Module 1 project instructions

D.Curreli

Module 1 Project Instructions
NPRE 598 Computational Plasma Physics

Module 1 project instructions

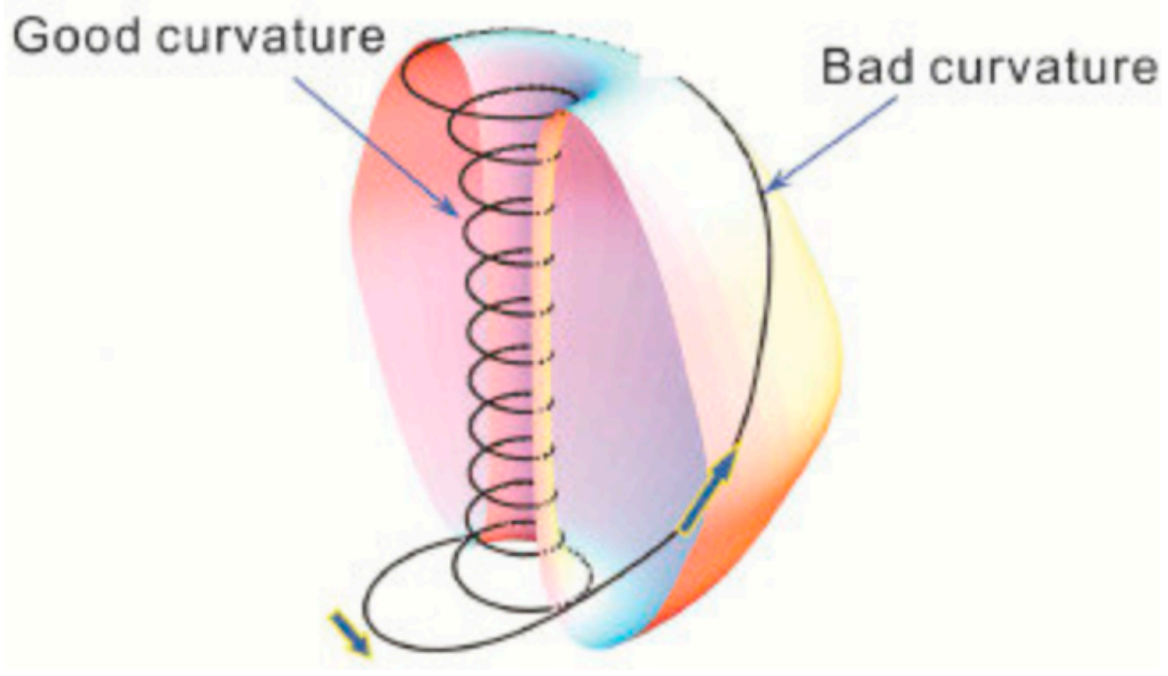
- This project will count 20% of the final score
- The project is individual, no group project will be accepted.
- The goal of this project is twofold:
  - Practice on preparing good scientific reports: this will always be required in your professional life, either in the form of technical reports, or in the form of scientific publications
  - Making practice at solving a plasma physics problem using the computational tools you learned in class
- The output of the computational project is made of two parts:
  - Your Code
  - Written Report: a brief document describing your activity and the output of your code
- The written report is a concise and detailed description of the scientific activity that you have done. The technical content will have to be minimum 800 words, plus figures, references, and the pages reporting the code produced. The references and the code will not count to the total number of words, but they will have to be included in the report. The grading of the written report will be based on the following sections of the document:
  - title, abstract, introduction and literature review, model derivation, model discretization, implementation, numerical analysis, production runs, scientific results, conclusions, references, and code produced.
  - Furthermore the overall scientific quality, the coding quality, clarity, and style, and the technical writing will also be considered in the evaluation process.
- Important. Each of your Computational Projects will have to produce at least one cool, publication-quality figure.
- A typical project requires approx 2-3 days of full-time work
- Deadline for turn-in: 10 days after the assignment date

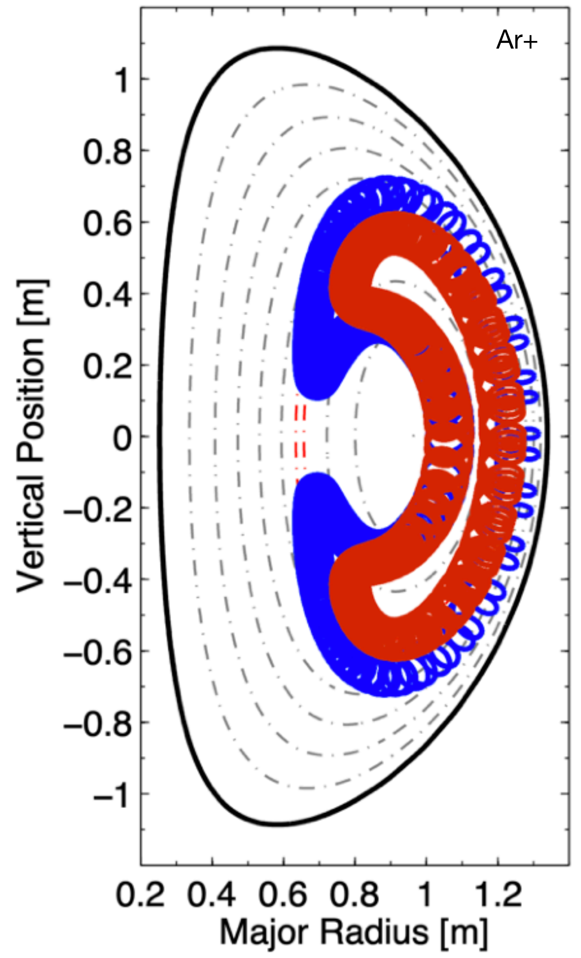
Project M1-1 Impurity transport in the MAST spherical tokamak with strongly-sheared electric fields

- First, read this work: https://arxiv.org/abs/1106.1981 an old preprint on work related to the MAST spherical tokamak. While reading, consider the following: (1) the model magnetic field, (2) the model equations of the single impurity, (3) the approach used to reconstruct the density. The goal of this project is to perform a similar work and reproduce the results of the paper.
- Here there is a suggestion on the steps you could follow:
  - First, write a routine returning the 3 components of the equilibrium field of the MAST tokamak, using Eq. (4) in the paper; e.g.:

```
BR, BZ, Bphi = bfield.mast(R,Z)
```

  - Sample the B-field on a regular R,Z grid, and write a function interpolating the 3 components of the B-field at a generic point P. This will greatly improve speed w.r.t. a direct evaluation of the B-field.
  - Similarly, write a routine returning the E-field (Eq. 7)
  - Solve the Lorentz-Langevin equation of an impurity ion moving in the grid (Eq. 1); do not solve the B-field at every point, but rather use the interpolator; initialize ions at the magnetic axis and integrate their trajectory for ~O(10^2) milli-seconds.
  - Sample the density on each magnetic flux surface, and compare He^{2+}, Ne^{10+}, W^{20+}
  - Optional: Estimate the diffusion coefficients
- Finally, discuss the shortcomings of this work, and suggest what you would do next to extend it.





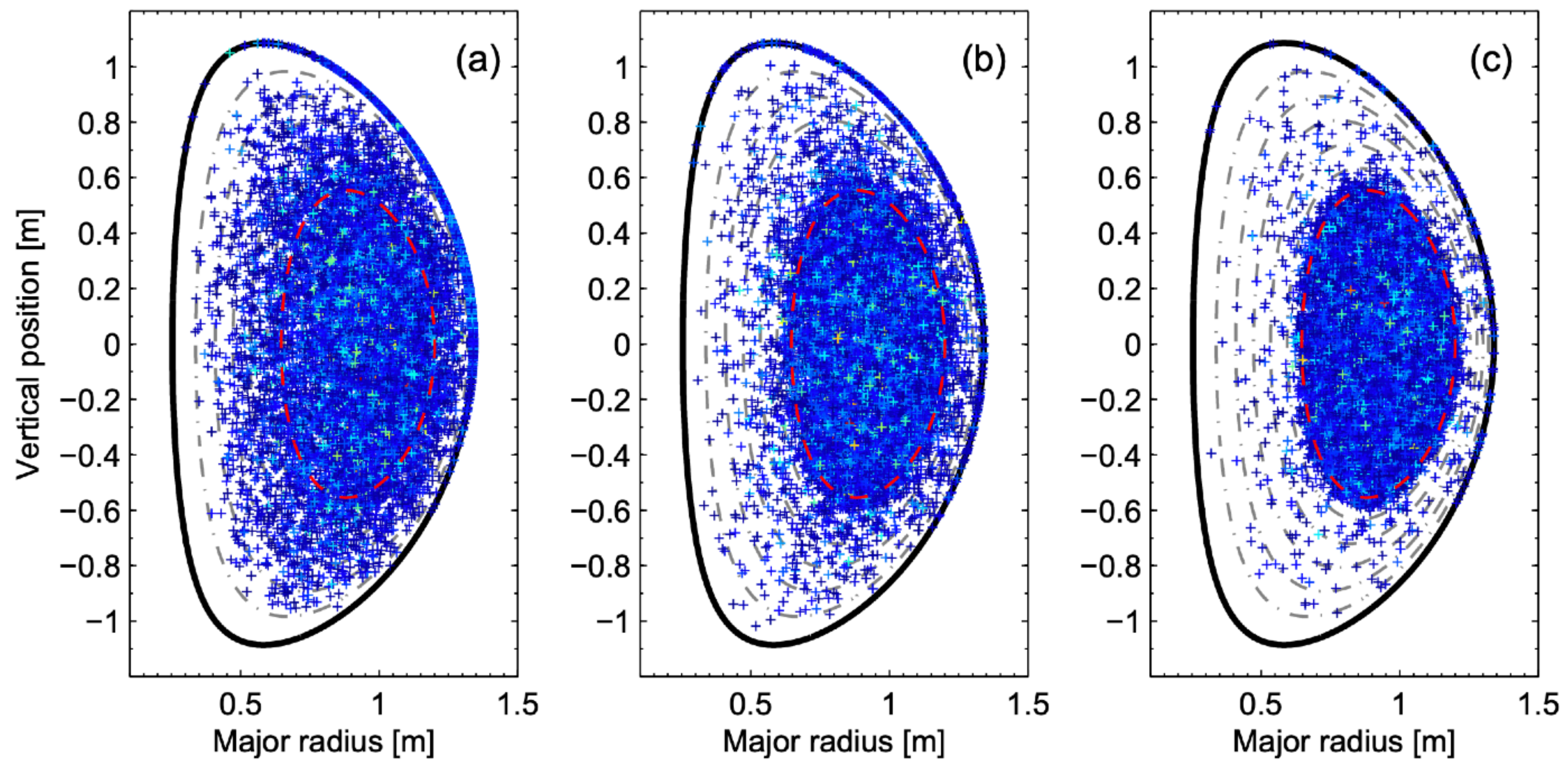


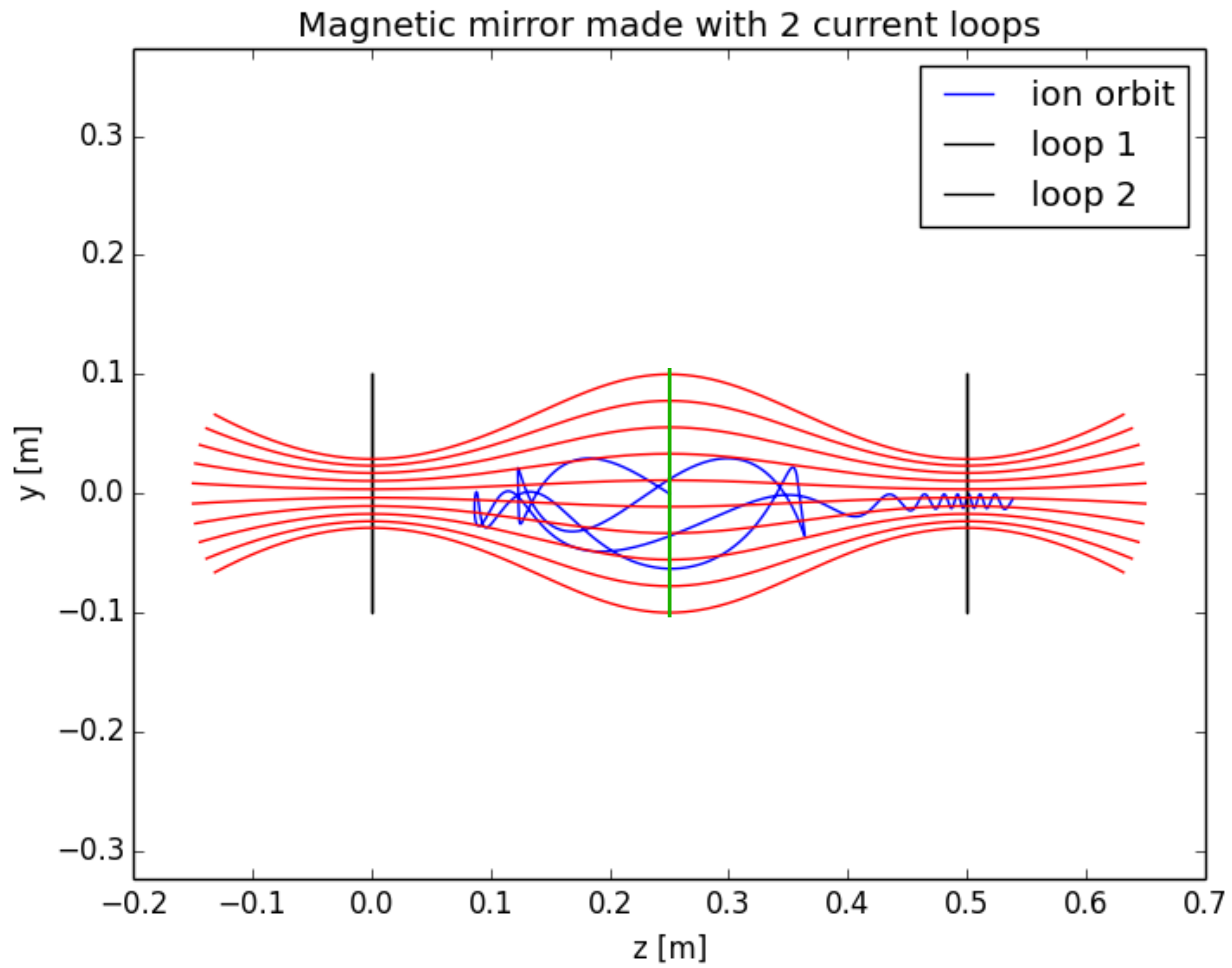
Figure 4. (Colour online) Poloidal distribution of (a) He^{2+} (b) Ne^{10+} and (c) W^{20+} ions in a MAST-like equilibrium with a radially sheared electric field, the position of which is indicated by a dashed (red online) line.

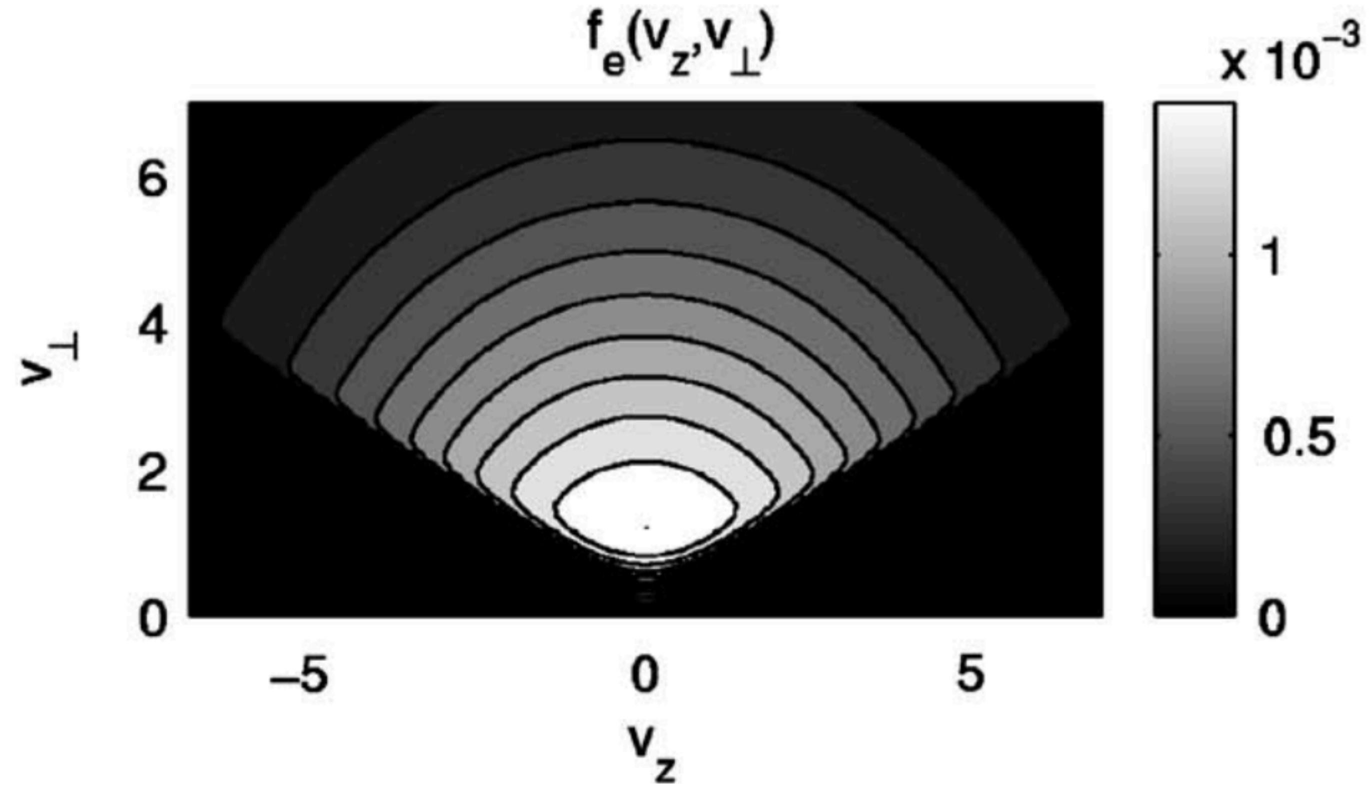
Project M1-2 Loss cone of a magnetic mirror for a Maxwell-Boltzmann ion population

- First, prepare a routine returning the 3 components of the B-field of a 2-loop magnetic mirror, e.g.:

```
Bx, By, Bz = bfield.mirror(x,y,z)
```

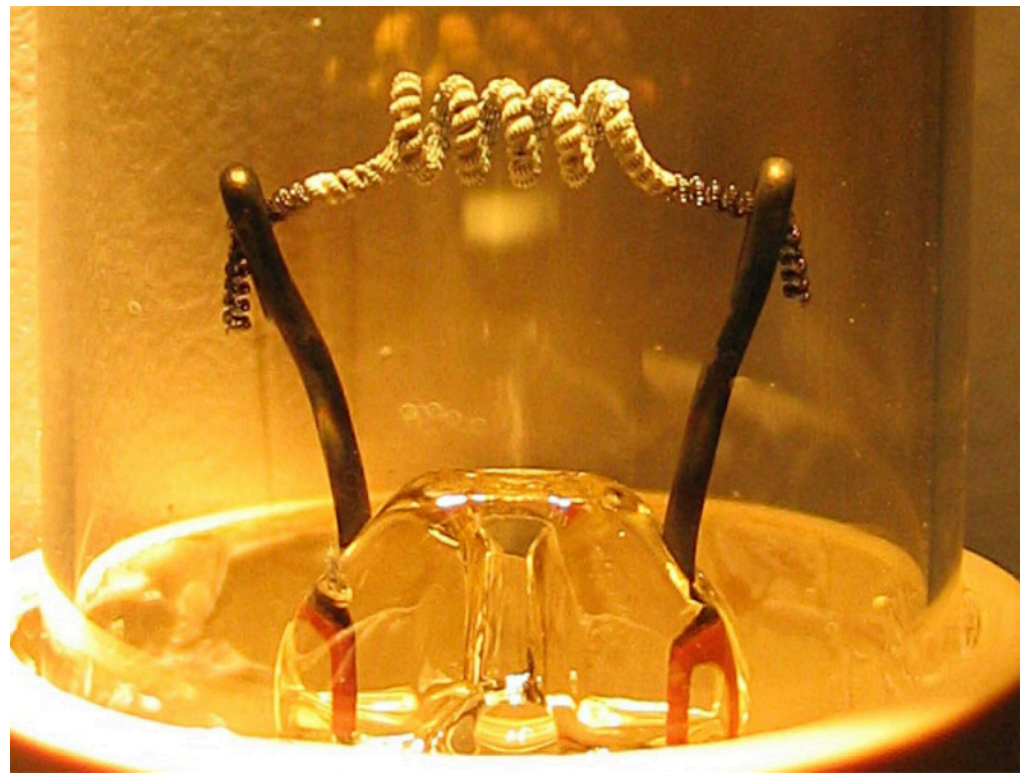
- Sample the B-field on the R,Z plane of the mirror, and write a function returning the 3 interpolated components of the B-field at a generic point P(x,y,z). This will greatly improve computational speed w.r.t. a direct B-field calculation.
- Solve the Newton-Lorentz equation of an ion moving in the grid; do not solve the B-field at every point, but rather use the interpolator; initialize ions using a Maxwell-Boltzmann distribution at the midplane (green line) and integrate their trajectory over time.
- Establish a criterion to decide automatically if the particle is ‘trapped’ within the mirror, or ‘lost’ into the loss cone.
- Repeat the calculation for a large number of particles, $O(10^4)$, and reconstruct the distribution function
- Compare the loss-cone you find from your calculation against the theoretical value.
- Optional: repeat the same calculation adding an electrostatic perturbation near the cyclotron frequency, $e\varphi(z, t) = e\varphi_0 \sin(k_\perp x - \omega_0 t)$, $\omega_0 = \Omega(\pm z_0)$, where Ω is the cyclotron frequency at the midpoint, and study the onset of chaotic motion as a function of the ES wave amplitude.

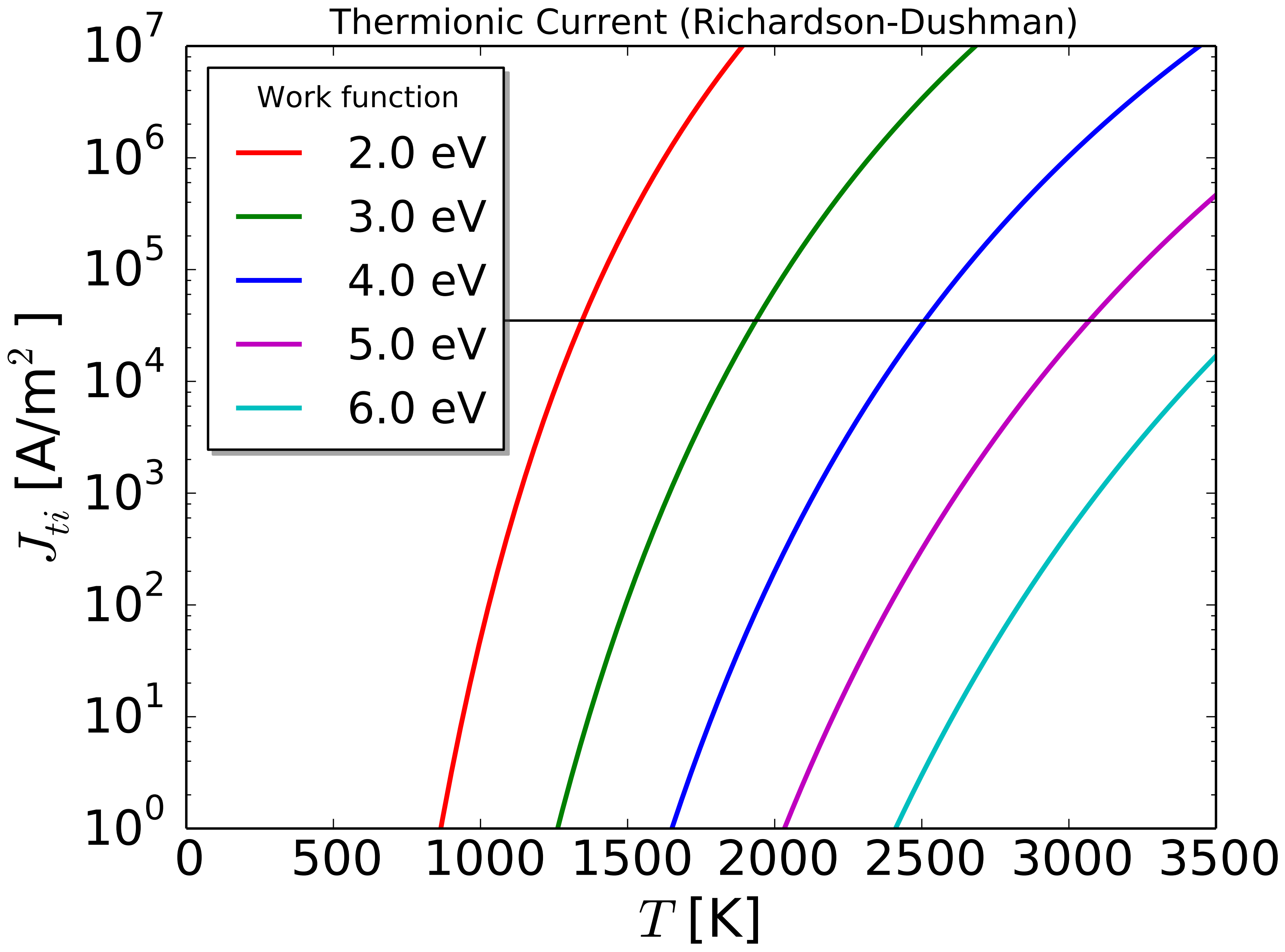




Project M1-3 A study on inverse sheaths: Thermionic emission from a solid surface immersed in a B-field

- A solid surface is heated to high temperature, T_s = 1000K~3000K, and emits cold thermal electrons ($T_e = T_s$) via thermionic emission (Richardson-Dushman). The electrons come out from the surface as a half Maxwell-Boltzmann population. The solid surface is immersed in a strong magnetic field B=5 Tesla, inclined at an angle ψ = 88 deg with respect to the normal to the surface. The goal of this project is calculating:
  1. The fraction of electrons promptly redeposited in a time shorter than one electron gyroperiod ("local" or "prompt" redeposition)
  2. The fraction of electrons non-locally redeposited (electrons re-deposited after more than one full gyro-orbit)
  3. The total redeposition fraction (prompt + nonlocal)
  4. The fraction of electron population escaping the surface
  5. The distribution function of electrons escaping the surface
- Solve numerically the Newton-Lorentz equation of one thermionic electron emitted by the surface.
- Establish a criterion to decide if each electron is "escaped", or "promptly" redeposited, or "nonlocally" redeposited.
- Repeat the calculation for a large number of electrons, $O(10^4)$, and evaluate the quantities 1-5 reported above. Do you notice any similarity between your distribution function and the loss-cone of a magnetic mirror?
- Perform a simple parametric study as a function of the magnetic field, for magnitudes between B=1-10 Tesla and inclinations in the interval ψ = 80-89 deg.
- Finally, find the surface temperature necessary to maintain a desired current density, for the two cases with and without B-field. Assume work functions between 2-5 eV. Discuss your results.



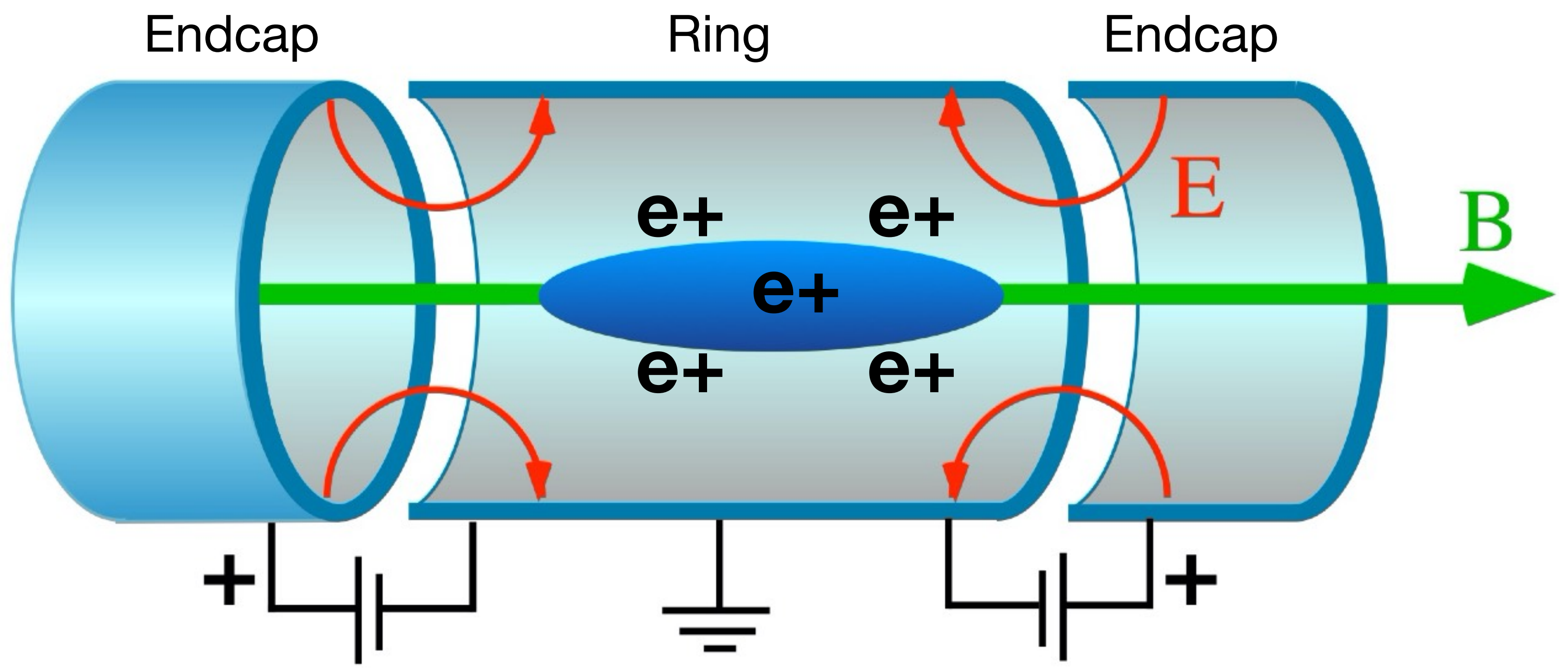


Project M1-4 Antimatter confinement using a Penning trap

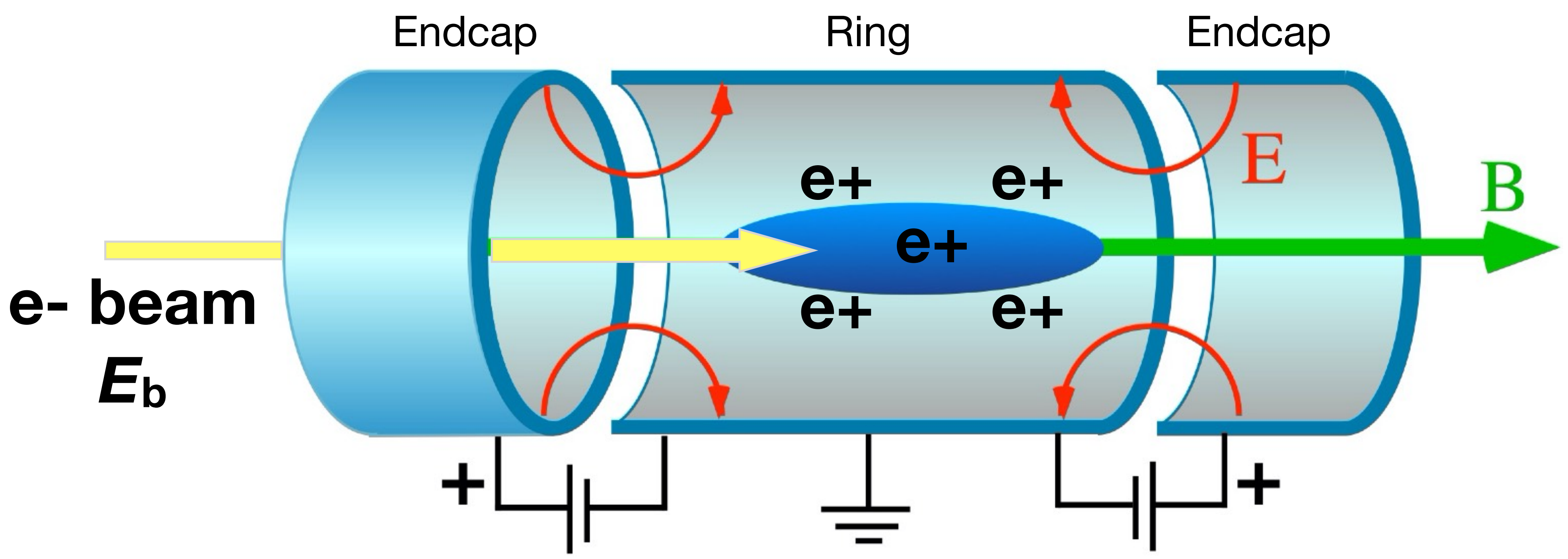
- Matter-antimatter collision is the best way to convert *all* the rest mass of a particle into energy via mutual annihilation.
- The goal of this project is designing an antimatter confinement chamber for a thermal population of positrons using a Penning trap.
- First, read what is a Penning trap, eg.: https://en.wikipedia.org/wiki/Penning_trap and pay particular attention to the E,B fields. This paper might also be a good reading: [https://doi.org/10.1103/RevModPhys.58.233]
- Prepare a routine returning the 3 components of the E-field of the trap, e.g.:

```
Ex, Ey, Ez = penningtrap(x,y,z)
```

- Sample the E-field on the R,Z plane, and write a function returning the 3 interpolated components of the E-field at a generic point P(x,y,z). This will greatly improve computational speed w.r.t. a direct E-field calculation.
- Solve the Newton-Lorentz (classical) dynamics of positrons confined inside the trap; do not solve the E-field at every point, but rather use the interpolator;
- Initialize positrons using a Maxwell-Boltzmann distribution at room temperature (T=300K) at the center of the trap and integrate their trajectory over time.
- Design the E,B fields allowing confinement of the positrons in the trap.
- Which particle drifts do you recognize?
- Is there a loss cone at the two endcaps of the trap?
- Finally, add a high energy electron beam of energy E_b at one end of the trap (see figure). The beam must have enough energy to overcome the endcap E-field barrier and hit the positrons at the center of the trap. Find the energy and the angles of e-beam to maximize the focusing of the beam on the positron cloud.



Penning trap



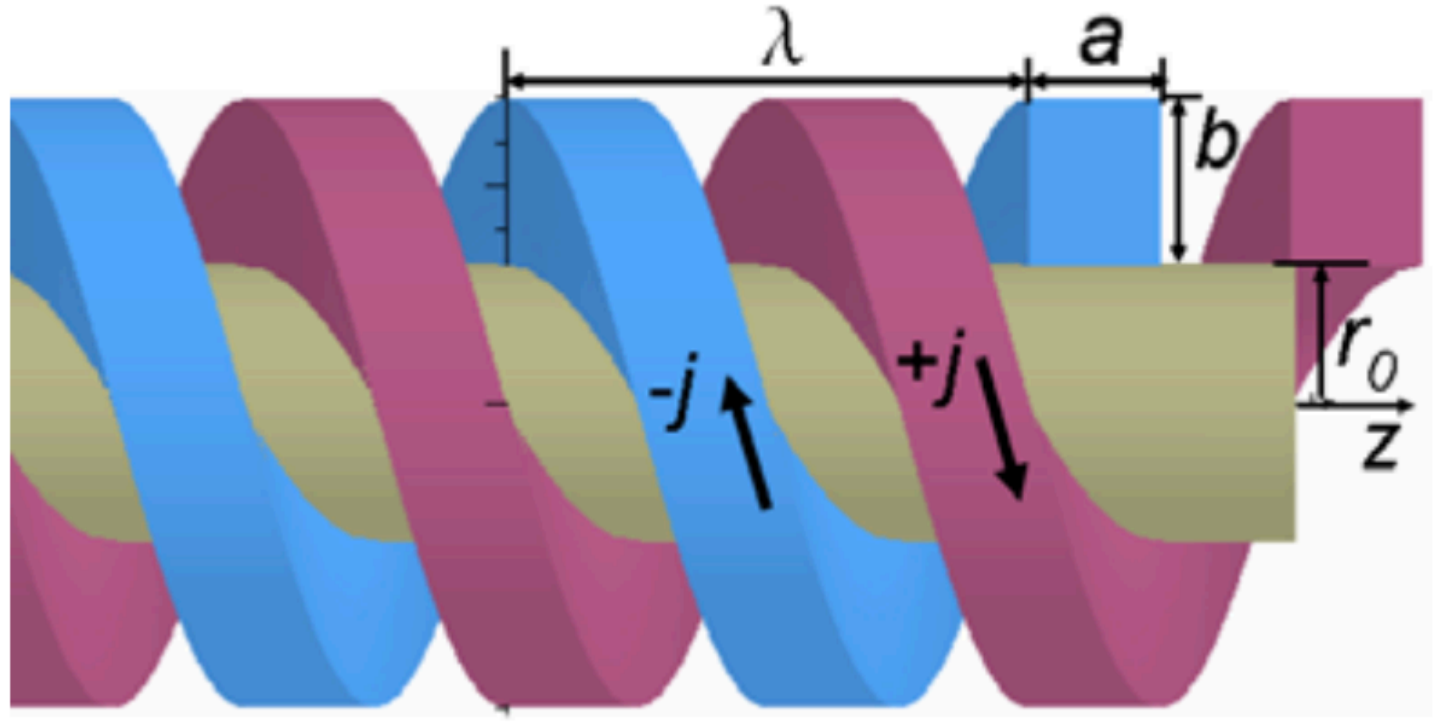
Penning trap plus electron beam

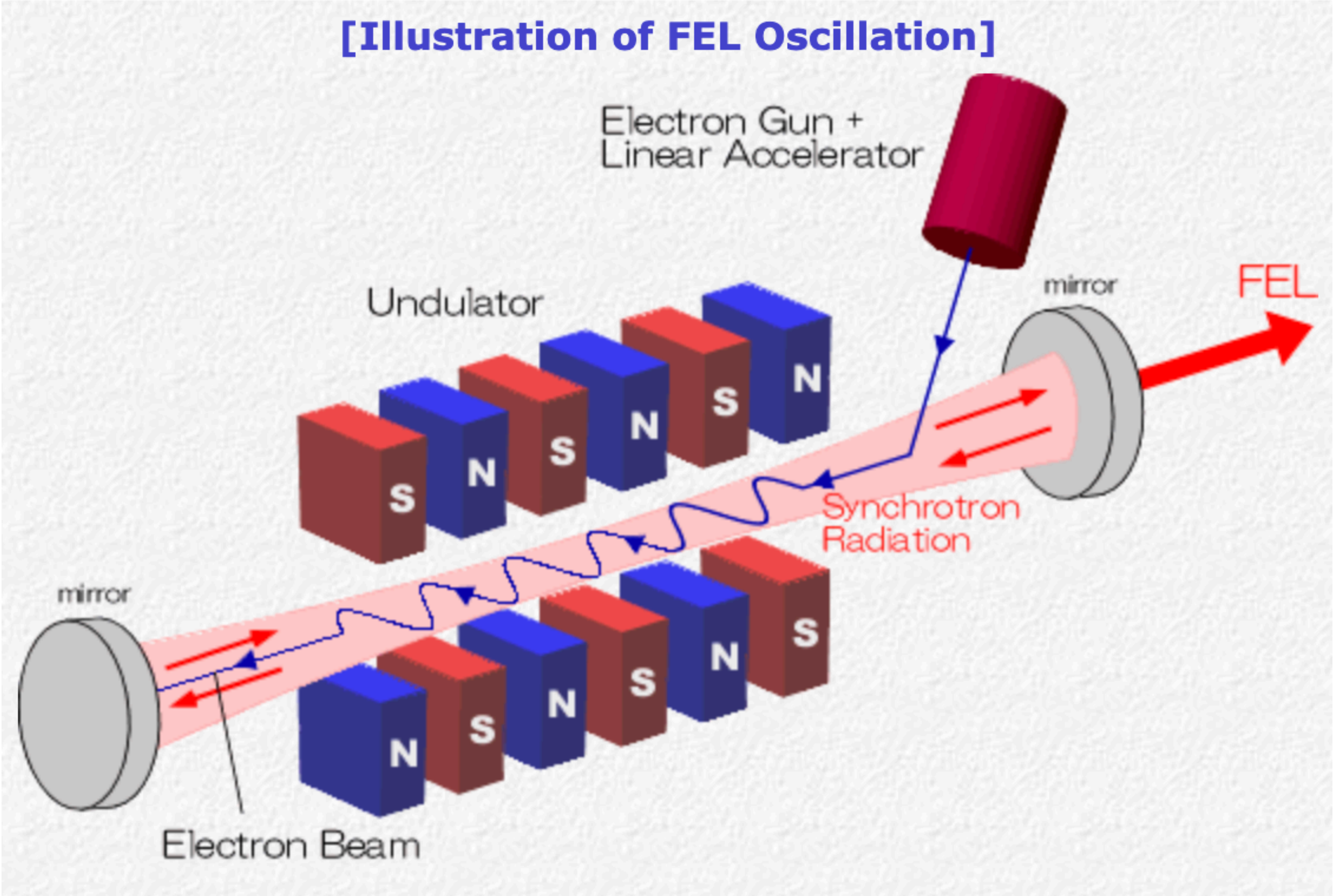
Project M1-5 Electron Orbits in a Helical Undulator

- Undulators are used for the production of coherent radiation (from X-Ray to visible light and down to millimeter waves) from the synchrotron radiation of a relativistic electron beam. Undulators are used in a variety of synchrotron radiation facilities and advanced photon sources, including free-electron lasers. For generalities on undulators, read the following wiki page: https://en.wikipedia.org/wiki/Undulator
- A helical undulator consists of a double helix carrying currents in opposite directions in each helix. Read about the magnetic field produced by an undulator at: Week 1 > Suggested Readings > 3 - Magnetic Field Analysis of Helical Undulators
- First, prepare a routine returning the the B-field of a helical undulator, e.g.:

```
Bx, By, Bz = bfield.undulator(x,y,z)
```

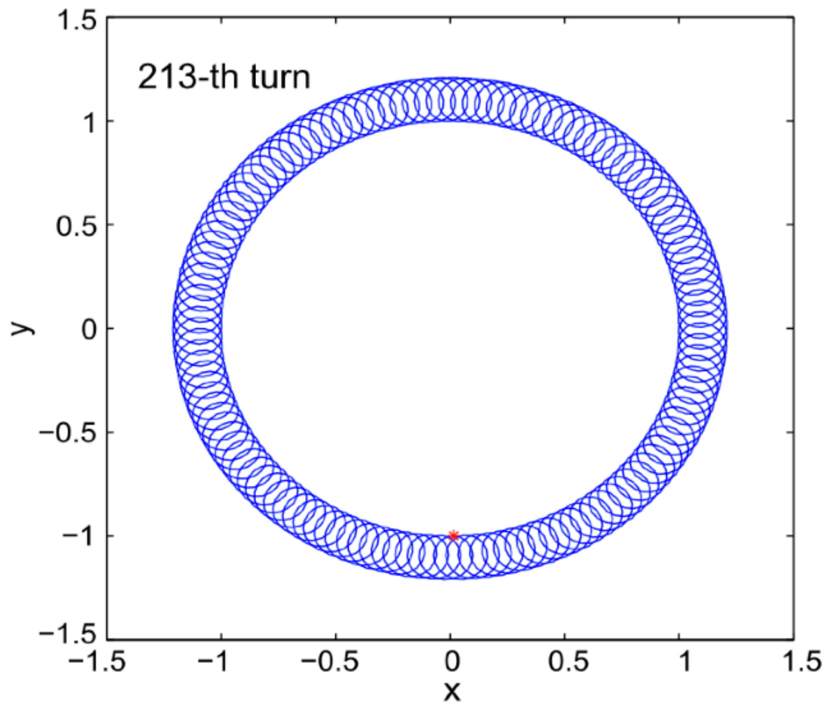
- Sample the B-field on the internal cylindrical path of the electrons, and write a function returning the 3 interpolated components of the B-field at a generic point P within the cylinder. This will greatly improve computational speed w.r.t. a direct B-field calculation.
- Solve the Newton-Lorentz equation of an electron moving in the undulator; do not solve the B-field at every point, but rather use the interpolator;
- As initial conditions, consider mono energetic electrons entering the undulatory at 40 MeV. Integrate the trajectory of few test electrons from the entrance to the exit of the undulator.
- From the acceleration of the electrons along the trajectory, derive amount of radiated power using the Larmor formula (in the limit v<<c).

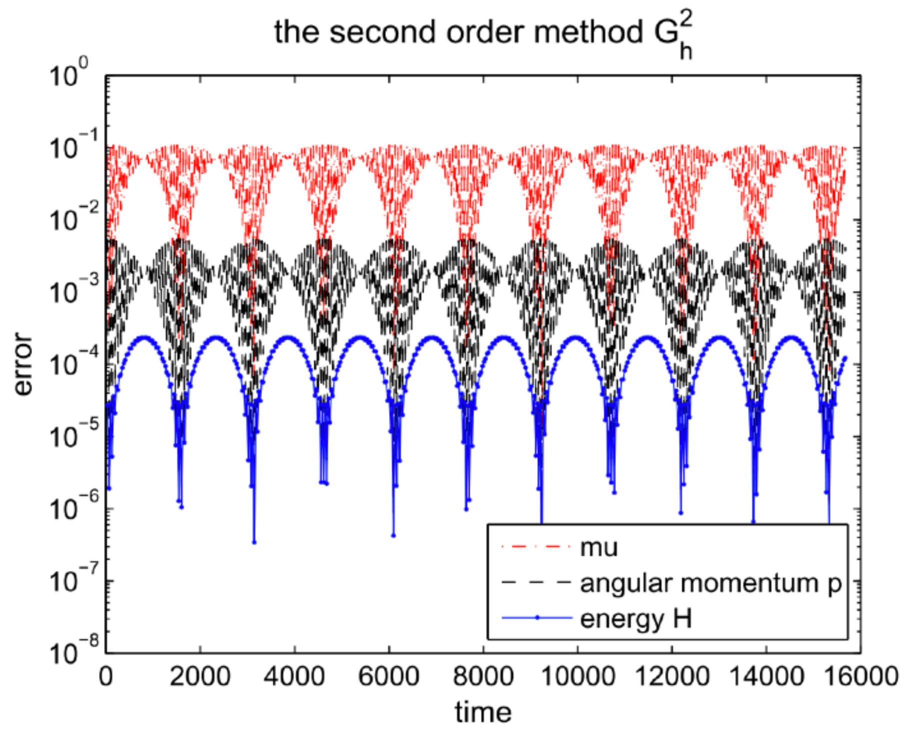




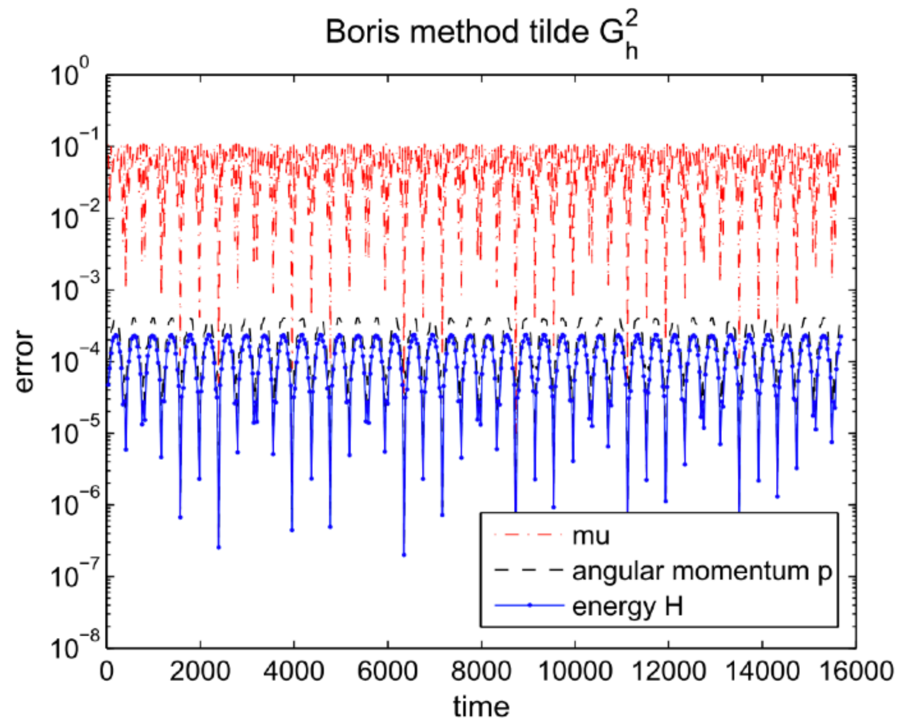
Project M1-6 A study on symplectic integrators

- Can we do better than the Boris-Bunemann algorithm? People have tried for decades to beat Boris, and many schemes have been proposed. However, to date there is no real consensus on any version better than Boris. The goal of this paper is scratching the tip of the iceberg, and learning more on this important topic, comparing the results from Boris against more recent schemes.
- First, read the following 2 papers, paying particular attention to the second one:
    1. H.Qin *et al.* Why is Boris algorithm so good? (2013) [http://dx.doi.org/10.1063/1.4818428]
    2. Y.He *et al.* Volume-preserving algorithms for charged particle dynamics [http://dx.doi.org/10.1016/j.jcp.2014.10.032] (2015)
- Focus on understanding Eqs. (14)-(15)-(16), which provide a generalization of 2nd-order volume-preserving numerical schemes.
- Implement the two 2nd-order schemes of Eq. (15) and Eq. (16), and test them using the same examples of Section 4.1 (2D static electromagnetic field) and Section 4.2 (2D ideal axisymmetric tokamak).
- Analyze the trend of the error of the two 2nd-order schemes, and compare against the results reported in the paper.
- Which method do you think is the most appropriate for long time integrations of O(10^5-10^6) gyro-orbits? Consider both accuracy and computational cost. Discuss.

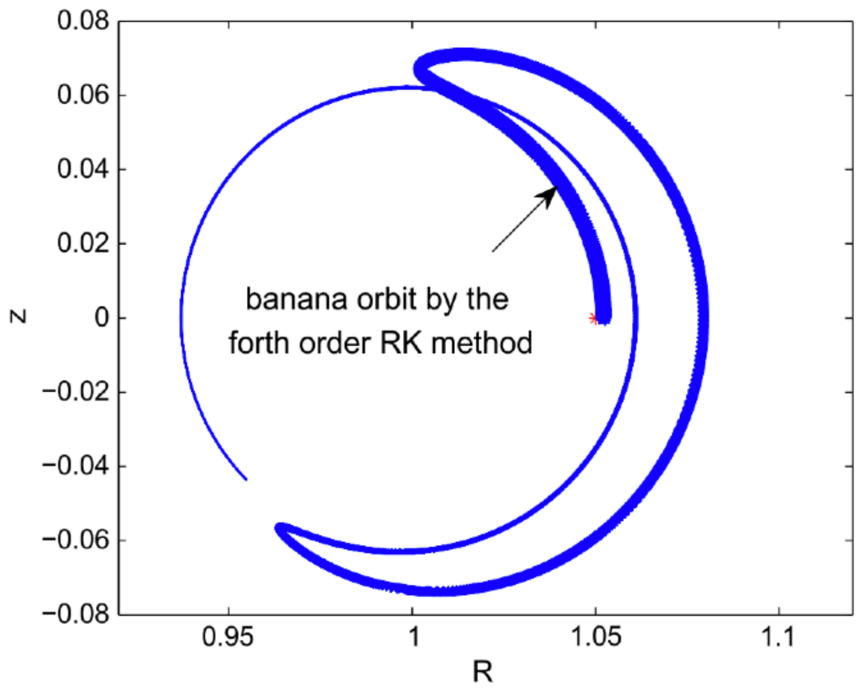




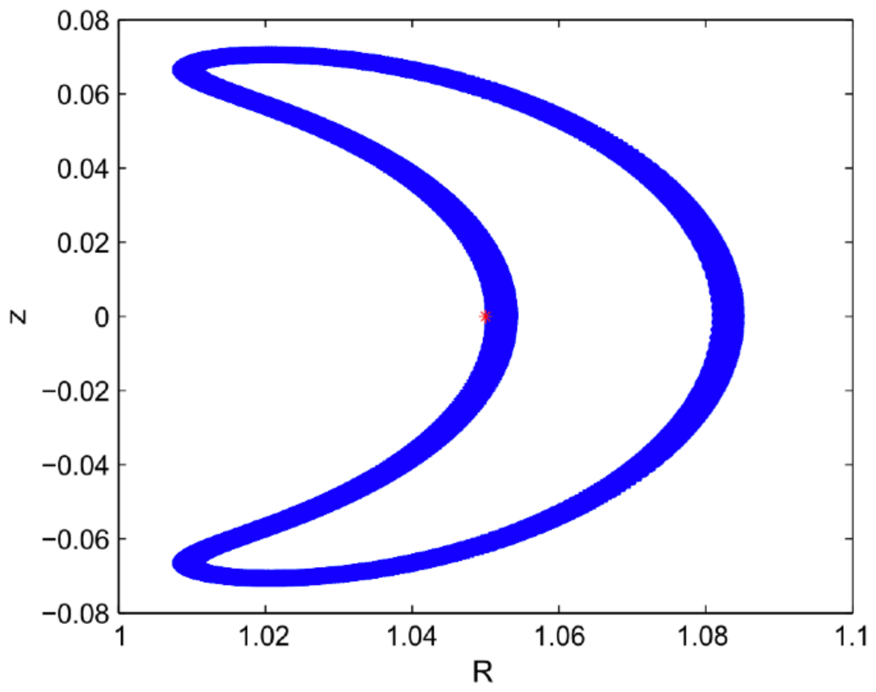
(a) Second order method G_h^2.



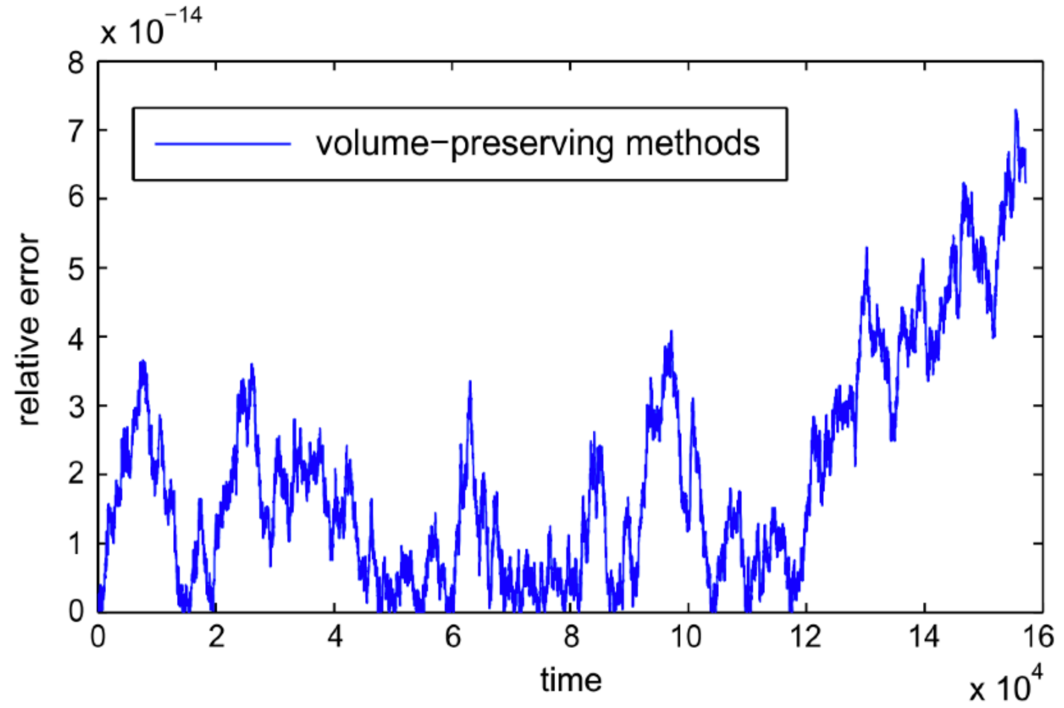
(b) The Boris method



(a)



(c)



List of other suggested M1 projects

1. Impurity transport in the MAST spherical tokamak with strongly-sheared electric fields
2. Loss cone of a magnetic mirror for a Maxwell-Boltzmann ion population
3. A study on inverse sheaths: Thermionic emission from a solid surface immersed in a B-field
4. Antimatter confinement using a Penning trap
5. Electron Orbits in a Helical Undulator
6. A study on Symplectic Integrators
7. Banana orbits in a tokamak and the banana loss cone
8. Electron orbits in HIDRA in the guiding center approximation
9. Particle beam in a synchrotron accelerator
10. Ion optics in an NBI grid extractor
11. Electrostatic accelerator of an ion implanter
12. Ion motion in a magnetic dipole (Earth's magnetic field)