[Braids and picture by Rosalie Neilson]

Twist, Turn, and Shout

The Symmetries of Braided Cords

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Land Acknowledgement

This talk is being broadcast from land that is part of the traditional territories of the Očeti Šakówiŋ (Sioux), Kiikaapoi (Kickapoo), and Myaamia (Miami) nations. These peoples, and many others, are still fighting for the rights promised them by treaties with the United States government.

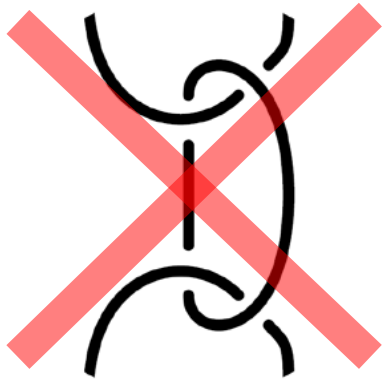
BIPOC lives and heritages matter.

What is a braid?

[Wikipedia User: Chris 73]

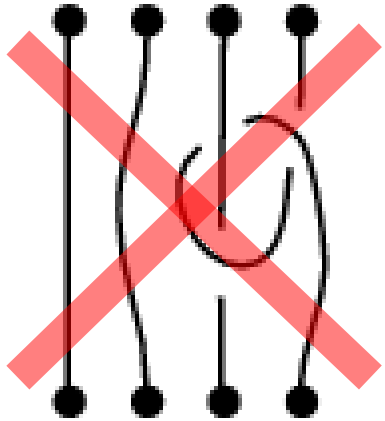
What is a braid? A nontechnical definition:

- Strands don't intersect.
- Strands go from bottom to top.



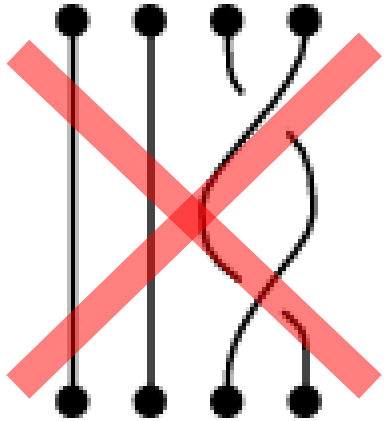
[Watson, 2004]

- Strands don't double back on themselves.



[Axel Boldt, Wikipedia]

- The braid doesn't fall apart into two or more pieces. (Otherwise: "pre-braid")



[Axel Boldt, Wikipedia]

Braids can be roughly categorized as "round" (polygonal) or "flat".

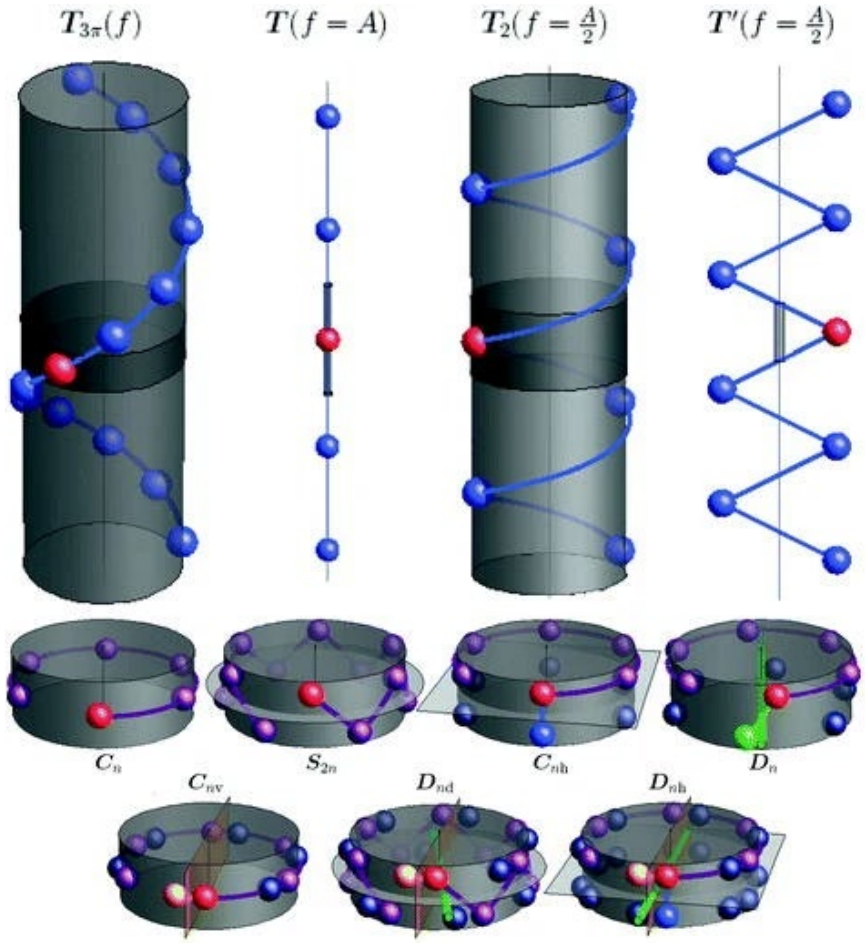
Rotational symmetry of degree > 2

[Frederick Brennan, Wikipedia]

No symmetry of degree > 2

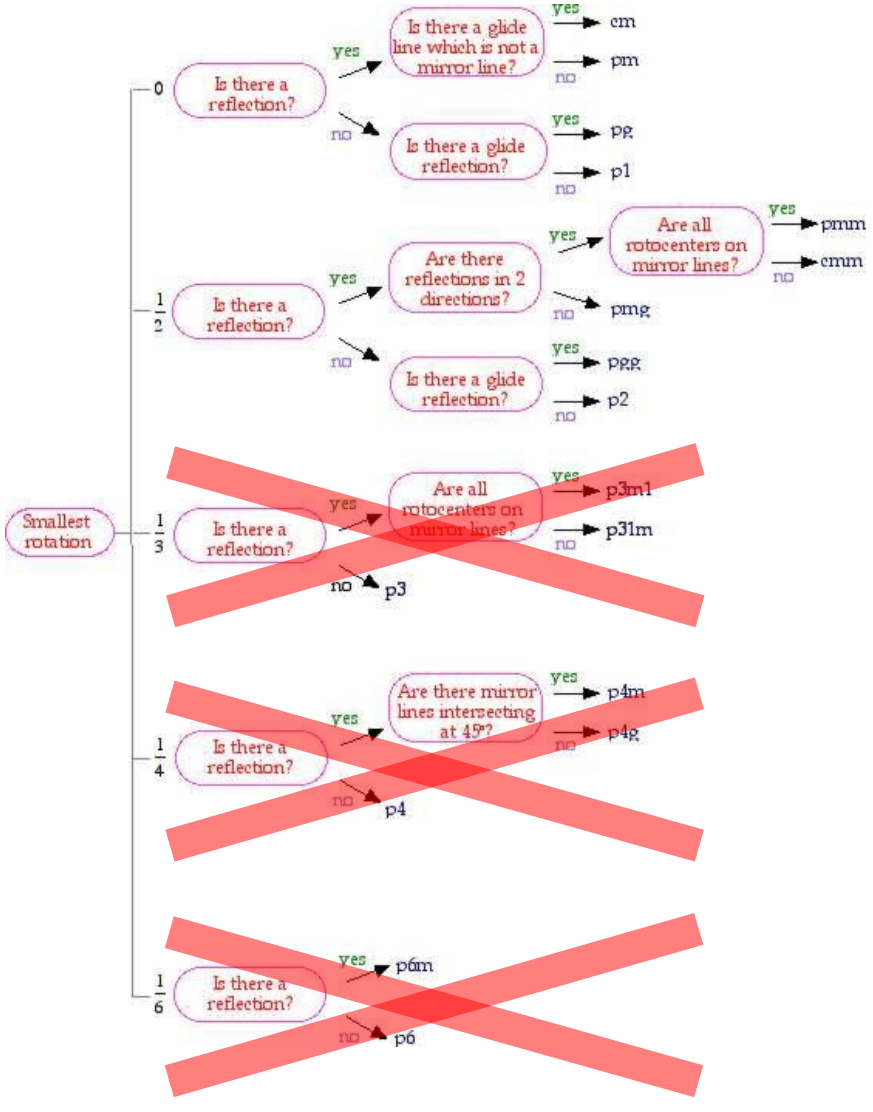
[Wikipedia User: Chris 73]

The group of symmetries of a round braid is basically a wallpaper group wrapped around a cylinder.



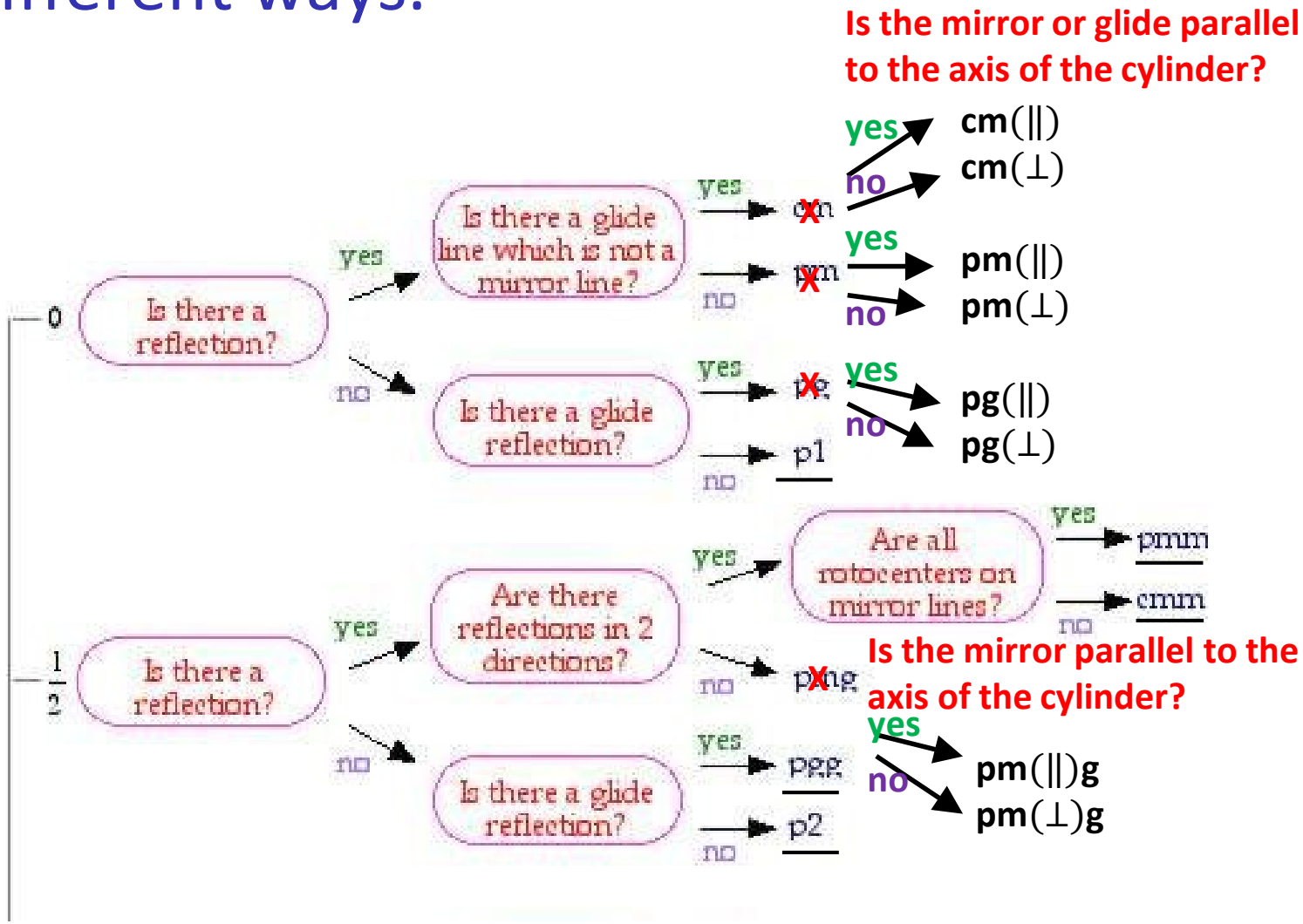
[Damnjanović and Milošsević, 2010]

Not all of the 17 wallpaper groups can be wrapped around a cylinder.



[After Gallian, 1998]

And some of the wallpaper groups can be wrapped in two different ways.

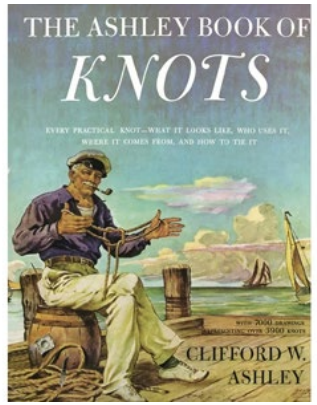


[After Gallian, 1998]

My examples will be primarily from four traditions:

- Japanese kumihimo (*Comprehensive Treatise of Braids,* Makiko Tada)

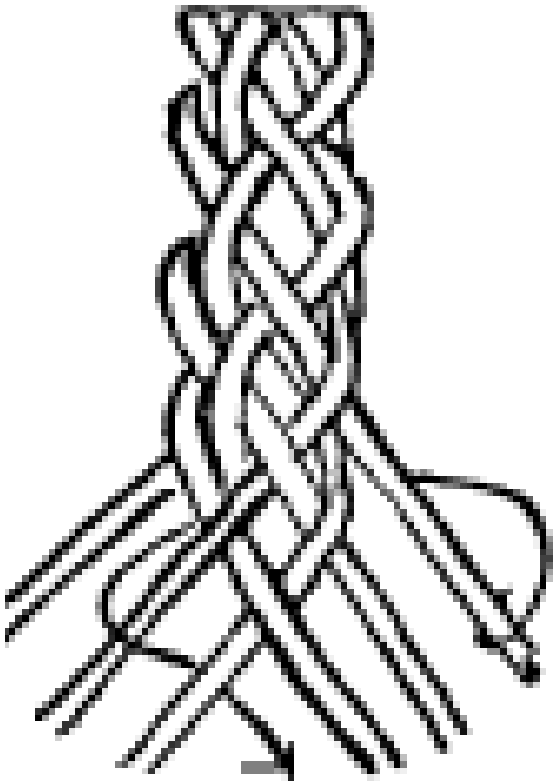
- British knot-tying (*The Ashley Book of Knots,* Clifford Ashley)



- Andean sling braiding (*Comprehensive Treatise of Braids,* Makiko Tada)

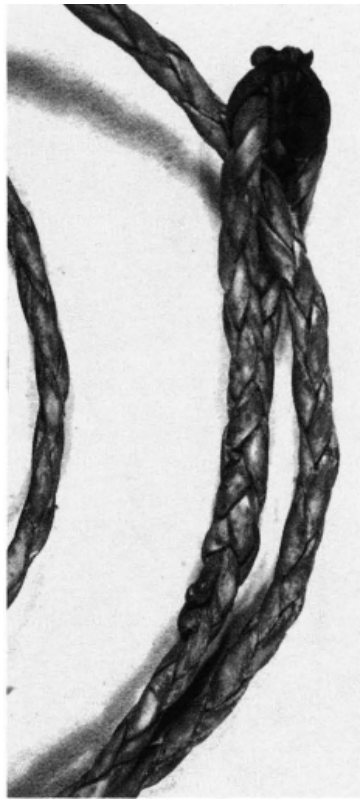
- Tejano rawhide and horsehair braiding (*Hecho en Tajas*, Joe S. Graham)

pgg is possibly the most common symmetry group.



[Six-Strand Round Sinnet, Ashley Book of Knots (ABOK)]

[Japanese Naiki, Rosalie Neilson]



[Rawhide *reata* (lariat) made by Félix Leal]

Breaking the symmetry of pgg gives pg(||) or pg(⊥).

[Tsuri-ito Kaku Yatsu, author photo]

[Tsuri-ito Edo Yatsu, author photo]

(Maybe this is cheating, but the Tsuri-ito technique is documented in Tado.)

p2 is also a very common symmetry group.

[Japanese Kongō Gumi, author photos]

[Andean Sling Braids,
Terry Newhouse Flynn]

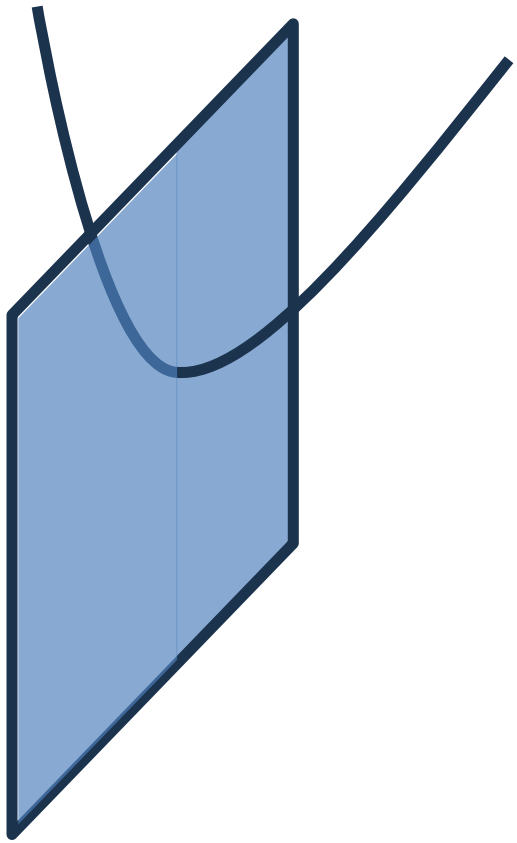
(Note that I am not basing the symmetries on the color pattern, only on the structure.)

p2 can be altered to pm(⊥)g by reversing directions.

[Kongo Gumi Kaeshi, Carolyn Oliver, topcenter.typepad.org]

A more complicated pattern of reversing directions could give pm(⊥). I have not yet found this documented.

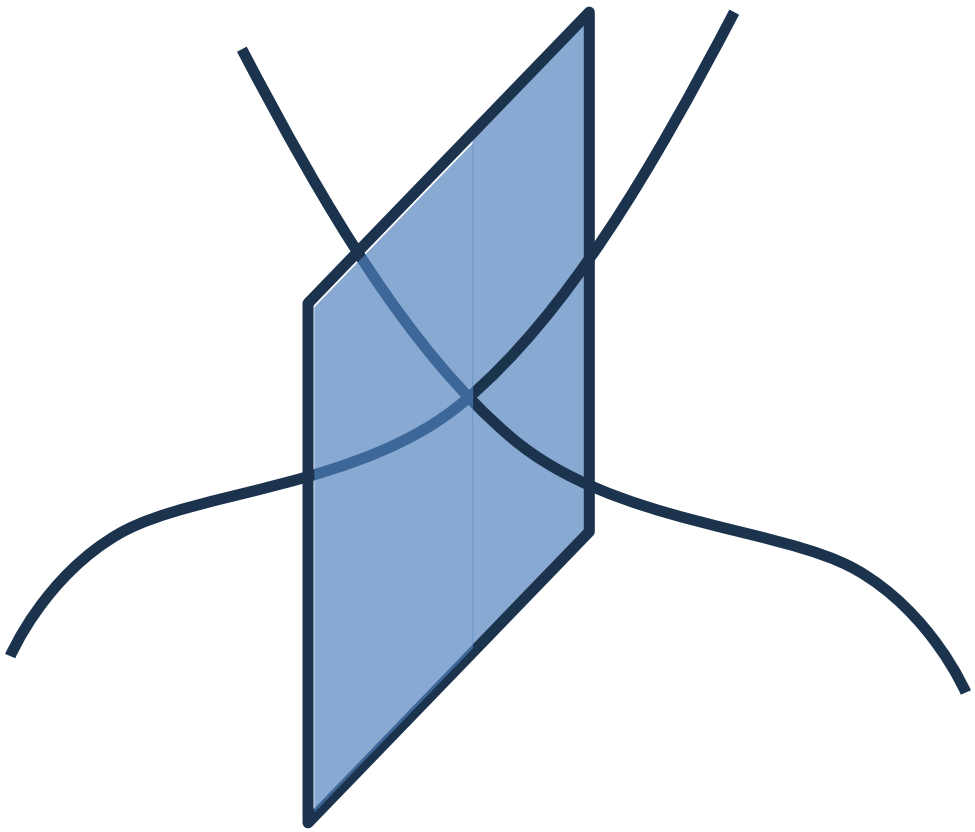
It is not possible for the complete structure of a braid to have a mirror parallel to the axis.



If a strand touches the mirror:

- it could double back ✕

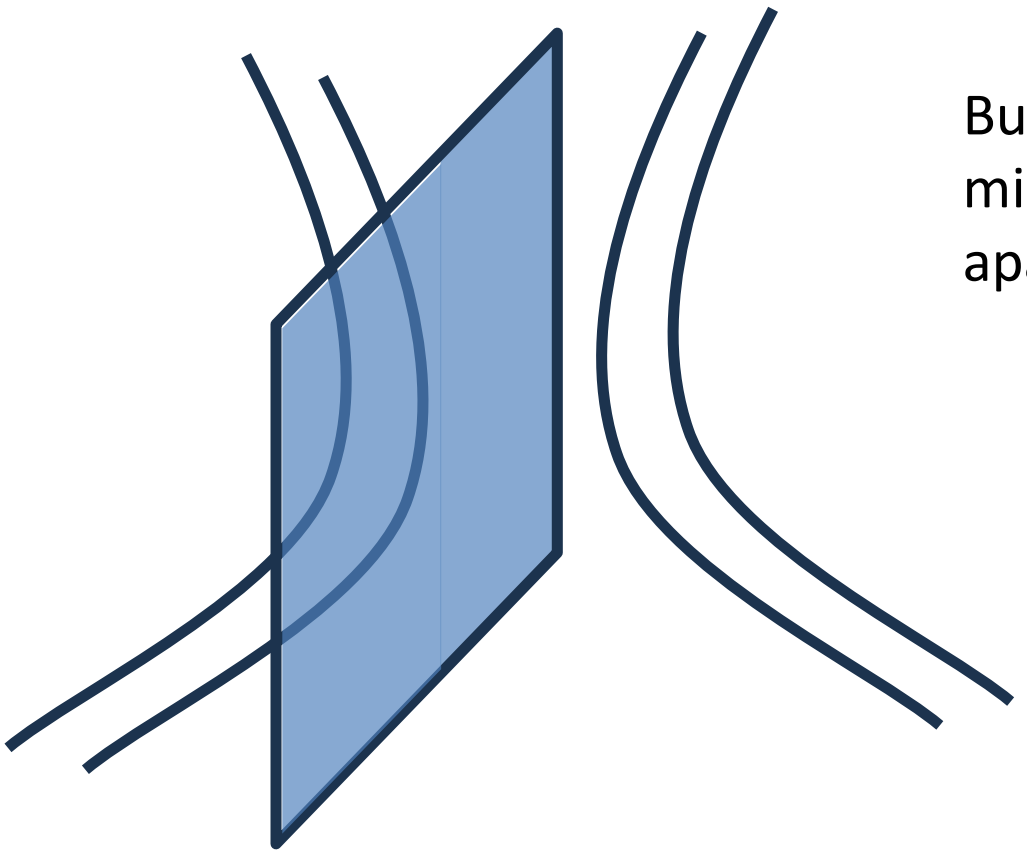
It is not possible for the structure of a braid to have a mirror parallel to the axis.



If a strand touches the mirror:

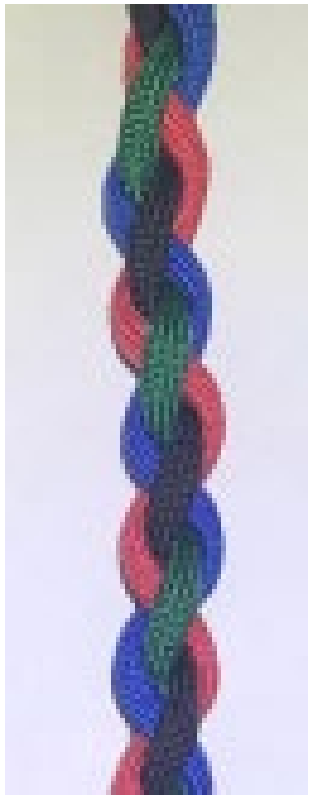
- it could double back, ✗
- or it could intersect another copy of itself. ✗

It is not possible for the structure of a braid to have a mirror parallel to the axis.



But if no strand touches the mirror, then the pre-braid falls apart. ✕

Nevertheless, many braids appear to be, e.g., cmm.

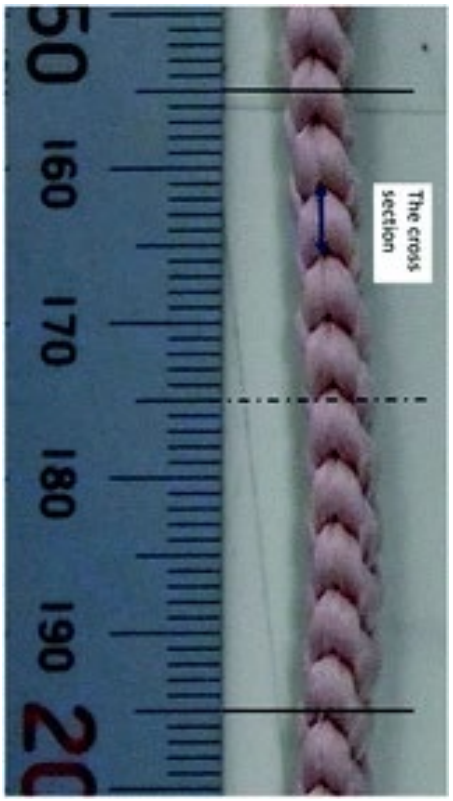


[Square Sinnet of four strands, a.k.a Maru-Yotsu, Brian Chan]

[Horsehair *cabresto* made by Mariano Martínez et al.]

[Ishibumi, author photo]

cm(||) and pmm can also be documented.

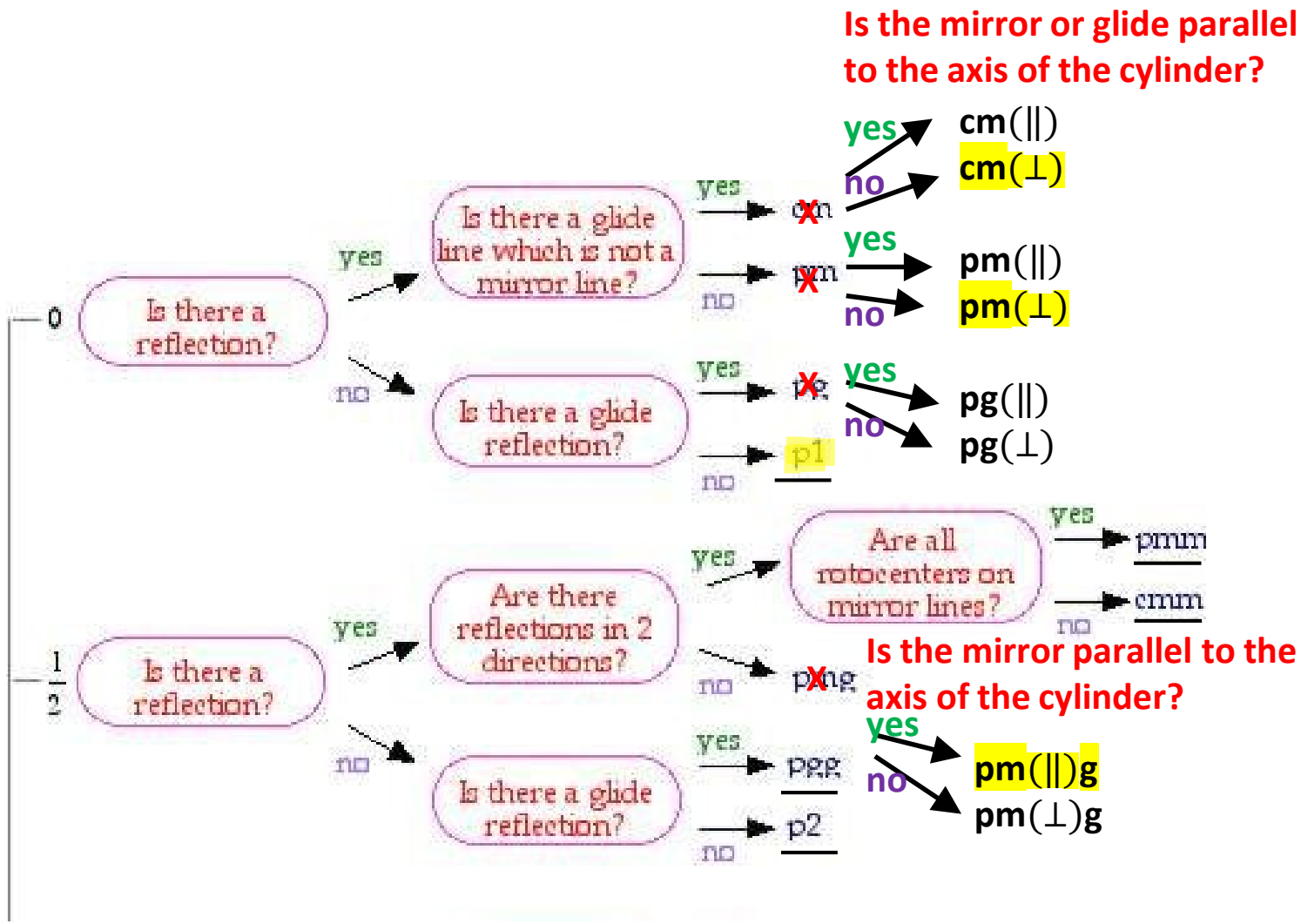


[Kusari Kaku-Yatsu, Kimura et al, 2019]

[Maru Genji, Aiko De Vleeschouwer, aizidiaries.blogspot.com]

[Tsukushi, *Comprehensive Treatise*]

What's missing (so far)?



[After Gallian, 1998]

Future work:

- Missing groups
- More cultures
- Flat braids
    - “Layer groups” of frieze patterns

Thanks for listening, and happy braiding!