Course 2: Supervised Learning

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Bretagne-Pays de la Loire
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Summary

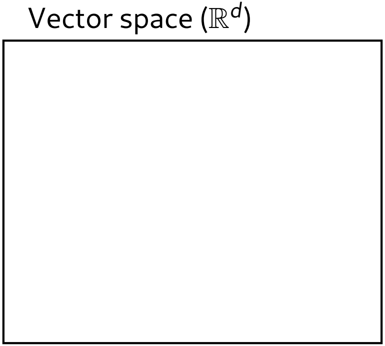
Last session

1. AI definition
2. Applications & Open Issues
3. Deep learning
4. Foundation models

Today's session

- Learning from labeled examples
- Challenges of supervised learning

Notations



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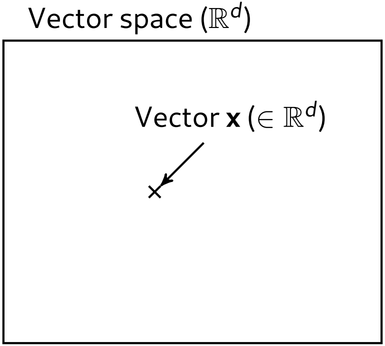

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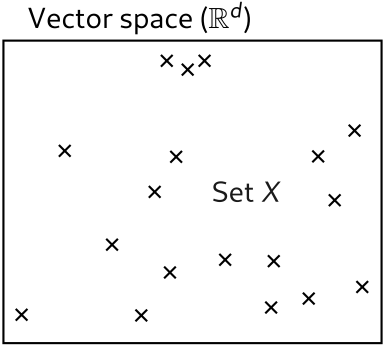
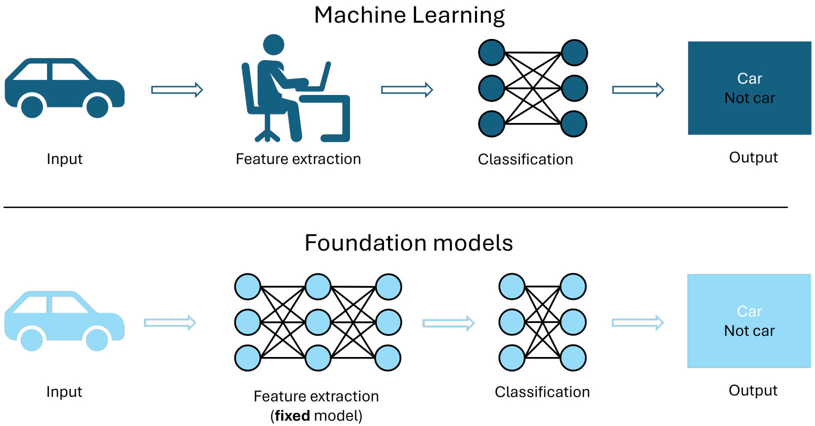


What is the vector x? (1/2)



What is the vector x? (2/2)

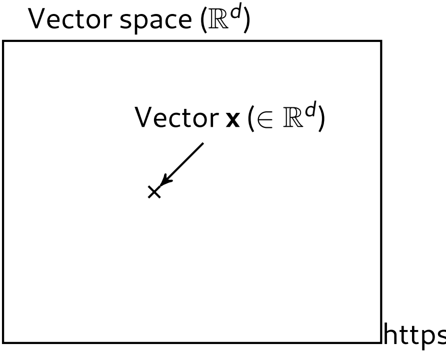
Traditional Machine Learning

x is the data, or a small transformation of the data
Ex: images, or edges in the image

The era of Foundation models

x is the projection of data in an **embedding** space

- Advantage: richer semantically than the original image
- Disadvantage: Not interpretable nor easily understandable



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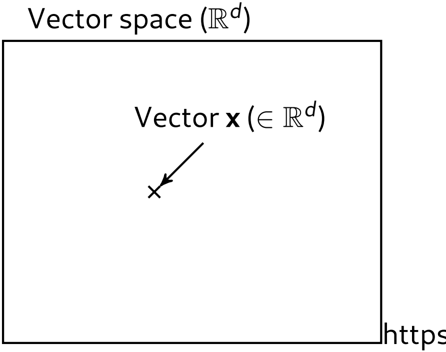

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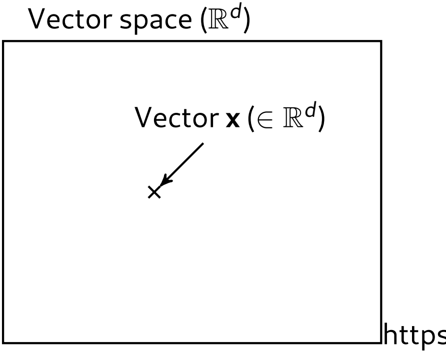

In this class, we will illustrate the concepts using images... BUT in the lab, you will use embedding spaces (the future is probably there)

Supervised learning

Definition

Given:

- $\mathbf{x}$: inputs (raw signals or feature vectors (e.g. embeddings))
- $\hat{\mathbf{y}}$: **labels** (annotated by humans)

Learn:

- a function $f()$ such that $\hat{\mathbf{y}} \approx \mathrm{f}(\mathbf{x})$
  $\Rightarrow$ $f()$ is **learned** by the Machine Learning algorithm
- Ideally, $f()$ should **generalize** ($\neq$ memorize) to unlabeled examples.

$\mathrm{f}(\mathbf{x})$: 

$\hat{\mathbf{y}}$: "cat"

Supervised learning

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f(**x**):  Learn *f*() → $\hat{\mathbf{y}}$: "cat"

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f($\mathbf{x}$):

Learn $f()$ $\longrightarrow$ $\hat{\mathbf{y}}$: "cat"

Supervised learning: in practice

x: 

$$\xrightarrow{f()} \quad f(\mathbf{x}) = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}$$

Supervised learning: in practice

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$$\xrightarrow{\quad f() \quad} \mathrm{f}(\mathbf{x}) = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} \neq \hat{\mathbf{y}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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Loss

- Here, labels are encoded as one-hot-bit vectors,
- We compute a **loss** $\mathcal{L}(f(\mathbf{x}), \hat{\mathbf{y}})$
- Training consist in minimizing the loss!

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- We compute a **loss** $\mathcal{L}(\mathrm{f}(\mathbf{x}), \hat{\mathbf{y}})$
  - Euclidean distance $\mathcal{L}(\mathrm{f}(\mathbf{x}), \hat{\mathbf{y}}) = \sum_{i=1}^{D} (\mathrm{f}(\mathbf{x})_i - \hat{\mathbf{y}}_i)^2$
  - Cross-entropy: $\mathcal{L}(\mathrm{f}(\mathbf{x}), \hat{\mathbf{y}}) = -\sum_{i=1}^{D} \hat{\mathbf{y}}_i \log(\mathrm{f}(\mathbf{x})_i)$
    $\Rightarrow$ To prevent the model to classify everything as one, outputs are **softmaxed**:
    $\mathrm{f}(\mathbf{x})_i = \frac{e^{\mathrm{f}(\mathbf{x})_i}}{\sum_{j=1}^{D} e^{\mathrm{f}(\mathbf{x})_j}}$
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Supervised learning: in practice

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  $\Rightarrow$ Here, one can use gradient descent (see class 1.)

Supervised learning: in practice

$\mathbf{x}$:

$$\xrightarrow{f()} \quad \mathrm{f}(\mathbf{x}) = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix} \neq \hat{\mathbf{y}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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Examples

- **Classification** ($\hat{y}$ is categorical)
- **Regression** ($\hat{y}$ is scalar)
- Tons of applications:
  - Pattern recognition,
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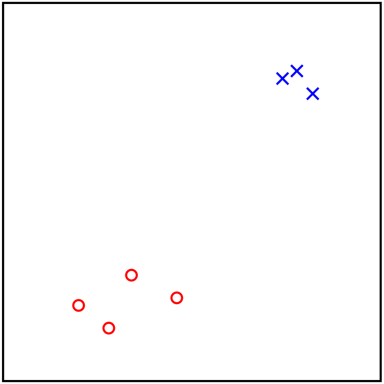

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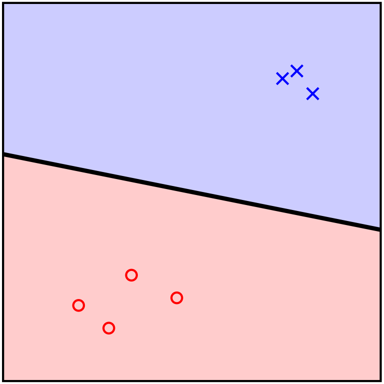

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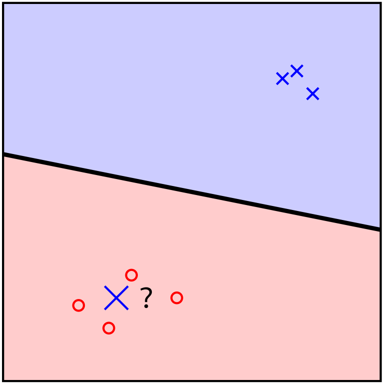

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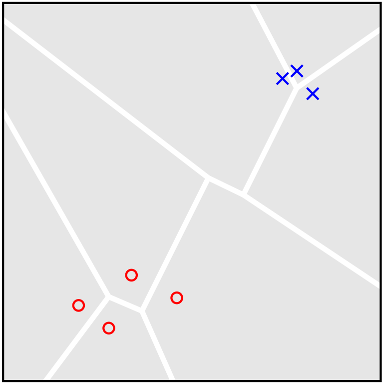
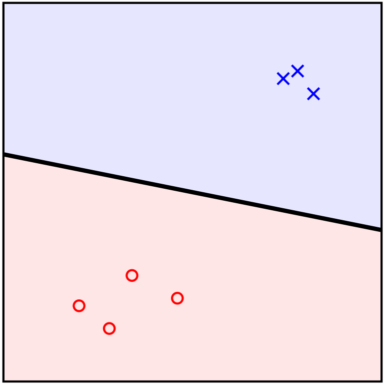


Challenges of supervised learning (1/5)

An ill-defined problem

- An infinity of potential solutions, one must be the “best one” but is unreachable,
- $\Rightarrow$ requires **priors or constraints**.



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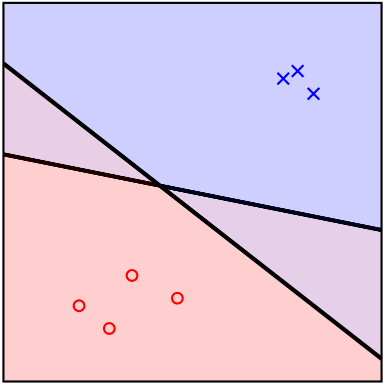

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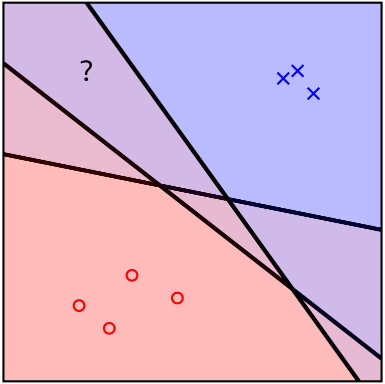
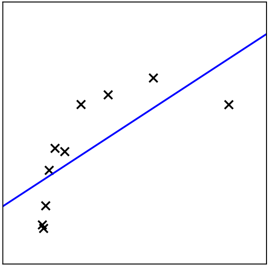


Bias/variance trade-off

- A **simple** solution that almost matches is better than a complex one that fully matches,
- Mimicking is not learning: **overfitting** problem.

- Bias: Error from **erroneous assumptions in the learning algorithm**.
- Variance: Error from **sensitivity to small fluctuations** in the training set.

Polynomial regresssion,
d = 1 (under-fit; high bias)



Challenges of supervised learning (2/5)

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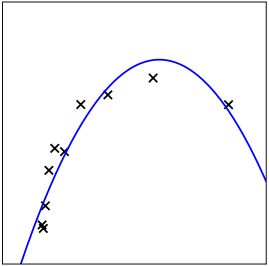

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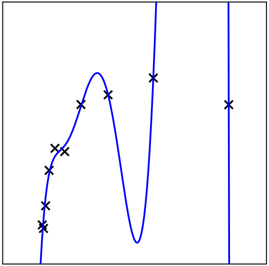

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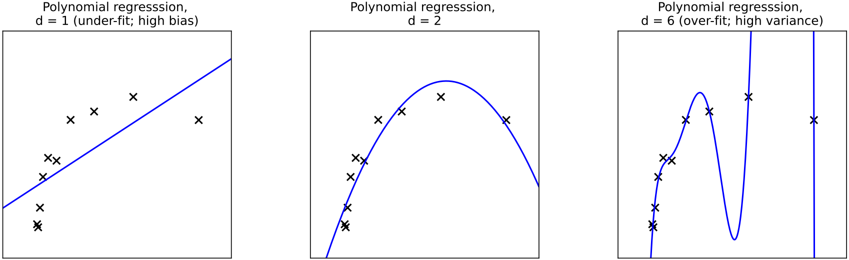

Challenges of supervised learning (2/5)

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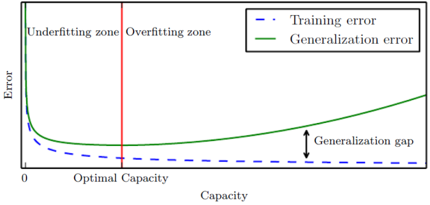
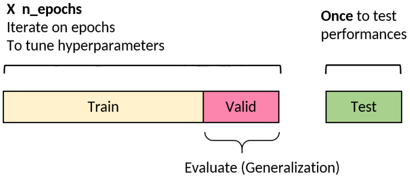
Crossvalidation

To detect overfitting, split training dataset in two parts, the first used to train, the second part to validate (Validation Set)





Challenges of supervised learning (3/5)

Curse of dimensionality

- Geometry is not intuitive in **high dimension**,
- Efficient methods in 2D are not necessarily still valid.

$$V_d^s = \frac{\pi^{d/2} R^d}{\Gamma(d/2+1)} \text{ versus } V_d^c = (2R)^d$$

see https://youtu.be/dZrGXYty3qc?t=533

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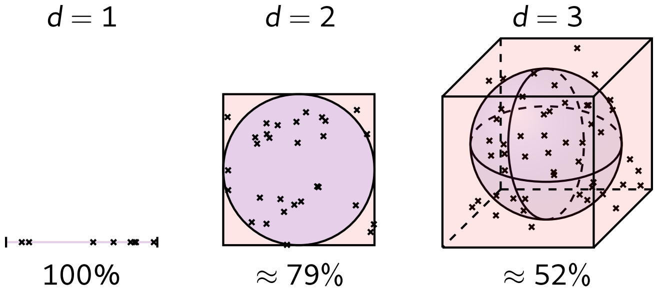

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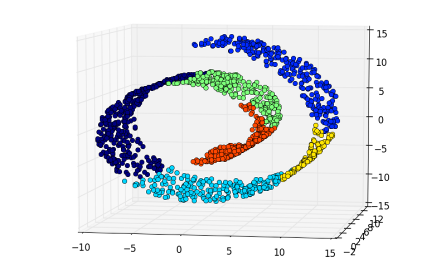
Challenges of supervised learning (4/5)

Riemannian manifolds



The natural space of data may not always be suited to represent data!
$\Rightarrow$ Part of the reason why embeddings are richer semantically.

Challenges of supervised learning (5/5)

Computation time

Example on ImageNet, simply going through all images:

- $n = 10.000.000$, $d \approx 1.000.000$,
- $\approx 10^{13}$ elementary operations,
- $\approx$ 2h45 on a modern processor.

Scalability

- Finding the best solution to a problem would be feasible with unlimited computation time,
- But searching through the space of possible functions is often **untractable**,
- Solutions must be computationally reasonable, which is the true challenge today.

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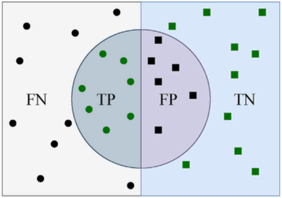
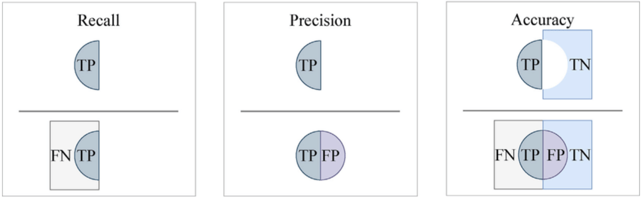
Metrics

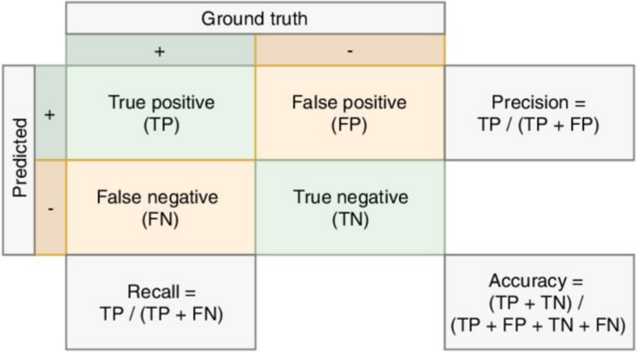
Accuracy, Precision and Recall





https://www.researchgate.net/publication/346129022_Overview_of_Machine_Learning_Part_1/figures

A useful tool: the confusion matrix



https://www.researchgate.net/publication/334840641_A_cloud_detection_algorithm_for_satellite_imagery_based_on_deep_learning/figures?lo=1

Lab Session 2 and assignments for Session 3

Lab Supervised Learning

- Basics of machine learning using sklearn (including new definitions / concepts)
- Tests on the modality chosen in Lab 1 (text, vision or audio), based on the same foundation model than in Lab 1.

Project 1 (P1)

You will choose a supervised learning method among those available (see Lab 2). You will present

- A brief description of the theory behind the method,
- Basic tests on this technique for your modality.

During Session 3 you will have 7 minutes to present.

Lab Session 2 and assignments for Session 3

List of Supervised Learning Methods

- Support Vector Machines (SVM)
- Decision Trees
- Multi-layer Perceptrons (MLP)
- Random Forest classifiers
- Logistic Regression
- Adaboost