International Journal of Infrared and Millimeter Waves, Vol. 15, No. 9, 1994

ELECTROMAGNETIC REFLECTION AND TRAMSISSION FOR A DIELECTRIC-Ω INTERFACE AND AN Ω SLAB

Martin Norgren and Sailing He

Department of Electromagnetic Theory
Royal Institute of Technology, S-100 44 Stockholm, Sweden

Received June 3, 1994

Abstract

A time-harmonic electromagnetic plane wave obliquely incident on a half-space or a slab consisting of a so-called Ω medium is considered. The up- and down-going eigenmodes in the Ω medium are derived and used to calculate the reflection and transmission coefficients for TE and TM modes. The Brewster angles for an Ω half-space are computed. Numerical results for the co- and cross-polarized reflection and transmission coefficients for an Ω slab are presented.

1 Introduction

Wave propagation, radiation and guidance in complex media have received considerable attention recently in view of its potential usefulness in applications. A complex medium provides a cross coupling between the electric and magnetic fields. Among these complex media, chiral media have been studied extensively in the past few years and led to a wide variety of applications in the areas of shielding, antenna design, microwave and optical devices, etc. [1]–[9].

A new class of complex materials, called Ω media, was introduced a few years ago [10], [11]. The properties of an Ω medium can be envisaged as

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arising from a distribution of metal half-loop with two extended arms. The metal wire elements of the Ω medium resemble the Greek letter Ω, hence the name. In the present paper, we consider an Ω medium which has the following constitutive relations in a xyz cartesian coordinate system

$$\vec{\mathbf{D}} = \bar{\bar{\varepsilon}}\, \vec{\mathbf{E}} + \bar{\bar{\xi}}\, \vec{\mathbf{H}}, \tag{1}$$

$$\vec{\mathbf{B}} = \bar{\bar{\mu}}\, \vec{\mathbf{H}} + \bar{\bar{\zeta}}\, \vec{\mathbf{E}}, \tag{2}$$

with the following parameter tensors

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix}, \qquad \bar{\bar{\mu}} = \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix}, \tag{3}$$

$$\bar{\bar{\xi}} = -j\sqrt{\varepsilon_0\mu_0} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \Omega \\ 0 & 0 & 0 \end{bmatrix}, \qquad \bar{\bar{\zeta}} = j\sqrt{\varepsilon_0\mu_0} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \Omega & 0 \end{bmatrix} \tag{4}$$

(note that unlike the one given in [11], the constitutive relations for the Ω medium used here are expressed in terms of $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$ fields), where ε_0, μ_0 are the permittivity and permeability in vacuum, respectively. Such a medium can be manufactured by putting small Ω elements in a host medium with all the half-loops lying in the xy planes and pointing to the $-x$ direction, and all the extended arms parallel to y axis (cf. Fig. 1). The particular forms for $\bar{\bar{\xi}}$ and $\bar{\bar{\zeta}}$ express the fact that in such a medium only the z (or y) component of the applied magnetic (or electric) field can produce electric (or magnetic) polarization parallel to the y (or z) axis. Such a medium is reciprocal, since $\bar{\bar{\varepsilon}} = \bar{\bar{\varepsilon}}^T$, $\bar{\bar{\mu}} = \bar{\bar{\mu}}^T$ and $\bar{\bar{\xi}} = -\bar{\bar{\zeta}}^T$, cf. [12]. For practical cases, the dimensionless parameter Ω will be small and satisfy $\Omega^2 < (\varepsilon\mu)/(\varepsilon_0\mu_0) - 1$, where ε (μ) is the minimum value of ε_i (μ_i), $i = 1, 2, 3$ (this condition will be satisfied in all the numerical examples in the present paper; if this condition is not satisfied, numerical results show that some of the scattering coefficients will be greater than 1, which is not physical; an on-going research effort is being conducted to investigate this condition from physical laws). For a lossless medium, Ω has a real value.

In the present paper, we consider a time-harmonic electromagnetic plane wave obliquely incident on such an Ω half-space or slab. A wave splitting technique is used to solve the reflection and transmission problem. Wave splitting refers to the decomposition of the total wave into two components which propagate in opposite directions, i.e., forward- and backward-moving waves. The wave-splitting approach has been applied to a variety of scattering problems (cf. [13]–[20] and earlier references given there). In particular, applications to scattering from plane-stratified complex media can be found in [4], [8]. We note that this approach is not limited to piecewise constant stratifications and it can be applied effectively to stratified media whose properties vary in a general way as a function of depth [4]. The paper is organized as follows. In Sec. 2 the problem is formulated and Maxwell's equations are rewritten in terms of the tangential electric and magnetic fields. In Sec. 3 the up- and down-going eigenmodes in the Ω medium are identified. These eigenmodes are used to calculate the reflection from an Ω half-space for TE and TM modes in Sec. 4, where the Brewster angles for an Ω half-space are also computed. The co- and cross-polarized reflection and transmission coefficients for an Ω slab are given in Sec. 5. Numerical results are presented in Sec. 6.

2 Problem formulation

We consider an electromagnetic plane wave with harmonic time dependence $\exp(j\omega t)$ obliquely impinging on an Ω half-space or an Ω slab from the upper side $z < 0$ (the Ω medium is described by the equations in the introduction). The medium above the surface $z = 0$ is vacuum.

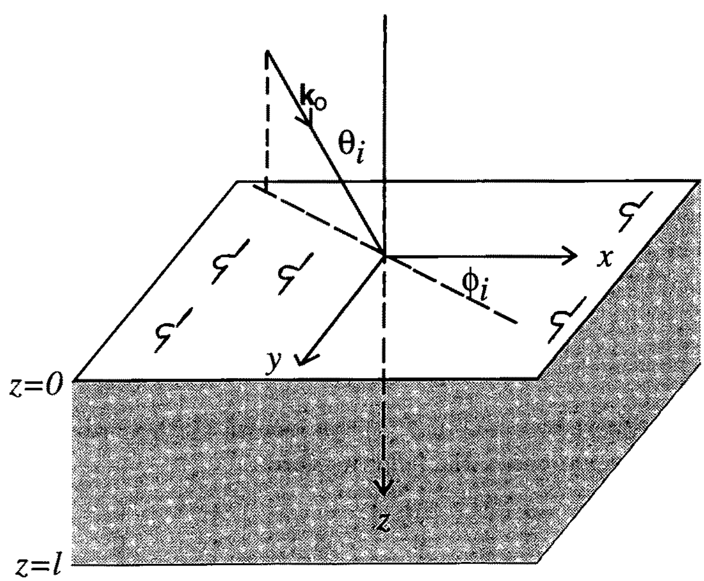


Figure 1. The scattering configuration.

Maxwell's equations in the xyz coordinate system are

$$\nabla \times \vec{\mathbf{E}} = -j\omega\vec{\mathbf{B}}, \qquad \nabla \times \vec{\mathbf{H}} = j\omega\vec{\mathbf{D}}, \tag{5}$$

where

$$\vec{\mathbf{E}} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}, \quad \text{etc.}$$

Assume that the x, y and t dependence of the electromagnetic fields is $\exp[j(wt - k_1x - k_2y)]$, where the propagation constants k_1 and k_2 are given by

$$k_1 = k_0 \sin\theta_i \cos\varphi_i, \tag{6}$$

$$k_2 = k_0 \sin\theta_i \sin\varphi_i, \tag{7}$$

and where $k_0 = \omega/c_0 = \sqrt{\varepsilon_0\mu_0}\omega$ is the wave number of the incident field (ε_0, μ_0 are the permittivity and permeability in vacuum, respectively), θ_i is the incident angle and $\varphi_i \in [0, 2\pi]$ is the angle between the $x-$axis and the projection of the incident wave vector $\mathbf{k_0}$ on the xy plane (cf. Fig. 1). Thus from the constitutive relations (1) and (2) and the third components of Maxwell's equations, we can express the third components of $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$ in terms of the tangential fields as

$$\begin{bmatrix} E_3 \\ H_3 \end{bmatrix} = P \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix}, \tag{8}$$

where

$$\mathbf{E} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \qquad \mathbf{H} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix},$$

$$P = \frac{1}{\varepsilon_3\mu_3} \begin{bmatrix} 0 & 0 & \mu_3 s_2 & -\mu_3 s_1 \\ -\varepsilon_3 s_2 & \varepsilon_3(s_1 - js_0\Omega) & 0 & 0 \end{bmatrix}, \tag{9}$$

and where

$$s_i = (\frac{k_i}{\omega}), \quad i = 0, 1, 2. \tag{10}$$

Using the constitutive relations (1) and (2), we rewrite the first two components of Maxwell's equations in the following form:

$$\partial_z \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} = j\omega\{ \begin{bmatrix} 0 & 0 & 0 & -\mu_2 \\ 0 & 0 & \mu_1 & 0 \\ 0 & \varepsilon_2 & 0 & 0 \\ -\varepsilon_1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} - \begin{bmatrix} s_1 & 0 \\ s_2 & 0 \\ 0 & s_1 + js_0\Omega \\ 0 & s_2 \end{bmatrix} \begin{bmatrix} E_3 \\ H_3 \end{bmatrix} \}. \tag{11}$$

It thus follows from Eqs. (8) and (11) that

$$\partial_z \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} = W \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix}, \tag{12}$$

where

$$W = j\omega \begin{bmatrix} 0 & 0 & -b & a_1 \\ 0 & 0 & a_2 & b \\ c_1 & a_3 & 0 & 0 \\ a_4 & c_2 & 0 & 0 \end{bmatrix}, \tag{13}$$

and where

$$a_1 = -\mu_2 + \frac{s_1^2}{\varepsilon_3}, \qquad a_2 = \mu_1 - \frac{s_2^2}{\varepsilon_3}, \tag{14}$$

$$a_3 = \varepsilon_2 - \frac{s_1^2 + s_0^2\Omega^2}{\mu_3}, \qquad a_4 = -\varepsilon_1 + \frac{s_2^2}{\mu_3}, \tag{15}$$

$$b = \frac{s_1 s_2}{\varepsilon_3}, \tag{16}$$

$$c_1 = \frac{s_2(s_1 + j s_0 \Omega)}{\mu_3}, \qquad c_2 = \frac{s_2(-s_1 + j s_0 \Omega)}{\mu_3}. \tag{17}$$

Eq. (12) is equivalent to Maxwell's equations for the Ω medium in terms of the tangential electric and magnetic fields.

3 Up- and down-going eigenmodes

In this section the down-going modes $\mathbf{E}^+$ (the z component of the wave vector is in the positive z direction) and up-going modes $\mathbf{E}^-$ (the z component of the wave vector is in the negative z direction) in the Ω half-space $z > 0$ will be identified. Let

$$\begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix} = T \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix}, \tag{18}$$

where the transformation matrix T will be determined later and

$$\mathbf{E}^\pm = \begin{bmatrix} E_1^\pm \\ E_2^\pm \end{bmatrix}.$$

Differentiating Eq. (18) with respect to z and using Eq. (12), yields

$$\partial_z \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix} = TWT^{-1} \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}, \tag{19}$$

where T^{-1} is the inverse of T. Diagonalizing the matrix W yields

$$\partial_z \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix} = \begin{bmatrix} -j\omega p_1 & 0 & 0 & 0 \\ 0 & -j\omega p_2 & 0 & 0 \\ 0 & 0 & j\omega p_1 & 0 \\ 0 & 0 & 0 & j\omega p_2 \end{bmatrix} \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}, \tag{20}$$

where

$$p_1 = \sqrt{\frac{1}{2}(p + \sqrt{q})}, \qquad p_2 = \sqrt{\frac{1}{2}(p - \sqrt{q})}, \tag{21}$$

and where

$$p = a_1 a_4 + a_2 a_3 - b(c_1 - c_2), \tag{22}$$

$$\begin{aligned} q = {} & (a_1 a_4 - a_2 a_3)^2 + b^2 (c_1 + c_2)^2 \\ & -2b(a_1 a_4 + a_2 a_3)(c_1 - c_2) + 4(a_1 a_2 c_1 c_2 - a_3 a_4 b^2). \end{aligned} \tag{23}$$

In order to allow a physical interpretation of $\mathbf{E}^+$ and $\mathbf{E}^-$ as down-going and up-going eigenmodes, respectively, we should take the square roots in Eq. (21) that give p_i, $i = 1, 2$, positive real parts ($\sqrt{q}$ has non-negative real part). The matrix T^{-1} consists of the eigenvectors of the matrix W and is given by

$$T^{-1} = \begin{bmatrix} M & M \\ N & -N \end{bmatrix}, \tag{24}$$

where the 2×2 matrices M, N and their inverses are given by

$$M = \begin{bmatrix} 1 & e_2 \\ e_1 & 1 \end{bmatrix}, \qquad M^{-1} = \frac{1}{1 - e_1 e_2} \begin{bmatrix} 1 & -e_2 \\ -e_1 & 1 \end{bmatrix}, \tag{25}$$

$$N = \begin{bmatrix} f_1 & f_2 \\ g_1 & g_2 \end{bmatrix}, \qquad N^{-1} = \frac{1}{f_1 g_2 - f_2 g_1} \begin{bmatrix} g_2 & -f_2 \\ -g_1 & f_1 \end{bmatrix}, \tag{26}$$

and where

$$e_1 = \begin{cases} \dfrac{a_2 c_1 + a_4 b}{p_1^2 - a_2 a_3 - b c_2}, & p_1^2 - a_2 a_3 - b c_2 \neq 0, \\ 0, & p_1^2 - a_2 a_3 - b c_2 = 0, \end{cases} \tag{27}$$

$$e_2 = \begin{cases} \dfrac{a_1 c_2 - a_3 b}{p_2^2 - a_1 a_4 + b c_1}, & p_2^2 - a_1 a_4 + b c_1 \neq 0, \\ 0, & p_2^2 - a_1 a_4 + b c_1 = 0, \end{cases} \tag{28}$$

$$f_1 = -\frac{c_1 + a_3 e_1}{p_1}, \qquad f_2 = -\frac{c_1 e_2 + a_3}{p_2}, \tag{29}$$

$$g_1 = -\frac{a_4 + c_2 e_1}{p_1}, \qquad g_2 = -\frac{a_4 e_2 + c_2}{p_2}. \tag{30}$$

The matrix T is given by

$$T = \frac{1}{2}\begin{bmatrix} M^{-1} & N^{-1} \\ M^{-1} & -N^{-1} \end{bmatrix}. \tag{31}$$

Thus we can decompose the total tangential electromagnetic fields into down- and up-going eigenmodes in the Ω medium as follows

$$\begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} = T^{-1}\begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}. \tag{32}$$

4 Reflection from a dielectric–Ω interface

As shown in the Appendix, the down- and up-going modes in the vacuum are given by the following expression

$$\begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}(0^-) = T_0\begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix}(0^-), \tag{33}$$

where

$$T_0 = \frac{1}{2}\begin{bmatrix} R & ZR \\ R & -ZR \end{bmatrix}, \tag{34}$$

$$R = \begin{bmatrix} \cos\varphi_i & \sin\varphi_i \\ -\sin\varphi_i & \cos\varphi_i \end{bmatrix}, \qquad Z = Z_0\begin{bmatrix} 0 & \cos\theta_i \\ -\dfrac{1}{\cos\theta_i} & 0 \end{bmatrix}, \tag{35}$$

and where the free-space impedance $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$. As can easily be checked, on the surface $z = 0^-$ the E_1^+, E_1^- modes are down- and up-going TM modes, respectively, and the E_2^+, E_2^- modes are down- and up-going TE modes, respectively ($E_i^\pm$, $i = 1, 2$, give the amplitudes of the *tangential* components of the electric field for these modes).

The reflection coefficient matrix is defined as

$$\mathbf{E}^-(0^-) = \mathbf{r}\mathbf{E}^+(0^-) \equiv \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}\mathbf{E}^+(0^-). \tag{36}$$

The coefficients of $\mathbf{r}$ relate the amplitudes of the *tangential* components of the reflected and incident electric field to each other for TM and TE modes. As is customary, the reflection coefficients for TM and TE modes are defined as

$$r_{XY} = \frac{E^r_{TX}}{E^i_{TY}}, \quad X, Y = E \text{ or } M, \tag{37}$$

where E^r_{TE} (E^i_{TE}) is the amplitudes of the reflected (incident) electric field for TE mode, etc.. r_{MM}, r_{EM} are the co- and cross-polarized reflection coefficients for TM mode incidence, respectively, and r_{EE}, r_{ME} are the co- and cross-polarized reflection coefficients for TE mode incidence, respectively. It is easy to see that

$$r_{MM} = r_{11}, \quad r_{ME} = \frac{r_{12}}{\cos\theta_i}, \quad r_{EM} = r_{21}\cos\theta_i, \quad r_{EE} = r_{22}. \tag{38}$$

By the continuity of the tangential fields $\mathbf{E}$ and $\mathbf{H}$ across the interface $z = 0$, we have

$$\begin{aligned} \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}(0^+) &= T \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix}(0^+) = T \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix}(0^-) \\ &= TT_0^{-1} \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}(0^-) = TT_0^{-1} \begin{bmatrix} \mathbf{E}^+(0^-) \\ \mathbf{r}\mathbf{E}^+(0^-) \end{bmatrix} \\ &= \begin{bmatrix} n & m \\ m & n \end{bmatrix} \begin{bmatrix} \mathbf{E}^+(0^-) \\ \mathbf{r}\mathbf{E}^+(0^-) \end{bmatrix}, \end{aligned} \tag{39}$$

where

$$m = \frac{1}{2}\left(M^{-1}R^{-1} - N^{-1}R^{-1}Z^{-1}\right) \tag{40}$$

$$n = \frac{1}{2}\left(M^{-1}R^{-1} + N^{-1}R^{-1}Z^{-1}\right). \tag{41}$$

Since $\mathbf{E}^-(0^+) = 0$ (which is due to the fact that there is no up-going mode throughout the Ω half-space $z > 0$), from Eq. (39) it follows that

$$n\mathbf{r} + m = 0, \tag{42}$$

i.e.,

$$\mathbf{r} = -n^{-1}m = -\frac{1}{n_{11}n_{22} - n_{12}n_{21}} \begin{bmatrix} n_{22} & -n_{12} \\ -n_{21} & n_{11} \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \tag{43}$$

Eq. (43) gives the reflection coefficient matrix at a planar interface between a dielectric medium and a Ω medium for an obliquely incident plane wave. One can easily check that Eq. (43) can be reduced to the well-known reflection coefficients for the TE and TM modes when $\Omega = 0$.

The Brewster angle is the incident angle at which there is no reflected power for a certain polarized mode, cf. [12], [3]. In the vacuum region $z < 0$, we can choose the down- and up-going eigenmodes as

$$\begin{bmatrix} \tilde{\mathbf{E}}^+ \\ \tilde{\mathbf{E}}^- \end{bmatrix} = \begin{bmatrix} \Gamma & 0 \\ 0 & \Gamma \end{bmatrix} \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}, \quad z < 0, \tag{44}$$

(Γ is a 2×2 matrix) and define the corresponding reflection coefficient matrix as

$$\tilde{\mathbf{E}}^-(0^-) = \tilde{\mathbf{r}}\tilde{\mathbf{E}}^+(0^-) \equiv \begin{bmatrix} \tilde{r}_{11} & \tilde{r}_{12} \\ \tilde{r}_{21} & \tilde{r}_{22} \end{bmatrix} \tilde{\mathbf{E}}^+(0^-). \tag{45}$$

In particular, if Γ is such that $\Gamma\mathbf{r}\Gamma^{-1}$ is diagonal, then the eigenmodes $\tilde{\mathbf{E}}^{\pm}$ defined by Eq. (44) are the eigenmodes with *eigenpolarizations*. For the eigenpolarizations, we have [9]

$$\tilde{r}_{11} = \frac{1}{2}[(r_{11} + r_{22}) + \sqrt{(r_{11} + r_{22})^2 - 4(r_{11}r_{22} - r_{12}r_{21})}], \tag{46}$$

$$\tilde{r}_{22} = \frac{1}{2}[(r_{11} + r_{22}) - \sqrt{(r_{11} + r_{22})^2 - 4(r_{11}r_{22} - r_{12}r_{21})}], \tag{47}$$

$$\tilde{r}_{12} = \tilde{r}_{21} = 0, \tag{48}$$

and

$$\Gamma = \begin{bmatrix} 1 & r_{12}/(r_{11} - \tilde{r}_{22}) \\ r_{21}/(r_{22} - \tilde{r}_{11}) & 1 \end{bmatrix}. \tag{49}$$

If there is a certain incident angle $\theta_i = \theta_B$ (for a certain fixed φ_i) that satisfies $\tilde{r}_{11} = 0$ or $\tilde{r}_{22} = 0$, the angle θ_B is called a Brewster angle for the eigenpolarization.

5 Reflection and transmission for an Ω slab

In this section we consider a homogeneous Ω slab. The slab occupies the region $0 < z < l$, and the regions $z < 0$ and $z > l$ are vacuum. A plane wave is obliquely incident on the slab from the region $z < 0$.

The split fields at $z = 0^+$ and $z = 0^-$ are related to each other by Eq. (39). In an analogous way we can obtain the following equation at $z = l$

$$\begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}(l^+) = T_0 T^{-1} \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}(l^-) = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}(l^-), \quad (50)$$

where

$$\alpha = \frac{1}{2}(RM + ZRN), \qquad \beta = \frac{1}{2}(RN - ZRN). \quad (51)$$

Inside the slab, we solve the system (20) of ordinary differential equations and obtain

$$\begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}(l^-) = \begin{bmatrix} L^{-1} & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}(0^+), \quad (52)$$

where

$$L = \begin{bmatrix} \exp(j\omega p_1 l) & 0 \\ 0 & \exp(j\omega p_2 l) \end{bmatrix}. \quad (53)$$

It thus follows from Eqs. (39), (50) and (52) that

$$\begin{aligned} \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}(l^+) &= T_0 T^{-1} \begin{bmatrix} L^{-1} & 0 \\ 0 & L \end{bmatrix} T T_0^{-1} \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}(0^-) \\ &\equiv \begin{bmatrix} \tilde{\alpha} & \tilde{\beta} \\ \tilde{m} & \tilde{n} \end{bmatrix} \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}(0^-), \end{aligned} \quad (54)$$

where

$$\tilde{\alpha} = \alpha L^{-1} n + \beta L m, \qquad \tilde{\beta} = \alpha L^{-1} m + \beta L n, \quad (55)$$

$$\tilde{m} = \beta L^{-1} n + \alpha L m, \qquad \tilde{n} = \beta L^{-1} m + \alpha L n. \quad (56)$$

Eq. (54) relates the split fields at two sides of the slab to each other. The reflection coefficient matrix for the Ω slab is defined by Eq. (36), and the transmission coefficient matrix is defined as follows

$$\mathbf{E}^+(l^+) = \mathbf{t}\mathbf{E}^+(0^-) \equiv \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \mathbf{E}^+(0^-). \quad (57)$$

Since there is no up-going mode in the region $z > l$, we have $\mathbf{E}^-(l^+) = 0$, which gives (cf. Eqs. (36), (54) and (57))

$$\begin{bmatrix} \mathbf{t}\mathbf{E}^+(0^-) \\ 0 \end{bmatrix} = \begin{bmatrix} \tilde{\alpha} & \tilde{\beta} \\ \tilde{m} & \tilde{n} \end{bmatrix} \begin{bmatrix} \mathbf{E}^+(0^-) \\ \mathbf{r}\mathbf{E}^+(0^-) \end{bmatrix}. \quad (58)$$

Since there is no up-going mode in the region $z > l$, we have $\mathbf{E}^-(l^+) = 0$, which gives (cf. Eqs. (36), (54) and (57))

$$\begin{bmatrix} \mathbf{t}\mathbf{E}^+(0^-) \\ 0 \end{bmatrix} = \begin{bmatrix} \tilde{\alpha} & \tilde{\beta} \\ \tilde{m} & \tilde{n} \end{bmatrix} \begin{bmatrix} \mathbf{E}^+(0^-) \\ \mathbf{r}\mathbf{E}^+(0^-) \end{bmatrix}. \tag{58}$$

From Eq. (58) we have

$$\mathbf{r} = -\tilde{n}^{-1}\tilde{m} \tag{59}$$

and

$$\mathbf{t} = \tilde{\alpha} + \tilde{\beta}\mathbf{r}. \tag{60}$$

The reflection coefficients for the TE and TM modes can thus be obtained through Eq. (38). The transmission coefficients for the TE and TM modes are

$$t_{MM} = t_{11}, \quad t_{ME} = \frac{t_{12}}{\cos\theta_i}, \quad t_{EM} = t_{21}\cos\theta_i, \quad t_{EE} = t_{22}, \tag{61}$$

where t_{MM}, t_{EE} are the co-polarized transmission coefficients for TE and TM incident modes, respectively, and t_{EM}, t_{ME} are the cross-polarized transmission coefficients for TM and TE incident modes, respectively.

6 Numerical results

If the host material is isotropic before the insertion of the Ω-shaped metal loops, the parameter tensors $\bar{\bar{\varepsilon}}$ and $\bar{\bar{\mu}}$ will become (approximately) uniaxial with symmetry axes given by the directions of the electric and magnetic dipoles, respectively (and furthermore with $\varepsilon_2 > \varepsilon_1 = \varepsilon_3$, $\mu_3 > \mu_1 = \mu_2$, see [10]). Based on the results reported in [21] and [22], we choose the following three sets of parameters for the Ω medium used in our numerical calculation:
Ω medium 1: $\Omega = 0.1$, $\varepsilon_2/\varepsilon_0 = 3.1$, $\mu_3/\mu_0 = 1.12$;
Ω medium 2: $\Omega = 0.3$, $\varepsilon_2/\varepsilon_0 = 4$, $\mu_3/\mu_0 = 1.12$;
Ω medium 3: $\Omega = 0.9$, $\varepsilon_2/\varepsilon_0 = 10$, $\mu_3/\mu_0 = 1.12$.
The other parameters are assumed to be $\varepsilon_1/\varepsilon_0 = \varepsilon_3/\varepsilon_0 = 3$, $\mu_1/\mu_0 = \mu_2/\mu_0 = 1$ for all the three Ω media. One notes that the larger the parameter Ω is, the more uniaxial the tensor $\bar{\bar{\varepsilon}}$ should become (the tensor $\bar{\bar{\mu}}$ is not much affected when the parameter Ω changes, cf. [22]; also note that the parameter estimation based on the electromagnetic analysis of loop antennas for the Ω medium should be similar as the one for the chiral medium [21]).

The reflection for the Ω half-space is illustrated in Fig. 2. Fig. 2(a) refers to the incidence in the xz-plane. In this case there is no cross-polarized reflection. The absolute values of the co-polarized reflection coefficients for the TE and TM modes, i.e., $|\, r_{EE} \,|$ and $|\, r_{MM} \,|$, are plotted as functions of the incident angle θ_i in Fig. 2(a) for the three Ω media. Fig 2(a) shows that $|\, r_{EE} \,|$ increases as Ω increases. (the Ω parameter has no effect for TM mode for this plane of incidence). Fig 2(b) illustrates the case when the plane of incidence is the yz-plane. Cross-polarized reflection appears but is very small in this case. Numerical results show that

$$|\, r_{ME} \,| = |\, r_{EM} \,| \tag{62}$$

(in fact this holds for any plane of incidence and for both an Ω half-space and an Ω slab), which is due to the reciprocity of the medium. It also shows that the co-polarized reflection for TM modes increases as Ω increases for small incident angle θ_i (the co-polarized reflection for TE modes is almost independent of the Ω parameter for any θ_i for incidence in the yz plane). Fig. 3 gives the Brewster angle θ_B as a function of the azimuthal angle φ_i (each φ_i corresponds to one plane of incidence) for the three Ω media. Due to the reciprocity of the material we only need to consider the interval $0° \leq \varphi_i \leq 90°$. Fig. 3 shows that the Brewster angle increases for any φ_i as Ω increases.

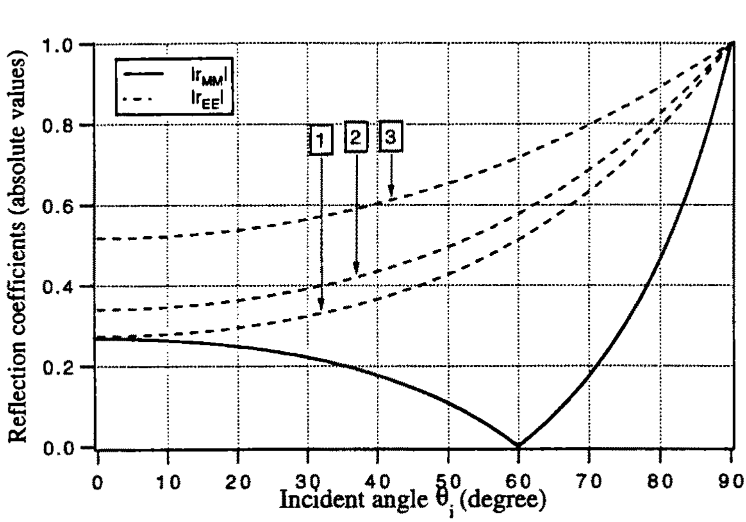


Figure 2(a). The co-polarized reflection coefficients for the three Ω half-spaces (consisting of the Ω media 1,2,3,respectively) as functions of the incident angle θ_i in the $xz-$plane of incidence.

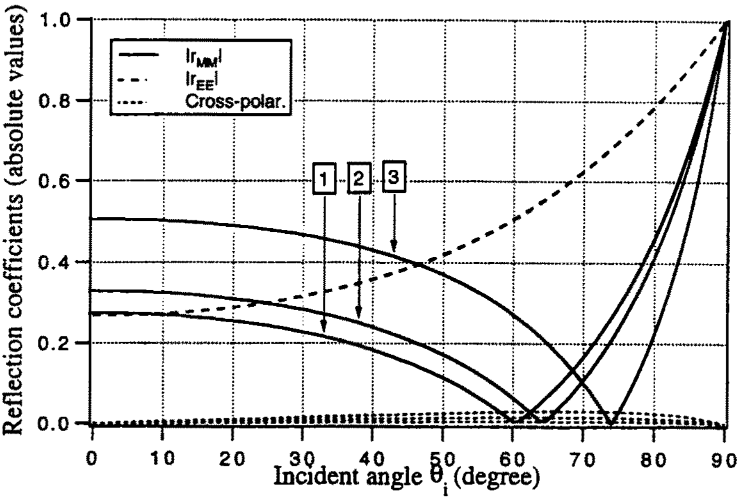


Figure 2(b). The co- and cross-polarized reflection coefficients for the three Ω half-spaces (consisting of the Ω media 1,2,3,respectively) as functions of the incident angle θ_i in the yz−plane of incidence.

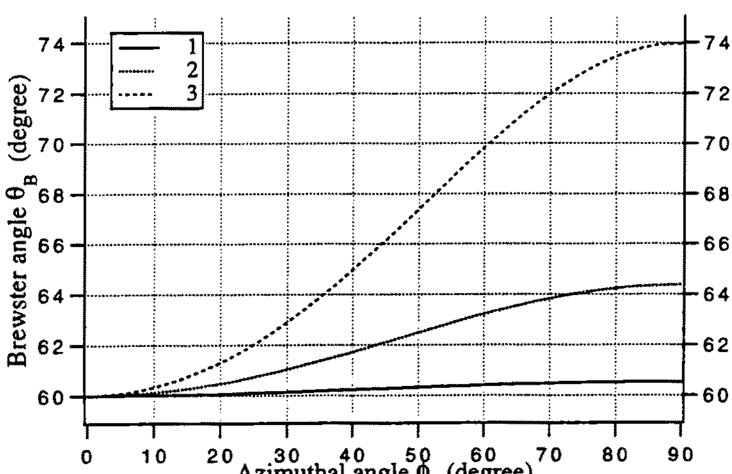
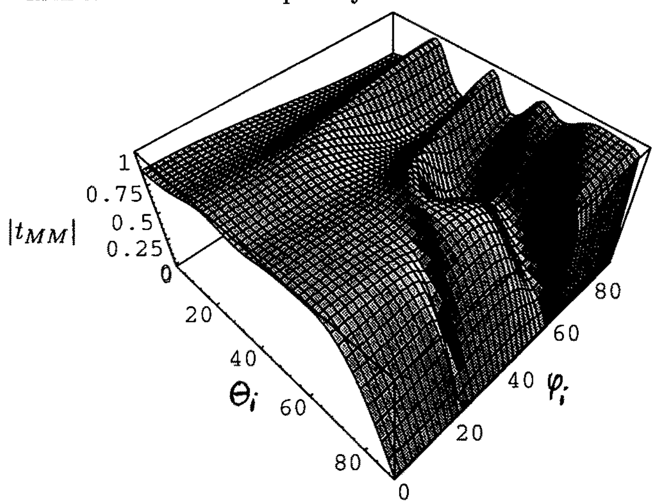


Figure 3. The Brewster angle for the Ω media 1,2,3, respectively, for different planes of incidence.

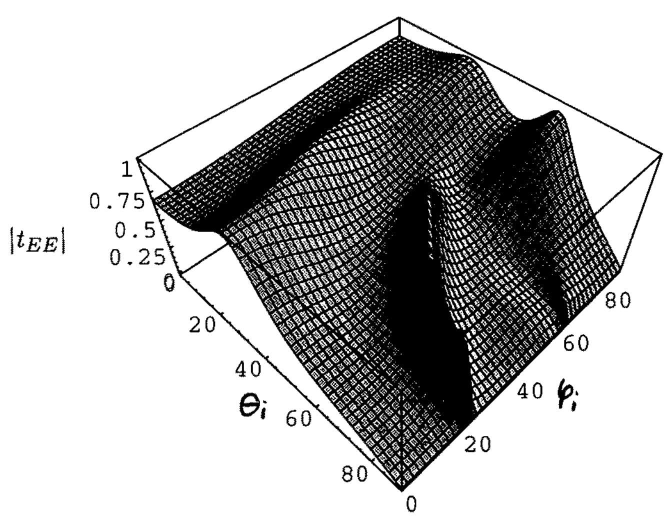
The reflection and transmission coefficients for an Ω slab have been calculated and plotted in Fig. 4 as functions of the incident angles θ_i and φ_i. The thickness of the slab, l, is given in the unit of the internal wavelength when $\Omega = 0$, i.e., 1 wavelength $= 2\pi/(\sqrt{\varepsilon_1\mu_1}\omega)$. In this example we choose the Ω medium 3 and l = 5.2 wavelengths. Figs. 4(a) and 4(b) show the co-polarized transmission coefficients for TM and TE modes, respectively. Figs. 4(c) and 4(d) show the cross-polarized reflection and transmission coefficients. Numerical results show that

$$| r_{EM} | = | r_{ME} |, \qquad | t_{EM} | = | t_{ME} |, \tag{63}$$

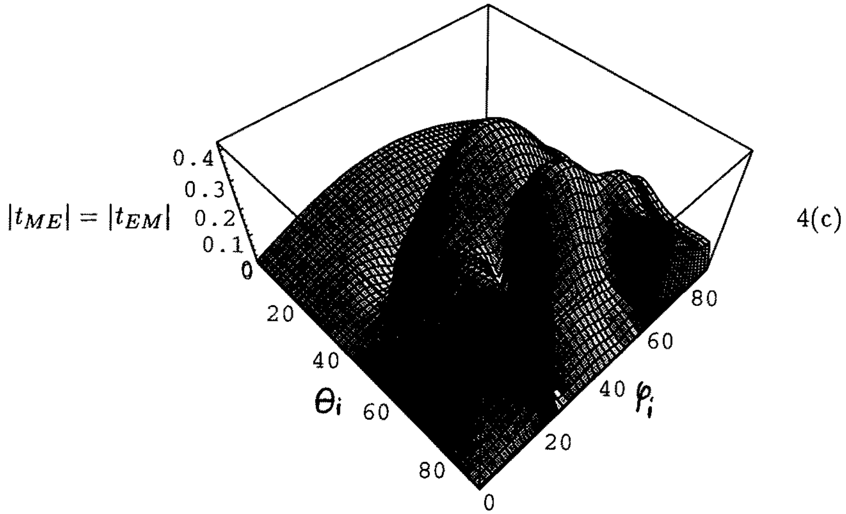
which is due to the reciprocity of the medium.



4(a)



4(b)



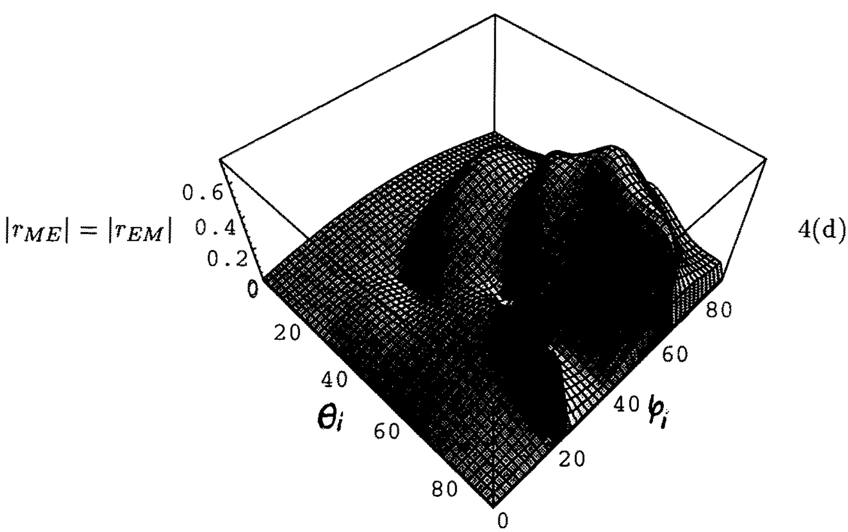


Figure 4. Reflection and transmission coefficients for an Ω slab (consisting of the Ω medium 3) as functions of the incident angles θ_i and φ_i. The thickness of the slab is $l = 5.2$ wavelengths. (a) co-polarized transmission coefficient for TM mode; (b) co-polarized transmission coefficient for TE mode; (c) cross-polarized transmission coefficient; (d) cross-polarized reflection coefficient.

Appendix: TE and TM modes in vacuum

Since the medium in the region $z < 0$ is vacuum, we have $p_1 = p_2 = \sqrt{\varepsilon_0\mu_0}\cos^2\theta_i$, and any linear combination of eigenmodes will still be an eigenmode in the region $z < 0$. To obtain the eigenmodes corresponding to TE and TM modes, we introduce a new coordinate system $x'y'z'$ in which the plane of incidence coincides with the $x'z'$ plane, and we have

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = R \begin{bmatrix} x \\ y \end{bmatrix}, \qquad z' = z, \tag{A1}$$

where R is given by Eq. (35). Let

$$\begin{bmatrix} \mathbf{E}^+ \\ \mathbf{E}^- \end{bmatrix}(0^-) = T_0' \begin{bmatrix} \mathbf{E}' \\ \mathbf{H}' \end{bmatrix}(0^-) \tag{A2}$$

($\mathbf{E}'$ is the tangential electric fields in the $x'y'z'$ system) where

$$T_0' = T(\varepsilon_j = \varepsilon_0, \mu_j = \mu_0, \Omega = 0, \varphi_i = 0) = \frac{1}{2}\begin{bmatrix} I & Z \\ I & -Z \end{bmatrix}, \quad j = 1, 2, 3, \tag{A3}$$

and where I is the 2×2 identity matrix and Z is defined in Eq. (35).

The amplitudes of the tangential fields in different coordinate systems are related to each other through

$$\begin{bmatrix} \mathbf{E}' \\ \mathbf{H}' \end{bmatrix}(0^-) = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix}(0^-). \tag{A4}$$

Thus we obtain Eq. (33) from Eq. (A2) and the above equation with $T_0 = T_0' R$ (which is identical to the expression (34)).

Acknowledgments

The authors are grateful to S. Ström for valuable discussions. The partial support of the Swedish Research Council for Engineering Sciences is gratefully acknowledged.

References

[1] D. L. Jaggard and N. Engheta, "Chirality in electrodynamics: Modelling and applications," in

Directions in Electromagnetic Wave Modelling, H. L. Bertoni and L. B. Felsen, eds. (Plenum Publishing Co., New York, 1993).

[2] A. H. Sihvola and I. V. Lindell, "Bi-isotropic constitutive relations," *Microwave and Optical Technology Letters*, Vol. 4, No. 8, 295-297 (1991).

[3] S. Bassiri, C. H. Papas and N. Engheta, "Electromagnetic wave propagation through a dielectric-chiral interface and through a chiral slab", *J. Opt. Soc. Am.*, 5(9), 1450-1459 (1986).

[4] S. He, "A time-harmonic Green function technique and wave propagation in a stratified nonreciprocal chiral slab with multiple discontinuities", *J. Math. Phys.*, 33(12), 4103-4110 (1992).

[5] I. V. Lindell and A. J. Viitanen, "Plane wave propagation in a uniaxial bianisotropic medium," *Electronics Letters*, 29(2), 150-152 (1993).

[6] A. J. Viitanen and I. V. Lindell, "Plane wave propagation in a uniaxial bianisotropic medium with an application to a polarization transformer," *Intern. J. Infrared Millim. Waves*, 14(10), 1993-2010 (1993).

[7] F. Olyslager and D. De Zutter, "Rigourous full-wave analysis of electric and dielectric waveguides embedded in a multilayered bianisotropic medium," *Radio Science*, 28(5), 937-946 (1993).

[8] S. He and Y. Hu, "Electromagnetic scattering for a stratified biisotropic (nonreciprocal chiral) slab: numerical calculations", *IEEE Trans. Antennas Propagat.*, 41(8),1057-1062 (1993).

[9] S. He, "Wave propagation through a dielectric-uniaxial bianisotropic interface and the computation of Brewster angles", *J. Opt. Soc. Am. A*, 10(11), 2402-2409 (1993).

[10] N. Engheta and M. M. I. Saadoun, "Noval pseudochiral or Ω medium and its application," *Proc. Progress in Electromagnetic Research Symposium PIERS 1991*, p. 339, Cambridge, Massachusetts, USA, July 1-5 (1991).

[11] M. M. I. Saadoun and N. Engheta, "A reciprocal phase shifter using noval pseudochiral or Ω medium," *Microwave Opt. Tech. Lett.*, 5(4), 184-187 (1992).

[12] J. A. Kong, Electromagnetic Wave Theory, Chap. 5, 403; Chap. 2, p.117 (John Wiley & Sons, 1986).

[13] G. Kristensson and R. J. Krueger, "Direct and inverse scattering in the time domain for a dissipative wave equation. Part I. Scattering operators," *J. Math. Phys.* 27, 1667-1682 (1986); "Part II. Simultaneous reconstruction of dissipation and phase velocity profiles," *J. Math. Phys.* 27, 1683-1693 (1986).

[14] G. Kristensson and S. Rikte, "Scattering of transient electromagnetic waves in bi-isotropic media," *J. Electro. Waves Applic.* vol. 6, p.1517, 1992.

[15] J. P. Corones and A. Karlsson, "Transient direct and inverse scattering for inhomogeneous viscoelastic media: obliquely incident SH mode,", *Inverse Problems*, 4(3), 643-660 (1988).

[16] R. J. Krueger and R. L. Ochs, "A Green's function approach to the determination of internal fields," *Wave Motion*, 11, 525-543 (1989).

[17] S. He and S. Ström, "The electromagnetic scattering problem in the time domain for a dissipative slab and a point source using invariant imbedding", *J. Math. Phys.*, 32(12), 3529-3539 (1991).

[18] S. He, "Factorization of a dissipative wave equation and the Green functions technique for axially symmetric fields in a stratified slab," *J. Math. Phys.*, 33(3), 953-966 (1992).

[19] S. He and V. H. Weston, "Inverse problem for the dissipative wave equation in a stratified half-space and linearization of the imbedding equation," *Inverse Problems*, 8(3), 435-455 (1992).

[20] V. H. Weston, "Time-domain wave splitting of Maxwell's equations," *J. Math. Phys.*, 34,(4), 1370-1392 (1993).

[21] S. A. Tretyakov, F. Mariotte, C. R. Simovski, T.G. Kharina, and S. Bolioli, "Antenna model for individul chiral and omega scatterers," *Chiral94, 3rd International Workshop on chiral, bi-isotropic and bi-anisotropic media*, pp. 41-48, F. Mariotte and J-P. Parneix, eds., Perigueux, France, May 18-20, 1994.

[22] T. Robin, B. Souillard, R. Accolas, P. Theret, and S. Bolioli, "Modelling of electromagnetic properties of helix-loaded chiral media," *Chiral94, 3rd International Workshop on chiral, bi-isotropic and bi-anisotropic media*, pp. 103-108, F. Mariotte and J-P. Parneix, eds., Perigueux, France, May 18-20, 1994.