivalently, $e^x = 2$ and $e^x = 1$. This gives us two solutions in x, $x = \ln(2)$ and $x = \ln(1) = 0$.

6.3*.26 We need to solve $x = \frac{e^y}{1+2e^y}$ for y. Cross-multiplying the equation leads to $x + 2xe^y = e^y$. Thus $e^y = x/(1-2x)$ or $y = \ln(x/(1-2x))$, which is the inverse function.

6.3*.30 As $x \to 2^-$, the exponent $3/(2-x) \to +\infty$; thus, $e^{3/(2-x)} \to +\infty$ as well.

6.3*.36 Using the product rule and simplifying our result, we obtain

$$y' = \frac{e^x}{(e^x - 1)^2}$$

6.3*.50 By the chain rule,

$$y' = \frac{e^{-2x}(1-2x)}{2\sqrt{e^{-2x}x + 1}}.$$

6.3*.68 We have $g'(x) = \frac{e^x(x-1)}{x^2}$. Making a sign chart for the derivative (for $x > 0$), we find that g is decreasing on $(0, 1]$ and increasing on $[1, \infty)$. This shows that $g(1) = e$ is the absolute minimum value of $g(x)$ for $x > 0$.

6.4*.8 We have $\log_{10}(\sqrt{10}) = \frac{1}{2}\log_{10}(10) = \frac{1}{2}$.

6.4*.10

1. We have $\log_a(1/a) = \log_a(a^{-1}) = -1$.
2. We have
$$10^{\log_{10}(4)+\log_{10}(7)} = 10^{\log_{10}(28)} = 28.$$

6.4*.32 It is a bit easier if we use properties of logarithms first; thus,

$$f(x) = \log_5(x) + x\log_5(e).$$

The derivative is

$$f'(x) = \frac{1}{x\ln(5)} + \log_5(e).$$

6.4*.38 First we will rewrite the function as

$$y = e^{\ln(\sqrt{x}^x)} = e^{x\ln(x^{1/2})} = e^{\frac{x}{2}\ln(x)}.$$

The derivative is easy:

$$y' = e^{\frac{x}{2}\ln(x)}\left(\frac{1}{2}\ln(x) + \frac{1}{2}\right).$$

6.4*.47 Let $u = \log_{10}(x)$ and $du = \dfrac{1}{x\ln(10)}$. The integral becomes

$$\ln(10)\int udu = \ln(10)\frac{u^2}{2} + C = \frac{\ln(10)}{2}\left(\log_{10}(x)\right)^2 + C.$$

6.5.6

1. Let $P(t)$ denote the population t years after 1951. Then $P(0) = 361$ and $P(10) = 439$. Thus, our growth constant k must satisfy

   $$439 = 361e^{10k}.$$

   Solving for k, we find $k \approx .0195621$. We would predict the population for 2001 as

   $$P(50) = 361e^{50(.0195621)} = 960.05 \text{ million}$$

   Our exponential model under approximates the actual population.

2. Let $P(t)$ denote the population t years after 1961. Then $P(0) = 439$ and $P(20) = 683$. Thus, our growth constant k must satisfy

   $$683 = 439e^{20k}.$$

   Solving for k, we find $k \approx 0.0220998$. We would predict the population for 2001 as

   $$P(40) = 439e^{40(.0220998)} = 1062.62 \text{ million},$$

   which is closer to the actual population in 2001.

6.5.8 1. From the half-life formula, we find

$$k = \frac{\ln(.5)}{28} \approx -0.0247553$$

After t days, the mass will be $A(t) = 50e^{-0.0247553t}$.

2. After 40 days, $A(40) \approx 18.57$ mg of the sample remains.

3. We must solve $A(t) = 2$ for the variable t, that is,

$$50e^{-0.0247553t} = 2.$$

Solving for t, we obtain $t = \ln(2/50)/(-0.0247553) = 130$ days.

4. Sketch omitted.

6.5.10 1. The decay constant must satisfy $k = \ln(.945) \approx -0.0565704$. Thus the half-life is

$$T = \frac{\ln(.5)}{-0.0565704} = 12.2528 \text{ years.}$$

2. We need to solve the equation $.2 = e^{-0.0565704t}$ for t. This yields $t = \ln(.2)/(-0.0565704) \approx 28.45$ years.

6.5.18 (a) We use the compound interest formula in each case.

1. $A(3) = 1259.71$
2. $A(3) = 1268.24$
3. $A(3) = 1270.24$
4. $A(3) = 1271.01$
5. $A(3) = 1271.22$
6. $A(3) = 1271.25$
7. $A(3) = 1271.25$