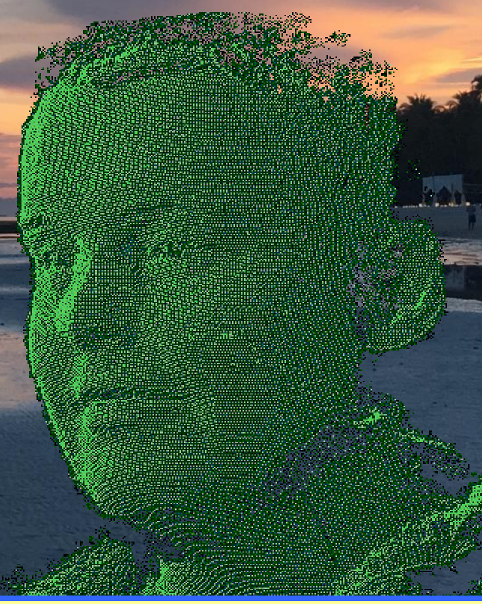
T8C: Guided light waves in a thin film.

’TFY4195 Optikk’ H2020



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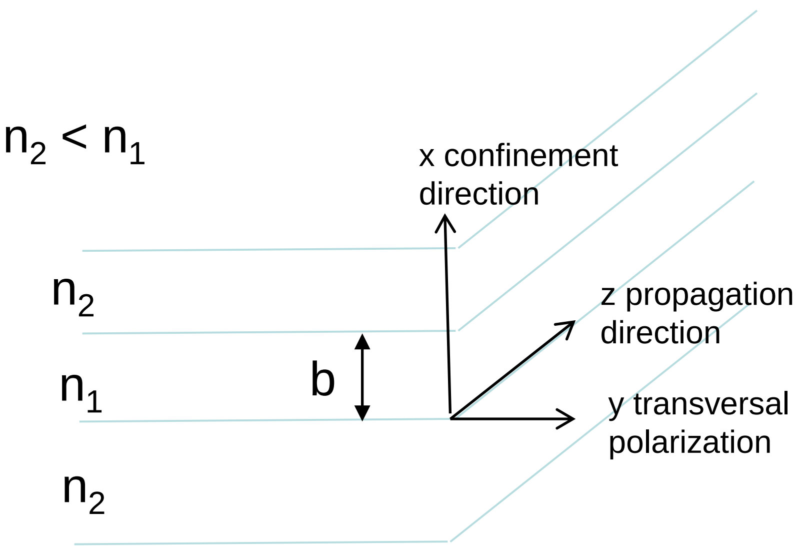
As alluded to last lecture the total internal reflection along with evanescent fields are central to many applications, such as high-resolution microscopy and fiber optics technology.

The objective of todays lecture is to understand the phenomenon of optical waveguiding. An introduction to fiber optics based on the ray model is given in PP Ch10 – so read it through.

Here we will focus on the electromagnetic field approach, solving the coupled Maxwell equations for a thin slab geometry. Such theory is very suitable to use in computer simulations; so the knowledge from the lecture is to be used to simulate optical waveguiding modes in Lab 2 – Computer simulation of optical waveguiding’

We will later in the course (after Lab 2/Assignment 4 is completed) have a follow-up discussion about the results, and discuss more about the fiber optics in PP Ch10.

SLAB WAVEGUIDE



Here, $n_1 > n_2$ so there can be a situation with total internal reflection. Thus, the light cannot escape and propagates along *z and we anticipate 'evanescent fields' into the cladding x regions.*

Let the optical wave propagate along z,

$$\bar{E} = \overline{E}_0 e^{i(\beta z - \omega t)}; \bar{H} = \overline{H}_0 e^{i(\beta z - \omega t)} \qquad (*)$$

With propagation constant β.

Maxwell's curl equations always apply:

$$\bar{\nabla} \times \bar{E} = -\frac{d\bar{B}}{dt} = -\mu_0 \frac{d\bar{H}}{dt}; \qquad \bar{\nabla} \times \bar{H} = \frac{d\bar{D}}{dt} = \varepsilon_0 n^2 \frac{d\bar{E}}{dt}$$

We will have evanescent waves in the x-direction, propagation along z, no variation along 'y', thus: $\frac{d}{dy} = 0$

There will be no 'changes' along the y-direction.

Differentiate assumed fields in eq. (*)

$$\bar{\nabla} \times \bar{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{d}{dx} & 0 & \frac{d}{dz} \\ E_x & E_y & E_z \end{vmatrix} = \begin{pmatrix} -\frac{dE_y}{dz} \\ \frac{dE_x}{dz} - \frac{dE_z}{dx} \\ \frac{dE_y}{dx} \end{pmatrix} = -\mu_0 \frac{d}{dt} \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} i\beta E_y \\ i\beta E_x - \frac{dE_z}{dx} \\ \frac{dE_y}{dx} \end{pmatrix} = i\omega\mu_0 \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}$$

$$\bar{\nabla} \times \bar{H} = \ldots \Rightarrow \begin{pmatrix} i\beta H_y \\ i\beta H_x - \frac{dH_z}{dx} \\ \frac{dH_y}{dx} \end{pmatrix} = -i\omega\varepsilon_0 n^2 \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

The eqs decouple in two sets of eqs, TE mode (transversal electric) and TM mode (transversal magnetic).
:

$TE: E_y, H_x and H_z$

$TM: H_y, E_x and E_z$

$$(1): -i\beta E_y = i\omega\mu_0 H_x$$

$$(2): i\beta E_x - \frac{dE_z}{dx} = i\omega\mu_0 H_y$$

$$(3): \frac{dE_y}{dx} = i\omega\mu_0 H_z$$

$$(4): -i\beta H_y = -i\omega\varepsilon_0 n^2 E_x$$

$$(5): i\beta H_x - \frac{dH_z}{dx} = -i\omega\varepsilon_0 n^2 E_y$$

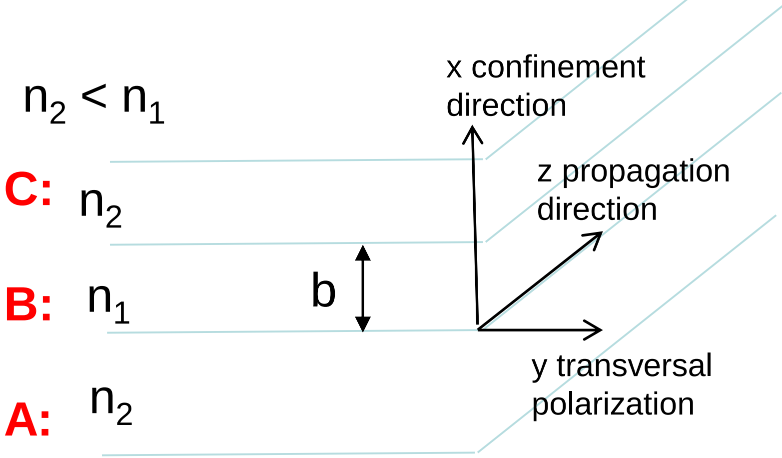
$$(6): \frac{dH_y}{dx} = -i\omega\varepsilon_0 n^2 E_z$$

These 'modes' are independent and correspond to the transverse electric and magnetic case we had for the Fresnel reflection and transmission coeffients

TE-mode

Now take H_x from eq. (1) and H_z from eq. (3) into eq. (5), giving:

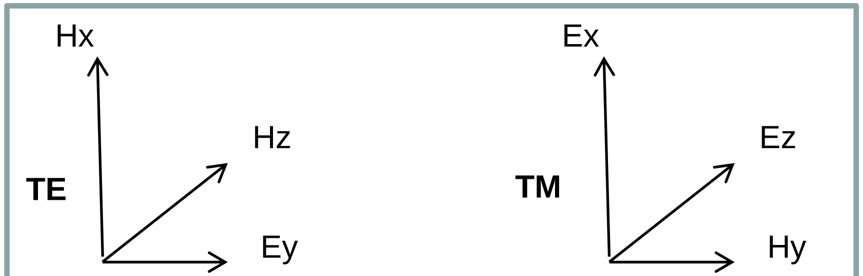
$$\frac{d^2E_y}{dx^2} = \left(\beta^2 - \frac{\omega^2}{c^2}n^2\right)E_y = -(k_{vac}^2 n^2 - \beta^2)E_y$$



which has general solutions:

$$\frac{d^2E_y}{dx^2} = -K^2E_y \Rightarrow E_y = C_1\cos K\,x + C_2\sin K\,x$$

$$\frac{d^2E_y}{dx^2} = \gamma^2E_y \Rightarrow E_y = C_3e^{\gamma x} + C_4e^{-\gamma x}$$



N.b. there are longitudinal components along the propagation direction (z) !

We may chose solutions depending on the physical situation and assume physical relevant solutions for the regions A, B and C:

We expect 'evanescent fields' into the top and lower levels since we must be in the total reflection regime to have wave-guiding.

$$E_{yC} = De^{-\gamma(x-b)}; \gamma^2 = (\beta^2 - k^2n_2^2)\ \ for\ \ x > b\,;\ \ \beta > kn_2$$

$$E_{yB} = B\cos K\,x + C\sin K\,x; K^2 = (k^2n_1^2 - \beta^2);\ for\ \ 0 < x < b\,;\ \beta < kn_1$$

$$E_{yA} = Ae^{\gamma x}; \gamma^2 = (\beta^2 - k^2n_2^2)\ \ for\ \ x < 0\,;\ \ \beta > kn_2$$

We find H_z from eq. (3)

$$(3): \frac{dE_y}{dx} = i\omega\mu_0H_z$$

$$H_{zC} = \frac{1}{i\omega\mu_0}(-\gamma)De^{-\gamma(x-b)}; for\ x > b$$

$$H_{zB} = \frac{1}{i\omega\mu_0}(-BK\sin K\,x + KC\cos K\,x);\ for\ \ 0 < x < b$$

$$H_{zA} = \frac{1}{i\omega\mu_0}A\gamma e^{\gamma x}; for\ \ x < 0$$

TE-mode; Boundary conditions, fields parallell interfaces must be continuous… E_y continuous gives:

$$x = 0: Ae^0 = B\cos 0 + C\sin 0 \Rightarrow A = B$$

$$x = b: De^0 = B\cos K\,b + C\sin K\,b$$

E_y and H_z continuous gives:

$$x = 0: A\gamma e^0 = -BK\sin 0 + CK\cos 0 \Rightarrow \gamma A = KC$$

$$x = b: -KB\sin K\,b + KC\cos K\,b = -\gamma D$$

The boundary conditions can be summarized in a matrix equation.

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & \cos K\,b & sinKb & -1 \\ \gamma & 0 & -K & 0 \\ 0 & -K\sin K\,b & K\cos K\,b & \gamma \end{pmatrix}\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The matrix equation has solutions only if its determinant = 0.

$$\begin{vmatrix} 1 & -1 & 0 & 0 \\ 0 & \cos K\,b & sinKb & -1 \\ \gamma & 0 & -K & 0 \\ 0 & -K\sin K\,b & K\cos K\,b & \gamma \end{vmatrix} = 0$$

$$\Rightarrow 1\cdot\begin{vmatrix} \cos K\,b & \sin K\,b & -1 \\ 0 & -K & 0 \\ -K\sin K\,b & K\cos K\,b & \gamma \end{vmatrix} + \gamma\begin{vmatrix} -1 & 0 & 0 \\ \cos K\,b & \sin K\,b & -1 \\ -K\sin K\,b & K\cos K\,b & \gamma \end{vmatrix} = 0$$

$$\Rightarrow -K\gamma\cos K\,b + K^2\sin K\,b + \gamma(-\gamma\sin K\,b - K\cos K\,b) = 0$$

divide with $\cos Kb$

$$\Rightarrow 2K\gamma = (K^2-\gamma^2)\tan K\,b = 0$$

$$\Rightarrow \tan K\,b = \frac{\left(\frac{\gamma}{K}\right)+\left(\frac{\gamma}{K}\right)}{1-\left(\frac{\gamma}{K}\right)\left(\frac{\gamma}{K}\right)}$$

This we recognize as

$$\tan(r\pm q) = \frac{\tan r \pm \tan q}{1 \mp r\tan q}$$

with $\quad r = q = \tan^{-1}\left(\frac{\gamma}{K}\right)$

$$Kb = r + q + m\pi = 2\tan^{-1}\left(\frac{\gamma}{K}\right) + m\pi; \qquad m = \pm 0,1,2,$$

Define the 'effective'refractive index' N, we had

$$K = \sqrt{n_1^2k^2 - \beta^2} = \sqrt{n_1^2k^2 - N^2k^2} = \frac{2\pi}{\lambda}\sqrt{n_1^2 - N^2}$$

$$\gamma = \sqrt{\beta^2 - n_2^2k^2} = \sqrt{N^2k^2 - n_2^2k^2} = \frac{2\pi}{\lambda}\sqrt{N^2 - n_2^2}$$

Mode dispersion relation for the TE mode:

$$\frac{2\pi b}{\lambda}\sqrt{n_1^2 - N^2} = 2\cdot\tan^{-1}\sqrt{\frac{N^2 - n_2^2}{n_1^2 - N^2}} + m\cdot\pi$$

Given the film parameters b, n_1 and n_2 at a certain wavelength, we can calculate the 'allowed' N values that satisfies this equation.

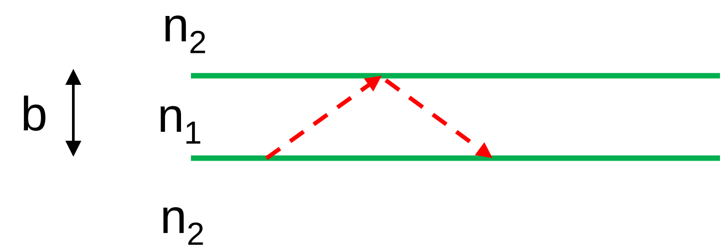
β was introduced as the 'k-vector' for the guided wave,

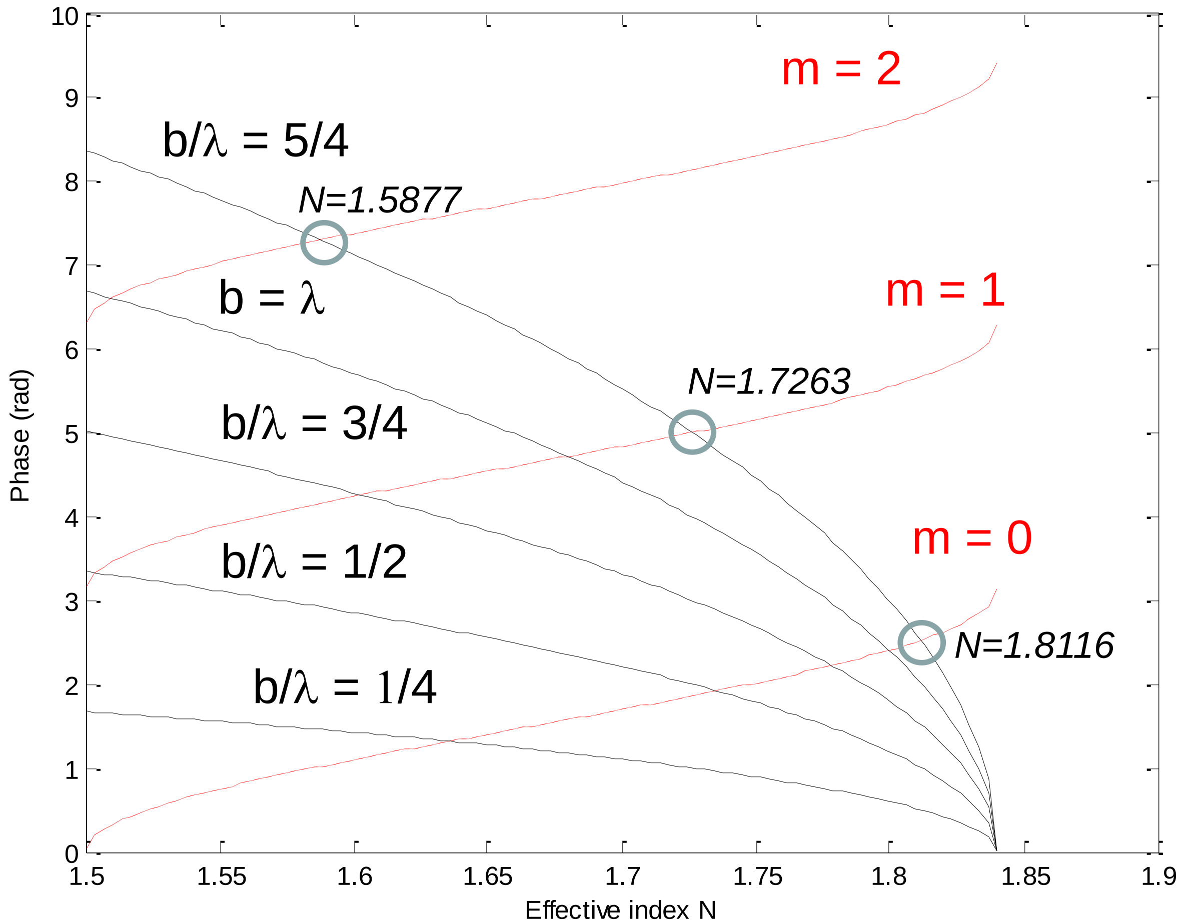
$$\bar{E} = \overline{E}_0 e^{i(\beta z-\omega t)};\ \bar{H} = \overline{H}_0 e^{i(\beta z-\omega t)}$$

But we found it by solving the differential equation in the x-direction (the confined direction).

Let's see it graphically:

Slab waveguide TE mode dispersion



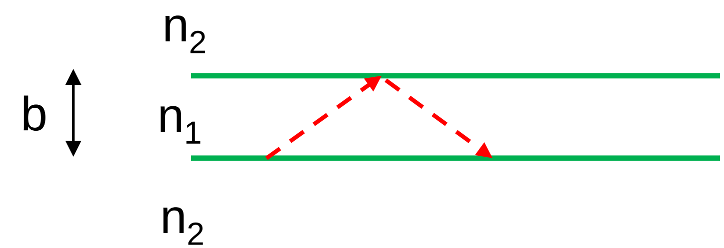


n_1 = 1.84 (flint glass)
n_2 = 1.50 (regular glass)

$$\frac{2\pi b}{\lambda}\sqrt{n_1^2 - N^2} \quad = \quad 2 \cdot \tan^{-1}\sqrt{\frac{N^2 - n_2^2}{n_1^2 - N^2}} + m \cdot \pi$$

Slab waveguide mode distribution

When we know the allowed *N*-values along with the parameters *b*, n_1 and n_2 at a certain wavelength, we can calculate the the field distribution in the x-direction. We get 'standing waves' oscillating between the low index surfaces as the mode is propagating along the z-direction at a speed *c/N*. So each mode has its own speed. The speed of the modes must be between c/n_1 and c/n_2.



$$\frac{2\pi b}{\lambda}\sqrt{n_1^2 - N^2} \;=\; 2\cdot\tan^{-1}\sqrt{\frac{N^2 - n_2^2}{n_1^2 - N^2}} + m\cdot\pi$$

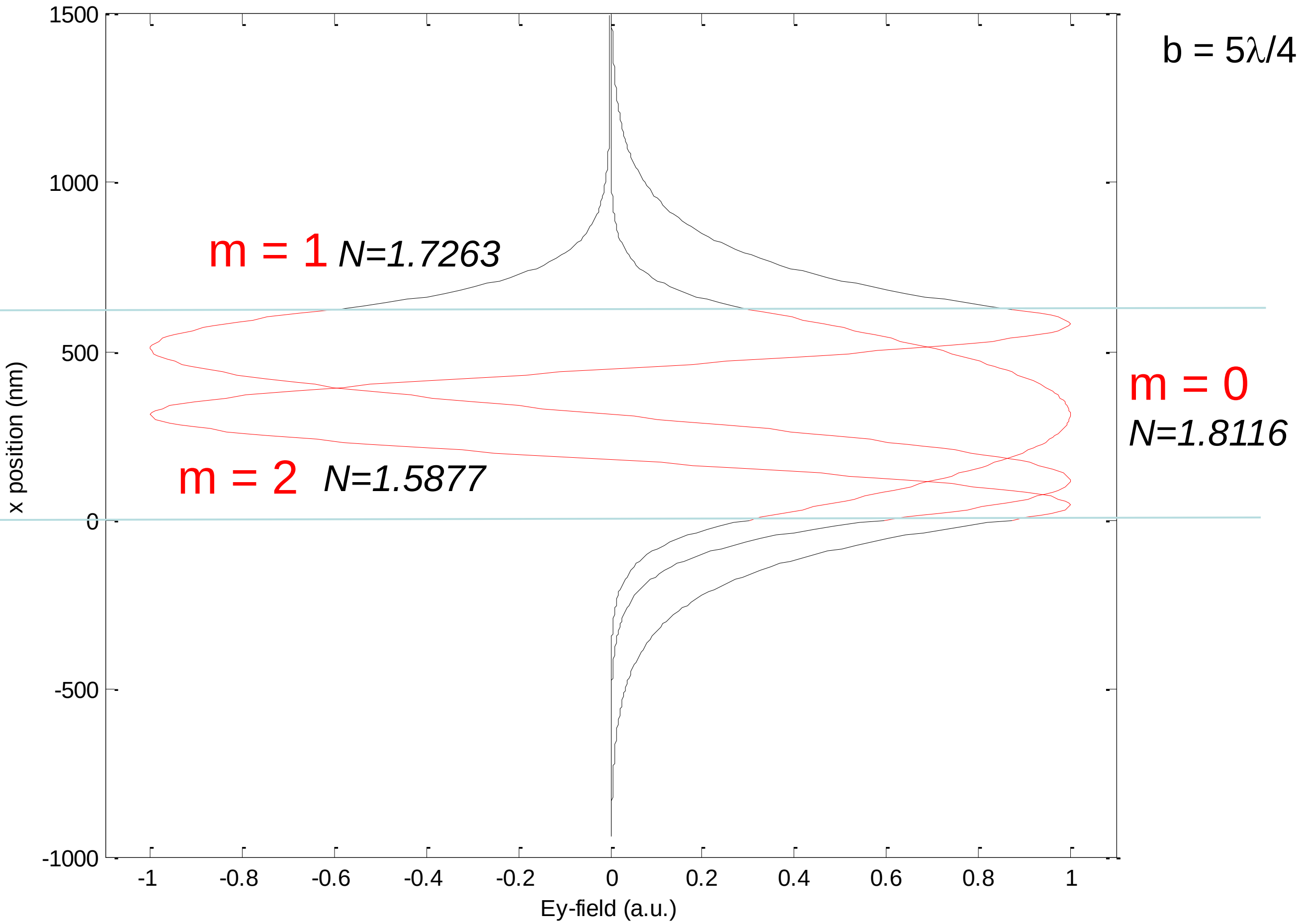
n_1 = 1.84 (flint glass)
n_2 = 1.50 (regular glass)

b = 5λ/4 (λ = 500 nm)

$$De^{-\gamma(x-b)}$$

$$B\cos(Kx) + C\sin(Kx)$$

$$Ae^{\gamma x}$$



Computer simulation lab – LAB 2 – Assignment 4

1) Make computer program(s) that calculate(s) the mode dispersion and Ey field component as discussed in the lecture. Use the common communication wavelength 1.55 μm. Take the refractive indices of the layers to be 1.4 and 1.7. Find solutions and plot the Ey-field for at least two situations:

a) A slab thickness giving only one allowed mode (single mode). Find a suitable thickness.
b) A slab waveguide with several modes (say 3 – 5). Find relevant parameters (thickness, N, etc)

2) Find the TM mode dispersion from the Maxwell eqs as for the TE case. Repeat a) and b) for this situation (calculate the Hy-field). Are the effective indices the same for the same thickness?

Challenge: calculate also the Ex and Ez fields for the TM mode. What can we say about the continuity of the calculated Ex field at the interfaces x = 0 and x = b for the TM mode?