Introduction to Computational and Biological Vision - Assignment 2

Brahan Wassan, Dan Zlotnikov

Question 1 - Edge Detection

Section A

Given $I(x, y) = x + siny$
And the following Edge Detectors:
E_1 = edge dector based on the local maximum of $|\nabla I|$
E_2 = Edge detector based on finding the ZC of the Laplacian ΔI
Compute analytically the edge points detected by each of the edge detectors:
For E1:
first eval $|\nabla I|$:
$|\nabla I| = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) = \|(1, cosy)\| = \sqrt{1^2 + cos^2(y)}$
we got an expression that independent of x , so we will assume that any x will work, and find the extramum for y:

$$\begin{aligned}
\frac{\partial}{\partial y}|\nabla I| =& \frac{\partial}{\partial y}\left(\sqrt{1^2 + cos^2(y)}\right) = -\frac{\sin(2y)}{2\sqrt{1 + \cos^2(y)}} \\
\frac{\partial}{\partial y}|\nabla I| =& 0 \\
-\frac{\sin(2y)}{2\sqrt{1 + \cos^2(y)}} =& 0 \\
sin(2y) =& 0 \\
y =& \frac{\pi}{2} + \pi k\ , \pi k\quad , \forall k \in \mathbb{N}
\end{aligned}$$

and now for the second derviative:

$$\begin{aligned}
\frac{\partial^2}{\partial y^2}|\nabla I| &= -\frac{4\cos(2y)\left(1 + \cos^2(y)\right) + \sin^2(2y)}{4\left(1 + \cos^2(y)\right)\sqrt{1 + \cos^2(y)}} \\
\left(\frac{\partial^2}{\partial y^2}|\nabla I|\right)_{y=\frac{\pi}{2}} &= -\frac{4\cos\left(2\left(\frac{\pi}{2}\right)\right)\left(1 + \cos^2\left(\left(\frac{\pi}{2}\right)\right)\right) + \sin^2\left(2\left(\frac{\pi}{2}\right)\right)}{4\left(1 + \cos^2\left(\left(\frac{\pi}{2}\right)\right)\right)\sqrt{1 + \cos^2\left(\left(\frac{\pi}{2}\right)\right)}} \\
&= -\frac{4\cos(\pi)\left(1 + \cos^2\left(\left(\frac{\pi}{2}\right)\right)\right) + \sin^2(\pi)}{4\left(1 + \cos^2\left(\left(\frac{\pi}{2}\right)\right)\right)\sqrt{1 + \cos^2\left(\left(\frac{\pi}{2}\right)\right)}} \\
&= 1 \\
\left(\frac{\partial^2}{\partial y^2}|\nabla I|\right)_{y=\pi} &= -\frac{4\cos(2(\pi))\left(1 + \cos^2((\pi))\right) + \sin^2(2(\pi))}{4\left(1 + \cos^2((\pi))\right)\sqrt{1 + \cos^2((\pi))}} \\
&= -\frac{\sqrt{2}}{2}
\end{aligned}$$

for the first assignment of y we got positive value, so its not an extramum point.

so we got that $(x, \pi k) \quad \forall k \in \mathbb{N}$ is our extramum points.
for E_2:
lets calc the laplacian:

$$\begin{aligned}
\Delta I =& \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} \\
=& \frac{\partial^2}{\partial x^2}(x + siny) + \frac{\partial^2}{\partial y^2}(x + siny) \\
=& \frac{\partial}{\partial x}(1) + \frac{\partial}{\partial y}(cos(y)) \\
=& 0 - sin(y) \\
=& sin(y)
\end{aligned}$$

lets find all the points where $\Delta I = 0$:

$$\begin{aligned}
\Delta I =& 0 \\
sin(y) =& 0 \\
y =& \pi k \quad \forall k \in \mathbb{N}
\end{aligned}$$

so our edge points according to E_2 are: $(x, \pi k) \quad \forall k \in \mathbb{N}$.
The result are identical, but its clearly that the computations of E_2 are much more simpler.

Question 2 - Parametric Curves

Section A

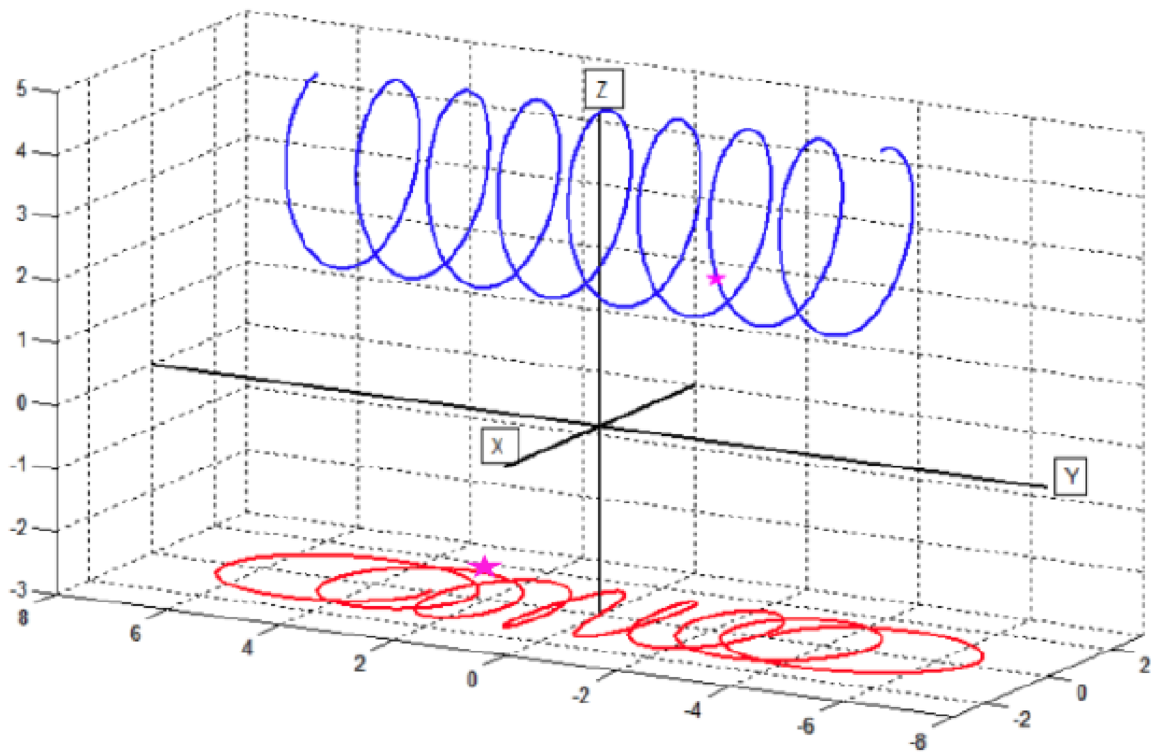
The parametric equation of a straight line in 3D is $a(t) = (x(t), \ y(t), \ z(t))$.
The parametric equation of a straight line in 2D is $a_{2d}(t) = (x(t), \ y(t))$.
Without loss of generality, we assume that the center of projection is at the origin.
Upon projection onto the plane $z = f$, the image of the point $(x_0, \ y_0, \ z_0)$ is:

$$\begin{aligned}
x_i =& \frac{f}{z(t)} \cdot x(t) \\
y_i =& \frac{f}{z(t)} \cdot y(t)
\end{aligned}$$

Dividing both equation will evaluate: $\frac{x_i}{y_i}$ we get: $\frac{f \cdot x(t)}{z(t)} \cdot \frac{z(t)}{f \cdot y(t)} = \frac{x(t)}{y(t)}$
And we know that $a(t)$ is a straight line in 3D, so for every t, $\frac{x(t)}{y(t)} = m$ for some constant m which is the slope of $a(t)$. So, the **projection also has a constant slope, hence it is a straight line.**

Section B

add picture:



1. Explantion: lets assume our point is $(-2, -1, 2.5)$ now we will compute the presepctive projection of the point.

$$u = \frac{-3 * -2}{2.5} + 0 = 2.4$$
$$v = \frac{-3 * -1}{2.5} + 0 = 1.2$$

so the projected point on the image plane is $(2.4, 1.2)$