Quick note about Jupyter cells

When you are editing a cell in Jupyter notebook, you need to re-run the cell by pressing `<Shift> + <Enter>`. This will allow changes you made to be available to other cells.

Use `<Enter>` to make new lines inside a cell you are editing.

Code cells

Re-running will execute any statements you have written. To edit an existing code cell, click on it.

Markdown cells

Re-running will render the markdown text. To edit an existing markdown cell, double-click on it.

Common Jupyter operations

Near the top of the https://try.jupyter.org (https://try.jupyter.org) page, Jupyter provides a row of menu options (`File` , `Edit` , `View` , `Insert` , ...) and a row of tool bar icons (disk, plus sign, scissors, 2 files, clipboard and file, up arrow, ...).

Inserting and removing cells

- Use the "plus sign" icon to insert a cell below the currently selected cell
- Use "Insert" -> "Insert Cell Above" from the menu to insert above

Clear the output of all cells

- Use "Kernel" -> "Restart" from the menu to restart the kernel
    - click on "clear all outputs & restart" to have all the output cleared

Save your notebook file locally

- Clear the output of all cells
- Use "File" -> "Download as" -> "IPython Notebook (.ipynb)" to download a notebook file representing your https://try.jupyter.org (https://try.jupyter.org) session

Load your notebook file in try.jupyter.org

1. Visit https://try.jupyter.org (https://try.jupyter.org)
2. Click the "Upload" button near the upper right corner
3. Navigate your filesystem to find your `*.ipynb` file and click "open"
4. Click the new "upload" button that appears next to your file name
5. Click on your uploaded notebook file

References

- https://try.jupyter.org (https://try.jupyter.org)
- https://docs.python.org/3/tutorial/index.html (https://docs.python.org/3/tutorial/index.html)
- https://docs.python.org/3/tutorial/introduction.html (https://docs.python.org/3/tutorial/introduction.html)
- https://daringfireball.net/projects/markdown/syntax (https://daringfireball.net/projects/markdown/syntax)

Chapter 2 - Introduction to Python

We will be using the Jupyter notebook for many activities this semester. Every notebook has an associated language called the "kernel". We will be using in the Python 3 kernel from the IPython project.

For more information on how to use the notebook, please read the following (which is also a notebook written by a Bryn Mawr student):

- https://athena.brynmawr.edu/jupyter/hub/dblank/public/Jupyter%20Notebook%20Users%20Manual.ipynb (https://athena.brynmawr.edu/jupyter/hub/dblank/public/Jupyter%20Notebook%20Users%20Manual.ipynb)

1. Python

Python is a programming language that has been under development for over 25 years [1].

This Chapter will not cover everything in Python. If you would like, please consider the following resources:

Getting Started with Python:

- https://www.codecademy.com/learn/python (https://www.codecademy.com/learn/python)
- http://docs.python-guide.org/en/latest/intro/learning/ (http://docs.python-guide.org/en/latest/intro/learning/)
- https://learnpythonthehardway.org/book/ (https://learnpythonthehardway.org/book/)
- https://www.codementor.io/learn-python-online (https://www.codementor.io/learn-python-online)

Learning Python in Notebooks:

- http://mbakker7.github.io/exploratory_computing_with_python/ (http://mbakker7.github.io/exploratory_computing_with_python/)

This is handy to always have available for reference:

Python Reference:

- https://docs.python.org/3.5/reference/ (https://docs.python.org/3.5/reference/)

1.1 Statements

Python is an imperative language (https://en.wikipedia.org/wiki/Imperative_programming) based on statements (https://en.wikipedia.org/wiki/Statement_(computer_science)). That is, programs in Python consists of lines composed of statements. A statement can be:

- a single expression
- an assignment
- a function call
- a function definition
- a statement; statement

1.1.1 Expressions

Everything in Python is an **object** and every object in Python has a **type**. Some of the basic types include:

- Numbers
  - integers
  - floating-point
  - complex numbers
- strings
- boolean values
- Nonetype
- base containers

1.1.1.1 Numbers

`int` (integer; a whole number with no decimal place)

- `10`
- `-3`
  - `float` (float; a number that has a decimal place)
- `7.41`
- `-0.006`

In [4]:
```
7//3
```
Out[4]: 2

In [ ]:
```
2
```

In [ ]:
```
-3
```

In [5]:
```
3.14
```
Out[5]:
```
3.14
```

1.1.1.2 Strings

- `str` (string; a sequence of characters enclosed in single quotes, double quotes, or triple quotes)
  - `'this is a string using single quotes'`
  - `"this is a string using double quotes"`
  - `'''this is a triple quoted string using single quotes'''`
  - `"""this is a triple quoted string using double quotes"""`

In [11]:
```
"apple"
```
Out[11]:
```
'apple'
```

In [ ]:
```
'apple'
```

Notice that the Out might not match exactly the In. In the above example, we used double-quotes but the representation of the string used single-quotes. Python will default to showing representations of values using single-quotes, if it can.

In [12]:
```
'"asdas'
"'asdas"
```
Out[12]:
```
"'asdas"
```

You can use single-quotes inside double-quotes and vise versa.

1.1.1.3 Boolean Values

`bool` (boolean; a binary value that is either true or false)

- `True`
- `False`

In [13]:
```
True
```
Out[13]:
```
True
```

In [14]:
```
False
```
Out[14]:
```
False
```

Basic operators

In Python, there are different types of **operators** (special symbols) that operate on different values. Some of the basic operators include:

- arithmetic operators
  - `+` (addition)
  - `-` (subtraction)
  - `*` (multiplication)
  - `/` (division)
  - `**` (exponent)
  - `%` (modulus)
- assignment operators
  - `=` (assign a value)
  - `+=` (add and re-assign; increment)
  - `-=` (subtract and re-assign; decrement)
  - `*=` (multiply and re-assign)
- comparison operators (return either `True` or `False`)
  - `==` (equal to)
  - `!=` (not equal to)
  - `<` (less than)
  - `<=` (less than or equal to)
  - `>` (greater than)
  - `>=` (greater than or equal to)

When multiple operators are used in a single expression, **operator precedence** determines which parts of the expression are evaluated in which order. Operators with higher precedence are evaluated first (like PEMDAS in math). Operators with the same precedence are evaluated from left to right.

- `()` parentheses, for grouping
- `**` exponent
- `*` , `/` multiplication and division
- `+` , `-` addition and subtraction
- `==` , `!=` , `<` , `<=` , `>` , `>=` comparisons

> See https://docs.python.org/3/reference/expressions.html#operator-precedence (https://docs.python.org/3/reference/expressions.html#operator-precedence)

In [26]:
```python
# Assigning some numbers to different variables
num1 = 10
num2 = -3
num3 = 7.41
num4 = -.6
num5 = 7
num6 = 3
num7 = 11.11
```

In [19]:
```
num4=-0.6
```

In [20]:
```
# Addition
num1 + num2
```

Out[20]: 7

In [21]:
```
# Subtraction
num2 - num3
```

Out[21]: -10.41

In [22]:
```
# Multiplication
num3 * num4
```

Out[22]: -4.446

In [24]:
```
# Division
num4 // num5
```

Out[24]: -1.0

In [25]:
```
# Exponent
num5 ** num6
```

Out[25]: 343

In [30]:
```
#bit wise XOR
num5^num6
```

Out[30]: 4

In [29]:
```
# Modulus
num1%num6
```

Out[29]: 1

In [31]:
```
# Increment existing variable
num7 += 4
num7
```

Out[31]: 15.11

In [32]:
```
# Decrement existing variable
num6 -= 2
# num6 = num6-2
num6
```

Out[32]: 1

In [ ]:
```
# Multiply & re-assign
num3 *= 5
num3
```

In [34]:
```
# Assign the value of an expression to a variable
num8 = num1 + (num2 * num3)
num8
```

Out[34]: -12.23

Exercise 1, First Python code

Compute the value of the polynomial $y = ax^2 + bx + c$ at $x = -2$, $x = 0$, and $x = 2$ using $a = 1$, $b = 1$, $c = -6$.

In [ ]:
```
```

Answer for Exercise 1

In [36]:
```
# Are these two expressions equal to each other?
num1 + num2 == num5
```

Out[36]: True

In [37]:
```
# Are these two expressions not equal to each other?
num3 != num4
```

Out[37]: True

In [38]:
```
# Is the first expression less than the second expression?
num5 < num6
```

Out[38]: False

In [40]:
```
# Is this expression True?
5 > 3 > 1
```

Out[40]: True

In [42]:
```
# Is this expression True?
5 > 3 > 4 == 3 + 1
```

Out[42]: False

In [45]:
```
# Assign some strings to different variables
simple_string1 = 'an example'
simple_string2 = "oranges "
```

In [48]:
```
# Addition
simple_string3=simple_string1 + ' of using the + operator'
```

In [49]:
```
simple_string3
```
Out[49]: 'an example of using the + operator'

In [47]:
```
# Notice that the string was not modified
simple_string1
```
Out[47]: 'an example'

In [50]:
```
# Multiplication
simple_string2 * 4
```
Out[50]: 'oranges oranges oranges oranges '

In [51]:
```
# This string wasn't modified either
simple_string2
```
Out[51]: 'oranges '

In [52]:
```
# Are these two expressions equal to each other?
simple_string1 == simple_string2
```
Out[52]: False

In [53]:
```
'a'=='a'
```
Out[53]: True

In [ ]:
```

```

In [54]:
```
# Are these two expressions equal to each other?
simple_string1 == 'an example'
```
Out[54]: True

In [55]:
```
# Add and re-assign
simple_string1 += ' that re-assigned the original string'
simple_string1
```
Out[55]: 'an example that re-assigned the original string'

In [56]:
```
# Multiply and re-assign
simple_string2 *= 3
simple_string2
```
Out[56]: 'oranges oranges oranges '

Note: Subtraction, division, and decrement operators do not apply to strings.

1.1.1.4 Special Values

NoneType (a special type representing the absence of a value)

- None

In [ ]:
```
None
```

1.1.1.5 Lists and Dicts

> Note: **mutable** objects can be modified after creation and **immutable** objects cannot.

Containers are objects that can be used to group other objects together. The basic container types include:

- **str** (string: immutable; indexed by integers; items are stored in the order they were added)
- **list** (list: mutable; indexed by integers; items are stored in the order they were added)
  - `[3, 5, 6, 3, 'dog', 'cat', False]`
- **tuple** (tuple: immutable; indexed by integers; items are stored in the order they were added)
  - `(3, 5, 6, 3, 'dog', 'cat', False)`
- **set** (set: mutable; not indexed at all; items are NOT stored in the order they were added; can only contain immutable objects; does NOT contain duplicate objects)
  - `{3, 5, 6, 3, 'dog', 'cat', False}`
- **dict** (dictionary: mutable; key-value pairs are indexed by immutable keys; items are NOT stored in the order they were added)
  - `{'name': 'Jane', 'age': 23, 'fav_foods': ['pizza', 'fruit', 'fish']}`

When defining lists, tuples, or sets, use commas (,) to separate the individual items. When defining dicts, use a colon (:) to separate keys from values and commas (,) to separate the key-value pairs.

Strings, lists, and tuples are all **sequence types** that can use the `+`, `*`, `+=`, and `*=` operators.

In [25]:
```
# Assign some containers to different variables
list1 = [3, 5, 6, 3, 'dog', 'cat', False]
tuple1 = (3, 5, 6, 3, 'dog', 'cat', False)
set1 = {3, 5, 6, 3, 'dog', 'cat', False}
dict1 = {'name': 'Jane', 'age': 23, 'fav_foods': ['pizza', 'fruit', 'fish']}
```

In [59]:
```
# Items in the list object are stored in the order they were added
list1
```

Out[59]:
```
[3, 5, 6, 3, 'dog', 'cat', False]
```

In [60]:
```
# Items in the tuple object are stored in the order they were added
tuple1
```

Out[60]:
```
(3, 5, 6, 3, 'dog', 'cat', False)
```

In [58]:
```
# Items in the set object are not stored in the order they were added
# Also, notice that the value 3 only appears once in this set object
set1
```

Out[58]:
```
{3, 5, 6, False, 'cat', 'dog'}
```

In [61]:
```
# Items in the dict object are not stored in the order they were added
dict1
```

Out[61]:
```
{'name': 'Jane', 'age': 23, 'fav_foods': ['pizza', 'fruit', 'fish']}
```

In [63]:
```
# Add and re-assign
list1= [5,'grapes']+list1
list1
```

Out[63]:
```
[5, 'grapes', 3, 5, 6, 3, 'dog', 'cat', False, 5, 'grapes']
```

In [64]:
```
# Add and re-assign
tuple1 += (5, 'grapes')
tuple1
```

Out[64]:
```
(3, 5, 6, 3, 'dog', 'cat', False, 5, 'grapes')
```

In [70]:
```
# Multiply
[1, 2, 3, 4]*2
```

Out[70]:
```
[1, 2, 3, 4, 1, 2, 3, 4]
```

In [ ]:
```
# Multiply
(1, 2, 3, 4) * 3
```

Accessing data in containers

For strings, lists, tuples, and dicts, we can use **subscript notation** (square brackets) to access data at an index.

- strings, lists, and tuples are indexed by integers, **starting at 0** for first item
  - these sequence types also support accesing a range of items, known as **slicing**
  - use **negative indexing** to start at the back of the sequence
- dicts are indexed by their keys

> Note: sets are not indexed, so we cannot use subscript notation to access data elements.

In [71]:
```
# Access the first item in a sequence
list1[0]
```

Out[71]:
```
5
```

In [73]:
```
# Access the last item in a sequence
print(tuple1)
tuple1[-1]
```

```
(3, 5, 6, 3, 'dog', 'cat', False, 5, 'grapes')
```

Out[73]:
```
'grapes'
```

In [78]:
```
# Access a range of items in a sequence
simple_string1[3:8]
```

Out[78]:
```
'examp'
```

In [84]:
```
# Access a range of items in a sequence
print(tuple1)
tuple1[::2]
```

```
(3, 5, 6, 3, 'dog', 'cat', False, 5, 'grapes')
```

Out[84]:
```
(3, 6, 'dog', False, 'grapes')
```

In [85]:
```
# Access a range of items in a sequence
list1[4:]
```

Out[85]:
```
[6, 3, 'dog', 'cat', False, 5, 'grapes']
```

In [88]:
```
# Access an item in a dictionary
dict1['name']
```

Out[88]:
```
'Jane'
```

In [89]:
```
# Access an element of a sequence in a dictionary
print(dict1)
dict1['fav_foods'][2]
```

```
{'name': 'Jane', 'age': 23, 'fav_foods': ['pizza', 'fruit', 'fish']}
```

Out[89]:
```
'fish'
```

1.1.2 `for` loops and `if/else` statements

The `for` loop

Loops are used to execute a command repeatedly. The syntax for a loop is as follows

In [2]:
```
for i in [0, 1, 2, 3, 4]:
    print('Hello world', i)
print (i)
```

```
Hello world 0
Hello world 1
Hello world 2
Hello world 3
Hello world 4
4
```

In the code above, the variable `i` loops through the five values in the list `[0, 1, 2, 3, 4]`. The first time through, the value of `i` is equal to `0`, the second time through, its value is `1`, and so on till the last time when its value is `4`. Note the syntax of a `for` loop. At the end of the `for` statement you need to put a colon (`:`) and after that you need to indent. It doesn't matter how many spaces you indent, as long as you keep using the same number of spaces for the entire `for` loop. Jupyter Notebooks automatically indent 4 spaces, which is considered good Python style, so use that. You can have as many lines of code inside the `for` loop as you want. To end the `for` loop, simply stop indenting.

In [3]:
```
for x in [0, 1, 2, 3]:
    xsquared = x ** 2
    print('x, xsquare', x, xsquared)
print('We are done with the loop')
```

```
x, xsquare 0 0
x, xsquare 1 1
x, xsquare 2 4
x, xsquare 3 9
We are done with the loop
```

The list of values to loop through can be anything. It doesn't even have to be numbers. The `for` loop simply goes through all the values in the list one by one:

In [ ]:
```
for x in ["Golabi", 23, 3.14, 'ahmad', '0010']:
    print("The value of x is",x)
```

It is, of course, rather inconvenient to have to specify a list to loop through when the list is very long. For example, if you want to do something 100 times, you don't want to type a list of values from 0 up to 100. But Python has a convenient function for that called `range`. You can loop through a `range` just like you can loop through a list. To loop 10 times, starting with the value `0`:

In [5]:
```
for i in range(11):
    print('the value of i is:', i)
```

```
the value of i is: 0
the value of i is: 1
the value of i is: 2
the value of i is: 3
the value of i is: 4
the value of i is: 5
the value of i is: 6
the value of i is: 7
the value of i is: 8
the value of i is: 9
the value of i is: 10
```

A `range` can be converted to a list with the `list` function (but we will not use that option very often). You can call `range` with just one argument, in which case it will generate a range from 0 up to but not including the specified number. Note that `range(10)` produces 10 numbers from 0 up to and including 9. You can optionally give a starting value and a step, similar to the `np.arange` function.

In [7]:
```
type(range(9))
print (list(range(9)))
```

```
[0, 1, 2, 3, 4, 5, 6, 7, 8]
```

In [11]:
```
print('a range with 10 values:', list(range(10)))
print('a range from 10 till 20', list(range(10, 20)))
print('a range from 10 till 20 with steps of 2:', list(range(10, 20, 2)))
```

```
a range with 10 values: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
a range from 10 till 20 [10, 11, 12, 13, 14, 15, 16, 17, 18, 19]
a range from 10 till 20 with steps of 2: [10, 12, 14, 16, 18]
```

A loop can be used to fill an array. Let's compute $y = x^2$ where x is an array that varies from 0 to 10. Sometimes this is not possible, however, and we need to fill an array with a loop. First we have to create the array `y` and then fill it with the correct values by looping through all values of `x`, so that the index goes from `0` to the length of the `x` array. The counter in the loop (the variable `i` in the code below) is used as the index of the array that is filled.

In [13]:
```
for i in range(11):
    print(i**2, end=', ')
```

```
0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,
```

In [14]:
```
y=[x**2 for x in range(11)]
print(y)
```

```
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
```

Exercise 2. First `for` loop

Create a list with the names of the months. Create a second list with the number of days in each month (for a regular year). Create a `for` loop that prints:

`The number of days in MONTH is XX days`

where, of course, you print the correct name of the month for `MONTH` and the correct number of days for `XX` .

In [ ]:

```

```

Answer for Exercise 2

The `if` statement

An `if` statement lets you perform a task only when the outcome of the `if` statement is true. For example

In [15]:

```
data = 4
print('starting value:', data)
if data < 6:
    print('changing data in the first if-statement')
    data += 2
print('value after the first if-statement:', data)
if data > 20:
    print('changing data in the second if-statement')
    data = 200
print('value after the second if-statement:', data)  # data hasn't changed as
 data is not larger than 20
```

```
starting value: 4
changing data in the first if-statement
value after the first if-statement: 6
value after the second if-statement: 6
```

Notice the syntax of the `if` statement. It starts with `if` followed by a statement that is either `True` or `False` and then a colon. After the colon, you need to indent and the entire indented code block (in this case 2 lines of code) is executed if the statement is `True` . The `if` statement is completed when you stop indenting. Recall from Notebook 2 that you can use larger than `>` , larger than or equal `>=` , equal `==` , smaller than or equal `<=` , smaller than `<` or not equal `!=` .

The `if` / `else` statement

The `if` statement may be followed by an `else` statement, which is executed when the condition after `if` is `False` . For example

In [16]:
```
a = 4
if a < 3:
    print('a is smaller than 3')
else:
    print('a is not smaller than 3')
```

```
a is not smaller than 3
```

You can even extend the `else` by adding one or more conditions with the `elif` command which is short for 'else if'

In [20]:
```
a = 4
if a <= 4:
    print('a is smaller than 4')
if a==4:
    print('a is 4')

elif a > 4:
    print('a is larger than 4')
else:
    print('a is equal to 4')
print('end process')
```

```
a is smaller than 4
end process
```

Rather than specifying the value of a variable at the top of the code cell, you can ask the user to enter a value and store that value in the variable using the `input` function. The `input` function returns a string that can be converted into a number with the `float` function. Run the code cell below and test that it works when the entered value is larger than 4, smaller than 4, or equal to 4.

In [ ]:
```
for i in range(3):
    a = float(input('Enter a value: '))
    if a < 4:
        print('the entered value is smaller than 4')
    elif a > 4:
        print('the entered value is larger than 4')
    else:
        print('the entered value is equal to 4')
```

Exercise 3. Combination of `for` loop with `if` statement

Consider the function

$$y = x^2 \qquad \text{for} \qquad x < 0$$
$$y = -x^2 \qquad \text{for} \qquad x \geq 0$$

Compute y for x going from -20 to 20 with 40 points.

In [ ]:

```

```

Answer for Exercise 2

Looping and summation

One application of a loop is to compute the sum of all the values in an array. Consider, for example, the array `data` with 8 values. We will compute the sum of all values in `data` . We first define a variable `datasum` and assign it the initial value 0. Next, we loop through all the values in `data` and add the value to `datasum` :

In [21]:

```
data = [1, 3, 2, 5, 7, 3, 4, 2]
datasum = 0
for i in range(len(data)):
    datasum = datasum + data[i]
    # datasum+= data[i]
    print( i,', datasum: ', datasum)
print('total sum of data: ', datasum)
```

```
0 , datasum:  1
1 , datasum:  4
2 , datasum:  6
3 , datasum:  11
4 , datasum:  18
5 , datasum:  21
6 , datasum:  25
7 , datasum:  27
total sum of data:  27
```

Note that the statement

`datasum = datasum + data[i]`

means that `data[i]` is added to the current value of `datasum` and that the result is assigned to `datasum` . There is actually a shorter syntax for the same statement:

`datasum += data[i]`

The `+=` command means: add whatever is on the right side of the `+=` sign to whatever is on the left side. You can use whichever syntax you are most comfortable with (although `+=` is considered to be better and in some cases more efficient).

we could also compute this sum using sum function but we don't always have predefined functions to help us!

In [22]:

```
sum([1, 3, 2, 5, 7, 3, 4, 2])
```

Out[22]: 27

Exercise 4. Running total

For the data of the previous example, compute the running total and store it in an array using a loop. Hence, the result should be an array with the same length as `data` where item `i` is the sum of all values in the array `data` up to and including `data[i]` . Print both the array `data` and the array with the running total to the screen. Finally, check your answer by using the `cumsum` function of `numpy` , which should give the same answer as your loop.

In [ ]:

```

```

Answer for Exercise 4

Finding the maximum value the hard way

Next, let's find the maximum in the array `data` and the index of the maximum value. For illustration purposes, we will do this the hard way by using a loop and an if statement. First, we create a variable `maxvalue` that contains the maximum value and set it initially to a very small number, and we create a variable `maxindex` that is the index of the maximum value and is initially set to `None` . Then we loop through all values in `data` and update the `maxvalue` and `maxindex` everytime we find a larger value than the current `maxvalue`

In [39]:

```python
maxvalue = -1e8
maxindex = None
for i in range(len(data)):
    if data[i] > maxvalue:
        maxvalue = data[i]
        maxindex = i
print('the maximum value is ', maxvalue)
print('the index of the maximum value is ', maxindex)
```

```
the maximum value is  7
the index of the maximum value is  4
```

For this example, it is easy to check whether these numbers are correct by looking at the `data` array, but that becomes more difficult when the `data` array is large. There are, of course, functions available in the `numpy` package to find the maximum value and the index of the maximum value: `np.max` returns the maximum value of an array, and `np.argmax` returns the index of the maximum of the array. There are similar functions for the mimimum value.

In [40]:

```python
print('the maximum value is ', max(data))
print('the index of the maximum value is ', data.index(max(data)))
```

```
the maximum value is  7
the index of the maximum value is  4
```

Nested loops

It is also possible to have loops inside loops. These are called nested loops. For example, consider the array `data` with 3 rows and 4 columns shown below. We want to compute the sum of the values in each row (so we sum the columns) and we are going to do this using a double loop. First, we make an array of zeros called `rowtotal` of length 3 (one value for each row of the array `data`). Next, we loop through each row. For each row inside the loop, we start another loop that goes through all the columns and adds the value to the array `rowtotal` for that row.

In [42]:
```
data = [[1, 2, 3, 5],
                 [4, 8, 6, 4],
                 [3, 5, 4, 6]]
rowtotal = [0]*3
for irow in range(3):
    for jcol in range(4):
        rowtotal[irow] += data[irow][jcol]
        #longer alternative:
        #rowtotal[irow] = rowtotal[irow] + data[irow, jcol]
print(rowtotal)
```

```
[11, 22, 18]
```

`break` and `while`

A common task is to find the position of a value in a sorted table (e.g., a list or array). For example, determine between which two numbers the number 6 falls in the ordered sequence `[1, 4, 5, 8, 9]` . I know, it is between `5` and `8` , but what if the list is long? To find the position in the list, we need to loop through the list and break out of the loop once we have found the position. For this, Python has the command `break` .

In [23]:
```
x = [1, 4, 5, 8, 9]
a = 6
for i in range(len(x)):
    if a < x[i]:
        break
print('a is between', x[i-1], 'and', x[i])
```

```
a is between 5 and 8
```

There is another way of coding this using a `while` loop as shown below

In [24]:
```
x = [1, 4, 5, 8, 9]
a = 6
i = 0
while a >= x[i]:
    i = i + 1
print('a is between', x[i-1], 'and', x[i])
```

```
a is between 5 and 8
```

In the `while` loop, the comparison is done at the beginning of the loop, while the counter (in this case `i`) is updated inside the loop. Either a loop with a `break` or a `while` loop with a counter works fine, but `while` loops may be tricky in some cases, as they can result in infinite loops when you have an error in your code. Once you are in an infinite loop (one that never stops), click on the [Kernel] menu item at the top of the window and select [Restart]. This will end your Python session and start a new one. When you print something to the screen in your `while` loop, it may not be possible to break out of the loop and you may need to end your Jupyter session (and potentially lose some of your work). Because of these problems with errors in `while` loops, it is recommended to use a loop with a break rather than a while loop when possible.

1.1.3 Python built-in functions and callables

A **function** is a Python object that you can "call" to **perform an action** or compute and **return another object**. You call a function by placing parentheses to the right of the function name. Some functions allow you to pass **arguments** inside the parentheses (separating multiple arguments with a comma). Internal to the function, these arguments are treated like variables.

Python has several useful built-in functions to help you work with different objects and/or your environment. Here is a small sample of them:

- **`type(obj)`** to determine the type of an object
- **`len(container)`** to determine how many items are in a container
- **`callable(obj)`** to determine if an object is callable
- **`sorted(container)`** to return a new list from a container, with the items sorted
- **`sum(container)`** to compute the sum of a container of numbers
- **`min(container)`** to determine the smallest item in a container
- **`max(container)`** to determine the largest item in a container
- **`abs(number)`** to determine the absolute value of a number
- **`repr(obj)`** to return a string representation of an object

> Complete list of built-in functions: https://docs.python.org/3/library/functions.html (https://docs.python.org/3/library/functions.html)

There are also different ways of defining your own functions and callable objects that we will explore later.

In [28]:
```
# Use the type() function to determine the type of an object
s='asdasd'
type(s)
```

Out[28]: str

In [29]:
```
# Use the len() function to determine how many items are in a container
len(dict1)
```

Out[29]: 3

In [30]:
```
# Use the len() function to determine how many items are in a container
s2='1234567'
len(s2)
```

Out[30]: 7

In [37]:
```
# Use the callable() function to determine if an object is callable
callable(dict1.values)
```

Out[37]: True

In [32]:
```
# Use the callable() function to determine if an object is callable
callable(dict1)
```

Out[32]: False

In [38]:
```
# Use the sorted() function to return a new list from a container, with the it
ems sorted
sorted([10, 1, 3.6, 7, 5, 2, -3])
```

Out[38]: [-3, 1, 2, 3.6, 5, 7, 10]

In [40]:
```
# Use the sorted() function to return a new list from a container, with the it
ems sorted
# - notice that capitalized strings come first
sorted(['dogs', 'cats', 'zebras', 'Chicago', 'California', 'ants', 'mice'])
```

Out[40]: ['California', 'Chicago', 'ants', 'cats', 'dogs', 'mice', 'zebras']

In [41]:
```
# Use the sum() function to compute the sum of a container of numbers
sum([10, 1, 3.6, 7, 5, 2, -3])
```

Out[41]: 25.6

In [42]:
```
# Use the min() function to determine the smallest item in a container
min([10, 1, 3.6, 7, 5, 2, -3])
```

Out[42]: -3

In [45]:
```
# Use the min() function to determine the smallest item in a container
min(['g', 'z', 'a', 'y','Z'])
```

Out[45]: 'Z'

In [ ]:
```
# Use the max() function to determine the largest item in a container
max([10, 1, 3.6, 7, 5, 2, -3])
```

In [46]:
```
# Use the max() function to determine the largest item in a container
max('gibberish')
```

Out[46]: 's'

In [47]:
```
# Use the abs() function to determine the absolute value of a number
abs(10)
```

Out[47]: 10

In [48]:
```
# Use the abs() function to determine the absolute value of a number
abs(-12)
```

Out[48]: 12

In [50]:
```
# Use the repr() function to return a string representation of an object
repr(set1)
```

Out[50]: str

In [47]:
```
repr(list1)
```

Out[47]: "[3, 5, 6, 3, 'dog', 'cat', False]"

Python object attributes (methods and properties)

Different types of objects in Python have different **attributes** that can be referred to by name (similar to a variable). To access an attribute of an object, use a dot (`.`) after the object, then specify the attribute (i.e. `obj.attribute`)

When an attribute of an object is a callable, that attribute is called a **method**. It is the same as a function, only this function is bound to a particular object.

When an attribute of an object is not a callable, that attribute is called a **property**. It is just a piece of data about the object, that is itself another object.

The built-in `dir()` function can be used to return a list of an object's attributes.

Some methods on string objects

- **`.capitalize()`** to return a capitalized version of the string (only first char uppercase)
- **`.upper()`** to return an uppercase version of the string (all chars uppercase)
- **`.lower()`** to return an lowercase version of the string (all chars lowercase)
- **`.count(substring)`** to return the number of occurences of the substring in the string
- **`.startswith(substring)`** to determine if the string starts with the substring
- **`.endswith(substring)`** to determine if the string ends with the substring
- **`.replace(old, new)`** to return a copy of the string with occurences of the "old" replaced by "new"

In [51]:
```
# Assign a string to a variable
a_string = 'tHis is a sTriNg'
```

In [52]:
```
# Return a capitalized version of the string
a_string.capitalize()
```
Out[52]: 'This is a string'

In [55]:
```
# Return an uppercase version of the string
a_string.upper()
```
Out[55]: 'THIS IS A STRING'

In [56]:
```
# Return a lowercase version of the string
a_string.lower()
```
Out[56]: 'this is a string'

In [57]:
```
# Notice that the methods called have not actually modified the string
a_string
```
Out[57]: 'tHis is a sTriNg'

In [61]:
```
# Count number of occurences of a substring in the string
a_string.count('iN')
```
Out[61]: 1

In [59]:
```
# Count number of occurences of a substring in the string after a certain posi
tion
a_string.count('i', 7)
```
Out[59]: 1

In [62]:
```
# Count number of occurences of a substring in the string
a_string.count('is')
```
Out[62]: 2

In [63]:
```
# Does the string start with 'this'?
a_string.startswith('this')
```
Out[63]: False

In [64]:
```
# Does the lowercase string start with 'this'?
a_string.lower().startswith('this')
```
Out[64]: True

In [65]:
```
# Does the string end with 'Ng'?
a_string.endswith('Ng')
```
Out[65]: True

In [68]:
```
# Return a version of the string with a substring replaced with something else
a_string.replace('is', 'XYZ',1)
```

Out[68]: 'tHXYZ is a sTriNg'

In [69]:
```
# Return a version of the string with a substring replaced with something else
a_string.replace('i', '!')
```

Out[69]: 'tH!s !s a sTr!Ng'

In [70]:
```
# Return a version of the string with the first 2 occurences a substring repla
ced with something else
a_string.replace('i', '!', 2)
```

Out[70]: 'tH!s !s a sTriNg'

Some methods on list objects

- **`.append(item)`** to add a single item to the list
- **`.extend([item1, item2, ...])`** to add multiple items to the list
- **`.remove(item)`** to remove a single item from the list
- **`.pop()`** to remove and return the item at the end of the list
- **`.pop(index)`** to remove and return an item at an index

In [71]:
```
list2=[1,'2',3,'yolo']
```

In [72]:
```
# add one item to the end of the list
list2.append('golabi')
```

In [73]:
```
list2
```

Out[73]: [1, '2', 3, 'yolo', 'golabi']

In [74]:
```
# doesn't work!
list2.append('golabi',4)
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
<ipython-input-74-c1a3f96900d4> in <module>
      1 # doesn't work!
----> 2 list2.append('golabi',4)

TypeError: append() takes exactly one argument (2 given)
```

In [75]:
```
# adds one item to the end of the list!
list2.append(['golabi',4])
```

In [76]:
```
list2
```

Out[76]: [1, '2', 3, 'yolo', 'golabi', ['golabi', 4]]

In [77]:
```
# add multiple items to the end of the list
list2.extend(['golabi',4])
```

In [78]:
```
list2
```

Out[78]:
```
[1, '2', 3, 'yolo', 'golabi', ['golabi', 4], 'golabi', 4]
```

In [79]:
```
# removes the first instance of an item
list2.remove('golabi')
print(list2)
```

```
[1, '2', 3, 'yolo', ['golabi', 4], 'golabi', 4]
```

In [80]:
```
# removes one item from the end of the list and returns it
list3=list2.pop()
print(list2,'pop=', list3)
```

```
[1, '2', 3, 'yolo', ['golabi', 4], 'golabi'] pop= 4
```

In [81]:
```
# removes the index and returns it
list3=list2.pop(1)
print(list2,'pop=',list3)
```

```
[1, 3, 'yolo', ['golabi', 4], 'golabi'] pop= 2
```

Some methods on set objects

- **`.add(item)`** to add a single item to the set
- **`.update([item1, item2, ...])`** to add multiple items to the set
- **`.update(set2, set3, ...)`** to add items from all provided sets to the set
- **`.remove(item)`** to remove a single item from the set
- **`.pop()`** to remove and return a random item from the set
- **`.difference(set2)`** to return items in the set that are not in another set
- **`.intersection(set2)`** to return items in both sets
- **`.union(set2)`** to return items that are in either set
- **`.symmetric_difference(set2)`** to return items that are only in one set (not both)
- **`.issuperset(set2)`** does the set contain everything in the other set?
- **`.issubset(set2)`** is the set contained in the other set?

In [82]:
```
set1
```

Out[82]:
```
{3, 5, 6, False, 'cat', 'dog'}
```

In [83]:
```
# add a single item to the set
set1.add('Golabi')
print(set1)
```

```
{False, 3, 5, 6, 'cat', 'Golabi', 'dog'}
```

In [84]:
```
# add multiple items to the set
set1.update(['a','b','c','d'])
print(set1)
```

```
{False, 3, 'a', 5, 6, 'c', 'b', 'd', 'cat', 'Golabi', 'dog'}
```

In [85]:
```
# add items from all provided sets to the set
set1.update({'a','b','c'},{'c'},{'d','e'})
print(set1)
```

```
{False, 3, 'a', 5, 6, 'c', 'e', 'b', 'd', 'cat', 'Golabi', 'dog'}
```

In [ ]:
```
#
```

In [ ]:
```
#
```

In [66]:
```
set2={1,2,3,4,5,6,7,8,9,0,10,11,12,13}
```

In [67]:
```
# returns items in the set1 that are not in set2
set1.difference(set2)
```

Out[67]:
```
{'Golabi', 'a', 'b', 'c', 'cat', 'd', 'dog', 'e'}
```

In [ ]:
```
#
```

In [ ]:
```
#
```

In [ ]:
```
#
```

In [ ]:
```
#
```

In [ ]:
```
#
```

Some methods on dict objects

- **`.update([(key1, val1), (key2, val2), ...])`** to add multiple key-value pairs to the dict
- **`.update(dict2)`** to add all keys and values from another dict to the dict
- **`.pop(key)`** to remove key and return its value from the dict (error if key not found)
- **`.pop(key, default_val)`** to remove key and return its value from the dict (or return default_val if key not found)
- **`.get(key)`** to return the value at a specified key in the dict (or None if key not found)
- **`.get(key, default_val)`** to return the value at a specified key in the dict (or default_val if key not found)
- **`.keys()`** to return a list of keys in the dict
- **`.values()`** to return a list of values in the dict
- **`.items()`** to return a list of key-value pairs (tuples) in the dict

In [86]:
```
dict1
```

Out[86]:
```
{'name': 'Jane', 'age': 23, 'fav_foods': ['pizza', 'fruit', 'fish']}
```

In [87]:
```
# add multiple key-value pairs to the dict or update old value
dict1.update([('fav_foods',['pizza','lasagna','fish']),('last name','daugles'
)])
print(dict1)
```

```
{'name': 'Jane', 'age': 23, 'fav_foods': ['pizza', 'lasagna', 'fish'], 'last
name': 'daugles'}
```

In [88]:
```
# update value
dict1['name']='ali'
print(dict1)
```

```
{'name': 'ali', 'age': 23, 'fav_foods': ['pizza', 'lasagna', 'fish'], 'last n
ame': 'daugles'}
```

In [89]:
```
# add key-value
dict1['blood type']='O+'
print(dict1)
```

```
{'name': 'ali', 'age': 23, 'fav_foods': ['pizza', 'lasagna', 'fish'], 'last n
ame': 'daugles', 'blood type': 'O+'}
```

In [89]:
```
# remove key and return its value from the dict (error if key not found)
last_name=dict1.pop('last name')
print(dict1)
print('last name:',last_name)
```

```
{'age': 23, 'fav_foods': ['pizza', 'lasagna', 'fish'], 'blood type': 'O+', 'f
ull name': 'ali daugles'}
last name: daugles
```

In [88]:
```
# update key
dict1['full name']=dict1.pop('name')+' '+ last_name
print(dict1)
```

```
{'age': 23, 'fav_foods': ['pizza', 'lasagna', 'fish'], 'last name': 'daugle
s', 'blood type': 'O+', 'full name': 'ali daugles'}
```

In [ ]:
```
#
```

In [ ]:
```

```

In [94]:
```
# return the value at a specified key in the dict (or None if key not found)
print (dict1.get('1'))
```

```
-1
```

In [95]:
```
# return the value at a specified key in the dict (or default_val if key not f
ound)
print (dict1.get('1',-1))
```

```
-1
```

In [ ]:

In [ ]:

In [ ]:

1.1.4 Functions

In this Notebook we learn how to write our own functions, but we start out with a bit about Python packages.

A bit about packages

A package is a set of Python functions. When we want to use a function from a package, we need to import it. There are many different ways to import packages. The most basic syntax is

`import numpy`

after which any function in `numpy` can be called as `numpy.function()`. If you don't like the name of the package, for example because it is long, you can change the name. The `numpy` package is renamed to `np` by typing

`import numpy as np`

all functions in `numpy` can be called as `np.function()`.

Packages can also have subpackages. For example, the `numpy` package has a subpackage called `random`, which has a bunch of functions to deal with random variables. If the `numpy` package is imported with `import numpy as np`, functions in the `random` subpackage can be called as `np.random.function()`.

If you only need one specific function, you don't have to import the entire package. For example, if you only want the cosine function of the numpy package, you may import it as `from numpy import cos`, after which you can simply call the cosine function as `cos()`. You can even rename functions when you import them. For example, after `from numpy import cos as newname`, you can call the function `newname()` to compute the cosine (I know, pretty silly, but this is can become handy).

In the previous Notebooks we always imported `numpy` and called it `np` and we imported the plotting part of `matplotlib` and called it `plt`. Both are standard names in the Python community. The statement we added before importing `matplotlib` is `%matplotlib inline`. This latter command is an IPython command and not a Python command. It will only work in IPython and is called a magic command. All magic commands are preceded with a `%`. The statement `%matplotlib inline` puts all figures in the Notebook rather than in a separate window.

Enough about packages for now. Let's start the way we always start.

In [2]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
```

Defining Functions

In [91]:
```
def plus(a, b):
    return a + b
```

In [93]:
```
a=plus(3, 4)
a
```

Out[93]: 7

In [102]:
```
# doesn't return anything
def plus(a, b):
    print(a + b)
    return
```

In [103]:
```
a=plus(3, 4)
print(a)
```

```
7
None
```

What happened? All functions return *something*, even if you don't specify it. If you don't specify a return value, then it will default to returning `None`.

In [ ]:
```
"a" + 1 #def __plus(a,b):
```

In [ ]:
```
# only prints the result but not return it
def plus(a, b):
    print(a + b)
```

In [ ]:
```
a=plus(2,4)
```

Functions are an essential part of a programming language. You already used many functions like `plot`, `loadtxt`, or `linspace`. But you can also define your own functions. To define a new function, use the `def` command. After `def` follows the name of the function and then between parentheses the arguments of the function and finally a colon. After the colon you indent until you are done with the function. The last line of the function should be `return` followed by what you want to return. For example, consider the following function of x:

$$f(x) = \cos(x) \qquad x < 0$$

$$f(x) = \exp(-x) \qquad x \geq 0$$

Let's implement $f(x)$ in a function called `func`. There is one input argument: x.

In [4]:
```
def func(x):
    if x < 0:
        f = np.cos(x)
    else:
        f = np.exp(-x)

    print(1)
    print(2)
    print("golabi")
    return f
print(func(3))
```

```
1
2
golabi
0.049787068367863944
```

Once you define a function in Python, you can call it whenever you want during the session. So we can call it again

In [ ]:
```
print(func(-2))
```

If you type

`func(` and then hit [shift-tab]

and wait a split second, the input arguments of the function pop-up in a little window, just like for other functions we already used. You can also provide additional documentation of your function. Put the documentation at the top of the indented block and put it between triple double quotes (`"""`). Run the code below to define the function `func` with the additional documentation, then in the code cell below that type

`func(`

and hit [shift][tab] to see the additional documentation. Warning: don't leave a code cell with just `func(` or `func()` as you will get an error on [Kernel][Restart & Run All].

In [100]:
```
def func(x):
    """First Python function
    written by Student X"""
    if x < 0:
        f = np.cos(x)
    else:
        f = np.exp(-x)
    return f
```

In [ ]:
```
func(
```

The names of the arguments of a function are the names used inside the function. They have no relationship to the names used outside the function. When using a variable as the argument of a function, only the *value* gets passed to the function. In the example below, the *value* of `y` is passed as the first argument ot the function `func`. Inside the function, this value is used for the variable `x`.

In [101]:
```
y = 2
print('func(2):', func(y))
```

```
func(2): 0.1353352832366127
```

Keyword arguments

Functions may have multiple input arguments followed by keyword arguments. Arguments *must* be entered and must be entered in the order defined. Keyword arguments don't need to be entered. When they are not entered, the default value is used. Keyword arguments may be given in any order as long as they come after the regular arguments. If you specify the keyword arguments in the order they are defined in the argument list, you don't even need to preceed them with the keyword, but it is saver to write the keywords out and it makes your code easier to read. For example, the function $f(x) = A\cos(\pi x + \theta)$ can be written with keyword arguments for A and θ as follows.

In [9]:
```
def testfunc(x, A=1, theta=0):
    return A * np.cos(np.pi * x + theta)
print(testfunc(1))  # Uses default A=1, theta=0: cos(pi)
print(testfunc(1, A=2))  # Now A=2, and theta is still 0: 2*cos(pi)
print(testfunc(1, A=2, theta=np.pi / 4))  # Now A=2, theta=pi/4: 2*cos(5pi/4)
print(testfunc(1, theta=np.pi / 4, A=2))  # Same as above: 2*cos(5pi/4)
print(testfunc(1, theta=np.pi / 4))  # Now theta=pi/4, and A is still 1: cos(5
pi/4)
```

```
-1.0
-2.0
-1.4142135623730954
-1.4142135623730954
-0.7071067811865477
```

In [8]:
```
for i in range(10):
    print(i, end='*')
```

```
0*1*2*3*4*5*6*7*8*9*
```

Positional arguments and keyword arguments to callables

You can call a function/method in a number of different ways:

- `func()` : Call `func` with no arguments
- `func(arg)` : Call `func` with one positional argument
- `func(arg1, arg2)` : Call `func` with two positional arguments
- `func(arg1, arg2, ..., argn)` : Call `func` with many positional arguments
- `func(kwarg=value)` : Call `func` with one keyword argument
- `func(kwarg1=value1, kwarg2=value2)` : Call `func` with two keyword arguments
- `func(kwarg1=value1, kwarg2=value2, ..., kwargn=valuen)` : Call `func` with many keyword arguments
- `func(arg1, arg2, kwarg1=value1, kwarg2=value2)` : Call `func` with positonal arguments and keyword arguments
- `obj.method()` : Same for `func` .. and every other `func` example

When using **positional arguments**, you must provide them in the order that the function defined them (the function's **signature**).

When using **keyword arguments**, you can provide the arguments you want, in any order you want, as long as you specify each argument's name.

When using positional and keyword arguments, positional arguments must come first.

In [ ]:
```
def testfunc(x, A=1, theta=0):
    return A * np.cos(np.pi * x + theta)
print(testfunc(1))  # Uses default A=1, theta=0: cos(pi)
print(testfunc(1, A=2))  # Now A=2, and theta is still 0: 2*cos(pi)
print(testfunc(1, A=2, theta=np.pi / 4))  # Now A=2, theta=pi/4: 2*cos(5pi/4)
print(testfunc(1, theta=np.pi / 4, A=2))  # Same as above: 2*cos(5pi/4)
print(testfunc(1, theta=np.pi / 4))  # Now theta=pi/4, and A is still 1: cos(5pi/4)
```

Plotting

In [ ]:
```
%matplotlib inline
```

After the magic, we then need to import the matplotlib library:

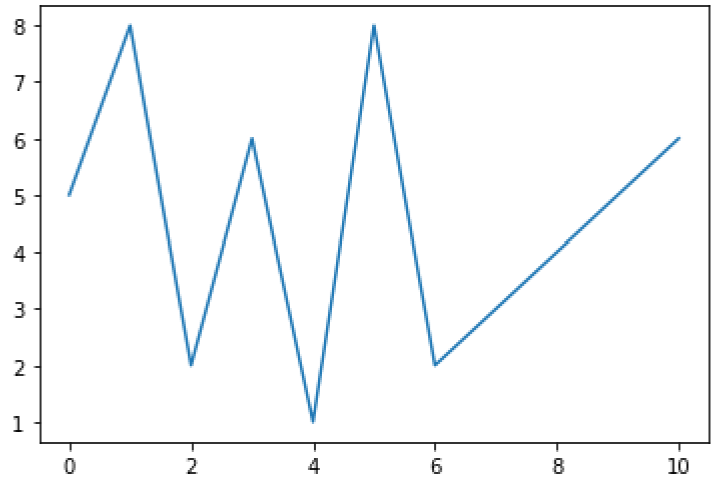
In [ ]:
```
import matplotlib.pyplot as plt
```

Python has many, many libraries. We will use a few over the course of the semester.

To create a simple line plot, just give a list of y-values to the function plt.plot().

In [10]:
```
plt.plot([5, 8, 2, 6, 1, 8, 2, 3, 4, 5, 6])
```

Out[10]:
```
[<matplotlib.lines.Line2D at 0xb033030>]
```



But you should never create a plot that doesn't have labels on the x and y axises, and should always have a title. Read the documentation on matplotlib and add labels and a title to the plot above:

http://matplotlib.org/api/pyplot_api.html (http://matplotlib.org/api/pyplot_api.html)

Another commonly used library (especially with matplotlib is numpy). Often imported as:

Exercise 5. First function

Write a Python function for the following function:

$f(x) = e^{-\alpha x}\cos(x)$

The function should take `x` and `alpha` as input arguments and return the function value. Give your function a unique name (if you also call it `func` it will overwrite the `func` function that we defined above). Make a plot of `f` vs. `x` for `x` going from 0 to 10π using two different values of `alpha` : 0.1 and 0.2. Add a legend and label the axes.

In [ ]:
```

```

Answer to Exercise 5

Local variables

Variables declared inside a function can only be used inside that function. The outside of a function doesn't know about the variables used inside the function, except for the variables that are returned by the function. In the code below, remove the `#` before `print(a)` and you will get an error message, as `a` is a local variable inside the function `localtest` (then put the `#` back, else you get an error when running [Kernel][Restart & Run All]).

In [14]:
```
import numpy as np
sin = np.sin
a=3
def localtest(x):

    b = 5
    return a * x + b
print(localtest(4))
print(a)
#print(a)  # Will cause an error, as 'a' is not known outside function
```

```
17
3
```

In [16]:
```
def a():
    def b():
        return 1
    return b()

a()
```

Out[16]:
```
1
```

Three types of variables inside a function

There are actually three types of variables inside a function. We already learned about two of them: variables passed to the function through the argument list, like `x` in the function above, and local variables, like `a` and `b` in the function above. The third type are variables defined outside the function but not passed to the function through the argument list. When a variable is used inside a Python function, Python first checks whether the variable has been defined locally. If not, it checks whether the variable is passed to the function through the argument list. And if that is not the case, Python checks whether the variable is defined outside the function, from the place the function was called. If that is not the case either, it will throw an error message. It is considered good coding practice to pass variables to a function when they are needed inside a function, rather than counting on Python to *find* the variable outside the function; it will likely lead to fewer coding errors as well.

Note that when a variable is defined locally, Python will not check whether that variable is also declared outside the function. It will happily create a new variable with the same name inside the function. It is important to realize the difference between these different types, so let's do a few examples.

In [ ]:
```
# This function works properly
def test1(x):
    a = 3
    b = 5
    return a * x + b
print(test1(4))

# This function also works, but it is sloppy coding
# since variable a is defined outside the function
a = 3
def test2(x):
    b = 5
    return a * x + b
print(test2(4))
```

In the following function, we define variable `var1` outside the function `test3` . The function `test3` doesn't take any input arguments (but it still needs the parentheses, else Python doesn't know it is a function!), and it creates a local variable `var1` . This local `var1` variable is only known inside the function `test3` and doesn't effect the value of `var1` outside function `test3` .

In [ ]:
```
var1 = 8
def test3():
    var1 = 4
    print('Inside the functino test3, var1 equals:', var1)
test3()
print('Value of var1 outside test3:', var1)
```

Functions are building blocks that need to be tested separately

Functions are the building blocks of a computer code. They represent a well-defined functionality, which means they can *and should* be tested separately. So make it a habit to test whether your function does what you intended it to do. Sometimes it is easy to test a function: you can compare the value to a hand-calculation, for example. Other times it is more difficult, and you need to write some additional code to test the function. It is always worthwhile to do that. If you test your functions well, it will aid you in debugging your code, because you know that the error is not inside the function.

Exercise 6, Stream function for flow around a cylinder

Consider two-dimensional inviscid fluid flow (potential flow) around a cylinder. The origin of the coordinate system is at the center of the cylinder. The stream function is a function that is constant along stream lines. The stream function ψ is a function of polar coordinates r and θ. The stream function is constant and equal to zero on the cylinder and doesn't really exist inside the cylinder, so let's make it zero there, like it is on the cylinder.

$$\psi = 0 \qquad r \leq R$$
$$\psi = U(r - R^2/r)\sin(\theta) \qquad r \geq R$$

where U is the flow in the x-direction, r is the radial distance from the center of the cylinder, θ is the angle, and R is the radius of the cylinder. You may recall it is not always easy to compute the correct angle when given a value of x and y, as the regular arctan function returns a value between $-\pi/2$ and $+\pi/2$ (radians), while if $x = -2$ and $y = 2$, the angle should be $3\pi/4$. `numpy` has a very cool function to compute the correct angle between $-\pi$ and $+\pi$ given the x and y coordinates. The function is `arctan2(y,x)`. Note that the function takes as its *first* argument `y` and as its *second* argument `x`.

Write a function that computes the stream function for flow around a cylinder. The function should take two arguments, `x` and `y`, and two keyword arguments, `U` and `R`, and should return the stream function value. If you write the function correctly, it should give `psi(2, 4, U=2, R=1.5) = 7.1`, and `psi(0.5, 0, U=2, R=1.5) = 0` (inside the cylinder).

In [ ]:

```

```

Answer to Exercise 6

Vectorization of a function

Not all functions can be called with an array of values as input argument. For example, the function `func` defined at the beginning of this notebook doesn't work with an array of `x` values. Remove the `#` and try it out. Then put the `#` back

In [ ]:

```
def func(x):
    if x < 0:
        f = np.cos(x)
    else:
        f = np.exp(-x)
    return f
x = np.linspace(-6, 6, 100)
#y = func(x) # Run this line after removing the # to see the error that occur
s. Then put the # back
```

The reason this doesn't work is that Python doesn't know what to do with the line

```
if x < 0
```

when `x` contains many values. Hence the error message

```
The truth value of an array with more than one element is ambiguous
```

For some values of `x` the `if` statement may be `True`, for others it may be `False`. A simple way around this problem is to vectorize the function. That means we create a new function, let's call it `funcvec`, that is a vectorized form of `func` and can be called with an array as an argument (this is by far the easiest but not necessarily the computationally fastest way to make sure a function can be called with an array as an argument)

In [ ]:
```
funcvec = np.vectorize(func)
x = np.linspace(-6, 6, 100)
y = funcvec(x)
plt.plot(x, y);
```

Back now to the problem of flow around a clinder. Contours of the stream function represent stream lines around the cylinder. To make a contour plot, the function to be contoured needs to be evaluated on a grid of points. The grid of points and an array with the values of the stream function at these points can be passed to a contouring routine to create a contour plot. To create a grid of points, use the function `meshgrid` which takes as input a range of `x` values and a range of `y` values, and returns a grid of `x` values and a grid of `y` values. For example, to have 5 points in the x-direction from -1 to +1, and 3 points in y-direction from 0 to 10:

In [ ]:
```
x,y = np.meshgrid( np.linspace(-1,1,5), np.linspace(0,10,3) )
print('x values')
print(x)
print('y values')
print(y)
```

Exercise 7, Contour plot for flow around a cylinder

Evaluate the function for the stream function around a cylinder with radius 1.5 on a grid of 100 by 100 points, where `x` varies from -4 to +4, and `y` varies from -3 to 3; use $U=1$. Evaluate the stream function on the entire grid (you need to create a vectorized version of the function you wrote to compute the stream function). Then use the `np.contour` function to create a contour plot (find out how by reading the help of the `contour` function or go to [this demo (http://matplotlib.org/examples/pylab_examples/contour_demo.html)](http://matplotlib.org/examples/pylab_examples/contour_demo.html)) of the `matplotlib` gallery. You need to use the command `plt.axis('equal')`, so that the scales along the axes are equal and the circle looks like a circle rather than an ellipse. Finally, you may want to add a nice circular patch using the `fill` command and specifying a bunch of x and y values around the circumference of the cylinder.

In [ ]:
```
```

[Answer to Exercise 7](#)

Return multiple *things*

An assignment can assign values to multiple variables in one statement, for example

In [ ]:
```
a, b = 4, 3
print('a:', a)
print('b:', b)
a, b, c = 27, np.arange(4), 'hello'
print('a:', a)
print('b:', b)
print('c:', c)
d, e, f = np.arange(0, 11, 5)
print('d:', d)
print('e:', e)
print('f:', f)
```

Similarly, a function may return one value or one array. Of a function may return multiple values, multiple arrays, or whatever the programmer decides to return (including nothing, of course). When multiple *things* are returned, they are returned as a tuple. They can be stored as a tuple, or, if the user knows how many *things* are returned, they can be stored in individual variables right away, as in the example above.

In [ ]:
```
def newfunc():
    dump = 4 * np.ones(5)
    dump[0] = 100
    return 33, dump, 'this works great!'
test = newfunc()
print(type(test))
print(test[1])
a, b, c = newfunc()
print('a:', a)
print('b:', b)
print('c:', c)
```

Exercise 8, Streamplot of flow around a cylinder

The radial and tangential components of the velocity vector $\vec{v} = (v_r, v_\theta)$ for inviscid fluid flow around a cylinder are given by

$$v_r = U(1 - R^2/r^2)\cos(\theta) \qquad r \geq R$$
$$v_\theta = -U(1 + R^2/r^2)\sin(\theta) \qquad r \geq R$$

and is zero otherwise. The x and y components of the velocity vector may be obtained from the radial and tangential components as

$$v_x = v_r\cos(\theta) - v_\theta\sin(\theta)$$
$$v_y = v_r\sin(\theta) + v_\theta\cos(\theta)$$

Write a function that returns the x and y components of the velocity vector for fluid flow around a cylinder with $R = 1.5$ and $U = 2$. Test your function by making sure that at $(x, y) = (2, 3)$ the velocity vector is $(v_x, v_y) = (2.1331, -0.3195)$. Compute the x and y components of the velocity vector (vectorization won't help here, as your function returns two values, so you need a double loop) on a grid of 50 by 50 points where `x` varies from -4 to +4, and `y` varies from -3 to 3. Create a stream plot using the cool function `plt.streamplot`, which takes four arguments: `x`, `y`, `vx`, `vy`.

In [ ]:

```

```

Answer to Exercise 8

Exercise 9, Derivative of a function

The function `func`, which we wrote earlier in this notebook, implements the following function

$$f(x) = \cos(x) \qquad x < 0$$

$$f(x) = \exp(-x) \qquad x \geq 0$$

Derive an analytic expression (by hand) for the first derivative of $f(x)$ and implement it in a Python function. Test your function by comparing its output to a numerical derivative using a central difference scheme

$\frac{\mathrm{d}f}{\mathrm{d}x} \approx \frac{f(x+d)-f(x-d)}{2d}$

where d is a small number. Test your function for both $x < 0$ and $x > 0$.

In [ ]:

```

```

Answer to Exercise 9

Using a function as the argument of another function

So far, we have used single values or arrays as input arguments of functions. But we can also use a function as one of the input arguments of another function. Consider, for example, a function called `takesquare` that takes two input arguments: a function `finput` and a value `x`, and it returns the function `finput` evaluated at `x` and then squared.

In [21]:
```
def takesquare(function_input, x):
    return function_input(x) ** 2
```

We can now call `takesquare` with any function f that can be called as $f(x)$ and returns one value. For example, we can call it with the cosine function, and we can test right away whether we got the right answer

In [22]:
```
print('takesquare result:', takesquare(np.cos, 2))
print('correct value is: ', np.cos(2) ** 2)
```

```
takesquare result: 0.17317818956819406
correct value is:  0.17317818956819406
```

Finding the zero of a function

Finding the zero of a function is a common task in exploratory computing. The value where the function equals zero is also called the *root* and finding the zero is referred to as *root finding*. There exist a number of methods to find the zero of a function varying from robust but slow (so it always finds a zero but it takes quite a few function evaluations) to fast but not so robust (it can find the zero very fast, but it won't always find it). Here we'll use the latter one.

Consider the function $f(x) = 0.5 - \text{e}^{-x}$. The function is zero when $x = -\ln(0.5)$, but let's pretend we don't know that and try to find it using a root finding method. First, we need to write a Python function for $f(x)$.

In [ ]:
```
def f(x):
    return 0.5 - np.exp(-x)
```

We will use the method `fsolve` to find the zero of a function. `fsolve` is part of the `scipy.optimize` package. `fsolve` takes two arguments: the function for which we want to find the zero, and a starting value for the search (not surpisingly, the closer the starting value is to the root, the higher the chance that `fsolve` will find it).

In [ ]:
```
from scipy.optimize import fsolve
xzero = fsolve(f, 1)
print('result of fsolve:', xzero)
print('f(x) at xzero:   ', f(xzero))
print('exact value of xzero:', -np.log(0.5))
```

What now if you want to find the value of x for which $f(x) = 0.3$ (I know, it is $-\ln(0.2)$). We could, of course, create a new function $f_2(x) = f(x) - 0.3$ and then try to find the zero of f_2. But if we do that, we might as well make it more generic. Let's try to find $f(x) = a$, so we create a function $f_2 = f(x) - a$

In [ ]:
```
xroot = fsolve(f2, 1, args=(0.3))
print('fsolve result:', xroot)
print('f(xroot):     ', f(xroot))
print('exact value:  ', -np.log(0.2))
```

Exercise 10

The cumulative density distribution $F(x)$ of the Normal distribution is given by

$$F(x) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{x-\mu}{\sqrt{2\sigma^2}}\right)\right]$$

where μ is the mean, σ is the standard deviation, and erf is the error function. Recall the definition of a cumulative density distribution: When a random variable has a Normal distribution with mean μ and standard deviation σ, $F(x)$ is the probability that the random variable is less than x. Write a Python function for $F(x)$. The fist input argument should be x, followed by keyword arguments for μ and σ. The error function can be imported as

```
from scipy.special import erf
```

Test your function, for example by making sure that when $x = \mu$, F should return 0.5, and when $x = \mu + 1.96\sigma$, F should return 0.975 (remember that from your statistics class?).

Next, find the value of x for which $F(x) = p$, where p is a probablity of interest (so it is between 0 and 1). Check you answer for $\mu = 3$, $\sigma = 2$, and find x for $p = 0.1$ and $p = 0.9$. Substitute the roots you determine with `fsolve` back into $F(x)$ to make sure your code works properly.

In [ ]:

Answer to Exercise 10

Exercise 11. Numerical integration

Numerical integration of a function is a common engineering task. The `scipy` package has a specific subpackage called `integrate` with a number of numerical integration functions. We will use the `quad` function. Use the `quad` function to integrate the function $f(x)=\mathrm{e}^{-x}$ from 1 till 5. Check that you did it right by doing the integration by hand (which is easy for this function).

Next, compute the following integral:

$$\int_1^5 \frac{\mathrm{e}^{-x}}{x}\mathrm{d}x$$

This integral is more difficult to do analytically. Perform the integration numerically with the `quad` function and check your answer, for example, at the [wolframalpha website (https://www.wolframalpha.com)](https://www.wolframalpha.com) where you can simply type: `integrate exp(-x)/x from 1 to 5`.

In [ ]:

```
```

Answer to Exercise 11

1.1.5 Scope of variables

Is not always clear:

In [4]:

```
y = 0
for x in range(10):
    y = x
```

In [ ]:

```
x
```

In [ ]:

```
[x for x in range(10, 20)]
```

In [ ]:

```
x
```

Python follows the LEGB Rule (after [https://www.amazon.com/dp/0596513984/ (https://www.amazon.com/dp/0596513984/)](https://www.amazon.com/dp/0596513984/)):

- L, Local: Names assigned in any way within a function (def or lambda)), and not declared global in that function.
- E, Enclosing function locals: Name in the local scope of any and all enclosing functions (def or lambda), from inner to outer.
- G, Global (module): Names assigned at the top-level of a module file, or declared global in a def within the file.
- B, Built-in (Python): Names preassigned in the built-in names module : open, range, SyntaxError,...

In [ ]:
```
x = 3
def foo():
    x=4
    def bar():
        print(x)  # Accesses x from foo's scope
    bar()  # Prints 4
    x=5
    bar()  # Prints 5
```

In [ ]:
```
foo()
```

Recursive Functions

Recursion has something to do with infinity. I know recursion has something to do with infinity. I think I know recursion has something to do with infinity. He is sure I think I know recursion has something to do with infinity. We doubt he is sure I think I know ... We think that you think that we convinced you now that we can go on forever with this example of a recursion from natural language. Recursion is not only a fundamental feature of natural language, but of the human cognitive capacity. Our way of thinking is based on a recursive thinking processes. Even with a very simple grammar, like "An English sentence contains a subject and a predicate, and a predicate contains a verb, an object and a complement", we can demonstrate the infinite possibilities of the natural language. The cognitive scientist and linguist Stephen Pinker phrases it like this: "With a few thousand nouns that can fill the subject slot and a few thousand verbs that can fill the predicate slot, one already has several million ways to open a sentence. The possible combinations quickly multiply out to unimaginably large numbers. Indeed, the repertoire of sentences is theoretically infinite, because the rules of language use a trick called recursion. A recursive rule allows a phrase to contain an example of itself, as in *She thinks that he thinks that they think that he knows* and so on, ad infinitum. And if the number of sentences is infinite, the number of possible thoughts and intentions is infinite too, because virtually every sentence expresses a different thought or intention."[1]

We have to stop our short excursion to the use of recursion in natural language to come back to recursion in computer science and programs and finally to recursion in the programming language Python.

The adjective "recursive" originates from the Latin verb "recurrere", which means "to run back". And this is what a recursive definition or a recursive function does: It is "running back" or returning to itself. Most people who have done some mathematics, computer science or read a book about programming will have encountered the factorial, which is defined in mathematical terms as

```
n! = n * (n-1)!, if n > 1 and 0! = 1
```

It's used so often as an example for recursion because of its simplicity and clarity. We will come back to it in the following.

Definition of Recursion

Recursion is a method of programming or coding a problem, in which a function calls itself one or more times in its body. Usually, it is returning the return value of this function call. If a function definition satisfies the condition of recursion, we call this function a recursive function.

Termination condition:
A recursive function has to fulfil an important condition to be used in a program: it has to terminate. A recursive function terminates, if with every recursive call the solution of the problem is downsized and moves towards a base case. A base case is a case, where the problem can be solved without further recursion. A recursion can end up in an infinite loop, if the base case is not met in the calls.

Example:

```
 4! = 4 * 3!
3! = 3 * 2!
2! = 2 * 1
```

Replacing the calculated values gives us the following expression

```
 4! = 4 * 3 * 2 * 1
```

Generally we can say: Recursion in computer science is a method where the solution to a problem is based on solving smaller instances of the same problem.

Recursive Functions in Python

Now we come to implement the factorial in Python. It's as easy and elegant as the mathematical definition.

In [42]:
```
def factorial(n):
    if n==1:
        return 1
    return n * factorial(n-1)
```

In [43]:
```
factorial(6)
```

Out[43]: 720

We can track how the function works by adding two print() functions to the previous function definition:

In [44]:
```
def factorial(n):
    print("factorial has been called with n = " + str(n))
    if n == 1:
        return 1
    else:
        res =n * factorial(n-1)
        print("intermediate result for ", n, " * factorial(" ,n-1, "): ",res)
        return res

print(factorial(5))
```

```
factorial has been called with n = 5
factorial has been called with n = 4
factorial has been called with n = 3
factorial has been called with n = 2
factorial has been called with n = 1
intermediate result for  2  * factorial( 1 ):  2
intermediate result for  3  * factorial( 2 ):  6
intermediate result for  4  * factorial( 3 ):  24
intermediate result for  5  * factorial( 4 ):  120
120
```

Let's have a look at an iterative version of the factorial function.

In [46]:
```
def iterative_factorial(n):
    result = 1
    for i in range(2,n+1):
        result *= i
    return result
```

It is common practice to extend the factorial function for 0 as an argument. It makes sense to define 0! to be 1, because there is exactly one permutation of zero objects, i.e. if nothing is to permute, "everything" is left in place. Another reason is that the number of ways to choose n elements among a set of n is calculated as n! divided by the product of n! and 0!.

All we have to do to implement this is to change the condition of the if statement:

In [45]:
```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)
```

In [50]:
```
factorial(10000)
```

```
---------------------------------------------------------------------------
RecursionError                            Traceback (most recent call last)
<ipython-input-50-58ab6b4e4a58> in <module>
----> 1 factorial(10000)

<ipython-input-45-02d9dc3605ba> in factorial(n)
      3         return 1
      4     else:
----> 5         return n * factorial(n-1)

... last 1 frames repeated, from the frame below ...

<ipython-input-45-02d9dc3605ba> in factorial(n)
      3         return 1
      4     else:
----> 5         return n * factorial(n-1)

RecursionError: maximum recursion depth exceeded in comparison
```

In [53]:
```
iterative_factorial(10000)
```

Out[53]:
```
2824229407960347874293421578024535518477494926091224850578918086542977950901
0
```

```
13665101131732865864329899368280582245417913586091281699426094875817458791889
```

```
82457947188435844333911772650430739095603250584481989958329152600842552729469
```

```
7148103843935544685011194934426280207090501376094908281450495676289923397938
```

```
93671131798440720514473071297671469256288529100871799135638819225378199739371
43961376073483722162825069521958888685523752741195529255056447754584800957007
73844879917578962739191515473215419449197855985644689246313903321716261658125
27651961991670987967027258250054203785178205728322883905548431754835790462358
06720522045103597127249480152437571026147673644930777276562020511523029284419
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73101972918319962732206511982068325231045067378341315839847599175001241948565
53402607893365990184683446139189141856836990116104871762395562029508914512264
85948051459625315054424557221324159699262856548588485732280119157332672242005
15175908637256493289757792370056711274692872630482509776206369518369027530541
24587479474598126535380577543494702133674918802593292720010853303698216585004
68211760265218150080785463038718658026724539643720747902353481639951968223160
25289090216584277797121778165468204418956742274346514808262341295207878951014
19869206407565471932124762091780942977322365769951855693751716329387037799465
50770892864786425750906004646891799861859487446420617023957418144845936846268
81949479706950259056855766094587748183345513992459267484795688457052252083403
37514146759302885369542838495394769063960988036771410633020041794578934406750
80536839815628590015226198809716210213323208794327899135980687221262515045870
72634476165027560095435307181703326432343240554190459777340006883595403370024

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08645687804128256068584031493830103631492848943029513253592732304639037194978
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30375851116567403646683413529868641780751028095938437726495400953340132206429
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46006274089009790870176743101384022360049071281392050855918432817170744618788
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80886365714956327181016953628012348417970375183329288481782729228908763098847
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88248240854597693670082456890007791742936188187895265446238052925888262720428
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54917273623566745118137648954248898063397860776893782168794355794581495796791
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In [ ]:
```
def fib(n):
#    print("fib has been called with n = " + str(n))
    if n == 0:
        return 1
    elif n==1:
        return 1
    else:
        res =fib(n-1)+fib(n-2)
#        print("intermediate result for ", n, " * fib(" ,n-1, "): ",res)
        return res

print(fib(100))
```

The Pitfalls of Recursion

Fibonacci Squares

This subchapter of our tutorial on recursion deals with the Fibonacci numbers. What do have sunflowers, the Golden ratio, fir tree cones, The Da Vinci Code, the song "Lateralus" by Tool, and the graphic on the right side in common. Right, the Fibonacci numbers.

The Fibonacci numbers are the numbers of the following sequence of integer values:

```
0,1,1,2,3,5,8,13,21,34,55,89, ...
```

The Fibonacci numbers are defined by:
$F_n = F_{n-1} + F_{n-2}$
with $F_0 = 0$
and $F_1 = 1$

The Fibonacci sequence is named after the mathematician Leonardo of Pisa, who is better known as Fibonacci. In his book "Liber Abaci" (publishes 1202) he introduced the sequence as an exercise dealing with bunnies. His sequence of the Fibonacci numbers begins with $F_1 = 1$, while in modern mathematics the sequence starts with $F_0 = 0$. But this has no effect on the other members of the sequence.

The Fibonacci numbers are the result of an artificial rabbit population, satisfying the following conditions:

- a newly born pair of rabbits, one male, one female, build the initial population
- these rabbits are able to mate at the age of one month so that at the end of its second month a female can bring forth another pair of rabbits
- these rabbits are immortal
- a mating pair always produces one new pair (one male, one female) every month from the second month onwards

The Fibonacci numbers are the numbers of rabbit pairs after n months, i.e. after 10 months we will have F_{10} rabits.

The Fibonacci numbers are easy to write as a Python function. It's more or less a one to one mapping from the mathematical definition:

In [ ]:
```python
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-1) + fib(n-2)
```

An iterative solution is also easy to write, though the recursive solution looks more like the definition:

In [ ]:
```
def fibi(n):
    old, new = 0, 1
    if n == 0:
        return 0
    for i in range(n-1):
        old, new = new, old + new
    return new
```

If you check the functions fib() and fibi(), you will find out that the iterative version fibi() is a lot faster than the recursive version fib(). To get an idea of how much this "a lot faster" can be, we have written a script, which uses the timeit module, to measure the calls. To do this, we save the function definitions for fib and fibi in a file fibonacci.py, which we can import in the program (fibonacci_runit.py) below:

In [ ]:
```
from timeit import Timer

t1 = Timer("fib(10)","from fibonacci import fib")

for i in range(1,41):
    s = "fib(" + str(i) + ")"
    t1 = Timer(s,"from fibonacci import fib")
    time1 = t1.timeit(3)
    s = "fibi(" + str(i) + ")"
    t2 = Timer(s,"from fibonacci import fibi")
    time2 = t2.timeit(3)
    print("n=%2d, fib: %8.6f, fibi:  %7.6f, percent: %10.2f" % (i, time1, time
2, time1/time2))
```

time1 is the time in seconds it takes for 3 calls to fib(n) and time2 respectively the time for fibi(). If we look at the results, we can see that calling fib(20) three times needs about 14 milliseconds. fibi(20) needs just 0.011 milliseconds for 3 calls. So fibi(20) is about 1300 times faster then fib(20). fib(40) needs already 215 seconds for three calls, while fibi(40) can do it in 0.016 milliseconds. fibi(40) is more than 13 millions times faster than fib(40).

In [ ]:
```
n= 1, fib: 0.000004, fibi:  0.000005, percent:       0.81
n= 2, fib: 0.000005, fibi:  0.000005, percent:       1.00
n= 3, fib: 0.000006, fibi:  0.000006, percent:       1.00
n= 4, fib: 0.000008, fibi:  0.000005, percent:       1.62
n= 5, fib: 0.000013, fibi:  0.000006, percent:       2.20
n= 6, fib: 0.000020, fibi:  0.000006, percent:       3.36
n= 7, fib: 0.000030, fibi:  0.000006, percent:       5.04
n= 8, fib: 0.000047, fibi:  0.000008, percent:       5.79
n= 9, fib: 0.000075, fibi:  0.000007, percent:      10.50
n=10, fib: 0.000118, fibi:  0.000007, percent:      16.50
n=11, fib: 0.000198, fibi:  0.000007, percent:      27.70
n=12, fib: 0.000287, fibi:  0.000007, percent:      41.52
n=13, fib: 0.000480, fibi:  0.000007, percent:      69.45
n=14, fib: 0.000780, fibi:  0.000007, percent:     112.83
n=15, fib: 0.001279, fibi:  0.000008, percent:     162.55
n=16, fib: 0.002059, fibi:  0.000009, percent:     233.41
n=17, fib: 0.003439, fibi:  0.000011, percent:     313.59
n=18, fib: 0.005794, fibi:  0.000012, percent:     486.04
n=19, fib: 0.009219, fibi:  0.000011, percent:     840.59
n=20, fib: 0.014366, fibi:  0.000011, percent:    1309.89
n=21, fib: 0.023137, fibi:  0.000013, percent:    1764.42
n=22, fib: 0.036963, fibi:  0.000013, percent:    2818.80
n=23, fib: 0.060626, fibi:  0.000012, percent:    4985.96
n=24, fib: 0.097643, fibi:  0.000013, percent:    7584.17
n=25, fib: 0.157224, fibi:  0.000013, percent:   11989.91
n=26, fib: 0.253764, fibi:  0.000013, percent:   19352.05
n=27, fib: 0.411353, fibi:  0.000012, percent:   34506.80
n=28, fib: 0.673918, fibi:  0.000014, percent:   47908.76
n=29, fib: 1.086484, fibi:  0.000015, percent:   72334.03
n=30, fib: 1.742688, fibi:  0.000014, percent:  123887.51
n=31, fib: 2.861763, fibi:  0.000014, percent:  203442.44
n=32, fib: 4.648224, fibi:  0.000015, percent:  309461.33
n=33, fib: 7.339578, fibi:  0.000014, percent:  521769.86
n=34, fib: 11.980462, fibi:  0.000014, percent:  851689.83
n=35, fib: 19.426206, fibi:  0.000016, percent: 1216110.64
n=36, fib: 30.840097, fibi:  0.000015, percent: 2053218.13
n=37, fib: 50.519086, fibi:  0.000016, percent: 3116064.78
n=38, fib: 81.822418, fibi:  0.000015, percent: 5447430.08
n=39, fib: 132.030006, fibi:  0.000018, percent: 7383653.09
n=40, fib: 215.091484, fibi:  0.000016, percent: 13465060.78
```

ANSWERS OF EXERCISES

Answer to Exercise 1

In [ ]:
```
print(1*0+1*0-6)
print(1*(2**2)+1*(2)-6)
```

Back to Exercise 1

Answer to Exercise 2

In [ ]:
```
months = ['January', 'February', 'March', 'April',\
          'May','June', 'July', 'August', 'September',\
          'October', 'November', 'December']
days = [31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31]
for i in range(12):
    print('The number of days in', months[i], 'is', days[i])
```

[Back to Exercise 2]()

Answer to Exercise 3

In [33]:
```
y=[]
for i in range(-20,21):
    if i <0:
        y.append(i**2)
    elif i>=0:
        y.append(-(i**2))

print(y)
```

```
[400, 361, 324, 289, 256, 225, 196, 169, 144, 121, 100, 81, 64, 49, 36, 25, 1
6, 9, 4, 1, 0, -1, -4, -9, -16, -25, -36, -49, -64, -81, -100, -121, -144, -1
69, -196, -225, -256, -289, -324, -361, -400]
```

[Back to Exercise 3]()

Answer to Exercise 4

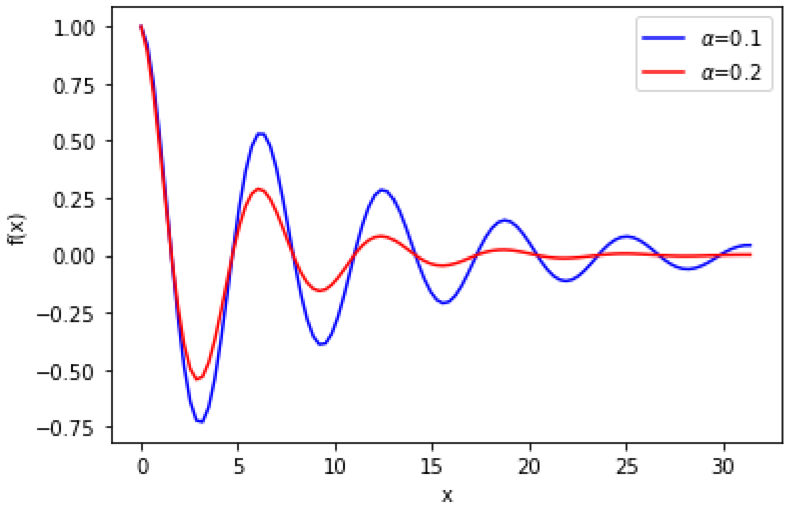
In [ ]:
```
data = np.array([1, 3, 2, 5, 7, 3, 4, 2])
runningtotal = np.zeros_like(data)
runningtotal[0] = data[0]
for i in range(1, len(data)):
    runningtotal[i] = runningtotal[i-1] + data[i]
print('data values:', data)
print('running total:', runningtotal)
print('running total with numpy:', np.cumsum(data))
```

[Back to Exercise 4]()

Answer to Exercise 5

In [13]:
```
def test(x, alpha):
    return np.exp(-alpha * x) * np.cos(x)

x = np.linspace(0, 10 * np.pi, 100)
y1 = test(x, 0.1)  # This function can be called with an array
y2 = test(x, 0.2)
plt.plot(x, y1,'b', label=r'$\alpha$=0.1') # if you specify a label, it will a
utomatically be used in the legend
plt.plot(x, y2,'r', label=r'$\alpha$=0.2')
plt.xlabel('x')
plt.ylabel('f(x)')
plt.legend();
```



[Back to Exercise 5]

Answer to Exercise 6

In [ ]:
```
def psi(x, y, U=1, R=1):
    r = np.sqrt(x ** 2 + y ** 2)
    if r < R:
        rv = 0.0
    else:
        theta = np.arctan2(y, x)
        rv = U * (r - R ** 2 / r) * np.sin(theta)
    return rv

print(psi(2, 4, U=2, R=1.5))
print(psi(0.5, 0, U=2, R=1.5))
```

[Back to Exercise 6]

Answer to Exercise 7

In [2]:
```
x,y = np.meshgrid(np.linspace(-4, 4, 100), np.linspace(-3, 3, 100))
psivec = np.vectorize(psi)
R = 1.5
p = psivec(x, y, U=2, R=R)
plt.contour(x, y, p, 50)
alpha = np.linspace(0, 2 * np.pi, 100)
plt.fill(R * np.cos(alpha), R * np.sin(alpha), ec='g', fc='g')
plt.axis('scaled')
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-2-8fd85ae290d7> in <module>
----> 1 x,y = np.meshgrid(np.linspace(-4, 4, 100), np.linspace(-3, 3, 100))
      2 psivec = np.vectorize(psi)
      3 R = 1.5
      4 p = psivec(x, y, U=2, R=R)
      5 plt.contour(x, y, p, 50)

NameError: name 'np' is not defined
```

[Back to Exercise 7]

Answer to Exercise 8

In [ ]:
```
def velocity(x, y, U=1, R=1):
    r = np.sqrt(x**2 + y**2)
    theta = np.arctan2(y, x)
    if r > R:
        vr =  U * (1 - R**2 / r**2) * np.cos(theta)
        vt = -U * (1 + R**2 / r**2) * np.sin(theta)
        vx = vr * np.cos(theta) - vt * np.sin(theta)
        vy = vr * np.sin(theta) + vt * np.cos(theta)
    else:
        vx,vy = 0.0,0.0
    return vx,vy

print('velocity at (2,3): ', velocity(2, 3, U=2, R=1.5))

x,y = np.meshgrid(np.linspace(-4, 4, 50), np.linspace(-3, 3, 50))
vx, vy = np.zeros((50, 50)), np.zeros((50, 50))
R = 1.5
for i in range(50):
    for j in range(50):
        vx[i,j], vy[i,j] = velocity(x[i,j], y[i,j], U=2, R=R)
alpha = np.linspace(0, 2 * np.pi, 100)
plt.fill(R * np.cos(alpha), R * np.sin(alpha), ec='g', fc='g')
plt.streamplot(x, y, vx, vy)
plt.axis('scaled')
plt.xlim(-4, 4)
plt.ylim(-3, 3);
```

[Back to Exercise 8]

Answer to Exercise 9

In [ ]:
```
def dfuncdx(x):
    if x < 0:
        rv = -np.sin(x)
    else:
        rv = -np.exp(-x)
    return rv

d = 1e-6
x = -1
dfdx = (func(x+d) - func(x-d)) / (2*d)
print('True value   ', dfuncdx(x))
print('Approx value ', dfdx)

x = 1
dfdx = (func(x+d) - func(x-d)) / (2*d)
print('True value   ', dfuncdx(x))
print('Approx value ', dfdx)
```

[Back to Exercise 9]

Answer to Exercise 10

In [ ]:
```
from scipy.special import erf
def F(x, mu=0, sigma=1, p=0):
    rv = 0.5 * (1.0 + erf((x - mu) / np.sqrt(2 * sigma ** 2)))
    return rv - p
print('x=mu gives F(x)=', F(2, mu=2, sigma=1))
print('x=mu+1.96sig gives:', F(2+1.96, mu=2, sigma=1))
x1 = fsolve(F, 3, args=(3, 2, 0.1))
x2 = fsolve(F, 3, args=(3, 2, 0.9))
print('x1, F(x1):', x1, F(x1, mu=3, sigma=2))
print('x2, F(x2):', x2, F(x2, mu=3, sigma=2))
```

[Back to Exercise 10]

Answer to Exercise 11

In [ ]:
```
def func1(x):
    return np.exp(-x)

def func2(x):
    return np.exp(-x) / x

from scipy.integrate import quad
print('func1:')
print('numerical integration:', quad(func1, 1, 5))
print('analytic integration:', -np.exp(-5) + np.exp(-1))

print('func2:')
print('numerical integration:', quad(func2, 1, 5))
print('wolframalpha result:', 0.218236)
```

Back to Exercise 11