CO_2 Flux Expansion Units

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Each term should be in mol/m²/yr F. -1

CESM1 Equ: $F = \underbrace{a\sqrt{\frac{660}{Sc}}\,u^2}_{PV} \cdot ff\,[atm.co2 \cdot atm\text{-}prs - pCO2SURF]$

Where $a = 6.972E-7\ s/m$, $sDIC = \frac{DIC}{S} \cdot S_0$, $sALK = \frac{ALK}{S} \cdot S_0$

Flux Expansion: $\Delta F = \frac{\partial F}{\partial u}\Delta u + \frac{\partial F}{\partial pCO2}\frac{\partial pCO2}{\partial T}\Delta T + \frac{\partial F}{\partial pCO2}\frac{\partial pCO2}{\partial S}\Delta S$

$+ \frac{\partial F}{\partial pCO2}\frac{\partial pCO2}{\partial DIC}\frac{S}{S_0}\Delta sDIC + \frac{\partial F}{\partial pCO2}\frac{\partial pCO2}{\partial ALK}\frac{S}{S_0}\Delta sALK + \frac{\partial F}{\partial pCO2}\frac{\partial pCO2}{\partial fW}\Delta fW$

I multiplied F so that + is outgassing

[1] $\frac{\partial F}{\partial u}\Delta u$: $-2a\sqrt{\frac{660}{Sc}}\,u \cdot dCO2STAR \cdot \Delta u$

$\frac{s}{m}$ $\frac{m}{s}$ $\frac{mmol}{m^3} \cdot \frac{m}{s}$

$\Big\} \cdot \frac{60 \cdot 60 \cdot 24 \cdot 365}{10^3} = mol/m^2/yr$

[2] $\frac{\partial F}{\partial pCO2}\frac{\partial pCO2}{\partial T}\Delta T$: $+\rho \cdot ff\, a\sqrt{\frac{660}{Sc}} \cdot u^2 \cdot 0.0423 \cdot pCO2 \cdot \Delta T$

$\frac{g}{cm^3} \cdot \frac{mol}{kg \cdot atm} \cdot \frac{\cancel{cm}}{\cancel{m}} \cdot \frac{m^2}{s^2} \cdot \frac{1}{\cancel{°C}} \cdot \mu atm \cdot \cancel{°C}$

$\Big\} \cdot \frac{60 \cdot 60 \cdot 24 \cdot 365}{10^3}$ ← ρ and ff conversion

[3] $\frac{\partial F}{\partial pCO2}\frac{\partial pCO2}{\partial S}\Delta S$: $+\rho \cdot ff \cdot a\sqrt{\frac{660}{Sc}} \cdot u^2 \cdot \frac{pCO2}{S} \cdot \Delta S$

$\frac{g}{cm^3} \cdot \frac{mol}{kg \cdot atm} \cdot \frac{\cancel{cm}}{\cancel{m}} \cdot \frac{m^2}{s^2} \cdot \frac{\mu atm}{PSU} \cdot \cancel{PSU}$

$\Big\} \cdot \frac{60 \cdot 60 \cdot 24 \cdot 365}{10^3}$ ← ρ and ff conversion

14 $\frac{\partial F}{\partial pCO2} \cdot \frac{\partial pCO2}{\partial DIC} \cdot \frac{S}{S_0} \Delta sDIC : + \rho \cdot ff \cdot a \sqrt{\frac{660}{Sc}} \cdot u^2 \cdot \frac{pCO2 \cdot \gamma DIC}{DIC} \cdot \frac{S}{S_0} \cdot \Delta sDIC \Big\} \frac{60 \cdot 60 \cdot 24 \cdot 365}{10^3}$ (2)

5 $\frac{\partial F}{\partial pCO2} \cdot \frac{\partial pCO2}{\partial AIK} \cdot \frac{S}{S_0} \Delta sAIK : + \rho \cdot ff \cdot a \sqrt{\frac{660}{Sc}} \cdot u^2 \cdot \frac{pCO2 \cdot \gamma AIK}{AIK} \cdot \frac{S}{S_0} \cdot \Delta sAIK \Big\} \frac{60 \cdot 60 \cdot 24 \cdot 365}{10^3}$

6a $\frac{\partial F}{\partial pCO2} \cdot \frac{\partial pCO2}{\partial DIC} \cdot \frac{sDIC}{S_0} \Delta S : + \rho \cdot ff \cdot a \sqrt{\frac{660}{Sc}} \cdot u^2 \cdot \frac{pCO2 \cdot \gamma DIC}{DIC} \cdot \frac{sDIC}{S_0} \cdot \Delta S \Big\} \frac{60 \cdot 60 \cdot 24 \cdot 365}{10^3}$

6b $\frac{\partial F}{\partial pCO2} \cdot \frac{\partial pCO2}{\partial AIK} \cdot \frac{sAIK}{S_0} \Delta S : + \rho \cdot ff \cdot a \sqrt{\frac{660}{Sc}} u^2 \frac{pCO2 \cdot \gamma AIK}{AIK} \frac{sAIK}{S_0} \cdot \Delta S \Big\} \frac{60 \cdot 60 \cdot 24 \cdot 365}{10^3}$

$$\gamma_{DIC} = \frac{3AIK * DIC - 2DIC^2}{(2DIC - AIK)(AIK - DIC)}$$

negative

$$\gamma_{AIK} = -\frac{AIK^2}{(2DIC - AIK)(AIK - DIC)}$$

$$F = \underbrace{a\sqrt{\frac{660}{Sc}}\,u^2}_{PV} \cdot \underbrace{ff\,[atm\_CO_2 \cdot atm\text{-}prs - pCO2surf]}_{dCO2Star}$$

$a = 6.972E-9$ s/cm

$$sDIC = \frac{DIC}{S} \cdot S_0 \qquad sAlk = \frac{Alk}{S} \cdot S_0$$ (35)

$$\Delta F = \frac{\partial F}{\partial u}\Delta u + \frac{\partial F}{\partial pCO2}\frac{\partial pCO2}{\partial T}\Delta T + \frac{\partial F}{\partial pCO2}\frac{\partial pCO2}{\partial S}\Delta S + \frac{\partial F}{\partial pCO2}\frac{\partial pCO2}{\partial DIC}\frac{S}{S_0}\Delta sDIC + \frac{\partial F}{\partial pCO2}\frac{\partial pCO2}{\partial Alk}\frac{S}{S_0}\Delta sAlk + \frac{\partial F}{\partial pCO2}\frac{\partial pCO2}{\partial fw}\Delta fw$$

$$\frac{\partial F}{\partial u} = -2a\sqrt{\frac{660}{Sc}}\,u\,dCO2Star$$

$$\frac{\partial F}{\partial pCO2} = -ff \cdot a\sqrt{\frac{660}{Sc}} \cdot u^2$$ Due to inverting Fco2

$$\frac{\partial pCO2}{\partial T} = 0.0423\,pCO2$$

$$\frac{\partial pCO2}{\partial S} = \frac{pCO2}{S}$$

$$\frac{\partial pCO2}{\partial DIC} = \frac{pCO2\,\gamma_{DIC}}{DIC}$$

$$\frac{\partial pCO2}{\partial Alk} = \frac{pCO2\,\gamma_{Alk}}{Alk}$$

$$\frac{\partial pCO2}{\partial fw}\Delta fw = \frac{sDIC}{S_0}\frac{\partial pCO2}{\partial DIC}\Delta S + \frac{sAlk}{S_0}\frac{\partial pCO2}{\partial Alk}\Delta S$$ ($\cdot\frac{\partial F}{\partial pCO2}$, $\cdot\frac{\partial F}{\partial pCO2}$)

$$\gamma_{DIC} = \frac{3Alk \cdot DIC - 2DIC^2}{(2DIC - Alk)(Alk - DIC)}$$

$$\gamma_{Alk} = -\frac{Alk^2}{(2DIC - Alk)(Alk - DIC)}$$