§11.11–Applications of Taylor Polynomials

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Outline

The Taylor Polynomial

Given a function f and a point a, recall that

$$f(x) = T_N(x) + R_N(x),$$

where T_N is the Taylor polynomial of order N centered at a point a and R_N is the remainder.

The remainder

1. The polynomial $T_N(x)$ approximates $f(x)$; the error in the approximation is

$$|f(x) - T_N(x)| = |R_N(x)| = \frac{|f^{(N+1)}(z)|}{(N+1)!}|x-a|^{N+1},$$

where z is a number between a and x.

2. If M is a number bounding $|f^{(N+1)}(z)|$, then

$$|f(x) - T_N(x)| \leqslant \frac{M}{(N+1)!}|x-a|^{N+1}.$$

Problem

Use a Taylor polynomial of order 2 centered at $a = 4$ to estimate $\sqrt{4.1}$. Use the remainder to bound the error in this approximation.

Problem

Use Newton's binomial theorem to estimate $\sqrt{4.1}$. Bound the error in your estimate.

Remark

1. Often it is not enough to approximate a function f by a Taylor polynomial at a single point. In certain applications we need to approximate f by a Taylor polynomial across an interval.
2. The corresponding estimate of the error in the approximation must hold throughout the interval. In other words we must bound $|f(x) - T_N(x)|$ simultaneously for all x in an interval.

The problem

Approximate the function $f(x)$ by the Taylor polynomial $T_N(x)$ centered at a. How accurate is this approximation when $x \in I$, where I is an interval containing a?

Problem

Approximate $f(x) = \sin(x)$ by a Taylor polynomial of order 6 centered at $a = 0$. Estimate the error in this approximation when $x \in [-.5, .5]$.

Problem

Approximate $f(x) = \sqrt{x}$ by a Taylor polynomial of order 3 centered at $a = 9$. Estimate the error in this approximation when $x \in [7.5, 9.5]$.