Brown University

Car #24

FSAE Michigan 2024

Brakes

BROWN FORMULA RACING

2023 Subsystem Design Binder

Subsystem Leads: Brady Garrison

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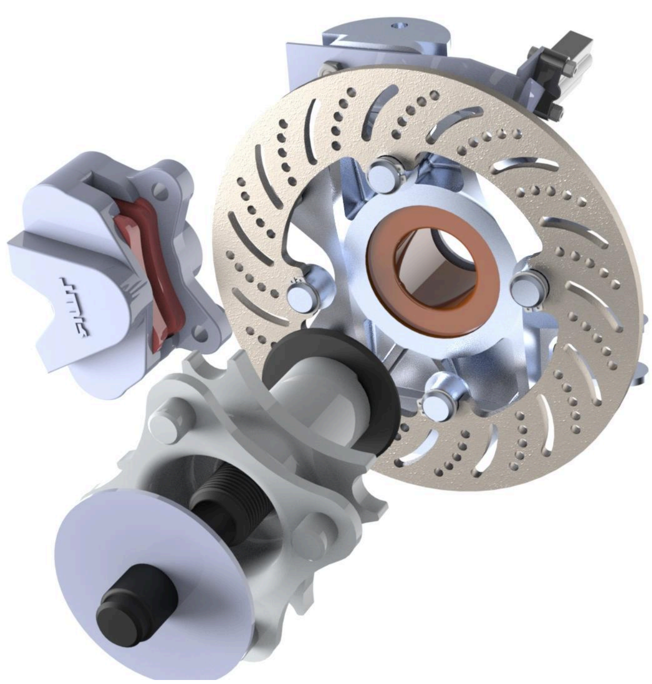
Introduction

The Braking system of Brown University's 2024 FSAE vehicle, Rhode Rage, has been designed with ease of manufacturing and braking performance as the primary considerations. This year's design includes a new brake rotor slotted vent pattern to optimize the thermal and structural loads upon the brake rotors. In addition, new master cylinder bore sizes were selected to increase the front bias of our braking system from previous years. Finally, the brake lines were custom made to route through the other subsystems in this year's vehicle.

The design process for this subsystem started with first principles calculations. By considering the weight distribution of the car, we can determine the pressures necessary for the brakes to stop the car. We can also select master cylinders, rotor diameters, brake bias, and pedal ratio that will allow the car to operate as expected. These calculations were performed in a matlab live script.

Following these calculations, the subsystem was designed in Solidworks. Ansys FEA was used to validate design decisions for the brake rotors. The design was then manufactured and assembled upon the car.

Thorough testing and sensor validation was performed to ensure that the subsystem is performing as expected.



BROWN FORMULA RACING

Rules and Constraints

The following are the key takeaways from this year's rules for our subsystem:

- Must have a braking system that acts on and is capable of locking all four wheels with one input, must have two hydraulic circuits with independent reservoirs
- Must have an aluminum or steel brake pedal that can withstand 2000N
- Must not have failure of non-load bearing components not interfere with the operation of the system
- Must have a brake over travel switch that stops the operation of the car if the brake pedal goes outside of the designed range of motion
- Must have brake light that is visible in sunlight and red on a black background.

The full rules are as follows:

T.3 BRAKE SYSTEM

T.3.1 Mechanical

T.3.1.1 The vehicle must have a braking system

T.3.1.2 The braking system must:

a. Act on all four wheels

b. Be operated by a single control

c. Be capable of locking all four wheels

T.3.1.3 The braking system must have two independent hydraulic circuits.

A leak or failure at any point in the system will maintain effective braking power on minimum two wheels.

T.3.1.4 Each hydraulic circuit must have its own fluid reserve using separate reservoirs or an OEM style reservoir

T.3.1.5 A single brake acting on a limited slip differential may be used.

T.3.1.6 "Brake by Wire" systems are prohibited.

T.3.1.7 Unarmored plastic brake lines are prohibited.

T.3.1.8 The braking systems must be protected with scatter shields from failure of the drive train (see **T.5.2**) or from minor collisions.

T.3.1.9 In side view any portion of the brake system that is mounted on the sprung part of the vehicle must not project below the lower surface of the chassis

T.3.1.10 Fasteners in the Brake system are **Critical Fasteners**, see **T.8.2**

T.3.1.11 The brake pedal must be one of:

- Fabricated from steel or aluminum
- Machined from steel, aluminum or titanium

T.3.1.12 The brake pedal and associated system components design must withstand a minimum force of 2000 N without any failure of the brake system, pedal box, chassis mounting, or pedal adjustment

This is not a design criteria. The brake system may be tested by pressing the pedal with the maximum force that can be exerted by any official when seated normally.

T.3.1.13 Failure of non-loadbearing components in the braking system or pedal box must not interfere with brake pedal operation or brake system function

T.3.1.14 (EV only) Additional requirements for Electric Vehicles:

a. The first 90% of the brake pedal travel may be used to regenerate brake energy without actuating the hydraulic brake system.

b. The remaining brake pedal travel must directly actuate the hydraulic brake system. Brake energy regeneration may remain active.

T.3.2 Brake Over Travel Switch - BOTS

T.3.2.1 The vehicle must have a Brake Over Travel Switch (BOTS). Brake pedal travel exceeding the normal range will actuate the switch

T.3.2.2 The BOTS must be a mechanical single pole, single throw (commonly known as a two position) switch (push-pull or flip type).

T.3.2.3 Operation of the BOTS to the OFF position must Open the Shutdown Circuit **IC.9.2.2 / EV.8.2.2**

T.3.2.4 Repeated operation of the switch must not reset or restore power

T.3.2.5 The driver must not be able to reset the BOTS.

T.3.2.6 The switch must be implemented with analog components, and not using programmable logic controllers, engine control units, or similar functioning digital controllers.

T.3.3 Brake Light

T.3.3.1 The vehicle must have a Brake Light that is clearly visible from the rear in very bright sunlight.

T.3.3.2 The Brake Light must be:

a. Red in color on a Black background

b. Rectangular, triangular or near round shape with a minimum shining surface of 15 cm^2

c. Mounted between the wheel centerline and driver's shoulder level vertically and approximately on vehicle centerline laterally.

T.3.3.3 When LED lights are used without a diffuser, they must not be more than 20 mm apart.

BROWN FORMULA RACING

Calculations

Overview

First principles calculations were performed for a braking system with two independent braking circuits for the front and rear to determine predicted pressures for the front and rear braking systems, master cylinder selection, pedal ratio, and bias bar adjustment.

The key steps, inputs, outputs, and equations for these calculations are shown in the following table.

Calculation Step	Inputs	Important Equations	Outputs	Explanation
Weight distribution	Length of car, L = 1.57 m Height of COM, h = 0.304 m Mass of car, m = 274.87 kg Tire CoF, µ = 1.2	$W_x = \frac{F h_{com}}{L}$ $W_{wheel} = \frac{W_{F/R} \pm W_x}{2}$ $W_x = \frac{F h_{com}}{L}$ $W_{wheel} = \frac{W_{F/R} \pm W_x}{2}$	Braking force F = 3235.8N Longitudinal Weight transfer W_x = 626.29 N Front Wheel Dynamic Weight F_F = 980.6 N, Rear Wheel Dynamic Weight F_R = 367.66 N	When a car breaks, a force is induced in the opposite direction of the acceleration, called longitudinal weight transfer, W_x. This induces a moment that causes more weight to be placed onto the front tires. We can calculate this dynamic weight and use it as inputs into the next steps of the calculation.
Braking Torque, Line Pressure	Caliper piston area, A_caliper = 0.99 in^2 Wheel radius, r_wheel = 6.93" Pad CoF, µ_pad = 0.45	$T_{wheel} = F_{wheel} \times r_{wheel}$ $F_{clamp} = \frac{T_{wheel}}{2 \times \mu_{pad} \times r_{wheel}}$ $P_{fluid} = F_{clamp} \times A_{caliper}$	Front Pressure= 42.53 bar Rear Pressure = 15.94 bar Clamping force = 2716.8 N (Input to Ansys)	We can find the torque exerted on each brake wheel,find the clamping force of the caliper, and then find the pressure in the front and rear brake lines
Master cylinder selection	Master Cylinder Bore Sizes, converted to areas, A_front and A_rear Pressure of front and rear lines P_front and P_rear	$P_{front} : P_{rear} = \frac{P_{front}}{P_{front} + P_{rear}}$ $A_{front} : A_{rear} = 1 - \frac{A_{front}}{A_{front} + A_{rear}}$ $F_m = P_{fluid} \times A_{mc}$	Front Bore Selection: 15.9mm Rear bore Selection: 25.4 mm	By using the fluid pressure to the front and rear lines calculated in the previous step, we can calculate the ratio of front to rear pressure. This is inversely proportional to the bore sizes of the master cylinders. So, by comparing these ratios we selected our bore sizes.
Brake Pedal Ratio and Bias Bar Position	Max driver force, estimated to be approximately 450 N	$F_{bias} = F_{mf} + F_{mr}$ $R_{pedal} = F_{bias}/F_{driver}$	Force on Bias Bar: Predicted bias bar adjustment: 6cm Minimum pedal ratio: 3.68	We can calculate the amount of force distributed across the bias bar and the pedal ratio necessary for

Matlab Live Script

First, input properties of the vehicle. These include the mass of the car, Center of Mass information, coefficient of friction of the brake pads and tires, and the radius of the wheels,

```
%% Car Input Variables
    m = 606* 0.453592 %Mass of car, kgs

    W_Car = m * 9.81 %Weight of car N (with driver?)

    Total_L = 62* 0.0254 %Car length m

    r_L = 30.693* 0.0254 ; %Length from rear axle to COM m
    f_L = Total_L - r_L; %Length from front axle to COM m
    h = 12.0* 0.0254 %Height of COM m

    W_R = W_Car * f_L/Total_L % Front Load

    W_F = W_Car - W_R %Rear Load

    mu = 1.2; %Coefficient of friction between tire and road
    mub = 0.5; %Coefficient of friction between pad and disc (assuming cast
iron/steel)
    rwheel = 8 * 0.0254; %Wheel rolling radius m
```

When a car breaks, the decceleration induces an inertial force in the opposite direction. This induces a moment that causes more weight to be placed onto the front tires.

We can calculate the total braking force for the car, weight on the front and rear, and longitudinal weight transfer using the following equations, where F is braking force, W is weight, and mu is the tire coeff

$$F = W \times \mu$$

$$F_{F/R} = F\frac{L_{F/R}}{L}$$

```
    %% Car Force Calculations
    F_brake = W_Car * mu %Braking Force
```

Then, we can find the weight transfer to the front and rear tires using the equation for longitudinal weight transfer:

$$W_x = \frac{F\,h_{com}}{L}$$

$$W_{wheel} = \frac{W_{F/R} \pm W_x}{2}$$

This is added to the force upon the front wheels and subtracted from the force on the rear wheels to account for weight transfer. We divide by two to consider the distribution of force across two tires.

```
Wx = F_brake*h/Total_L


W_Fwheels = (W_F+Wx)/2 %Braking Load Front Wheels

W_Rwheels = (W_R-Wx)/2 %Braking Load Rear Wheels
```

The ratio between these values will be the front and rear braking force ratios for the car, in this case 0.73:0.27

```
f_weight_ratio = W_Fwheels/(W_Fwheels + W_Rwheels)

r_weight_ratio = W_Rwheels/(W_Fwheels + W_Rwheels)
```

We can then find the front and rear braking forces and torques.

$$T_{wheel} = F_{wheel} \times r_{wheel}$$

```
F_Fwheels = W_Fwheels * mu; %Braking Force Front Wheels
F_Rwheels = W_Rwheels * mu; %Braking Force Rear Wheels

T_F = F_Fwheels*rwheel; %Front braking torque, Nm
T_R = F_Rwheels*rwheel; %Rear braking torque, Nm
```

Now, we can actually input the parameters that can be adjusted to determine braking pressure

$$F_{clamp} = \frac{T_{wheel}}{2 \times \mu_{pad} \times r_{wheel}}$$

$$P_{fluid} = F_{clamp} \times A_{caliper}$$

```
%% Input Rotor sizes Here
    %Constraints: Rotor Diameters must range from 6in to 7in in order to fit in
    %the caliper, _ in order to fit in the wheel
    rfront = 3.465 * 0.0254; %Front brake pad center radius m
    rrear = 3.465 * 0.0254; %Rear brake pad center radius m

    Fcf = T_F/(2*rfront*mub) %Front clamping force, N

    Fcr = T_R/(2*rrear*mub) %Rear clamping force, N
```

```
%% Input Caliper Sizes Here
Asf = 0.99 * 0.00064516; %Front caliper cylinder area m^2
Asr = 0.99 * 0.00064516; %Rear caliper cylinder area m^2

Pf = Fcf/Asf %Front fluid pressure, N/m^2, needs to be less than 7 * 10^6 for safety

Pr = Fcr/Asr %Rear fluid pressure, N/m^2, needs to be less than 7 * 10^6 for safety


Pf_bar = Pf/ 100000

Pr_bar = Pr/ 100000
```

The ratio of the front braking pressure to rear braking pressure is as follows:

$$P_{front} : P_{rear} = \frac{P_{front}}{P_{front} + P_{rear}}$$

We can compare this ratio to the inverse of the ratio between the master cylinder bore sizes, as this will approximate the difference in pressure between the master cylinders:

$$A_{front} : A_{rear} = 1 - \frac{A_{front}}{A_{front} + A_{rear}}$$

Using this relationship, we can do a comparative analysis of how different ratios of the available bore sizes for our master cylinder of choice, the AP Racing CP7855 Model, compare to the ratio of expected fluid pressures. By using a comparative metric, such as mean squared error, we can see which combinations result in the lowest difference between these ratios.

```
bore_sizes = [14, 15, 15.9, 16.8, 17.8, 19.1, 20.6, 22.2, 23.8, 25.4]/1000;
front_cylinder_areas = pi * (bore_sizes/2).^2; %m^2
rear_cylinder_areas = pi * (bore_sizes/2).^2; %m^2

xvalues = {'14', '15', '15.9', '16.8', '17.8', '19.1', '20.6', '22.2', '23.8', '25.4'};
yvalues =  {'14', '15', '15.9', '16.8', '17.8', '19.1', '20.6', '22.2', '23.8', '25.4'};

data = zeros(length(front_cylinder_areas),length(rear_cylinder_areas));
f_pressure_ratio = Pf/(Pf + Pr)

for f_area_index = 1:length(front_cylinder_areas)
    for r_area_index = 1:length(rear_cylinder_areas)
        f_cylinder_A = front_cylinder_areas(f_area_index);
        r_cylinder_A = rear_cylinder_areas(r_area_index);
```

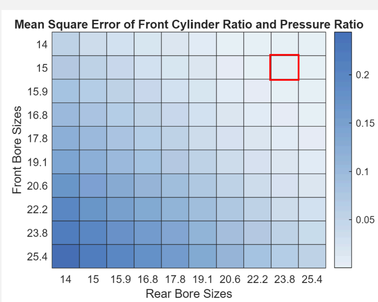
```
            f_cylinder_ratio = 1 - (f_cylinder_A/(f_cylinder_A +
r_cylinder_A));
            MSE = mean((f_cylinder_ratio - f_pressure_ratio).^2);
            data(f_area_index,r_area_index) = MSE;
        end
    end
    h = heatmap(xvalues,yvalues,data)

    h.Title = 'Mean Square Error of Front Cylinder Ratio and Pressure Ratio';

    h.XLabel = 'Rear Bore Sizes';
    h.YLabel = 'Front Bore Sizes';

    data(1,1)

    f_pressure_ratio = Pf/(Pf + Pr)

    f_cylinder_A = front_cylinder_areas(1)

    r_cylinder_A = rear_cylinder_areas(1)

    f_cylinder_ratio = 1 - (f_cylinder_A/(f_cylinder_A + r_cylinder_A))

    MSE = mean((f_cylinder_ratio - f_pressure_ratio).^2)
```

We can then calculate the force on a given master cylinder with our selected master cylinder sizes by mulitplying the fluid pressure by the area of the master cylinder:

$$F_m = P_{fluid} \times A_{mc}$$

```
    %% Input Master Cylinder Sizes Here
    %Constraints: Only the following bore sizes, and therefore areas are
    %available:

    bore_sizes = [14, 15, 15.9, 16.8, 17.8, 19.1, 20.6, 22.2, 23.8, 25.4]/1000;
    cylinder_areas = pi * (bore_sizes/2).^2; %m^2
    f_cylinder_A = cylinder_areas(3)

    r_cylinder_A = cylinder_areas(8)

    f_cylinder_bore  = bore_sizes(3)*1000

    r_cylinder_bore = bore_sizes(8)*1000
```

```
%1- because smaller area leads to greater pressure
%Choose a cylinder ratio slightly below pressure ratio so that the rest
%can be handled by the pedal and bias
f_pressure_ratio = Pf/(Pf + Pr)

r_pressure_ratio = Pr/(Pf + Pr)


f_cylinder_ratio = 1 - (f_cylinder_A/(f_cylinder_A + r_cylinder_A))

r_cylinder_ratio = 1 - (r_cylinder_A/(f_cylinder_A + r_cylinder_A))



Fmf = Pf * f_cylinder_A; %Force on front master cylinder
Fmr = Pr * r_cylinder_A; %Force on rear master cylinder
```



The force upon the bias bar is the sum of the forces upon the front and rear master cylinders. By changing the relative position of the bias bar, we can make small adjustments to the bias of the braking system:

$F_{bias} = F_{mf} + F_{mr}$

Fb = Fmf + Fmr %Force on bias bar

Then, we can find the minimum required pedal ratio by dividing the bias force by the force of the driver's leg.

$R_{pedal} = F_{bias}/F_{driver}$

```
%% Input Driver Force to find Minimum Pedal Ratio
Fdriver = 350; %Max Driver Force, N
```

```
    PedalRatioMinimum = Fb/Fdriver
```

Finally, we can consider the effect of the bias bar on brake bias by considering how much the bias bar is adjusted to favor a particular size

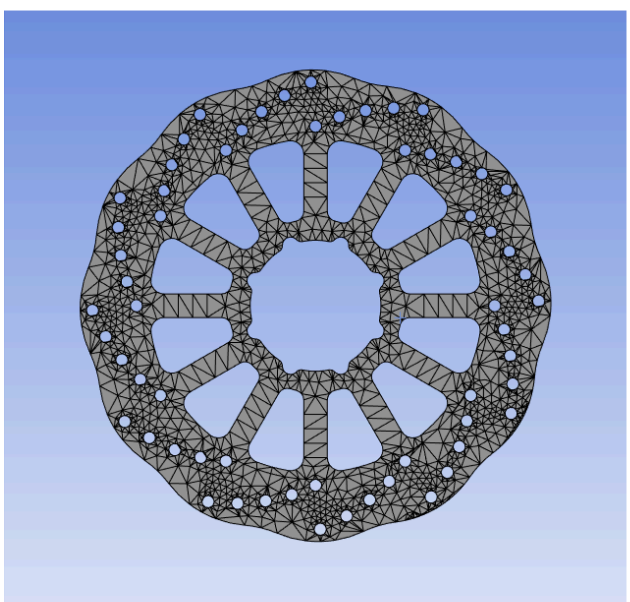
```
    %% Input Bias Bar Total Size To Find Bias Ratio
    Btotal = 4.75 * 0.0254; %Should be B1 + B2

    B1 = Btotal * Fmf/Fb %Bias bar length left m

    Btotal


    %B1/Btotal
    B2 = Btotal - B1; %Bias bar length right m
```

Past Designs

2022 Design

Flaws:

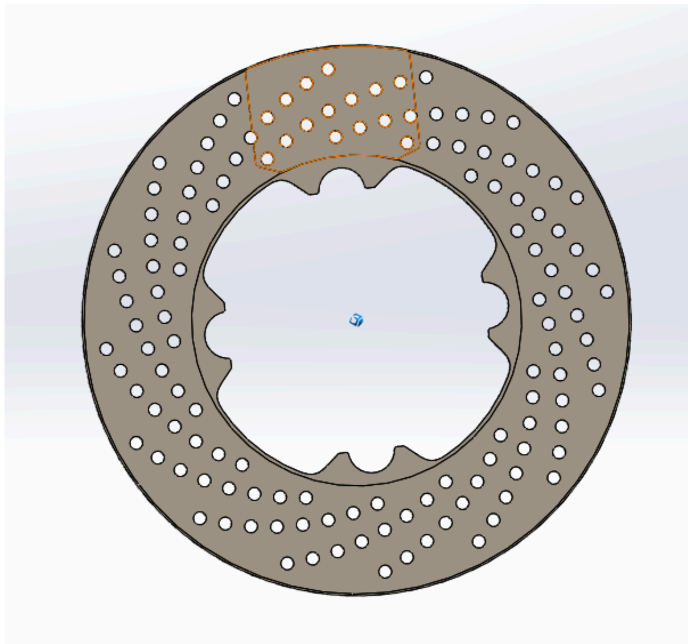
- Direct connection to wheel hub caused warping, issues with maintainability and removing brake rotors



2023 Design

Flaws:

- Poor management of brake lines, made placement into car and interaction with other subsystems difficult
- Poor justification of design with first principles, rotor hole design chosen arbitrarily



Current Design

Brake Rotors

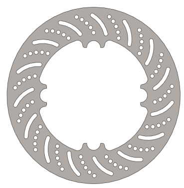
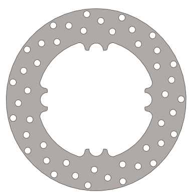
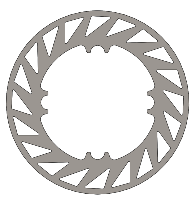
The Brake Rotors are the primary point of contact of the braking system. It is therefore extremely important to consider the materials selection and thermal/mechanical loads upon the brake rotors to ensure that they are designed in such a way that they can withstand the intense forces and temperatures of braking.

Selection of Brake Rotor Slots

Design constraints: 6.93” diameter in order to fit within wheels
Slots and Holes design chosen because it has the lowest temperature and highest factor of safety in Ansys simulations while maintaining a relatively low weight. Mounted by brake rotor pins to allow for thermal expansion

Design Philosophy: I wanted to test three designs with different surface area to volume ratios and masses. The first has a high SA:V ratio and middling mass. The second has a low SA:V ratio and high mass. The third has a high SA:V ratio and low mass.

Design Selection: The slots and holes design exhibited the lowest maximum temperature and highest factor of safety over the other options. It is also a weight reduction from previous designs.

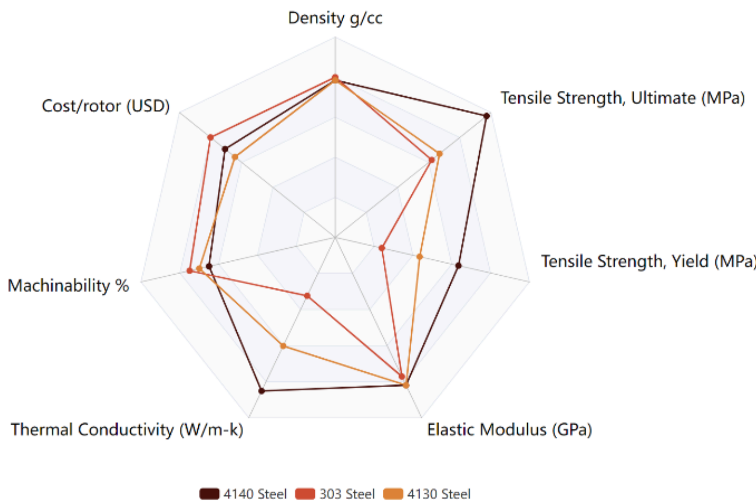
Design	Image	SA/V Ratio	Mass (g)	Max Temp (C)	Deformation (m)	Von Mises Stress (GPa)	Factor of Safety
Slots and Holes		0.65	449.80	282.23	1.429 E-6	11.72	0.018
Holes Only		0.56	492.04	437.22	3.277 E-6	22.72	0.036
Triangular Slots		0.65	383.59	398.8	6.37E-6	43.9	0.069

Brake Rotor Material Selection

Design Constraints: Lowest cost is best. Materials with high tensile strengths, elastic modulus, and thermal conductivity are best suited for rotor materials. This is because these materials best avoid the formation of cracks at higher temperatures and most likely to avoid failure.

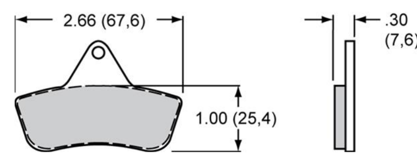
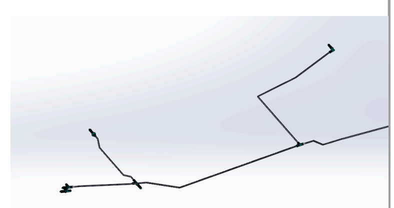
Design Selection: 4140’s higher tensile strength and thermal conductivity make it the clear choice. One drawback is that it is harder to machine than the other metals.

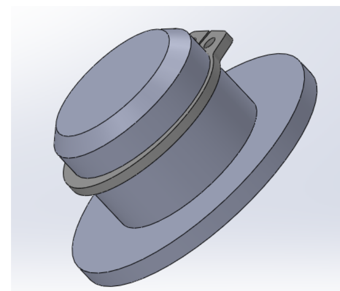
Source: Properties from Matweb Materials Database, costs from our source vendor Dix Metals for the size of our brake rotor design

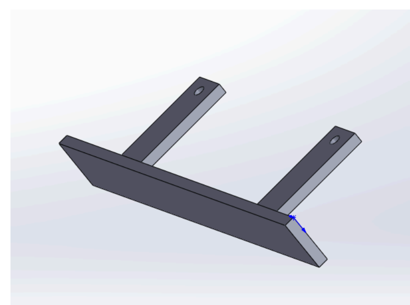


Brake Rotor Material			
Brake Rotor Material	4140 Steel	303 Steel	4130 Steel
Density g/cc	7.85	8.00	7.85
Tensile Strength, Ultimate (MPa)	972	620	670
Tensile Strength, Yield (MPa)	635	240	435
Elastic Modulus (GPa)	205	193	205
Thermal Conductivity (W/m-k)	42.6	16.2	30.1
Machinability %	65	75	70
Cost/rotor (USD)	176.80	200.20	161.00

Other Subsystem Components

Design Feature	Design Choice	Reasoning/Important Parameters	Image
Caliper / Brake Pad Selection	Wilwood PS1 Calipers Wilwood Disc Brake Pads 150-4091K	• Fits rotor diameters from 6”-9” and mounts well within wheel assembly • Two piston design provides necessary braking force • Cost effective compared to other options on the market and easily maintained and switched between wheels. Lightweight, 0.93 lbs. • Pad Coefficient of Friction: 0.45-0.52 • Caliper piston area 0.99 in^2	 
Master Cylinders	AP Racing CP7855	• Variety of bore sizes, allows for selection of ideal master cylinder ratio for pressure ratio of the system • Spherical bearing mounted to allow for easy mounting within the pedalbox	
Brake Lines	Aluminum Hardlines Dot4 Brake Fluid Steel AN -3 Connection Fittings	• AN -3 Flared to prevent leaks and allow for clean interfaces between brake lines • Hardlines chosen to minimize space, weight, and cost. • Hardlines allow for higher fluid pressure than softlines, ensure safe and fast braking • Custom fit to chassis specifications • Dot4 Brake Fluid - compatible with our calipers and lines, cheap.	 

Bias Bar	Tilton 72-250	● Allows for small adjustments to front and rear bias for a braking system using two master cylinders	
Brake Rotor Pins	Manufactured In- House	● Manufactured on the lathe with aluminum stock ● Groove radius: 0.26 in ● Head radius: 0.5 in ● Outer radius: 0.29 in ● Height: 0.38 in ● ● McMaster Retaining Snap Rings ● 0.55 in outer diameter ● 0.45 in inner diameter	

Brake Lights	RED LED Pressure Switch 3D printed Mounting	• Components chosen for rules compliance with minimum cost • 3D printed light mounting to ensure that brake lights are on a black background	  
Reservoirs	Plastic Reservoirs Remotely Mounted Tygon Tubing	• Reservoirs mounted directly to the chassis with custom fit mountings • Tygon plastic tubing chosen for strength, resistance to chemical erosion, and temperature resilience	

BROWN FORMULA RACING

Manufacturing

In House

- Brake rotors manufactured in house
    - Precision thickness stock courtesy Dix Metals. 0.19” stock with tolerances of +-0.001” chosen in order to ensure that rotors would have a smooth surface so that caliper braking force would be evenly distributed across the brake rotor.
    - Electric Discharge Machining manufacturing pipeline for high precision, +/- 0.0005”, outer edges necessary to ensure that brake rotors are perfectly round and do not get caught in calipers. Cost effective and reproducible.

Out of House

- Brake Lines
    - Necessary to ensure precision bending, make sure that the brake lines do not leak.

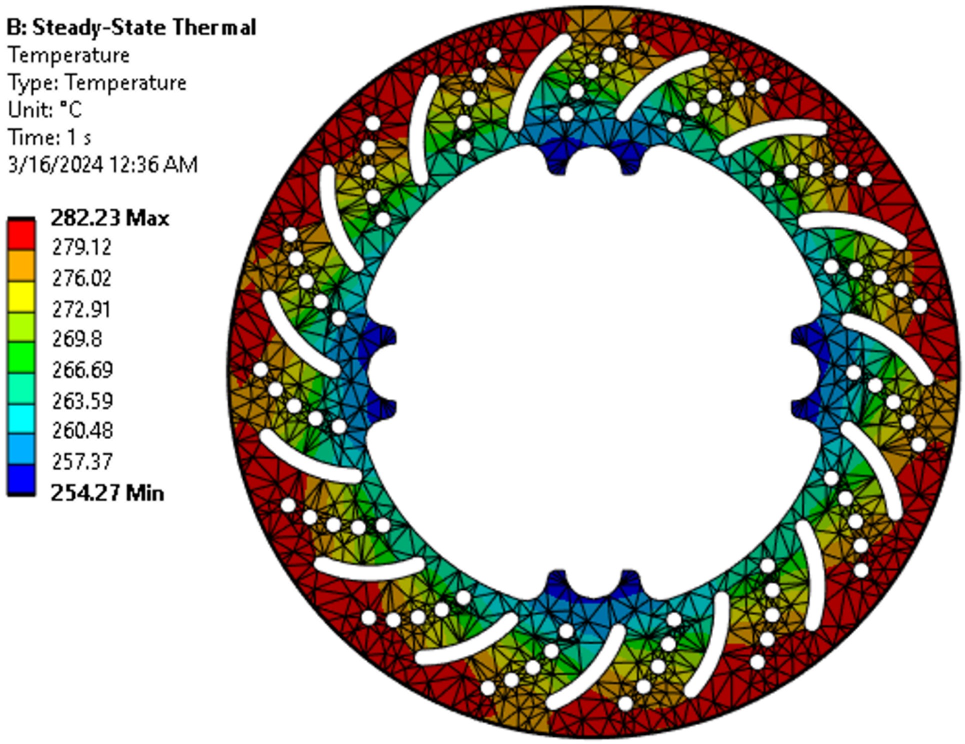
Drawings

Analysis / Finite Element Analysis

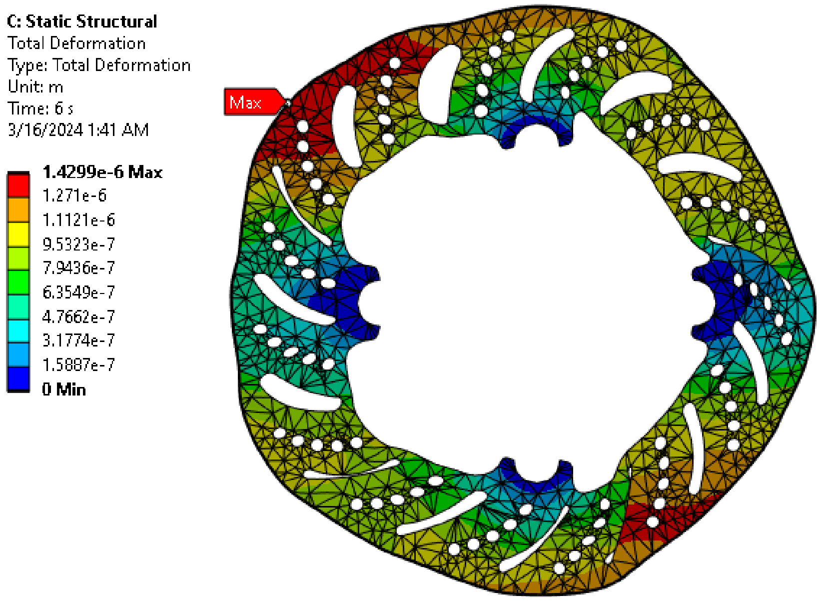
- FEA Analysis in Ansys for thermal and structural validation
    - Steady state thermal system
        - Heat Flux: 25000 W/m^2, ramped over the course of one second, chosen to simulate maximum temperature conditions. On the higher end of typical values to ensure that our brakes will maintain structural integrity under maximum temperature conditions. Convection: Through slots and holes. Film coefficient 5. W/m^2·°C. Ambient Temperature 22°C
    - Static structural simulation
        - Initial velocity 100 rads/s. 2716.8N forces from each brake pad and 1222.56 N (2716.8N * 0.45) force from friction.
        - Can determine von mises stress and deformation, used to calculate factor of safety: Factor of safety = Von mises stress/Yield Strength

Drilled and Slotted Vent Design (Final Design) Ansys Graphs

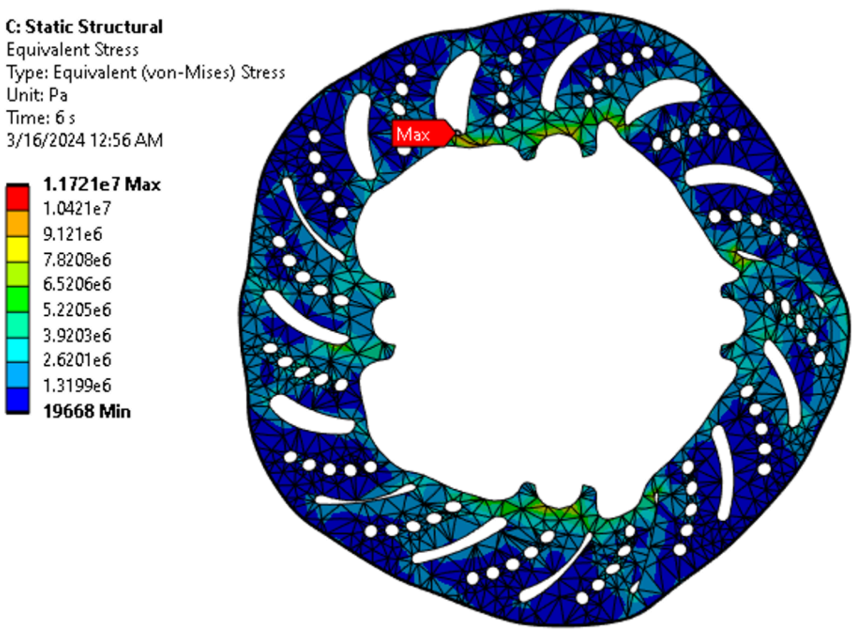
Steady State Thermal



Static Structural Simulation - Deformation

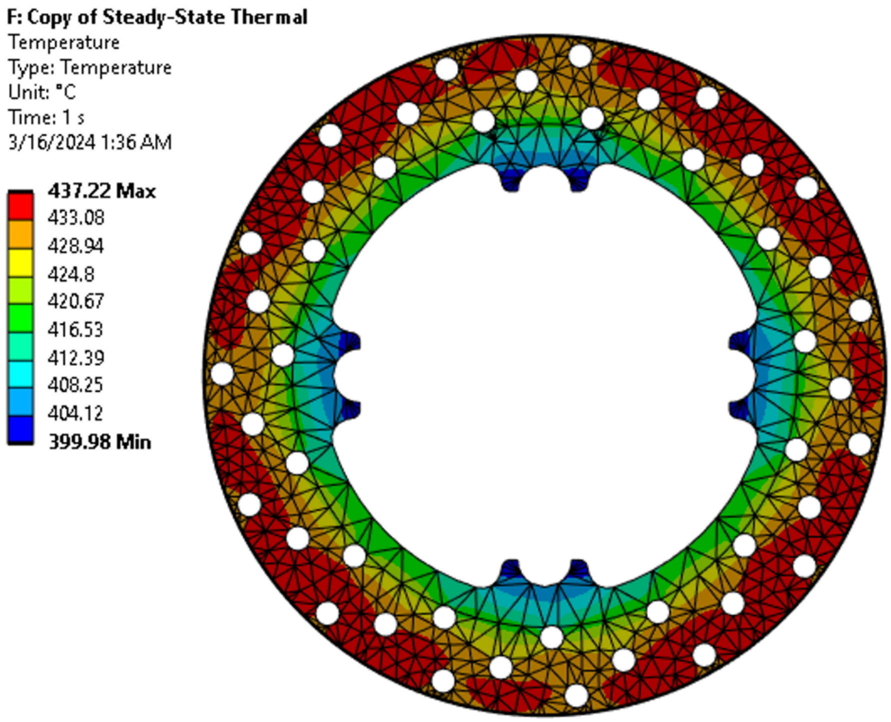


Static Structural Simulation - Von Mises Stress

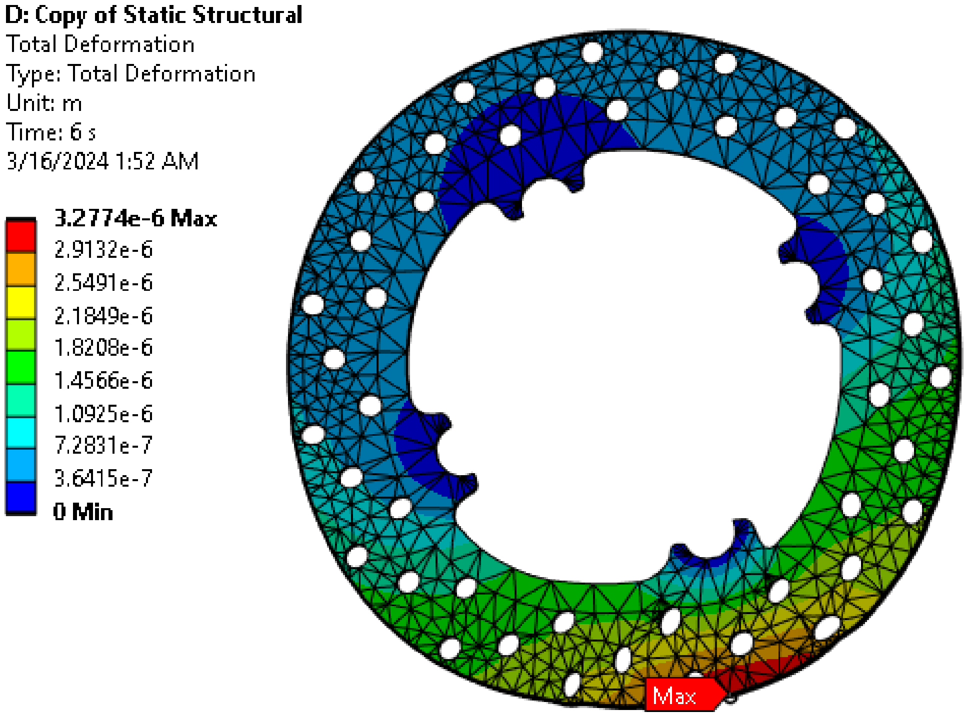


Drilled Vent Design Ansys Graphs

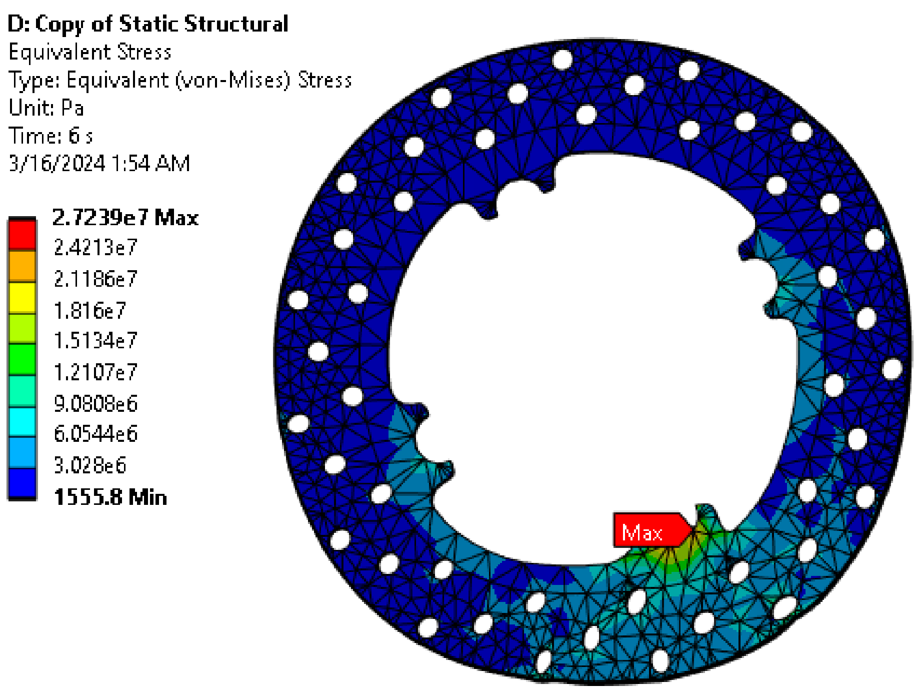
Steady State Thermal



Static Structural Simulation - Deformation

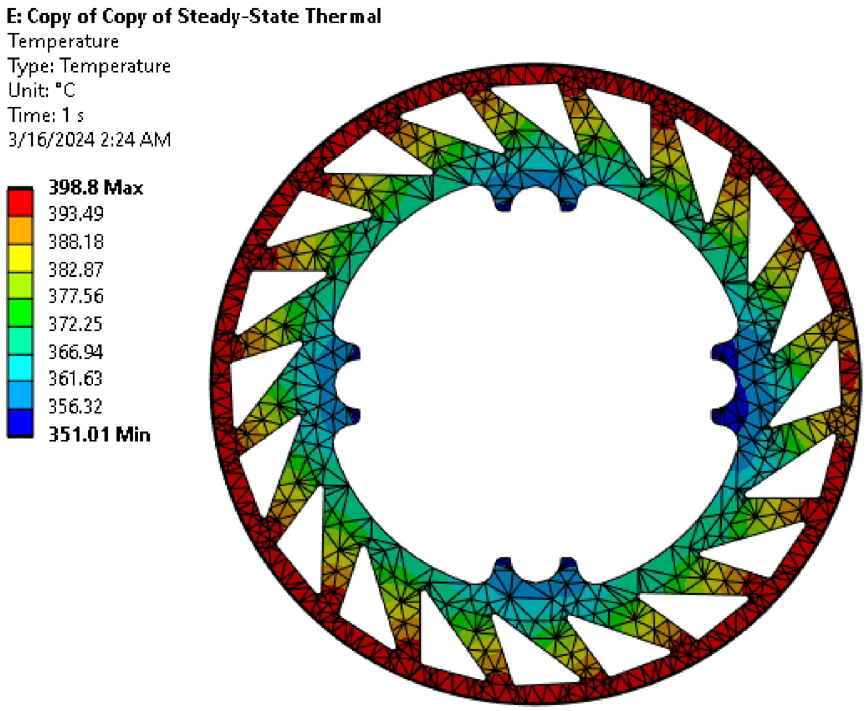


Static Structural Simulation - Von Mises Stress

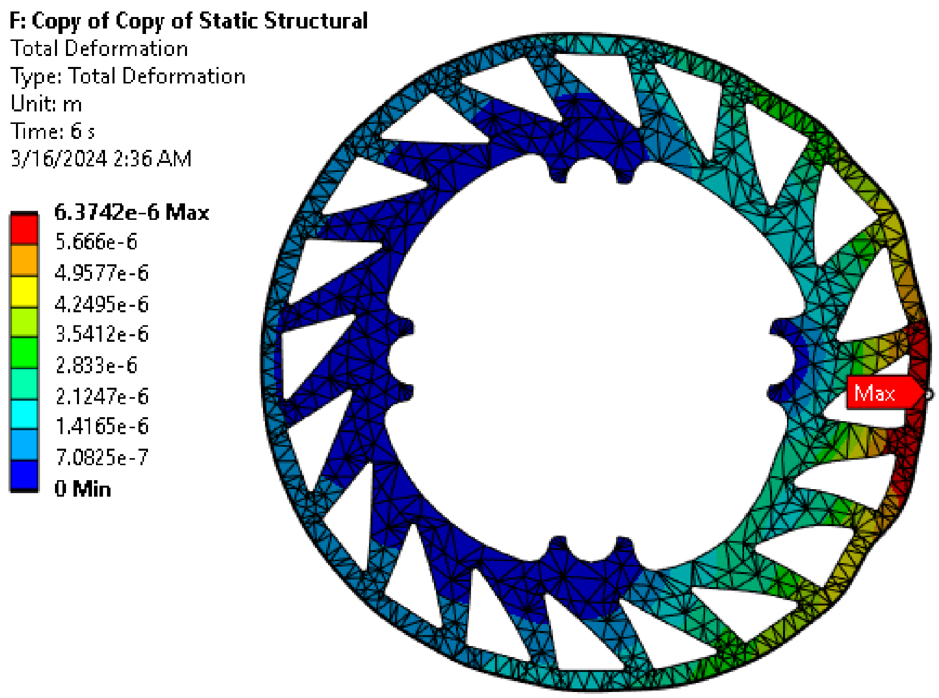


Triangular Vent Design Ansys Graphs

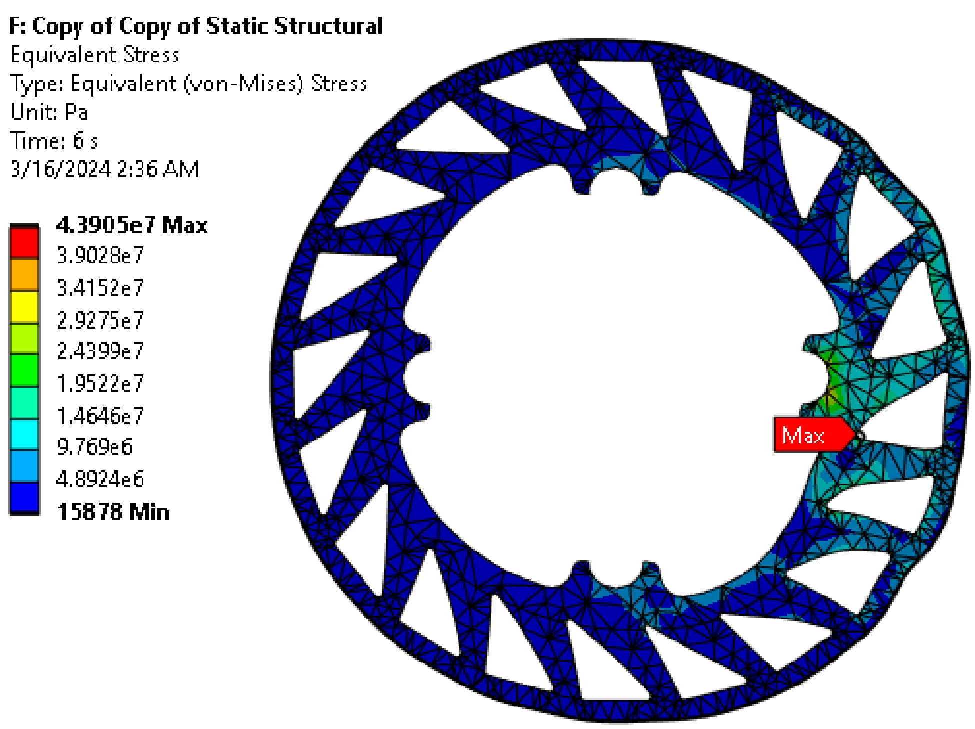
Steady State Thermal



Static Structural Simulation - Deformation

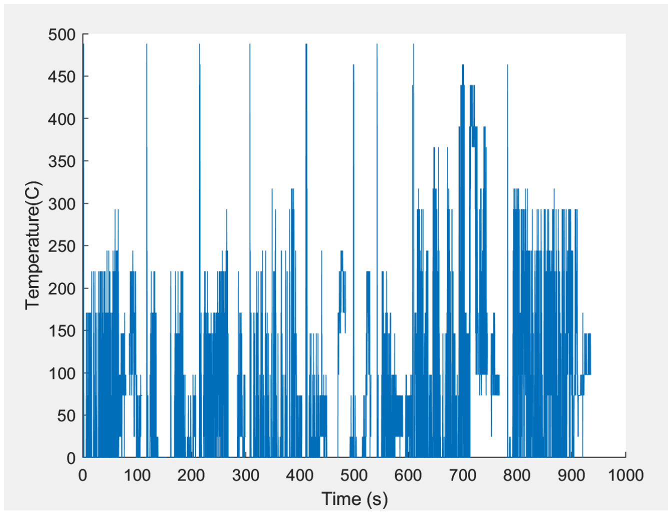


Static Structural Simulation - Von Mises Stress

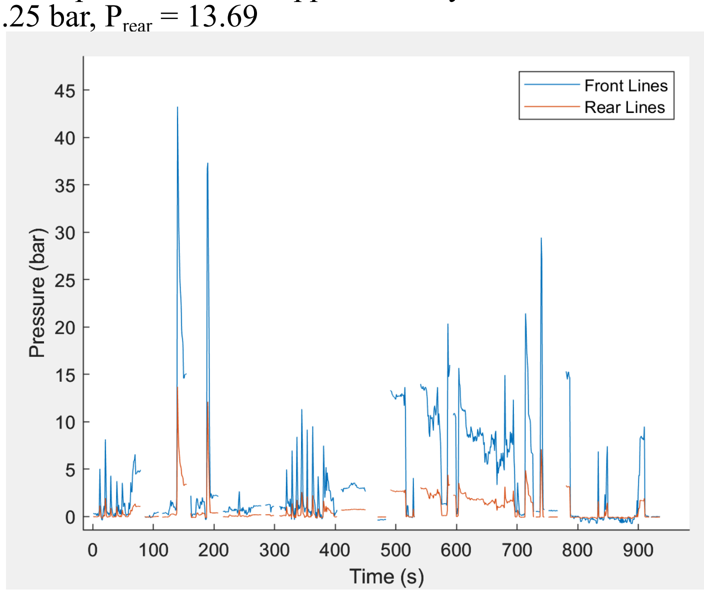


Validation

- Data collection to validate design matches simulation results
  - Thermal
    - Izzie IR Thermal Temperature Sensors and Type K Thermocouple Sensors. Data recorded over a short driving period (Figure 3)
    - Excluding outliers, temperatures stay in the 250-300 range, matching our estimates.



  - Pressure
    - AEM 2000 PSI Pressure Sensors for front and rear. Data recorded over a short driving period (Figure 4)
    - The maximum pressure values approximately match calculated values: P_{front} = 43.25 bar, P_{rear} = 13.69



  - Stopping distance - 26.82 m from 60 mph (from driving day footage)
    - 26.82 / 1,57 = 17.08 car lengths, typical is 18 from AA (https://www.theaa.com/breakdown-cover/advice/stopping-distances)

BROWN FORMULA RACING

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