Introduction to Computational and Biological Vision - Assignment 2

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Question 1 - Edge Detection

Section A

Given $I\left(x,y\right)=x+siny$
And the following Edge Detectors:
E_1 = edge dector based on the local maximum of $|\nabla I|$
E_2 = Edge detector based on finding the ZC of the Laplacian ΔI
Compute analytically the edge points detected by each of the edge detectors:
For E1:
first eval $|\nabla I|$:
$|\nabla I|=\left(\frac{\partial I}{\partial x},\frac{\partial I}{\partial y}\right)=\|(1,cosy)\|=\sqrt{1^2+cos^2(y)}$
we got an expression that independent of x , so we will assume that any x will work, and find the extramum for y:

$$\begin{aligned}
\frac{\partial}{\partial y}\left|\nabla I\right| =& \frac{\partial}{\partial y}\left(\sqrt{1^2+cos^2(y)}\right)=-\frac{\sin\left(2y\right)}{2\sqrt{1+\cos^2\left(y\right)}}\\
\frac{\partial}{\partial y}\left|\nabla I\right| =&0\\
-\frac{\sin\left(2y\right)}{2\sqrt{1+\cos^2\left(y\right)}} =&0\\
sin(2y) =&0\\
y =&\frac{\pi}{2}+\pi k\ ,\pi k\quad,\forall k\in\mathbb{N}
\end{aligned}$$

and now for the second derviative:

$$\begin{aligned}
\frac{\partial^2}{\partial y^2}\left|\nabla I\right| &= -\frac{4\cos\left(2y\right)\left(1+\cos^2\left(y\right)\right)+\sin^2\left(2y\right)}{4\left(1+\cos^2\left(y\right)\right)\sqrt{1+\cos^2\left(y\right)}}\\
\left(\frac{\partial^2}{\partial y^2}\left|\nabla I\right|\right)_{y=\frac{\pi}{2}} &= -\frac{4\cos\left(2\left(\frac{\pi}{2}\right)\right)\left(1+\cos^2\left(\left(\frac{\pi}{2}\right)\right)\right)+\sin^2\left(2\left(\frac{\pi}{2}\right)\right)}{4\left(1+\cos^2\left(\left(\frac{\pi}{2}\right)\right)\right)\sqrt{1+\cos^2\left(\left(\frac{\pi}{2}\right)\right)}}\\
&= -\frac{4\cos\left(\pi\right)\left(1+\cos^2\left(\left(\frac{\pi}{2}\right)\right)\right)+\sin^2\left(\pi\right)}{4\left(1+\cos^2\left(\left(\frac{\pi}{2}\right)\right)\right)\sqrt{1+\cos^2\left(\left(\frac{\pi}{2}\right)\right)}}\\
&=1\\
\left(\frac{\partial^2}{\partial y^2}\left|\nabla I\right|\right)_{y=\pi} &= -\frac{4\cos\left(2\left(\pi\right)\right)\left(1+\cos^2\left(\left(\pi\right)\right)\right)+\sin^2\left(2\left(\pi\right)\right)}{4\left(1+\cos^2\left(\left(\pi\right)\right)\right)\sqrt{1+\cos^2\left(\left(\pi\right)\right)}}\\
&= -\frac{\sqrt{2}}{2}
\end{aligned}$$

for the first assignment of y we got positive value, so its not an extramum point.

so we got that $(x, \pi k) \quad \forall k \in \mathbb{N}$ is our extramum points.
for E_2:
lets calc the laplacian:

$$\begin{aligned}\Delta I =& \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} \\ =& \frac{\partial^2}{\partial x^2}(x + siny) + \frac{\partial^2}{\partial y^2}(x + siny) \\ =& \frac{\partial}{\partial x}(1) + \frac{\partial}{\partial y}(cos(y)) \\ =& 0 - sin(y) \\ =& sin(y)\end{aligned}$$

lets find all the points where $\Delta I = 0$:

$$\begin{aligned}\Delta I =& 0 \\ sin(y) =& 0 \\ y =& \pi k \quad \forall k \in \mathbb{N}\end{aligned}$$

so our edge points according to E_2 are: $(x, \pi k) \quad \forall k \in \mathbb{N}$.
The result are identical, but its clearly that the computations of E_2 are much more simpler.

Question 2 – Parametric Curves

Section A

The parametric equation of a straight line in 3D is $a(t) = \big(x(t), y(t), z(t)\big)$.
The parametric equation of a straight line in 2D is $a_{2d}(t) = \big(x(t), y(t)\big)$.

Without loss of generality, we assume that the center of projection is at the origin. Upon projection onto the plane z = f, the image of the point (x_0, y_0, z_0) is:

$$x_i = \frac{f}{z(t)} * x(t)$$

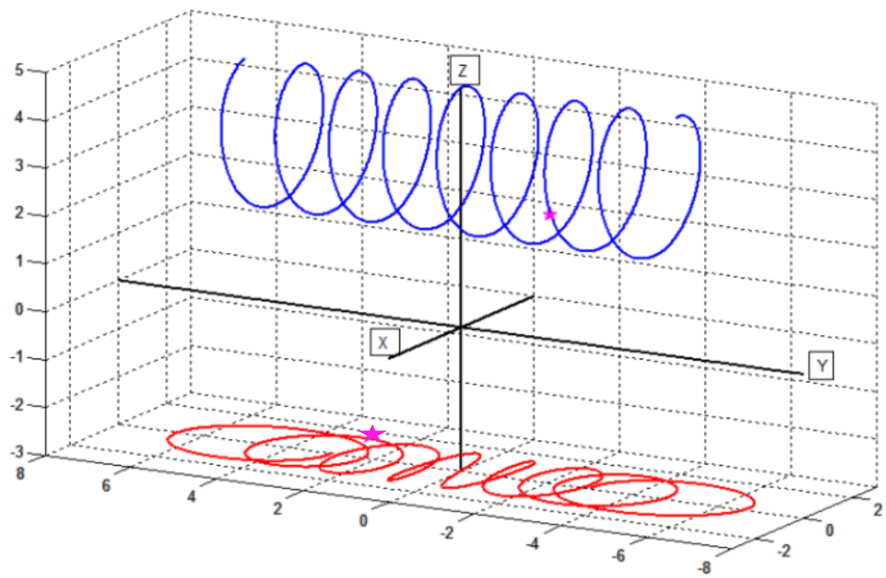
$$y_i = \frac{f}{z(t)} * y(t)$$

Dividing $\frac{x_i}{y_i}$ we get:

$$\frac{f * x(t)}{z(t)} * \frac{z(t)}{f * y(t)} = \frac{x(t)}{y(t)}$$

And we know that $a(t)$ is a straight line in 3D, so for every t, $\frac{x(t)}{y(t)} = m$ for some constant m which is the slope of $a(t)$**. So, the projection also has a constant slope, hence it is a straight line.**

Section B



1. The projection of the star is marked on the image plane as a pink star.
2. First, we have that:

$$a(0) = (0,0,5),$$
$$a(-8\pi) = \left(0, -\frac{8\pi}{5}, 5\right)$$
$$Z_{min} = 2, Z_{max} = 5$$

We can see that this trace is created by a circle which is continuously moved along the y axis, so the x and z coordinates will be some variation of cos and sin of some expression. We have

$$a(-8\pi) = \left(0, -\frac{8\pi}{5}, 5\right)$$

And so we know that when s = -8π, then x = 0.
We also know that when s = 0, then, x = 0. In order to get 8 full circles in the span of 16 π, we have to have the expression cos(s) and in order to adjust for the starting and 0 points, we add $\frac{\pi}{2}$ to the angle, leaving us with cos(s+$\frac{\pi}{2}$). The final adjustment is for the radius which is 1.5 (we have $Z_{min} = 2, Z_{max} = 5$), and so we get:
x = 1.5cos(s+$\frac{\pi}{2}$).

Y changes constantly and ranges from $-\frac{8\pi}{5}$ to $\frac{8\pi}{5}$, so we get:
y = $\frac{s}{5}$.

The definition of z comes similarly to x; the only difference is that the circle radius is at a distance of 3.5 from the XY plane, because $Z_{min} = 2, Z_{max} = 5$, and so we get:
z = 3.5 +1.5sin(s+$\frac{\pi}{2}$).

3. Ϭ(s) is the perspective projection of α(s), so for every point p we will define its projected point $p_i = \left(\frac{p_x * f}{p_z}, \frac{p_y * f}{p_z}\right)$, so we substitute the terms and get:

$$p_i = \left(\frac{6\cos\left(s + \frac{\pi}{2}\right)}{3.5 + 1.5\sin\left(s + \frac{\pi}{2}\right)}, \frac{3s}{3.5 + 1.5\sin\left(s + \frac{\pi}{2}\right)}\right)$$