§7.2–Trigonometric Integrals

Tom Lewis

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Some identities, part I Powers of sine and cosine Powers of secant and tangent

Outline

Essential identities

Here are the basic identities that we need:

- $\cos^2(x) + \sin^2(x) = 1$
- $\cos^2(x) = \dfrac{1 + \cos(2x)}{2}$
- $\sin^2(x) = \dfrac{1 - \cos(2x)}{2}$
- $1 + \tan^2(x) = \sec^2(x)$

Powers of sine and cosine, Case 1

- Consider an integral of the form $\displaystyle\int \cos^m(x)\sin^n(x)dx$: m **or** n is odd.
- We use the identity $\sin^2(x) + \cos^2(x) = 1$ and u-substitution.

Problem

Evaluate $I = \displaystyle\int \sin^4(x)\cos^7(x)dx$.

Powers of sine and cosine, Case 2

- Consider an integral of the form $\int \cos^m(x)\sin^n(x)dx$: m **and** n even.
- Use the the half-angle identities, repeatedly if necessary.

Problem

Solve the integral $\int_0^{\pi/2} \sin^2(x)dx$.

Problem

Evaluate $I = \int \sin^2(x)\cos^2(x)dx$.

Powers of tangent and secant, Case 1

- Consider an integral of the form $\int \tan^m(x)\sec^n(x)dx$ where n, the power of the secant, is even.
- In this case, keep a $\sec^2(x)$ and convert the remaining secants to tangents through $1+\tan^2(x)=\sec^2(x)$.
- Make a u-substitution: $u=\tan(x)$, $du=\sec^2(x)dx$.

Problem

Solve $I=\int_0^{\pi/4}\tan^4(x)\sec^6(x)dx$.

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Powers of tangent and secant, Case 2

- Consider an integral of the form $\int \tan^m(x)\sec^n(x)dx$ where m, the power of the tangent, is odd.
- In this case, keep one tangent and convert the remaining tangents to secants through $1+\tan^2(x)=\sec^2(x)$.
- Make a u-substitution: $u=\sec(x)$, $du=\sec(x)\tan(x)$.

Problem

Solve $\int_0^{\pi/3}\tan^7(x)\sec^5(x)dx$.

A note on powers of tangents and secants

Our analysis is *not* exhaustive:

- We do not have any direct methods for integrating powers of tangent alone and powers of secant alone, for example,

$$\int \tan^8(x)\,dx \quad \text{and} \quad \int \sec^4(x)\,dx.$$

  These integrals can be solved, but we will not address these cases by direct methods.
- We do not have any methods for integrating an even power of tangent times an odd power of secant, for example,

$$\int \tan^4(x)\sec^5(x)\,dx.$$

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Problem (Some ad hoc problems)

- *Show that* $\int \tan(x)\,dx = \ln|\sec(x)| + C$
- *Show that* $\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$
- *Evaluate* $\int \sec^2(x)\,dx$
- *Evaluate* $\int \tan^2(x)\,dx$
- *Evaluate* $\int \tan^3(x)\,dx$
- *Evaluate* $\int \sec^3(x)\,dx$