Assignment #1 **Name** Answer Key.

Due 16 January 2015

1. Let $f(x) = x^2 + 5$.

(a) Show that $f(x)$ is one-to-one on the restricted domain $D = [0, \infty)$.

$f'(x) = 2x$ $\quad - \;|\; +$ $\quad$ 0 $\quad f'(x) > 0$ on $(0, +\infty)$.

3 $\quad$ f is increasing on $[0, +\infty)$

thus f is one-to-one.

(b) What is the range of f on the set D? $\quad R = [5, +\infty)$.

(c) Let f^{-1} denote the inverse of f on the set D. Find an algebraic formula for $f^{-1}(x)$. What are the domain and range of f^{-1}.

$x = y^2 + 5$

$x - 5 = y^2$

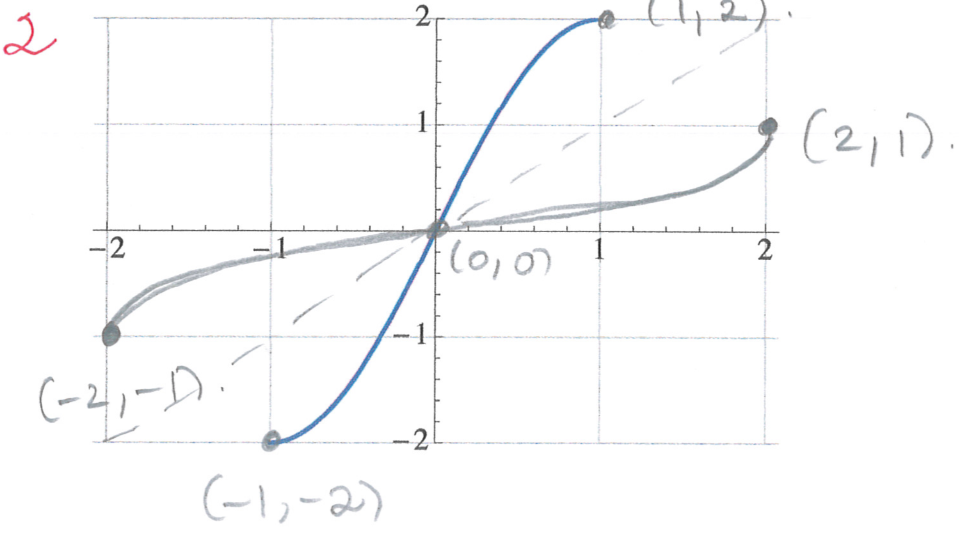
$y = \pm\sqrt{x-5}$.

The range of f^{-1} is $[0, +\infty)$, the domain of f; thus,

$f^{-1}(x) = \sqrt{x-5}$!

2. The graph of a function $y = f(x)$ is pictured below. Graph $y = f^{-1}(x)$ on the same axes.

2



3. Let f be a one-to-one and invertible function whose graph contains the point $P(1,3)$. If the tangent line to the curve $y = f(x)$ at P is given by the equation $y = \frac{1}{2}x + \frac{5}{2}$, then find $(f^{-1})'(3)$.

2 $(f^{-1})'(3) = \dfrac{1}{f'(1)}$, since $f(1) = 3$.

But $f'(1) = \frac{1}{2}$, according to the slope of the tangent line. Thus

$$(f^{-1})'(3) = \frac{1}{(1/2)} = 2.$$

4. Let $f(x) = x^5 + x^3 + 2x$.

3 (a) Show that f is invertible and find $f^{-1}(-4)$.

$$f'(x) = 5x^4 + 3x^2 + 2 \geq 2.$$

Thus f' is increasing on $(-\infty, \infty)$ hence one-to-one.

$$f(x) = x^5 + x^3 + 2x = -4$$

has $X = -1$ as its solution $\therefore$ $f^{-1}(-4) = -1$

(b) Evaluate $(f^{-1})'(-4)$.

$$\boxed{(f^{-1})'(-4) = \frac{1}{f'(-1)} = \frac{1}{10}.}$$

<u>side calculation:</u>

$$f'(-1) = 5(-1)^4 + 3(-1)^2 + 2$$

$$= 5 + 3 + 2$$

$$= 10$$