

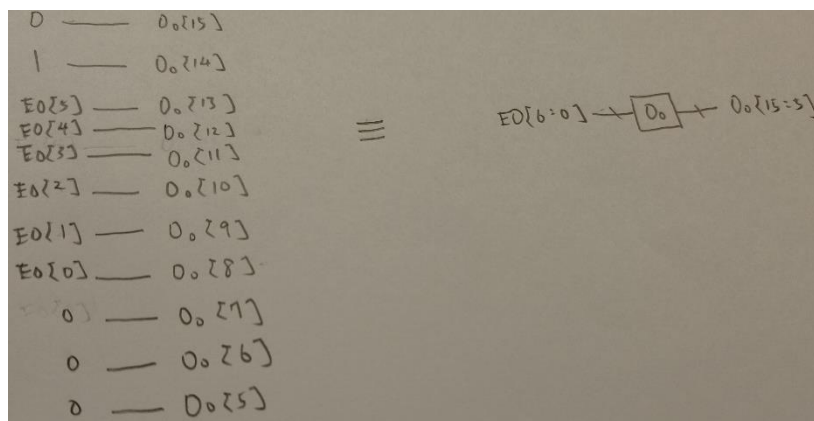
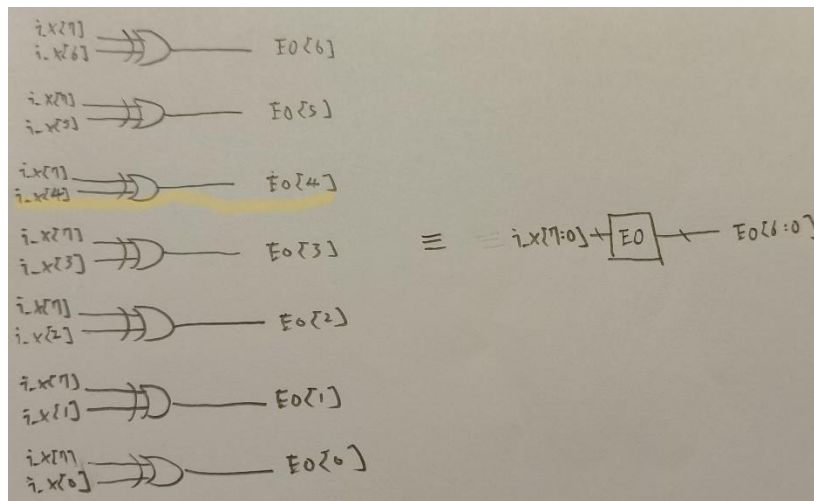
(a) Simulation

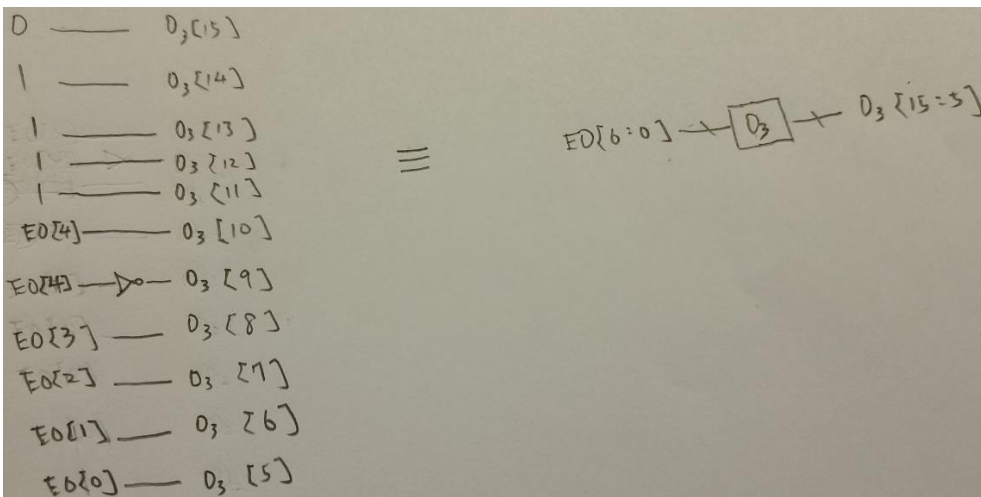
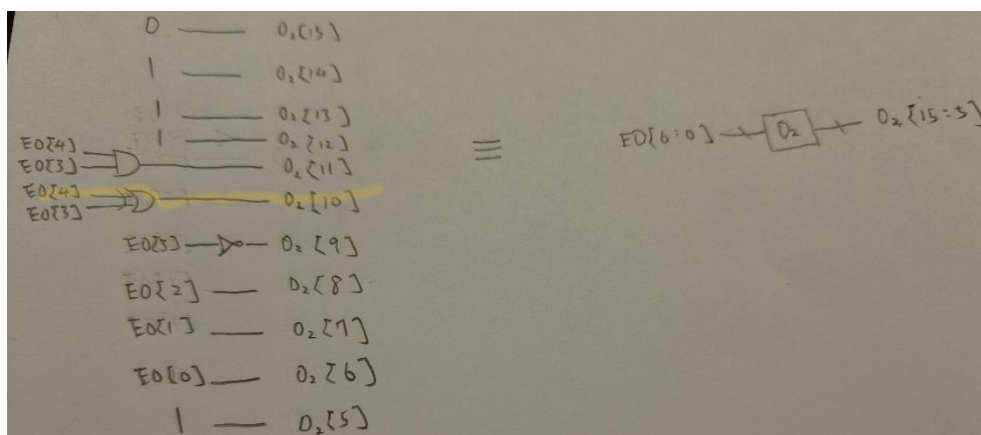
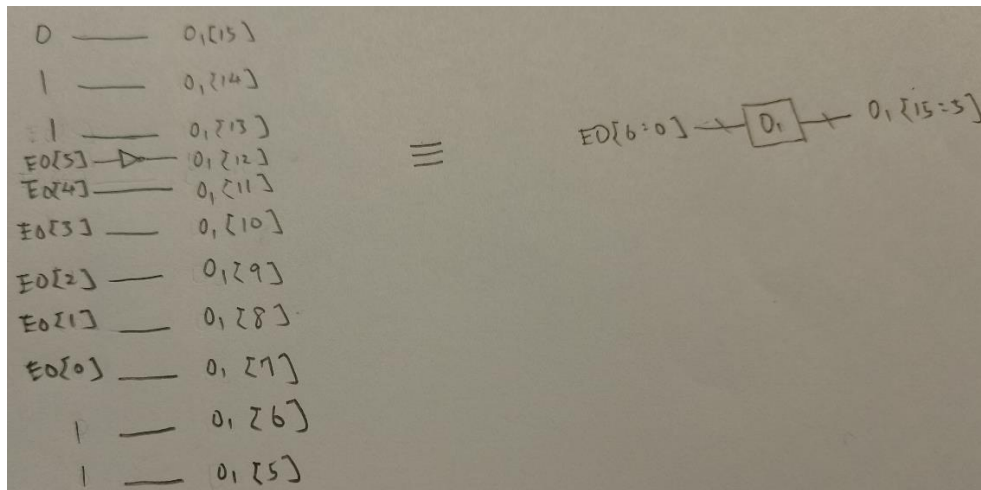
minimum cycle time is 2.8

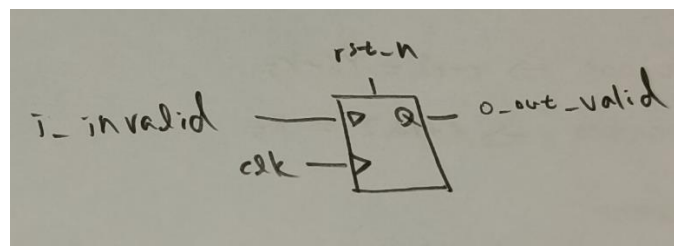
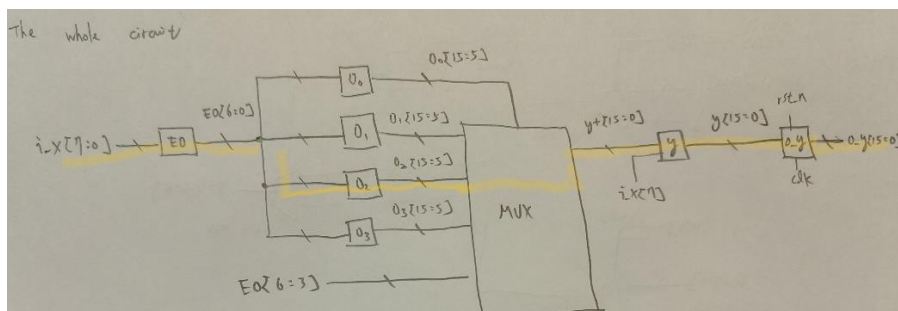
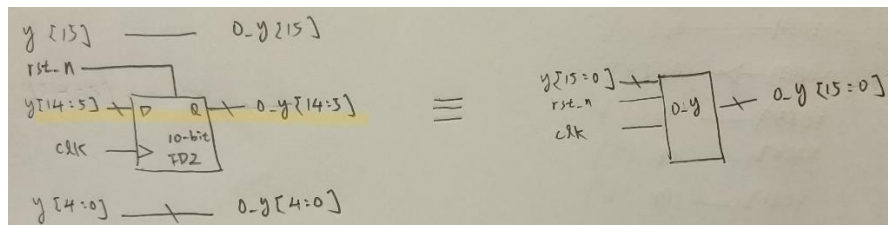
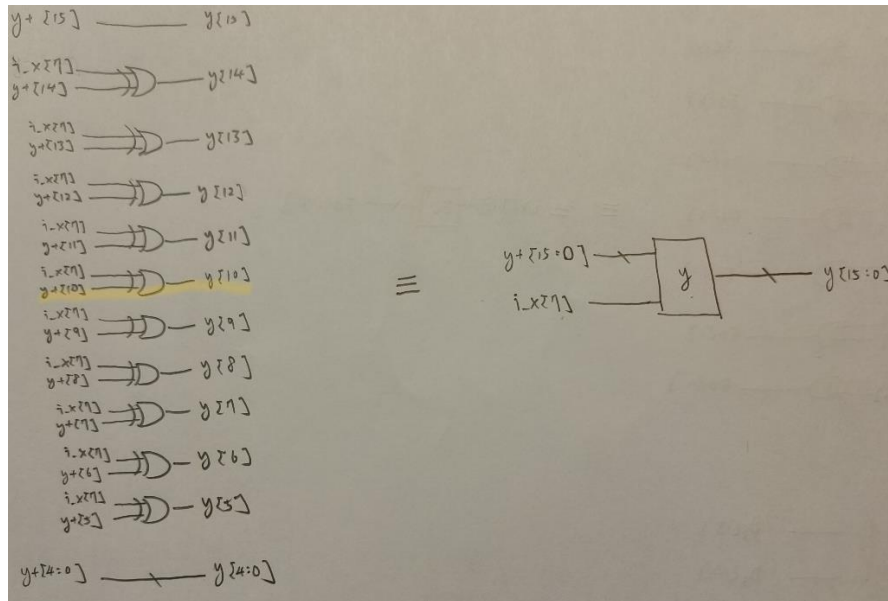
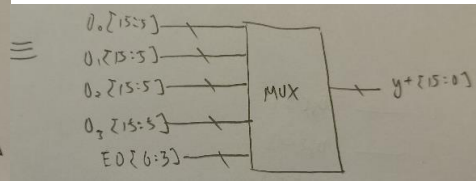
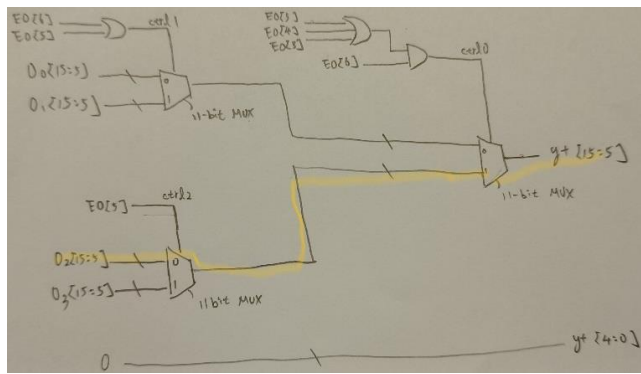
Simulation finished	
Summary	
Clock cycle:	2.8 ns
Number of transistors:	863
Total excution cycle:	256
Approximation Error Score:	30365.0
Performance Score:	618598.4

(b) Circuit diagram

Critical path is marked in yellow







### (c) Discussion

#### Introduce your design

Consider  $x \geq 0$   
 $\Rightarrow x_7 = 0$  . We approximate sigmoid by four functions

$\Rightarrow y_{[15:0]} = \text{sigmoid}(x) = \begin{cases} \frac{1}{4}x + \frac{1}{2} = 0_0 [15:5] , & x = 00.00000 \sim 00.11111 \\ \frac{1}{8}x + \frac{1}{2} + \frac{1}{8} + \frac{1}{512} + \frac{1}{1024} = 0_1 [15:5] , & x = 01.00000 \sim 10.00111 \\ \frac{1}{16}x + \frac{1}{2} + \frac{1}{4} + \frac{1}{64} + \frac{1}{1024} = 0_2 [15:5] , & x = 10.01000 \sim 10.11111 \\ \frac{1}{32}x + \frac{1}{2} + \frac{1}{4} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} = 0_3 [15:5] , & x \geq 11.00000 \end{cases}$

Let  $x$  be  $0 \quad x_6 x_5 \cdot x_4 x_3 x_2 x_1 x_0$

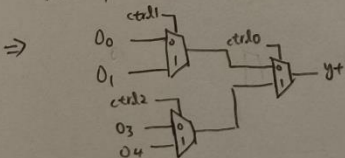
(i)  $D_0 = \frac{1}{4} \cdot x_6 x_5 \cdot x_4 x_3 x_2 x_1 x_0 + \frac{1}{2} = 0.00x_4 x_3 x_2 x_1 x_0$   
 $+ 0.1$   
 $= 0.10x_4 x_3 x_2 x_1 x_0 000 = 0_0 [15:5]$

(ii)  $D_1 = \frac{1}{8} \cdot x_6 x_5 \cdot x_4 x_3 x_2 x_1 x_0 + \frac{1}{2} + \frac{1}{8} + \frac{1}{512} + \frac{1}{1024} = 0.0x_6 x_5 x_4 x_3 x_2 x_1 x_0$   
 $+ 0.1010000011$   
 $= 0.11(x_5)(x_4 x_3 x_2 x_1 x_0 11) = 0_1 [15:5]$

(iii)  $D_2 = \frac{1}{16} \cdot x_6 x_5 \cdot x_4 x_3 x_2 x_1 x_0 + \frac{1}{2} + \frac{1}{4} + \frac{1}{64} + \frac{1}{1024} = 0.0010x_4 x_3 x_2 x_1 x_0$   
 $+ 0.1100010001$   
 $= 0.111(x_4 x_3)(x_4 x_3)(x_3^2) x_2 x_1 x_0 1 = 0_2 [15:5]$

(iv)  $D_3 = \frac{1}{32} \cdot x_6 x_5 \cdot x_4 x_3 x_2 x_1 x_0 + \frac{1}{2} + \frac{1}{4} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} = 0.00011x_4 x_3 x_2 x_1 x_0$   
 $+ 0.110111$   
 $= 0.1111(x_4)(x_4^2) x_3 x_2 x_1 x_0 = 0_3 [15:5]$

Then, we use two level of MUX to get  $y_{[15:5]}$

$\Rightarrow$  

(i)  $\text{ctrl}0 = 1$  when  $x \geq 10.01000 \Rightarrow \text{ctrl}0 = x_6(x_5 + x_4 + x_3)$

(ii) given  $\text{ctrl}0 = 0$ ,  $\text{ctrl}1 = 1$  when  $x \geq 01.00000 \Rightarrow \text{ctrl}1 = x_6 + x_5$

(iii) given  $\text{ctrl}0 = 1$ ,  $\text{ctrl}2 = 1$  when  $x \geq 11.00000 \Rightarrow \text{ctrl}2 = x_5$

As for  $y_{[4:0]}$ , we just set them to zero  
 because accuracy is meaningless here due to approximation

$\Rightarrow$  we get  $y_{[15:0]} = \text{sigmoid}(x)$  for  $x \geq 0$

for  $x < 0$ , we use the symmetry of sigmoid,  $f(-x) + f(x) = 1 \Leftrightarrow f(-x) = 1 - f(x)$

$\Rightarrow f(1 - x_6 x_5, x_6 x_5 x_2 x_1) = f(-(x_6 x_5, x_6 x_5 x_2 x_1 x_0 + 0.00001)) \approx 1 - f(\bar{x}_6 \bar{x}_5, \bar{x}_6 \bar{x}_5 \bar{x}_2 \bar{x}_1 \bar{x}_0)$

Therefore we set  $EO[i] = x_i \oplus x_i$  for  $i = 0 \sim 6$ , and use  $EO[6:0]$  to calculate  $y[15:0]$  by the method mentioned previously.

for the final answer  $y$ ,  $y = \begin{cases} y^+ & x_7 = 0 \\ 1 - y^+ & x_7 = 1 \end{cases} \approx 0.(\overline{y[14]})(\overline{y[13]})(\overline{y[12]}) \dots (\overline{y[5]})$

$\Rightarrow y[15] = 0$

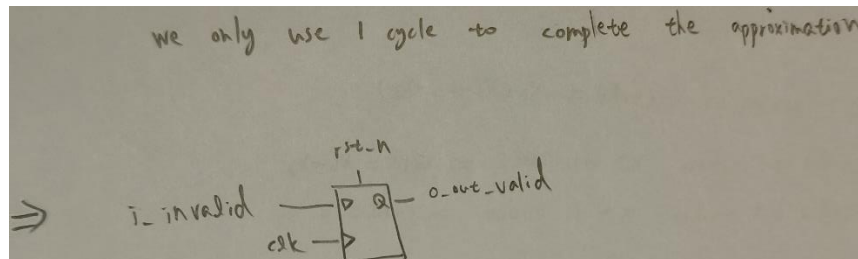
$y[i] = (y[i] \oplus x_i)$  for  $i = 5 \sim 14$

and we set  $y[4:0]$  to zero for the same reason as we set  $y[4:0]$  to zero.

$\Rightarrow$  our final approximation is  $y[15:0]$

$\Rightarrow$  we add flipflops before output to hold the output for a cycle

$\Rightarrow$  output  $o = y[15:0]$



How do you improve your critical path and the number of transistors

(i)

We use NDs, NRs instead of Ands and ORs, because the former requires fewer transistors and have less delay than the latter.

(ii)

$o\_y[15]$  and  $o\_y[4:0]$  are always zero, so we can save 6 flipflops to reduce the usage of transistors.

(iii)

For EO, the delay from A to Z is higher than that of B to Z, and for MUX21H, the delay from A to Z is lower than that of B to Z, so we make the critical path go the one with smaller delay, we improved our critical path by 0.1 using this method.

How do you trade-off between area and speed?

I considered using pipeline in my design; however, because flipflops requires a lot of transistors, the reduction in the cycle time is less than the increase in the usage transistors. Hence, I did not use pipeline. Other than that, I found little trade-off between time and area, the main trade-off is between accuracy and  $A \cdot T$ .

Compare with other architectures you have designed

Besides my original design, I also implemented another circuit using LUT based design. The simulation results are shown below. Though it has longer clock cycle, it has better error score and better performance score.

Simulation finished	
Summary	
Clock cycle:	3.1 ns
Number of transistors:	751
Total excution cycle:	256
Approximation Error Score:	25941.0
Performance Score:	595993.6