Assignment #3 Name Answer Key

Due 30 January 2015

1. Solve the equations or evaluate some integrals?

2. The half-life of Beryllium-11 (^{11}Be) is 13.81 seconds.

   (a) How much of a 150g sample of Beryllium-11 sample remains after 10 seconds?

$$k = \frac{\ln(.5)}{13.81} = -.0501917$$

$$A(10) = 150\, e^{(-.0501917)(10)}$$

$$= 90.8 \text{ g.}$$

   (b) How long will it take for a sample of Beryllium-11 to be reduced to one-third of its original mass?

$$50 = 150\, e^{kt}$$

$$\frac{1}{3} = e^{kt}$$

$$\ln(1/3) = kt$$

$$t = \frac{\ln(1/3)}{-.0501917}$$

$$t \approx 21.89 \text{ secs.}$$

3. You invest \$500.00 into an account paying an annual interest rate of 6%. Calculate how much money is in the account after 5 years under each of the following compounding schemes:

   (a) the interest is compounded monthly;

$$A(5) = 500\left(1 + \frac{.06}{12}\right)^{5\times 12} = 674.43.$$

   (b) the interest is compounded daily;

$$A(5) = 500\left(1 + \frac{.06}{365}\right)^{365\times 5} = 674.91$$

   (c) the interest is compounded continuously.

$$A(5) = 500\, e^{(.06)5} = 674.93.$$

4. Use properties of logs and exponential functions to evaluate the following:

(a) $\log_2(\sqrt{32}) + \log_2(16^{2/3})$ $= \log_2\left(2^{5/2}\right) + \log_2\left(2^{8/3}\right)$

$32 = 2^5$

$16 = 2^4$

$= \frac{5}{2} + \frac{8}{3} = \frac{31}{6}.$

(b) $100^{\log_{10}(3)}$ $= 10^{2\log_{10}(3)}$

$= 10^{\log_{10}(9)} = 9.$

5. Evaluate $\int_0^1 \frac{4^x}{4^x+1}dx.$

$u = 4^x + 1$

$du = 4^x \ln(4)\, dx.$

$= \frac{1}{\ln(4)} \int_2^5 \frac{1}{u}\, du = \frac{1}{\ln(4)} (\ln 5 - \ln 2).$

$= \frac{\ln(5/2)}{\ln(4)}.$

6. Rewrite the function $f(x) = x^{\ln(x)}$ and evaluate $f'(x)$.

$f(x) = e^{\ln\left(x^{\ln(x)}\right)} = e^{\ln(x)\ln(x)} = e^{(\ln(x))^2}$

$f'(x) = e^{(\ln x)^2} \cdot \left\{ 2\ln(x)\ \frac{1}{x} \right\}.$