Question 2 – Parametric Curves

Section A

The parametric equation of a straight line in 3D is $a(t) = (x(t), y(t), z(t))$.
The parametric equation of a straight line in 2D is $a_{2d}(t) = (x(t), y(t))$.

Without loss of generality, we assume that the center of projection is at the origin. Upon projection onto the plane z = f, the image of the point (x_0, y_0, z_0) is:

$$x_i = \frac{f}{z(t)} * x(t)$$

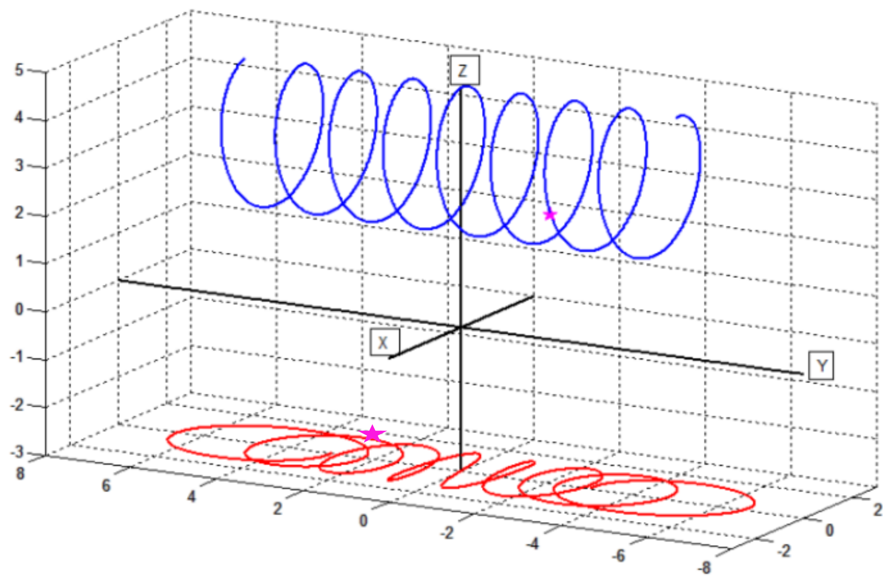
$$y_i = \frac{f}{z(t)} * y(t)$$

Dividing $\frac{x_i}{y_i}$ we get:

$$\frac{f * x(t)}{z(t)} * \frac{z(t)}{f * y(t)} = \frac{x(t)}{y(t)}$$

And we know that $a(t)$ is a straight line in 3D, so for every t, $\frac{x(t)}{y(t)} = m$ for some constant m which is the slope of $a(t)$**. So, the projection also has a constant slope, hence it is a straight line.**

Section B



1. The projection of the star is marked on the image plane as a pink star.
2. First, we have that:

$$a(0) = (0,0,5),$$
$$a(-8\pi) = \left(0, -\frac{8\pi}{5}, 5\right)$$
$$Z_{min} = 2, Z_{max} = 5$$

We can see that this trace is created by a circle which is continuously moved along the y axis, so the x and z coordinates will be some variation of cos and sin of some expression. We have

$$a(-8\pi) = \left(0, -\frac{8\pi}{5}, 5\right)$$

And so we know that when s = -8π, then x = 0.
We also know that when s = 0, then, x = 0. In order to get 8 full circles in the span of 16 π, we have to have the expression cos(s) and in order to adjust for the starting and 0 points, we add $\frac{\pi}{2}$ to the angle, leaving us with cos(s+$\frac{\pi}{2}$). The final adjustment is for the radius which is 1.5 (we have $Z_{min} = 2, Z_{max} = 5$), and so we get:
x = 1.5cos(s+$\frac{\pi}{2}$).

Y changes constantly and ranges from $-\frac{8\pi}{5}$ to $\frac{8\pi}{5}$, so we get:
y = $\frac{s}{5}$.

The definition of z comes similarly to x; the only difference is that the circle radius is at a distance of 3.5 from the XY plane, because $Z_{min} = 2, Z_{max} = 5$, and so we get:
z = 3.5 +1.5sin(s+$\frac{\pi}{2}$).

3. ϭ(s) is the perspective projection of α(s), so for every point p we will define its projected point $p_i = \left(\frac{p_x * f}{p_z}, \frac{p_y * f}{p_z}\right)$, so we substitute the terms and get:

$$p_i = \left(\frac{6\cos\left(s+\frac{\pi}{2}\right)}{3.5+1.5\sin\left(s+\frac{\pi}{2}\right)}, \frac{3s}{3.5+1.5\sin\left(s+\frac{\pi}{2}\right)}\right)$$