Factorization in Deep Neural Networks

Course organisation

Sessions

1. Deep Learning and Transfer Learning,
2. Data Augmentation and Self Supervised Learning,
3. Quantization,
4. Pruning,
5. Factorization,
6. Distillation,
7. Embedded Software and Hardware for DL.
8. Presentations for challenge.

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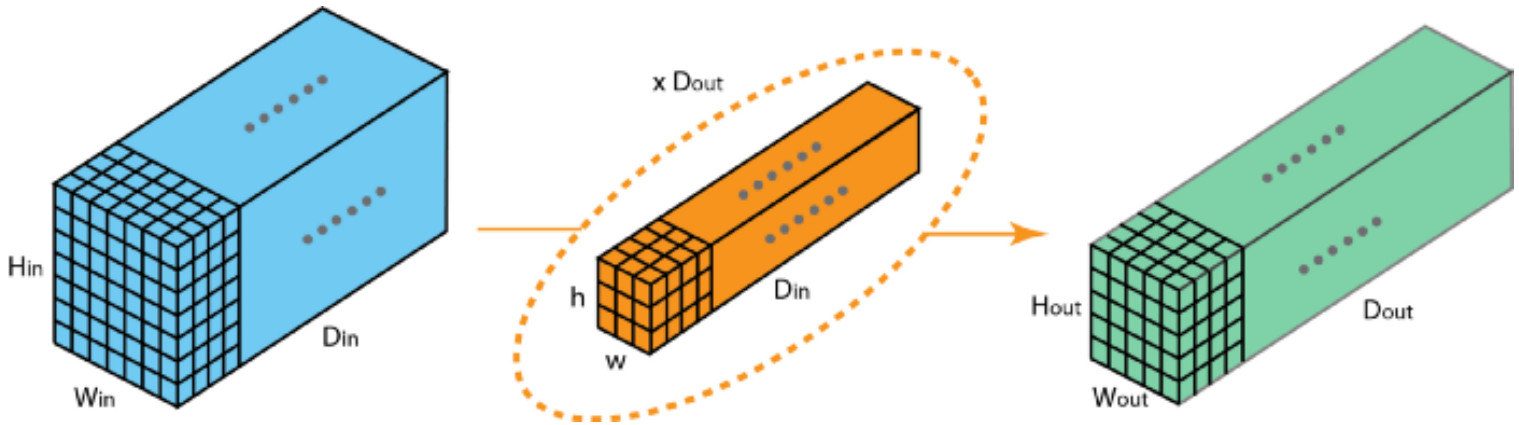
Factorization of Convolutional Networks

Why?

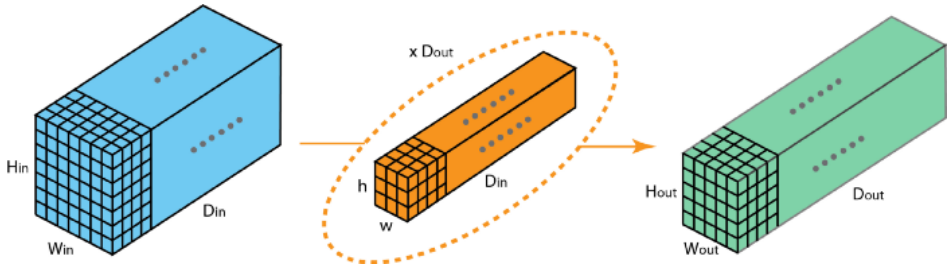
1. Reduce memory footprint
2. Reduce number of operations

How?

Modifying (decomposing, simplifying) convolutional filters structure



General principle



Complexity of 2D Convolutions

$N_{ops} = H_{out} \cdot W_{out} \cdot h \cdot w \cdot D_{in} \cdot D_{out}$
with kernel size (h, w), D_{in} the number of input feature maps, D_{out} the number of output feature maps of height H_{out} and width W_{out}.

To reduce the number of parameters, we can :

- Reduce the size of kernels
- Reduce the number of feature maps

Different strategies :

- Decompose kernels (Spatial separable convolutions)
- Depthwise Separable Convolutions
- Grouped Convolutions

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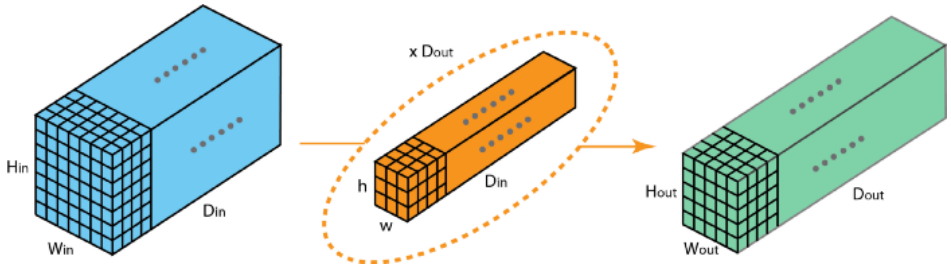

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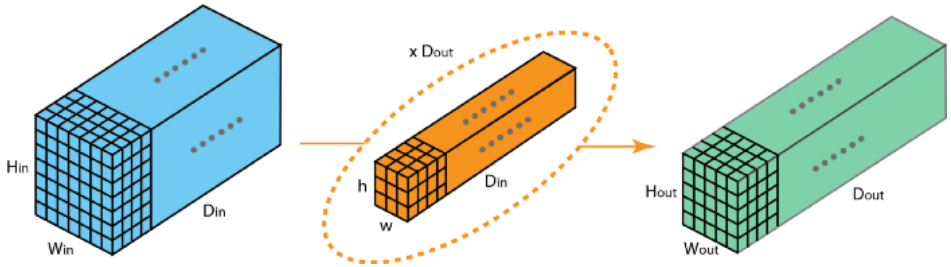

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Decompose Kernels

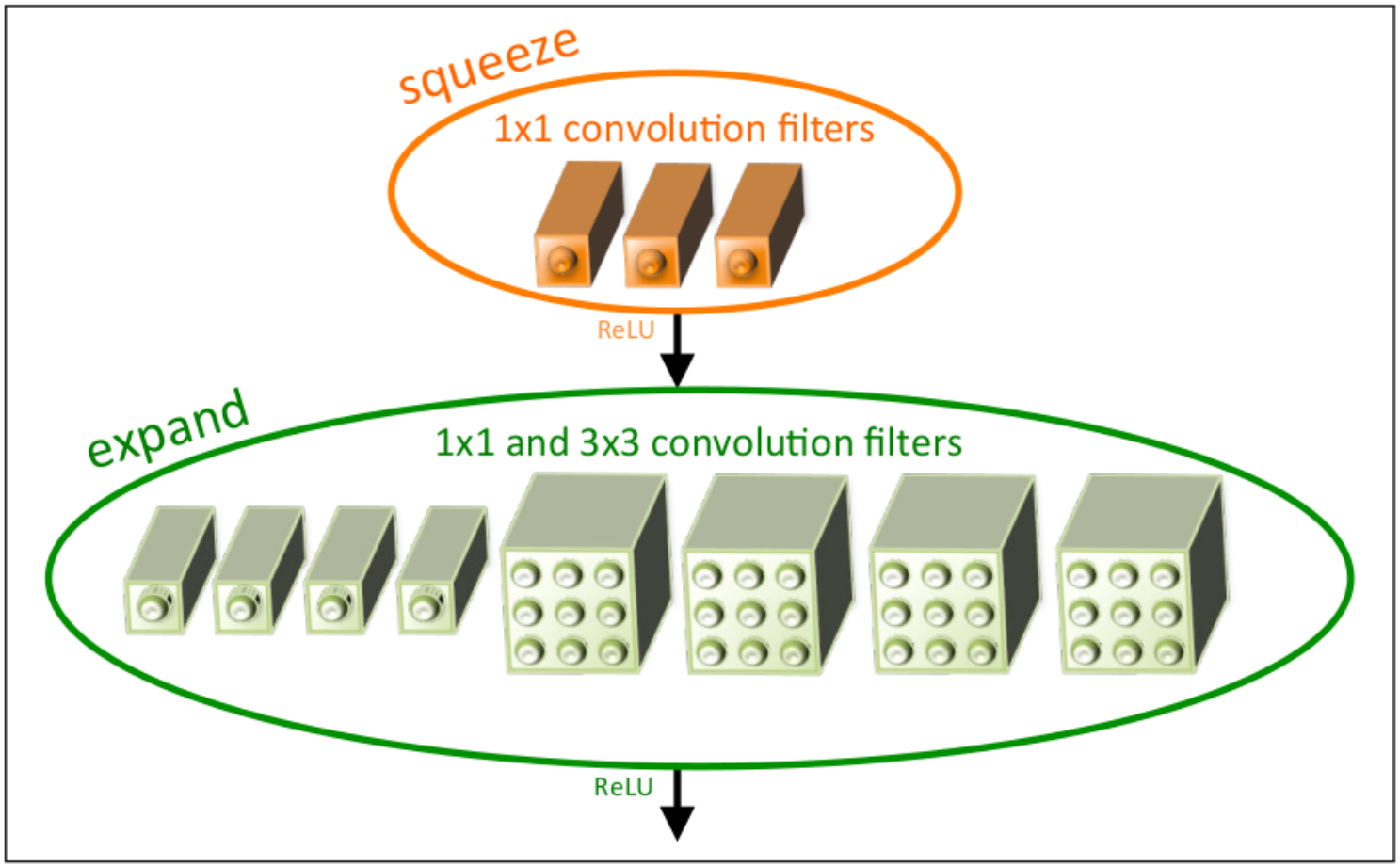
Spatially separable convolutions

To simplify, assuming $D_{in} = D_{out}$, decompose (h, w) kernel by $(h, 1)$ and $(1, w)$:
$N_{ops} = h \cdot 1 \cdot D_{in} + 1 \cdot w \cdot D_{in} = (h + w) \cdot D_{in}$
with kernel size (h, w), D_{in} input and out number of feature maps.

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

Example: SqueezeNet

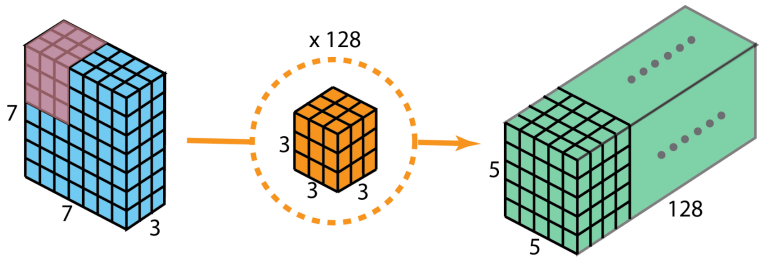
Introducing the Fire Module



Iandola et al. 2016, `https://arxiv.org/abs/1602.07360`

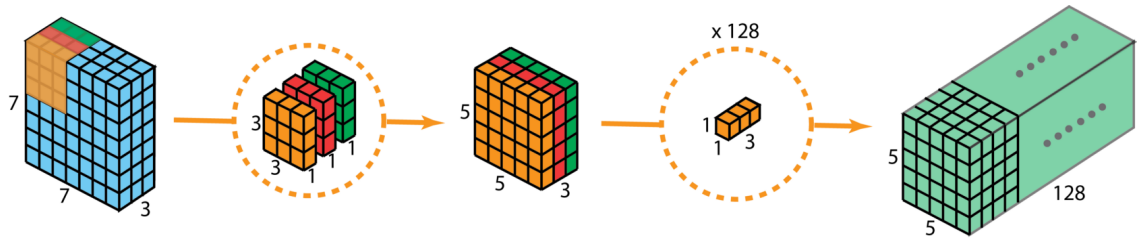
Depthwise separable convolutions

Instead of learning parameters that recombine all input feature maps to compute each output feature map:



$$N_{mul}^{N} = H_{out} \cdot W_{out} \cdot h \cdot w \cdot D_{in} \cdot D_{out} = 5 \cdot 5 \cdot 3 \cdot 3 \cdot 3 \cdot 128 = 86400$$

One can separate the operations into two steps:



$$N_{mul}^{D} = H_{out} \cdot W_{out} \cdot h \cdot w \cdot D_{in} \cdot 1 + H_{out} \cdot W_{out} \cdot 1 \cdot 1 \cdot D_{in} \cdot D_{out}$$

$$N_{mul}^{D} = 5 \cdot 5 \cdot 3 \cdot 3 \cdot 3 \cdot 1 + 5 \cdot 5 \cdot 1 \cdot 1 \cdot 3 \cdot 128 = 10275$$

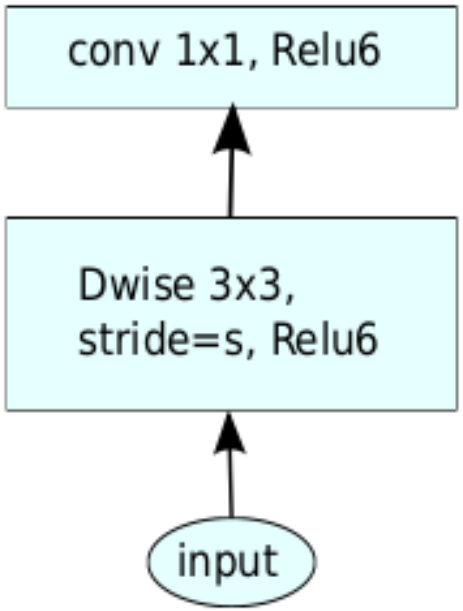
$$N_{mul}^{D} = (\frac{1}{D_{out}} + \frac{1}{h^2}) \cdot N_{mul}^{N}, h = w$$

`https://towardsdatascience.com/`

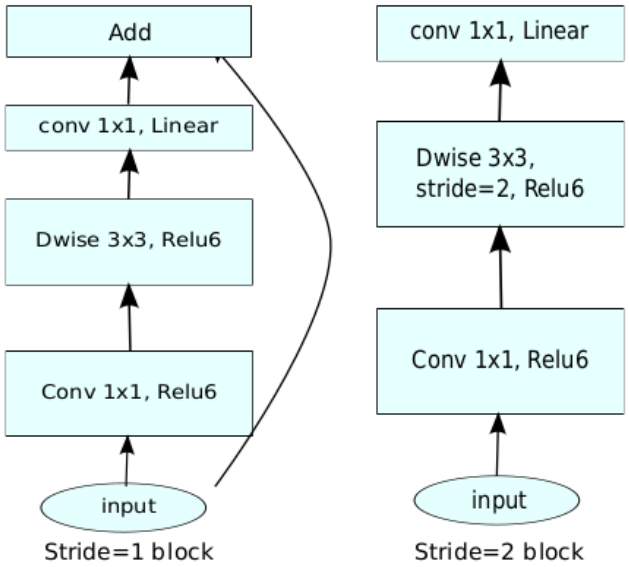
`a-comprehensive-introduction-to-different-types-of-convolutions-in-deep-learning-669281e58215`

Example: MobileNet

MobileNetV1 (2017)



MobileNetV2 (2017)



https://arxiv.org/abs/1704.04861, https://arxiv.org/abs/1801.04381

Example: MobileNet

Table 9. Smaller MobileNet Comparison to Popular Models

Model	ImageNet Accuracy	Million Mult-Adds	Million Parameters
0.50 MobileNet-160	60.2%	76	1.32
Squeezenet	57.5%	1700	1.25
AlexNet	57.2%	720	60

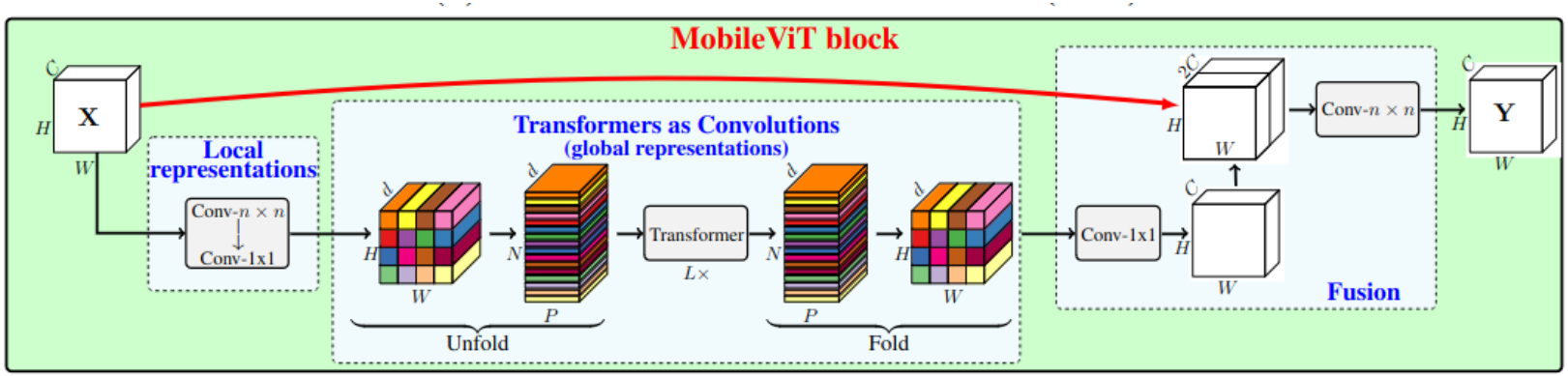
Table 10. MobileNet for Stanford Dogs

Model	Top-1 Accuracy	Million Mult-Adds	Million Parameters
Inception V3 [18]	84%	5000	23.2
1.0 MobileNet-224	83.3%	569	3.3
0.75 MobileNet-224	81.9%	325	1.9
1.0 MobileNet-192	81.9%	418	3.3
0.75 MobileNet-192	80.5%	239	1.9

`https://arxiv.org/abs/1704.04861`, `https://arxiv.org/abs/1801.04381`

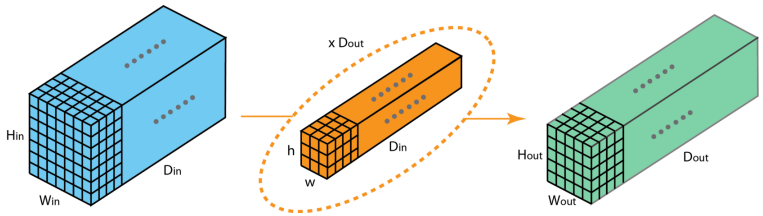
More recent examples: MobileVit (2022) and VMamba (2024)

- **MobileVit**, a Light weight Vision Transformer
  `https://arxiv.org/pdf/2110.02178.pdf`
- **VMamba**, another efficient alternative to Vision Transformer
  `https://arxiv.org/pdf/2401.10166.pdf`



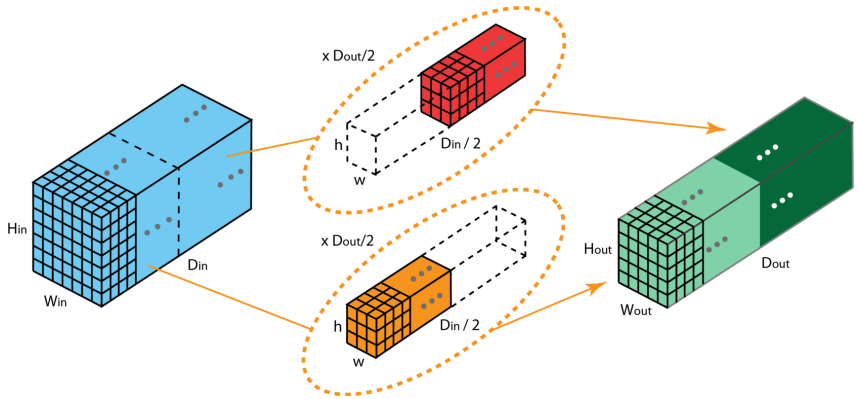
Grouped Convolutions

Instead of learning parameters that recombine all input feature maps to compute each output feature map:



$$N_{mul}^{N} = H_{out} \cdot W_{out} \cdot h \cdot w \cdot D_{in} \cdot D_{out}$$

One can divide the kernels into multiple groups:



$$N_{mul}^{G} = H_{out} \cdot W_{out} \cdot h \cdot w \cdot \frac{D_{in}}{2} \cdot \frac{D_{out}}{2} + H_{out} \cdot W_{out} \cdot h \cdot w \cdot \frac{D_{in}}{2} \cdot \frac{D_{out}}{2}$$

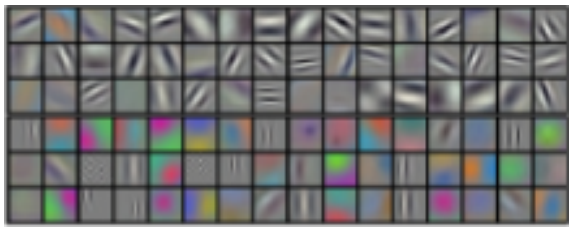
$$N_{mul}^{G} = \frac{N_{mul}^{N}}{2}$$

`https://towardsdatascience.com/`

`a-comprehensive-introduction-to-different-types-of-convolutions-in-deep-learning-669281e58215`

Examples

AlexNet filters



ResNeXt block

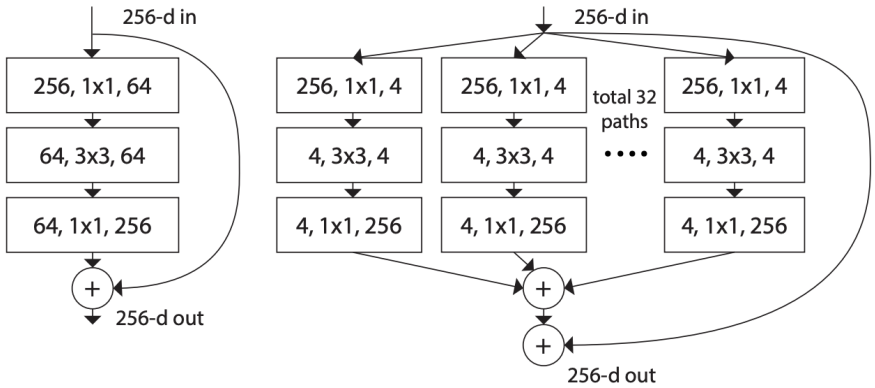


Figure 1. **Left**: A block of ResNet [14]. **Right**: A block of ResNeXt with cardinality = 32, with roughly the same complexity. A layer is shown as (# in channels, filter size, # out channels).

`https://arxiv.org/abs/1704.04861`, `https://arxiv.org/abs/1801.04381`,`https://arxiv.org/abs/1611.05431`

Combining Factorization with other Techniques: Attention based Pruning

Introducing Shift Attention Layer

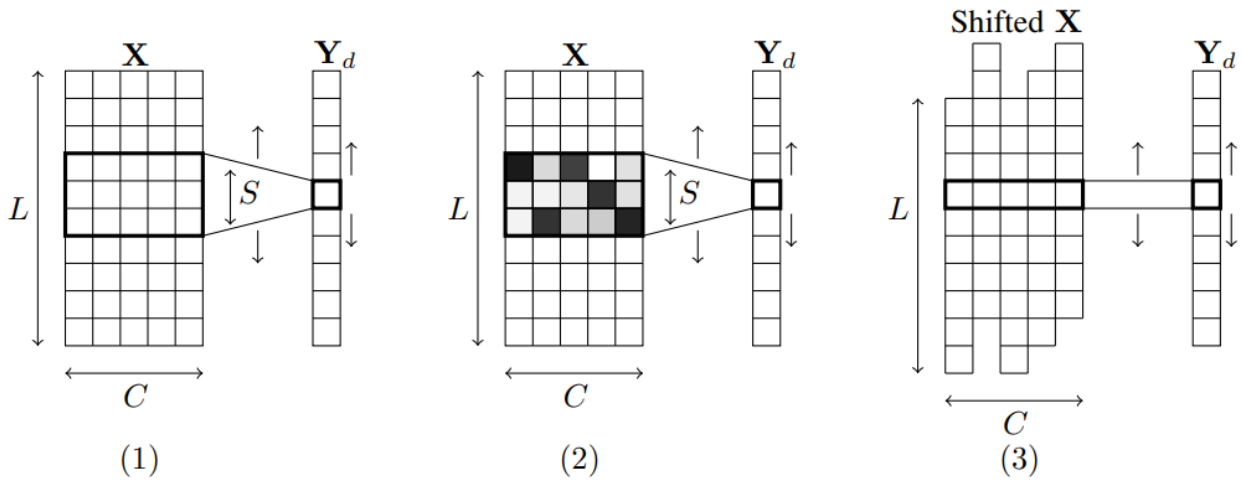


Figure 1: Overview of the proposed method: we depict here the computation for a single output feature map d, considering a 1d convolution and its associated shift version. Panel (1) represents a standard convolutional operation: the weight filter $\mathbf{W}_{d,\cdot,\cdot}$ containing SC weights is moved along the spatial dimension (L) of the input to produce each output in $\mathbf{Y}_d$. In panel (2), we depict the attention tensor $\mathbf{A}$ on top of the weight filter: the darker the cell, the most important the corresponding weight has been identified to be. At the end of the training process, $\mathbf{A}$ should contain only binary values with a single 1 per slice $\mathbf{A}_{d,c,\cdot}$. In panel (3), we depict the corresponding obtained shift layer: for each slice along the input feature maps (C), the cell with the highest attention is kept and the others are disregarded. As a consequence, the initial convolution with a kernel size S has been replaced by a convolution with a kernel size 1 on a shifted version of the input $\mathbf{X}$. As such, the resulting operation in panel (3) is exactly the same as the shift layer introduced in Wu et al. [2017], but here the shifts have been trained instead of being arbitrarily predetermined.

Hacene et al. 2019, `https://arxiv.org/abs/1905.12300`

Combining Factorization with other Techniques: Attention based Pruning

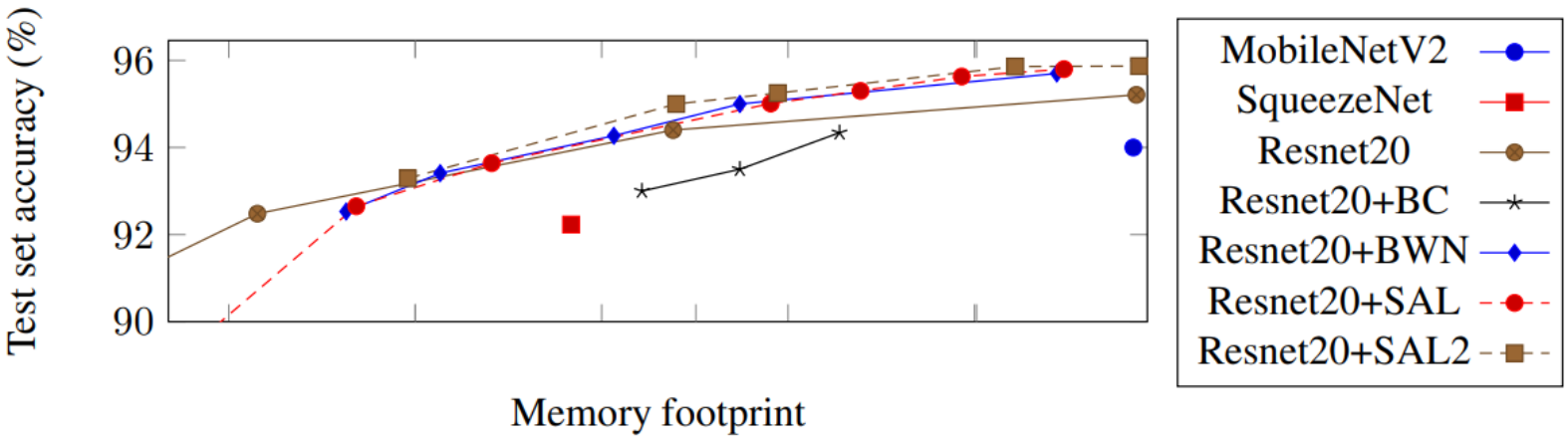
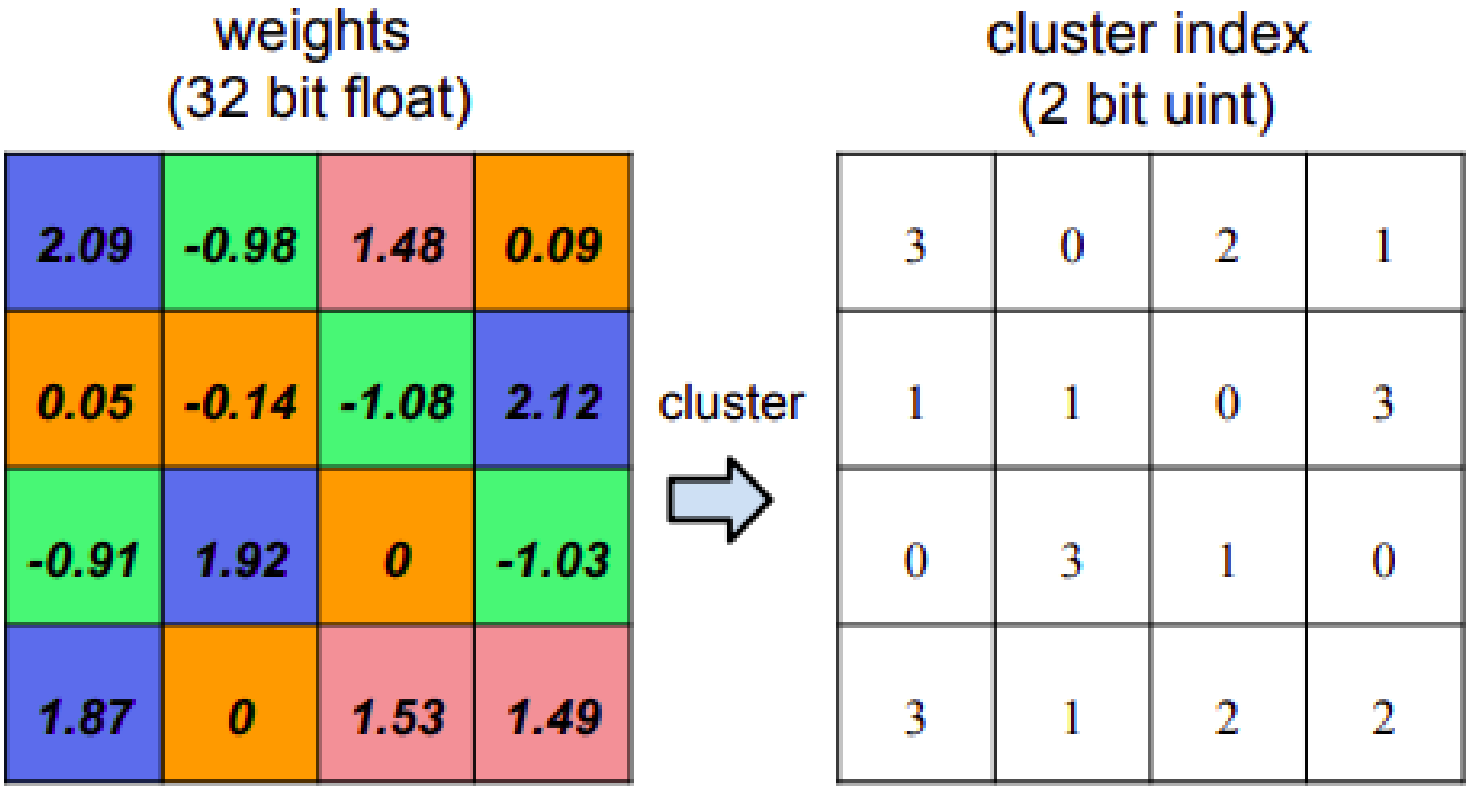


Figure 7: Evolution of accuracy when applying compression methods on different DNN architectures trained on CIFAR10.

Hacene et al. 2019, `https://arxiv.org/abs/1905.12300`

Combining Factorization with other Techniques: using clustering to share kernel weights



from `https://arxiv.org/abs/1510.00149`

Lab Session

Factorizing a CNNs using Pytorch

- Read carefully the documentation of **conv2d** (`https://pytorch.org/docs/stable/generated/torch.nn.Conv2d.html#conv2d`) and play with the parameters *in channels*, *out channels* and *groups* to implement factorised convolutions
- Have a look at the MobileNet implementation for CIFAR10 (`https://github.com/kuangliu/pytorch-cifar/blob/master/models/mobilenet.py`)

Work for your Long Project

- If you haven't done it yet, familiarise yourself with the micronet-resources folder (profile.py and challenge rules)
- Combine/test different strategies to improve your MicroNet score!

1 Overview of unsupervised learning

- Clustering
- Decomposition using Sparse Dictionary Learning

Unsupervised learning

Goal

Discover patterns/structure in X,

Unsupervised learning

- Unsupervised = no expert, no labels,
- Two main approaches:
  - Clustering = find a partition of X in K subsets,
  - Decomposition using K vectors.
- Applications :
  - Quantization,
  - Visualization...

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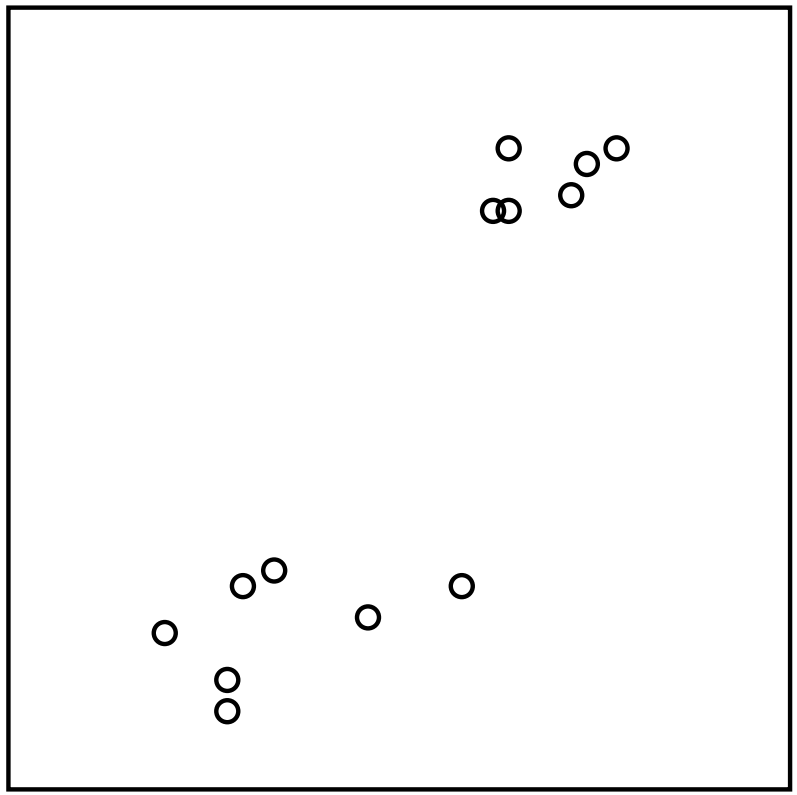

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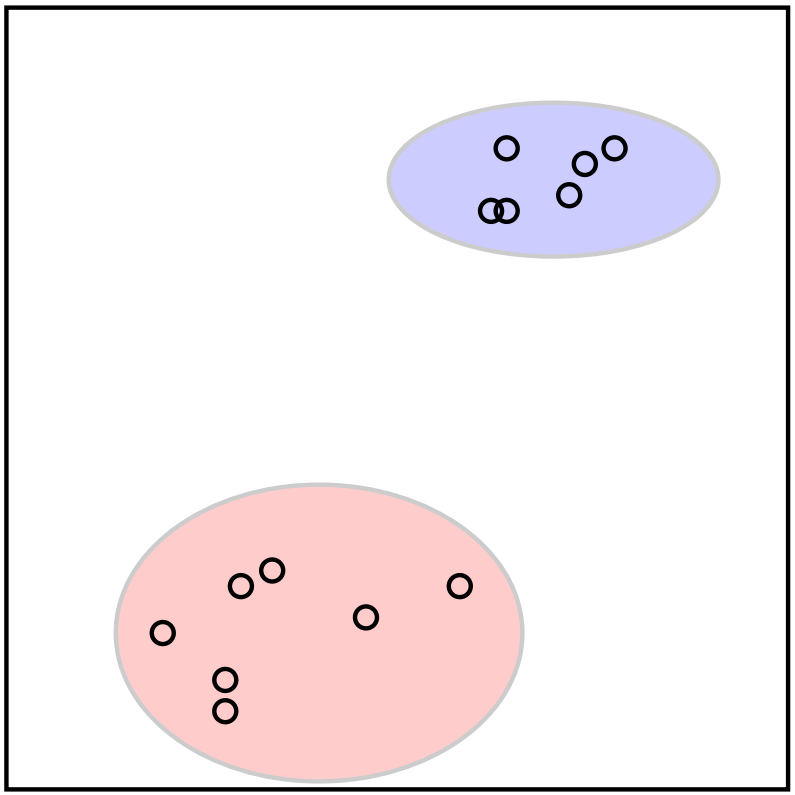

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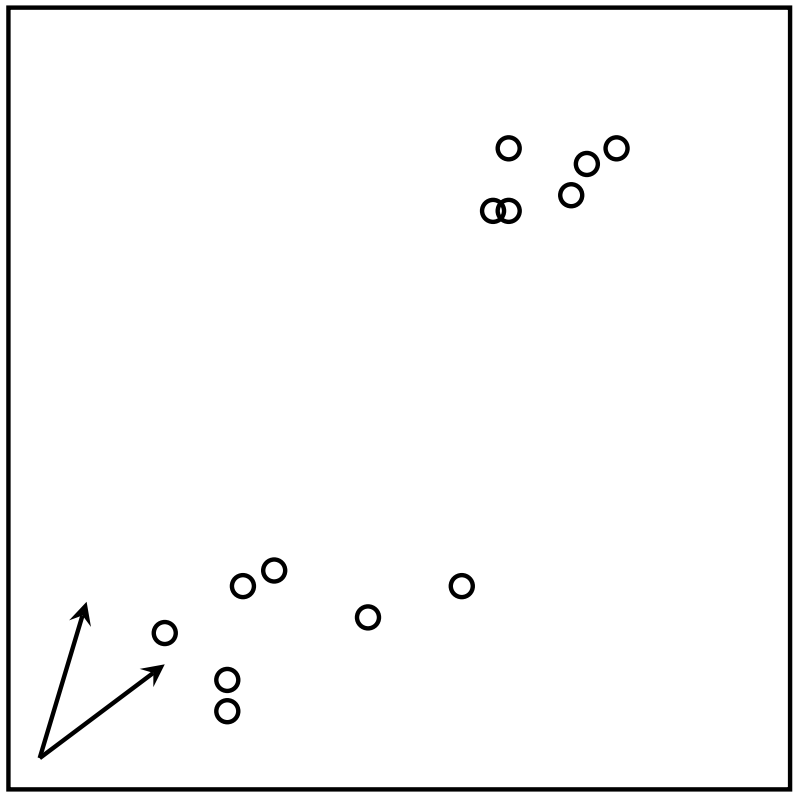

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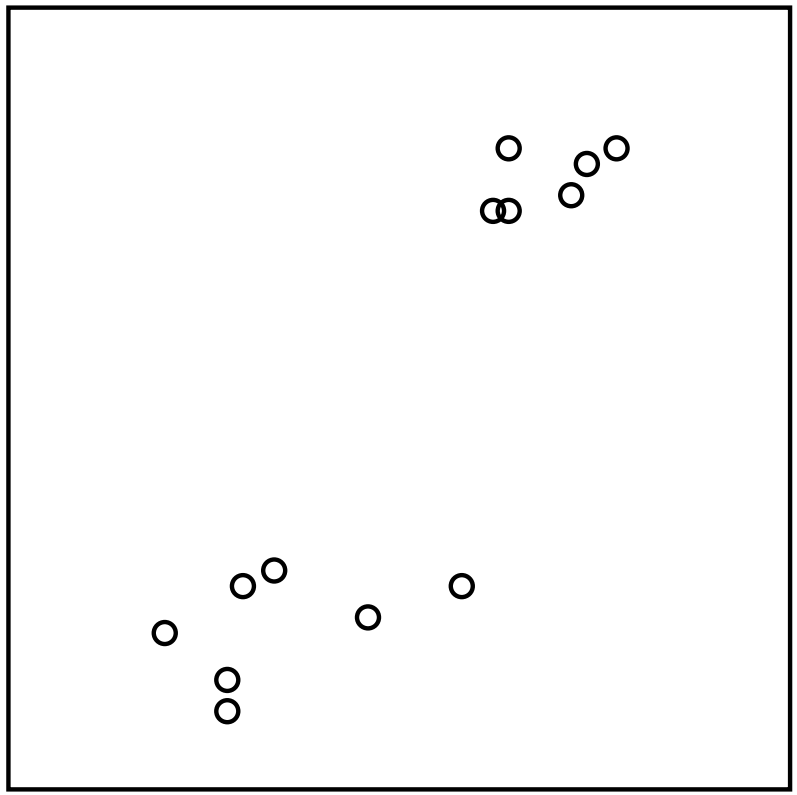


Example: clustering using L_2 norm

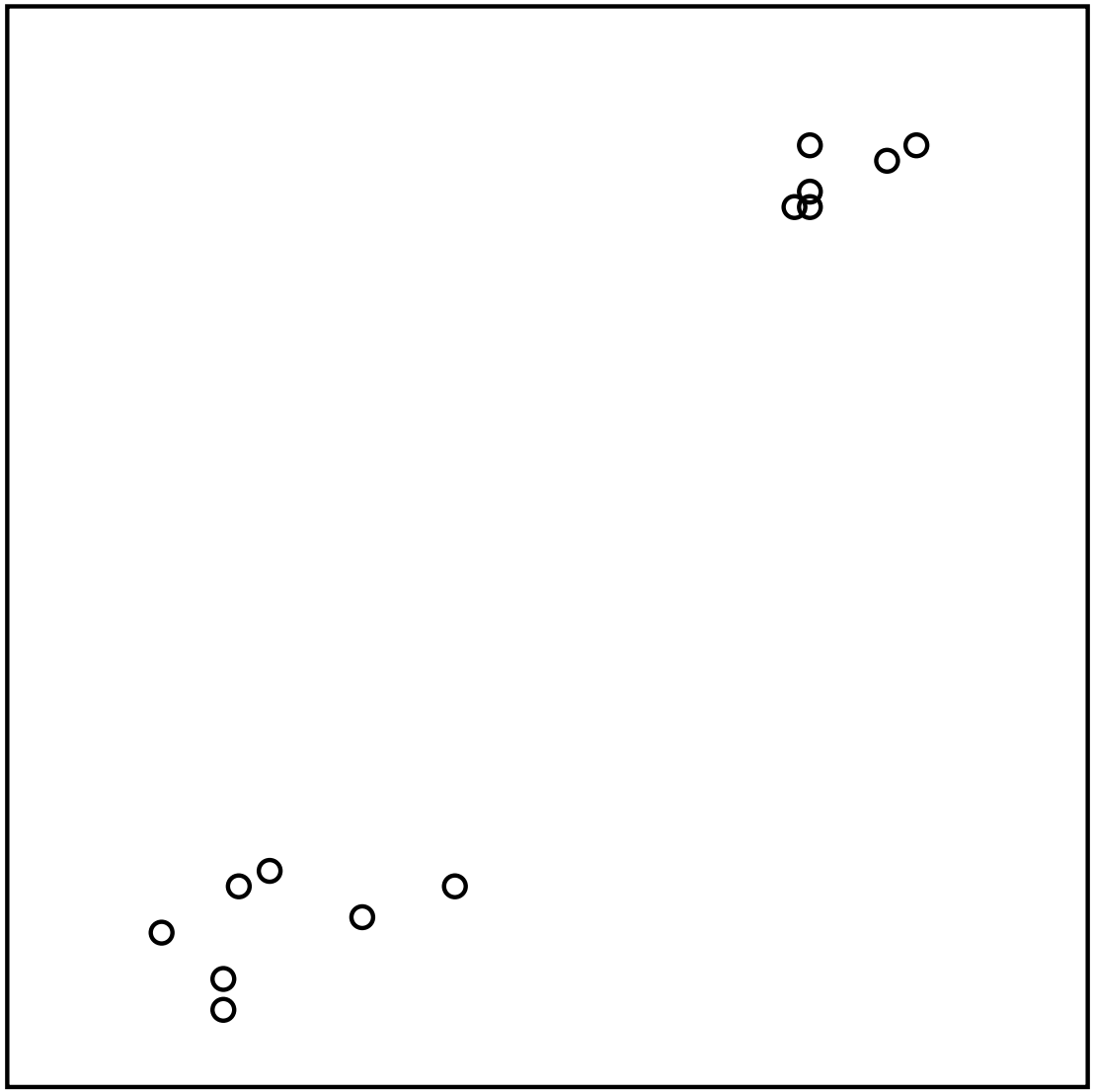
An example to perform clustering is to rely on distances to centroids. We define K *cluster centroids* $\Omega_k, \forall k \in [1..K]$

Definitions

We denote $q : \mathbb{R}^d \to [1..K]$ a function that associates a vector $\mathbf{x}$ with the index of (one of) its closest centroid $q(\mathbf{x})$. Formally:

- $\forall k \in [1..K], \Omega_k \in \mathbb{R}^d$
- $\forall \mathbf{x} \in X, \forall j \in [1..K], \|\mathbf{x} - \Omega_{q(\mathbf{x})}\|_2 \leq \|\mathbf{x} - \Omega_j\|_2$
- Error $E(q) \triangleq \sum_{\mathbf{x} \in X} \|\mathbf{x} - \Omega_{q(\mathbf{x})}\|_2$
- $X = \bigcup_k \underbrace{\{\mathbf{x} \in X, q(\mathbf{x}) = k\}}_{\text{cluster } k}$

Example: clustering using L_2 norm



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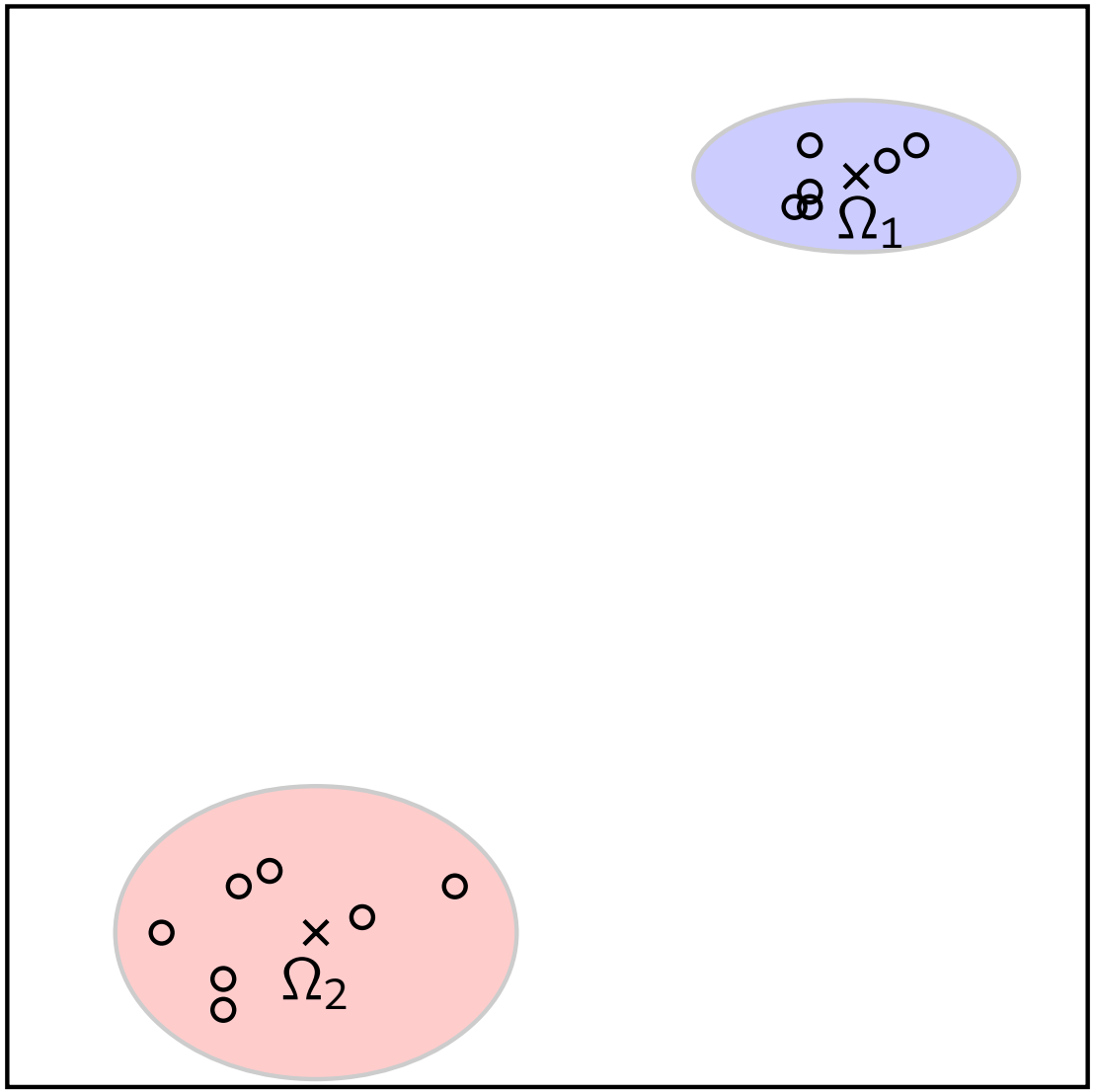


Clustering using L_2 norm

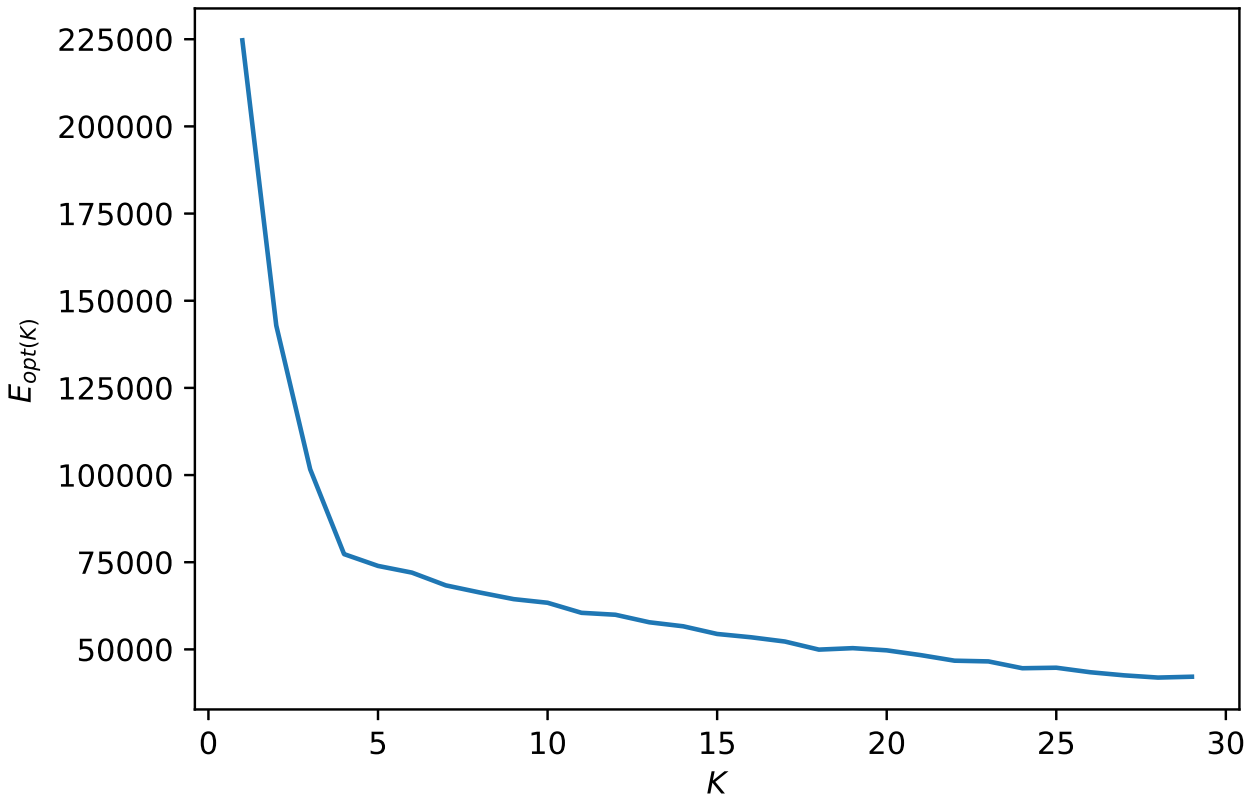
Quantizing MNIST

- Replace $\mathbf{x}$ by $\Omega_{k(\mathbf{x})}$
- Compression factor $\kappa = 1 - K/N$

Clustering using L_2 norm

Choosing K

- Finding a compromise between error and compression,
- Simple practical method : "elbow".



Sparse Dictionary Learning

Definitions

Dictionary learning solves the following matrix factorization problem:

- The set X is considered as a matrix $X \in \mathcal{M}_{N\times d}(\mathbb{R})$,
- We consider decompositions using a dictionary $V \in \mathcal{M}_{K\times d}(\mathbb{R})$ and a code $U \in \mathcal{M}_{N\times k}(\mathbb{R})$, with the lines of V being with norm 1,
- Error $E(U, V) \triangleq \|X - UV\|_2 + \alpha\|U\|_1$
- Training: find U^*, V^* that minimizes $E(U^*, V^*)$
- α is a sparsity control parameter that enforces codes with soft (ℓ_1) sparsity