Information and Communication
End Sem Exam, 4th July, 2022

$^{n}C_{100}(1-\epsilon)^{100}\epsilon^{N-100}$

Total: 50 marks

1. Consider a random variable X which takes values $x_1, x_2, \ldots, x_7$ with probabilities $0.5, 0.26, 0.11, 0.04, 0.04, 0.03,$ respectively. Design a Huffman over a ternary alphabet $\{0, 1, 2\}$ using similar rules as that of Huffman code over binary alphabet discussed in the class. What is the average length of the resulting code? (5 marks)

2. Prove the following chain rules on conditional entropy and mutual information:
   (a) $H(X_1, X_2, X_3|Y) = H(X_1|Y) + H(X_2|X_1, Y) + H(X_3|X_1, X_2, Y)$. (3 marks)
   (b) $I(X_1, X_2; Y) = I(X_1; Y) + I(X_2; Y|X_1)$ (Please note the position of semicolon carefully). (4 marks)

3. The Z-channel has binary input and alphabets and transition probabilities $p(y|x)$ given by the following matrix:
$$Q = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \quad x, y \in \{0, 1\}.$$
Find the capacity of the Z-channel and the maximizing input probability distribution. (6 marks)

4. Consider a code C with generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$. A code is said to be systematic if the every message is a part of the corresponding codeword.
   (a) Is the above code systematic? Why or why not? (2 marks)
   (b) Give another generator matrix for the same code such that it is systematic (Hint: perform row operations) (3 marks)
   (c) What is the minimum distance of this code? Justify. (3 marks)

5. Consider a 100-bit message that is to be sent from a source to a receiver through an 'erasure' channel in which each bit is erased (independently) with probability ϵ $(0 < \epsilon < 1)$. If a bit is received correctly, the receiver sends a ACK message to the source through an error-free reverse channel and the source only then transmits the next bit. If a bit is erased, the receiver sends a NACK message to the source, and then the source retransmits the erased bit once again. Let N denote the number of transmissions needed to send the entire 100-bit message.
   (a) Find a probability distribution for N and prove that it is infact a valid distribution. (Note (if you need): The Taylor series expansion of a function f near 0 is $f(x) = f(0) + \sum_{n=1}^{\infty} f^n(0)\frac{x^n}{n!}$, where f^n indicates the n^{th} derivative). (5 marks)
   (b) Find (approximately) the mean of N. (3 marks)

6. Give an example of 3 discrete random variables X, Y, Z which are all pair-wise independent but not jointly independent., Please justify carefully. (5 marks)

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7. (a) All the constellation which we discussed are all symmetric about origin in the complex plane. What could be the reason for the same? (2 marks)

(b) QPSK is a constellation which has 4 points equally spaced on a unit circle in the complex plane. Similarly, 8-PSK is a constellation which has 8 points equally spaced on a unit circle in the complex plane. Which of the two constellations has higher transmission rate in terms of bits? For a given E_b and N_0, which of the two constellations has lower probability of error? Please justify. (5 marks)

8. We have studied frequency modulation and demodulation. Similarly, we can talk about phase modulation and demodulation (briefly mentioned in the class). What is the expression for phase modulated signal in terms of message and carrier signal? Give a procedure to demodulate phase modulated signals. (4 marks)