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Decision Support

Variables sampling inspection scheme for resubmitted lots based on the process capability index C_{pk}

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ABSTRACT

This paper attempts to develop a sampling inspection scheme by variables based on process performance index for product acceptance determination, which examines the situation where resampling is permitted on lots not accepted on original inspection. The equations for plan parameters, the required sample size and the corresponding critical value, are derived based on the exact sampling distribution rather than an approximation approach hence the decisions made are more accurate and reliable. Moreover, the efficiency of the proposed variables resubmitted sampling plan is evaluated and compared with the existing variables single sampling plan. For illustrative purpose, an example is presented to demonstrate the use of the derived results for making a decision on product acceptance determination.

1. Introduction

Acceptance sampling is an important field of statistical quality control that was popularized by Dodge and Romig and originally applied by the US military to the testing of bullets during World War II. If every bullet was tested in advance, no bullets would be left to ship. If, on the other hand, none were tested, malfunctions might occur in the field of battle, with potentially disastrous results (Schilling, 1982; ISO 3951-1:2005, 2005; NIST, 2006). Acceptance sampling is "the middle of the road" approach between no inspection and 100% inspection. It has several advantages when acceptance sampling is contrasted with 100% inspection (see Montgomery (2009) for more details). Rather than evaluating all items, a specified sample is taken, inspected or tested, and a decision is made about accepting or rejecting the entire production lot. However, due to the lot disposition is based on sample results there is a probability of making an incorrect disposition of a lot. Incorrectly rejecting a lot that is really acceptable is called a Type I error. The risk of making a Type I error is called producer's risk or α-risk. Incorrectly accepting a lot that is really unacceptable is called a Type II error. The risk of making a Type II error is called consumer's risk or β-risk.

An acceptance sampling plan is a statement regarding the required sample size for product inspection and the associated acceptance or rejection criteria for lot disposition. It provides both producers and consumers to reserve their own rights by compromising on a rule to judge a batch of products. A well-designed sampling plan can significantly reduce the difference between the required and the actual supplied product quality. Vander Wiel and Vardeman (1994) provided a discussion of all-or-none inspection policies and examined the assumptions that lead to all-or-none optimality. This discussion can help analysts better understand the practical meaning and applicability of all-or-none results. In the literature, there are a number of acceptance sampling plans are developed for a wide variety of situations. Some recent references are Yeh Lam et al. (2006), Balamurali and Jun (2007), Chen et al. (2007), Aslam and Jun (2009), Seo et al. (2009) and Fernández et al. (2011).

The single acceptance sampling plan is the most popularly used plan due to simplicity in administration. In safety or consumer related testing of products or on preshipment inspection, usually a single sample is taken and tested. Whenever a lot is not accepted based on a single sample, the lot may be resubmitted after testing by the producer. In certain cases, the producer may dispute the first sample results and resampling the same number of units may be done under the provisions of the contract or statutes. In certain countries, tax on products is assessed based on a sample result and if the producer does not agree with the result, a second sample result will be used. If the producer disputes the first sample results, the second sample is tested and decision is taken discarding the first sample results. Even though it makes sense to combine the results of the two samples, as is done in the traditional double sampling plan, the practice in situations said above is to discard the

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first sample results. Often the producer has empirical knowledge about his product and he may like a resampling done (Govindaraju and Ganesalingam, 1997).

The ANSI/ASQC Standard A2 (1987) defined original inspection of a lot as the first inspection of a lot which has been resubmitted after previous nonacceptance. A resubmitted lot is defined in the standard as the one which has been designated as not-acceptable and which is submitted again for acceptance inspection after having been further tested, sorted, reprocessed, etc. If a lot is not accepted on original inspection, the producer may test it and may also resubmit it without sorting or reprocessing it for resampling. Govindaraju and Ganesalingam (1997) developed a resubmitted sampling scheme by attributes and examined the situation where resampling is permitted on lots not accepted on original inspection. It is assumed that during the course of resubmission, the quality of the lot is not improved by sorting or reprocessing.

Generally, numerical measurements of quality characteristics usually provide more information about the manufacturing process or lot than do attributes data. It is also to be pointed out that the sample size required by the attributes sampling plans is very large when acceptable quality levels (AQLs) for fraction of defectives are very small, such as measured in parts per million (PPM). Under these circumstances, there may be significant advantages in switching to variables sampling plans. Thus, as many manufacturers begin to emphasize allowable numbers of non-conforming parts per million (PPM), variables sampling plans become very attractive and necessary.

Recently, several variables single sampling plans have been developed by considering numerous process capability indices (PCIs) (see Arizono et al., 1997; Pearn and Wu, 2006a,b, 2007; Wu and Pearn, 2008; Yen and Chang, 2009; Negrin et al., 2009, 2011). However, to the best of our knowledge, the resubmitted sampling plan based on PCIs has not been well studied. Therefore, this paper attempts to develop a variables resubmitted sampling plan based on the most commonly capability index C_{pk} for product acceptance determination. That is, this paper examines the situation where resampling is permitted on lots not accepted on original inspection. Besides, it is worth to note that the proposed resubmitted sampling plan is developed based on the exact sampling distribution rather than an approximation approach hence the decisions made are more accurate and reliable.

The remainder of this paper is organized as follows. In Section 2, we first introduce the concept and definition of the process performance index C_{pk}, then the estimation of the index C_{pk} and its statistical properties are also presented. In Section 4, the operating procedure of the designed variables resubmitted sampling plan based on the index C_{pk} is presented. The operating characteristic (OC) function and the conditions for solving parameters of the proposed resubmitted sampling plan are also provided here. Furthermore, a detailed discussion and analysis is made in Section 5. In Section 6, an example is presented to demonstrate the use of the derived results for making a decision on product acceptance determination. Finally, some conclusions are given in the last section.

2. Process performance index

During the past decade, numerous capability indices, such as C_p, C_a, C_{pk} and C_{pm}, have been widely used to provide numerical measures on process performance in manufacturing industry and many applications. Kotz and Johnson (2002) presented a thorough review for the development of PCIs during the period 1992–2000. Wu et al. (2009) further discussed the more recent developments during the years 2002–2008 and provided an overview of theory and practice on PCIs for quality assurance from a different perspective. Recently, Yum and Kim (2011) summarized a bibliography of approximately 530 journal papers and books on PCIs for the period 2000–2009. The related literature is classified into four major categories, namely, books, review/overview papers, theory- and method-related papers, and special applications.

Assume that the quality characteristic X of a product item follows a normal distribution with mean μ and variance σ^2, denoted by $(N\mu\sigma^2)$. Let LSL and USL be the corresponding lower and upper specification limits. The capability index C_{pk} was developed by taking the magnitude of process variation as well as process location into consideration and defined as (Kane, 1986)

$$C_{pk} = \mathrm{Min}\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right\} = \frac{d-|\mu-M|}{3\sigma}, \quad (1)$$

where $d = (USL - LSL)/2$ and $M = (USL + LSL)/2$ are the half-length and the midpoint of the specification interval, respectively.

The index C_{pk} has been regarded as a yield-based index since it provides bounds on the process yield for a normally distributed process, i.e., $\Phi(3C_{pk}) - 1 \leqslant Yield < \Phi(3C_{pk})$ (Boyles, 1991), where $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard normal distribution. This implies one would expect that the process's fraction of defectives is at most 2700 PPM falling outside the specification limits with C_{pk} at level 1. At a C_{pk} level of 1.33, the fraction of defectives drops to 66 PPM. To achieve less than 0.544 PPM defect rate, a C_{pk} level of 1.67 is needed. At a C_{pk} level of 2.0, the likelihood of a defective part drops to 2 parts per billion (PPB). Thus, a minimum value of C_{pk} is usually specified in a purchasing contract. If the prescribed minimum C_{pk} fails to be met, the process is determined to be incapable. Otherwise, the process is considered capable.

The natural estimator of C_{pk}, denoted by $\widehat{C}_{pk}$ and defined below, is obtained by replacing the process mean μ and the process standard deviation σ by their sample estimators $\bar{x} = \sum_{i=1}^{n} x_i/n$ and $s = \left[\sum_{i=1}^{n}(x_i-\bar{x})^2/(n-1)\right]^{1/2}$. We note that the process must be demonstrably stable (under statistical control) in order to produce a reliable estimate of process capability

$$\widehat{C}_{pk} = \mathrm{Min}\left\{\frac{USL-\bar{x}}{3s}, \frac{\bar{x}-LSL}{3s}\right\} = \frac{d-|\bar{x}-M|}{3s}. \quad (2)$$

Under the assumption of normality, Lin and Pearn (2004) obtained an exact and explicit form of the CDF of the natural estimator $\widehat{C}_{pk}$ by taking the integration technique similar to that presented in Vännman (1997).

$$F_{\widehat{C}_{pk}}(y) = 1 - \int_0^{b\sqrt{n}} G\left(\frac{(n-1)(b\sqrt{n}-t)^2}{9ny^2}\right) \times \left[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right] \mathrm{d}t, \quad (3)$$

for $y > 0$, where $b = d/\sigma$, $\xi = (\mu - M)/\sigma$, $G(\cdot)$ is the CDF of the chi-square distribution with degrees of freedom $n-1, \chi^2_{n-1}$, and $\phi(\cdot)$ is the probability density function (PDF) of the standard normal distribution $N(0,1)$. It can bee seen that the CDF of $\widehat{C}_{pk}$ is expressed in terms of a mixture of the chi-square distribution and the normal distribution and it is noted that we would obtain an identical equation if we substitute ξ by $-\xi$ into Eq. (3) for fixed values of y and n.

3. Developing a sampling inspection scheme for resubmitted lots based on the C_{pk} index

Lot resubmissions are allowed in situations where original inspection results are suspected or when the supplier is allowed to opt for resampling as per the provisions of the contract or statute. Govindaraju and Ganesalingam (1997) developed a resubmitted sampling scheme by attributes and examined the situation where resampling is permitted on lots not accepted on original inspection. The usefulness and limitations of this resampling scheme for resubmitted lots are also discussed.

When a quality characteristic is measurable on a continuous scale and is known to have a distribution of a specified type, it may be possible to use it as a substitute for an attributes sampling plan based on sample measurements. Consider a variables sampling inspection scheme for resubmitted lots based on the most prevalent capability index C_{pk} to control the lot or process fraction defective (or nonconforming). Suppose that the quality characteristic of interest has two-sided specification limits (*LSL* and *USL*) and follows a normal distribution. It is assumed that during the course of resubmission, the quality of the lot is not improved by sorting or reprocessing. The operating procedure of the proposed variables resubmitted sampling plan based on C_{pk} index is stated as follows:

Step 1. Perform the inspection from the original sample, i.e., apply a reference sampling plan based on C_{pk} index (with a sample size n and critical acceptance value C_0). That is, accept the lot if $\widehat{C}_{pk} \geqslant C_0$ based on the sample of size n.

Step 2. For the case of nonacceptance on the submitted sample, apply the reference plan m times and reject the lot if it is not accepted on the $(m-1)$st resubmission.
Even though any type of sampling plan can be used as a reference plan, such as single, double or multiple sampling plan, etc., it is more practical to use only a single sampling plan because resubmission is allowed.

From the definition of the C_{pk} index, it may be rewritten as $C_{pk} = (d/\sigma - |\xi|)/3$, where $\xi = (\mu - M)/\sigma$. Thus, given $C_{pk} = C$, $b = d/\sigma$ can be expressed as $b = 3C + |\xi|$. Let $P_a(C)$ be the operating characteristic (OC) function when the submitted lot quality is $C_{pk} = C$, i.e.,

$$P_a(C) = P(\widehat{C}_{pk} \geqslant C_0) = 1 - P(\widehat{C}_{pk} < C_0) = 1 - F_{\widehat{C}_{pk}}(C_0)$$
$$= \int_0^{b\sqrt{n}} G\left(\frac{(n-1)(b\sqrt{n}-t)^2}{9n(C_0)^2}\right)\left(\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right)\mathrm{d}t. \tag{4}$$

For the resubmitted sampling plan, resubmission is allowed $m-1$ times and in case of nonacceptance in each inspection, resampling is done. In other words, sampling inspection of the lot with same quality may happen m times. The commonly preferred value for m is two. That implies that resampling is done on the resubmitted lot once if the lot is not accepted on the original inspection by a single sampling plan with parameters, sample size n and critical value C_0.

For the developed resubmitted sampling plan based on the C_{pk} index, the eventual probability of acceptance, $\pi_A(C_{pk})$, can be obtained as

$$\begin{aligned}\pi_A(C_{pk}) &= P_a(C_{pk}) + [1 - P_a(C_{pk})]P_a(C_{pk}) \\ &\quad + [1 - P_a(C_{pk})]^2 P_a(C_{pk}) + \cdots + [1 - P_a(C_{pk})]^{m-1} P_a(C_{pk}) \\ &= \frac{P_a(C_{pk})\left(1 - [1 - P_a(C_{pk})]^m\right)}{1 - [1 - P_a(C_{pk})]} = 1 - \left[1 - P_a(C_{pk})\right]^m. \end{aligned} \tag{5}$$

This expression is valid if the lot quality remains unaltered on sampling and for any type of reference plan. The average sampling number (ASN) for the resubmitted sampling plan based on C_{pk} index is given by

$$\begin{aligned}\mathrm{ASN}(C_{pk}) &= n + n[1 - P_a(C_{pk})] + n[1 - P_a(C_{pk})]^2 + \cdots \\ &\quad + n[1 - P_a(C_{pk})]^{m-1} = \frac{n\left(1 - [1 - P_a(C_{pk})]^m\right)}{1 - [1 - P_a(C_{pk})]} \\ &= \frac{n\left(1 - [1 - P_a(C_{pk})]^m\right)}{P_a(C_{pk})}. \end{aligned} \tag{6}$$

Acceptance sampling plans basically consist of a required sample size for inspection and the critical value for product acceptance determination. While any two points on the OC curve could be used to construct the sampling plan, it is customary in the industry to use the AQL and LTPD (lot tolerance percent defective) for this purpose. Both producers and consumers will lay down their requirements and set their own safeguard line to protect their benefits in the contract. Thus, two kinds of risks should be balanced by a well-designed sampling plan. Let C_{AQL} be the AQL based on C_{pk} index at which the producer desires a high probability of acceptance $1-\alpha$ and C_{LTPD} be the LTPD or limiting quality level (LQL) at which the consumer desires the probability of acceptance be no more than β. That is, the parameters (n, C_0) of the resubmitted sampling plan based on C_{pk} index should be satisfied with the following two conditions:

$$\pi_A(C_{\mathrm{AQL}}) = 1 - [1 - P_a(C_{\mathrm{AQL}})]^m \geqslant 1 - \alpha \tag{7}$$

and

$$\pi_A(C_{\mathrm{LTPD}}) = 1 - [1 - P_a(C_{\mathrm{LTPD}})]^m \leqslant \beta. \tag{8}$$

Therefore, the required inspection sample size n and critical acceptance value C_0 of the estimation $\widehat{C}_{pk}$ can be obtained by solving the following two non-linear simultaneous equations:

$$\left[1 - \left(\int_0^{b_{\mathrm{A}}\sqrt{n}} G\left(\frac{(n-1)(b_{\mathrm{A}}\sqrt{n}-t)^2}{9n(C_0)^2}\right)\left(\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right)\mathrm{d}t\right)\right]^m - \alpha \leqslant 0 \tag{9}$$

and

$$\left[1 - \left(\int_0^{b_{\mathrm{L}}\sqrt{n}} G\left(\frac{(n-1)(b_{\mathrm{L}}\sqrt{n}-t)^2}{9n(C_0)^2}\right)\left(\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right)\mathrm{d}t\right)\right]^m - (1-\beta) \geqslant 0, \tag{10}$$

where $b_{\mathrm{A}} = 3C_{\mathrm{AQL}} + |\xi|$ and $b_{\mathrm{L}} = 3C_{\mathrm{LTPD}} + |\xi|, C_{\mathrm{AQL}} > C_{\mathrm{LTPD}}$. It is noted that the required sample size n is the smallest possible value of n satisfying Eqs. (9) and (10), and determining the $\lceil n \rceil$ as sample size, where $\lceil n \rceil$ means the least integer greater than or equal to n. Further, from the Eqs. (9) and (10), it can be seen that the designed resubmitted sampling plan will reduce to the variables single sampling plan based on C_{pk} index proposed by Pearn and Wu (2007) if $m = 1$.

4. Analysis and discussion

For the convenience of applying the proposed results to practical applications, we calculate and tabulate the parameters (n, C_0) of the proposed resubmitted sampling plan based on C_{pk} index with commonly used levels of α, β, C_{AQL} and C_{LTPD} by solving Eqs. (9) and (10). As noted by Pearn and Wu (2007) that the process parameters μ and σ are unknown, then the parameter $\xi = (\mu - M)/\sigma$ is also unknown which has to be estimated in real applications. Such approach introduces additional sampling errors from estimating ξ in finding the critical acceptance values and the required sample sizes. To eliminate the need for estimating ξ, they performed extensive calculations to investigate the behavior of the critical acceptance value and sample size for various parameters and found that the required sample size and the critical acceptance value will be conservative by setting $\xi = 1.00$. Thus, we calculate the parameters (n, C_0) with the condition $\xi = 1.00$ to ensure that the decisions made are reliable.

Tables 1 and 2 display the values of n and C_0 for various producer's α-risk and consumer's β-risk = 0.01, 0.025(0.025)0.10 with selected benchmarking quality levels $(C_{\mathrm{AQL}}, C_{\mathrm{LPTD}}) = (1.33, 1.00)$, $(1.50, 1.33)$, $(1.67, 1.33)$ and $(2.00, 1.67)$. Based on the designed

Table 1
The required sample size and the critical acceptance value (n, C_0) for the resubmitted sampling plan based on C_{pk} index with $m = 2$.

α	β	$C_{AQL} = 1.33$, $C_{LTPD} = 1.00$		$C_{AQL} = 1.50$, $C_{LTPD} = 1.33$		$C_{AQL} = 1.67$, $C_{LTPD} = 1.33$		$C_{AQL} = 2.00$, $C_{LTPD} = 1.67$	
		n	C_0	n	C_0	n	C_0	n	C_0
0.01	0.01	115	1.2211	588	1.4437	168	1.5577	258	1.8912
	0.025	95	1.2113	488	1.4384	139	1.5474	212	1.8808
	0.05	80	1.2014	411	1.4332	117	1.5371	178	1.8707
	0.075	71	1.1942	365	1.4292	103	1.5295	158	1.8633
	0.10	64	1.1881	331	1.4260	94	1.5232	143	1.8570
0.025	0.01	101	1.2391	510	1.4527	147	1.5761	223	1.9086
	0.025	83	1.2303	417	1.4480	120	1.5669	182	1.8995
	0.05	68	1.2214	346	1.4431	99	1.5575	151	1.8903
	0.075	60	1.2147	304	1.4395	87	1.5506	132	1.8834
	0.10	54	1.2091	273	1.4364	78	1.5447	119	1.8776
0.05	0.01	90	1.2573	447	1.4618	130	1.5946	196	1.9265
	0.025	72	1.2498	361	1.4577	104	1.5868	158	1.9187
	0.05	59	1.2421	295	1.4535	85	1.5787	129	1.9107
	0.075	51	1.2364	256	1.4502	74	1.5727	112	1.9047
	0.10	46	1.2315	228	1.4475	66	1.5675	100	1.8996
0.075	0.01	83	1.2707	408	1.4686	119	1.6085	180	1.9397
	0.025	66	1.2645	326	1.4650	95	1.6018	143	1.9331
	0.05	53	1.2580	263	1.4613	77	1.5950	116	1.9263
	0.075	46	1.2532	226	1.4585	66	1.5898	100	1.9211
	0.10	41	1.2491	200	1.4561	59	1.5854	88	1.9167
0.10	0.01	77	1.2822	379	1.4743	111	1.6201	167	1.9510
	0.025	61	1.2771	300	1.4713	88	1.6146	132	1.9455
	0.05	49	1.2718	240	1.4681	70	1.6090	106	1.9398
	0.075	42	1.2679	205	1.4657	60	1.6048	91	1.9355
	0.10	37	1.2646	180	1.4637	53	1.6012	80	1.9318

Table 2
The required sample size and the critical acceptance value (n, C_0) for the resubmitted sampling plan based on C_{pk} index with $m = 3$.

α	β	$C_{AQL} = 1.33$, $C_{LTPD} = 1.00$		$C_{AQL} = 1.50$, $C_{AQL} = 1.33$		$C_{AQL} = 1.67$, $C_{AQL} = 1.33$		$C_{AQL} = 2.00$, $C_{LTPD} = 1.67$	
		n	C_0	n	C_0	n	C_0	n	C_0
0.01	0.01	99	1.2578	492	1.4622	143	1.5953	216	1.9271
	0.025	81	1.2511	405	1.4585	117	1.5882	178	1.9201
	0.05	68	1.2443	338	1.4548	98	1.5811	148	1.9132
	0.075	60	1.2394	299	1.4521	87	1.5759	131	1.9080
	0.10	54	1.2353	271	1.4498	78	1.5716	119	1.9037
0.025	0.01	87	1.2778	431	1.4722	126	1.6157	190	1.9468
	0.025	71	1.2727	349	1.4693	102	1.6103	154	1.9414
	0.05	58	1.2676	288	1.4664	84	1.6049	127	1.9360
	0.075	51	1.2639	251	1.4642	74	1.6009	111	1.9320
	0.10	46	1.2608	226	1.4623	66	1.5976	100	1.9286
0.05	0.01	78	1.2979	380	1.4823	112	1.6363	169	1.9666
	0.025	63	1.2949	304	1.4803	90	1.6329	135	1.9633
	0.05	51	1.2919	247	1.4784	73	1.6296	110	1.9598
	0.075	44	1.2897	213	1.4769	63	1.6272	95	1.9572
	0.10	39	1.2879	190	1.4757	56	1.6252	84	1.9551
0.075	0.01	72	1.3128	349	1.4897	103	1.6515	155	1.9813
	0.025	57	1.3116	276	1.4886	82	1.6500	123	1.9796
	0.05	46	1.3104	222	1.4875	66	1.6485	99	1.9779
	0.075	40	1.3097	190	1.4867	57	1.6475	85	1.9767
	0.10	35	1.3091	168	1.4860	50	1.6468	75	1.9757
0.10	0.01	68	1.3255	325	1.4960	97	1.6644	145	1.9937
	0.025	53	1.3259	255	1.4957	76	1.6646	114	1.9935
	0.05	43	1.3265	203	1.4954	61	1.6649	91	1.9936
	0.075	37	1.3272	173	1.4952	52	1.6653	78	1.9937
	0.10	32	1.3279	152	1.4951	46	1.6658	68	1.9939

resubmitted sampling plan, the practitioner can determine the number of production items (n) to be sampled for inspection and the corresponding critical value (C_0) for lot disposition. For instance, if the quality level (C_{AQL}, C_{LPTD}) is set to $(1.33, 1.00)$ with producer's α-risk $= 0.01$ and buyer's β-risk $= 0.05$, then the corresponding sample size and critical value for acceptance are obtained as $(n, C_0) = (80, 1.2014)$ for $m = 2$. This implies that the lot will be accepted if the 80 inspected production items yield measurements with $\widehat{C}_{pk} > 1.2014$. Otherwise, resubmission is allowed once if the lot is not accepted on original inspection and the lot will be rejected if the resubmission is also not accepted (i.e., $\widehat{C}_{pk} < 1.2014$). Moreover, plan parameters are obtained as

Table 3
The plan parameters (n, C_0) for various m under $(C_{AQL}, C_{LPTD}) = (1.33, 1.00)$ and $(\alpha, \beta) = (0.05, 0.05)$.

(C_{AQL}, C_{LPTD})	m	1	2	3	4	5	6	7	8	9	10
(1.33, 1.00)	n	80	59	51	46	43	41	40	38	37	36
	C_0	1.1669	1.2421	1.2919	1.3295	1.3599	1.3855	1.4077	1.4273	1.4448	1.4607
(1.50, 1.33)	n	418	295	247	220	203	190	181	173	167	162
	C_0	1.4154	1.4535	1.4784	1.4971	1.5122	1.5249	1.5358	1.5454	1.5540	1.5617

$(n, C_0) = (68, 1.2443)$ if $m = 3$, this implies the consumer allows resubmissions twice in the case of nonacceptance on the submitted sample of size 68, and reject the lot if two resubmissions with the same sample size are not accepted.

It can be observed from Tables 1 and 2 that the smaller of the risks which the producer and/or the consumer could suffer, the larger sample size n is required for inspection. This phenomenon can be explained intuitively: if we expect that the chance of wrongly concluding a bad process as good or good lots as bad ones is smaller, the more sample information need to take for judging the lots. For fixed m, α-risk, C_{AQL} and C_{LTPD}, the corresponding critical acceptance value becomes smaller as the β-risk increases. On the other hand, for fixed m, β-risk, C_{AQL} and C_{LTPD}, the corresponding critical acceptance value becomes larger as the α-risk increases. Further, for fixed m, α-risk, β-risk and C_{LTPD}, the required sample size becomes smaller when the C_{AQL} becomes larger. This can also be explained by similar reasoning as above, since the judgment will be relatively easier to make correct decisions with a larger difference between the values of C_{AQL} and C_{LTPD}.

Furthermore, in order to examine the behavior of the proposed resubmitted sampling plan based on C_{pk} index with different values of m, the required sample size n and the critical acceptance value C_0 for various m with $(C_{AQL}, C_{LPTD}) = (1.33, 1.00)$ and $(1.50, 1.33)$, α-risk = 0.05 and β-risk = 0.05 are shown in Table 3. From Table 3, it can be seen that the required sample size becomes smaller and the corresponding critical acceptance value becomes larger as the number of resampling times (m) increases. The resubmitted sampling plan based on C_{pk} index with different m can be developed so that they produce equivalent results. That is, this resampling scheme can be designed so that a lot of specified quality has exactly the same probability of acceptance under all resubmitted sampling plans with different number of resampling times.

Besides, the OC curves of the designed resubmitted sampling plan for $m = 1, 2, 3$ and 5 are considered. Fig. 1 shows the OC curve of the resubmitted sampling plan for $m = 1$ (i.e., single sampling plan) with parameters (sample size $n = 50$, critical value $c_0 = 1.17$), for $m = 2$ with plan parameters $(n, C_0) = (1.17, 1.00)$, for $m = 3$ with plan parameters $(n, C_0) = (1.17, 1.00)$ and for $m = 5$ with plan parameters $(n, C_0) = (1.17, 1.00)$.

From Fig. 1, the probability of acceptance will become larger as the value of C_{pk} increases for the variables single sampling plan and resubmitted sampling plans. In addition, the figure apparently shows that the developed resubmitted sampling plan has a better OC curve than the variables single sampling plan at good quality levels and also ensures protection against the consumer point of view at poor quality levels. This is also an important feature of the resubmitted sampling plans.

The average sample number (ASN) curves of the variables single sampling plan ($m = 1$) and resubmitted sampling plans with $m = 2$, 3, 5 under $(C_{AQL}, C_{LPTD}) = (1.33, 1.00)$ and $(\alpha, \beta) = (0.05, 0.05)$ are shown in Fig. 2. From these ASN curves, it is noted that the resampling scheme involves more sample size when the submitted quality is bad. This is logical since lots that are declared as not acceptable are always resampled even though the original inspection showed the evidence of poor quality. For higher values of quality levels based on C_{pk} index, the resampling scheme becomes more efficient. That is, the resampling scheme performs better at larger values of C_{pk} index, which is the most useful situation encountered in practice.

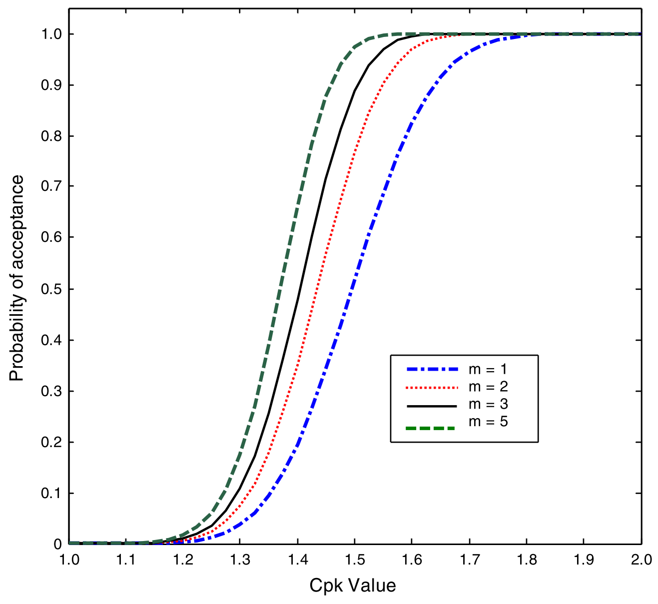

Fig. 1. OC curves of a variables single sampling plan ($m = 1$) and resubmitted sampling plans with $m = 2$, 3, 5.

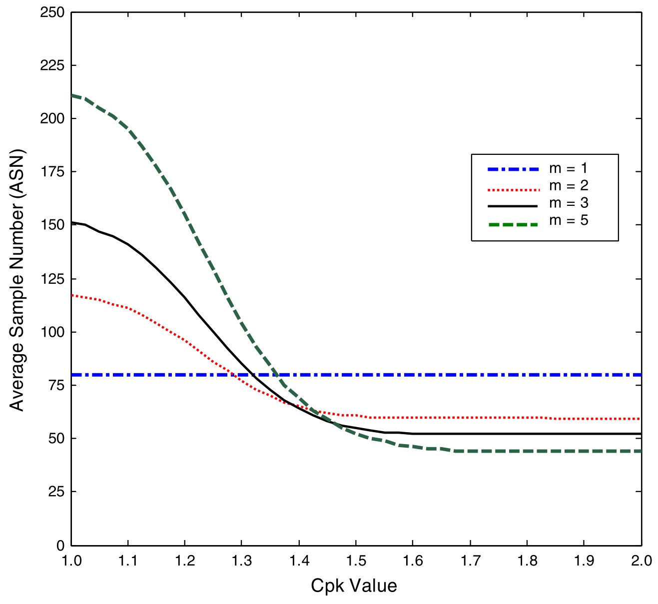

Fig. 2. ASN curves of a variables single sampling plan ($m = 1$) and resubmitted sampling plans with $m = 2$, 3, 5.

Table 4
The 80 sample measurements taken from the lot.

10.19	9.06	9.40	9.73	9.23	9.80	9.59	9.76	10.02	9.07
9.39	10.22	10.04	10.10	9.68	9.84	10.21	9.84	9.05	8.73
9.97	9.73	10.91	9.80	10.28	10.25	9.89	10.50	9.76	9.73
11.12	9.61	9.42	10.33	8.88	10.12	10.23	9.91	9.35	10.36
9.07	10.43	10.06	9.71	10.10	9.63	9.83	9.74	9.49	9.71
10.01	9.42	9.53	9.26	10.16	10.58	10.99	9.83	9.94	9.69
9.69	9.87	10.75	9.92	9.90	9.78	9.00	10.16	9.26	9.30
9.40	10.00	10.06	10.55	9.43	9.41	9.60	9.47	9.33	10.56

5. An example

To illustrate how the proposed sampling plan can be established and applied to actual data collected from factories, we consider the following example taken from an electronic component manufacturer. The factory manufactures various types of resistors. For a particular model of resistors investigated, the target value of thickness is set to T = 10.0 mil, and the tolerance is 2.0 mil, i.e., the lower and upper specification limit are set to LSL = 8.0 mil and USL = 12.0 mil. If the characteristic does not fall within the tolerance $[LSL, USL]$, the lifetime or reliability of the resistors will be discounted.

In the contract, suppose that the values of C_{AQL} and C_{LTPD} are set to 1.33 and 1.00 with producer's α-risk = 0.01 and consumer's β-risk = 0.05. In this case, the sample size required for inspection and the corresponding critical acceptance value then can be obtained as $(n, C_0) = (80, 1.2014)$ if m = 2 by checking Table 1. This implies that the entire lot will be accepted if the 80 inspected production items yield measurements with $\widehat{C}_{pk} > 1.2014$. Otherwise, resubmission is allowed once if the lot is not accepted on original inspection and the lot will be rejected if the resubmission is also not accepted (i.e., $\widehat{C}_{pk} < 1.2014$).

A sample of size 80 is taken from the lot randomly and the observed measurements are displayed in Table 4. These data are also shown to be fairly close to the normal distribution using Anderson–Darling normality test. The normal probability plot of the collected data with p-value for testing the normality is showed in Fig. 3. The histogram of the collected data with lower and upper specification limits is shown in Fig. 4.

The sample mean, sample standard deviation and the estimation of the C_{pk} calculated from the collected samples are $\bar{x} = 9.8215$ mil, s = 0.4843 mil and $\widehat{C}_{pk} = 1.2537$. Thus, in this case, the consumer would accept the entire lot since the sample estimator from these 80 measurements, 1.2537, is larger than the critical value 1.2014.

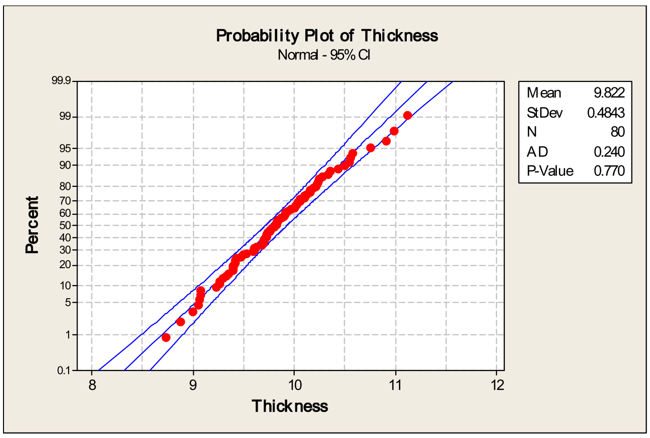

Fig. 3. The normal probability plot of the collected data.

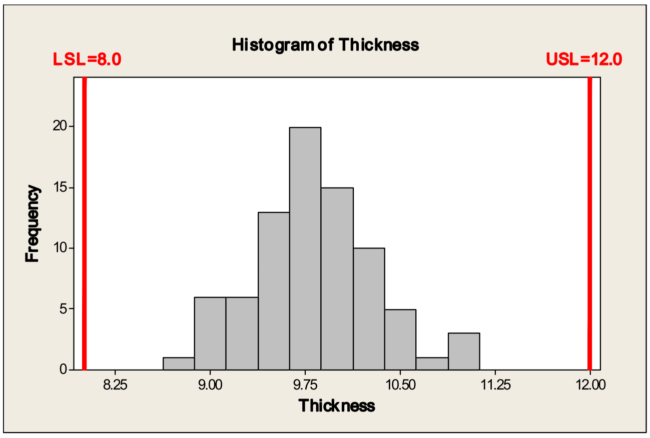

Fig. 4. The histogram of the collected data with specification limits.

We note that if the variables single sampling plan based on C_{pk} index is applied here, the sample size required for inspection is 112 under the same specified values of AQL, LTPD, α-risk and β-risk. This implies that the developed variables resubmitted sampling plan will give the desired protection with less inspection in this case so that the cost of inspection will be greatly reduced. Thus, the proposed sampling inspection scheme for resubmitted lots can provide an efficient and economic device for lot sentencing.

6. Conclusions

Acceptance sampling is a statistical procedure used for determining whether to accept or reject a production lot of material. It has been a common quality control technique used in industry and particularly the military for contracts and procurement. Due to the sampling cannot guarantee that every defective item in a lot will be inspected, the sampling involves risks of not adequately reflecting the quality conditions of the lot. Such risk is even more significant as the rapid advancement of the manufacturing technology and stringent customers demand is enforced. In particular, when the fraction of nonconforming products is required very low, such as measured in PPM, the required number of inspection items must be very large in order to adequately reflecting the actual lot quality. This paper developed a variables inspection scheme for resubmitted lots based on the commonly used capability index C_{pk} to deal with the product acceptance decision making problem, especially for situations with very low fraction of defectives. The proposed variables resubmitted sampling plan is developed based on the exact sampling distribution rather than an approximation approach. The performance of the proposed variables resubmitted sampling plan is examined and it will reduce to the existing variables single sampling plan if m = 1. The results show that the developed resubmitted sampling plan has a better OC curve than the variables single sampling plan at good quality levels and also ensures protection against the consumer point of view at poor quality levels. For practical purpose, an example is presented for illustration and tables of the required sample size and the corresponding critical value for product acceptance determination under selected parameters are also provided.

Finally, we have to stress that the capability index C_{pk} is designed to measure the performance for solely normal and near-normal processes with symmetric tolerances, and is shown to be inappropriate for processes with non-normally distributed. Therefore, the adequacy of the normality assumption must be checked before making the product acceptance decision based on the proposed sampling scheme. Otherwise, the actual product quality might be misrepresented and erroneous decisions might be made. It is suggested that further research, for instance, developing suitable variables resubmitted sampling plans for a specified distribution or non-normal distributions.

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