# §6.7–Hyperbolic Functions

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## Outline

## Question

What is the shape of a hanging chain?



## Definition

Let

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad \text{and} \quad \cosh(x) = \frac{e^x + e^{-x}}{2}.$$

$\sinh(x)$ and $\cosh(x)$ are called the *hyberbolic sine* and *hyperbolic cosine* function respectively.

## Definition

The other four hyperbolic trig functions are defined through the sinh and the cosh:

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} \qquad \operatorname{sech}(x) = \frac{1}{\sinh(x)}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} \qquad \coth(x) = \frac{\cosh(x)}{\sinh(x)}$$

The catenary problem | The six hyperbolic functions | Derivatives

## Problem

*Graph the* $\sinh(x)$, $\cosh(x)$, *and* $\tanh(x)$.

## Problem

- *Show that* $\cosh^2(x) - \sinh^2(x) = 1$
- *Show that* $\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$
- *Evaluate the limits as* $x \to \pm\infty$ *in the function*

$$f(x) = 3\cosh(x) + 2\sinh(x).$$



## Problem

- *Show that* $D_x \sinh(x) = \cosh(x)$
- *Show that* $D_x \cosh(x) = \sinh(x)$*.*

## Note

The formulas for the remaining four hyperbolic functions can be found on page 464.

## Problem

*Find $y'$ in each case:*

- $y = \cosh(\ln(x))$
- $y = 2^{\sinh(x)}$
- $y = \cosh(\sinh(x))$