E40M
Review - Part 1

Topics in Part 1 (Today): - KCL, KVL, Power

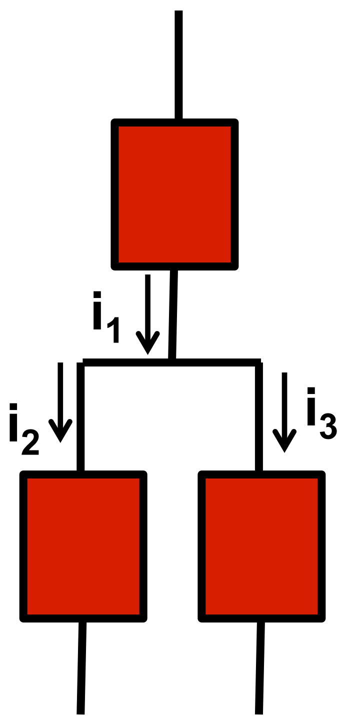
- Devices: V and I sources, R
- Nodal Analysis. Superposition
- Devices: Diodes, C, L
- Time Domain Diode, C, L Circuits

Topics in Part 2 (Wed): - MOSFETs, CMOS Circuits, Logic Gates

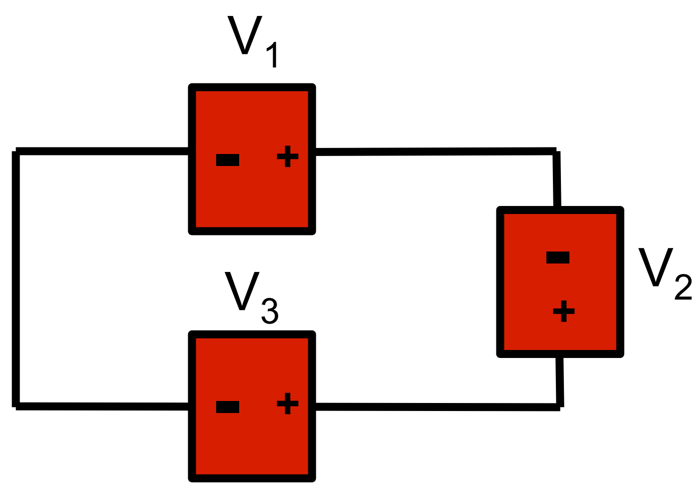
- Binary Numbers
- Time Division Multiplexing
- Frequency Domain Circuits, Impedance
- Filters, Bode Plots
- Op Amps

KCL and KVL

- KCL - Sum of current flowing into node or device is **zero**
- KVL - Sum of device voltages around any loop is **zero**



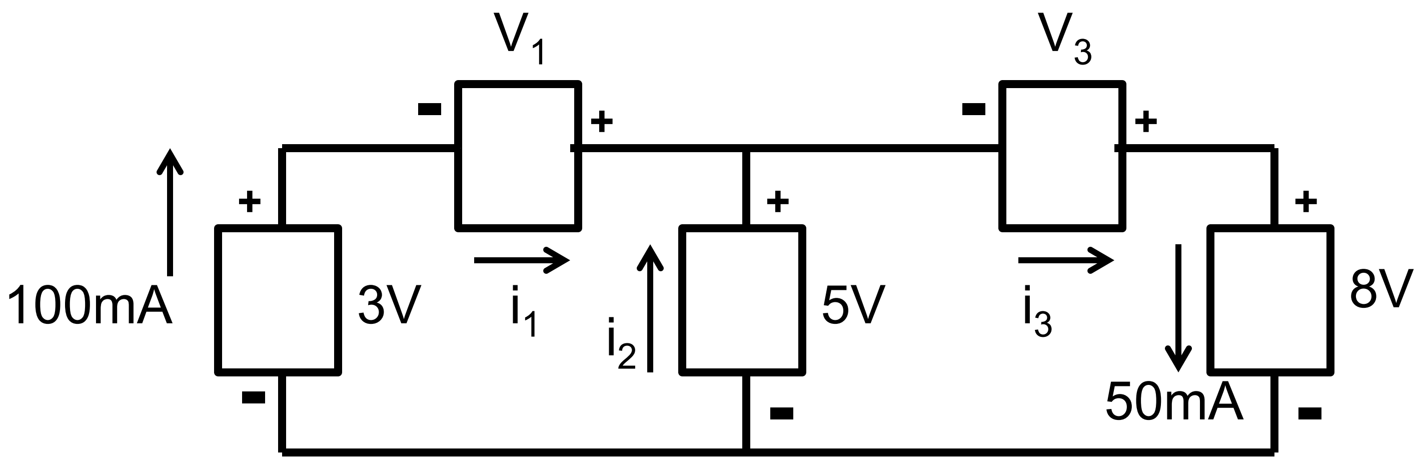
$i_1 = i_2 + i_3$



$V_1 + V_2 - V_3 = 0$

Using KCL and KVL

- Find the current, and voltages for the circuit below



Power

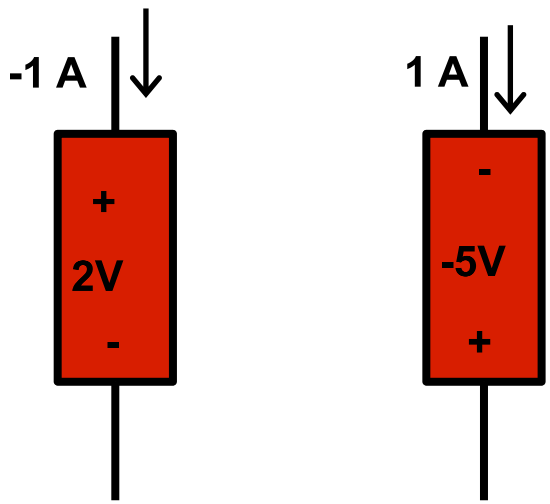
- Power = iV Measured in Watts (= Volt *Amp)
  – It is the flow of energy
    - Energy is measured in Joules
    - Watts = Joules/sec

- For devices that absorb energy power flows into device
  – Current flows from higher to lower voltage

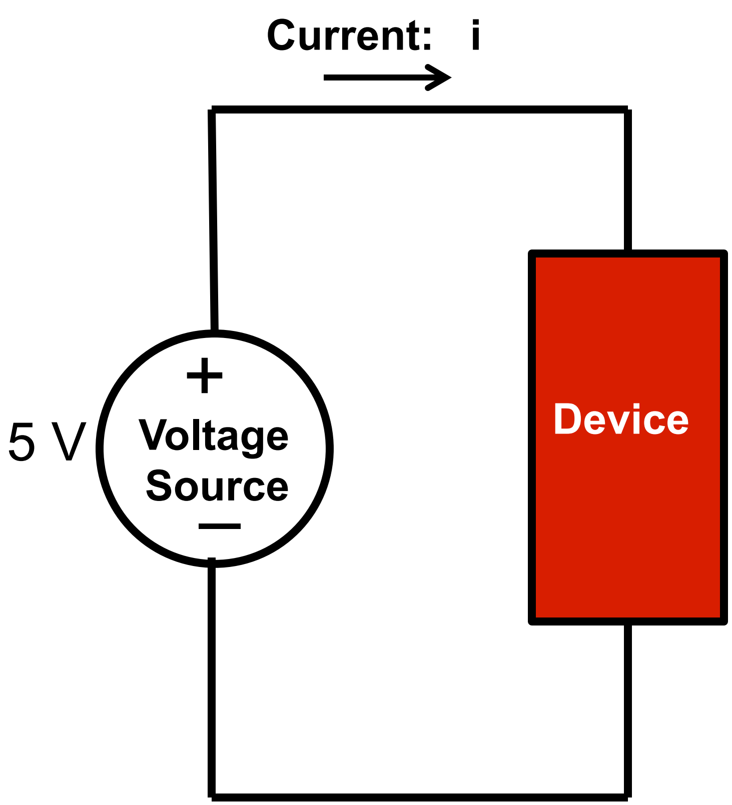
- For devices that supply energy, power flows out of device
  – Current flow from lower to higher voltage

Absorb or Provide

Power?



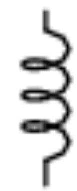
Electrical Devices



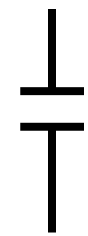
- We learned about many different electrical devices.

Resistors

Diodes

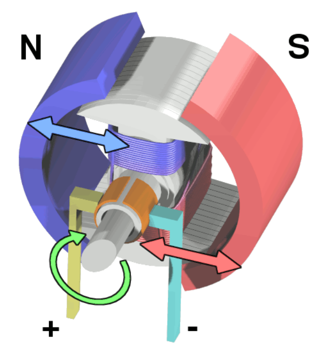


Inductors

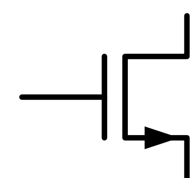


Capacitors

Light Emitting Diodes

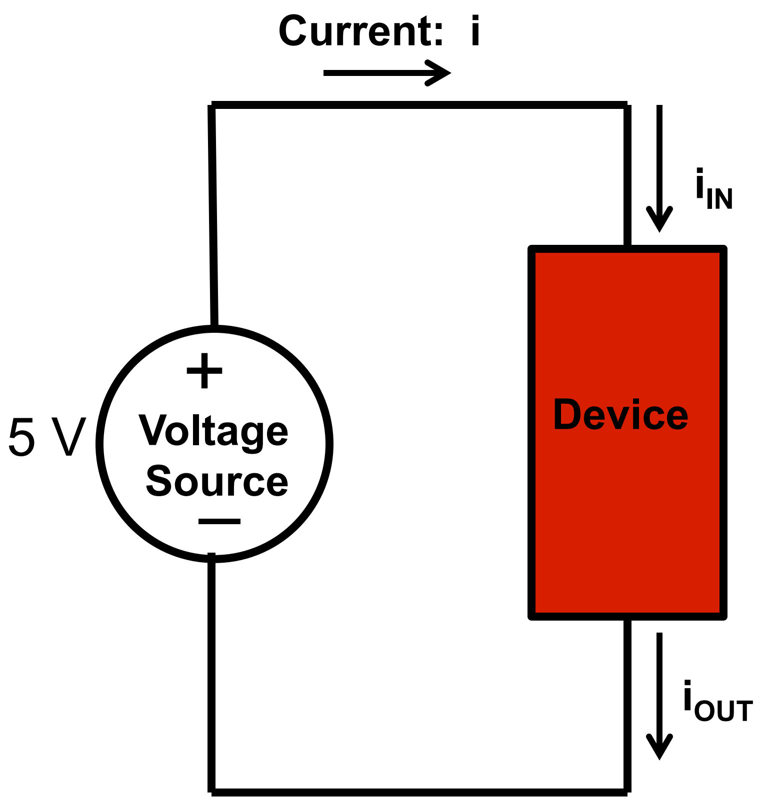


Motors



Transistors

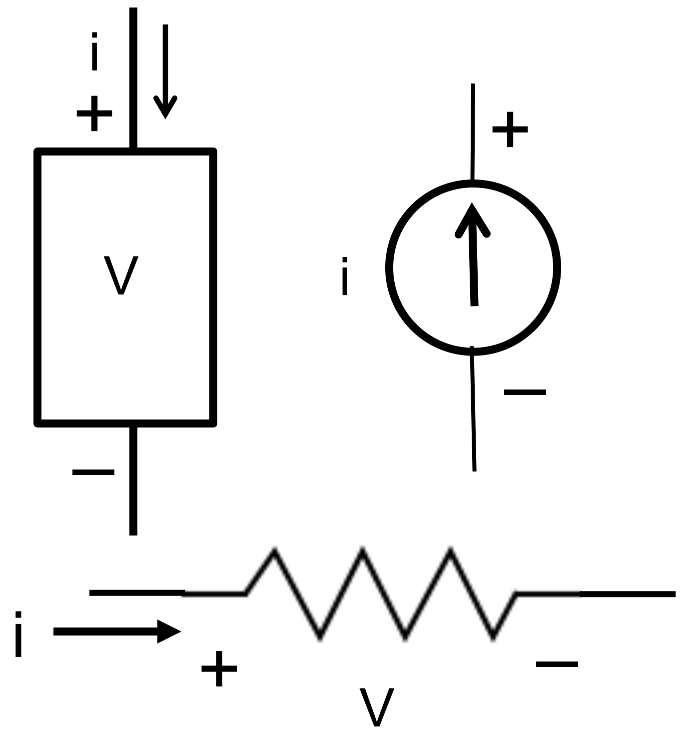
Electrical Devices – Some Properties

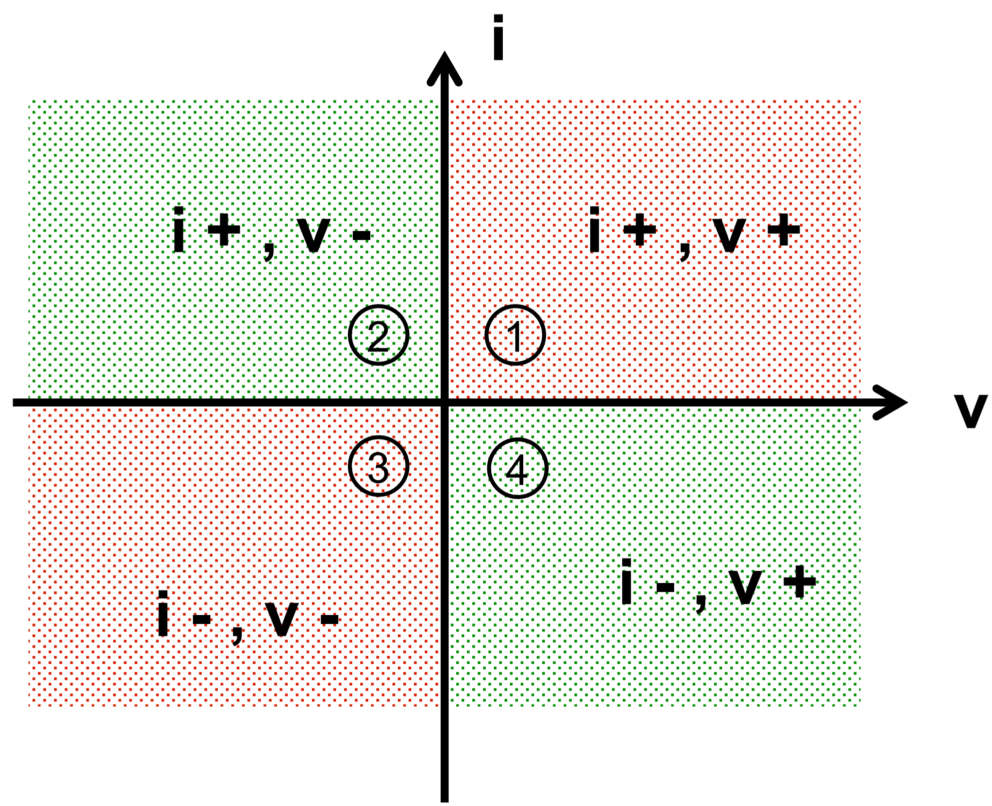


- Charge neutral; *i.e.*, charge entering = charge leaving
  - Batteries or power supplies separate charge but the overall device is still charge neutral
- The net current into any device is **always zero**, so $i_{IN} = i_{OUT}$
- Dissipate power ($P = i \cdot V$)

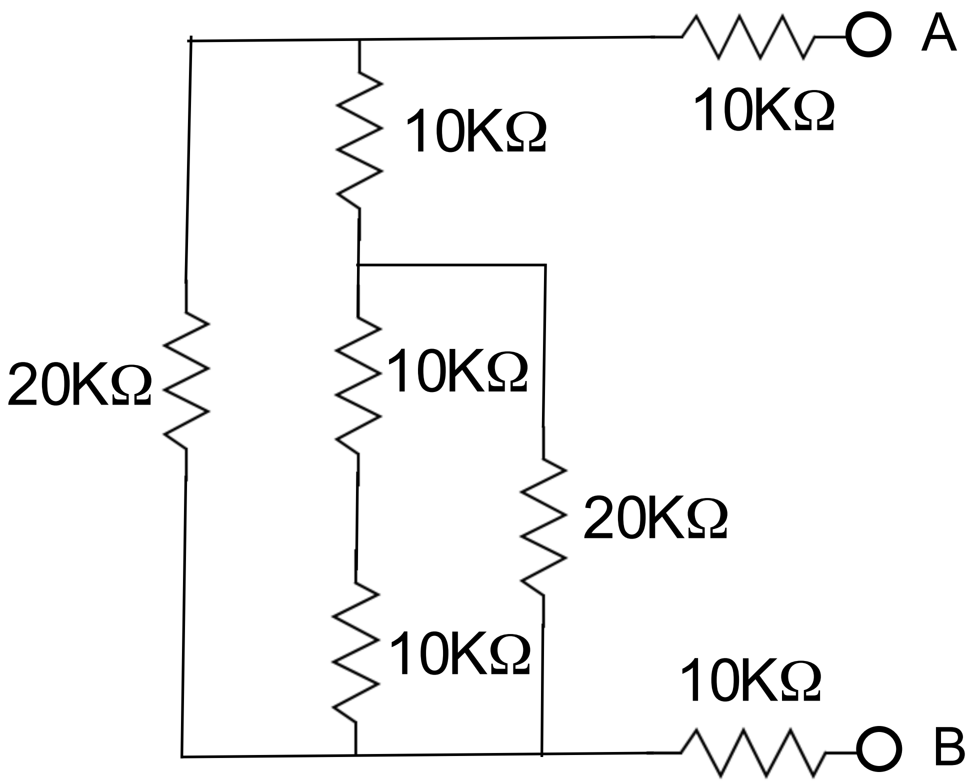
Electrical Devices –Voltage Source, Current Source, Resistor

- Note that the energy is dissipated by the device in quadrants 1 and 3, and power is generated by the device in quadrants 2 and 4.
- Sketch the i-V curves for these devices.



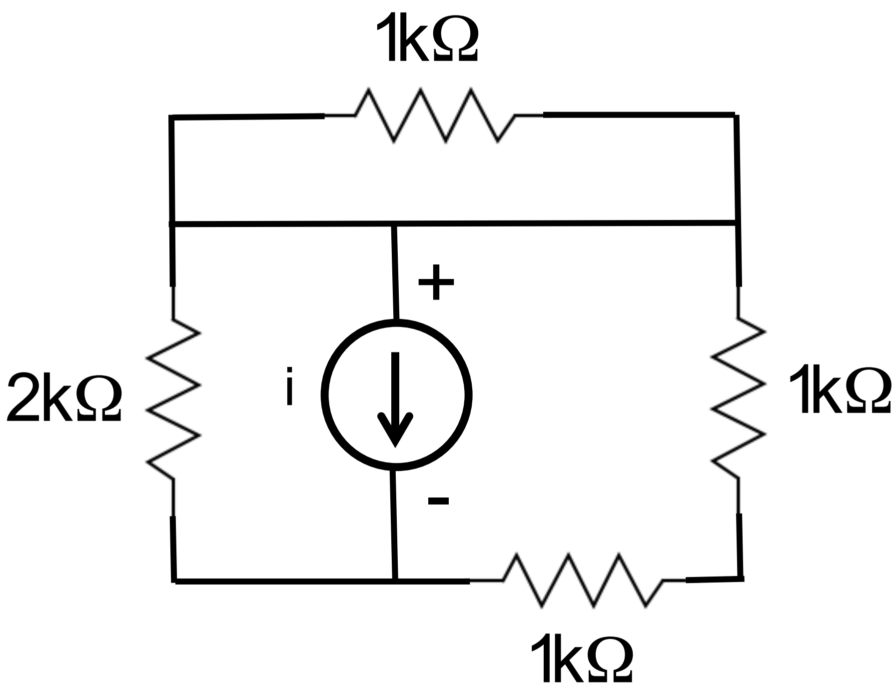


Resistor Circuits



What is the resistance between nodes A and B?

The Power of Redrawing a Circuit



Nodal Analysis: The General Solution Method

1. Label all the nodes (V_A, V_B, or V_1, V_2, etc.), after selecting the node you choose to be Gnd.

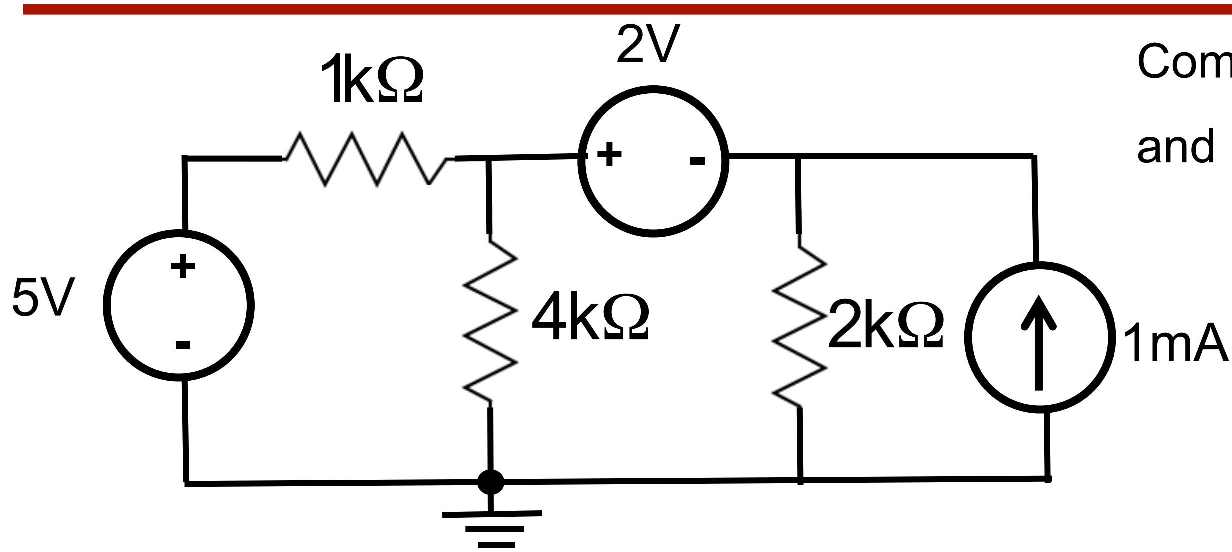
2. Label all the branch currents (i_1, i_2, etc.) and choose directions for each of them

3. Write the KCL equations for every node except the reference (Gnd)
   - Sum of the device currents at each node must be zero

4. Substitute the equations for each device's current as a function of the node voltages, when possible

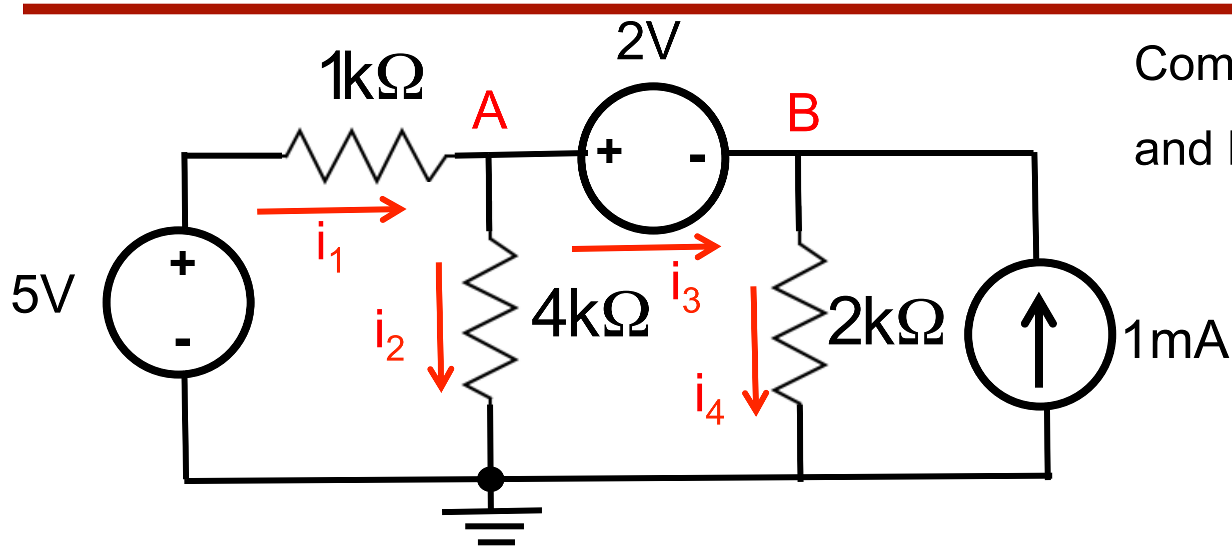
5. Solve the resulting set of equations

Example: Nodal Analysis



Compute the node voltages and branch currents.

Example: Nodal Analysis



Compute the node voltages and branch currents.

At node A: $i_1 = i_2 + i_3$

$$\therefore 1.\quad \frac{5 - V_A}{1k\Omega} = \frac{V_A}{4k\Omega} + i_3$$

At node B: $i_3 = i_4 - 1mA$

$$\therefore 2.\quad i_3 = \frac{V_B}{2k\Omega} - 1mA$$

$$3.\quad V_A = V_B + 2V$$

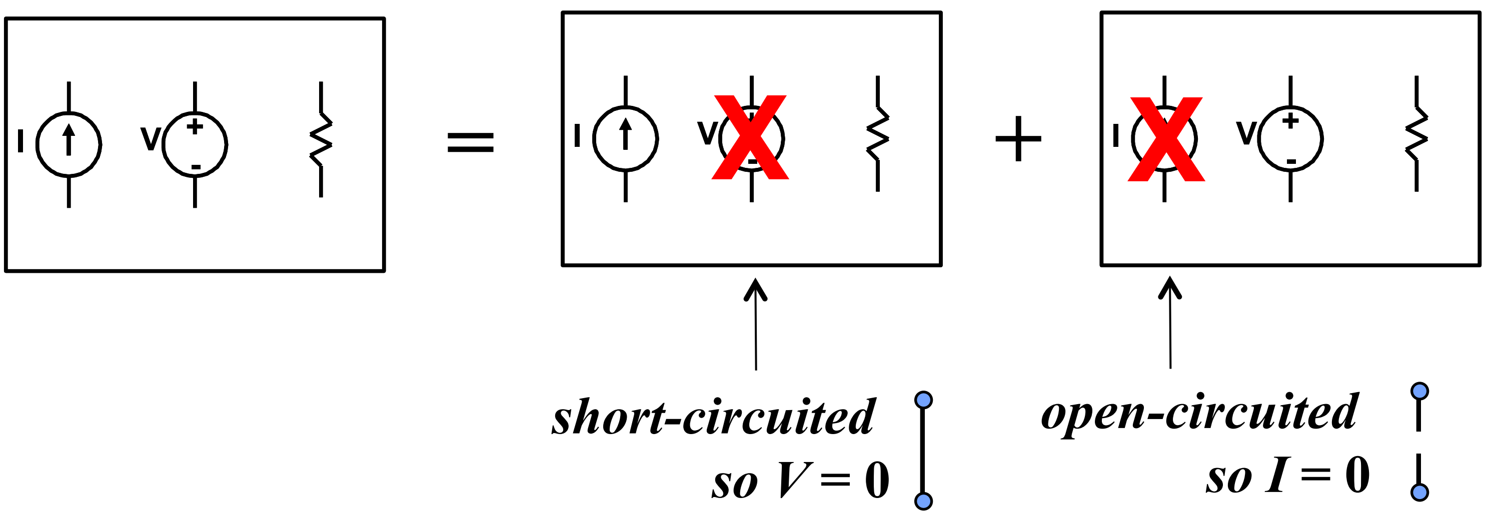
Substituting 2. into 1. $\dfrac{5 - V_A}{1k\Omega} = \dfrac{V_A}{4k\Omega} + \dfrac{V_B}{2k\Omega} - 1mA$

$$\frac{5 - V_A}{1k\Omega} = \frac{V_A}{4k\Omega} + \frac{V_A - 2V}{2k\Omega} - 1mA \quad \therefore\ V_A\left(\frac{7}{4k\Omega}\right) = 7mA \quad \therefore\ V_A = 4V$$

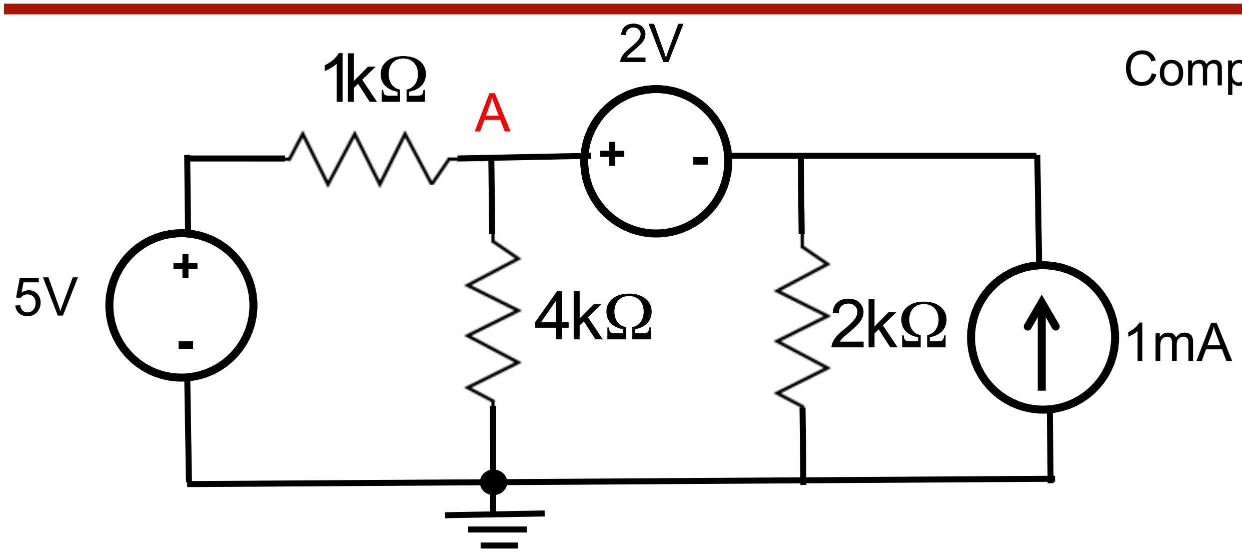
$$\therefore\ V_A = 4V,\ \ V_B = 2V,\ \ i_1 = 1mA,\ i_2 = 1mA,\ i_3 = 0,\ i_4 = 1mA$$

Superposition For Linear Circuits

- Calculate the response of the circuit for each independent source at a time, with the other's turned off
- What happens when we turn off a source?
  - Voltage sources: have 0 V (are ***shorted*** … replace by a wire)
  - Current sources: have 0 current (are ***opened*** … replace by a *broken* wire)

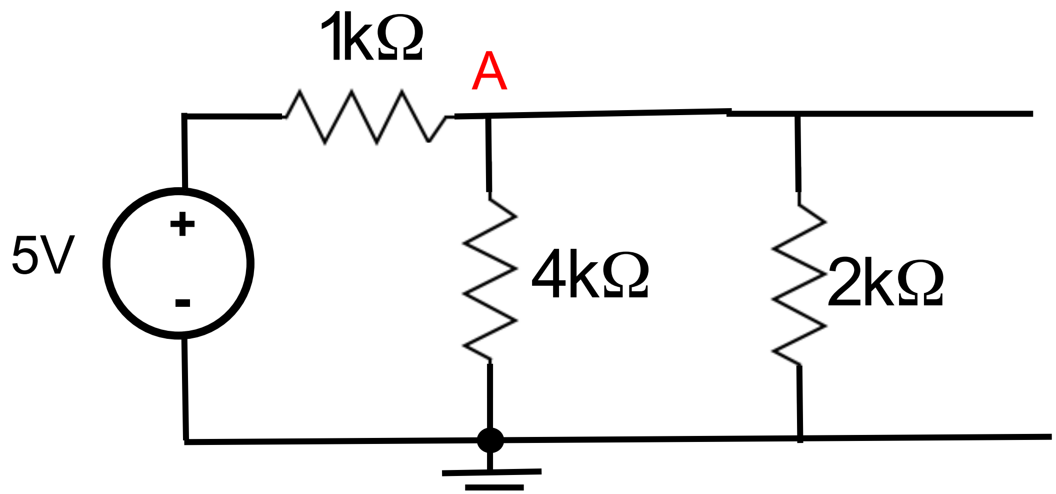


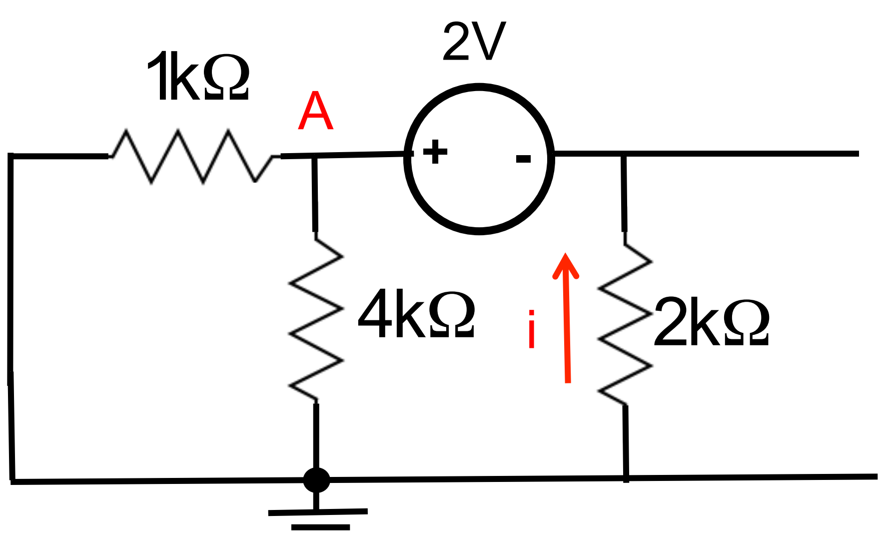
Example: Superposition



Compute V_A using superposition.

Example: Superposition





Compute V_A using superposition.

$$4K\Omega \,||\, 2k\Omega = \frac{4}{3}k\Omega$$

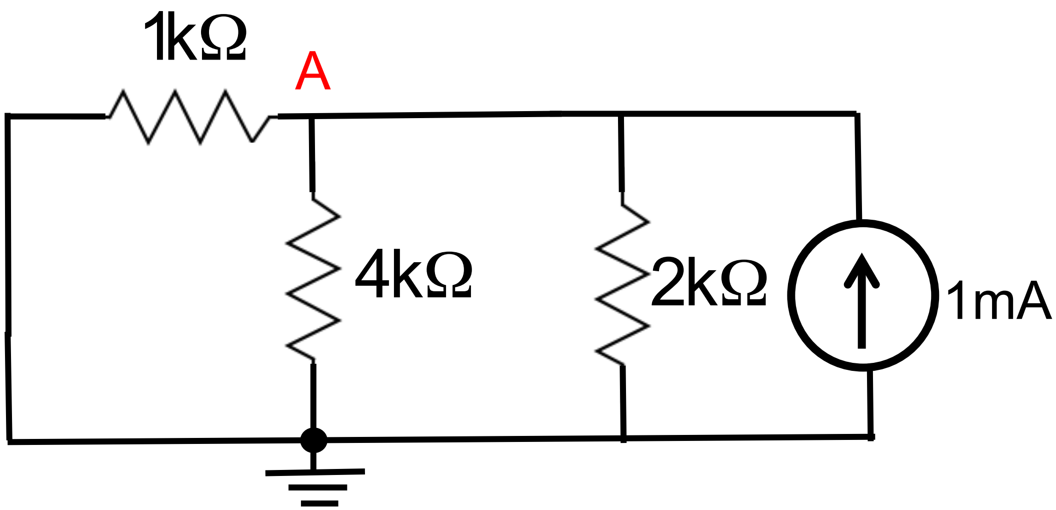
$$V_{A1} = 5V\frac{\frac{4}{3}k\Omega}{\frac{4}{3}k\Omega + 1k\Omega} = \frac{20}{7}V$$

$$4K\Omega \,||\, 1k\Omega = \frac{4}{5}k\Omega$$

$$i = \frac{2V}{2k\Omega + \frac{4}{5}k\Omega} = \frac{10}{14}mA = \frac{5}{7}mA$$

$$\therefore V_{A2} = \frac{5}{7}mA\left(\frac{4}{5}k\Omega\right) = \frac{4}{7}V$$

Example: Superposition



$$4K\Omega \,||\, 2k\Omega \,||\, 1k\Omega = \frac{4}{7}k\Omega$$

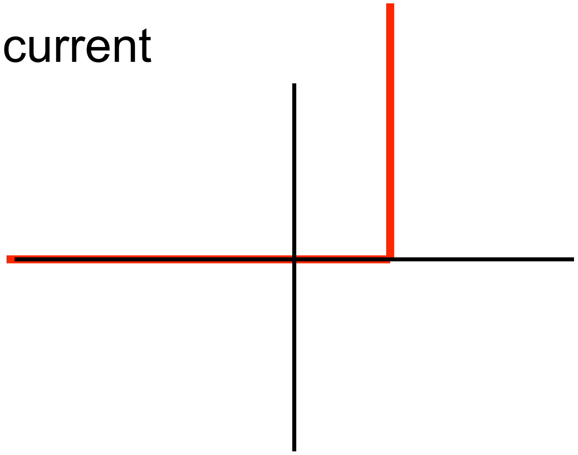
$$V_{A3} = 1mA\left(\frac{4}{7}k\Omega\right) = \frac{4}{7}V$$

$$\therefore V_A = \frac{4}{7}V + \frac{4}{7}V + \frac{20}{7}V = 4V$$

Electrical Devices – Diodes

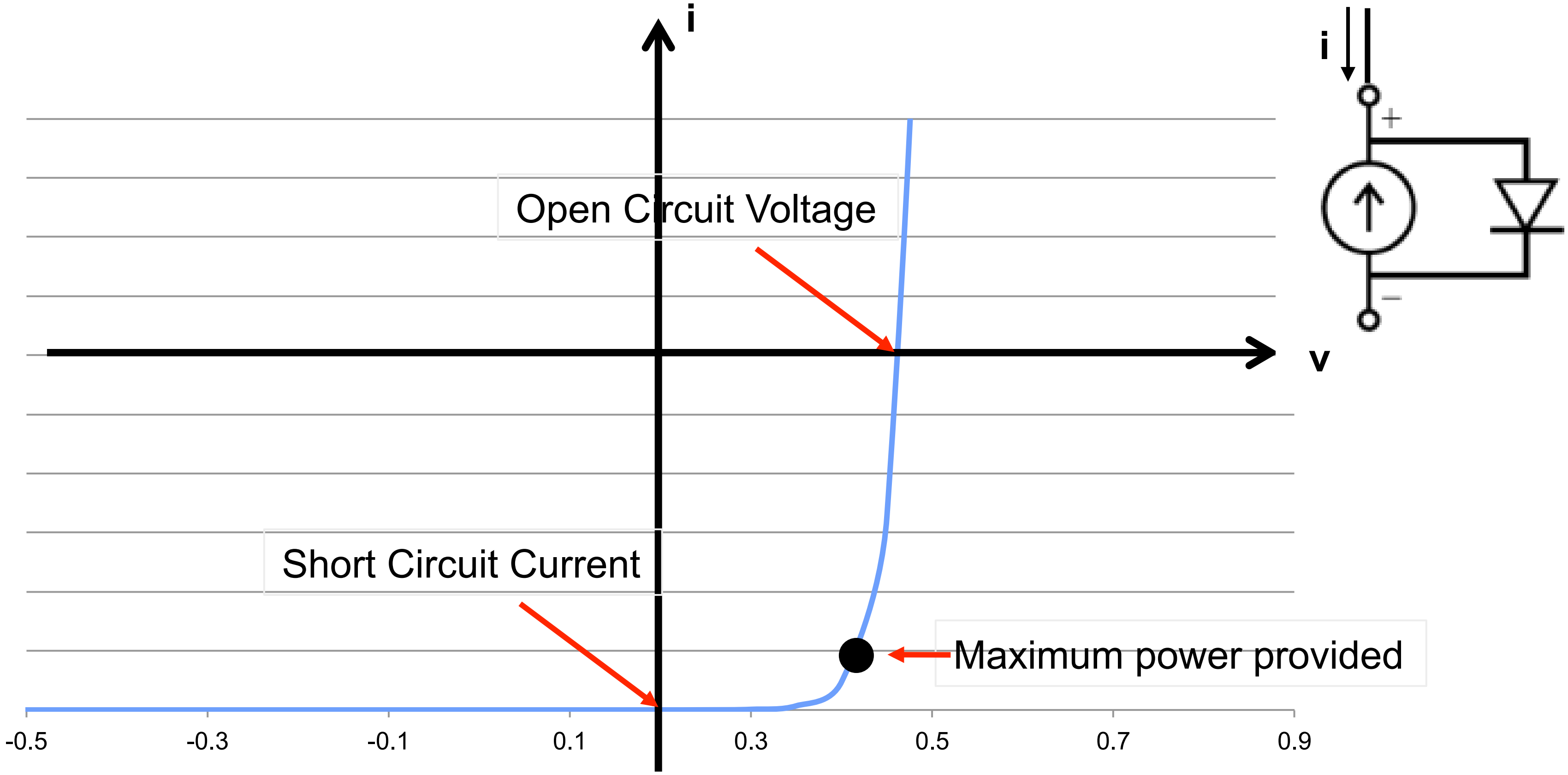
- Diode is a one-way street for current
  - Current can flow in only one direction

- An idealized diode model
  - Is a voltage source for positive current
    - Voltage drop is always equal to Vf for any current
  - Is an open circuit for negative current
    - Current is always zero for any voltage



Electrical Devices - Solar Cells

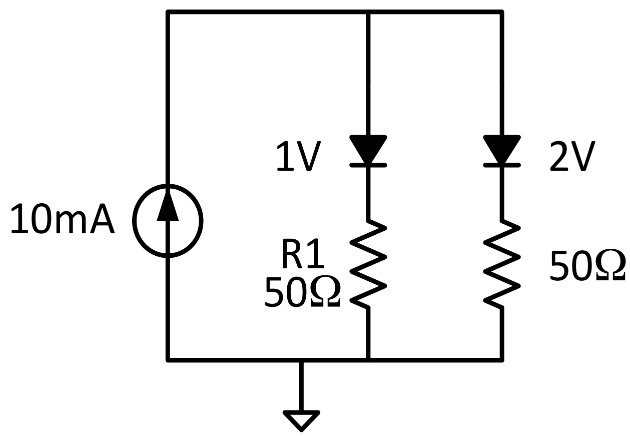
- Incoming photons create current.
- If no external current path (i = 0), current flows through diode.



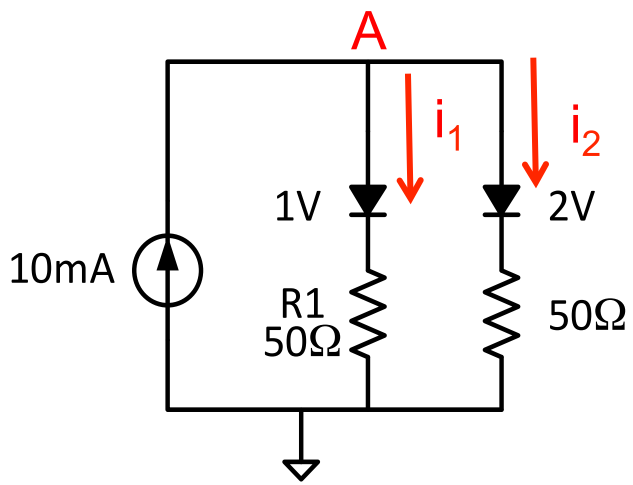
Solving Diode Circuits

- Look at the circuit, guess the voltages and/or currents
  - From this, guess whether the diode will be on or off
  - If you can't estimate anything, just guess the diode state(s)
- Assume your guess was right
  - Solve for the voltages in the circuit
- Then check your answer
  - If you guessed the diode was off,
    - Look at the resulting diode voltage
    - Check to make sure it is less than V_f
    - If you guessed that the diode was on
    - You fixed the voltage to be V_f
    - So check to make sure the current is positive
- If your guess was wrong, change the guess and resolve

Example: Diode Circuit



Example: Diode Circuit



The current source is in the right direction to forward bias the diodes, so we might "guess" that both diodes are ON.

$$\therefore 10\text{mA} = i_1 + i_2$$

$$V_A = 1\text{V} + i_1(50\Omega) = 2\text{V} + i_2(50\Omega)$$

$$\therefore i_1 = \frac{1\text{V} + i_2(50\Omega)}{50\Omega} = \frac{1\text{V}}{50\Omega} + i_2$$

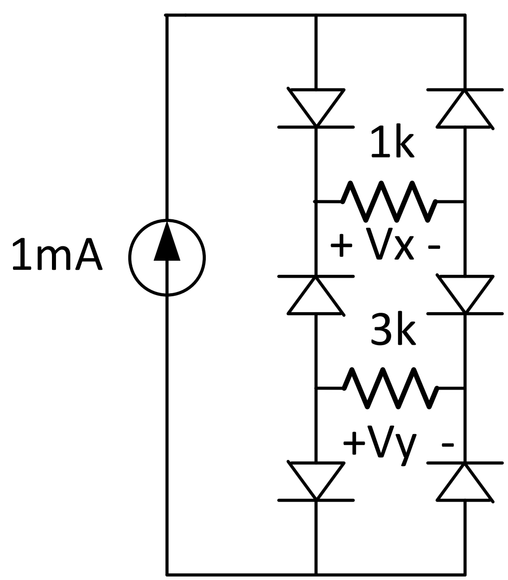
$$\therefore 10\text{mA} = \frac{1\text{V}}{50\Omega} + 2i_2 = 20\text{mA} + 2i_2 \quad \text{so that} \quad i_2 = -5\text{mA}$$

Clearly this is incorrect, so i_2 must = 0,

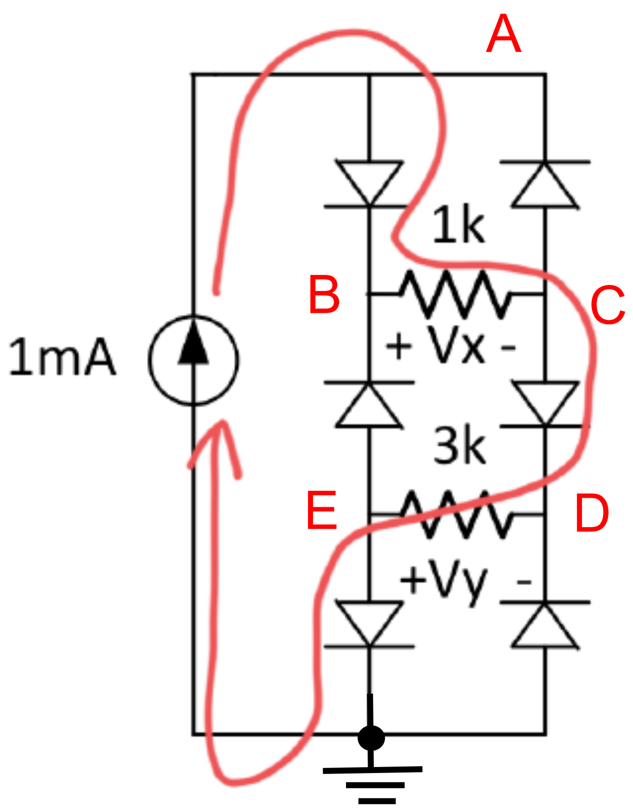
$i_1 = 10\text{mA}$ and $V_A = 1\text{V} + (10\text{mA})(50\Omega) = 1.5\text{V}.$

Example 2: Diode Circuit

- Don’t really want to randomly choose diode state in this case



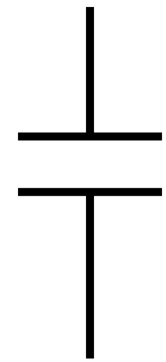
Example 2: Diode Circuit

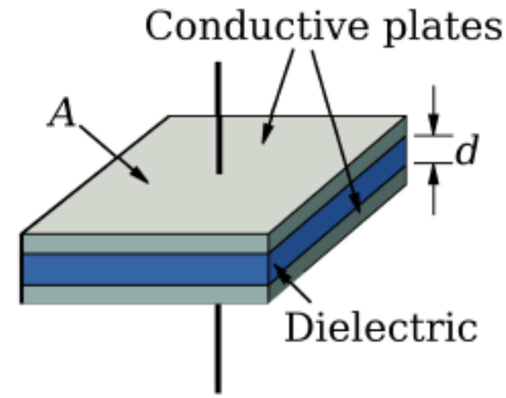


- Don't really want to randomly choose diode state in this case.
- The red line shows a reasonable guess as to the current path since this is through 3 forward biased diodes.
- If the forward voltage on each diode is 0.6V, then the node voltages are
- V_A = 5.8V, V_B = 5.2V, V_C = 4.2V, V_D = 3.6V, V_E = 0.6V.
- The other three diodes are all reverse biased so this is a self-consistent solution.

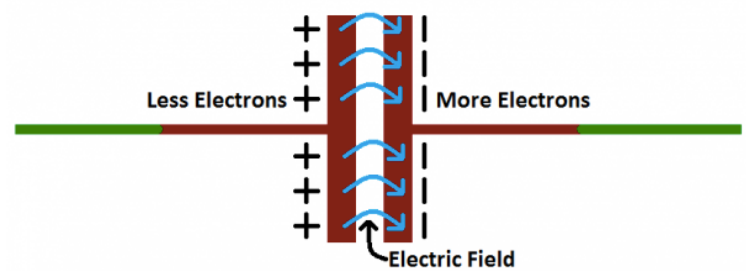
Electrical Devices - Capacitors

- What is a capacitor?
  - It is a new type of two terminal device
  - It is linear
  - It doesn't dissipate energy





- Rather than relating i and V
  - Relates Q, the charge stored on each plate, to Voltage
  - Q = CV

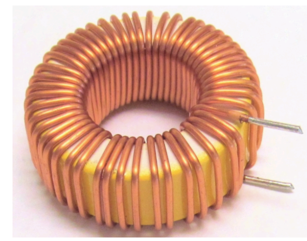


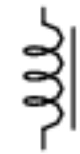
- So if Q = CV, and i=dQ/dt
  - i= C dV/dt

- $Z_C = 1/j*2\pi FC$ so at low F it is an open circuit, at high F it is a wire.

Electrical Devices - Inductors

- An inductor is a new type of two terminal device
  - It is linear – double V and you will double i
  - Like a capacitor, it stores energy
    - Ideal inductors don't dissipate energy



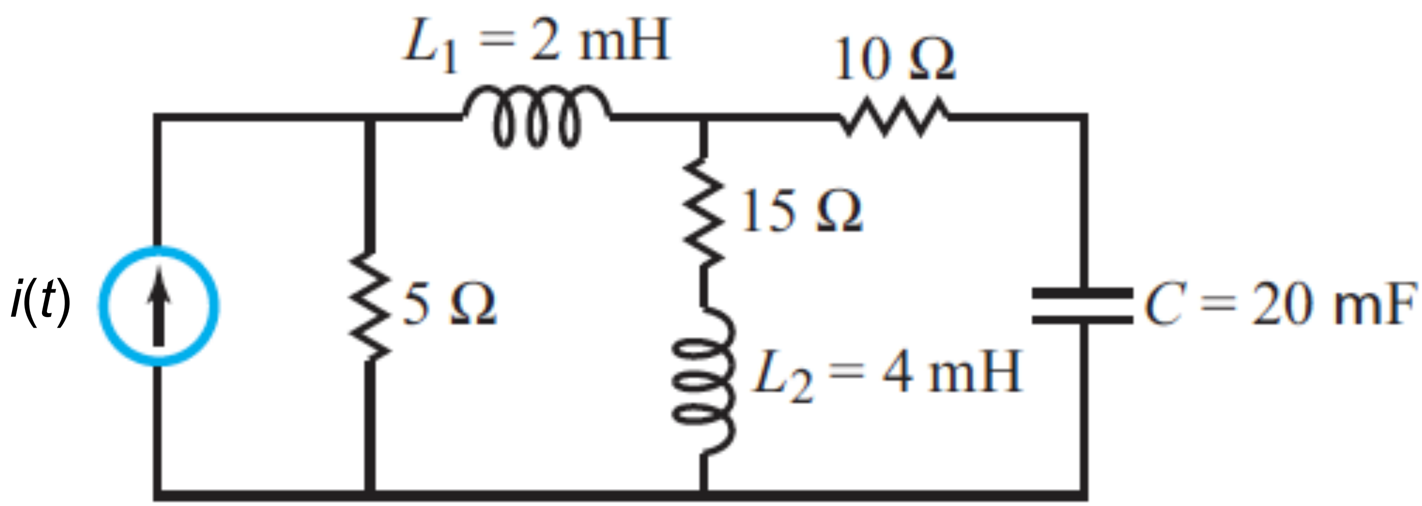


- Defining equation: $V = L\,di/dt$ L is inductance (in Henrys)

- For very small Δt inductors look like current sources
  - They can supply very large voltages (+ or -)
  - And not change their current

- $Z_L = j * 2\pi FL$ so at low F, it looks like a wire, at high F an open circuit.

Example: Asymptotic Circuit Analysis

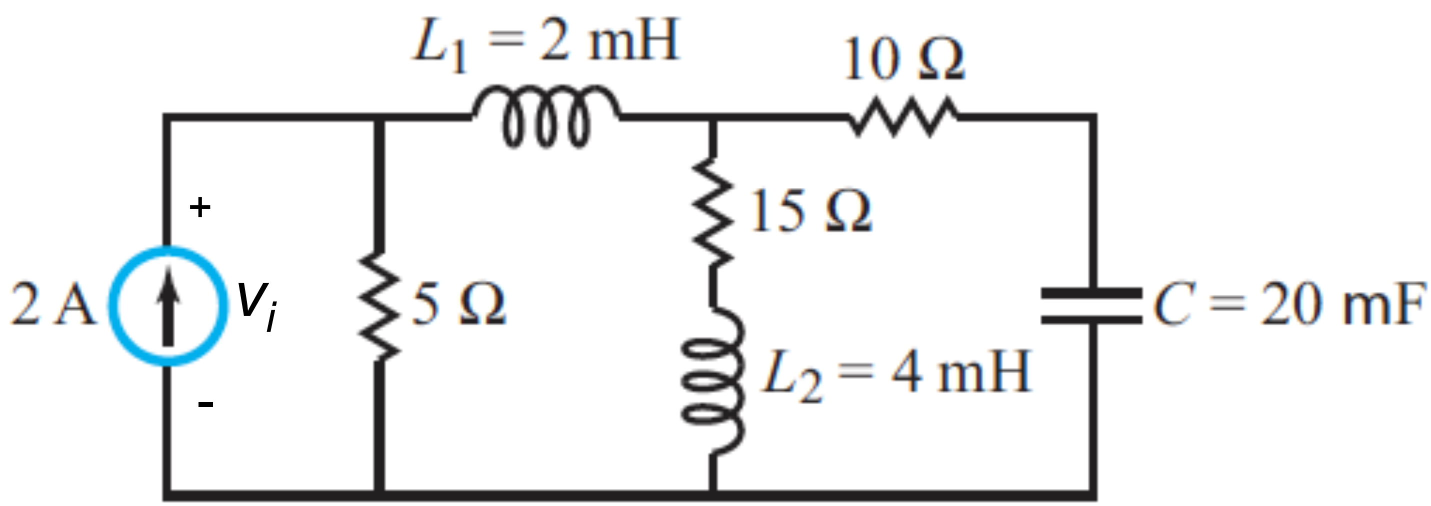


$i(t) = (2\text{ A})\ cos(2\pi Ft)$

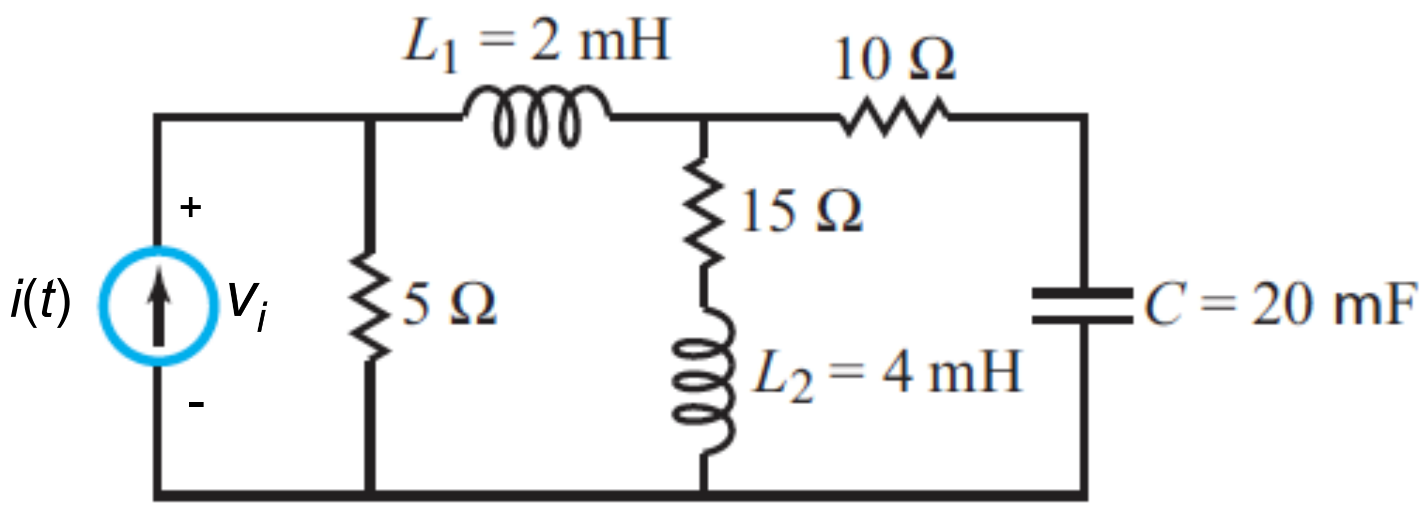
Find the power provided by the 2A current source when F = 0 Hz and when F → infinity

Example: Asymptotic Circuit Analysis: F = 0 Hz

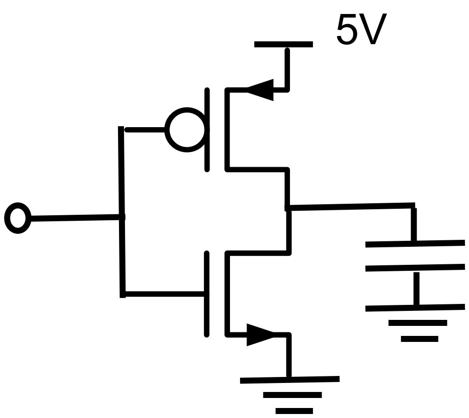
F = 0 Hz → current source is 2 A, DC



Example: Asymptotic Circuit Analysis: $F \rightarrow \infty$ Hz

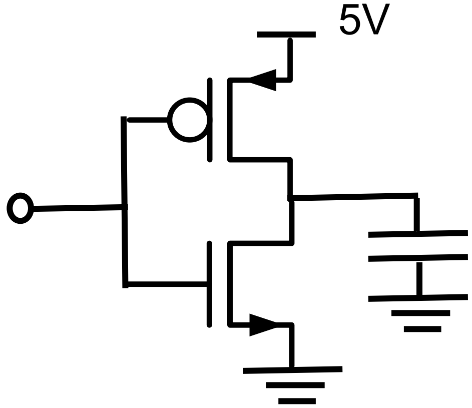


Example: RC Time Domain Analysis



A CMOS inverter is driven with a 1 GHz square wave input. Assume the transistor R_{on} = 250 Ω and C = 5 pF. Will the inverter produce “1” and “0” values at its output, if “1” means > 4V and “0” means < 1V?

Example: RC Time Domain Analysis



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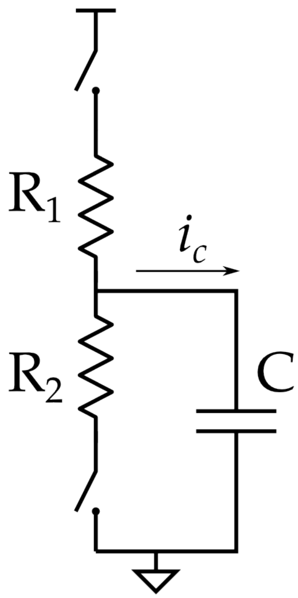


$i_{RES} = V_{out}/R_1$

$i_{CAP} = CdV_{out}/dt$

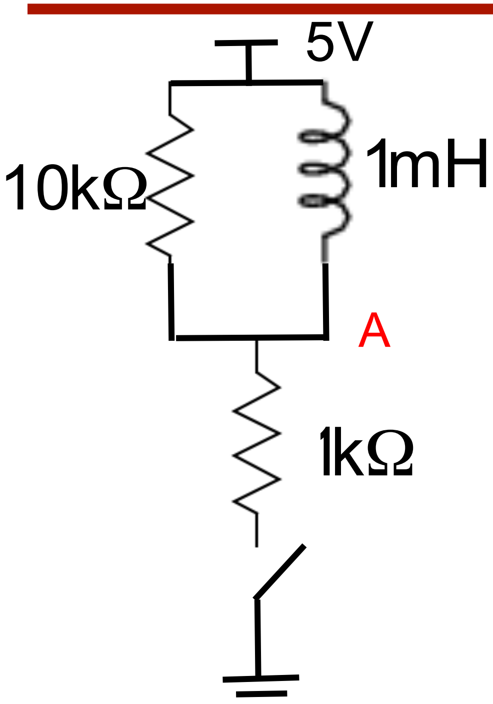
$$\frac{dV_{out}}{dt} = -\frac{V_{out}}{R_1C}$$

The output waveform will be symmetric. We need to see if it can reach 4V in ½ of the input period. F = 1 GHz, so ½ period = 0.5 nsec

$$V_{out} = 5V\left(1-e^{-t/R_1C}\right) = 5V\left(1-e^{-\frac{0.5x10^{-9}}{(250)(5x10^{-12})}}\right) = 1.64V$$

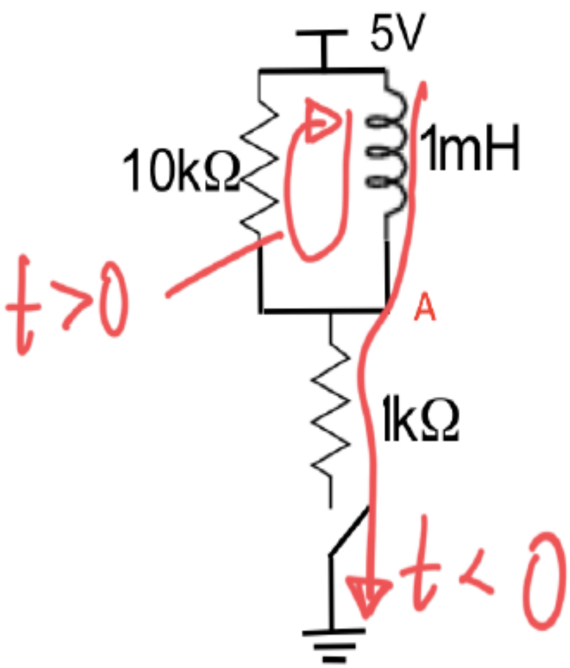
So the answer is NO.

Example: RL Time Domain Analysis



If the switch opens at t = 0 after being closed for a long time, what is the voltage at node A at $t = 0^+$?

Example: RL Time Domain Analysis



If the switch opens at t = 0 after being closed for a long time, what is the voltage at node A at $t = 0^+$?

When the switch is closed, the current flows through the 1kΩ resistor.

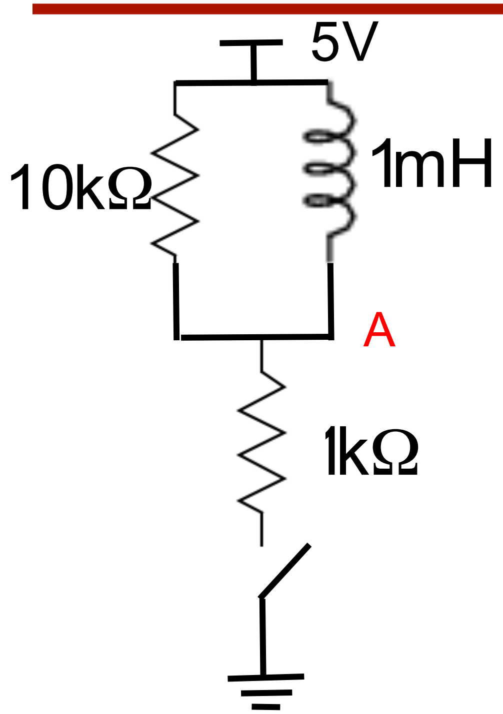
$$i = \frac{5V}{1k\Omega} = 5mA$$

When the switch opens this current has no where to go except through the 10kΩ resistor, so the voltage across the 10kΩ resistor will need to be

$$V_{10k\Omega}(t = 0^+) = \left(5mA\right)\left(10k\Omega\right) = 50V$$

So the voltage at node A will need to be 55 V.

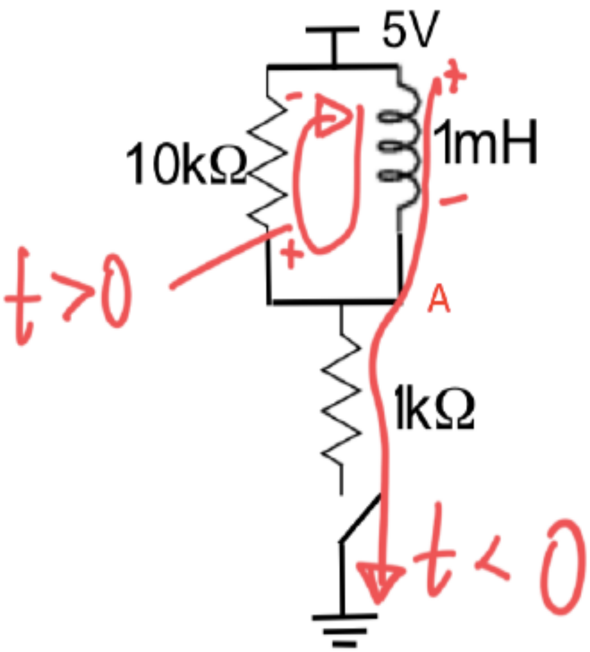
Example: RL Time Domain Analysis



After the switch opens at t = 0, how long does it take the voltage at node A to decrease to within 1V of its final value?

Example: RL Time Domain Analysis

After the switch opens at t = 0, how long does it take the voltage at node A to decrease to within 1V of its final value?



$$V_R + V_L = 0$$

$$Ri(t) + L\frac{di(t)}{dt} = 0 \quad \therefore \quad \frac{L}{R}\frac{di}{dt} + i(t) = 0$$

$$\therefore \ i(t) = I_0 e^{-\frac{R}{L}t} = 5e^{-\frac{R}{L}t} \text{ mA}$$

$$V_{10k\Omega} = 1\text{V when i = 0.1mA, so } 0.1 = 5e^{-\frac{10^4}{10^{-3}}t} \text{ or } \ln\frac{0.1}{5} = -\frac{10^4}{10^{-3}}t$$

$$\therefore = 1.609 = -10^{-7}t \quad \text{or} \quad t = 1.6\text{x}10^{-7}\text{sec} = 0.16\mu\text{sec}$$