University of Oslo

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Project 1

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Abstract

This paper describes the mixtures-of-trees model, a probabilistic model for discrete multidimensional domains. Mixtures-of-trees generalize the probabilistic trees of [1] in a different and complementary direction to that of Bayesian networks. We present efficient algorithms for learning mixtures-of-trees models in maximum likelihood and Bayesian frameworks. We also discuss additional efficiencies that can be obtained when data are "sparse," and we present data structures and algorithms that exploit such sparseness. Experimental results demonstrate the performance of the model for both density estimation and classification. We also discuss the sense in which tree-based classifiers perform an implicit form of feature selection, and demonstrate a resulting insensitivity to irrelevant attributes.

Keywords: Bayesian Networks, Mixture Models, Chow-Liu Trees

1. Introduction

Probabilistic inference has become a core technology in AI, largely due to developments in graph-theoretic methods for the representation and manipulation of complex probability distributions [3]. Whether in their guise as [2] directed graphs (Bayesian networks) or as undirected graphs (Markov random fields), *probabilistic graphical models* have a number of virtues as representations of uncertainty and as inference engines. Graphical models allow a separation between qualitative, structural aspects of uncertain knowledge and the quantitative, parametric aspects of uncertainty...

Remainder omitted in this sample. See http://www.jmlr.org/papers/ for full paper.

2. Ordinary Least Squares

2.1 Discussion on Scaling

```
    # sample design matrix fitting 1-dimensional polynomial of degree 5 (not scaled)
                   [[1.      0.      0.      0.      0.      0.     ]
                    [1.      0.25    0.0625  0.01562 0.00391 0.00098]
                    [1.      0.5     0.25    0.125   0.0625  0.03125]
                    [1.      0.75    0.5625  0.42188 0.31641 0.2373 ]
                    [1.      1.      1.      1.      1.      1.     ]]

    # sample design matrix fitting 1-dimensional polynomial of degree 5 (scaled)
              [[ 0.      -1.41421 -1.0171  -0.83189 -0.728   -0.66226]
               [ 0.      -0.70711 -0.84758 -0.7903  -0.71772 -0.65971]
               [ 0.       0.      -0.33903 -0.49913 -0.56348 -0.58075]
               [ 0.       0.70711  0.50855  0.29116  0.10488 -0.0433 ]
               [ 0.       1.41421  1.69516  1.83016  1.90431  1.94603]]

# unscaled beta
[  0.4097    7.55124   3.79304 -32.85696 -14.83669  -8.81645  45.45889
  43.33221  20.70625  -7.63623 -21.24552 -51.81866  -7.53731 -29.60175
  28.57282   0.73824  18.29253  10.60883  -5.52465  16.60259 -16.13743]

# scaled beta
[ 2.21481  1.1093  -9.93941 -3.32124 -2.6612  13.11848  8.70306  4.14949
 -2.19979 -5.79259 -9.48697 -1.28166 -5.40093  7.77825  0.19055  3.09519
  1.60557 -0.83485  2.797   -4.1589 ]
```

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Appendix A.

In this appendix we prove the following theorem from Section 6.2:

Theorem *Let u, v, w be discrete variables such that v, w do not co-occur with u (i.e., $u \neq 0 \;\Rightarrow\; v = w = 0$ in a given dataset $\mathcal{D}$). Let N_{v0}, N_{w0} be the number of data points for which $v = 0, w = 0$ respectively, and let I_{uv}, I_{uw} be the respective empirical mutual information values based on the sample $\mathcal{D}$. Then*

$$N_{v0} \;>\; N_{w0} \;\;\Rightarrow\;\; I_{uv} \;\leq\; I_{uw}$$

with equality only if u is identically 0. ■

Proof. We use the notation:

$$P_v(i) \;=\; \frac{N_v^i}{N}, \quad i \neq 0; \quad P_{v0} \;\equiv\; P_v(0) \;=\; 1 - \sum_{i \neq 0} P_v(i).$$

These values represent the (empirical) probabilities of v taking value $i \neq 0$ and 0 respectively. Entropies will be denoted by H. We aim to show that $\frac{\partial I_{uv}}{\partial P_{v0}} < 0$....

Remainder omitted in this sample. See http://www.jmlr.org/papers/ for full paper.

Bibliography

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