Introduction to Computational and Biological Vision - Assignment 2

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Question 1 - Edge Detection

Section A

Given $I\left(x,y\right)=x+siny$
And the following Edge Detectors:
$E_1=$ edge dector based on the local maximum of $\left|\nabla I\right|$
$E_2=$ Edge detector based on finding the ZC of the Laplacian ΔI
Compute analytically the edge points detected by each of the edge detectors:
For E1:
first eval $\left|\nabla I\right|$:
$\left|\nabla I\right|=\left(\frac{\partial I}{\partial x},\frac{\partial I}{\partial y}\right)=\left\Vert (1,cosy)\right\Vert =\sqrt{1^2+cos^2(y)}$
we got an expression that independent of x , so we will assume that any x will work, and find the extramum for y:

$$\begin{aligned}
\frac{\partial}{\partial y}\left|\nabla I\right| =&\frac{\partial}{\partial y}\left(\sqrt{1^2+cos^2(y)}\right)=-\frac{\sin\left(2y\right)}{2\sqrt{1+\cos^2\left(y\right)}}\\
\frac{\partial}{\partial y}\left|\nabla I\right| =&0\\
-\frac{\sin\left(2y\right)}{2\sqrt{1+\cos^2\left(y\right)}} =&0\\
sin(2y) =&0\\
y =&\frac{\pi}{2}+\pi k\ ,\pi k\ \ ,\forall k\in\mathbb{N}
\end{aligned}$$

and now for the second derviative:

$$\begin{aligned}
\frac{\partial^2}{\partial y^2}\left|\nabla I\right| =& -\frac{4\cos\left(2y\right)\left(1+\cos^2\left(y\right)\right)+\sin^2\left(2y\right)}{4\left(1+\cos^2\left(y\right)\right)\sqrt{1+\cos^2\left(y\right)}}\\
\left(\frac{\partial^2}{\partial y^2}\left|\nabla I\right|\right)_{y=\frac{\pi}{2}} =& -\frac{4\cos\left(2\left(\frac{\pi}{2}\right)\right)\left(1+\cos^2\left(\left(\frac{\pi}{2}\right)\right)\right)+\sin^2\left(2\left(\frac{\pi}{2}\right)\right)}{4\left(1+\cos^2\left(\left(\frac{\pi}{2}\right)\right)\right)\sqrt{1+\cos^2\left(\left(\frac{\pi}{2}\right)\right)}}\\
=& -\frac{4\cos\left(\pi\right)\left(1+\cos^2\left(\left(\frac{\pi}{2}\right)\right)\right)+\sin^2\left(\pi\right)}{4\left(1+\cos^2\left(\left(\frac{\pi}{2}\right)\right)\right)\sqrt{1+\cos^2\left(\left(\frac{\pi}{2}\right)\right)}}\\
=&1\\
\left(\frac{\partial^2}{\partial y^2}\left|\nabla I\right|\right)_{y=\pi} =& -\frac{4\cos\left(2\left(\pi\right)\right)\left(1+\cos^2\left(\left(\pi\right)\right)\right)+\sin^2\left(2\left(\pi\right)\right)}{4\left(1+\cos^2\left(\left(\pi\right)\right)\right)\sqrt{1+\cos^2\left(\left(\pi\right)\right)}}\\
=& -\frac{\sqrt{2}}{2}
\end{aligned}$$

for the first assignment of y we got positive value, so its not an extramum point.

so we got that $(x, \pi k) \quad \forall k \in \mathbb{N}$ is our extramum points.
for E_2:
lets calc the laplacian:

$$\begin{aligned}
\Delta I =& \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} \\
=& \frac{\partial^2}{\partial x^2}(x + siny) + \frac{\partial^2}{\partial y^2}(x + siny) \\
=& \frac{\partial}{\partial x}(1) + \frac{\partial}{\partial y}(cos(y)) \\
=& 0 - sin(y) \\
=& sin(y)
\end{aligned}$$

lets find all the points where $\Delta I = 0$:

$$\begin{aligned}
\Delta I =& 0 \\
sin(y) =& 0 \\
y =& \pi k \quad \forall k \in \mathbb{N}
\end{aligned}$$

so our edge points according to E_2 are: $(x, \pi k) \quad \forall k \in \mathbb{N}$.
The result are identical, but its clearly that the computations of E_2 are much more simpler.

Question 2 - Parametric Curves

Section A

The parametric equation of a straight line in 3D is $a(t) = (x(t),\ y(t),\ z(t))$.

The parametric equation of a straight line in 2D is $a_{2d}(t) = (x(t),\ y(t))$.

Without loss of generality, we assume that the center of projection is at the origin.

Upon projection onto the plane $z = f$, the image of the point $(x_0,\ y_0,\ z_0)$ is:

$$\begin{aligned}
x_i =& \frac{f}{z(t)} \cdot x(t) \\
y_i =& \frac{f}{z(t)} \cdot y(t)
\end{aligned}$$

Dividing both equation will evaluate: $\frac{x_i}{y_i}$ we get: $\frac{f \cdot x(t)}{z(t)} \cdot \frac{z(t)}{f \cdot y(t)} = \frac{x(t)}{y(t)}$

And we know that $a(t)$ is a straight line in 3D, so for every t, $\frac{x(t)}{y(t)} = m$ for some constant m which is the slope of $a(t)$. So, the **projection also has a constant slope, hence it is a straight line.**

Section B