§6.6–The Inverse Trigonometric Functions

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Outline

Definition

- The sine function is one-to-one on $[-\pi/2, \pi/2]$ and has range $[-1, 1]$ on this domain.
- We define $\sin^{-1}$ to be the inverse of sine on this domain on this domain. It follows that $\sin^{-1}$ has domain $[-1, 1]$ and range $[-\pi/2, \pi/2]$.

Cancellation equations

Because of these restrictions, we must be a little careful with the inverse relationships:

$$\sin\left(\sin^{-1}(x)\right) = x, \quad -1 \leqslant x \leqslant 1$$
$$\sin^{-1}\left(\sin(x)\right) = x, \quad -\pi/2 \leqslant x \leqslant \pi/2$$

Problem

Evaluate the following:

- $\sin\left(\sin^{-1}(.3)\right)$
- $\sin^{-1}\left(\sin(14\pi/3)\right)$.

Problem

Show that $\cos\left(\sin^{-1}(x)\right) = \sqrt{1-x^2}$.

Problem

Find $\cos\left(2\sin^{-1}(1/4)\right)$.

The inverse sine function The inverse cosine function The inverse tangent function The other inverse trig functions Miscellaneous pr

Theorem

$$D_x \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}.$$

Problem

Find the derivative of $y = x \sin^{-1}(x^2)$.

Definition

- The cosine function is one-to-one on the interval $[0, \pi]$ and has range $[-1, 1]$ on that domain.
- Let $\cos^{-1}$ denote the inverse of the cosine function restricted to the domain $[0, \pi]$. Thus the domain of $\cos^{-1}$ is $[-1, 1]$ and its range is $[0, \pi]$.

The cancellation equations

$$\cos\left(\cos^{-1}(x)\right) = x, \quad -1 \leqslant x \leqslant 1$$
$$\cos^{-1}\left(\cos(x)\right) = x, \quad 0 \leqslant x \leqslant \pi$$

Problem

- *Evaluate* $\cos^{-1}\left(\cos(14\pi/3)\right)$
- *Show that* $\sin\left(\cos^{-1}(x)\right) = \sqrt{1-x^2}$

Theorem

$$D_x \cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

Definition

- The tangent is one-to-one on the interval $(-\pi/2, \pi/2)$ and has range $(-\infty, \infty)$ on this domain.
- Let $\tan^{-1}$ be the inverse of the tangent function on this restricted domain. Thus the domain of $\tan^{-1}$ is $(-\infty, \infty)$ and its range is $(-\pi/2, \pi/2)$.

The cancellation equations

$$\tan\left(\tan^{-1}(x)\right) = x \quad -\infty < x < \infty$$
$$\tan^{-1}\left(\tan(x)\right) = x \quad -\pi/2 < x < \pi/2.$$

Problem

Show that $\sec^2\left(\tan^{-1}(x)\right) = 1 + x^2$

Theorem

$$D_x \tan^{-1}(x) = \frac{1}{1+x^2}, \quad x \in \mathbb{R}.$$

The other inverse trig functions

The remaining inverse trig functions ($\cot^{-1}$, $\sec^{-1}$, and $\csc^{-1}$) are defined in similar ways. See page 287 of the text for a summary of these functions, their definitions, and derivatives.

Problem

Find y' in each case:

- $y = \tan^{-1}(e^x)$
- $y = \sqrt{1-x^2}\sin^{-1}(x)$
- $y = \sin^{-1}(x) + \cos^{-1}(x)$

Basic integration formulas

- $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + C$
- $\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + C$

Problem

Why is there no formula involving $\cos^{-1}(x)$?

Problem

Evaluate the following integrals:

- $\int \frac{\tan^{-1}(x)}{1+x^2}dx$
- $\int_0^{\pi/2} \frac{\sin(x)}{1+\cos^2(x)}dx$
- $\int_0^{\pi/2} \frac{1}{\sqrt{4-t^2}}dt$
- $\int \frac{1}{a^2+x^2}dx$, *where* a *is any real number.*