Assignment 3

Brage Bøe Svendsen

October 8, 2020

1

1.1 a)

From the electric field

$$\vec{E} = (-6\hat{x} + 3\sqrt{5}\hat{y})\cos\left[\frac{1}{3}\left(\sqrt{5}x + 2y\right)\pi \cdot 10^7 - 9.42 \cdot 10^{15}t\right] \cdot 10^2 \tag{1}$$

we see that the field amplitude vector must be $\vec{E}_0 = (-6\hat{x} + 3\sqrt{5}\hat{y}) \cdot 10^2$, so the magnitude of the electric field amplitude is

$$E_0 = |\vec{E}_0| = \sqrt{(-600)^2 + (300\sqrt{5})^2} = \underline{\underline{900\text{V/m}}}.$$

The magnetic field amplitude is

$$B_0 = \frac{E_0}{c} = \frac{900\text{V/m}}{3 \cdot 10^8\text{m/s}} = 3 \cdot 10^{-6}\text{T} = \underline{\underline{3\mu T}}.$$

1.2 b)

The direction of the electric field amplitude is

$$\hat{E}_0 = \frac{\vec{E}_0}{|\vec{E}_0|} = \frac{-600\hat{x} + 300\sqrt{5}\hat{y}}{900} = \underline{\underline{-\frac{2}{3}\hat{x} + \frac{\sqrt{5}}{3}\hat{y}}}$$

Studying equation (1) we see that the wave propagation is along the xy-plane, with both positive x and y, while we just found that $\hat{E}_0$ is along xy-plane with negative x and positive y. From the right hand rule the magnetic field must therefore point directly in positive z-direction, thus

$$\hat{B}_0 = \underline{\underline{\hat{z}}}.$$

1.3 c)

From equation (1) we see that $\vec{k} \cdot \vec{r} = \frac{1}{3}(\sqrt{5}x + 2y)\pi \cdot 10^7$, where $\vec{r} =< x, y, z >$, so the wave vector must be

$$\vec{k} = \frac{\sqrt{5}}{3}\pi \cdot 10^7\hat{x} + \frac{2}{3}\pi \cdot 10^7\hat{y}$$

with unit vector

$$\hat{k} = \frac{\vec{k}}{|\vec{k}|}$$
$$= \frac{\left(\frac{\sqrt{5}}{3}\hat{x} + \frac{2}{3}\hat{y}\right)\pi \cdot 10^7}{\sqrt{\left(\frac{\sqrt{5}}{3}\right)^2 + \left(\frac{2}{3}\right)^2}\pi \cdot 10^7}$$
$$= \frac{\left(\frac{\sqrt{5}}{3}\hat{x} + \frac{2}{3}\hat{y}\right)}{\sqrt{\frac{5}{9} + \frac{4}{9}}}$$
$$= \underline{\underline{\frac{\sqrt{5}}{3}\hat{x} + \frac{2}{3}\hat{y}}}$$

Figure 1 shows the directions of $\hat{k}$, $\hat{E}_0$ and $\hat{B}_0$ in a xyz-coordinate system.

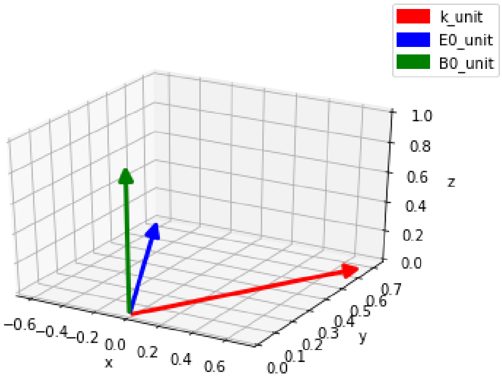


Figure 1: Unit vectors of $\vec{E}_0$, $\vec{B}_0$ and $\vec{k}$.

1.4 d)

We found the wave number during the above calculation,

$$k = |\vec{k}| = \underline{\underline{\pi \cdot 10^7 \text{m}^{-1}}}.$$

The wavelength is found from the angular wave number definition,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi \cdot 10^7} = 2 \cdot 10^{-7}\text{m} = \underline{\underline{200\text{nm}}},$$

while the frequency is found from

$$\nu = \frac{c}{\lambda} = \underline{\underline{1.5 \cdot 10^{15}\text{Hz}}}.$$

1.5 e)

The wave is travelling in air/vacuum, so the wave speed is $v = c$.

1.6 f)

The irradiance is

$$E_e = \frac{1}{2}\varepsilon_0 c E_0^2 = \underline{\underline{1075\mathrm{W/m}^2}}$$

2

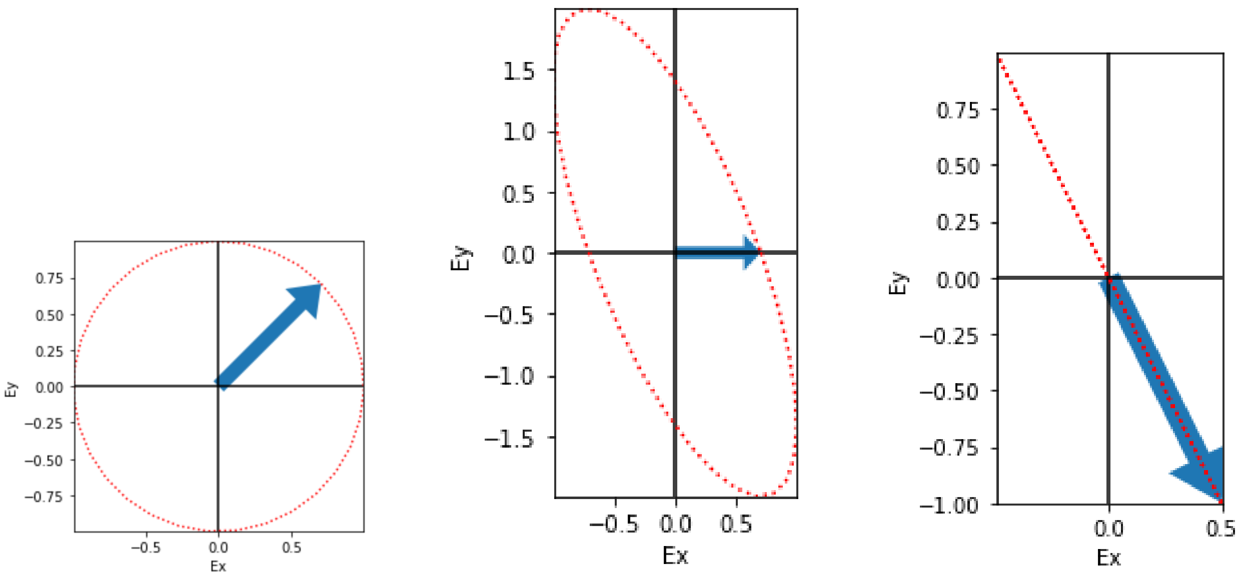
In the following polarization plots we have set $E_0 = 1$. At $z = 0$, these electromagnetic waves reduces to

$$\vec{E} = \cos(\omega t - \frac{\pi}{4})\hat{x} + \cos(\omega t + \frac{\pi}{4})\hat{y} \tag{2}$$

$$\vec{E} = \cos(\omega t - \frac{\pi}{4})\hat{x} - 2\sin(\omega t)\hat{y} \tag{3}$$

$$\vec{E} = \frac{1}{2}\cos(\omega t)\hat{x} - \sin(\omega t + \frac{\pi}{2})\hat{y} \tag{4}$$

Plots of these polarization states, where ωt is traced over a period of $[0, 2\pi]$, are shown in figure 2. The blue arrow indicates the starting position of $\omega t = 0$.



(a) Eq (2). (b) Eq (3). (c) Eq (4).

Figure 2: Polarization plots of the electric fields in equations (2)-(4). Red-dotted lines trace the outline of a polarization state at a given time ωt. Blue arrow indicates polarization state at $\omega t = 0$ for each wave.

The field in equation (2) is right-circularly polarized, meaning the blue arrow traces the red-dotted circle going clock-wise around the loop. Equation (3) is elliptically polarized with right-handedness. Lastly, equation (4) is linearly polarized. At $\omega t = 0$ it starts fully extended toward the lower right, tracing the line towards the opposite side at $\omega t = \pi$, before returning at its starting position at $\omega = 2\pi$.