

# Learning with Mixtures of Trees

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## Abstract

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## 1 Introduction

Probabilistic inference has become a core technology in AI, largely due to developments in graph-theoretic methods for the representation and manipulation of complex probability distributions [1]. Whether in their guise as directed graphs (Bayesian networks) or as undirected graphs (Markov random fields), *probabilistic graphical models* have a number of virtues as representations of uncertainty and as inference engines. Graphical models allow a separation between qualitative, structural aspects of uncertain knowledge and the quantitative, parametric aspects of uncertainty... *Remainder omitted in this sample. See <http://www.jmlr.org/papers/> for full paper.*

## Appendix A.

In this appendix we prove the following theorem from Section 6.2:

**Theorem** *Let  $u, v, w$  be discrete variables such that  $v, w$  do not co-occur with  $u$  (i.e.,  $u \neq 0 \Rightarrow v = w = 0$  in a given dataset  $\mathcal{D}$ ). Let  $N_{v0}, N_{w0}$  be the number of data points for which  $v = 0, w = 0$  respectively, and let  $I_{uv}, I_{uw}$  be the respective empirical mutual information values based on the sample  $\mathcal{D}$ . Then*

$$N_{v0} > N_{w0} \Rightarrow I_{uv} \leq I_{uw}$$

*with equality only if  $u$  is identically 0.*

**Proof.** We use the notation:

$$P_v(i) = \frac{N_v^i}{N}, \quad i \neq 0; \quad P_{v0} \equiv P_v(0) = 1 - \sum_{i \neq 0} P_v(i).$$

These values represent the (empirical) probabilities of  $v$  taking value  $i \neq 0$  and 0 respectively. Entropies will be denoted by  $H$ . We aim to show that  $\frac{\partial I_{uv}}{\partial P_{v0}} < 0 \dots$

*Remainder omitted in this sample. See <http://www.jmlr.org/papers/> for full paper.*

## References

- [1] Wessel N. van Wieringen. Lecture notes on ridge regression, 2023.