§6.6–The Inverse Trigonometric Functions

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The inverse sine function The inverse cosine function The inverse tangent function The other inverse trig functions Miscellaneous pr

Outline

The inverse sine function

The inverse cosine function

The inverse tangent function

The other inverse trig functions

Miscellaneous problems

Integrals

Definition

- The sine function is one-to-one on $[-\pi/2, \pi/2]$ and has range [-1, 1] on this domain.
- We define \sin^{-1} to be the inverse of sine on this domain on this domain. It follows that \sin^{-1} has domain [-1,1] and range $[-\pi/2,\pi/2]$.

The inverse sine function The inverse cosine function The inverse tangent function The other inverse trig functions Miscellaneous programmes of the inverse cosine function and the inverse tangent function.

Cancellation equations

Because of these restrictions, we must be a little careful with the inverse relationships:

$$\sin\left(\sin^{-1}(x)\right) = x, \quad -1 \leqslant x \leqslant 1$$

 $\sin^{-1}\left(\sin(x)\right) = x, \quad -\pi/2 \leqslant x \leqslant \pi/2$

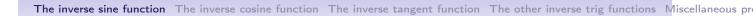
Evaluate the following:

- $\sin\left(\sin^{-1}(.3)\right)$
- $\sin^{-1} (\sin(14\pi/3))$.

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Problem

Show that $\cos\left(\sin^{-1}(x)\right) = \sqrt{1-x^2}$.

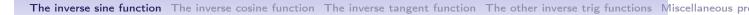


Find $\cos \left(2\sin^{-1}(1/4)\right)$.

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Theorem

$$D_x \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}.$$



Find the derivative of $y = x \sin^{-1}(x^2)$.

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Definition

- The cosine function is one-to-one on the interval $[0, \pi]$ and has range [-1, 1] on that domain.
- Let \cos^{-1} denote the inverse of the cosine function restricted to the domain $[0, \pi]$. Thus the domain of \cos^{-1} is [-1, 1] and its range is $[0, \pi]$.

The cancellation equations

$$\cos\left(\cos^{-1}(x)\right) = x, \quad -1 \leqslant x \leqslant 1$$

 $\cos^{-1}\left(\cos(x)\right) = x, \quad 0 \leqslant x \leqslant \pi$

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Problem

- Evaluate $\cos^{-1} \left(\cos(14\pi/3) \right)$
- Show that $\sin(\cos^{-1}(x)) = \sqrt{1-x^2}$

Theorem

$$D_x \cos^{-1}(x) = -\frac{1}{\sqrt{1 - x^2}}$$

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Definition

- The tangent is one-to-one on the interval $(-\pi/2, \pi/2)$ and has range $(-\infty, \infty)$ on this domain.
- Let \tan^{-1} be the inverse of the tangent function on this restricted domain. Thus the domain of \tan^{-1} is $(-\infty, \infty)$ and its range is $(-\pi/2, \pi/2)$.

The cancellation equations

$$\tan \left(\tan^{-1}(x)\right) = x \quad -\infty < x < \infty$$
$$\tan^{-1}\left(\tan(x)\right) = x \quad -\pi/2 < x < \pi/2.$$

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Problem

Show that
$$\sec^2\left(\tan^{-1}(x)\right) = 1 + x^2$$

Theorem

$$D_x an^{-1}(x) = rac{1}{1+x^2}, \quad x \in \mathbb{R}.$$

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The other inverse trig functions

The remaining inverse trig functions (\cot^{-1} , \sec^{-1} , and \csc^{-1}) are defined in similar ways. See page 287 of the text for a summary of these functions, their definitions, and derivatives.

Find y' in each case:

- $y = \tan^{-1}(e^x)$
- $y = \sqrt{1 x^2} \sin^{-1}(x)$
- $y = \sin^{-1}(x) + \cos^{-1}(x)$

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Basic integration formulas

•
$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + C$$

Problem

Why is there no formula involving $\cos^{-1}(x)$?

Evaluate the following integrals:

$$\bullet \int \frac{\tan^{-1}(x)}{1+x^2} dx$$

$$\bullet \int_0^{\pi/2} \frac{\sin(x)}{1 + \cos^2(x)} dx$$

$$\bullet \int_0^{\pi/2} \frac{1}{\sqrt{4-t^2}} dt$$

•
$$\int \frac{1}{a^2 + x^2} dx$$
, where a is any real number.