(Deemed to be University)

Probability and Random Processes

MA6.102, Monsoon-2022

Exam: Mid Semester
Total Marks: 50

Date: 19 Sept 2022
Time: 4:30 PM-6:00 PM

Instructions:

- This is a closed book exam.
- There are two questions and answering both is compulsory.
- Clearly state the assumptions (*if any*) made that are not specified in the questions.

1. Answer any four of the following questions. [Marks: 30 (7.5x4)]

   (a) A coin is tossed for N times independently and the probability of showing head in each toss is p. Find the correlation between the numbers of head and tail occur in the outcome.

   (b) A box contains two biased coins having probabilities of 0.4 and 0.6 of showing head. Consider you randomly select a coin and toss it 3 times. If the outcome is THT, then find the probability that the selected coin has biased probability equal to 0.4?

   (c) Derive the MGF of the sum of K independent binomial random variables with parameters p_k and N_k for $k = 1, \ldots, K$. Use the derived MGF to determine the mean and variance of the sum.

   (d) Consider two points are placed uniformly at random on the circumference of a circle having radius R. Find the pdf of the length of the segment connecting these two points.

   (e) Assume X follows a two-sided exponential distribution as

$$f_X(x) = \begin{cases} p\lambda \exp(-\lambda x) & \text{for } x \geq 0 \\ (1-p)\lambda \exp(\lambda x) & \text{for } x < 0, \end{cases}$$

   where $\lambda > 0$ and $p \in [0, 1]$. Find the mean and variance of X.

   (f) Let X and Y be the two random variables. Show that

$$\text{Var}[X] = \mathbb{E}[\text{Var}[X|Y]] + \text{Var}[\mathbb{E}[X|Y]].$$

2. A circle $\mathcal{C}$ of radius R contains N number of uniformly distributed points (over $\mathcal{C}$), where N is a Poisson random variable with mean λ. Let $N_{\mathcal{S}}$ denote the number of points falling within set $\mathcal{S} \subset \mathcal{C}$. Answer the following. [Marks: 20]

   (a) Find the pmf of $N_{\mathcal{A}}$.

   (b) For $\mathcal{A} \cap \mathcal{B} = \phi$, determine whether $N_{\mathcal{A}}$ and $N_{\mathcal{B}}$ are independent or not?

All the Best!