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Stage-independent multiple sampling plan by variables inspection for lot determination based on the process capability index C_{pk}

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ABSTRACT

Multiple sampling plan (MSP) has been proved that the sample units required for inspection at each stage are usually smaller than the conventional single or double sampling. However, it is more complex to administer and difficult to derive the corresponding operating characteristic function since the judgment on the submitted lot under the MSP is not only dependent on the result of current sampling but also on previous sampling results. Thus, this paper attempts to provide a relaxed type of conventional MSP by assuming the sampling inspection at each stage is independent which is called variables stage-independent multiple sampling plan and integrated with the most widely-used process capability index C_{pk}. For the cost-efficient purpose, the plan parameters are solved under an optimisation model that minimises the average sample number by satisfying the required quality levels and tolerated risks. Finally, the applicability of the proposed plan is illustrated in a case study.

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KEYWORDS
Acceptance sampling; average sample number; process capability indices; sampling plans; operating characteristic function

1. Introduction

The verification of high-quality standard products has become one of the success factors for maintaining a long-term supplier-buyer relationship, owing to the global competitive market today (Wiengarten et al. 2021). Before being shipped to the downstream process or terminal market, ordered items such as raw materials, components, or finished goods should be thoroughly examined. Since sophisticated and complex products become the trend of customer demand motivated by the current advanced manufacturing technology, the inspection of those items has consequently become high-cost and time-consuming. To avoid two events, (*i*) taking risky warehousing under zero inspection and (*ii*) generating a tremendous cost under full inspection in the real-life lot acceptance, an alternative way, called 'acceptance sampling' is more preferable to the quality engineers and has been widely used in many industries.

Acceptance sampling is a necessary defensive measure designed to protect against irregular degradation of submission quality levels that are considered permissible by the business partners (Schilling and Neubauer 2008). Acceptance sampling plans have been widely used in the industry as a dependable and cost-efficient inspection tool of incoming or outgoing lots to ensure the product quality that meets tolerable risks and desirable requirements (Wu, Shu, and Wu 2021). Because of its ease of administration and implementation, a single sampling plan (SSP) is a basic all sampling strategy that is commonly used in industries. SSP renders a one-time decision on submitted lots which may cause a rift in the vendors-buyers relationship. Furthermore, due to the smaller size of tested samples, it is more advantageous to implement sampling with progressive strategies for reducing inspection costs while maintaining the same level of protection (Genta, Galetto, and Franceschini 2020; Liu, Wu, and Tsai 2021; Wang, Hsu, and Shu 2021; Wu, Shu, and Wu 2021).

The double sampling plan (DSP) is an easy and straightforward extension of the SSP. If the submitted lot cannot be sentenced based on the first sample information, it allows for a second chance for inspection. From an economic standpoint, the DSP is generally better than the corresponding SSP because it can give an early decision at the first stage whenever lot quality is extremely good or bad. Thus, it not only could reduce the total amount of required inspection but also has the psychological advantage of giving a lot a second chance, which may have some appeal to the supplier. Sommer (1981) firstly tabulated the solved plan parameters of variables DSP under selected conditions, and the operating characteristic (OC) function was derived

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by using the approximation method (for more information, see Eisenhart, Hastay, and Wallis (1947) and Bowker and Goode (1952)) with consideration of the combined sample information. Unlike Sommer's method, Arizono, Yoshimoto, and Tomphiro (2020) proposed a concept of stage-independent DSP (SIDSP) to simplify the procedure of conventional variables DSP for the acceptance quality loss limit (AQLL) inspection scheme to overcome the complexity of OC function with conditional probability in variables DSP. The SIDSP has a feature that the judgment at the 2nd-stage sampling is not dependent on the result of the 1st-stage sampling.

Multiple sampling plan (MSP) is a more general extension of SSP and DSP. The primary advantage of MSP is that the sample required at each stage is usually smaller than those in single or double sampling; thus, some economic efficiency is connected with the use of the procedure. However, since the MSP considers the inspection result at every individual sampling is dependent so that the development of variables MSP becomes tricky due to the difficulty of calculating conditional probabilities of acceptance. Wilson and Burgess (1971), Hughes, Dickinson, and Chow (1973), Speevak and Yu (1987), Soundararajan and Kuralmani (1989), and Sathakathulla and Murthy (2012) were among the MSP researchers who focused on the attributes data type in recent decades. The MSP design becomes more complicated, especially when it comes to inspecting variables. Moreover, with the rapid advancement of manufacturing technology, consumers began to demand high-quality products with a very low fraction of nonconformities, which is usually less than 0.01% and often measured in parts per million (PPM). Because any sample of 'reasonable size' will almost certainly contain no defective items, traditional methods of determining the fraction nonconforming are no longer applicable. As a result, process capability indices (PCIs) are a popular and alternative method of determining the percentage of non-conforming products. Therefore, this paper extends Arizono, Yoshimoto, and Tomphiro (2020) concept of SIDSP and develops a variable 'stage-independent' MSP (SIMSP) based on the most popular index C_{pk}, which is suitable for today's high-yield process environments and products.

The rest of this paper is organised as follows. In Section 2, the definition of the index C_{pk} and its estimator's sampling distribution is briefly introduced. Section 3 describes the proposed plan's operating procedure and derives the OC function of the proposed plan based on the exact sampling distribution of the estimated C_{pk}, the mathematical model constructed for determining plan parameters is also provided here. Section 4 examines and discusses several analyses and comparisons, as well as the solved plan parameters for quick reference. The applicability of the proposed plan is demonstrated in Section 5 with an application example from the power distribution switch (PDS) industry. Finally, in Section 6, some conclusions are drawn.

2. Process capability indices

PCIs explain the level of the process itself or the produced items by bridging the relationship between actual process performance and specified specification limits. Readers can refer to Kotz and Johnson (2002), Spiring et al. (2003) for examples of PCIs that have been proven to be successful in quality assurance or improvement activities in real-world applications, Pearn and Kotz (2006), Wu, Pearn, and Kotz (2009), and Chen et al. (2022) for more details. Since the first PCI, $C_p = (\text{USL} - \text{LSL})/6\sigma = d/3\sigma$, the precision index, was introduced by Juran (1974), the relevant and extended studies never stop, where USL and LSL are the upper and lower specification limits, respectively, $d = (\text{USL} - \text{LSL})/2$ is the half-length of specification limits and σ represents process standard deviation. The index C_p tells the ratio between specification interval and natural tolerance, but due to the simplified design, it cannot reflect the tendency of process centreing. On the other hand, the index $C_a = 1 - (|\mu - M|/d)$ measures the degree of process centreing to alert the engineer if the process mean μ deviates from the centre of specification limits (i.e. $M = (\text{USL} + \text{LSL})/2$).

Kane (1986) further proposed a new capability index, C_{pk}, by considering both the process spread and process location, and is defined as follows:

$$C_{pk} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right\} = \frac{d - |\mu - M|}{3\sigma}. \tag{1}$$

In fact, several new types of PCIs have been designed to improve upon, out-perform, and provide a wider coverage area than traditional C_p and C_{pk} indices under a wider range of conditions. However, none of these new indices have as yet surpassed C_{pk} in popularity or ease of use. Due to its simplicity and ease of understanding, C_{pk} has withstood the test of time and new competitors to become the standard index for process capability quantification. Because it provides bounds on the process yield, the index C_{pk} has been referred to as a yield-based index, $[2\Phi(3C_{pk}) - 1] \times 100\% \leq$ process yield $\leq [\Phi(3C_{pk})] \times 100\%$, for a normally distributed process, where $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard normal distribution (Boyles 1991). For example, at a C_{pk} level of 1, it means that no more than 2700 PPM falls outside the specification limits. The defect rate drops to 66 PPM when the C_{pk} is set to 1.33. Montgomery (2009) provided some recommended C_{pk} index minimum values under different process statuses.

However, in the most practical applications, the true process mean and standard deviation are usually unknown in practical applications. To obtain a natural estimate of C_{pk}, both μ and σ can be estimated by the sample mean $\bar{X} = (\sum_{i=1}^{n} X_i)/n$ and the sample standard deviation $S = [\sum_{i=1}^{n} (X_i - \bar{X})/(n-1)]^{1/2}$ from the collected sample of size n. Thus, the estimator $\hat{C}_{pk}$ can be calculated by

$$\hat{C}_{pk} = \min\left\{\frac{\mathrm{USL} - \bar{X}}{3S}, \frac{\bar{X} - \mathrm{LSL}}{3S}\right\} = \frac{d - |\bar{X} - M|}{3S}. \tag{2}$$

For further applications, Pearn and Shu (2003) provided another exact form of CDF for $\hat{C}_{pk}$ using the integration techniques proposed by Vännman and Kotz (1995) and Vännman (1997) as follows:

$$F_{\hat{C}_{pk}}(y) = 1 - \int_0^{b\sqrt{n}} G\left[\frac{(n-1)(b\sqrt{n} - t)^2}{9ny^2}\right] \times \left[\phi(t - \xi\sqrt{n}) + \phi(t + \xi\sqrt{n})\right] \mathrm{d}t, \tag{3}$$

where $y > 0$, $b = 3C_{pk} + |\xi|$, $\xi = (\mu - M)/\sigma$, $G(\cdot)$ is the CDF of the chi-square distribution with $n-1$ degrees of freedom and $\phi(\cdot)$ is the probability density function of the standard normal distribution. Note that the parameter $\xi = 1$ is suggested to obtain a conservative and reliable lower confidence bound of C_{pk} (Pearn and Shu 2003). The unitless measure C_{pk} has been used to develop acceptance sampling plans for variables inspection with various strategies over the last decade, including Pearn and Wu (2007), Wu, Aslam, and Jun (2012), Aslam et al. (2013, 2014), Wu, Lee, and Chen (2016), Lee, Wu, and Chen (2016), Wu et al. (2017; Wu, Shu, et al. 2021) and so on.

3. The proposed plan

Both the producer and the consumer will set their requirements in a purchasing contract, along with the risks they may face. To meet these quality and risk requirements, a well-designed acceptance sampling plan should be created. The requirement that the OC curve passes through two designated points is a common way to design an acceptance sampling plan. It is usual to use the AQL (acceptable quality level) and LQL (limiting quality level) points on the OC curve for this purpose. That is, if the submitted lot's quality is at $C_{pk} = c_{\mathrm{AQL}}$ (in high quality), the probability of acceptance must be no less than $1 - \alpha$ (where α is usually called the producer's risk). And if the submitted lot's quality is at $C_{pk} = c_{\mathrm{LQL}}$ (in low quality), the probability of acceptance would be no more than β (where β is usually called the consumer's risk).

The operating procedure of the proposed plan can be organised as follows:

STEP 1. Determine the required settings in the contract, including tolerable producer's risk (α) and consumer's risk (β), acceptable quality level (c_{AQL}) and limiting quality level (c_{LQL}) in terms of C_{pk}, and the maximum allowable number of sampling (m). Let u denote the cumulative sampling times, and set $u = 1$.
STEP 2. Perform the uth stage sampling from the submitted lot. That is, take a sample of size n and compute the sample estimator $\hat{C}_{pk}$ from the collected sample units.
STEP 3. If $u < m$, then go on STEP 4, otherwise go to STEP 5.
STEP 4. Accept the submitted lot if $\hat{C}_{pk} \geq k_a$, and reject the submitted lot if $\hat{C}_{pk} < k_r$. If $k_r \leq \hat{C}_{pk} < k_a$, the resampling from the submitted lot is required for further judgment (i.e. go back to STEP 2 and $u = u + 1$).
STEP 5. If $\hat{C}_{pk} \geq k_a$, then accept the lot, otherwise reject the lot.

Some probability, functions are defined here to derive the OC function of the proposed plan using the above-mentioned procedures. When the submitted lot's quality is at a given level $C_{pk} = c$, the probability of acceptance at the uth stage sampling $P_a(c)$, the probability of rejection at the uth stage sampling $P_r(c)$, and the probability that resampling is required for further judgment $P_S(c)$ can be defined as follows:

$$P_a(c) = P(\hat{C}_{pk} \geq k_a | C_{pk} = c) = 1 - F_{\hat{C}_{pk}}(k_a), \tag{4}$$

$$P_r(c) = P(\hat{C}_{pk} < k_r | C_{pk} = c) = F_{\hat{C}_{pk}}(k_r), \tag{5}$$

$$\begin{aligned} P_S(c) &= P(k_r \leq \hat{C}_{pk} < k_a | C_{pk} = c) \\ &= 1 - P_a(c) - P_r(c) = F_{\hat{C}_{pk}}(k_a) - F_{\hat{C}_{pk}}(k_r). \end{aligned} \tag{6}$$

Thus, the eventual probability of acceptance (or called OC function) of the proposed plan under $C_{pk} = c$, $\pi_A(c)$, can be derived as follows:

$$\begin{aligned} \pi_A(c) &= P_a(c) + [P_S(c) \times P_a(c)] \\ &\quad + \cdots + \{[P_S(c)]^{m-2} \times P_a(c)\} \\ &\quad + \{[P_S(c)]^{m-1} \times P_a(c)\} \end{aligned}$$

$$= \sum_{u=1}^{m} \{[P_S(c)]^{u-1} \times P_a(c)\}$$

$$= \frac{P_a(c)\{1-[P_S(c)]^m\}}{1-P_S(c)}. \quad (7)$$

Note that the OC function of the proposed plan will reduce to the C_{pk} - based variables SSP developed by Pearn and Wu (2007) if $m = 1$.

According to the operating procedure, we know that at most $m - 1$ resampling is allowable to an individual submitted lot until the lot can be sentenced. Therefore, the average sample number (ASN) is a more appropriate measure of performance for the proposed plan, where ASN is the expected number of sample units required to make a lot of disposition decision. When the quality of the submitted lot is at a certain level $C_{pk} = c$, the ASN function of the proposed plan is used and can be calculated by:

$$\mathrm{ASN}(c) = n + [n \times P_S(c)] + \{n \times [P_S(c)]^2\} + \cdots + \{n \times [P_S(c)]^{m-1}\}$$

$$= \sum_{u=1}^{m} \{n \times [P_S(c)]^{u-1}\} = \frac{n\{1-[P_S(c)]^m\}}{1-P_S(c)}. \quad (8)$$

To ensure the proposed plan satisfies the desirable quality levels $(c_{\mathrm{AQL}}, c_{\mathrm{LQL}})$ and tolerable sampling risks (α, β) for both the consumer and the producer simultaneously; meanwhile, the required ASN should be minimised for the cost-efficient intention, we formulate a minimisation model as follows and the plan parameters can be obtained by solving the developed mathematical model. Here, the average value of ASN is evaluated at c_{AQL} and c_{LQL} is considered to be the objective function for the model.

$$\underset{n,k_a,k_r}{\mathrm{Min}} \ \frac{1}{2}[\mathrm{ASN}(c_{\mathrm{AQL}}) + \mathrm{ASN}(c_{\mathrm{LQL}})] \quad (9)$$

Subject to

$$\pi_{\mathrm{A}}(c_{\mathrm{AQL}}) = \frac{P_a(c_{\mathrm{AQL}})\{1-[P_S(c_{\mathrm{AQL}})]^m\}}{1-P_S(c_{\mathrm{AQL}})} \geq 1-\alpha, \quad (10)$$

$$\pi_{\mathrm{A}}(c_{\mathrm{LQL}}) = \frac{P_a(c_{\mathrm{LQL}})\{1-[P_S(c_{\mathrm{LQL}})]^m\}}{1-P_S(c_{\mathrm{LQL}})} \leq \beta, \quad (11)$$

$$k_a > k_r, n \geq 2.$$

As noted before, the distribution parameter ξ is usually unknown in the CDF of $\hat{C}_{pk}$, Lepore, Palumbo, and Castagliola (2018) provided an investigation of ξ on the plan parameters of C_{pk}-based variables SSP and recommended setting different values in Equations (10) and (11) of the mathematical model to ensure the two-point condition can be satisfied simultaneously. Therefore, we set $\xi = 0$ for $\pi_{\mathrm{A}}(c_{\mathrm{AQL}})$ in Equation (10) and $\xi = 1$ for $\pi_{\mathrm{A}}(c_{\mathrm{LQL}})$ in Equation (11) to determine the proposed plan's parameters (n, k_a, k_r) according to their suggestion.

To solve optimisation model with the given constraints expressed in Equations (9)–(11), an iterative method, called the sequential quadratic programming (SQP) algorithm, is applied. The SQP algorithm is one of the most effective methods for nonlinearly constrained optimisation problems which generates steps by solving quadratic subproblems, and was introduced by Nocedal and Wright (1999). The main concept SQP algorithm combines the objective and constraint functions into a merit function and attempts to minimise the merit function subject to relaxed constraints (more details of the SQP algorithm can be seen in Nocedal and Wright (2006) and Chapra (2012)). In particular, the SQP algorithm has been implemented by some numerical programming languages, such as Matlab, Maple and Mathematica. Several studies have utilised the SQP algorithm to solve such minimisation problems, such as Balamurali et al. (2005), Liu and Wu (2014), Wu, Liu, and Lee (2015), and Nadi, Gildeh, and Afshari (2020). The plan parameters of the proposed plan are obtained using Matlab R2019b software with the routine function 'fmincon'.

4. Analysis and comparison

To investigate the effect of m on the sample size required for inspection, the ASN curves of the proposed plan with m =1, 2, 3 and 4 under selected quality conditions $(c_{\mathrm{AQL}}, c_{\mathrm{LQL}}) = (1.33, 1.00)$ and $(2.00, 1.67)$ are displayed in Figures 1 and 2, respectively. Note that the proposed plan under $m = 1$ will reduce to the conventional variables SSP, in which the sample size required for inspection will be a constant, i.e. ASN= n, and the proposed method will turn into variables SIDSP proposed by Arizono, Yoshimoto, and Tomphiro (2020) if we set $m = 2$. Therefore, the proposed method can be considered as a generalised form of stage-independent sampling strategy with users determining the value of m based on their requirements. The following paragraph examines the analysis and comparison of the proposed method under various m.

It can be observed from Figures 1 and 2 that the ASN value of the proposed plan highly depends on the submitted lot's quality. If the lot quality is excellent (i.e. C_{pk} value is close to or larger than c_{AQL}) or bad (say, C_{pk} value is close to or less than c_{LQL}), then the proposed plan under $m = 2, 3$ or 4 would require a much smaller ASN than the

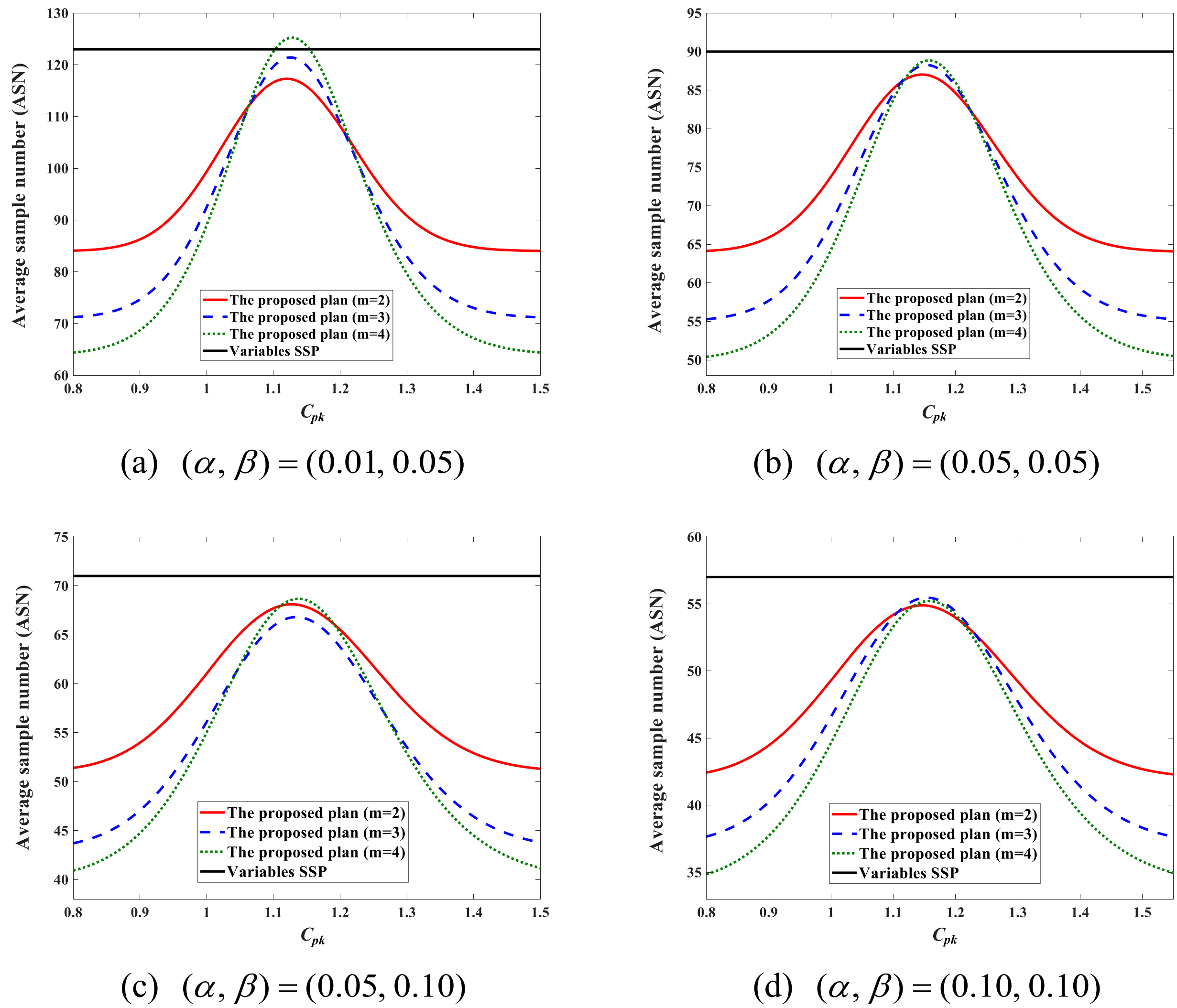


Figure 1. ASN curves of C_{pk}-based variables SSP and the proposed plan with $m = 2, 3$ and 4 under $(c_{\mathrm{AQL}}, c_{\mathrm{LQL}}) = (1.33, 1.00)$.

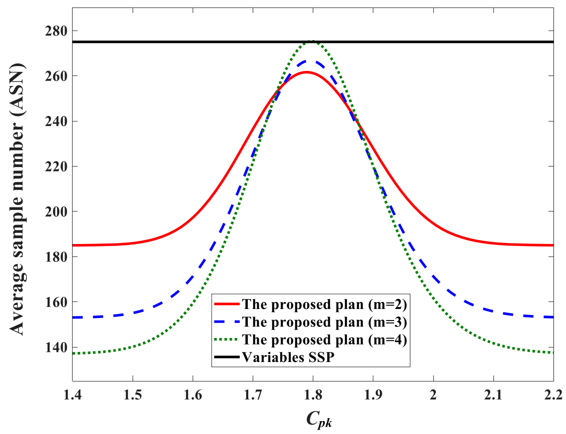
SSP does. On the other hand, the proposed plan might require a larger ASN when the submitted lot quality falls between c_{AQL} and c_{LQL} since the resampling procedure should be conducted if $k_r \le \hat{C}_{pk} < k_a$.

Moreover, we calculate n and the maximum value of ASN curve ($\mathrm{ASN_{Max}}$) with $m = 1(1)10$ under selected quality conditions and tabulated in Tables 1–4 for different combinations of (α, β). These tables elucidate the similar feature that the value of n decreases as m increases and the smallest $\mathrm{ASN_{Max}}$ mostly occurs at $m = 2$, 3 or 4. It explains the trade-off relation that setting a large value of m for the proposed plan can reduce the required sample size n in each stage, but the required ASN in the worst case becomes larger since it might need to conduct sampling m times. In real applications, the value m is suggested to be set at 2, 3, or 4 for cost-effective purposes.

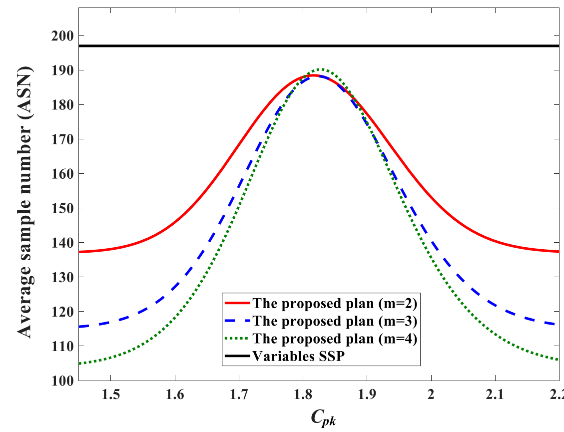
The OC curve is used to evaluate and compare the performance of various sampling plans because it explains their discriminatory power. Herein, the OC curves of the variables SSP and proposed plan with $m = 2$, 3, and 4 under $(c_{\mathrm{AQL}}, c_{\mathrm{LQL}}) = (1.33, 1.00)$ when $(\alpha, \beta) = (0.01, 0.05)$ and $(\alpha, \beta) = (0.05, 0.10)$ are plotted in Figure 3(a,b), respectively. These OC curves have

Table 1. The required n and $\mathrm{ASN_{Max}}$ with $m = 1(1)10$ under $(\alpha, \beta) = (0.01, 0.05)$.

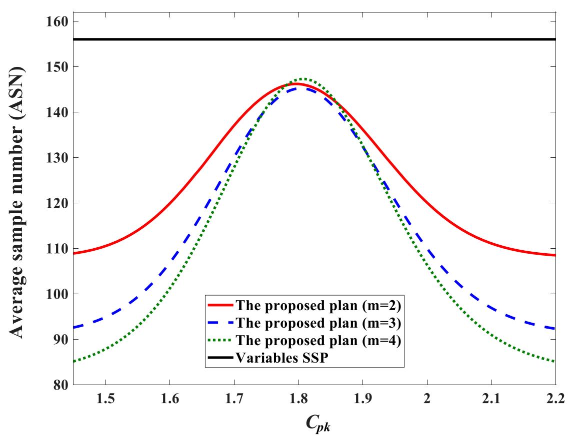
	$(c_{\mathrm{AQL}}, c_{\mathrm{LQL}})$							
	(1.33, 1.00)		(1.50, 1.00)		(1.67, 1.33)		(2.00, 1.67)	
m	n	$\mathrm{ASN_{Max}}$	n	$\mathrm{ASN_{Max}}$	n	$\mathrm{ASN_{Max}}$	n	$\mathrm{ASN_{Max}}$
1	123	123.00	60	60.00	179	179.00	275	275.00
2	84	117.31	42	58.43	122	171.63	185	248.96
3	71	121.41	35	59.24	102	176.56	153	244.45
4	64	125.25	32	62.07	91	180.68	137	245.33
5	60	128.85	30	63.80	86	188.78	129	250.21
6	58	132.78	29	65.79	82	191.34	124	253.08
7	57	135.73	28	65.60	81	197.42	121	253.91
8	56	135.49	28	67.41	80	198.84	120	255.34
9	56	137.13	28	68.33	80	201.36	120	257.07
10	56	137.93	28	68.78	80	202.60	120	257.74



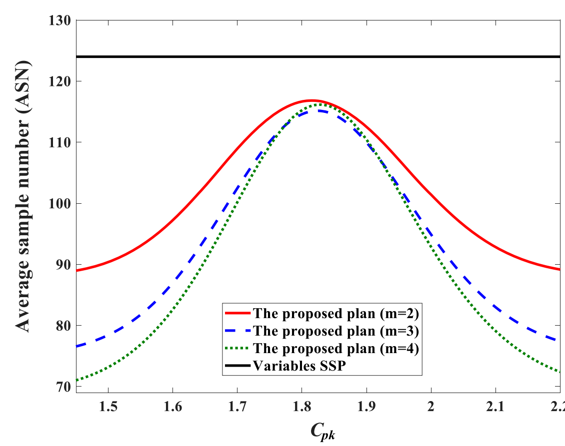
(a) $(\alpha, \beta) = (0.01, 0.05)$



(b) $(\alpha, \beta) = (0.05, 0.05)$



(c) $(\alpha, \beta) = (0.05, 0.10)$



(d) $(\alpha, \beta) = (0.10, 0.10)$

Figure 2. ASN curves of C_{pk}-based variables SSP and the proposed plan with $m = 2, 3$ and 4 under $(c_{\mathrm{AQL}}, c_{\mathrm{LQL}}) = (2.00, 1.67)$.

Table 2. The required n and $\mathrm{ASN}_{\mathrm{Max}}$ with $m = 1(1)10$ under $(\alpha, \beta) = (0.05, 0.05)$.

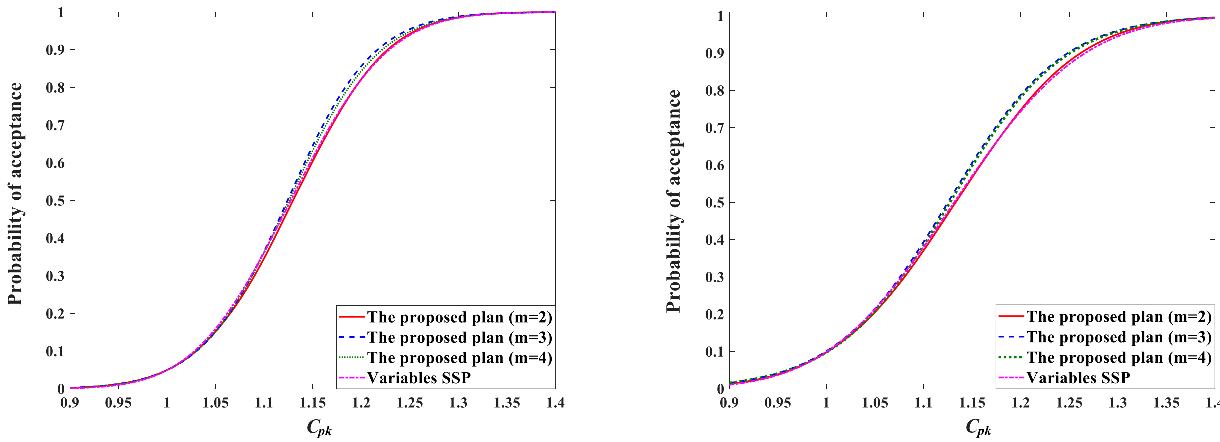
	$(c_{\mathrm{AQL}}, c_{\mathrm{LQL}})$							
	(1.33, 1.00)		(1.50, 1.00)		(1.67, 1.33)		(2.00, 1.67)	
m	n	$\mathrm{ASN}_{\mathrm{Max}}$	n	$\mathrm{ASN}_{\mathrm{Max}}$	n	$\mathrm{ASN}_{\mathrm{Max}}$	n	$\mathrm{ASN}_{\mathrm{Max}}$
1	90	90.00	44	44.00	130	130.00	197	197.00
2	64	87.01	32	43.32	91	124.52	137	188.48
3	55	88.28	28	44.75	77	124.86	115	188.22
4	50	88.85	25	43.85	70	126.43	104	190.22
5	47	89.06	24	45.03	66	128.00	98	193.30
6	46	91.32	24	47.45	64	130.29	95	197.46
7	45	91.50	23	46.07	63	132.29	93	199.57
8	45	93.18	23	46.84	62	132.05	92	201.21
9	45	94.05	23	47.23	62	133.55	91	200.54
10	45	94.47	23	47.42	62	134.29	91	201.84

Table 3. The required n and $\mathrm{ASN}_{\mathrm{Max}}$ with $m = 1(1)10$ under $(\alpha, \beta) = (0.05, 0.10)$.

	$(c_{\mathrm{AQL}}, c_{\mathrm{LQL}})$							
	(1.33, 1.00)		(1.50, 1.00)		(1.67, 1.33)		(2.00, 1.67)	
m	n	$\mathrm{ASN}_{\mathrm{Max}}$	n	$\mathrm{ASN}_{\mathrm{Max}}$	n	$\mathrm{ASN}_{\mathrm{Max}}$	n	$\mathrm{ASN}_{\mathrm{Max}}$
1	71	71.00	35	35.00	103	103.00	156	156.00
2	51	68.11	25	33.11	72	96.85	108	146.18
3	43	66.81	22	33.99	61	96.25	91	145.19
4	40	68.68	20	33.84	56	97.97	83	147.29
5	38	69.44	19	34.11	53	99.09	78	148.15
6	37	70.34	19	35.77	51	99.19	75	148.57
7	36	69.61	18	34.06	50	99.51	74	150.75
8	36	70.60	18	34.52	50	101.14	73	150.46
9	36	71.07	18	34.73	50	101.93	73	151.74
10	36	71.27	18	34.82	50	102.28	73	152.32

a nearly equivalent shape and satisfy the two-point condition, as shown in Figure 3(a,b). This implies that both the producer and the consumer would benefit from these plans, but the proposed plan requires a smaller ASN so that the cost of an inspection will be greatly reduced.

We perform comprehensive calculations for the proposed plan under various quality-and-risk settings to make it easy for practitioners to apply the proposed plan $(c_{\mathrm{AQL}}, c_{\mathrm{LQL}}, \alpha, \beta)$ and tabulate in Tables 5–7 for $m =$ 2, 3, and 4, respectively. For instance, if $(c_{\mathrm{AQL}}, c_{\mathrm{LQL}}) =$



(a) $(c_{\mathrm{AQL}}, c_{\mathrm{LQL}}) = (1.33, 1.00)$ and $(\alpha, \beta) = (0.01, 0.05)$

(b) $(c_{\mathrm{AQL}}, c_{\mathrm{LQL}}) = (1.33, 1.00)$ and $(\alpha, \beta) = (0.05, 0.10)$

Figure 3. The OC curve comparison under different conditions.

Table 4. The required n and $\mathrm{ASN}_{\mathrm{Max}}$ with m =1(1)10 under $(\alpha, \beta) = (0.10, 0.10)$.

	$(c_{\mathrm{AQL}}, c_{\mathrm{LQL}})$							
	(1.33, 1.00)		(1.50, 1.00)		(1.67, 1.33)		(2.00, 1.67)	
m	n	$\mathrm{ASN}_{\mathrm{Max}}$	n	$\mathrm{ASN}_{\mathrm{Max}}$	n	$\mathrm{ASN}_{\mathrm{Max}}$	n	$\mathrm{ASN}_{\mathrm{Max}}$
1	57	57.00	29	29.00	82	82.00	124	124.00
2	42	54.88	21	27.24	59	77.77	88	116.81
3	37	55.46	19	28.26	51	77.45	75	115.15
4	34	55.22	17	27.10	47	77.91	69	116.17
5	32	54.45	17	28.79	45	79.04	65	115.89
6	32	56.63	16	27.54	43	77.78	63	116.67
7	31	55.48	16	28.08	43	79.86	62	117.56
8	31	56.12	16	28.36	42	78.42	61	116.83
9	31	56.43	16	28.49	42	78.97	61	117.82
10	31	56.57	16	28.55	42	79.23	61	118.30

(1.33, 1.00) and $(\alpha, \beta) = (0.05, 0.10)$, the corresponding plan parameters are (n, k_a, k_r) =(51, 1.1812, 1.0750) for $m = 2$, (n, k_a, k_r) =(43, 1.2063, 1.0666) for $m = 3$ and (n, k_a, k_r) =(40, 1.2223, 1.0594) for $m = 4$. From the solved plan parameters in Tables 5–7, it can be observed that the required n increases as α and/or β decreases. This phenomenon explains why a smaller sample is required to estimate the actual process performance for making the correct submission decision if the producer and/or consumer allow higher risks for making an incorrect judgment. On the other hand, when the difference between c_{AQL} and c_{LQL} gets closer, the required sample size becomes larger. If the specified acceptable and rejectable quality levels are close, it means that a larger sample is required to assess the submitted lot's quality.

As described before, the sample size required for inspection of SSP is always constant, whereas the required ASN in the proposed plan is highly dependent on the quality level of the submitted lot. A desirable sampling plan is the one that requires a minimal ASN inspected from the lot while offering the same protection for both the producer and the consumer (Balamurali and Jun 2009; Wu and Liu 2018; Wu, Lee, and Huang 2021).

The required ASN value of the conventional variables SSP (Pearn and Wu 2007) is summarised in Tables 8 and 9, as well as the proposed plan for $m = 2$, 3, and 4 under the same conditions when the submitted lot's quality level is varied. Regardless of whether m is 2, 3, or 4, the proposed plan requires fewer ASN than the variables SSP. The ASN comparison also points out that the ASN reduction would be more significant when the lot's quality is extremely good or bad. For instance, under $(c_{\mathrm{AQL}}, c_{\mathrm{LQL}}) = (1.33, 1.00)$ and $(\alpha, \beta) = (0.01, 0.05)$, assuming the submitted lot's quality level is at $C_{pk} = 1.30$, the required sample size (n =123) is needed if the conventional variables SSP is employed. But, if the examiner utilises the proposed plan for m =2, the required sample size (n =84) and the corresponding ASN is only 90.82 (the ASN reduction is around 25%), and the ASN reduction is around 30% for m =3 (ASN = 82.99) and m =4 (ASN = 79.61).

Finally, to ensure the actual α - risk (denoted α^*) and β - risk (denoted by β^*) under the solved plan parameters (n, k_a, k_r) can satisfy the required risk conditions (i.e. $\alpha^* \le \alpha$ and $\beta^* \le \beta$), we also calculate and tabulate α^* and β^* with different combinations of $(c_{\mathrm{AQL}}, c_{\mathrm{LQL}})$ and m under $(\alpha, \beta) = (0.01, 0.05)$ and $(0.05, 0.10)$ in Tables 10 and 11, respectively. It can be seen that both α^* and β^* are all very close to and no more than the given tolerable risks (α, β), this implies the proposed plan is reliable and can provide the required protection to both the producer and the consumer.

Table 5. The plan parameters of the proposed plan for $m = 2$.

		$c_{AQL} = 1.33$ $c_{LQL} = 1.00$			$c_{AQL} = 1.50$ $c_{LQL} = 1.00$			$c_{AQL} = 1.67$ $c_{LQL} = 1.33$			$c_{AQL} = 2.00$ $c_{LQL} = 1.67$		
α	β	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r
0.01	0.01	119	1.1969	1.0999	59	1.3000	1.1552	171	1.5345	1.4317	260	1.8697	1.7676
	0.05	84	1.1701	1.0720	42	1.2599	1.1152	122	1.5070	1.4019	185	1.8430	1.7378
	0.10	69	1.1529	1.0515	34	1.2341	1.0863	99	1.4892	1.3800	151	1.8258	1.7158
0.05	0.01	94	1.2252	1.1222	48	1.3445	1.1894	135	1.5645	1.4548	202	1.8993	1.7903
	0.05	64	1.1989	1.0955	32	1.3056	1.1517	91	1.5376	1.4263	137	1.8732	1.7616
	0.10	51	1.1812	1.0750	25	1.2793	1.1229	72	1.5195	1.4041	108	1.8557	1.7391
0.10	0.01	83	1.2436	1.1384	42	1.3735	1.2147	117	1.5842	1.4715	174	1.9188	1.8065
	0.05	54	1.2185	1.1130	28	1.3368	1.1792	77	1.5587	1.4443	114	1.8942	1.7790
	0.10	42	1.2011	1.0928	21	1.3113	1.1514	59	1.5410	1.4224	88	1.8772	1.7567

Table 6. The plan parameters of the proposed plan for $m = 3$.

		$c_{AQL} = 1.33$ $c_{LQL} = 1.00$			$c_{AQL} = 1.50$ $c_{LQL} = 1.00$			$c_{AQL} = 1.67$ $c_{LQL} = 1.33$			$c_{AQL} = 2.00$ $c_{LQL} = 1.67$		
α	β	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r
0.01	0.01	100	1.2204	1.0919	50	1.3375	1.1449	143	1.5593	1.4226	215	1.8940	1.7581
	0.05	71	1.1942	1.0612	35	1.2982	1.1010	102	1.5324	1.3898	153	1.8680	1.7253
	0.10	58	1.1778	1.0396	29	1.2736	1.0705	83	1.5159	1.3666	125	1.8521	1.7018
0.05	0.01	81	1.2487	1.1184	41	1.3819	1.1858	114	1.5895	1.4500	171	1.9240	1.7848
	0.05	55	1.2231	1.0884	28	1.3444	1.1429	77	1.5636	1.4180	115	1.8990	1.7527
	0.10	43	1.2063	1.0666	22	1.3194	1.1124	61	1.5466	1.3945	91	1.8828	1.7288
0.10	0.01	72	1.2660	1.1371	37	1.4089	1.2154	101	1.6085	1.4691	150	1.9431	1.8032
	0.05	47	1.2424	1.1074	24	1.3750	1.1730	66	1.5847	1.4376	97	1.9202	1.7717
	0.10	37	1.2260	1.0858	19	1.3511	1.1431	51	1.5683	1.4143	75	1.9046	1.7479

Table 7. The plan parameters of the proposed plan for $m = 4$.

		$c_{AQL} = 1.33$ $c_{LQL} = 1.00$			$c_{AQL} = 1.50$ $c_{LQL} = 1.00$			$c_{AQL} = 1.67$ $c_{LQL} = 1.33$			$c_{AQL} = 2.00$ $c_{LQL} = 1.67$		
α	β	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r
0.01	0.01	90	1.2362	1.0857	45	1.3628	1.1371	128	1.5760	1.4154	193	1.9104	1.7506
	0.05	64	1.2103	1.0527	32	1.3243	1.0897	91	1.5495	1.3804	137	1.8849	1.7155
	0.10	52	1.1945	1.0305	26	1.3004	1.0582	75	1.5336	1.3565	113	1.8697	1.6913
0.05	0.01	74	1.2629	1.1156	38	1.4041	1.1834	105	1.6051	1.4462	155	1.9396	1.7804
	0.05	50	1.2386	1.0821	25	1.3692	1.1351	70	1.5804	1.4108	104	1.9158	1.7450
	0.10	40	1.2223	1.0594	20	1.3453	1.1031	56	1.5641	1.3862	83	1.9003	1.7201
0.10	0.01	67	1.2785	1.1360	35	1.4277	1.2158	94	1.6226	1.4671	138	1.9575	1.8005
	0.05	44	1.2572	1.1021	22	1.3983	1.1668	61	1.6010	1.4313	89	1.9368	1.7648
	0.10	34	1.2415	1.0794	17	1.3760	1.1351	47	1.5855	1.4068	69	1.9221	1.7398

Table 8. ASN comparison between the variables SSP and the proposed plan under $(c_{AQL}, c_{LQL}) = (1.33, 1.00)$.

	$(\alpha, \beta) = (0.01, 0.05)$				$(\alpha, \beta) = (0.05, 0.10)$			
		The proposed plan				The proposed plan		
C_{pk}	Variables SSP	$m = 2$	$m = 3$	$m = 4$	Variables SSP	$m = 2$	$m = 3$	$m = 4$
0.80	123	84.09	71.24	64.41	71	51.40	43.70	40.91
0.85	123	84.54	72.07	65.55	71	52.22	44.82	42.22
0.90	123	86.27	74.65	68.71	71	53.98	47.06	44.75
0.95	123	90.89	80.90	75.85	71	57.01	50.84	48.98
1.00	123	99.40	92.52	89.03	71	61.07	56.11	55.04
1.05	123	109.72	107.91	107.37	71	65.12	61.80	61.95
1.10	123	116.61	119.67	122.54	71	67.71	65.90	67.29
1.15	123	115.80	119.81	123.63	71	67.83	66.62	68.52
1.20	123	108.13	108.76	110.43	71	65.51	63.75	65.16
1.25	123	98.41	94.33	93.10	71	61.78	58.74	59.12
1.30	123	90.82	82.99	79.61	71	57.94	53.51	52.89
1.35	123	86.58	76.29	71.46	71	54.89	49.30	47.91
1.40	123	84.79	73.02	67.20	71	52.91	46.43	44.50
1.45	123	84.19	71.66	65.22	71	51.82	44.71	42.39
1.50	123	84.04	71.19	64.41	71	51.31	43.78	41.18

Table 9. ASN comparison between the variables SSP and the proposed plan under $(c_{\text{AQL}}, c_{\text{LQL}}) = (2.00, 1.67)$.

	$(\alpha,\beta) = (0.01, 0.05)$				$(\alpha,\beta) = (0.05, 0.10)$			
		The proposed plan				The proposed plan		
C_{pk}	Variables SSP	$m = 2$	$m = 3$	$m = 4$	Variables SSP	$m = 2$	$m = 3$	$m = 4$
1.50	275	185.94	155.08	140.14	156	110.52	94.87	87.85
1.55	275	188.94	159.84	146.13	156	113.97	99.33	92.88
1.60	275	197.28	171.33	159.44	156	119.91	106.86	101.21
1.65	275	213.80	193.39	184.41	156	128.15	117.64	113.30
1.70	275	236.50	225.38	221.86	156	137.04	130.12	127.89
1.75	275	255.99	255.80	259.90	156	143.86	140.67	140.97
1.80	275	261.18	266.58	275.25	156	146.15	145.19	147.18
1.85	275	249.20	250.75	256.95	156	143.13	141.83	143.68
1.90	275	227.82	219.98	219.83	156	136.12	132.33	132.54
1.95	275	207.74	190.83	185.05	156	127.65	120.56	118.69
2.00	275	194.67	171.21	161.57	156	120.03	109.91	106.29
2.05	275	188.31	160.60	148.31	156	114.47	101.99	97.05
2.10	275	185.91	155.75	141.67	156	111.07	96.86	90.92
2.15	275	185.20	153.86	138.71	156	109.28	93.87	87.18
2.20	275	185.04	153.23	137.55	156	108.48	92.29	85.06

Table 10. The calculated α^* and β^* under $(\alpha,\beta) = (0.01, 0.05)$.

		Plan parameters				
m	$(c_{\text{AQL}}, c_{\text{LQL}})$	n	k_a	k_r	α^*	β^*
2	(1.33, 1.00)	84	1.1701	1.0720	0.00997	0.04992
	(1.50, 1.00)	42	1.2599	1.1152	0.00939	0.04812
	(1.67, 1.33)	122	1.5070	1.4019	0.00983	0.04949
	(2.00, 1.67)	185	1.8430	1.7378	0.00987	0.04960
3	(1.33, 1.00)	71	1.1942	1.0612	0.00965	0.04895
	(1.50, 1.00)	35	1.2982	1.1010	0.00968	0.04900
	(1.67, 1.33)	102	1.5324	1.3898	0.00968	0.04902
	(2.00, 1.67)	153	1.8680	1.7253	0.00993	0.04980
4	(1.33, 1.00)	64	1.2103	1.0527	0.00966	0.04899
	(1.50, 1.00)	32	1.3243	1.0897	0.00926	0.04765
	(1.67, 1.33)	91	1.5495	1.3804	0.00993	0.04979
	(2.00, 1.67)	137	1.8849	1.7155	0.00998	0.04995

Table 11. The calculated α^* and β^* under $(\alpha,\beta) = (0.05, 0.10)$.

		Plan parameters				
m	$(c_{\text{AQL}}, c_{\text{LQL}})$	n	k_a	k_r	α^*	β^*
2	(1.33, 1.00)	51	1.1812	1.0750	0.04824	0.09748
	(1.50, 1.00)	25	1.2793	1.1229	0.04944	0.09912
	(1.67, 1.33)	72	1.5195	1.4041	0.04921	0.09884
	(2.00, 1.67)	108	1.8557	1.7391	0.04929	0.09896
3	(1.33, 1.00)	43	1.2063	1.0666	0.04971	0.09959
	(1.50, 1.00)	22	1.3194	1.1124	0.04731	0.09572
	(1.67, 1.33)	61	1.5466	1.3945	0.04941	0.09913
	(2.00, 1.67)	91	1.8828	1.7288	0.04943	0.09915
4	(1.33, 1.00)	40	1.2223	1.0594	0.04811	0.09731
	(1.50, 1.00)	20	1.3453	1.1031	0.04836	0.09743
	(1.67, 1.33)	56	1.5641	1.3862	0.04869	0.09809
	(2.00, 1.67)	83	1.9003	1.7201	0.04898	0.09851

5. Application example

Nowadays, many electronic devices such as a tablet, laptop or set-top box, etc., which have become an essential part for people in daily life. As the demand for these products grows, manufacturers face a serious challenge in maintaining the customer's safety when using the devices. To prevent device damage or even a fire disaster caused by a short circuit, the above-mentioned electronic devices are typically equipped with a unique power distribution switch (PDS).

When a short circuit occurs, a heavy current called short-circuits current flows through the circuit. It causes the system's current to spike to an abnormally high level while the voltage drops to a low level. The high current was caused by a short circuit, which resulted in excessive heating, which could result in a fire or explosion. Short-circuits can sometimes takes the form of an arc and causes considerable damage to the system.

The PDS restricts the output current within a safe range and provides a stable current to ensure that the devices operate normally. The short-circuit threshold (SCT) is a critical quality characteristic of PDS, and the value of SCT will be measured to confirm if the PDS meets the required specifications or not. A particular 4-pins type of PDS (see Figure 4) with the bilateral specification limits $(\text{LSL}, \text{USL}) = (0.5\text{A}, 2.0\text{A})$ is studied for illustration. Suppose that the required quality levels $(c_{\text{AQL}}, c_{\text{LQL}}) = (1.33, 1.00)$, tolerable risks $(\alpha,\beta) = (0.05, 0.10)$ and allowable numbers of sampling $(m = 3)$ are specified in the business contract. That is, if the submitted lot's quality is at $C_{pk} = 1.33$, the lot acceptance probability must be at least 0.95, and if the submitted lot's quality is at $C_{pk} = 1.00$, the probability of acceptance would be no more than 0.10. Based on the given settings, the corresponding plan parameters $(n, k_a, k_r) = (43, 1.2063, 1.0666)$ can be obtained by checking Table 13. This implies a sample of 43 PDSs from the submitted lot should be taken for inspection, the measured SCT values of collected PDSs are shown in Table 12. We should check the normality assumption

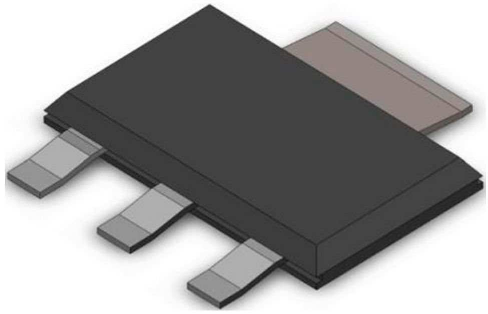


Figure 4. A 4-pins type of PDS.

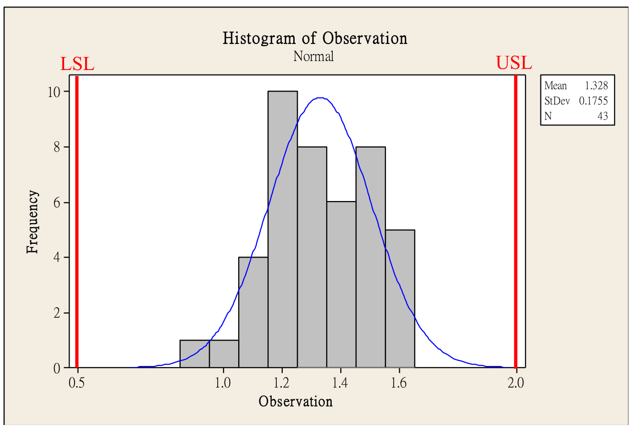


Figure 5. The histogram of the sample measurements.

Table 12. The measured SCT values of collected PDSs (Unit: Ampere (A)).

1.33	0.89	1.56	1.12	1.19	1.32	1.13	1.40	1.21
1.22	1.21	1.52	1.34	1.32	1.21	1.31	1.41	1.33
1.33	0.99	1.43	1.10	1.53	1.40	1.17	1.50	1.08
1.62	1.49	1.20	1.38	1.45	1.17	1.59	1.50	
1.57	1.48	1.20	1.37	1.53	1.27	1.58	1.16	

of the collected data before calculating the estimator. Figure 5 depicts the histogram of the collected 43 sample measurements with specification limits, and the data can be confirmed to be normally distributed by the normal probability plot and Anderson–Darling test with p - value $= 0.353$ (see Figure 6). The sample estimator is calculated as $\hat{C}_{pk} = 1.2813$, which is greater than the critical value for acceptance $k_a = 1.2063$. Hence, the submitted lot can be accepted in this case. On the contrary, the lot will be rejected if $\hat{C}_{pk} < 1.0666$, and if $\hat{C}_{pk}$ falls into the resampling region $[1.0666, 1.2063)$, the 2nd stage sampling from the submitted lot is required to perform and the lot sentencing will be made following the same procedure in the 1st stage. If the 3rd stage (the final stage) is needed, the lot will be accepted only if $\hat{C}_{pk} \geq 1.2063$; otherwise the entire lot should be rejected.

To further examine the performance in the long-term lot inspection of this application example, a sensitivity

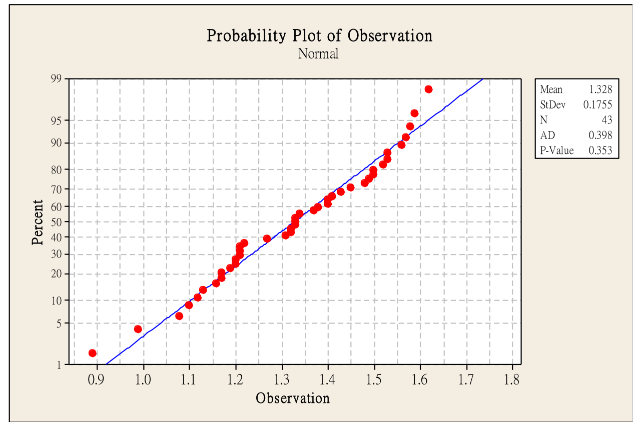


Figure 6. Normal probability plot of the sample measurements.

Table 13. The required ASN for C_{pk}-based SSP and SIMSP under the given conditions, $(c_{AQL}, c_{LQL}) = (1.33, 1.00)$ and $(\alpha, \beta) = (0.05, 0.10)$.

C_{pk}	Process yield (lower bound)	C_{pk}-based SSP	The proposed C_{pk}-based SIMSP	
		n	n	ASN
0.80	98.3605%	71	43	43.70
0.90	99.3066%	71	43	47.06
1.00	99.7300%	71	43	56.11
1.14	99.9374%	71	43	66.78
1.18	99.9600%	71	43	65.26
1.20	99.9682%	71	43	63.75
1.24	99.9801%	71	43	59.81
1.28	99.9877%	71	43	55.53
1.30	99.9904%	71	43	53.51
1.34	99.9942%	71	43	50.04

analysis on ASN is conducted by using the proposed C_{pk}-based SIMSP with $m = 3$, and the variables C_{pk}-based SSP (Pearn and Wu 2007) under the given conditions. When the actual quality level of the submitted lot in terms of C_{pk} ranges from 0.80 to 1.34 under the given conditions, Table 13 summarises the ASN required for inspection. It is discovered that, regardless of the quality of the submitted lot, the proposed SIMSP consumes less ASN for inspection. This implies that the proposed C_{pk}-based SIMSP will give the desired protection to both the producer and the consumer with a smaller ASN so that the cost of the inspection will be reduced.

6. Conclusions

The inspection cost is directly fluctuated by the required number of sample items for inspection to ensure or estimate the quality level of submitted lots. Numerous sampling plans with advanced strategies have been designed to increase efficiency, flexibility and discriminatory power for lot acceptance determination. In particular, multiple sampling plan (MSP) has been proved to be an efficient scheme for lot sentencing while ensuring requirement and protection for the producer and the consumer. However, the complication and difficulty in the design of the MSP increases more especially for variables inspection. To remedy this, this paper proposed a new variables sampling plan, stage-independent multiple sampling plan (SIMSP), by assuming the inspections at each stage are independent which can be regarded as a relax type of the conventional MSP. The proposed SIMSP is integrated with the most widely-used process capability index C_{pk}, so it is more suitable for today's high-yield process environments and products.

For the cost-efficient purpose, the plan parameters are obtained by solving the formulated optimisation model with minimised ASN as the objective function and constrained by certain quality-and-risk conditions. To examine the performance of the proposed plan and the conventional C_{pk}-based variables SSP, a comparison study on several performance measures including the OC curve, sample size (n) and average sample number (ASN) required for inspection are conducted.

The results indicated that the proposed plan can ensure both benefits of saving inspection cost and reducing the complexity of traditional MSP. The solved plan parameters have also been confirmed to provide desirable protection to both the producer and the consumer simultaneously. It is worthy noting that the OC curve of the proposed plan is derived based on exact sampling distribution rather than approximation hence the decisions made would be more accurate and reliable. Besides, such a designated operating procedure may avoid the breakdown of a business partnership. Additionally, a PDS application example is illustrated in a case study to demonstrate the feasibility of the proposed plan.

As for future research, two possible extensions of this article are provided here. (*i*) there are various types of PCIs developed from different perspectives, it is worth integrating other advanced indices with the proposed new sampling strategy for special occasions in the real application (*ii*) since the C_{pk} index can only be used when the collected data is normally distributed, it is suggested to extend the concept of this study to develop suitable sampling plans for specified or non-normal distributions.

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Data availability statement

Data sharing is not applicable to this article as no new data were created or analysed in this study.

Disclosure statement

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