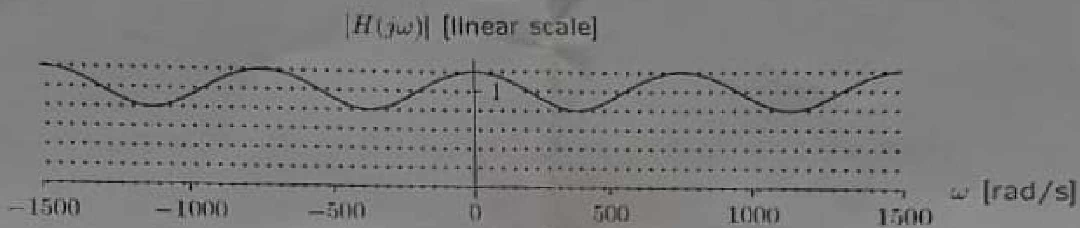
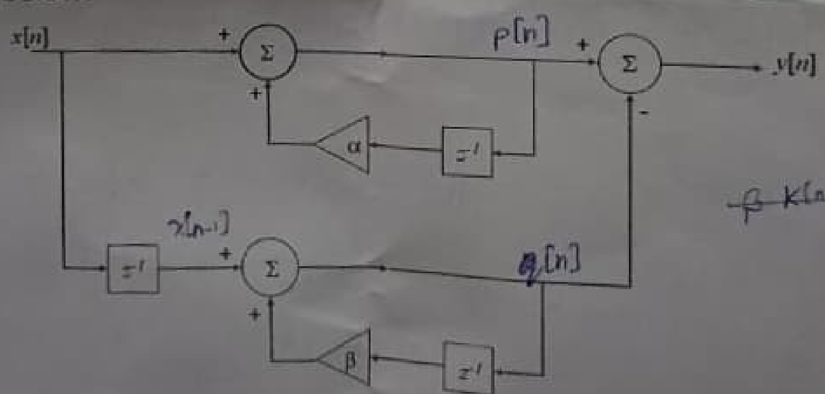
Signal Processing   Mid-semester exam   Monsoon 2022

1. An LTI system is represented by $h[n] = \delta[n - n_0] + \alpha\, \delta[n - n_1]$ with $n_1 > n_0$. The plot below shows the magnitude of the $H(z)$ when evaluated on the unit circle, i.e $|z| = 1, or\ z = e^{j\omega}$ where $\omega = \frac{2\pi}{T}$ is the angular frequency.
   a. Assume $\alpha < 1$ and sketch $h[n]$. Find the system function $H(z)$.
   b. Justify the oscillatory pattern in $|H(e^{j\omega})|$ by evaluating $H(z)$on the unit circle. Relate the variables α, n_0, n_1 in $h[n]$ to the oscillations. If $n_0 = 0$, find n_1 and α.
   c. Draw the block diagram of this system.

   Bonus question (3 points): what real life problem can be modeled by this system? Explain clearly.



2. Consider the system below.



$-\beta\, k[n-1] - x[n-1] - k$

   a. Find the impulse response $h[n]$ and system function of this system. Plot the poles and zeros on the z-plane.
   b. Specify the condition on α, β and the relation between them required for
      i. $h[n]$ to be of finite duration
      ii. System to be stable
      iii. System to be causal

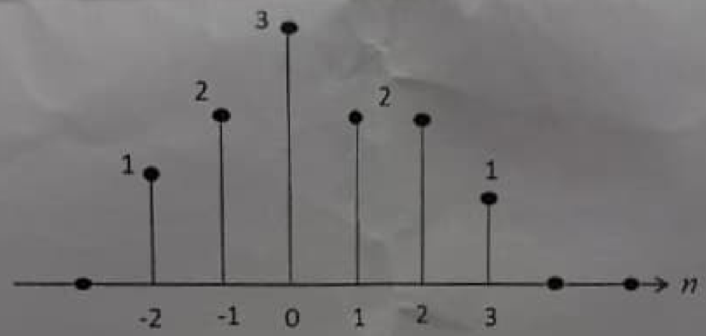
      You can if required, give the answer 'not possible'.
   c. If $\alpha = \beta = 1$, describe the signal processing function done by this system.

3. An LTI system initially at rest is described by the difference equation:

$$y[n] + 2y[n-1] = x[n] + 2x[n-2].$$

a. What is the impulse response of this system?

b. If x[n] is as shown below, find the output of this system using convolution.



4. Let $x[n] = \delta(n+3) - \delta(n+1) + 2\delta(n) + 3\delta(n-2)$ with DTFT as $X(e^{j\omega}) = X_R(e^{j\omega}) + jX_I(e^{j\omega})$ then

a. Compute $X_R(e^{j\omega})$ and $\int_{-\pi}^{\pi} X_I(e^{j\omega})\, d\omega$

b. $DTFT\ (y[n]) = X_R(e^{j\omega})e^{j2\omega} + jX_I(e^{j\omega})$, find $y[n]$ without explicitly considering DTFT?

c. Find $X_R(k); 0 < k < N$, which is discretized $X_R(e^{j\omega})$ at $\omega = \frac{2\pi k}{N}$ for $N = 5$

d. Compute $x_1[n] =$ IDFT of $X_R(k)$ and find the relation between $x[n]$ and $x_1[n]$.