Introduction to Computational and Biological Vision - Assignment 2

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Question 1 - Edge Detection

Section A

Given $I(x,y) = x + sin y$

And the following Edge Detectors:

$E_1 =$ edge dector based on the local maximum of $|\nabla I|$

$E_2 =$ Edge detector based on finding the ZC of the Laplacian ΔI

Compute analytically the edge points detected by each of the edge detectors:

For E1:

first eval $|\nabla I|$:

$|\nabla I| = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) = \|(1, cos y)\| = \sqrt{1^2 + cos^2(y)}$

we got an expression that independent of x , so we will assume that any x will work, and find the extramum for y:

$$\frac{\partial}{\partial y}|\nabla I| = \frac{\partial}{\partial y}\left(\sqrt{1^2 + cos^2(y)}\right) = -\frac{\sin(2y)}{2\sqrt{1 + \cos^2(y)}}$$

$$\frac{\partial}{\partial y}|\nabla I| = 0$$

$$-\frac{\sin(2y)}{2\sqrt{1 + \cos^2(y)}} = 0$$

$$sin(2y) = 0$$

$$y = \frac{\pi}{2} + \pi k \ , \pi k \ \ , \forall k \in \mathbb{N}$$

and now for the second derviative:

$$\frac{\partial^2}{\partial y^2}|\nabla I| = -\frac{4\cos(2y)\left(1 + \cos^2(y)\right) + \sin^2(2y)}{4\left(1 + \cos^2(y)\right)\sqrt{1 + \cos^2(y)}}$$

$$\left(\frac{\partial^2}{\partial y^2}|\nabla I|\right)_{y=\frac{\pi}{2}} = -\frac{4\cos\left(2\left(\frac{\pi}{2}\right)\right)\left(1 + \cos^2\left(\left(\frac{\pi}{2}\right)\right)\right) + \sin^2\left(2\left(\frac{\pi}{2}\right)\right)}{4\left(1 + \cos^2\left(\left(\frac{\pi}{2}\right)\right)\right)\sqrt{1 + \cos^2\left(\left(\frac{\pi}{2}\right)\right)}}$$

$$= -\frac{4\cos(\pi)\left(1 + \cos^2\left(\left(\frac{\pi}{2}\right)\right)\right) + \sin^2(\pi)}{4\left(1 + \cos^2\left(\left(\frac{\pi}{2}\right)\right)\right)\sqrt{1 + \cos^2\left(\left(\frac{\pi}{2}\right)\right)}}$$

$$= 1$$

$$\left(\frac{\partial^2}{\partial y^2}|\nabla I|\right)_{y=\pi} = -\frac{4\cos(2(\pi))\left(1 + \cos^2((\pi))\right) + \sin^2(2(\pi))}{4\left(1 + \cos^2((\pi))\right)\sqrt{1 + \cos^2((\pi))}}$$

$$= -\frac{\sqrt{2}}{2}$$

for the first assignment of y we got positive value, so its not an extramum point.

so we got that $(x, \pi k)$ $\forall k \in \mathbb{N}$ is our extramum points.

for E_2:

lets calc the laplacian:

$$
\begin{aligned}
\Delta I =& \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} \\
=& \frac{\partial^2}{\partial x^2}\left(x + siny\right) + \frac{\partial^2}{\partial y^2}\left(x + siny\right) \\
=& \frac{\partial}{\partial x}\left(1\right) + \frac{\partial}{\partial y}\left(cos(y)\right) \\
=& 0 - sin(y) \\
=& sin(y)
\end{aligned}
$$

lets find all the points where $\Delta I = 0$:

$$
\begin{aligned}
\Delta I =& 0 \\
sin(y) =& 0 \\
y =& \pi k \quad \forall k \in \mathbb{N}
\end{aligned}
$$

so our edge points according to E_2 are: $(x, \pi k)$ $\forall k \in \mathbb{N}$.

The result are identical, but its clearly that the computations of E_2 are much more simpler.

Question 2 - Parametric Curves

Section A

The parametric equation of a straight line in 3D is $a(t) = (x(t),\ y(t),\ z(t))$.

The parametric equation of a straight line in 2D is $a_{2d}(t) = (x(t),\ y(t))$.

Without loss of generality, we assume that the center of projection is at the origin.

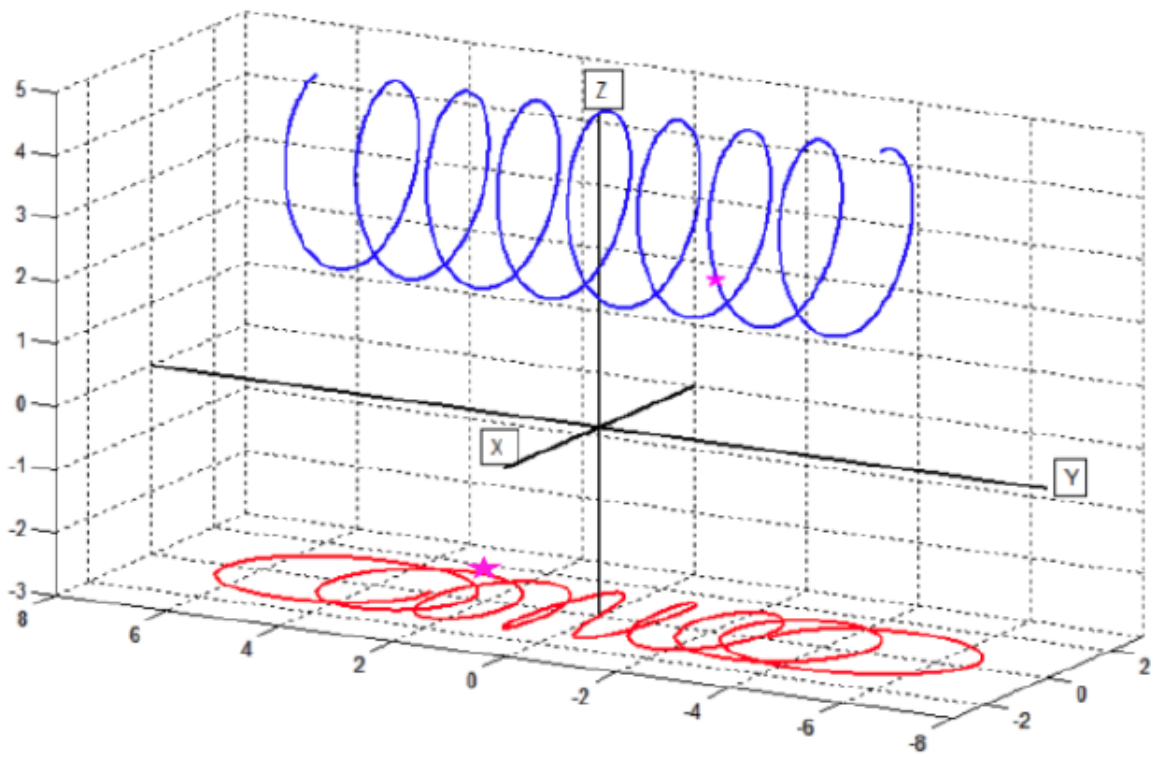Upon projection onto the plane $z = f$, the image of the point $(x_0,\ y_0,\ z_0)$ is:

$$
\begin{aligned}
x_i =& \frac{f}{z(t)} \cdot x(t) \\
y_i =& \frac{f}{z(t)} \cdot y(t)
\end{aligned}
$$

Dividing both equation will evaluate: $\frac{x_i}{y_i}$ we get: $\frac{f \cdot x(t)}{z(t)} \cdot \frac{z(t)}{f \cdot y(t)} = \frac{x(t)}{y(t)}$

And we know that $a(t)$ is a straight line in 3D, so for every t, $\frac{x(t)}{y(t)} = m$ for some constant m which is the slope of $a(t)$. So, the **projection also has a constant slope, hence it is a straight line.**

Section B

add picture:

1. lets assume our point is $(-2, -1, 2.5)$ now we will compute the presepctive projection of the point.

$$u = \frac{3 * -2}{2.5} + 0 = -2.4$$
$$v = \frac{3 * -1}{2.5} + 0 = -1.2$$