# **Weekly Summary Template**

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```
library(tidyverse)
-- Attaching packages ----- tidyverse 1.3.2 --
v ggplot2 3.4.0 v purrr 1.0.1
v tibble 3.1.8
           v dplyr 1.1.0
v tidyr 1.3.0 v stringr 1.5.0
     2.1.3 v forcats 1.0.0
v readr
-- Conflicts ----- tidyverse conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag() masks stats::lag()
 library(ISLR2)
 library(cowplot)
 library(kableExtra)
Attaching package: 'kableExtra'
The following object is masked from 'package:dplyr':
  group_rows
```

## Tuesday, February 7th

## ! TIL

Include a  $very\ brief$  summary of what you learnt in this class here. Today, I learnt the following concepts in class:

- 1. What the interpretation of  $\beta_0$  and  $\beta_1$  is (regression coefficients)
- 2. Categorical covariates
- 3. How to reorder factors

## Interpretation of $\ 0$ and $\beta_1$ (regression coefficients)

 $\bullet\,$  The regression model is...

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

•  $\beta_0$  and  $\beta_1$  are the regression coefficients with  $\beta_0$  being the intercept and  $\beta_1$  being the slope

```
library(ggplot2)
attach(mtcars)
```

The following object is masked from package:ggplot2:

mpg

```
x <- mtcars$hp
y <- mtcars$mpg

model <- lm(y ~ x)
summary(model)</pre>
```

#### Call:

```
lm(formula = y \sim x)
```

## Residuals:

```
Min 1Q Median 3Q Max -5.7121 -2.1122 -0.8854 1.5819 8.2360
```

```
Estimate Std. Error t value Pr(>|t|)

(Intercept) 30.09886    1.63392    18.421 < 2e-16 ***

x         -0.06823    0.01012    -6.742    1.79e-07 ***

---

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.863 on 30 degrees of freedom

Multiple R-squared: 0.6024, Adjusted R-squared: 0.5892

F-statistic: 45.46 on 1 and 30 DF, p-value: 1.788e-07
```

Based on the summary model...

- The intercept means that a car with 'hp=0' would have 'mpg=30.09' ## Thursday, February 9th
- From this, if we have some co-variate  $x_0$ , then the expected value for  $y(x_0)$  is given by:

$$y(x_0) = \beta_0 + \beta_1 x_0$$

• Using this we can find the expected value for  $x_0 + 1$ , which is...

$$y(x_0+1) = \beta_0 \, + \, \beta_1 \, \times \, (x_0+1) = \beta_0 \, + \, \beta_1 \, \, x_0 \, + \, \beta_1 = y(x_0) \, + \, \beta_1$$

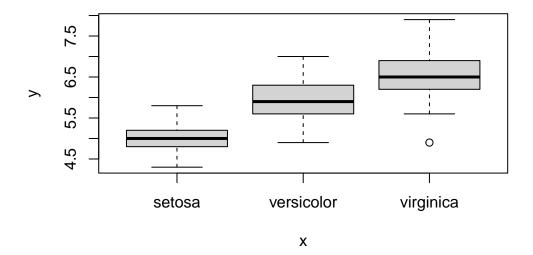
• This implies that  $\beta_1 = y(x_0+1) - y(x_0)$ 

## Categorical covariates

In class, we looked at categorical covariates in the iris dataset, and created a boxplot and a summary of the model for the categorical covariates

```
x <- iris$Species
y <- iris$Sepal.Length

# boxplot(Sepal.Length ~ Species, df)
boxplot(y ~ x)</pre>
```



## Call:

lm(formula = Sepal.Length ~ Species, data = iris)

## Coefficients:

Even if x is categorical, we can still write down the regression model as follows:

$$y_i = \beta_0 + \beta_1 x_i$$

where  $x_i \in \{setosa, versicolor, virginica\}$ . This means that we end up with, three different models with each one having a different intercept.

- $\begin{aligned} &1. \ y_i = \beta_0 + \beta_1(x_i == 'setosa') \\ &2. \ y_i = \beta_0 + \beta_1(x_i == 'versicolor') \\ &3. \ y_i = \beta_0 + \beta_1(x_i == 'virginica') \end{aligned}$
- The interpretation of the intercept  $(\beta_0)$  is the expected y value when x belongs to the base category

- The slope  $(\beta_1)$  with the name 'Species.versicolor' represents the following:
- '(Intercept)' = y(x = setosa)
- 'Species.versicolor' = y(x = versicolor) y(x = setosa)
- 'Species.virginica' =  $y(x = \mathtt{virginica}) y(x = \mathtt{setosa})$

## Reordering factors

Lets say that we didn't want 'setosa' to be the baseline level, and instead, we wanted 'virginica' to be the baseline level. How would we do this?

First we reorder/relevel the categorical covariate

```
# before
iris$Species
```

[1]	setosa	setosa	setosa	setosa	setosa	setosa
[7]	setosa	setosa	setosa	setosa	setosa	setosa
[13]	setosa	setosa	setosa	setosa	setosa	setosa
[19]	setosa	setosa	setosa	setosa	setosa	setosa
[25]	setosa	setosa	setosa	setosa	setosa	setosa
[31]	setosa	setosa	setosa	setosa	setosa	setosa
[37]	setosa	setosa	setosa	setosa	setosa	setosa
[43]	setosa	setosa	setosa	setosa	setosa	setosa
[49]	setosa	setosa	${\tt versicolor}$	${\tt versicolor}$	${\tt versicolor}$	versicolor
[55]	${\tt versicolor}$	versicolor				
[61]	${\tt versicolor}$	versicolor				
[67]	${\tt versicolor}$	versicolor				
[73]	${\tt versicolor}$	versicolor				
[79]	${\tt versicolor}$	versicolor				
[85]	${\tt versicolor}$	versicolor				
[91]	${\tt versicolor}$	versicolor				
[97]	${\tt versicolor}$	${\tt versicolor}$	${\tt versicolor}$	${\tt versicolor}$	virginica	virginica
[103]	virginica	virginica	virginica	virginica	virginica	virginica
[109]	virginica	virginica	virginica	virginica	virginica	virginica
[115]	virginica	virginica	virginica	virginica	virginica	virginica
[121]	virginica	virginica	virginica	virginica	virginica	virginica
[127]	virginica	virginica	virginica	virginica	virginica	virginica
[133]	virginica	virginica	virginica	virginica	virginica	virginica
[139]	virginica	virginica	virginica	virginica	virginica	virginica
[145]	virginica	virginica	virginica	virginica	virginica	virginica
Levels: setosa versicolor virginica						

```
iris$Species <- relevel(iris$Species, "virginica")
# after
iris$Species</pre>
```

```
[1] setosa
              setosa
                        setosa
                                 setosa
                                           setosa
                                                     setosa
 [7] setosa
              setosa
                        setosa
                                 setosa
                                           setosa
                                                     setosa
 [13] setosa
              setosa
                        setosa
                                 setosa
                                           setosa
                                                     setosa
              setosa
[19] setosa
                        setosa
                                 setosa
                                           setosa
                                                   setosa
[25] setosa
             setosa
                        setosa
                                setosa
                                          setosa
                                                   setosa
 [31] setosa
              setosa
                        setosa
                                 setosa
                                           setosa
                                                    setosa
 [37] setosa
              setosa setosa
                                setosa
                                           setosa
                                                   setosa
[43] setosa
              setosa
                                           setosa
                                                     setosa
                        setosa
                                 setosa
 [49] setosa
              setosa
                        versicolor versicolor versicolor versicolor
 [55] versicolor versicolor versicolor versicolor versicolor
 [61] versicolor versicolor versicolor versicolor versicolor
 [67] versicolor versicolor versicolor versicolor versicolor
 [73] versicolor versicolor versicolor versicolor versicolor
 [79] versicolor versicolor versicolor versicolor versicolor
 [85] versicolor versicolor versicolor versicolor versicolor
 [91] versicolor versicolor versicolor versicolor versicolor
 [97] versicolor versicolor versicolor virginica virginica
[103] virginica virginica virginica virginica virginica
[109] virginica virginica virginica virginica virginica virginica
[115] virginica virginica virginica virginica virginica
[121] virginica virginica virginica virginica virginica virginica
[127] virginica virginica virginica virginica virginica virginica
[133] virginica virginica virginica virginica virginica virginica
[139] virginica virginica virginica virginica virginica virginica
[145] virginica virginica virginica virginica virginica
Levels: virginica setosa versicolor
```

Once we do the re-leveling, we can now run the regression model:

```
new_cat_model <- lm(Sepal.Length ~ Species, iris)
new_cat_model</pre>
```

```
Call:
lm(formula = Sepal.Length ~ Species, data = iris)
```

```
Coefficients:

(Intercept) Speciessetosa Speciesversicolor
6.588 -1.582 -0.652
```

## Tuesday, February 9th

## ! TIL

Include a *very brief* summary of what you learnt in this class here. Today, I learnt the following concepts in class:

- 1. How to make a plot for a model that incorporates more than 1 quantitative covariate
- 2. The impact of noise and  $\beta$  values on  $\mathbb{R}^2$
- 3. Multiple regression with categorical covariates

```
library(tibble)
  library(ISLR2)
  attach(Credit)
  df <- Credit %>%
    tibble()
  colnames(df) <- tolower(colnames(df))</pre>
  df
# A tibble: 400 x 11
   income limit rating cards
                                                      student married region balance
                                 age educat~1 own
                  <dbl> <dbl> <dbl>
                                                               <fct>
                                                                        <fct>
    <dbl> <dbl>
                                         <dbl> <fct> <fct>
                                                                                  <dbl>
 1
     14.9
           3606
                    283
                             2
                                  34
                                            11 No
                                                      No
                                                               Yes
                                                                        South
                                                                                    333
   106.
            6645
                    483
                             3
                                  82
                                            15 Yes
                                                      Yes
                                                               Yes
                                                                        West
                                                                                    903
 3
   105.
           7075
                    514
                             4
                                  71
                                            11 No
                                                      No
                                                               No
                                                                        West
                                                                                    580
 4
   149.
           9504
                    681
                             3
                                  36
                                            11 Yes
                                                      No
                                                               No
                                                                        West
                                                                                    964
 5
     55.9
           4897
                    357
                             2
                                  68
                                            16 No
                                                      No
                                                               Yes
                                                                        South
                                                                                    331
 6
     80.2
           8047
                    569
                             4
                                  77
                                            10 No
                                                               No
                                                                        South
                                                                                   1151
                                                      No
7
     21.0
                    259
                             2
                                  37
                                                                                    203
           3388
                                            12 Yes
                                                      No
                                                               No
                                                                        East
                             2
8
     71.4
                    512
                                  87
                                             9 No
                                                               No
                                                                        West
                                                                                    872
           7114
                                                      No
9
     15.1
           3300
                    266
                             5
                                  66
                                            13 Yes
                                                      No
                                                               No
                                                                        South
                                                                                    279
10
     71.1
           6819
                    491
                             3
                                  41
                                            19 Yes
                                                      Yes
                                                               Yes
                                                                        East
                                                                                   1350
# ... with 390 more rows, and abbreviated variable name 1: education
```

Plotting a model with more than 1 quantitative covariate

We will look at the following 3 columns: 'income, rating, limit'.

```
df3 <- df %>%
    select('income', 'rating', 'limit')
  df3
# A tibble: 400 x 3
  income rating limit
   <dbl>
          <dbl> <dbl>
1
    14.9
            283 3606
2 106.
            483 6645
3 105.
            514 7075
4 149.
            681 9504
            357 4897
5
   55.9
6
   80.2
            569 8047
7
    21.0
            259 3388
8
    71.4
            512 7114
9
    15.1
            266 3300
    71.1
10
            491 6819
# ... with 390 more rows
```

If we want to see how the credit limit is related to income and credit rating, we can visualize the following plot

```
library(plotly)

Attaching package: 'plotly'

The following object is masked from 'package:ggplot2':
    last_plot

The following object is masked from 'package:stats':
    filter

The following object is masked from 'package:graphics':
    layout
```

```
fig <- plot_ly(df3, x=~income, y=~rating, z=~limit)
fig %>% add_markers()
```

This models a linear relationship, which in 3-dimensions is a plane (a hyperplane)

• Was shown in class what the hyperplane looked like but weren't given the code, so couldn't include that in the summary

The regression model is as follows:

```
model <- lm(limit ~ income + rating, df3)
model

Call:
lm(formula = limit ~ income + rating, data = df3)

Coefficients:
(Intercept) income rating</pre>
```

• Have 3 different coefficients

-532.4711

• 2nd/3rd numbers are the slopes associated with the income and rating

14.7711

What is the interpretation for the coefficients?

0.5573

- 1.  $\beta_0$  is the expected value of y when income = 0 and rating = 0 (the credit limit with 0 income and 0 rating is -532) -> this is an extrapolation
- 2.  $\beta_1$  is saying that if rating is held constant and income changes by 1 unit, then the corresponding change in the 'limit' is 0.5573.
- 3.  $\beta_2$  is saying that if *income* is held constant and *rating* changes by 1 unit, then the corresponding change in the 'limit' is 14.7711.

What about the significance?

```
summary(model)

Call:
lm(formula = limit ~ income + rating, data = df3)

Residuals:
    Min    1Q Median    3Q Max
-420.97 -121.77    14.97    126.72    485.48
```

```
Estimate Std. Error t value Pr(>|t|)
(Intercept) -532.47115
                       24.17283 -22.028
                                          <2e-16 ***
                        0.42349 1.316
income
              0.55727
                                          0.189
             14.77115
                        0.09647 153.124 <2e-16 ***
rating
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Residual standard error: 182.3 on 397 degrees of freedom
Multiple R-squared: 0.9938,
                              Adjusted R-squared: 0.9938
F-statistic: 3.18e+04 on 2 and 397 DF, p-value: < 2.2e-16
```

The p-value for 'rating' is very significant, while its not significant for 'income'

- Multi-colinearity issue with the 3D model
  - 1. Not idealisitic that if we hold rating constant and income increases, by one unit then limit increase by 0.5573. -> You can't change income by 1 without impacting rating

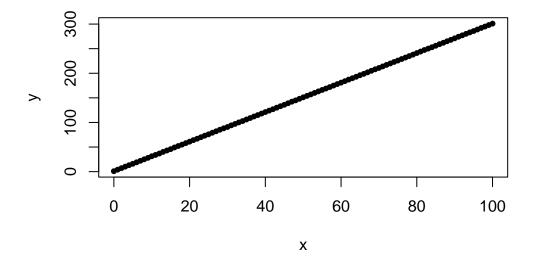
## The impact of noise and $\beta$ values on $\mathbb{R}^2$

This first code chunk shows the model with very little noise and the output model summary to demonstrate the very high  $R^2$  and p-value that occurs with very low noise

```
x <- seq(0,100,1)
b0 <- 1.0
b1 <- 3.0
y <- b0 + b1 * x + rnorm(100) * 0.1
```

Warning in b0 + b1 \* x + rnorm(100) \* 0.1: longer object length is not a multiple of shorter object length

```
plot(x, y, pch = 20)
```



```
model <- lm(y ~ x)
summary(model)</pre>
```

#### Call:

 $lm(formula = y \sim x)$ 

#### Residuals:

Min 1Q Median 3Q Max -0.259432 -0.066147 -0.005918 0.065873 0.264160

## Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) 1.0141490 0.0203853 49.75 <2e-16 \*\*\*

x 2.9999584 0.0003522 8517.65 <2e-16 \*\*\*

--
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1032 on 99 degrees of freedom Multiple R-squared: 1, Adjusted R-squared: 1 F-statistic: 7.255e+07 on 1 and 99 DF, p-value: < 2.2e-16

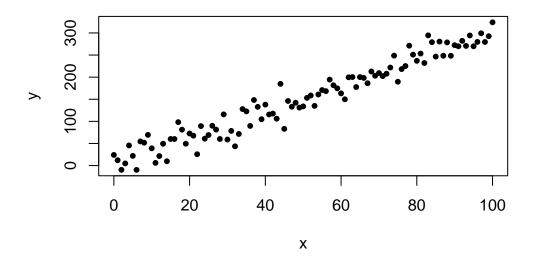
This next code chunk shows what happens with a much greater noise value. The intercept

p-value is much high than the previous model and the  $R^2$  value decreases as well.

```
x <- seq(0,100,1)
b0 <- 1.0
b1 <- 3.0
y <- b0 + b1 * x + rnorm(100) * 20
```

Warning in b0 + b1 \* x + rnorm(100) \* 20: longer object length is not a multiple of shorter object length

```
plot(x, y, pch = 20)
```



```
model <- lm(y ~ x)
summary(model)</pre>
```

Call:

 $lm(formula = y \sim x)$ 

Residuals:

```
Min 1Q Median 3Q Max -54.056 -13.653 -1.043 12.971 51.625
```

```
Estimate Std. Error t value Pr(>|t|)

(Intercept) 2.78546  4.16550  0.669  0.505

x        2.96371  0.07197  41.180  <2e-16 ***
---

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

Residual standard error: 21.09 on 99 degrees of freedom

Multiple R-squared: 0.9448, Adjusted R-squared: 0.9443

F-statistic: 1696 on 1 and 99 DF, p-value: < 2.2e-16
```

#### To summarize:

- Lower noise increases the  $R^2$  value
- If you make  $\beta_0$ ,  $\beta_1$ , and noise values super small, then p-value will be super high and lead to a low  $R^2$  value
- Can have high p-value with low  $\mathbb{R}^2$  value, but CANT have low p-value with high  $\mathbb{R}^2$  value

## Multiple regression with categorical covariates

To demonstrate how to incorporate both categorical and quantitative covariates into a regression model, we will use the rating and marital status from the Credit dataset to try and predict the limit.

To create the models, you need to run 2 separate models using income, rating, and marital status to predict limit. 1. once for when student is yes 1. once for when student is no

• By adding the categorical variable, you are adding an additional intercept term

```
model <- lm(limit ~ rating + married, df)
summary(model)

Call:
lm(formula = limit ~ rating + married, data = df)

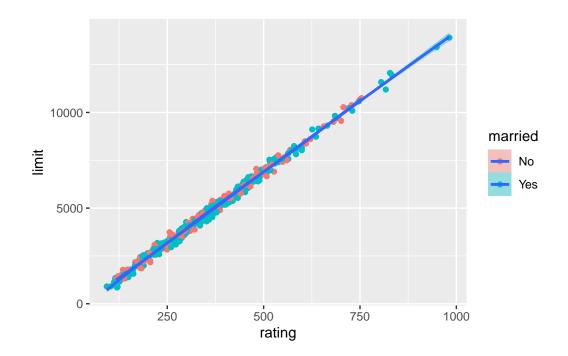
Residuals:
    Min    1Q Median    3Q Max</pre>
```

```
-404.83 -126.56 20.86 131.80 456.19
```

Residual standard error: 182.2 on 397 degrees of freedom Multiple R-squared: 0.9938, Adjusted R-squared: 0.9938 F-statistic: 3.181e+04 on 2 and 397 DF, p-value: < 2.2e-16

```
ggplot(df) +
  geom_point(aes(x=rating, y=limit, color=married)) +
  geom_smooth(aes(x=rating, y=limit, fill=married))
```

 $\ensuremath{\tt `geom\_smooth()`}\ using method = 'loess' and formula = 'y ~ x'$ 



The model above does have 2 different lines, they are just so close together, that you can't really see it, which implies that they have the same regression line. From the model summary, you can see that the p-value for marital status is very high, which indicates that it is not a good predictor of someones credit limit.