Course 2: Supervised Learning

IMT Atlantique
Bretagne-Pays de la Loire
École Mines-Télécom

Summary

Last session

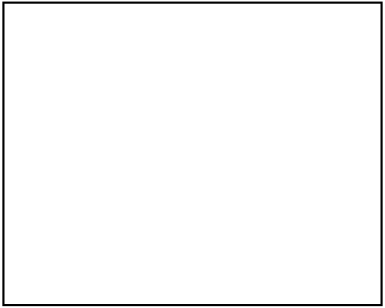
1. AI definition
2. Applications & Open Issues
3. Deep learning
4. Foundation models

Today's session

- Learning from labeled examples
- Challenges of supervised learning

Notations

Vector space ($\mathbb{R}^d$)



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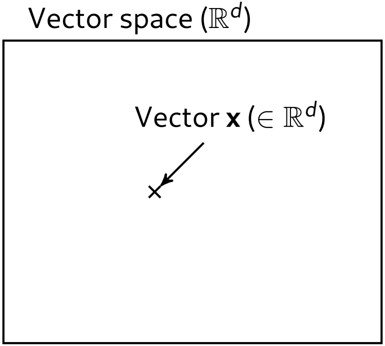

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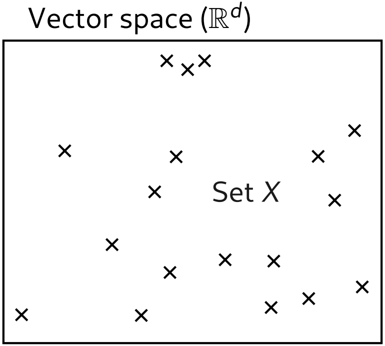
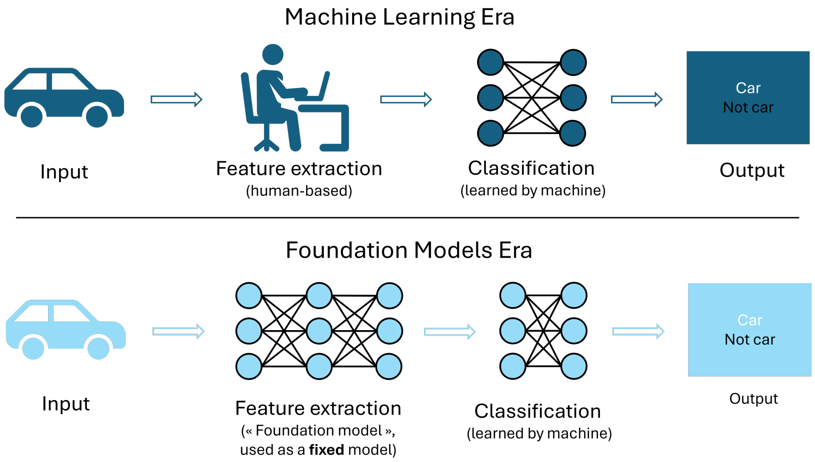


What is the vector *x*? (1/2)



What is the vector x? (2/2)

Traditional Machine Learning

x is the data, or a small transformation of the data

Ex: images, or edges in the image

The era of Foundation models

x is the projection of data in an **embedding** space

- Advantage: richer semantically than the original image
- Disadvantage: Not interpretable nor easily understandable

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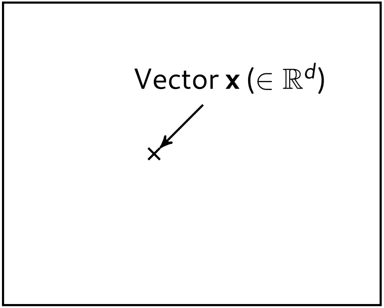

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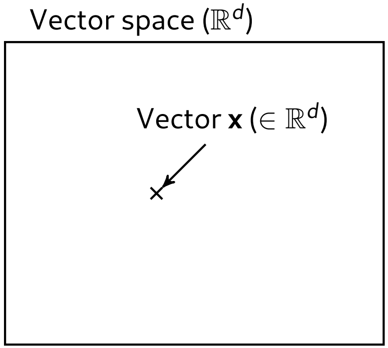

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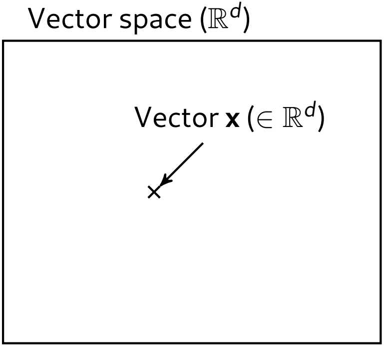

In this class, we will illustrate the concepts using images... BUT in the lab, you will use embedding spaces (the future is probably there)

Learning: Training and Test sets

The Training Set

Data used by the model to **learn** from examples.

The Test Set

A smaller, hold-out portion that the model has **never seen**. Used to **evaluate** its performance.

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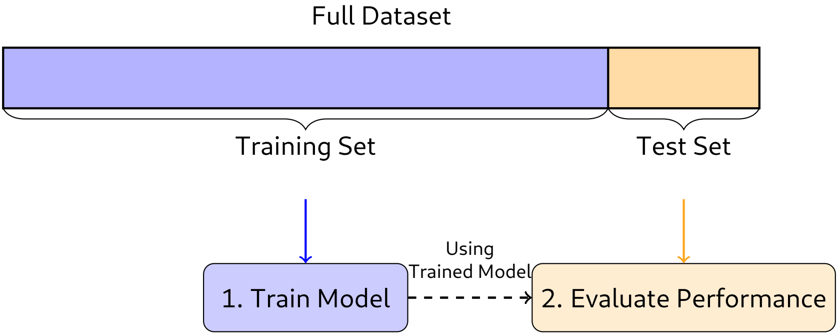

Supervised learning

Definition

Given a training set, composed of:

- $\mathbf{x}$: inputs (raw signals or feature vectors (e.g. embeddings))
- $\hat{\mathbf{y}}$: **labels** (annotated by humans)

Learn:

- a function $f()$ such that $\hat{\mathbf{y}} \approx \mathrm{f}(\mathbf{x})$
  $\Rightarrow$ $f()$ is **learned** by the Machine Learning algorithm
- Ideally, $f()$ should **generalize** ($\neq$ memorize) to unlabeled examples.

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x:  *f*() (to be learned) → **ŷ**: "cat"

Supervised learning: in practice

x: 

$$\xrightarrow{f()} \quad f(\mathbf{x}) = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}$$

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- We compute a **loss** $\mathcal{L}(f(\mathbf{x}), \hat{\mathbf{y}})$
- Training consist in minimizing the loss!

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$\Rightarrow$ Here, one can use gradient descent (see class 1.)

Supervised learning: in practice

$\mathbf{x}$:  $\xrightarrow{f()}$ $\mathrm{f}(\mathbf{x}) = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix} \neq \hat{\mathbf{y}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

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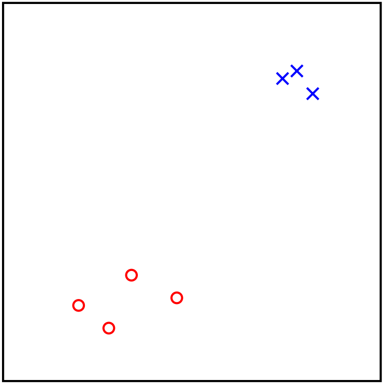

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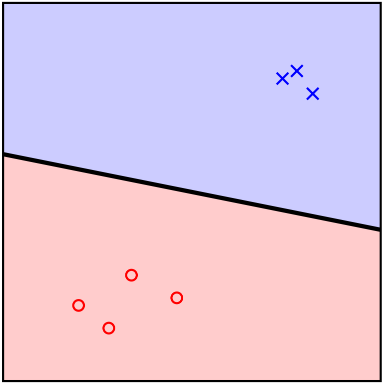
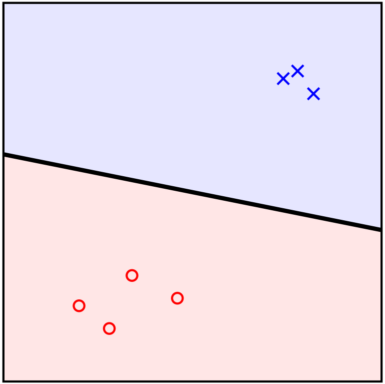


An ill-defined problem

- An infinity of potential solutions, one must be the "best one" but is unreachable,
- $\Rightarrow$ requires **priors or constraints**.



Challenges of supervised learning (1/6)

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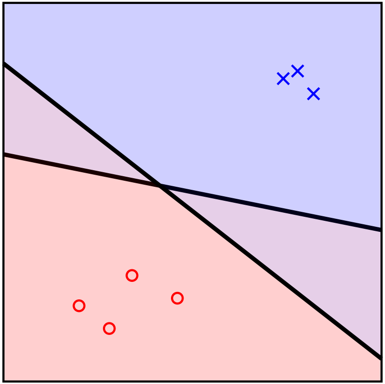

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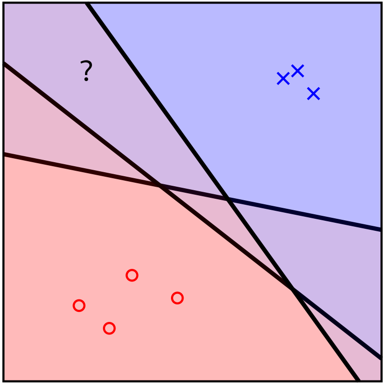
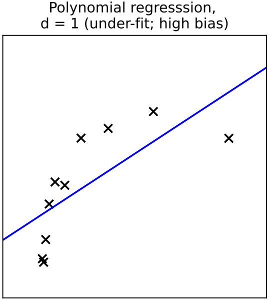


Challenges of supervised learning (2/6)

Bias/variance trade-off

A **simple** solution that almost matches is better than a complex one that fully matches!

- Bias: Error from **erroneous assumptions in the learning algorithm**.
- Variance: Error from **sensitivity to small fluctuations** in the training set.



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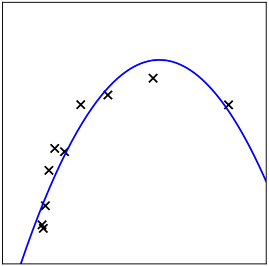

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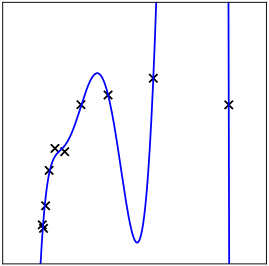

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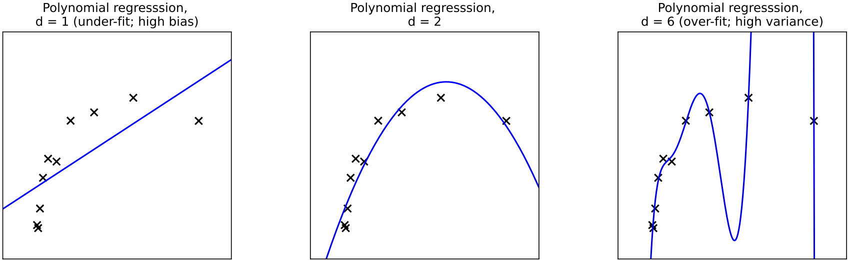
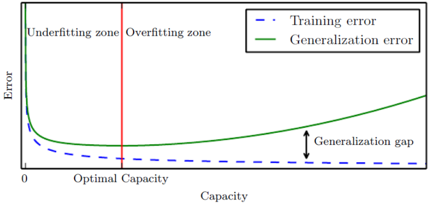
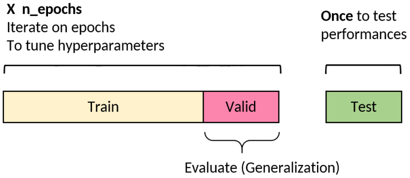


Overfitting

Mimicking is not learning: **overfitting** problem.





To detect overfitting, you may use a validation set:

- Split the **training** dataset into two parts:
- The first part is used to train,
- The second part is used to validate (Validation Set), i.e. check for overfitting.

Curse of dimensionality

- Geometry is not intuitive in **high dimension**,
- Efficient methods in 2D are not necessarily still valid.

$$V_d^s = \frac{\pi^{d/2} R^d}{\Gamma(d/2+1)} \text{ versus } V_d^c = (2R)^d$$

see https://youtu.be/dZrGXYty3qc?t=533

Challenges of supervised learning (4/6)

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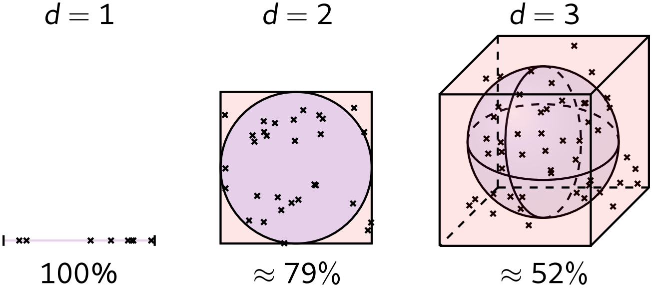

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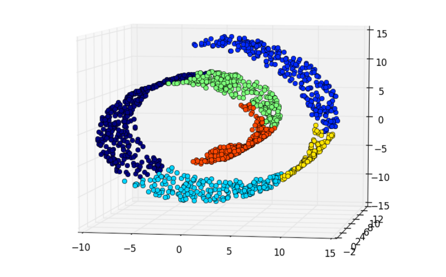
Challenges of supervised learning (5/6)

Riemannian manifolds



The natural space of data may not always be suited to represent data!
$\Rightarrow$ Part of the reason why embeddings are richer semantically.

Challenges of supervised learning (6/6)

Computation time

Example on ImageNet, simply going through all images:

- $n = 10.000.000$, $d \approx 1.000.000$,
- $\approx 10^{13}$ elementary operations,
- $\approx$ 2h45 on a modern processor.

Scalability

- Finding the best solution to a problem would be feasible with unlimited computation time,
- But searching through the space of possible functions is often **untractable**,
- Solutions must be computationally reasonable, which is the true challenge today.

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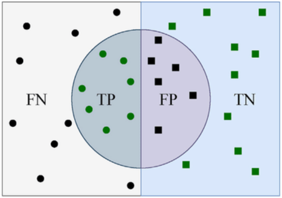
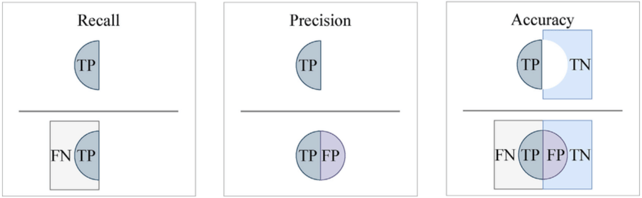
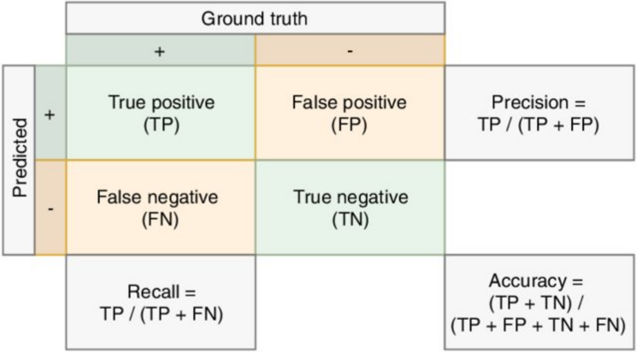
Accuracy, Precision and Recall





https://www.researchgate.net/publication/346129022_Overview_of_Machine_Learning_Part_1/figures

A useful tool: the confusion matrix



https://www.researchgate.net/publication/334840641_A_cloud_detection_algorithm_for_satellite_imagery_based_on_deep_learning/figures?lo=1

Lab Session 2 and presentation in Session 3

Lab Supervised Learning

- Basics of machine learning using sklearn (including new definitions / concepts)
- Tests on the modality chosen in Lab 1 (text, vision or audio), based on the same foundation model than in Lab 1.

Project 1 (P1)

Odd binome number must choose one supervised learning method among those available (see Lab 2). You will present

- A description of the theory behind both methods,
- Basic tests on toy datasets and on your modality.

During Session 3 you will have 7 minutes to present.
Careful, **7min is very short!**
Your presentation should be **educational** and addressed to the rest of the class.

Lab Session 2 and assignments for Session 3

List of Supervised Learning Methods

- Adaboost
- Support Vector Machines (SVM)
- Decision Trees
- Random Forest classifiers
- Logistic Regression
- Ridge Classifier