Introduction to Computational and Biological Vision - Assignment 2

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Question 1 - Edge Detection

Section A

Given $I\left(x,y\right)=x+siny$
And the following Edge Detectors:
E_1 = edge dector based on the local maximum of $\left|\nabla I\right|$
E_2 = Edge detector based on finding the ZC of the Laplacian ΔI
Compute analytically the edge points detected by each of the edge detectors:
For E1:
first eval $\left|\nabla I\right|$:
$\left|\nabla I\right|=\left(\frac{\partial I}{\partial x},\frac{\partial I}{\partial y}\right)=\left\Vert (1,cosy)\right\Vert =\sqrt{1^{2}+cos^{2}(y)}$
we got an expression that independent of x , so we will assume that any x will work, and find the extramum for y:

$$\begin{aligned}
\frac{\partial}{\partial y}\left|\nabla I\right| = & \frac{\partial}{\partial y}\left(\sqrt{1^{2}+cos^{2}(y)}\right)=-\frac{\sin\left(2y\right)}{2\sqrt{1+\cos^{2}\left(y\right)}}\\
\frac{\partial}{\partial y}\left|\nabla I\right| = & 0\\
-\frac{\sin\left(2y\right)}{2\sqrt{1+\cos^{2}\left(y\right)}} = & 0\\
sin(2y) = & 0\\
y = & \frac{\pi}{2}+\pi k\ ,\pi k\ \ ,\forall k\in\mathbb{N}
\end{aligned}$$

and now for the second derviative:

$$\begin{aligned}
\frac{\partial^{2}}{\partial y^{2}}\left|\nabla I\right| = & -\frac{4\cos\left(2y\right)\left(1+\cos^{2}\left(y\right)\right)+\sin^{2}\left(2y\right)}{4\left(1+\cos^{2}\left(y\right)\right)\sqrt{1+\cos^{2}\left(y\right)}}\\
\left(\frac{\partial^{2}}{\partial y^{2}}\left|\nabla I\right|\right)_{y=\frac{\pi}{2}} = & -\frac{4\cos\left(2\left(\frac{\pi}{2}\right)\right)\left(1+\cos^{2}\left(\left(\frac{\pi}{2}\right)\right)\right)+\sin^{2}\left(2\left(\frac{\pi}{2}\right)\right)}{4\left(1+\cos^{2}\left(\left(\frac{\pi}{2}\right)\right)\right)\sqrt{1+\cos^{2}\left(\left(\frac{\pi}{2}\right)\right)}}\\
= & -\frac{4\cos\left(\pi\right)\left(1+\cos^{2}\left(\left(\frac{\pi}{2}\right)\right)\right)+\sin^{2}\left(\pi\right)}{4\left(1+\cos^{2}\left(\left(\frac{\pi}{2}\right)\right)\right)\sqrt{1+\cos^{2}\left(\left(\frac{\pi}{2}\right)\right)}}\\
= & 1\\
\left(\frac{\partial^{2}}{\partial y^{2}}\left|\nabla I\right|\right)_{y=\pi} = & -\frac{4\cos\left(2\left(\pi\right)\right)\left(1+\cos^{2}\left(\left(\pi\right)\right)\right)+\sin^{2}\left(2\left(\pi\right)\right)}{4\left(1+\cos^{2}\left(\left(\pi\right)\right)\right)\sqrt{1+\cos^{2}\left(\left(\pi\right)\right)}}\\
= & -\frac{\sqrt{2}}{2}
\end{aligned}$$

for the first assignment of y we got positive value, so its not an extramum point.

so we got that $(x, \pi k) \quad \forall k \in \mathbb{N}$ is our extramum points.
for E_2:
lets calc the laplacian:

$$\begin{aligned}
\Delta I =& \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} \\
=& \frac{\partial^2}{\partial x^2}(x + siny) + \frac{\partial^2}{\partial y^2}(x + siny) \\
=& \frac{\partial}{\partial x}(1) + \frac{\partial}{\partial y}(cos(y)) \\
=& 0 - sin(y) \\
=& sin(y)
\end{aligned}$$

lets find all the points where $\Delta I = 0$:

$$\begin{aligned}
\Delta I =& 0 \\
sin(y) =& 0 \\
y =& \pi k \quad \forall k \in \mathbb{N}
\end{aligned}$$

so our edge points according to E_2 are: $(x, \pi k) \quad \forall k \in \mathbb{N}$.
The result are identical, but its clearly that the computations of E_2 are much more simpler.