DE Assignment

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1 Part I

1.1 Exact solution (4th variant)

$$2x^3 + 2\frac{y}{x} = y'$$

We can rewrite it as:

$$y' - 2\frac{y}{x} = 2x^3$$

It is a linear F.O. O.D.E with non-homogeneous coefficients. Let's solve a complementary equation:

$$y' - 2\frac{y}{x} = 0$$

After some mathematical operations we get:

$$\frac{y'}{y} = \frac{2}{x}$$

(we assume that $y \neq 0$ Also, $y = 0$ is not a trivial solution). Using the property of differential we find:

$$\frac{dy}{y} = 2\frac{dx}{x}$$

Integrating it we get

$$y = C_1 x^2, C_1 \in \mathbb{R} \text{ and } C_1 \neq 0$$

$$C_1 \to C_1(x), \text{ so}$$

$$y' = C_1'(x)x^2 + 2C_1(x)x$$

Substituting obtained values of y and y' into the rewritten original equation we get

$$C_1'(x)x^2 + 2C_1(x)x - 2\frac{C_1x^2}{x} = 2x^3$$

From this we get

$$C_1'(x) = 2x$$

Integrating this we obtain

$$C_1(x) = x^2 + C, x^2 \neq C \text{ (as } C_1 \neq 0)$$

Now we can find y:

$$y = x^4 + x^2 C, x^2 \neq C \text{ and } x \neq 0 \text{ (since } y \neq 0)$$

This is the exact solution.
Now we need to express C in terms of x_0, y_0. We have

$$y_0 = y(x_0) = x_0^4 + x_0^2 C$$

so

$$C = \frac{y_0}{x_0^2} - x_0^2$$

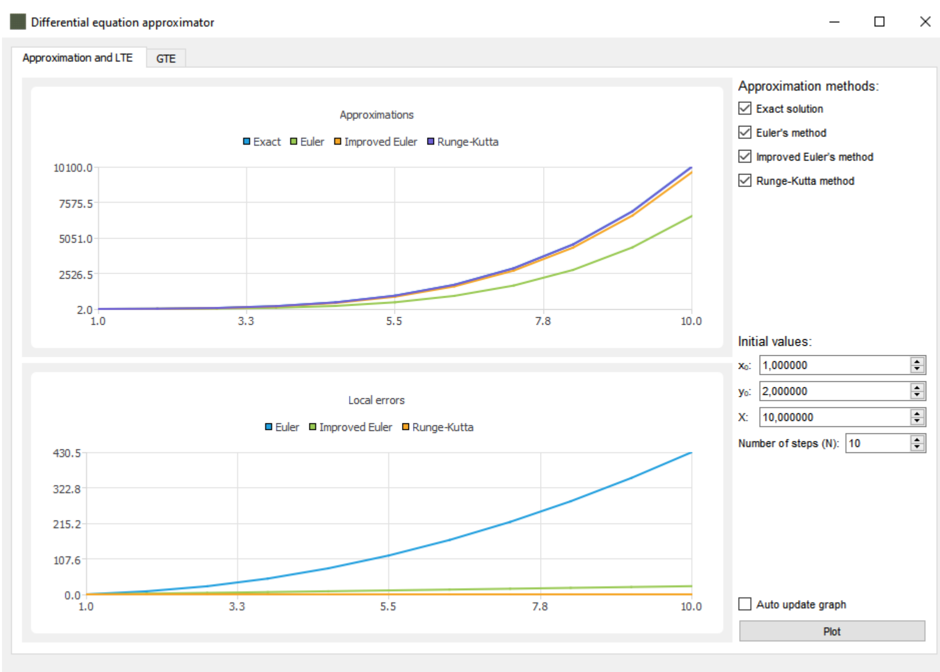
Substituting it into the exact solution we find:

$$y = x^2(x^2 + \frac{y_0}{x_0^2} - x_0^2)$$

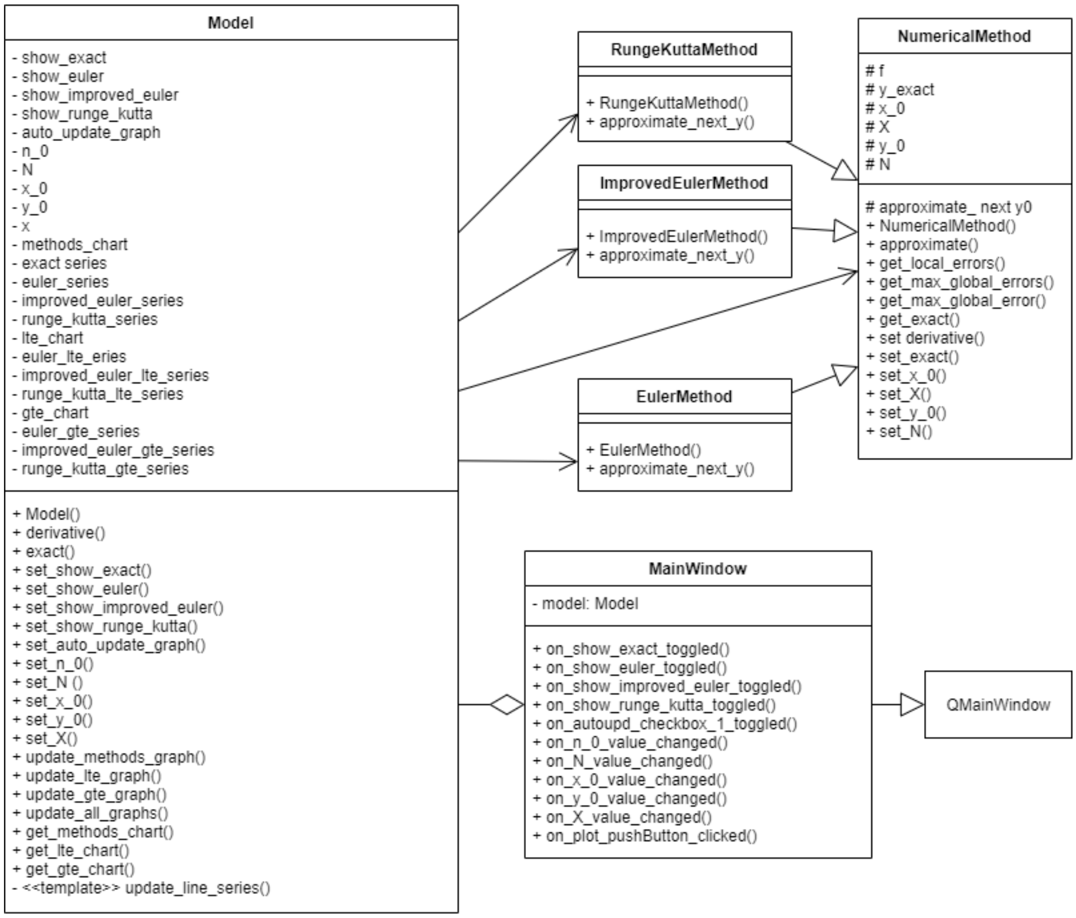
This formula is ready for our application.

2 Part II

2.1 Plots created by the program (approximations and LTEs)



2.2 UML Diagram



2.3 The most interesting parts of source code

These are the main methods for calculating approximations

```
QVector<QPointF> NumericalMethod::approximate() const{
    QVector<QPointF> result(N+1);
    result[0].rx() = x_0;
    result[0].ry() = y_0;
    double step = (X - x_0)/N;
    for (unsigned int i = 1; i < N+1; i++) {
        double x_prev = result[i-1].rx();
        double y_prev = result[i-1].ry();
        double x = x_0 + step*i;
        double y = approximate_next_y(x_prev, y_prev, step, f);
        result[i].rx() = x;
```

```
        result[i].ry() = y;
    }
    return result;
}

QVector<QPointF> NumericalMethod::get_local_errors() const {
    QVector<QPointF> result(N+1);
    result[0].rx() = x_0;
    result[0].ry() = 0;
    double step = (X - x_0)/N;
    for (unsigned int i = 1; i < N+1; i++) {
        double x_prev = result[i-1].rx();
        double x = x_0 + step*i;
        double error = fabs(y_exact(x, x_0, y_0) - approximate_next_y(x_prev, y_exact(x_prev
        result[i].rx() = x;
        result[i].ry() = error;
    }
    return result;
}

QVector<QPointF> NumericalMethod::get_max_global_errors(unsigned int n_0)
{
    // use N_orig as N will change within the method
    // keep original value of N to restore it later
    unsigned int N_orig = N;

    QVector<QPointF> result(N_orig - n_0 + 1);
    for (unsigned int i = n_0; i <= N_orig; i++) {
        N = i;
        result[i-n_0].rx() = i;
        result[i-n_0].ry() = get_max_global_error();
    }

    N = N_orig;
    return result;
}

double NumericalMethod::get_max_global_error() const {
    auto approximation = approximate();

    double max_error = 0;
    double step = (X - x_0)/N;
    for (unsigned int i = 1; i < N+1; i++) {
        double x = x_0 + step*i;
        double error = fabs(y_exact(x, x_0, y_0) - approximation[i].ry());
        if (error > max_error) {
```

```
            max_error = error;
        }
    }
    return max_error;
}

QVector<QPointF> NumericalMethod::get_exact() const
{
    QVector<QPointF> result(N+1);
    result[0].rx() = x_0;
    result[0].ry() = y_0;
    double step = (X - x_0)/N;
    for (unsigned int i = 1; i < N+1; i++) {
        double x = x_0 + step*i;
        double y = y_exact(x, x_0, y_0);
        result[i].rx() = x;
        result[i].ry() = y;
    }
    return result;
}
```

This is one of the overloadings of approximate_next_y (for Euler method)

```
double EulerMethod::approximate_next_y(double x_prev, double y_prev, double h, double (*f)(d
    return y_prev + h*f(x_prev, y_prev);
}
```

3 Part III

3.1 Plot created by the program (global errors)