

Information and Communication
EC5.102 Spring 2022
Mid Sem Exam

Please note:

- No source material is to be used.
- Go through every question carefully before answering.

Maximum Marks: 25

1. The signal $y(t)$ is generated by multiplying a band-limited signal $x_1(t)$ with another band-limited signal $x_2(t)$

$$y(t) = x_1(t)x_2(t),$$

Where

$$\begin{aligned} X_1(\omega) &= 0 \quad \text{for } |\omega| > 1000\pi \\ X_2(\omega) &= 0 \quad \text{for } |\omega| > 2000\pi. \end{aligned}$$

What is the Nyquist rate at which $y(t)$ has to be sampled, for perfectly reconstructing $y(t)$ from samples? Please justify. (3 marks)

2. Consider an experiment in which a coin is tossed until r heads occur where r is some fixed number. The coin tosses are all independent and identically distributed with probability of head being p . Let X denote the number of tosses in the experiment.
- (a) Determine the probability mass function of X . (3 marks)
- (b) Determine the mean of X . (3 marks)
3. Consider a signal which is quantized with a step size of 2Δ . We can model the quantization noise as a uniform random variable say X . Then,
- (a) Determine the pmf of random variable X . (2 marks)
- (b) Determine the mean and variance of X . (4 marks)
4. Consider three random variables X, Y, Z , which form a Markov chain, i.e., the joint pmf of X, Y, Z factorizes as follows:

$$p_{X,Y,Z}(x, y, z) = p_X(x)p_{Y|X}(y|x)p_{Z|Y}(z|y).$$

For this joint pmf, expand the joint entropy $H(X, Y, Z)$ in terms of marginal entropies and conditional entropies (in the simplest possible form). How is the joint entropy of the Markov chain different from that of the most general case? (5 marks)

5. (a) Show that the relative entropy $D(p||q)$ is always non-negative. (2 marks)
- (b) Suppose that X is a random variable which takes values from alphabet \mathcal{X} and M denotes the number of elements in set \mathcal{X} . Then, using part (a), show that $H(X) \leq \log_2(M)$. (3 marks)