E40M

Solving Circuits: Nodal Analysis and EveryCircuit™

Circuit Debugging

- For future labs you will be building more complicated circuits
  - You will build these circuits using breadboards

- These circuits will contain many different components
  - Including transistors with three connections

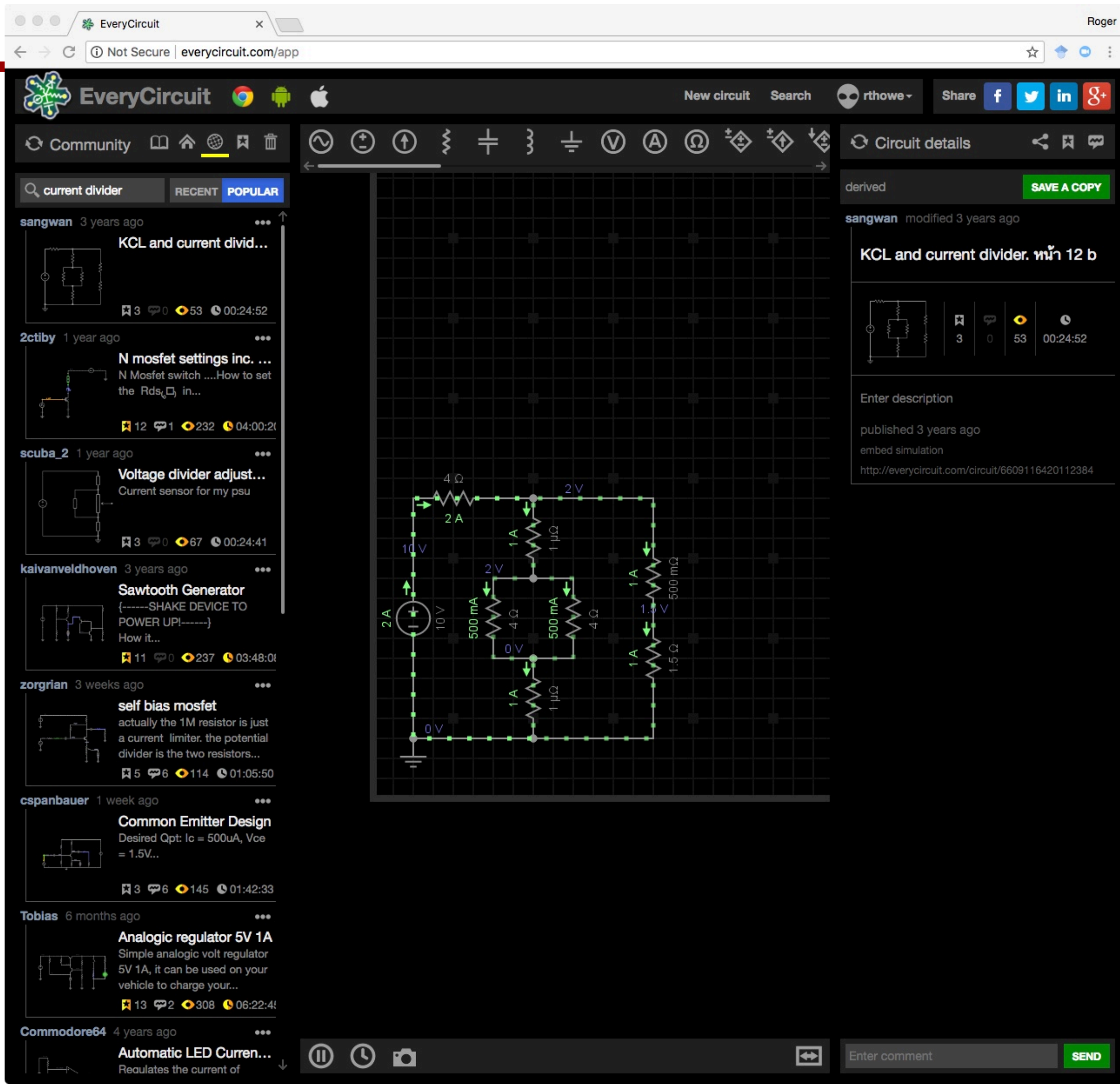
- Many times, these circuits won't work the way you expect
  - Perhaps your circuit is wrong
  - Perhaps a component isn't working properly
  - Perhaps you just wired it up incorrectly
  - How do you find bugs in circuits and get rid of them?

Circuit Simulators Can Help!

- We create a program to estimate how our circuit will behave
- The program shows the wiring in a nice way
  - and makes it easy to probe (virtually) voltages and currents
- It also makes it easy to change component values
  - So you can tune/play with your circuit
- It's also useful for building your intuition on circuits and checking your homework problems, too.
- You are going to use an easy-to-use simulator: EveryCircuit™

EverycircuitTM

http://everycircuit.com/app/



For the spring quarter in E40M, your license key for all platforms is:

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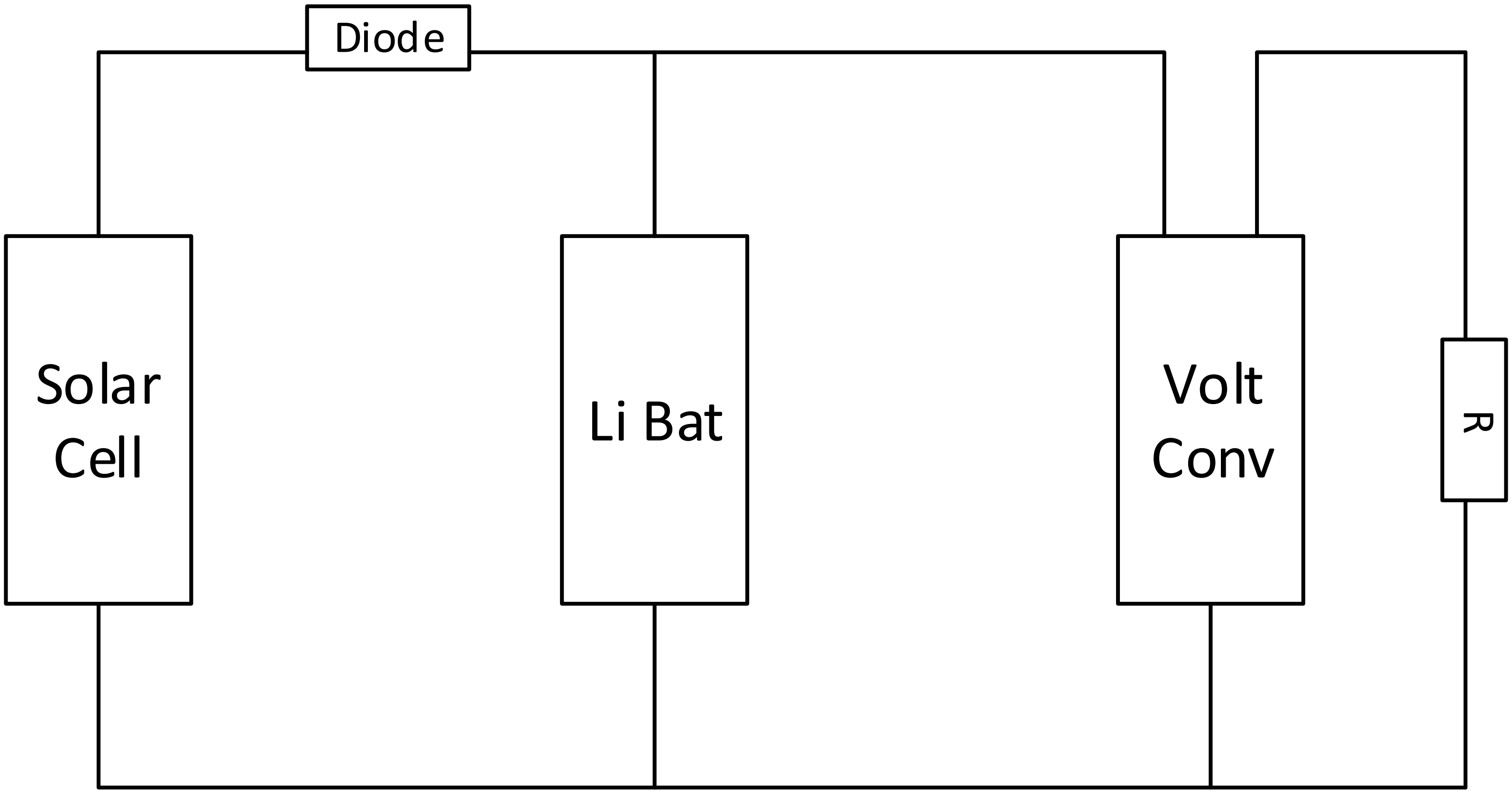
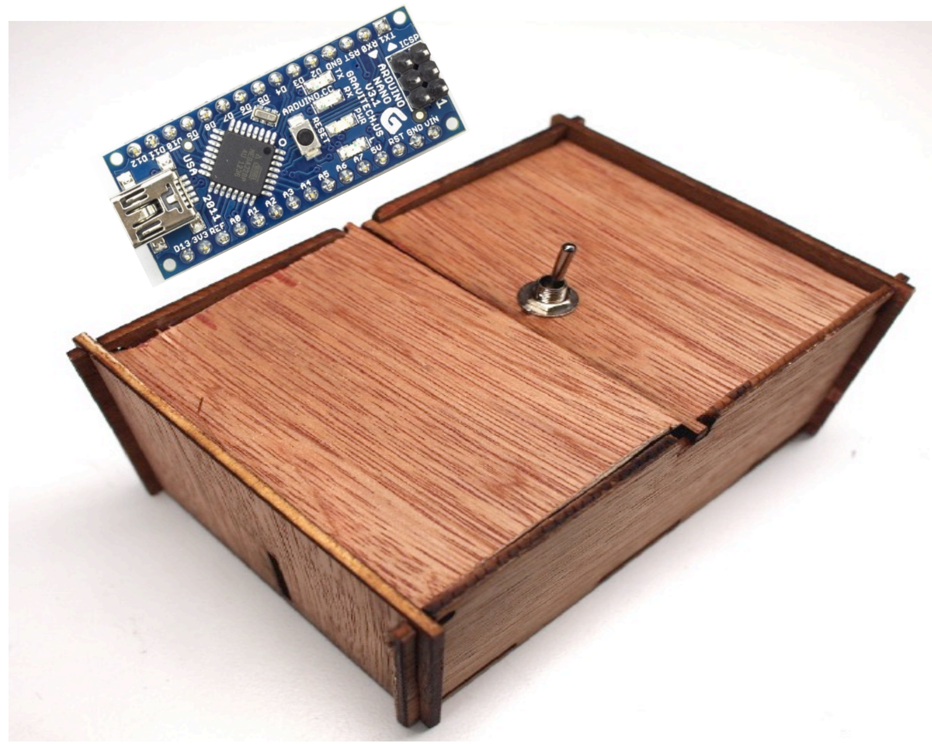
How Do We Figure Out the Voltages and Currents?



In this set of lecture notes we'll develop methods to analyze circuits.

Useless Box: Lab 2

- Concepts
  - Finite State Machines
  - Digital Logic
  - Binary numbers
  - CMOS Gate
  - Programming
- Devices
  - Motors
  - Switches
  - nMOS
  - pMOS



In Lab 2, you'll build much more complex circuits involving switches, motors and transistors.

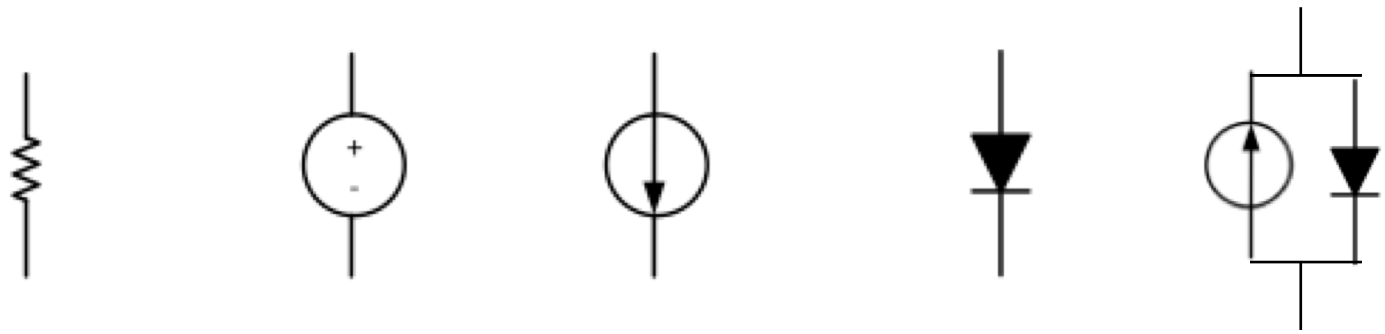
Reading For These Topics

- Reader, Chapter 3 (except 3.5)

- A&L
  - 3.1 3.2 – Node voltages
  - 3.3/3.3.1 – Nodal analysis
  - 3.5 – Superposition

Key Ideas From The Last Few Lectures - Review

- Devices you should know and symbols you should recognize



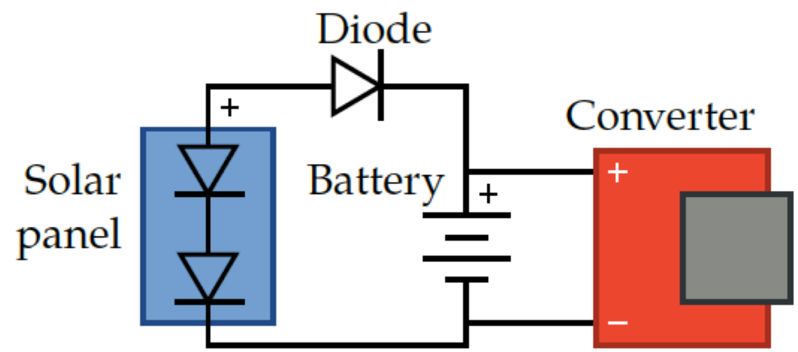
- You should understand the device i - v curves of these devices

- You should understand KCL, KVL and power: $P = i \cdot v$

- You should be comfortable using a DMM to measure voltage, current and resistance

Solving For Voltages and Currents

- Given a circuit, and device models
  - Want to solve for device voltages and currents



- Be ~~lazy~~ or efficient
  - With the least work possible
- KVL means
  - Not all device voltages are independent
  - Can we formulate the problem differently
    - Reduce the number of variables we need to deal with?

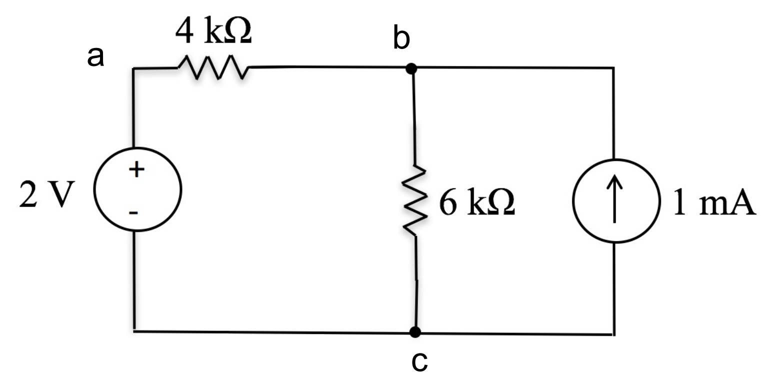
The Ground (GND): Setting a Reference Voltage

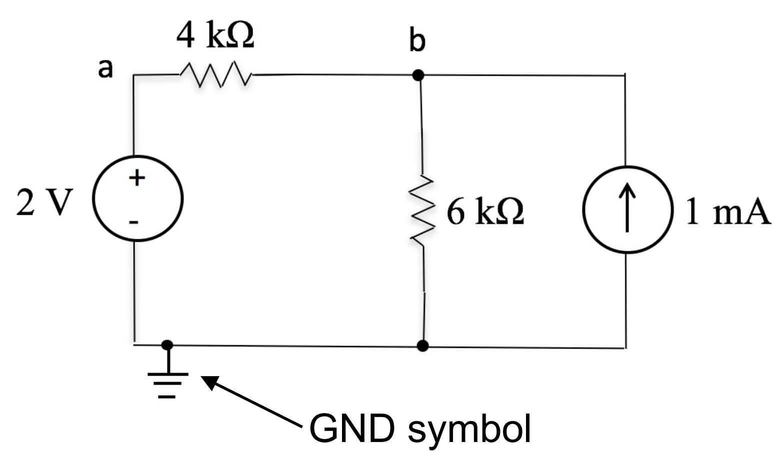
- KVL forces the voltage across a device to not depend on the path you take

- So
  - We can make *any node* a reference node and define (select) its voltage to be **zero** … we choose the lower node, c
  - And we measure the voltages of other nodes with respect to the reference node's voltage, which is zero:

$$V_b - V_c \quad \rightarrow \quad V_b - 0 = V_b$$

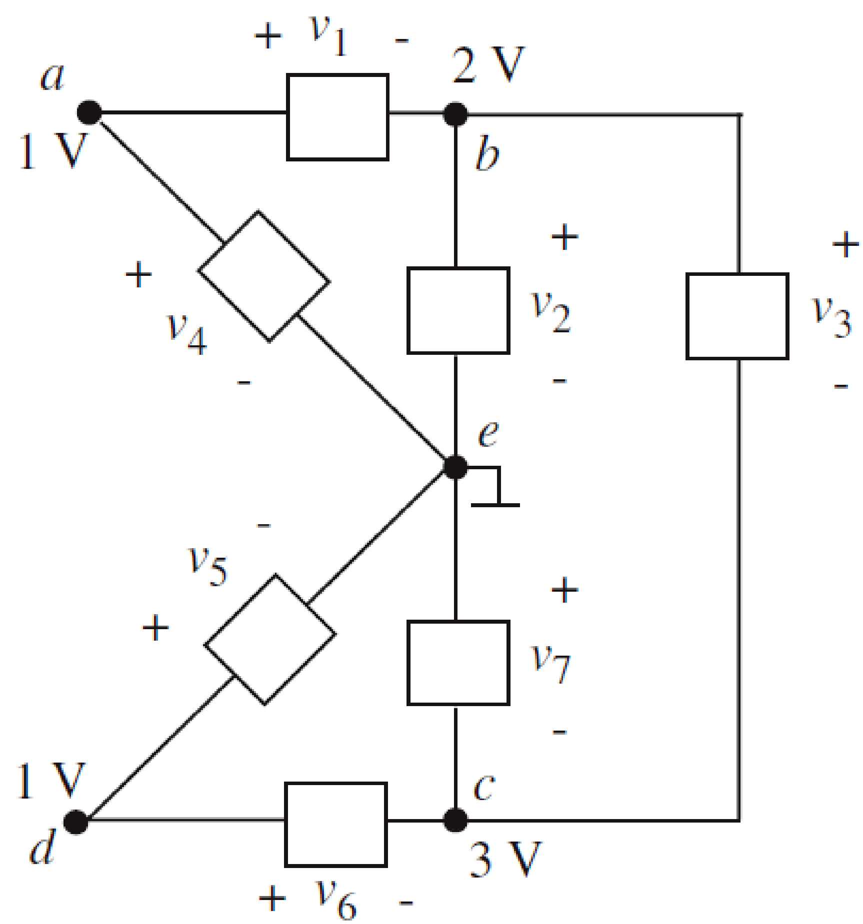
  - Device voltages are found from differences of the relevant node voltages, accounting for the polarity of course.





What are the Device Voltages?

- Why doesn't "e" have a voltage next to it?



Beware of GND (Ground)

- Since voltages are all relative
  - Often designs will declare one voltage to be the reference
  - We generally call this voltage ground (GND)

- There is also something called “earth ground”
  - And this is the voltage of a metal pipe running through the earth
  - This should be the voltage of the rounded hole on 3 pronged grounded power outlet used in North America

- Not all nodes labeled GND are connected to earth ground
  - And not all earth GNDs are at exactly the same potential

Nodal Analysis: The General Solution Method

1. Label all the nodes (V_A, V_B, or V_1, V_2, etc.), after selecting the node you choose to be GND.

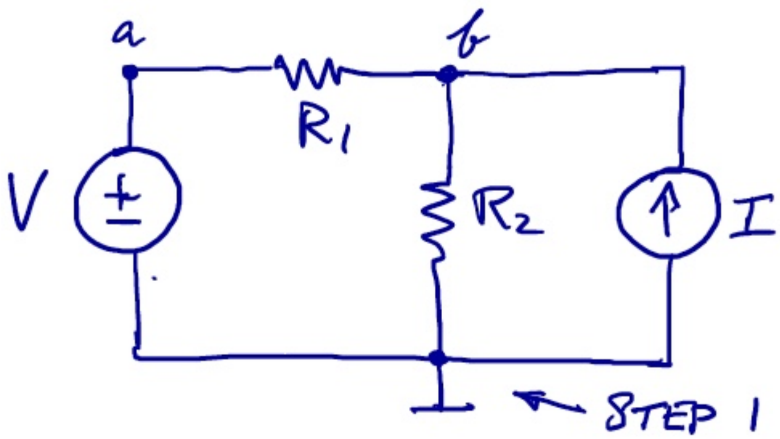
2. Label all the branch currents (i_1, i_2, etc.) and choose directions for each of them

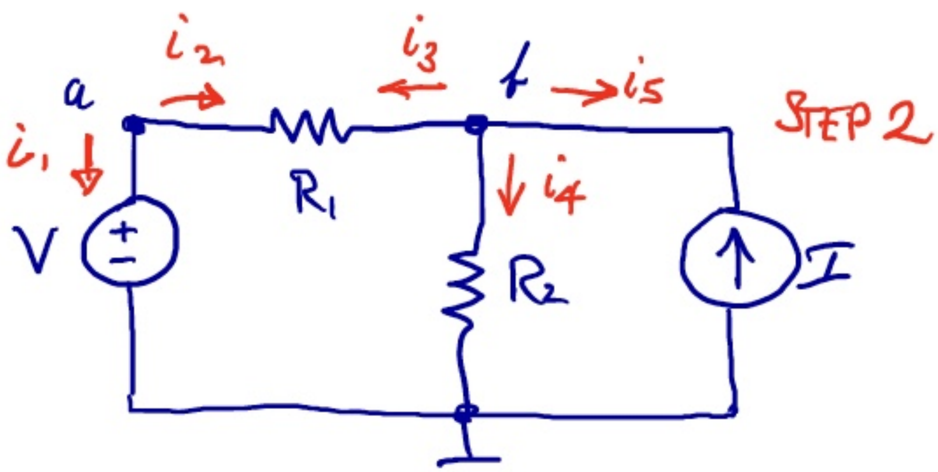
3. Write the KCL equations for every node except the reference (Gnd)
   - Sum of the device currents at each node must be zero

4. Substitute the equations for each device's current as a function of the node voltages, when possible

5. Solve the resulting set of equations

Nodal Analysis: An Example





STEP 3

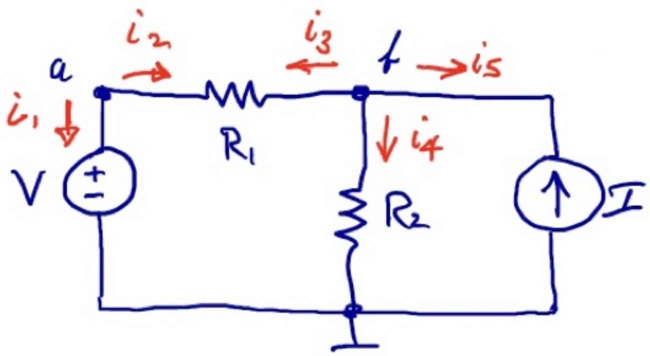
KCL @ NODE a

$$i_1 + i_2 = 0$$

KCL @ NODE b

$$i_3 + i_4 + i_5 = 0$$

An Example (cont.)



STEP 4

$$i_2 = \frac{V_a - V_b}{R_1} \qquad i_4 = \frac{V_b - 0}{R_2}$$

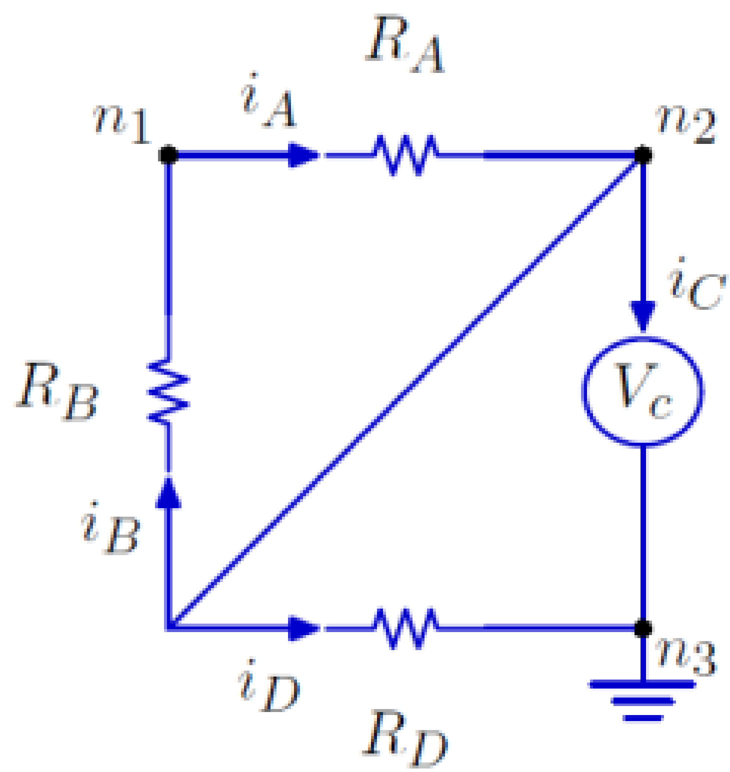
$$i_3 = \frac{V_b - V_a}{R_1} \qquad i_5 = -I \qquad \text{ALSO, } V_a = V$$

STEP 5

$$i_3 = \frac{V_b - V}{R_1}; \qquad \frac{V_b - V}{R_1} + \frac{V_b}{R_2} - I = 0$$

$$V_b \left[\frac{1}{R_1} + \frac{1}{R_2}\right] = I + \frac{V}{R_1} \Rightarrow V_b = \frac{I + V/R_1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

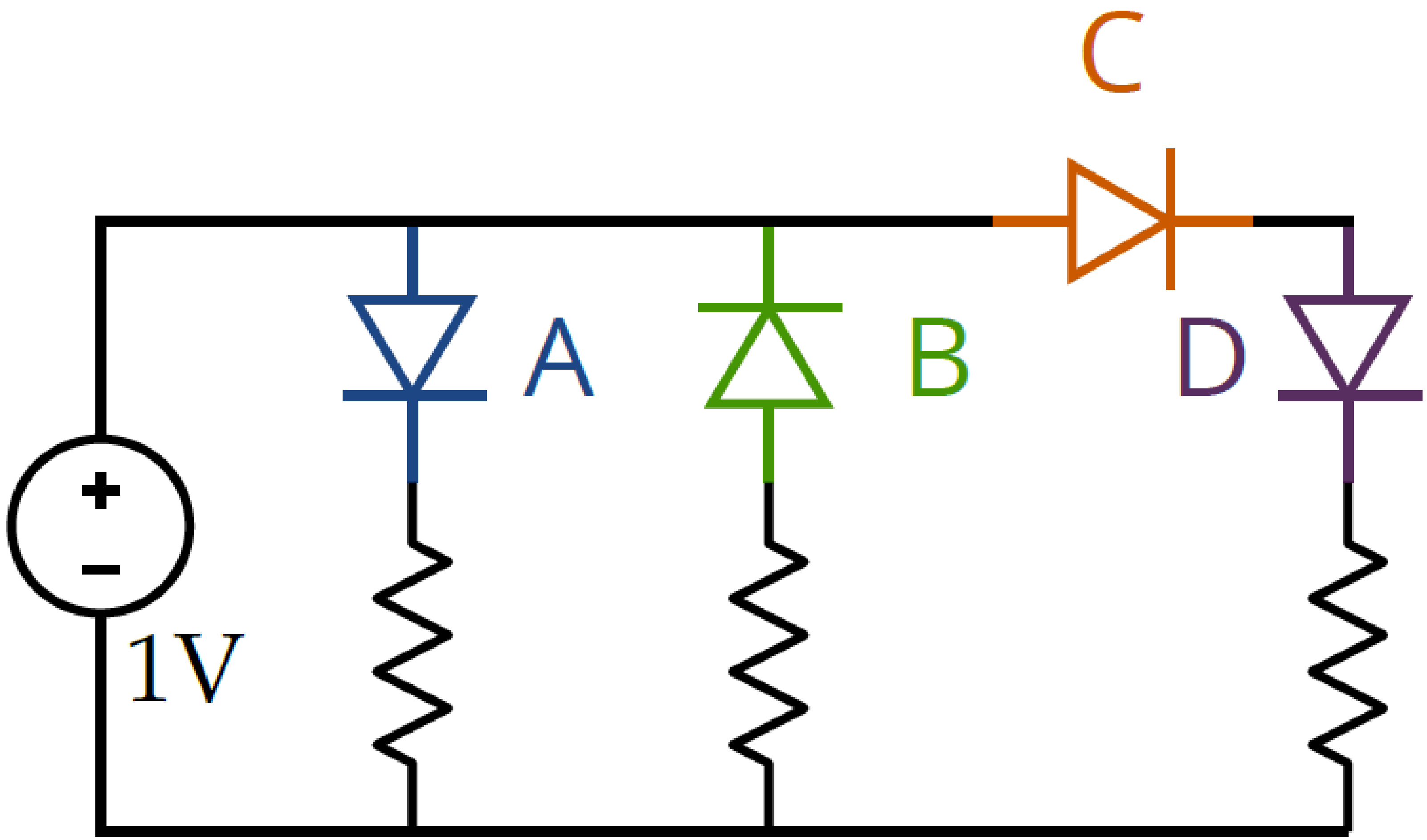
Another Nodal Analysis Example



Another Example (cont.)

Which diodes turn on?

Assume that $V_f = 0.6V$



How To Reduce Circuit Complexity

- Fewer variables is better
  - Could be fewer nodes
  - Could be fewer devices

- Can we break the circuit into pieces
  - Look at a **sub-circuit**
  - Replace that sub-circuit with a simpler equivalent

- We'll look at several examples

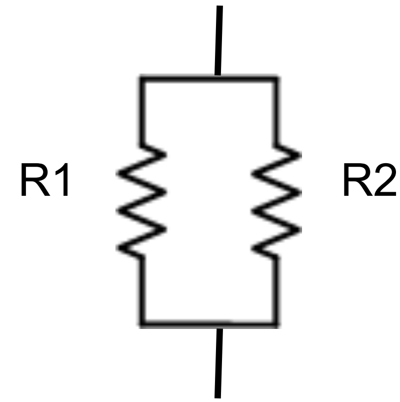
Series Combinations

- Two resistors in series ("share a current")
  - The voltage across the combination is the sum of the device voltages
  - The current through the devices is the same
  - So the effective resistance of the series is R = R1 + R2

- So we can replace series resistors
  - With a single equivalent resistor
  - Removes a node voltage and device from our equations!

Parallel Combinations

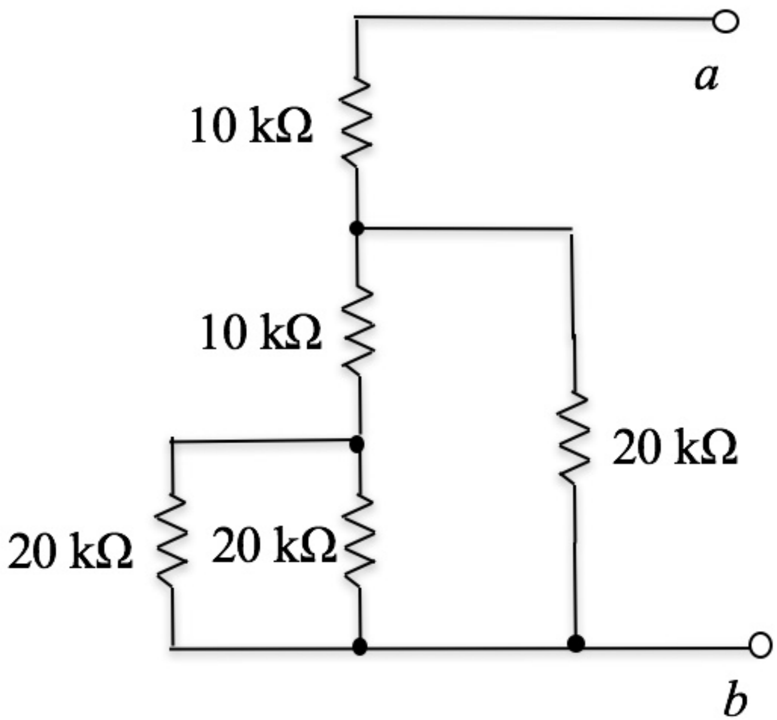
- Two resistors in parallel
  - The total current through parallel resistors is the sum of the currents through the two resistors
  - The voltage across each resistor is the same … they “share a voltage”
  - So the effective resistance of parallel resistors is:



$$1/R = 1/R1 + 1/R2$$

$$R = (R1 \cdot R2) / (R1+R2)$$

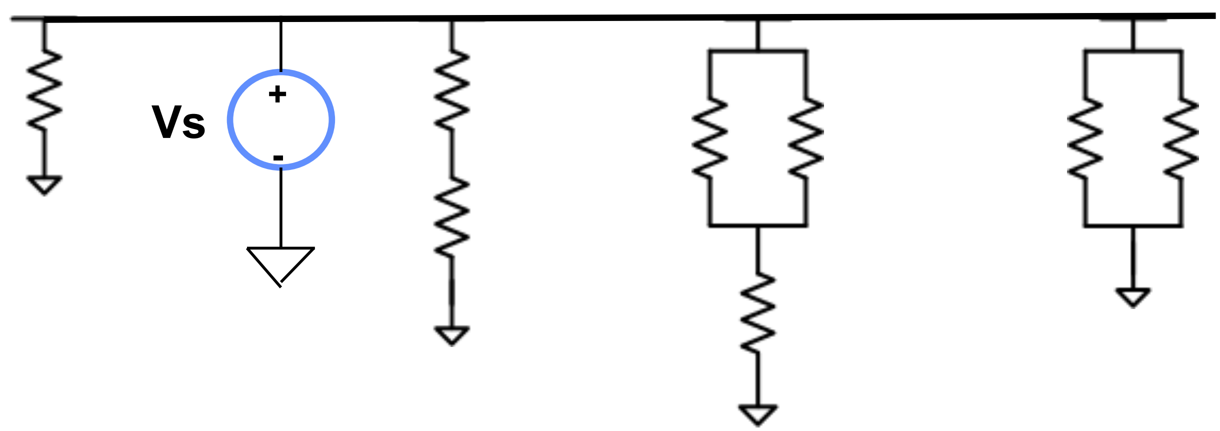
Using Series and Parallel Combinations to Simplify Circuits



Find the resistance between node a and node b

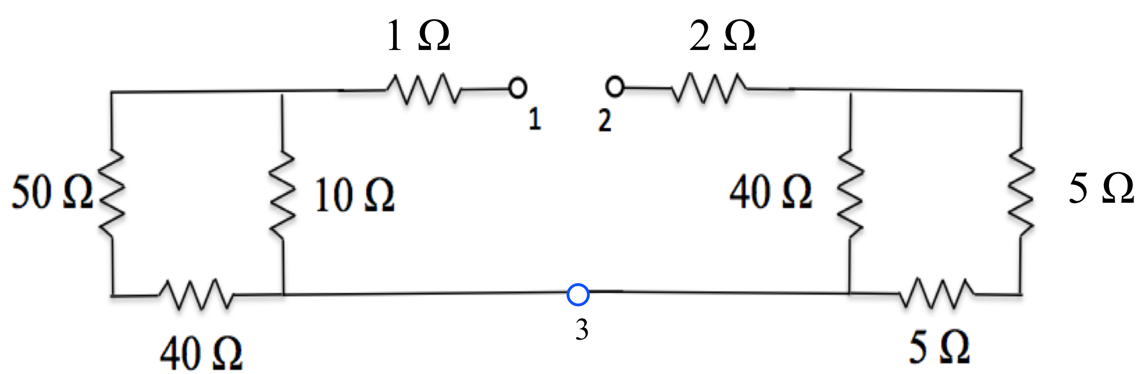
Another Example

- Notice that some circuits have multiple connections to gnd.
  - This just means that they are all connected together.
- Look at the circuit to see if there are new simplifications
- Assume R = 1 kΩ



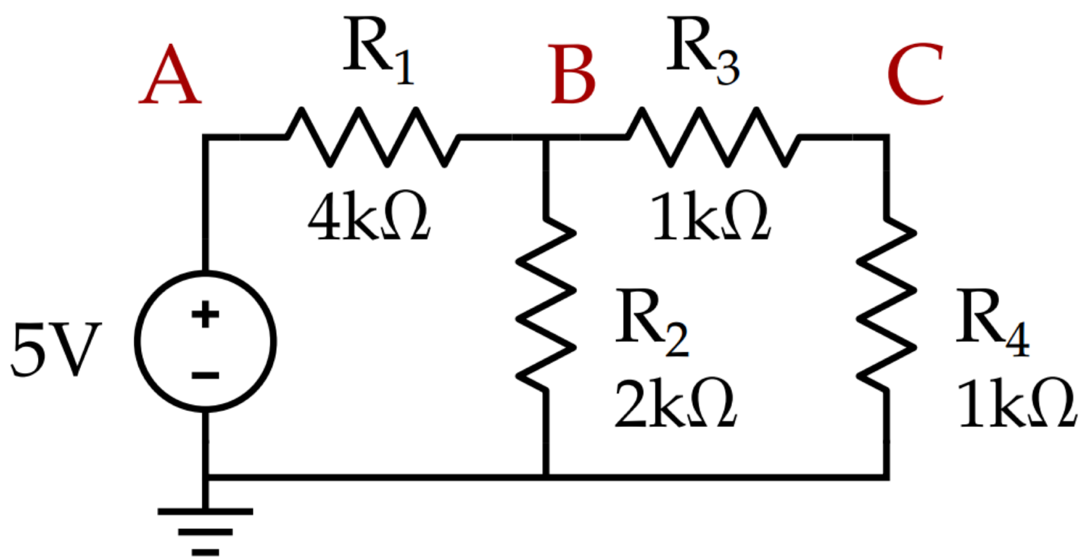
Series-Parallel Reduction Practice

We connect a 2 V battery between nodes 1(+) and 2 (-).
What current flows through the battery?
What is the voltage difference between node 2 and node 3?

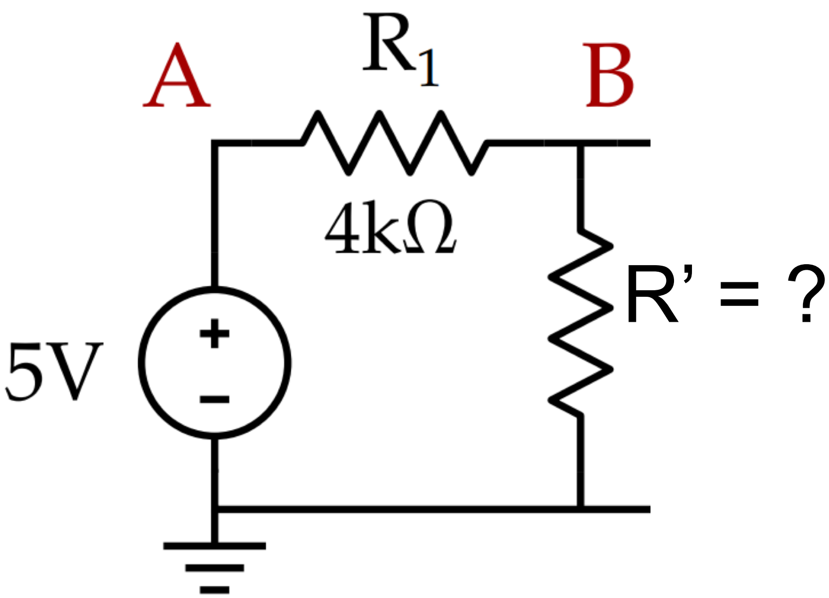


But What About This Circuit?

- R_3 and R_4 are in series
  - But I need to find the voltage at the node I will eliminate
  - “collapse and then expand”
- First eliminate the node to simplify the circuit



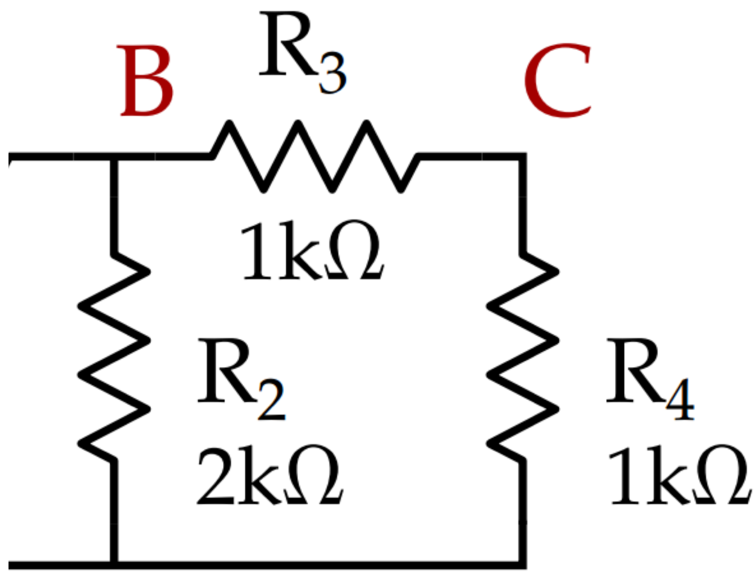
First Solve for the Voltage at Node B



Node A

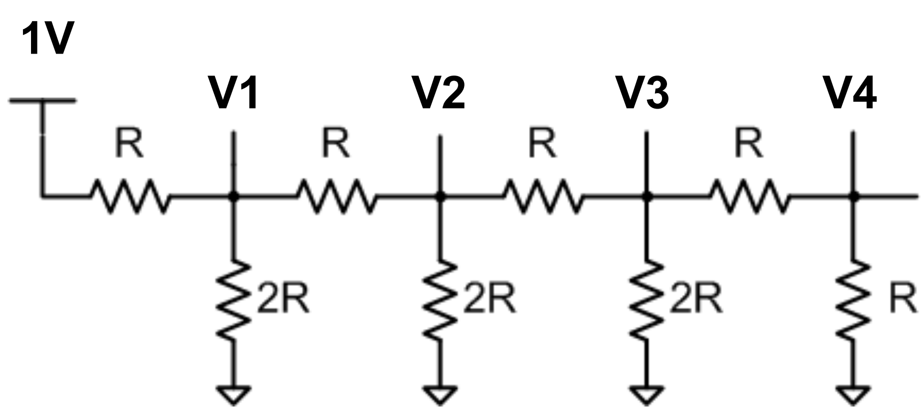
Node B

Then Solve for the Voltage at Node C



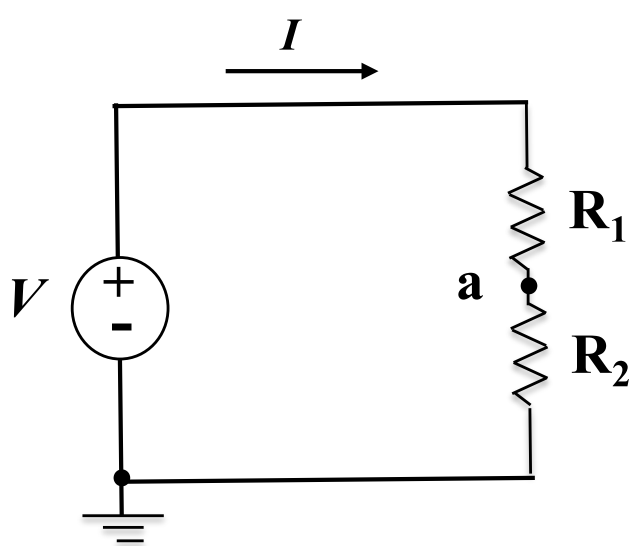
Challenge Problem

- Want to find the voltage at each output.
  - Assume left-most resistor is driven to 1V
- Doesn't look series or parallel, or is it?
  - Can we reduce it to a single resistor with our rules? Yes!



Voltage Divider

- First simplify circuit to a single resistor and find the current I
- Then use the current to find the voltage V_a

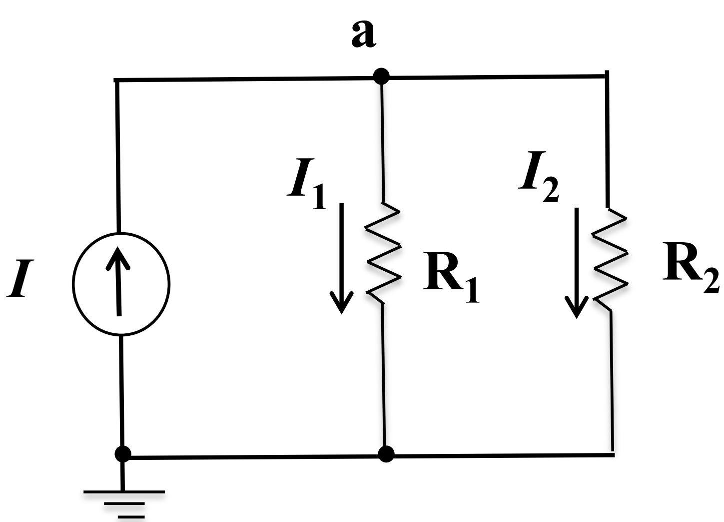


$I =$

$V_a =$

Current Divider

- In this case simplify the circuit to a single resistor, then find voltage across each resistor and use it to find the current through each resistor

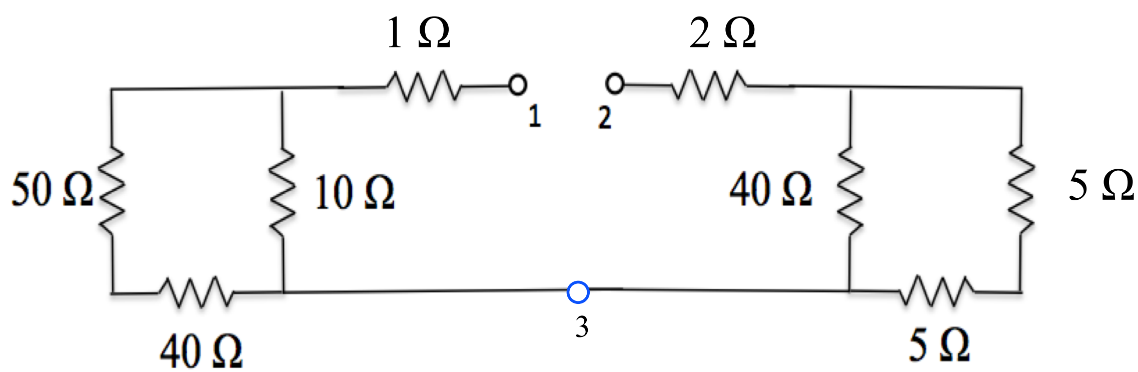


Intuition on Dividers

- Voltage divider: $R_2 = 10\ R_1$ … what is $V_a = V_{R2}$ in terms of V?

- Current divider: $R_2 = 10\ R_1$ … what is I_2 in terms of I?

Series-Parallel Reduction - Review



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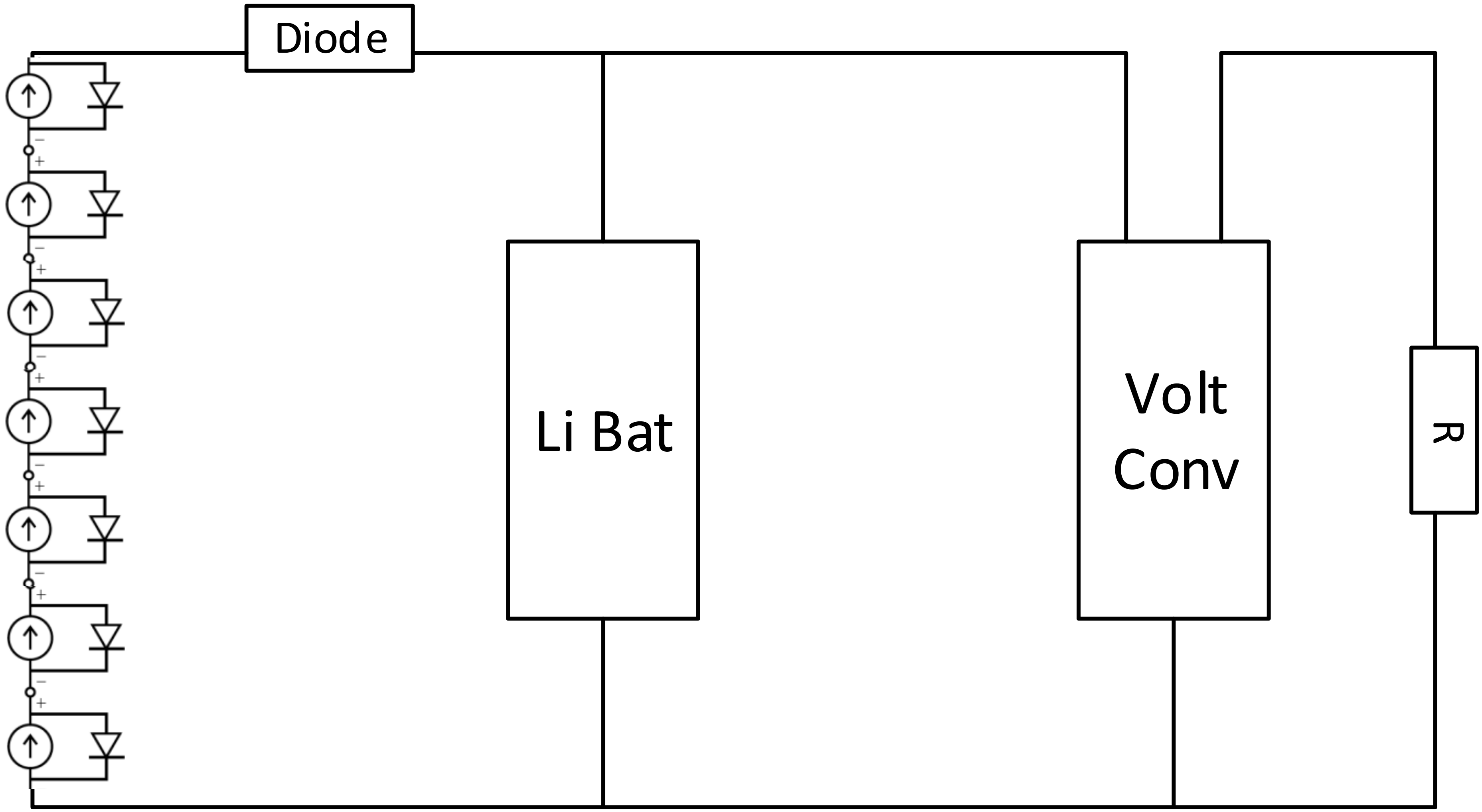


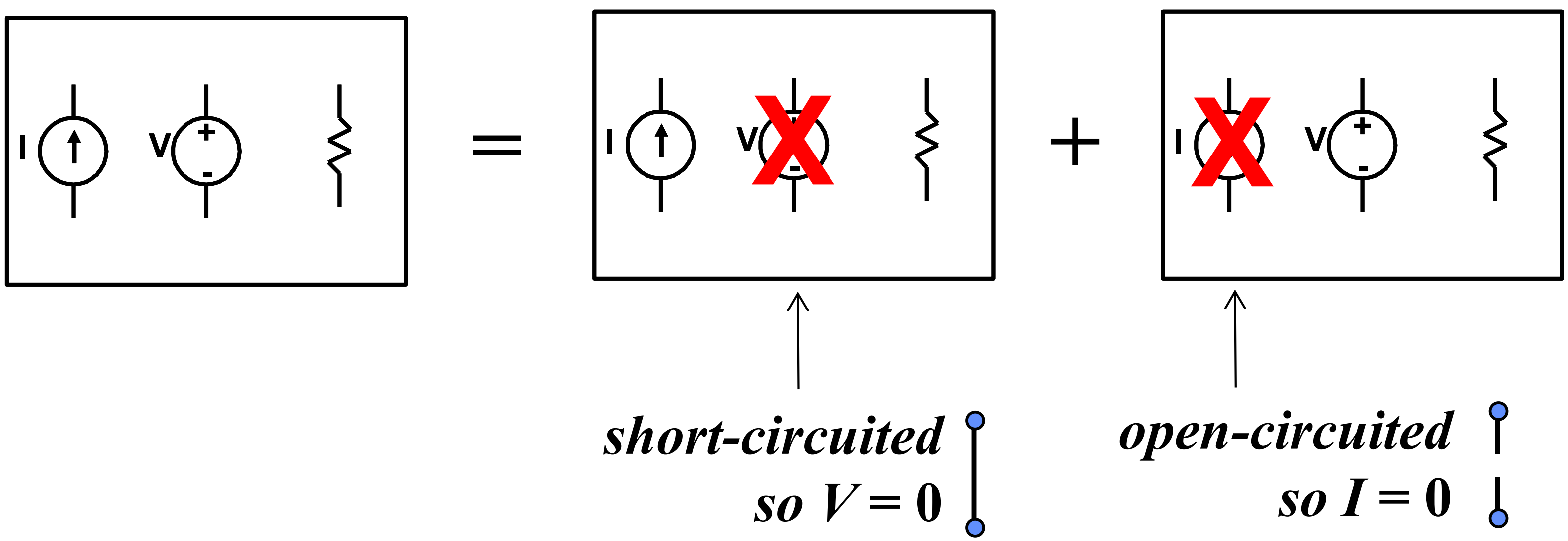
Superposition For Linear Circuits

If a circuit contains more than one independent source, the voltage (or current) response of any element in the circuit is equal to the algebraic sum of the individual responses associated with the individual independent sources, as if each had been acting alone.

- Reason:
  - Resistors, voltage, and current sources are linear
  - Resulting equations are linear
- What's the benefit?
  - Superposition enables the analysis of several simpler circuits in place of one complicated circuit

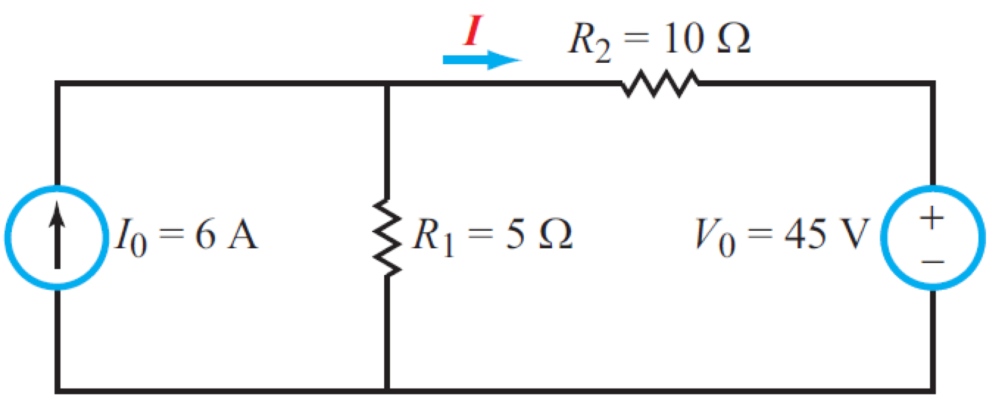
Applying Superposition

- Calculate the response of the circuit for each independent source at a time, with the other's turned off
- What happens when we turn off a source?
  - Voltage sources: have 0 V (are ***shorted*** … replace by a wire)
  - Current sources: have 0 current (are ***opened*** … replace by a *broken* wire)



Applying Superposition

- We need to “zero-out” sources into order to find the sub-circuits (one per source)
- Find the current ***I***



F. T. Ulaby and M. M. Maharbiz, *Circuits*, NSTP, 2009, p. 97.

Applying Superposition

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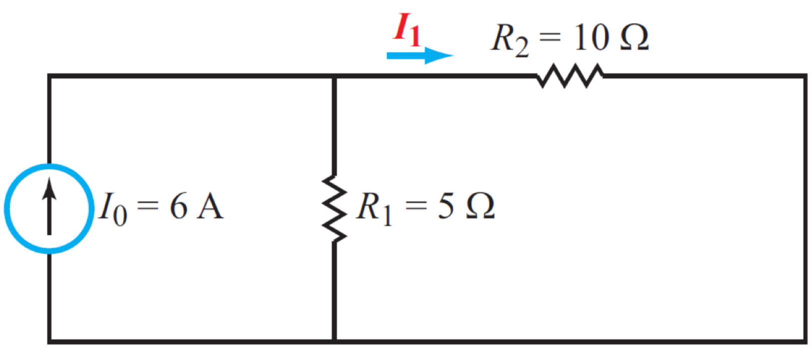
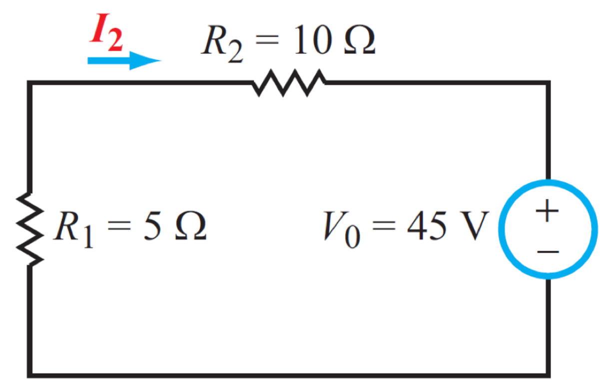


Sub-circuit 1: V_0 *shorted*



Sub-circuit 2: I_0 *opened*

$I_1 =$

$I_2 =$

$\boxed{I = I_1 + I_2 =}$

F. T. Ulaby and M. M. Maharbiz, *Circuits*, NSTP, 2009, p. 97.

Learning Objectives

- EveryCircuit™ can solve your circuits, so you can be sure your homework and prelab answers are correct!*

- Understand how to solve for device voltage and currents
  - First label node voltages (KVL)
  - Solve current equations at each node (KCL)
  - Called nodal analysis

* Not allowed on the midterm or final, needless to say!

Learning Objectives

- Recognize some common circuit patterns
  - Reduce the complexity of the circuit you need to solve!
  - Series and parallel resistors, expand/contract with dividers

- Superposition: a powerful tool for handling multiple sources
  - We end up doing more (one per source), but simpler circuits
  - We are becoming proficient at single-source circuits – doing them quickly and knowing we're right
  - Add up the results from the sub-circuits to find the voltage or current we're looking for in the complicated circuit
  - Break up a large circuit into smaller circuits – the standard "divide and conquer" approach

Need to See More Examples

- Two tutorials will be posted as recordings on the website
  - Nodal Analysis
  - Superposition