Assignment 1

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Middle lense negative: $f_1 = 10$ cm, $f_2 = -15$ cm, $f_3 = 20$ cm.

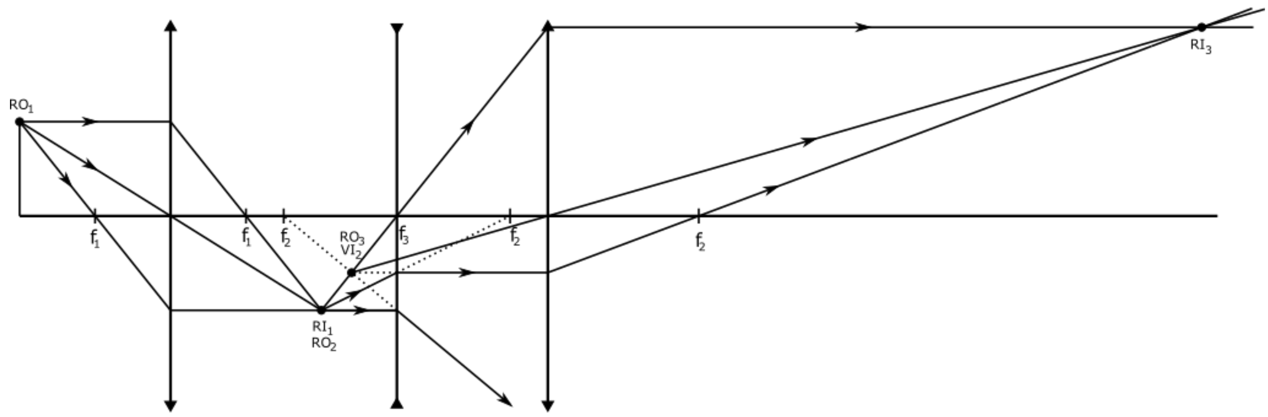

Figure 1: Ray tracing showing the essential rays from object RO_1 to final image RI_3.

- Image formed from the first lens:

$$s_i^{(1)} = \left(\frac{1}{f_1} - \frac{1}{s_o^{(1)}}\right)^{-1} = \left(\frac{1}{10} - \frac{1}{20}\right)^{-1} = 20\text{cm},$$

  with magnification

$$m_1 = -\frac{s_i^{(1)}}{s_o^{(1)}} = -\frac{20}{20} = -1.$$

- This image RI_1 works as a real object RO_2 placed to the left of the second lens at a distance $s_o^{(2)} = 30\text{cm} - 20\text{cm} = 10\text{cm}$.

  Virtual image formed from the second lense:

$$s_i^{(2)} = \left(\frac{1}{f_2} - \frac{1}{s_o^{(2)}}\right)^{-1} = \left(\frac{1}{-15} - \frac{1}{10}\right)^{-1} = -6.0\text{cm}$$

  with minification

$$m_1 = -\frac{s_i^{(2)}}{s_o^{(2)}} = -\frac{-6}{10} = \frac{3}{5}.$$

- Distance from RO_3 to third lens is $s_o^{(3)} = 20\text{cm} + 6\text{cm} = 26\text{cm}$.

  Its image is then located at

$$s_i^{(2)} = \left(\frac{1}{f_3} - \frac{1}{s_o^{(3)}}\right)^{-1} = \left(\frac{1}{20} - \frac{1}{26}\right)^{-1} = \underline{\underline{86.67\text{cm}}}$$

  with magnification

$$m_1 = -\frac{s_i^{(3)}}{s_o^{(3)}} = -\frac{87}{26} = -\frac{10}{3}.$$

  The final image is located 87 cm to the right of the third lens, and the total magnification is

$$m = m_1 m_2 m_3 = (-1)\frac{3}{5}\left(-\frac{10}{3}\right) = \underline{\underline{2}}$$

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From Eq (2-7) in PP we have

$$n_o\sqrt{x^2 + y^2} + n_i\sqrt{y^2 + (s_o + s_i - x)^2} = n_o s_o + n_i s_i \tag{1}$$

with the notation given in figure 2-12. Values for refractive indices and object/image distances are $n_o = 1$, $n_i = 1.5$, $s_o = 20$ cm, $s_i = 10$ cm. Inserted into eq 1 we get

$$\begin{aligned}\sqrt{x^2 + y^2} + 1.5\sqrt{y^2 + 900 - 60x + x^2} &= 35\\ 1.5\sqrt{y^2 + 900 - 60x + x^2} &= 35 - \sqrt{x^2 + y^2}.\end{aligned}$$

Squaring both sides we get

$$\frac{9}{4}(x^2 + y^2) + 70\sqrt{x^2 + y^2} - 135x + 800 = 0,$$

which we can solve for $\sqrt{x^2 + y^2}$:

$$\begin{aligned}\sqrt{x^2 + y^2} &= \frac{-70 \pm \sqrt{70^2 - 4 \cdot 1.25 \cdot (-135x + 800)}}{2 \cdot 1.25}\\ &= \frac{-70 \pm \sqrt{675x + 900}}{2.5}.\end{aligned}$$

Solving the above equation with respect to y results in

$$y = \sqrt{\frac{-70 \pm \sqrt{675x + 900}}{2.5}^2 - x^2} \tag{2}$$

Equation 2 can be solved for a x that will result in a point $P(x, y)$ that intersects the refracting Cartesian oval. Table 1 demonstrates a small selection (x, y)-coordinates found by using equation 2. In figure 2 the Cartesian oval has been plotted for $x\epsilon[20, 30]$ cm, together with a half circle of radius s_i centered at the image point (rightmost black dot).

x (cm)	20	20.5	21	21.5	22	22.5
y (cm)	0	1.56	2.17	2.62	2.97	3.26

Table 1: Numerical values of y for given values of x, using equation 2.

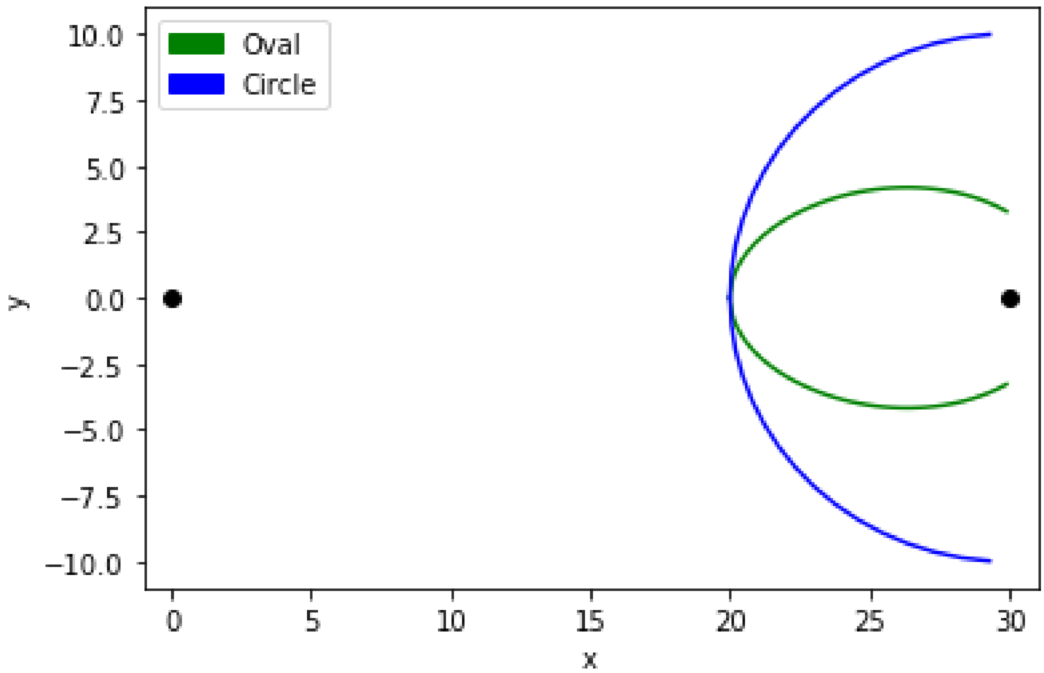

Figure 2: Green: a plot of equation 2 for a selection x-values, with y-values mirrored over the x-axis to illustrate the oval. Blue: a half circle of radius s_i centered at the image point. Left dot: object point. Right dot: image point.